TDHF計算による多核子移行反応の研究

Time-dependent Hartree-Fock Theory for Multi-nucleon Transfer Reactions

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1. Introduction

Application of TDHF to multi-nucleon transfer reaction

Time-dependent Hartree-Fock Theory for Multi-nucleon Transfer Reactions

TDHF:
- Microscopic theory for nuclear dynamics: GDR, heavy ion collision.
- Skyrme force reproduce static properties of nuclei.

There is no extra parameter to describe nuclear collision.

Purpose of this talk

1) To explain how I calculate nucleon transfer probabilities.

2) To compare our calculation with measurements for $^{40,48}Ca+^{124}Sn$.

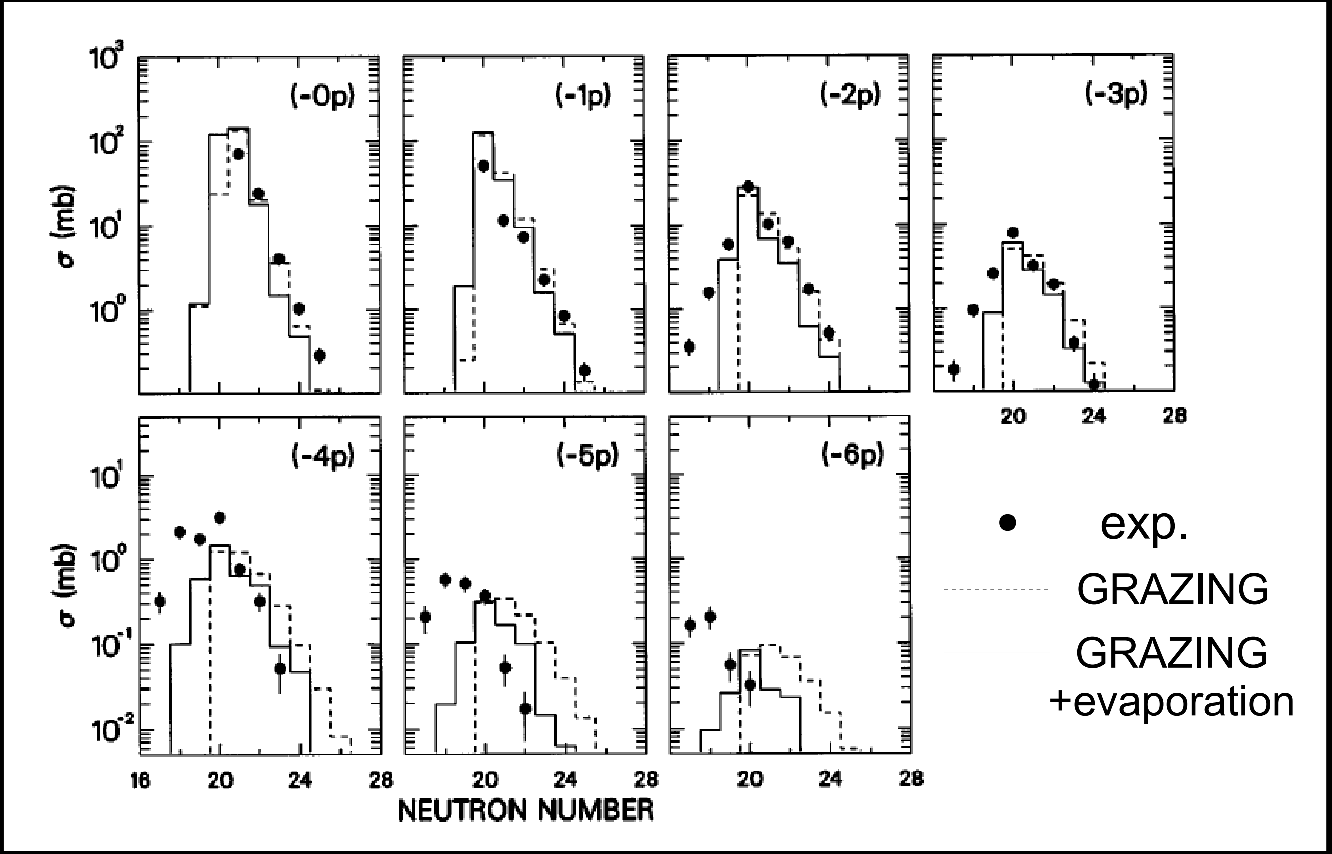
1. Introduction

Experiment:
L. Corradi *et al.*, Phys. Rev. C **54**, 201 (1996)

Total integrated cross sections for the transfer products

$^{40}Ca+^{124}Sn$, E_{lab}=170 [MeV]

- Energy and angle integrated cross section
- Up to 6 proton transfer has been observed



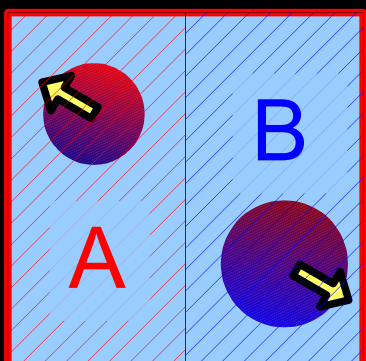
I'll Compare the result of TDHF calculation with this measurement.

2. Formulation Nucleon transfer probability; Definition

Probability: n nucleons in A and N-n nucleons in B.

✔ $$P_n \equiv \sum_{\{\tau_i;A^n B^{N-n}\}} \int_{\tau_1} d\vec{r}_1 \cdots \int_{\tau_N} d\vec{r}_N \, |\Phi_f(\vec{r}_1,\cdots,\vec{r}_N)|^2$$

τ_i = A or B All combinations: A appears n times and B appears N-n times.



Final state Φ_f

N; total # of nucleons

$$\sum_{n=0}^{N} P_n = 1$$

Normalization of many-body wave function

$$\int d\vec{r}_1 \cdots \int d\vec{r}_N |\Phi_f(\vec{r}_1,\cdots,\vec{r}_N)|^2 = 1$$

$$\int d\vec{r} = \int_{\mathrm{A}} d\vec{r} + \int_{\mathrm{B}} d\vec{r}$$ Divide spacial integral into two parts

$$\sum_{\tau_1,\cdots,\tau_N} \int_{\tau_1} d\vec{r}_1 \cdots \int_{\tau_N} d\vec{r}_N \, |\Phi_f(\vec{r}_1,\cdots,\vec{r}_N)|^2 = 1$$

τ_i = A or B All permutations: each tau has value either A or B.

