

Spectroscopy and correlations probed via two-nucleon knockout reactions

DCEN Workshop
25th October 2011

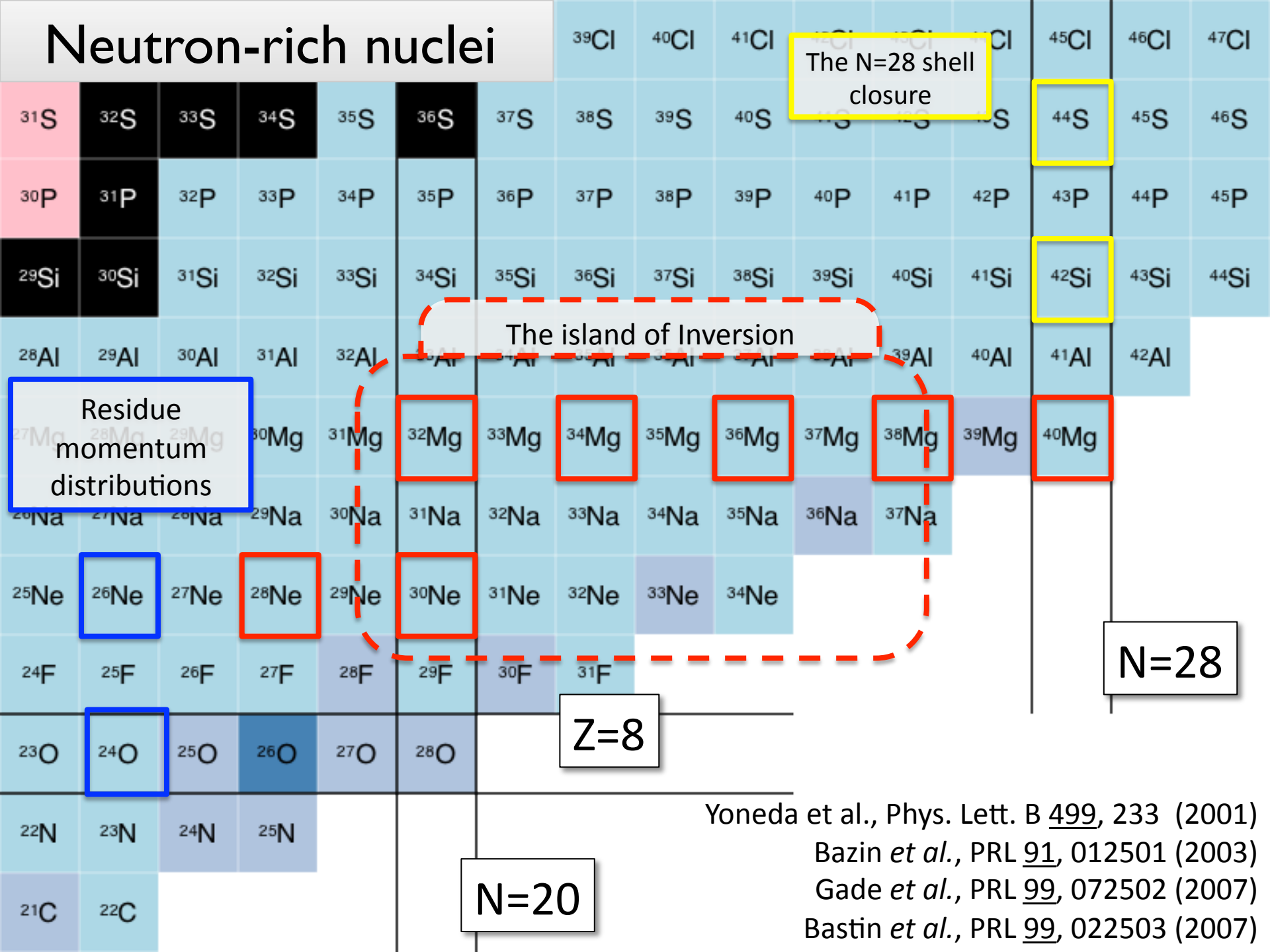
Edward Simpson
University of Surrey



Outline

- Introduction
- Two-nucleon removal reactions
- How can we learn about structure?
 - Final state spins from momentum distributions
 - Two-nucleon spatial correlations
 - Tests of underlying structure
- $N=Z$ nuclei
 - Stable nuclei
 - np-correlations?
- Regions of changing structure
- Conclusions