Transfer probability, Pn, can be derived from normalization relation.

2. Formulation Nucleon transfer probability; For TDHF

Probability: n nucleons in A and N-n nucleons in B.

✔
$$P_n \equiv \sum_{\{\tau_i; A^n B^{N-n}\}} \int_{\tau_1} d\vec{r}_1 \cdots \int_{\tau_N} d\vec{r}_N \, |\Phi_f(\vec{r}_1, \cdots, \vec{r}_N)|^2$$

$$= \sum_{\{\tau_i; \mathrm{A}^n \mathrm{B}^{N-n}\}} \det\{<\phi_i|\phi_j>_{\tau_i}\}$$ ***for Slater determinant [1], [2]***

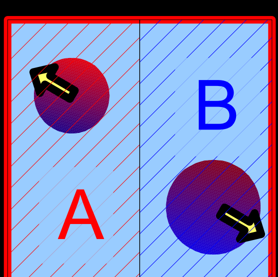
τ_i = A or B   All combinations: A appears n times and B appears N-n times.

Φ_f of TDHF: a Slater determinant

$$\Phi_f(\vec{r}_1, \cdots, \vec{r}_N) = \frac{1}{\sqrt{N!}} \det\{\phi_i(\vec{r}_j)\}$$

$\phi_i(\vec{r})$: Single particle wave function $i = 1, \cdots, N$

$$<\phi_i|\phi_j> = <\phi_i|\phi_j>_{\mathrm{A}} + <\phi_i|\phi_j>_{\mathrm{B}} = \delta_{ij}$$



Final state Φ_f

To calculate all patterns of *Pn* (n=0, 1, …, N)

$$\sum_{n=0}^{N} {}_N C_n = 2^N$$ times calculations of the determinant

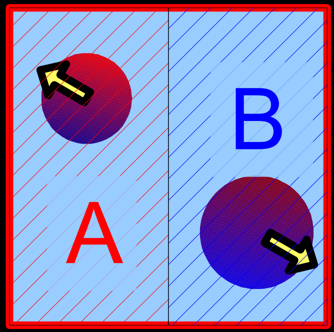
N;	order of 2^N
10;	10^3
50;	10^{15}
100;	10^{30}

[1] H J Lüdde and R M Dreizler, J. Phys. B **16**, 3973 (1983)
[2] R. Nagano, K. Yabana, T. Tazawa, and Y. Abe, Phys. Rev. A **62**, 062721 (2000)

2. Formulation Nucleon transfer probability; Projection

Alternative approach: *Particle number projection technique* [3]

[3] C. Simenel, Phys. Rev. Lett. **105**, 192701 (2010)



Final state Φ_f

Particle number projection operator

$$\delta(n-\hat{N}_{\mathrm{A}}) = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, e^{i(n-\hat{N}_{\mathrm{A}})\theta}$$

$$\hat{N}_{\mathrm{A}} = \int_{\mathrm{A}} d\vec{r} \sum_{i=1}^{N} \delta(\vec{r}-\vec{r}_i)$$: Number operator in the region A

Probability: n nucleons in A and N-n nucleons in B

$$P_n = <\Phi_f|\delta(n-\hat{N}_{\mathrm{A}})|\Phi_f> = \sum_{\{\tau_i;\mathrm{A}^n\mathrm{B}^{N-n}\}} \det\{<\phi_i|\phi_j>_{\tau_i}\}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\theta\, e^{in\theta} \det\{<\phi_i|\phi_j>_{\mathrm{B}} + e^{-i\theta}<\phi_i|\phi_j>_{\mathrm{A}}\}$$

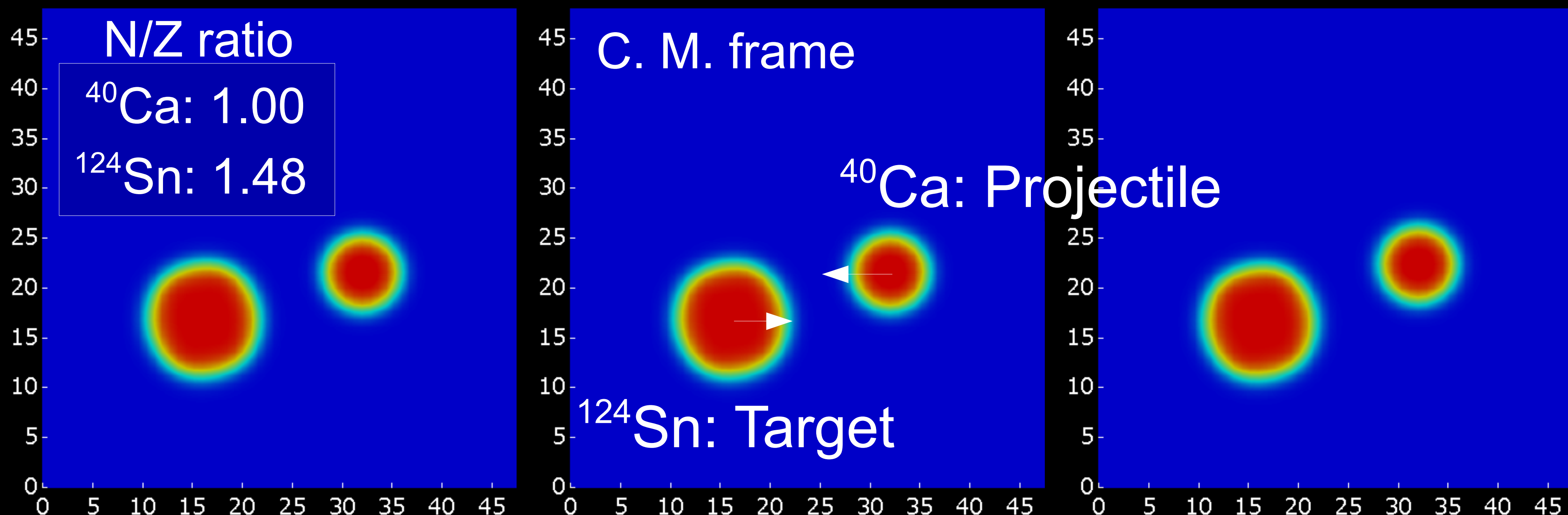
Discretization of the integral ➡ ~100 times calculations of the determinant

This method is equivalent to previous one analytically, but it reduce the computational cost considerably.

3. Results: ^{40}Ca+^{124}Sn, E_{lab}=170 [MeV]

TDHF time evolution of densities of the two colliding nuclei

b=3.69 [fm] b=3.70 [fm] b=4.50 [fm]



Skyrme: SLy5 Grid size: 60×60×26 (48×48×20.8 fm) Initial distance: 16 fm

Mesh spacing: 0.8 fm Time step: 0.2 fm/c L. Corradi *et al.*, Phys. Rev. C **54**, 201, (1996)

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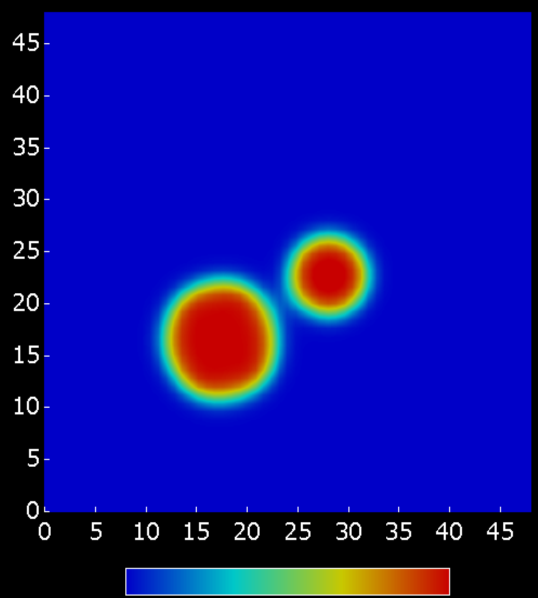
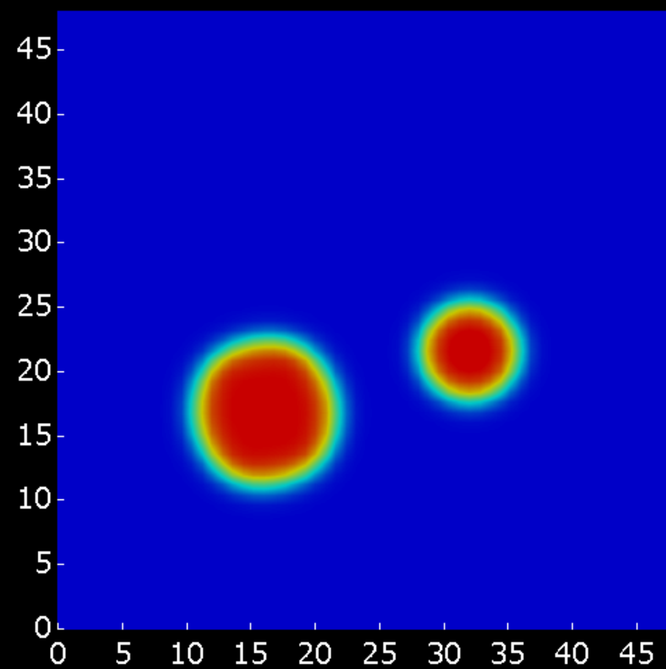
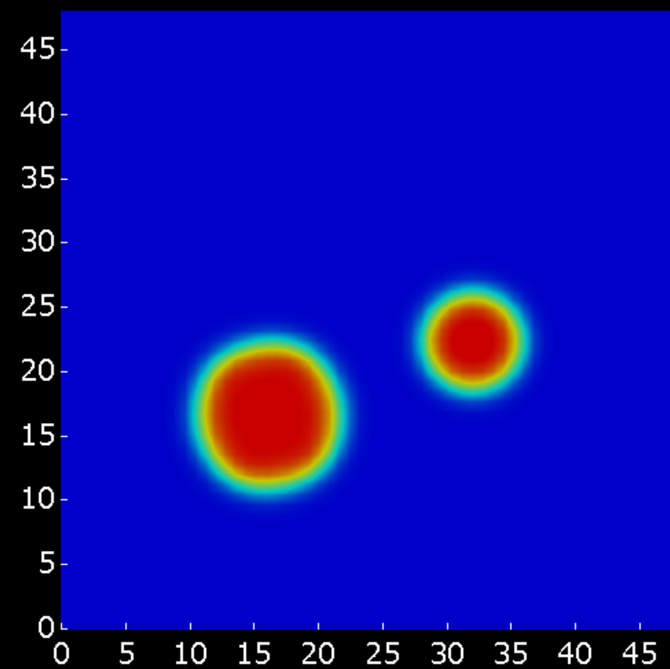


b=3.70 [fm]



b=4.50 [fm]



Skyrme: SLy5 | Grid size: 60×60×26 (48×48×20.8 fm) | Initial distance: 16 fm

Mesh spacing: 0.8 fm | Time step: 0.2 fm/c | L. Corradi *et al.*, Phys. Rev. C **54**, 201, (1996)