Neutron-rich nuclei



The N=28 shell closure

The island of Inversion

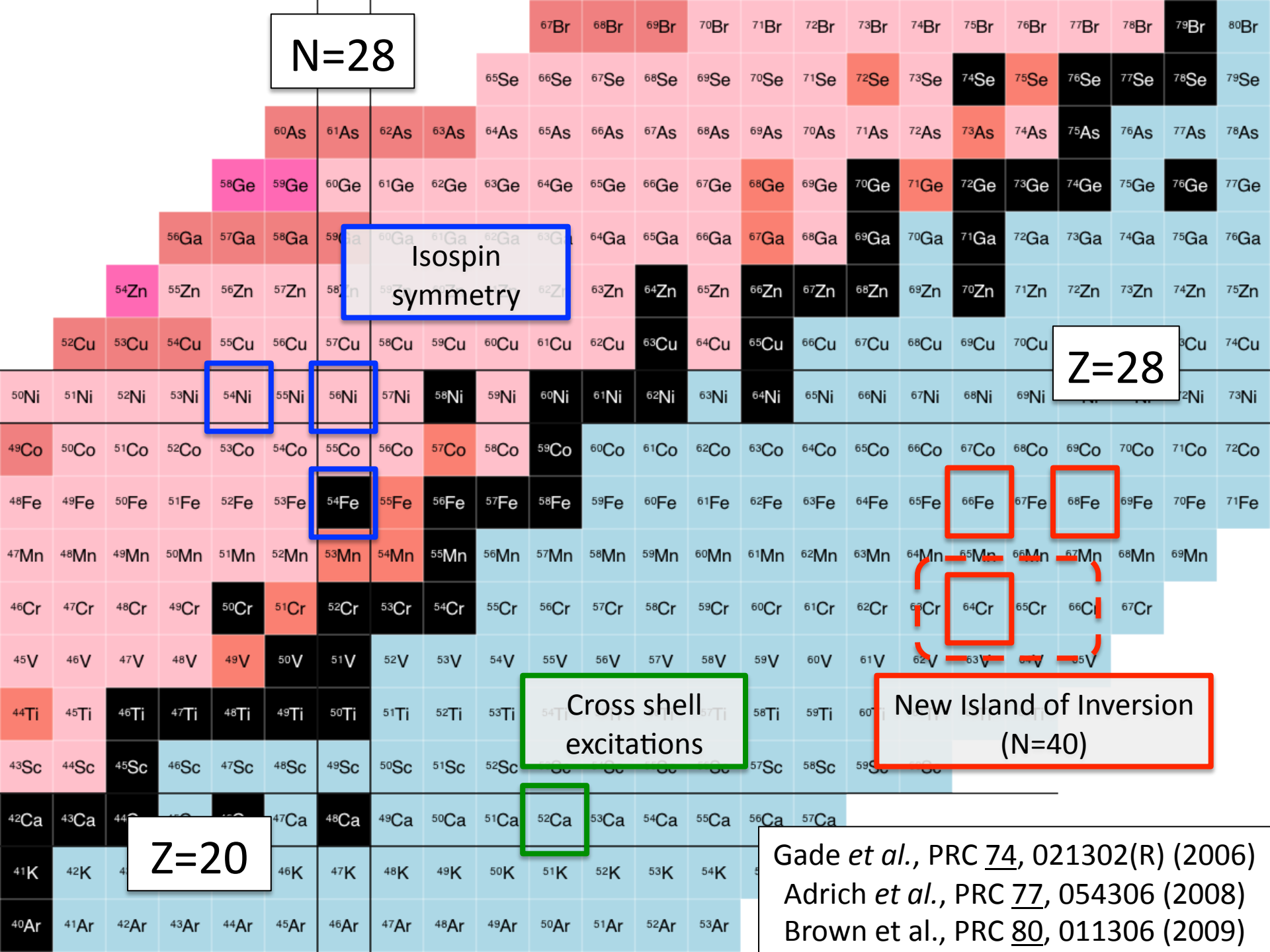
Residue momentum distributions

N=28

Z=8

N=20

Yoneda et al., Phys. Lett. B 499, 233 (2001)
Bazin *et al.*, PRL 91, 012501 (2003)
Gade *et al.*, PRL 99, 072502 (2007)
Bastin *et al.*, PRL 99, 022503 (2007)



N=28

Isospin
symmetry

Z=28

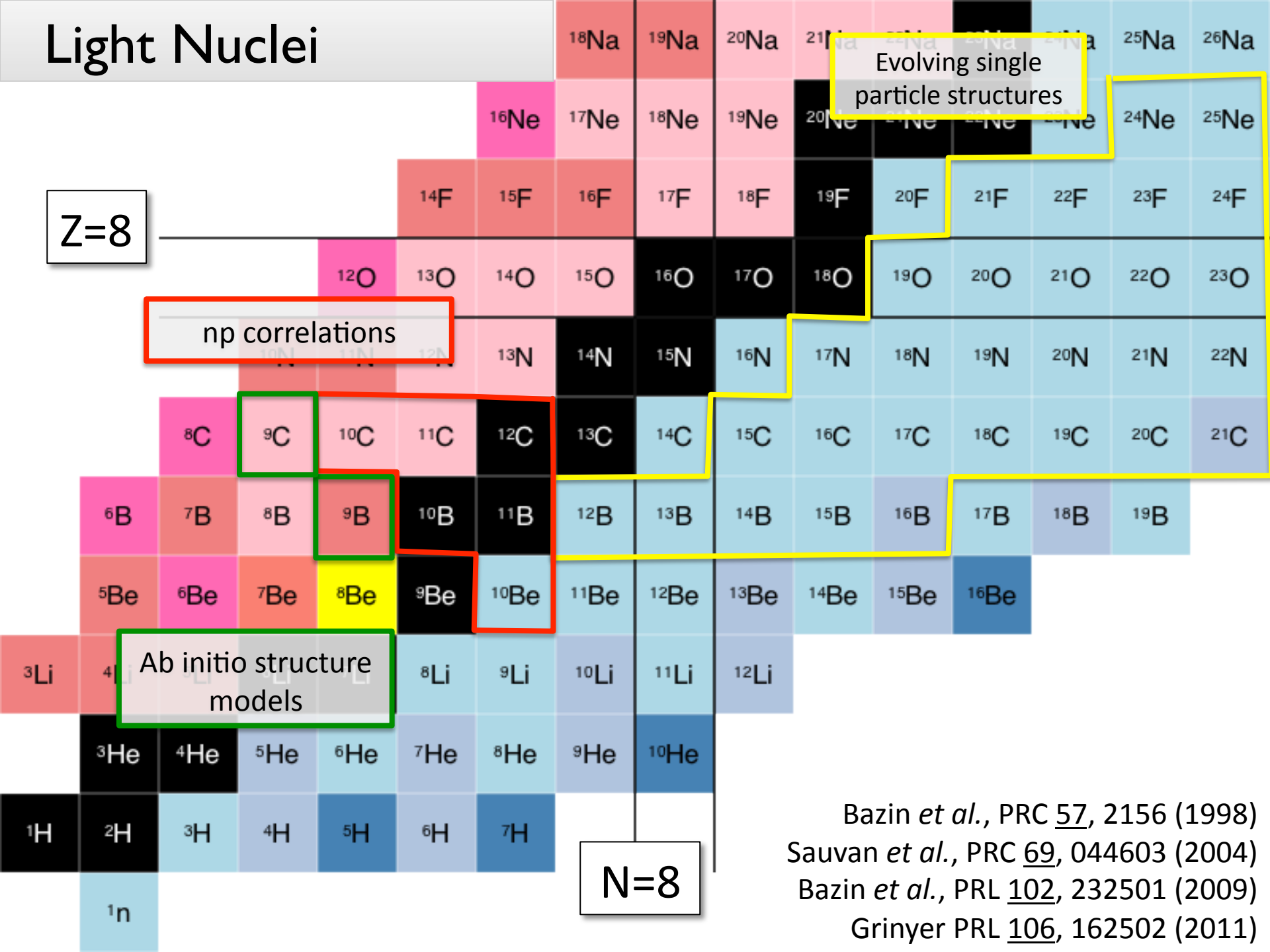
Z=20

Cross shell
excitations

New Island of Inversion
(N=40)

Gade *et al.*, PRC 74, 021302(R) (2006)
 Adrich *et al.*, PRC 77, 054306 (2008)
 Brown *et al.*, PRC 80, 011306 (2009)

Light Nuclei



Evolving single particle structures

Z=8

np correlations

Ab initio structure models

N=8

Bazin *et al.*, PRC 57, 2156 (1998)
 Sauvan *et al.*, PRC 69, 044603 (2004)
 Bazin *et al.*, PRL 102, 232501 (2009)
 Grinyer PRL 106, 162502 (2011)

Two-nucleon removal reactions

Single-nucleon removal reactions

Removal of nucleons from a (secondary radioactive) beam at energies >80 MeV/nucleon on a light nuclear target (Be, C)