3. Results: ^{40}Ca+^{124}Sn, E_{lab}=170 [MeV]

TDHF time evolution of densities of the two colliding nuclei

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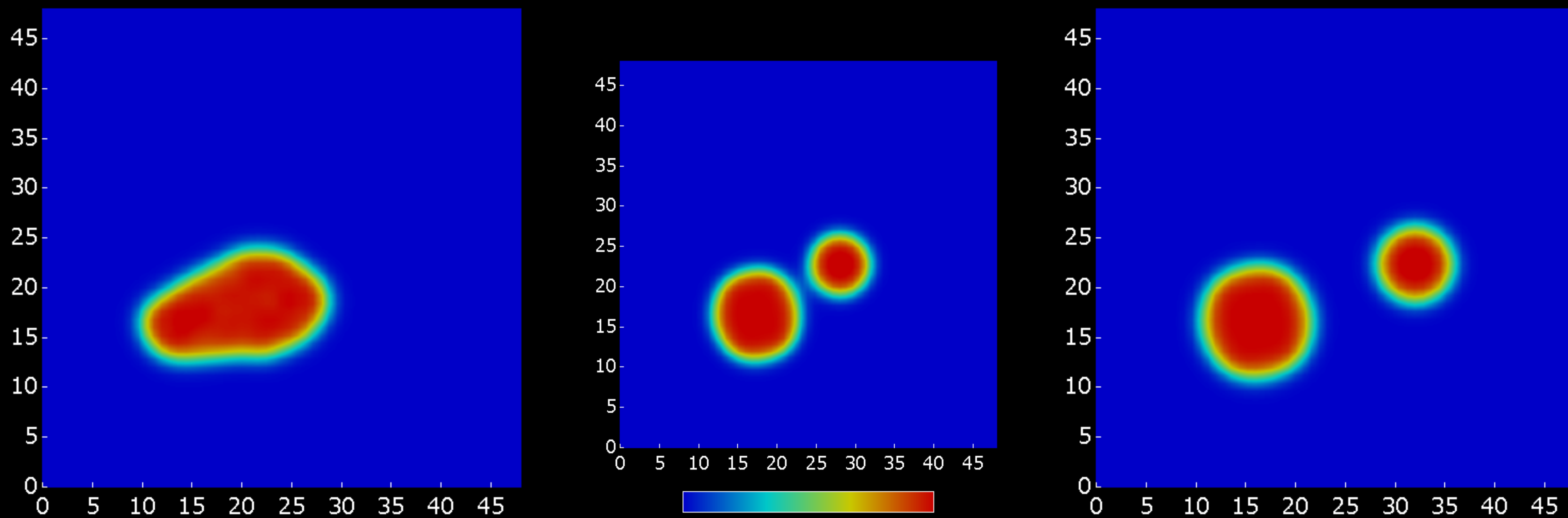

Skyrme: SLy5   Grid size: 60×60×26 (48×48×20.8 fm)   Initial distance: 16 fm

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3. Results: $^{40}Ca + ^{124}Sn$, E_{lab}=170 [MeV]

TDHF time evolution of densities of the two colliding nuclei

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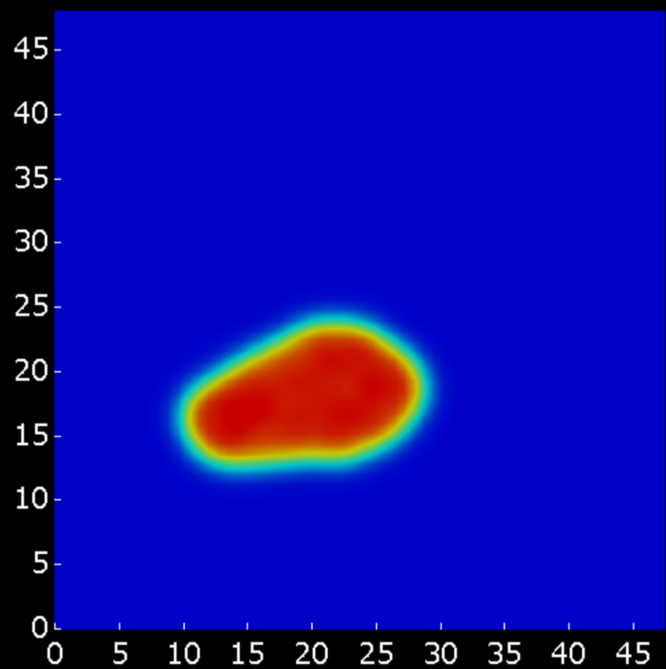
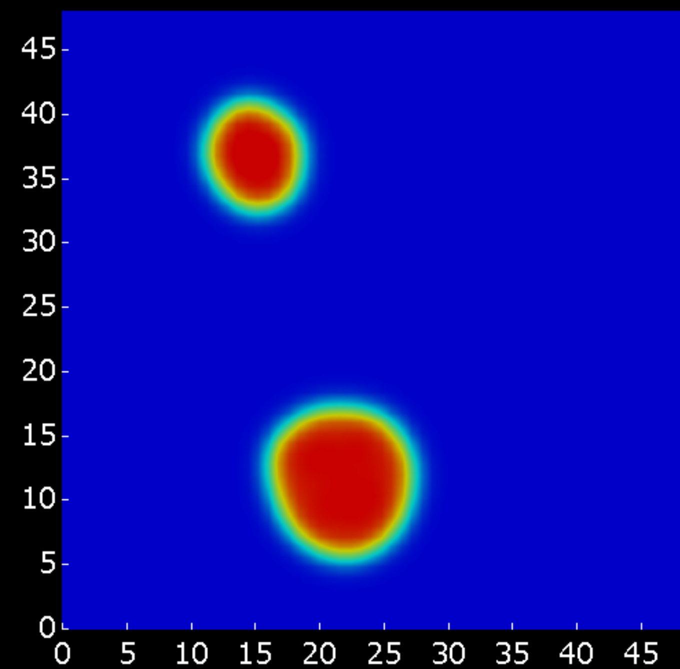
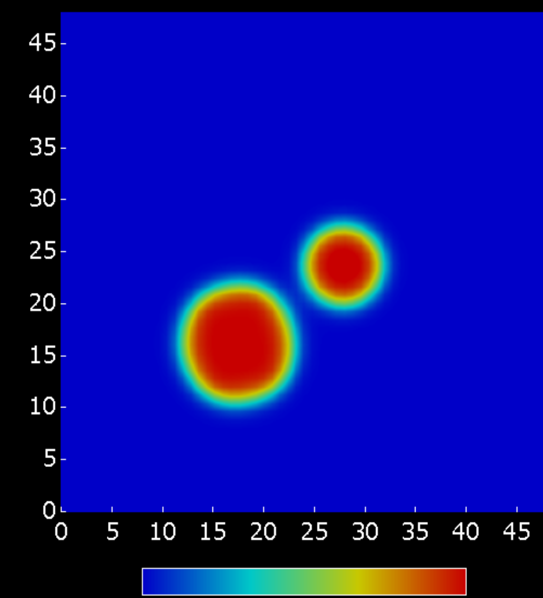


b=3.70 [fm]



b=4.50 [fm]



Skyrme: SLy5 Grid size: 60×60×26 (48×48×20.8 fm) Initial distance: 16 fm

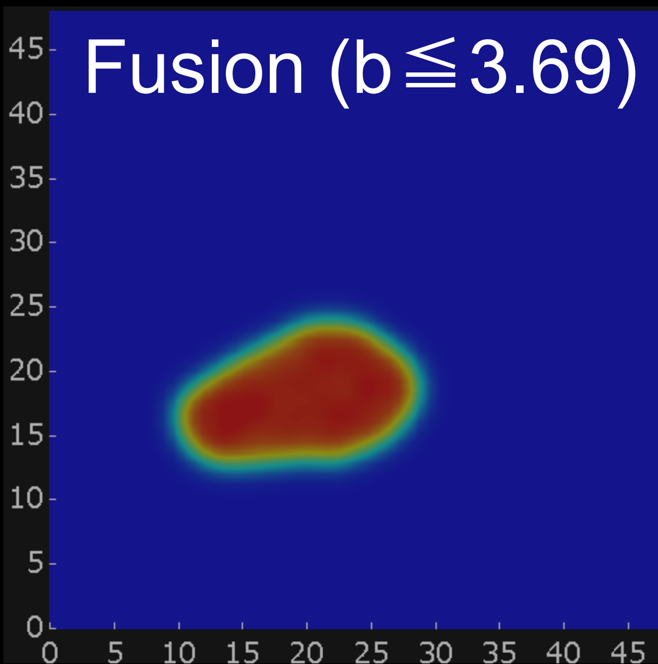
Mesh spacing: 0.8 fm Time step: 0.2 fm/c L. Corradi *et al*., Phys. Rev. C **54**, 201, (1996)

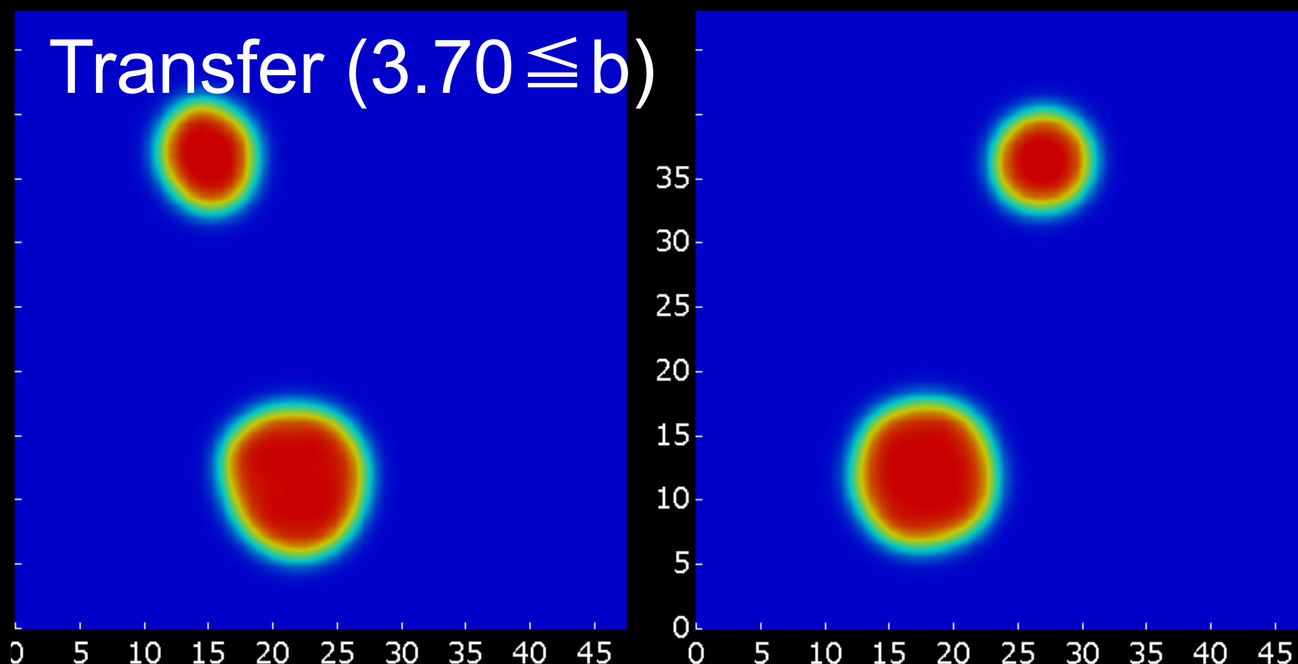
3. Results: $^{40}Ca+^{124}Sn$, E_{lab}=170 [MeV]

We have applied the number projection technique to these final w.f.s.

b= 0, ･･･, b= 3.69 fm

b= 3.70 fm, ･･･, bcut ≡ 10.00 fm





<ΔN>: 2.93　σ_n: 1.11
<ΔZ>: 1.55　σ_p: 1.06

<ΔN>: 0.45　σ_n: 0.63
<ΔZ>: 0.37　σ_p: 0.56

Skyrme: SLy5　　Grid size: 60×60×26 (48×48×20.8 fm)　　Initial distance: 16 fm

Mesh spacing: 0.8 fm　　Time step: 0.2 fm/c　　L. Corradi *et al.*, Phys. Rev. C **54**, 201, (1996)