Halos: ^{15}C , ^{19}C , ^{27}P , ^{31}Ne

Magic numbers: ^{24}O , ^{42}Si

Exotic R_s : ^{23}Al , ^{23}Si , ^{27}P , ^{27}S

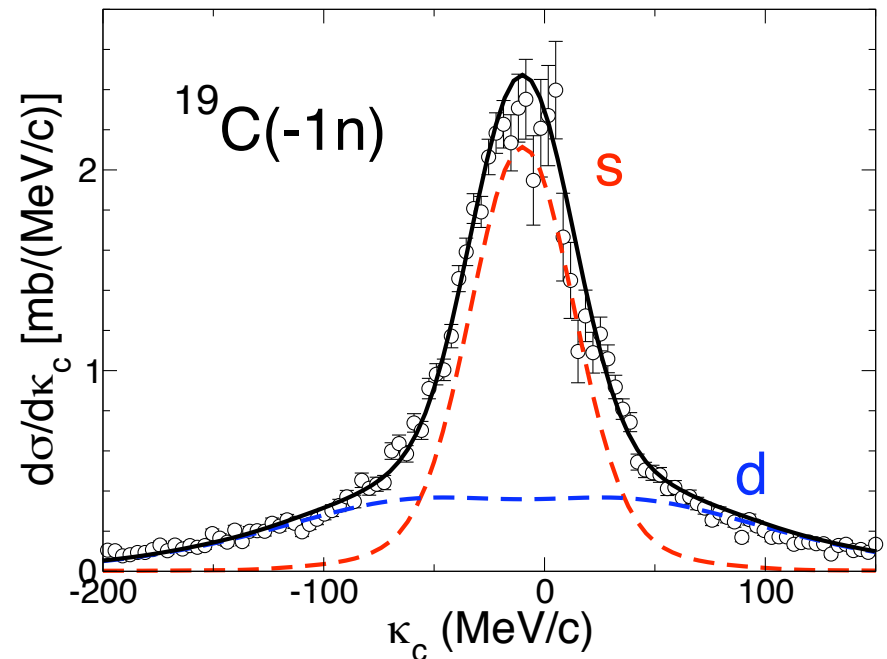
Absolute cross sections

- Cross section proportional to spectroscopic strength
- Suppression of spectroscopic strengths in asymmetric systems

$$\sigma_{-1n} = \sum_{nlj} C^2 S_{nlj} \sigma_{sp}$$

Beam directional momentum distributions

- Width \rightarrow Orbital angular momentum (final state spins, evolution of shell ordering)

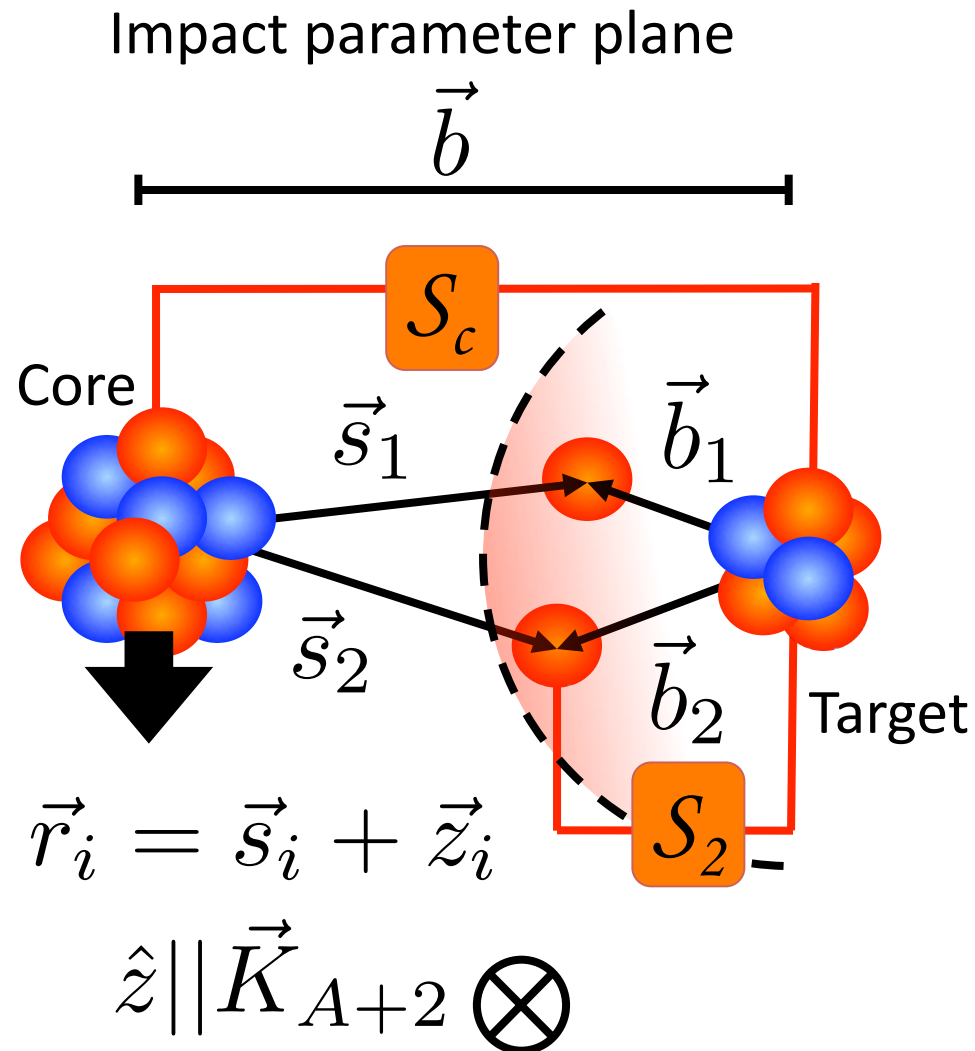


Hansen and Tostevin, Annu. Rev. Nucl. Part. Sci. 53, 219 (2003)

Bertulani and Hansen, PRC 70 034609 (2004)

Surface structure

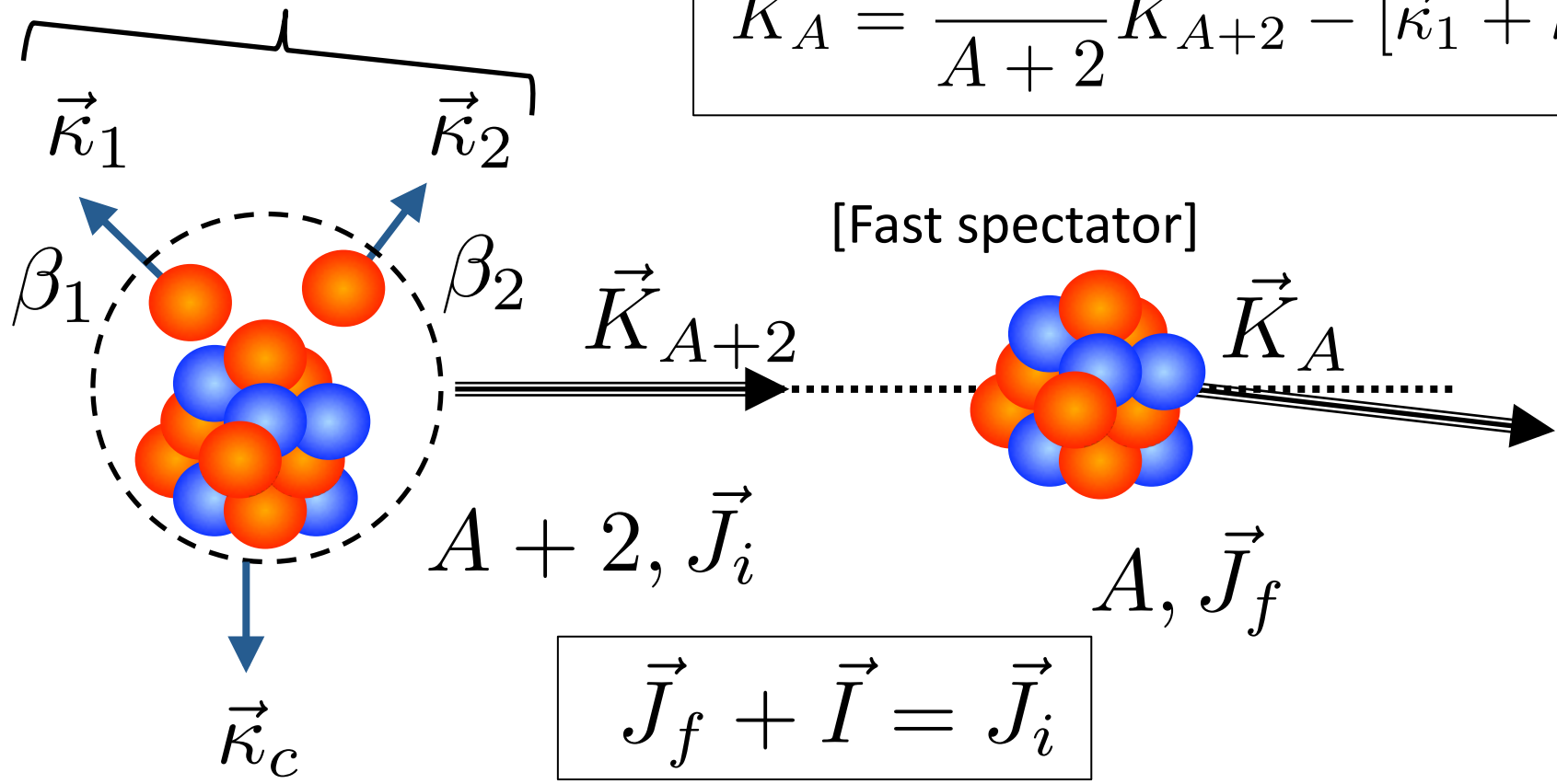
- Eikonal reaction dynamics (straight line trajectory)
- Projectile internal co-ordinates assumed fixed for the duration of the (fast) interaction
- Core assumed to act as spectator during the fast interaction
- Reaction probes the projection of the (two-) nucleon wave function on the impact parameter plane
- Final state of the valence nucleons and target unobserved



Momentum distributions

$$\vec{j}_1 + \vec{j}_2 = \vec{I}$$

$$\vec{K}_A = \frac{A}{A+2} \vec{K}_{A+2} - [\vec{\kappa}_1 + \vec{\kappa}_2]$$



$$\vec{J}_f + \vec{I} = \vec{J}_i$$

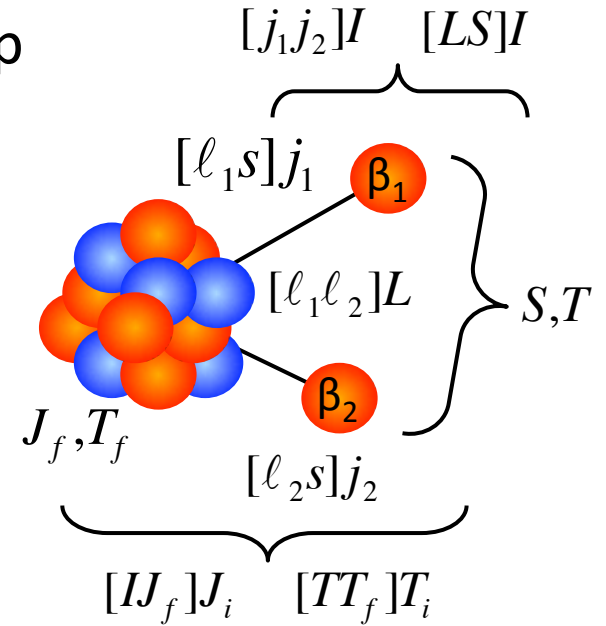
K_A distribution characteristic of $\kappa_1 + \kappa_2$

Two-nucleon overlap

Shell-model LS-coupled two-nucleon overlap

$$\begin{aligned} \Psi_i^{(F)}(1, 2) &= \langle \Phi^{(F)}(A) | \Psi_i(A+2) \rangle \\ &= \sum_{I\mu T\alpha} C_\alpha^{IT} (T\tau T_f \tau_f | T_i \tau_i) (I\mu J_f M_f | J_i M_i) \\ &\quad \overline{[\psi_{\beta_1}(1) \otimes \psi_{\beta_2}(2)]_{I\mu}^{T\tau}} \end{aligned}$$

$$\alpha \equiv (\beta_1, \beta_2) \quad \beta \equiv (nlj)$$



Two-nucleon wave function

$$\begin{aligned} \overline{[\psi_{\beta_1}(1) \otimes \psi_{\beta_2}(2)]_{I\mu}^{T\tau}} &= D_\alpha \hat{j}_1 \hat{j}_2 \sum_{\substack{L\Lambda S\Sigma \\ \lambda_1 \lambda_2}} (\ell_1 \lambda_1 \ell_2 \lambda_2 | L\Lambda) (L\Lambda S\Sigma | I\mu) \hat{L} \hat{S} \chi_{S\Sigma}(1, 2) \chi_{T\tau}(1, 2) \\ &\quad \times \begin{Bmatrix} \ell_1 & s & j_1 \\ \ell_2 & s & j_2 \\ L & S & I \end{Bmatrix} [\psi_{\beta_1}^{\lambda_1}(\vec{r}_1) \psi_{\beta_2}^{\lambda_2}(\vec{r}_2) - (-)^{S+T} \psi_{\beta_1}^{\lambda_1}(\vec{r}_2) \psi_{\beta_2}^{\lambda_2}(\vec{r}_1)] \end{aligned}$$

Single-nucleon wave function

$$\psi_\beta^\lambda(\vec{r}) = u_\beta(r) Y_{\ell\lambda}(\hat{r})$$

Absorption cross section

Absorption cross section, three-body projectile

$$\sigma_{abs} = \frac{1}{\hat{j}_i^2} \sum_{M_i} \int d\vec{b} \langle \Psi_i | 1 - |\mathcal{S}_f \mathcal{S}_1 \mathcal{S}_2|^2 | \Psi_i \rangle$$