3. Results: $^{40}Ca+^{124}Sn$, E_{lab}=170 [MeV]

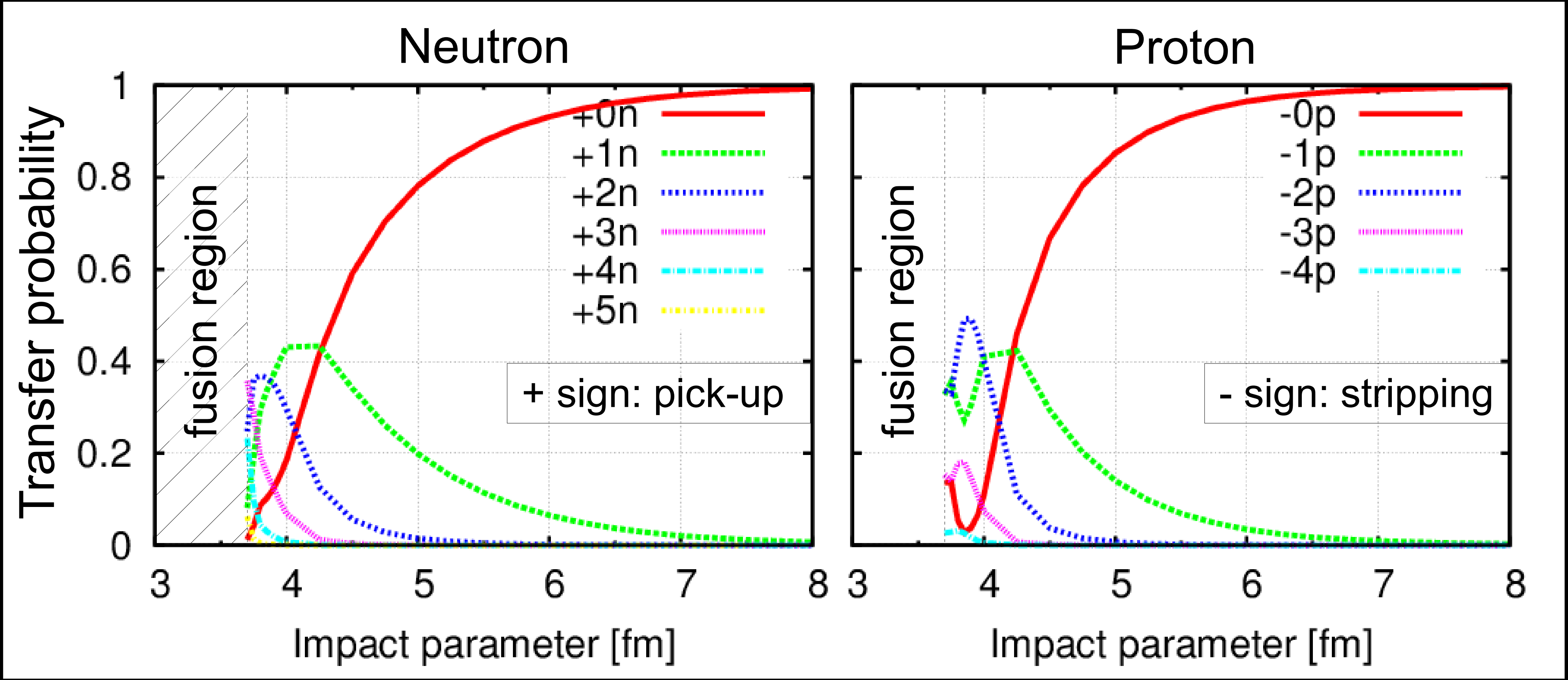
N/Z ratio

^{40}Ca: 1.00

^{124}Sn: 1.48

Nucleon transfer probabilities

$$P_n = <\Phi_f|\delta(n-\hat{N}_A)|\Phi_f> = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, e^{in\theta} \det\{<\phi_i|\phi_j>_B + e^{-i\theta}<\phi_i|\phi_j>_A\}$$



Neutron: ^{40}Ca ← ^{124}Sn

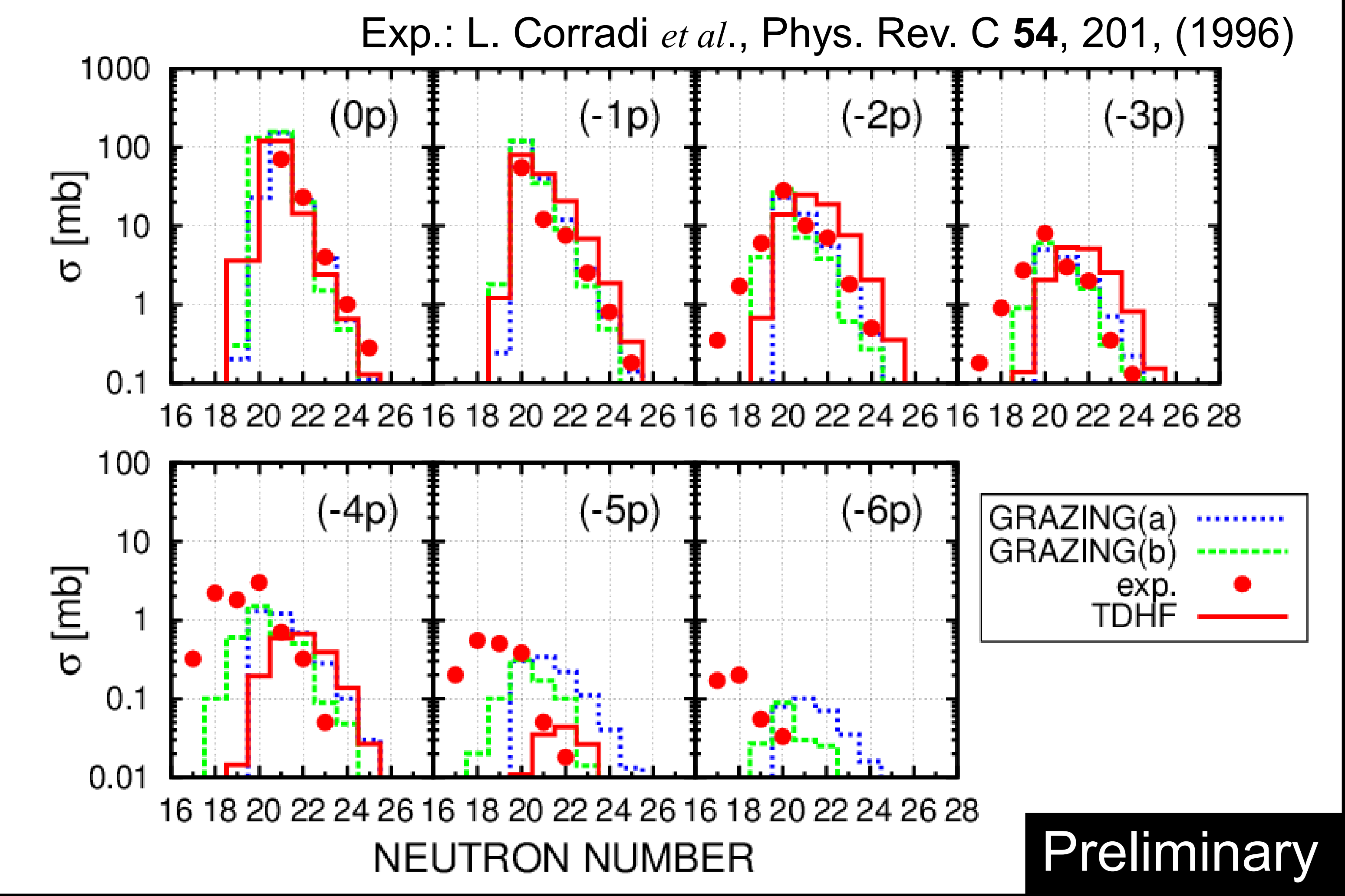
Proton: ^{40}Ca → ^{124}Sn

Nucleons are transferred toward the direction of charge equilibration.