Two-nucleon stripping cross section

$$\sigma_{str} = \frac{1}{\hat{j}_i^2} \sum_{M_i} \int d\vec{b} \langle \Psi_i | |\mathcal{S}_f|^2 (1 - |\mathcal{S}_1|^2) (1 - |\mathcal{S}_2|^2) | \Psi_i \rangle$$

Also contributions to diffractive-stripping – one nucleon removed via an elastic interaction; momentum distributions expected to be very similar to pure stripping

Core assumed to be spectator – no dynamic excitation

$$\langle \Phi^{(F')} (A) | |\mathcal{S}_f|^2 | \Phi^{(F)} (A) \rangle = |\mathcal{S}_c|^2 \delta_{FF'}$$

Stripping momentum distributions

Differential two-nucleon stripping cross section

$$\begin{aligned} \frac{d\sigma_{str}^{(f)}}{d\kappa_c} &= \sum_{LST} \frac{d\sigma_{LST}^{(f)}}{d\kappa_c} = \sum_T (T\tau T_f \tau_f | T_i \tau_i)^2 \sum_{LSI\alpha\alpha'} \frac{2\mathfrak{C}_{\alpha LS}^{IT} \mathfrak{C}_{\alpha' LS}^{IT} D_\alpha D_{\alpha'}}{\hat{L}^2} \\ &\int d\kappa_1 \int d\kappa_2 \delta(\kappa_c + \kappa_1 + \kappa_2) \int d\vec{b} |\mathcal{S}_c(b)|^2 \\ &\sum_{\Lambda \lambda_1 \lambda_2 \lambda'_1 \lambda'_2} (\ell_1 \lambda_1 \ell_2 \lambda_2 | L\Lambda) (\ell'_1 \lambda'_1 \ell'_2 \lambda'_2 | L\Lambda) \\ &\int ds_1 s_1 \int ds_2 s_2 [direct - exchange] \end{aligned}$$

$$direct = \left\{ \mathcal{H}_{\lambda_1 \lambda'_1}(1) \mathcal{R}_{\beta_1}^{\lambda_1}(1) \mathcal{R}_{\beta'_1}^{\lambda'_1}(1)^* \mathcal{H}_{\lambda_2 \lambda'_2}(2) \mathcal{R}_{\beta_2}^{\lambda_2}(2) \mathcal{R}_{\beta'_2}^{\lambda'_2}(2)^* \right\}$$

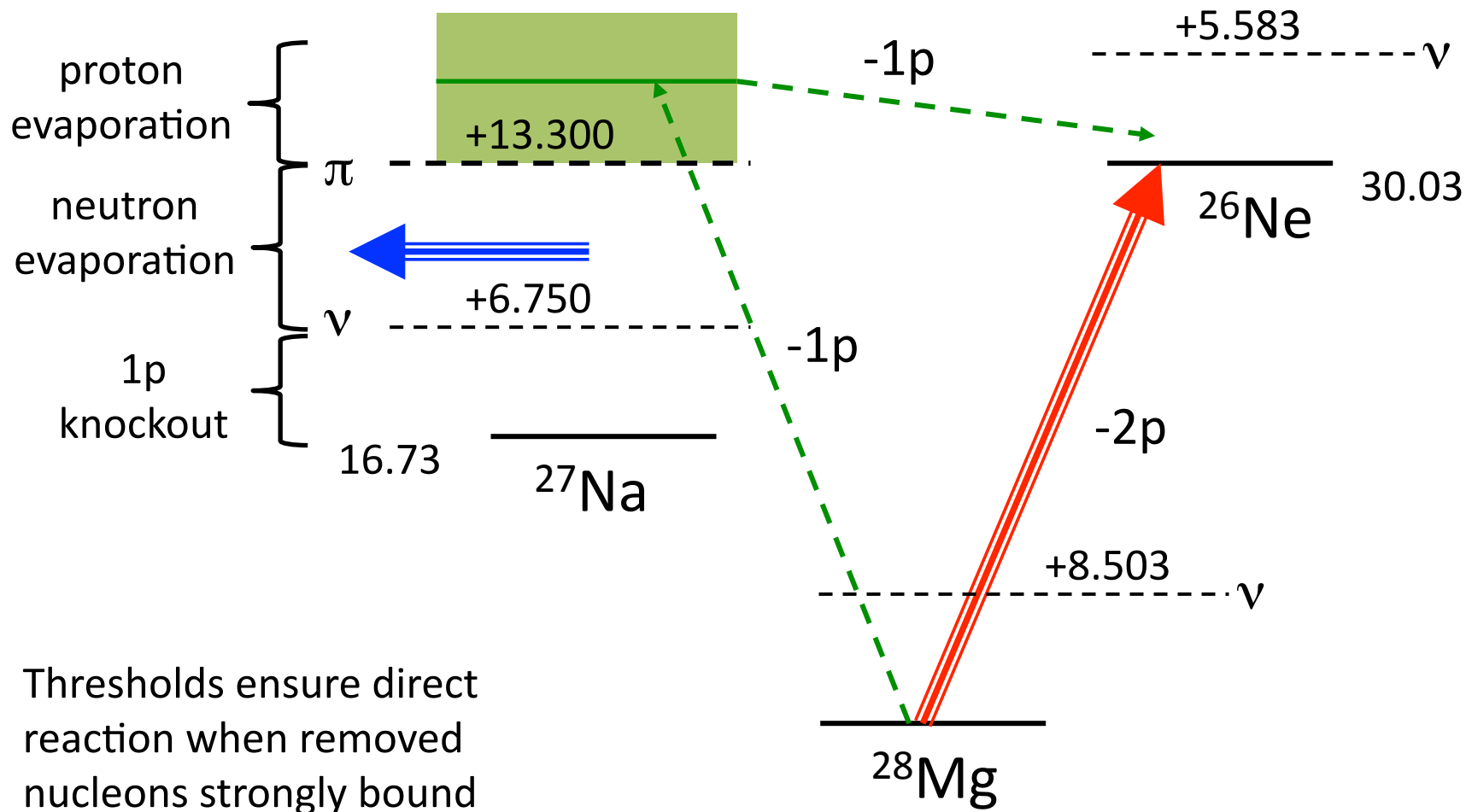
$$exchange = (-1)^{S+T} \left\{ \mathcal{H}_{\lambda_2 \lambda'_1}(1) \mathcal{R}_{\beta_2}^{\lambda_2}(1) \mathcal{R}_{\beta'_1}^{\lambda'_1}(1)^* \mathcal{H}_{\lambda_1 \lambda'_2}(2) \mathcal{R}_{\beta_1}^{\lambda_1}(2) \mathcal{R}_{\beta'_2}^{\lambda'_2}(2)^* \right\}$$

LS-coupled amplitude $\mathfrak{C}_{\alpha LS}^{IT} = \hat{j}_1 \hat{j}_2 \hat{L} \hat{S} \left\{ \begin{array}{ccc} \ell_1 & s & j_1 \\ \ell_2 & s & j_2 \\ L & S & I \end{array} \right\} C_\alpha^{IT}$

Spectroscopy and structure sensitivities

Example 1: $^{28}\text{Mg}(-2p)$ thresholds

$^{28}\text{Mg}(-2p)$



Thresholds ensure direct reaction when removed nucleons strongly bound

Example 1: $^{28}\text{Mg}(-2p)$ calculation input

Radial wave functions calculated in a Woods-Saxon, the geometry of which is fitted to Hartree-Fock rms radii and binding energies.