3. Results: ^{40}Ca+^{124}Sn, E_{lab}=170 [MeV]

Nucleon transfer cross section

$$\sigma_{\mathrm{tr}}(n) = \int_{\mathrm{b_{min}}}^{\mathrm{b_{max}}} 2\pi b P_n(b) db$$



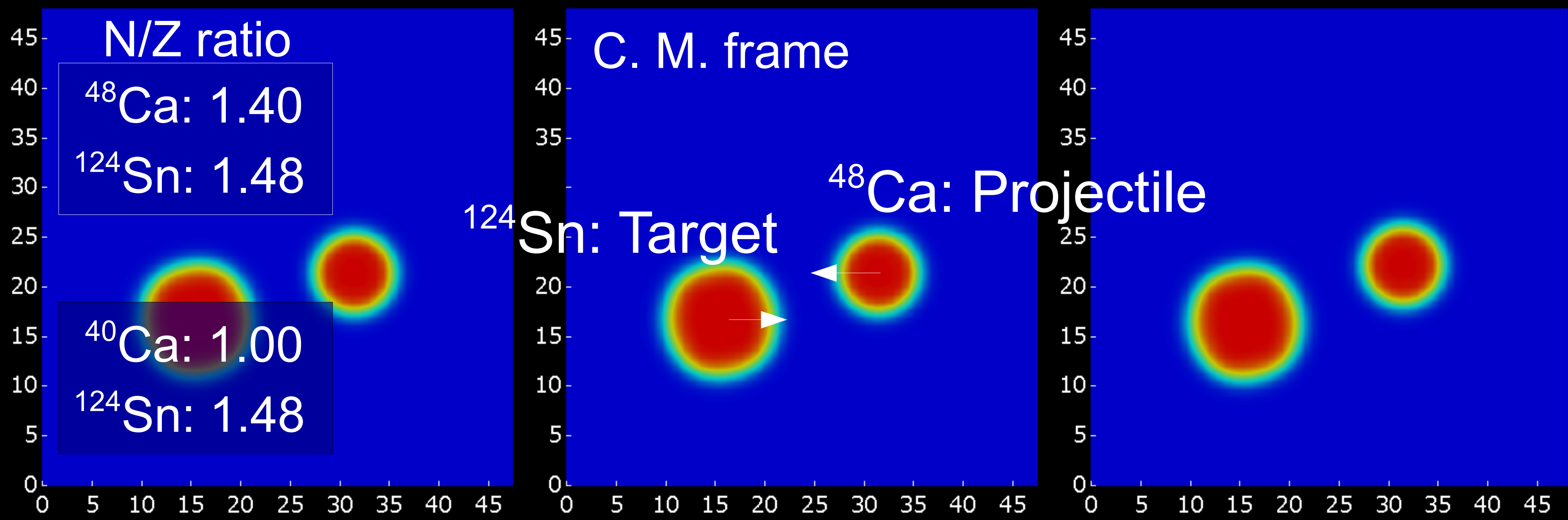
Preliminary

Overall agreement is good when number of transferred proton is small.

3. Results: ^{48}Ca+^{124}Sn, E_{lab}=174 [MeV]

We have also calculate final wave functions for each impact parameter.

Fusion	Transfer
b= 0, ･･･, b= 3.62 fm	b= 3.63 fm, ･･･, bcut ≡ 10.00 fm



<ΔN>: 0.79 σ_n: 1.02
<ΔZ>: 0.19 σ_p: 0.50

<ΔN>: 0.10 σ_n: 0.40
<ΔZ>: 0.01 σ_p: 0.15

Skyrme: SLy5 Grid size: 60×60×26 (48×48×20.8 fm) Initial distance: 16 fm

Mesh spacing: 0.8 fm Time step: 0.2 fm/c L. Corradi *et al.*, Phys. Rev. C **56**, 938, (1997)

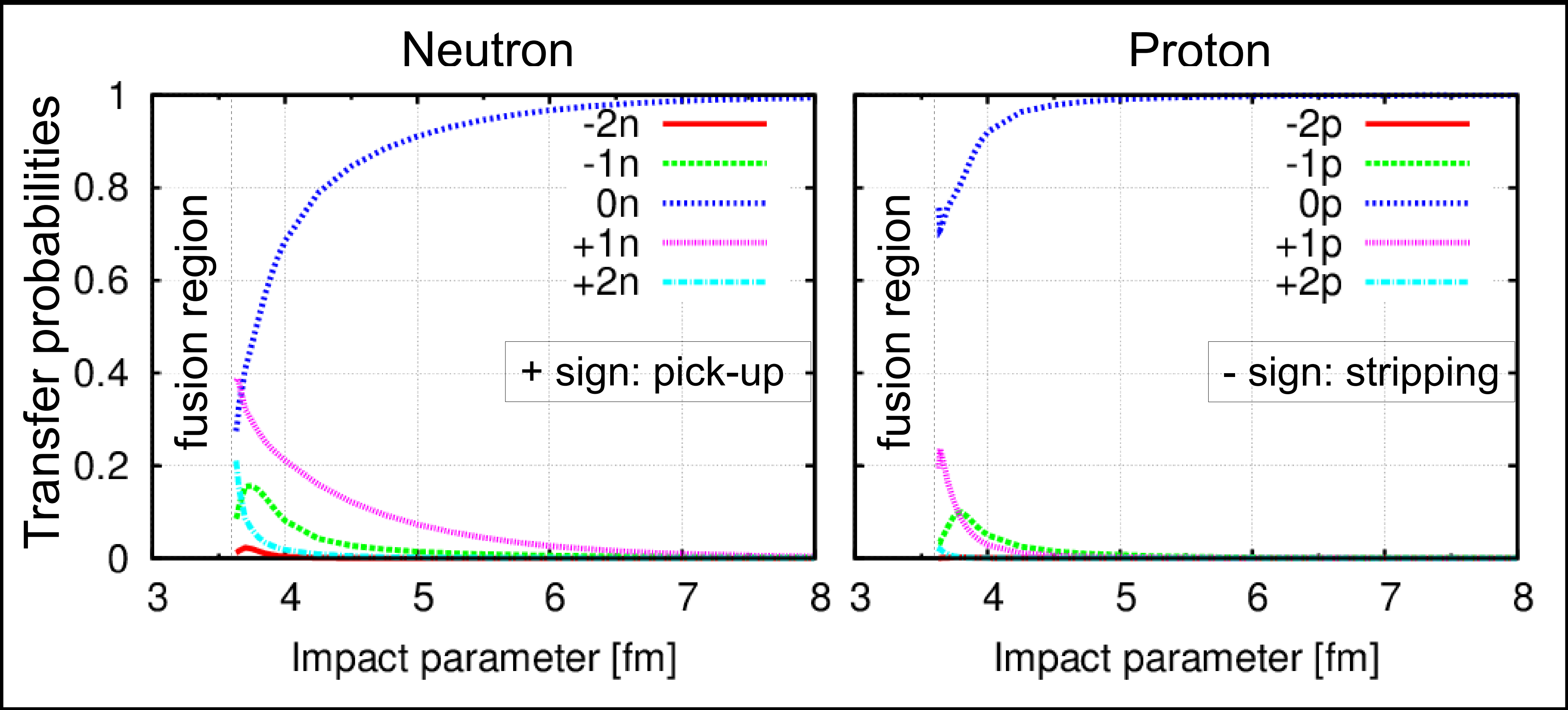
3. Results: ^{48}Ca+^{124}Sn, E$_{lab}$=174 [MeV]