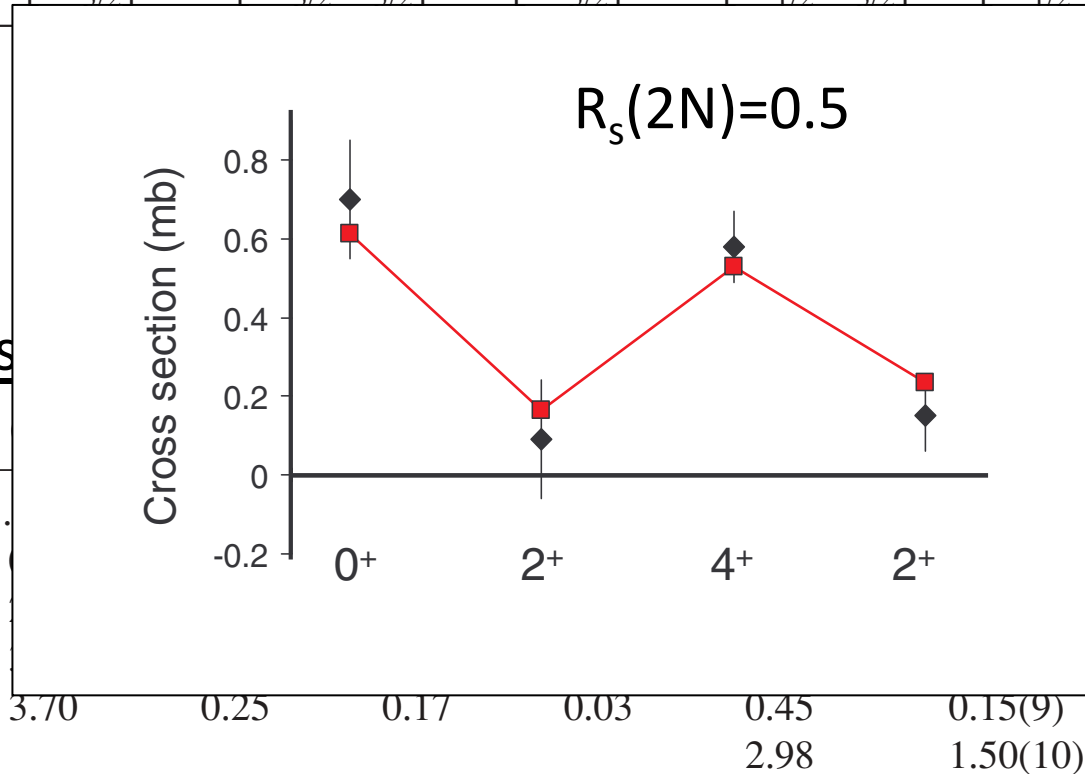
Hartree-Fock density used to calculate residue-target S-matrix.

Full sd-shell USD two-nucleon amplitudes

J_f^π	E^* (MeV)	$[0d_{3/2}]^2$	$[0d_{3/2}0d_{5/2}]$	$[0d_{5/2}]^2$	$[1s_{1/2}0d_{3/2}]$	$[1s_{1/2}0d_{5/2}]$	$[1s_{1/2}]^2$
0_1^+	0.0						-0.30496
2_1^+	2.02						0.916
4_1^+	3.50						0.90
2_2^+	3.70						0.90

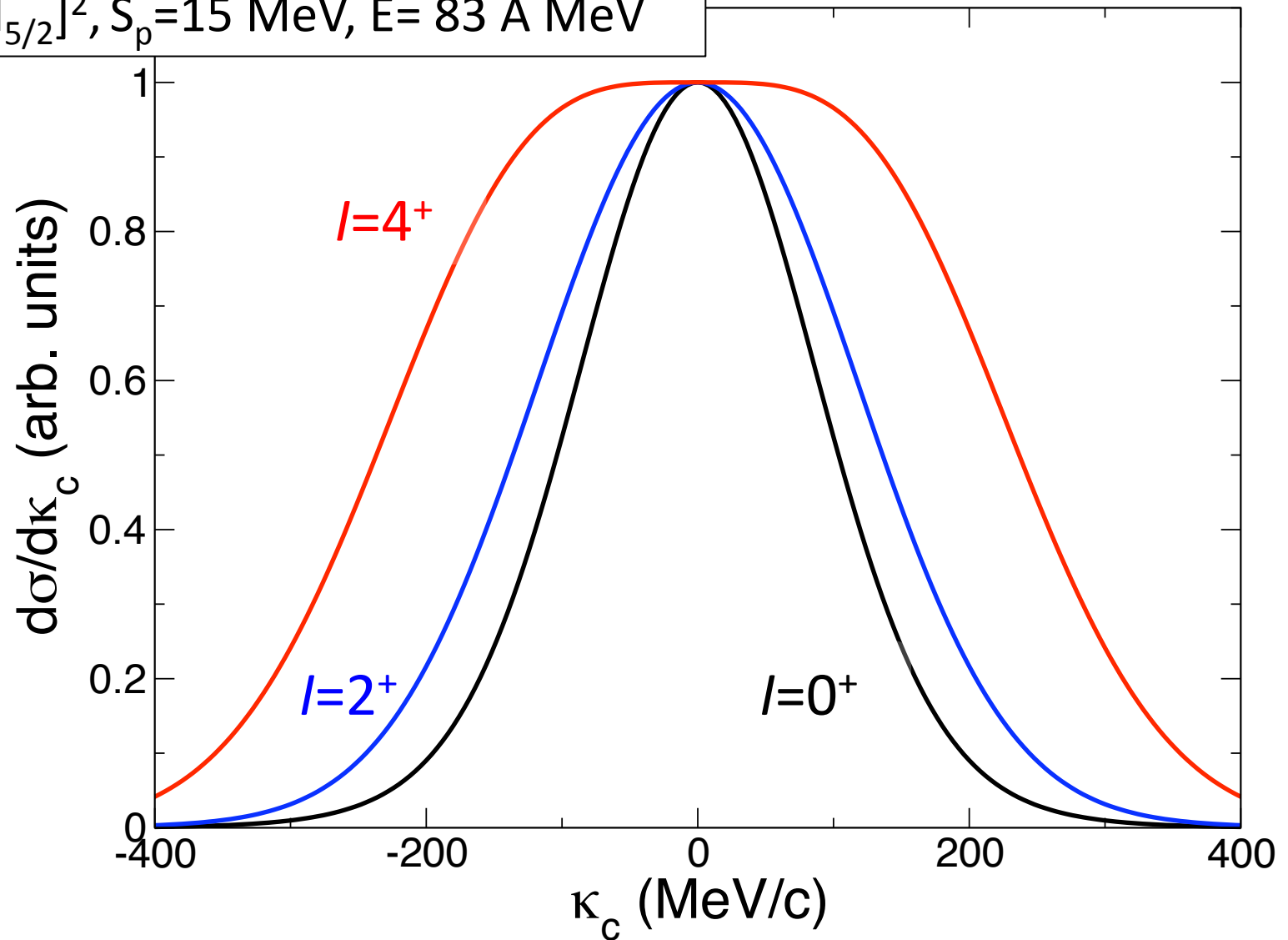
Cross sections

J_f^π	E	$R_s(2N)$
$^{28}\text{Mg} \rightarrow ^{26}\text{Ne}$	83.0	
0^+		0.59(13)
2_1^+		0.28(47)
4^+		0.57(9)
2_2^+	3.70	0.33(20)
Incl.		0.50(3)