N/Z ratio

^{48}Ca: 1.40
^{124}Sn: 1.48

Nucleon transfer probabilities

$$P_n = < \Phi_f | \delta(n - \hat{N}_A) | \Phi_f > = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, e^{in\theta} \det \{ < \phi_i | \phi_j >_B + e^{-i\theta} < \phi_i | \phi_j >_A \}$$



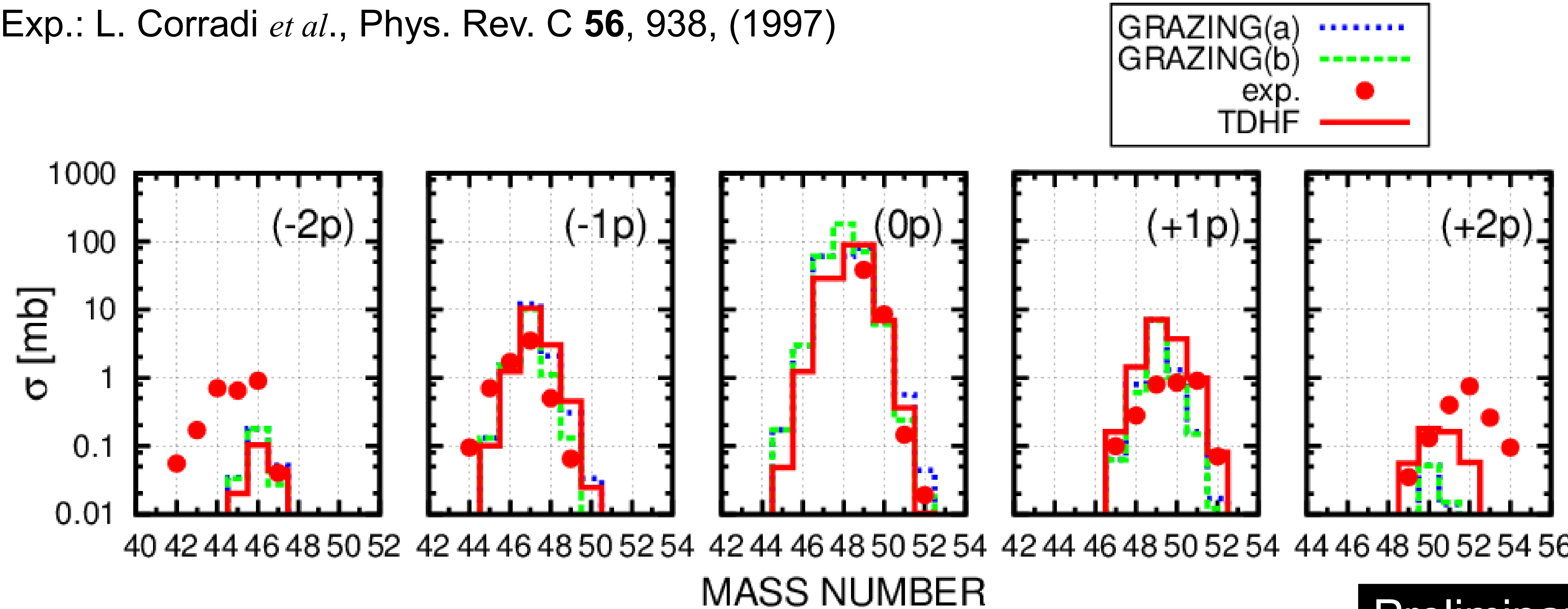
Nucleon transfer depends not only on the N/Z ratio but also on the Q-value.

3. Results: ^{48}Ca+^{124}Sn, E_{lab}=174 [MeV]

Nucleon transfer cross section

$$\sigma_{tr}(n) = \int_{b_{min}}^{b_{max}} 2\pi b P_n(b) db$$

Exp.: L. Corradi *et al.*, Phys. Rev. C **56**, 938, (1997)



Preliminary

➔ ***TDHF is promising for multi-nucleon transfer reaction.***

4. Summary and Outlook

Summary

- ✔ The methods to calculate nucleon transfer probabilities from many-body wave function have been presented.
- ✔ We have carried out $^{40, 48}Ca+^{124}Sn$ *(E_{lab}=170, 174 MeV)* TDHF calculations and compare the results with the measurements.

  *Exp.: L. Corradi et al., PRC **54**, 201, (1996), PRC **56**, 938 (1997)*
- ✔ Overall agreement is good when transferred proton number is small.

➔ *We consider TDHF is promising for multi-nucleon transfer reaction.*

Outlook

- ➢ To estimate the evaporation's effect.
- ➢ We'll elucidate the mechanism of multi-nucleon transfer reaction by performing systematic calculation.