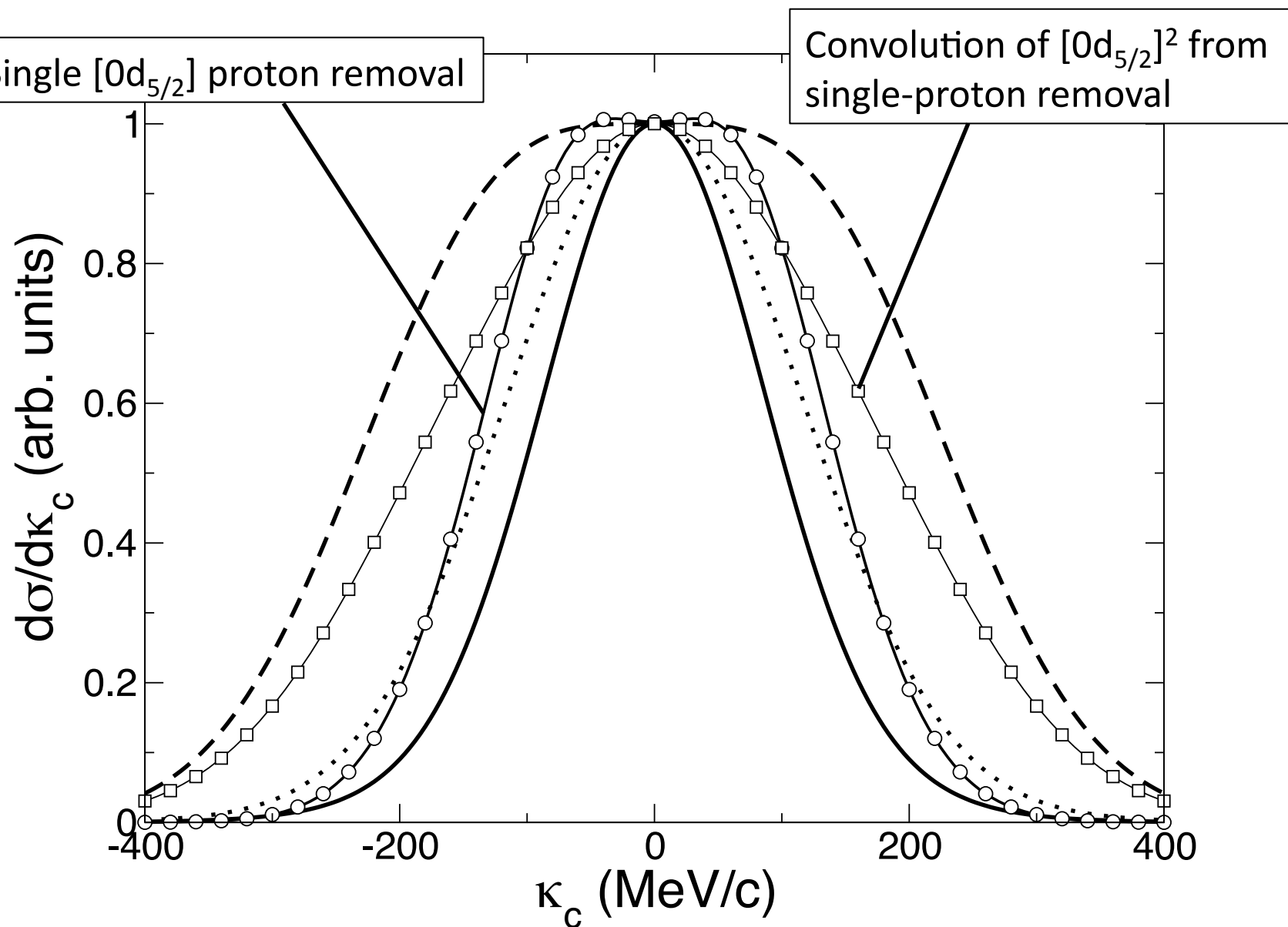


Example 1: $^{28}\text{Mg}(-2p) [0d_{5/2}]^2$

$[0d_{5/2}]^2, S_p=15 \text{ MeV}, E=83 \text{ A MeV}$



Example 1: Single/Uncorrelated



Example 1: $^{28}\text{Mg}(-2p)$ momentum distributions

$^{28}\text{Mg}(-2p)$

Beam energy

$E = 82.3 \text{ A MeV}$

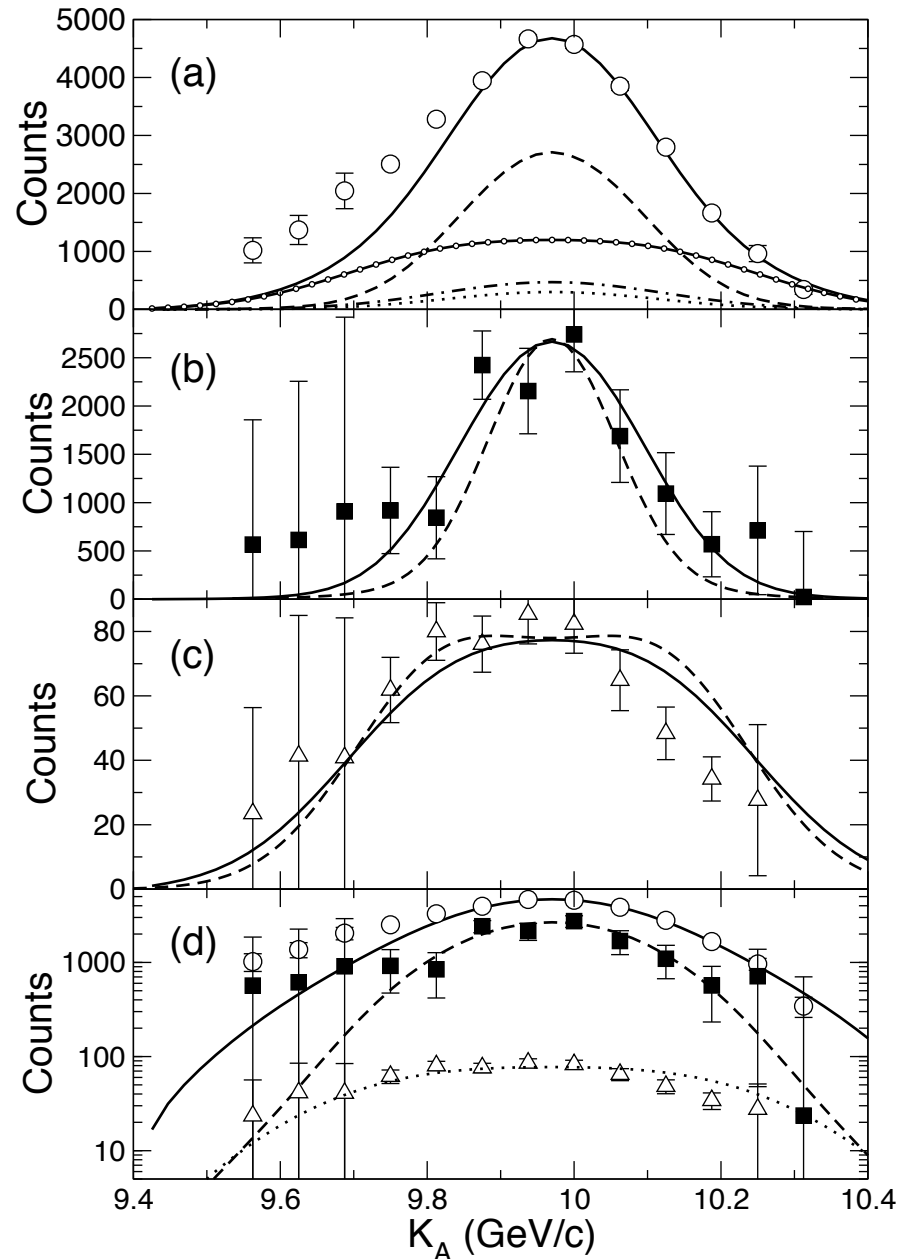
$S_p = 16.8 \text{ MeV}$

$S_n = 8.5 \text{ MeV}$

Broadening in thick
reaction target

$^9\text{Be } 375 \text{ mg/cm}^2$

$\Delta K_A = 0.29 \text{ GeV}/c$

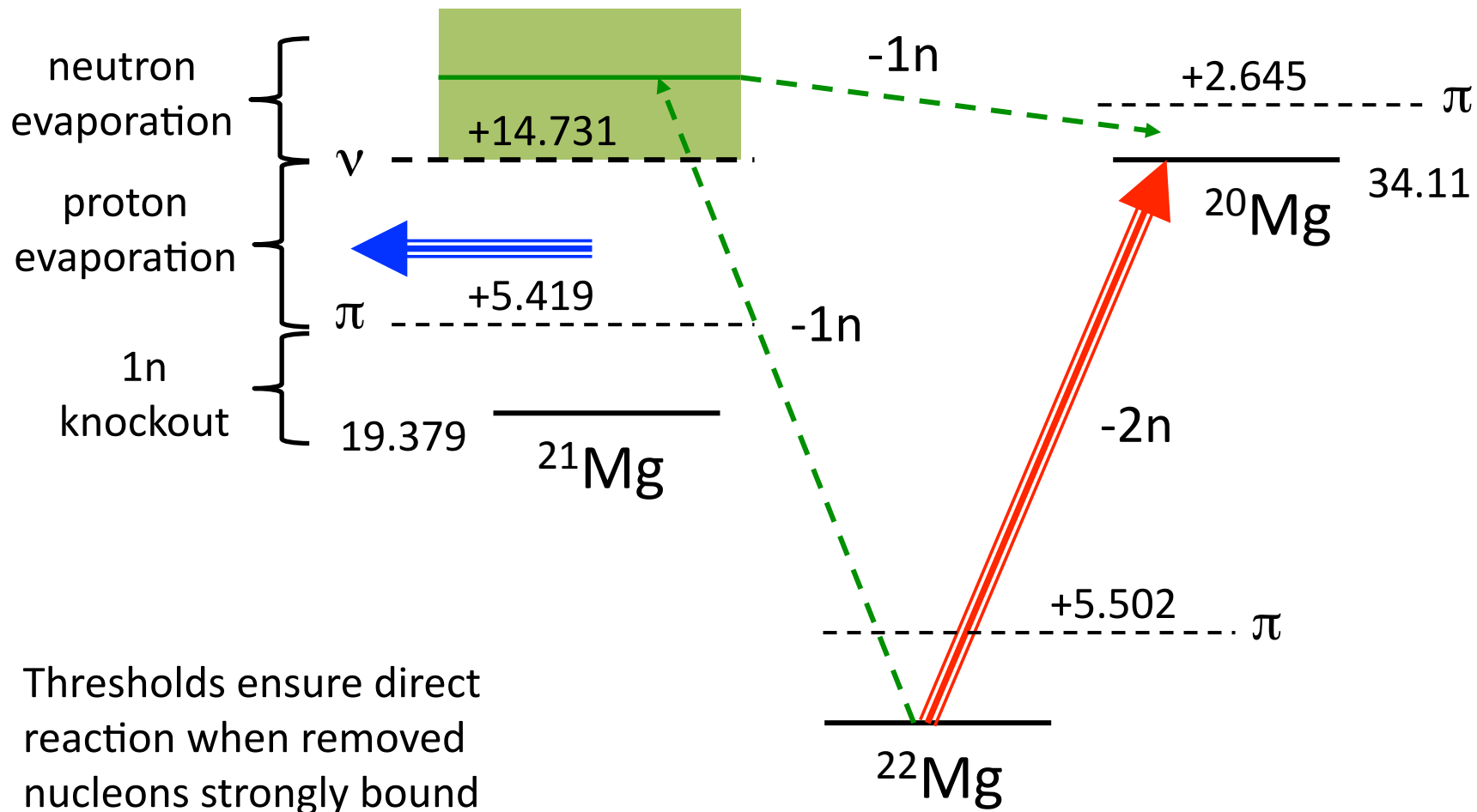


Bazin *et al.*, PRL 91 012501 (2003)

Simpson *et al.*, PRL 102, 132502 (2009)

Example 2: $^{22}\text{Mg}(-2n)$ thresholds

$^{22}\text{Mg}(-2n)$



Example 2: $^{22}\text{Mg}(-2n)$ structure input

USD two-nucleon amplitudes

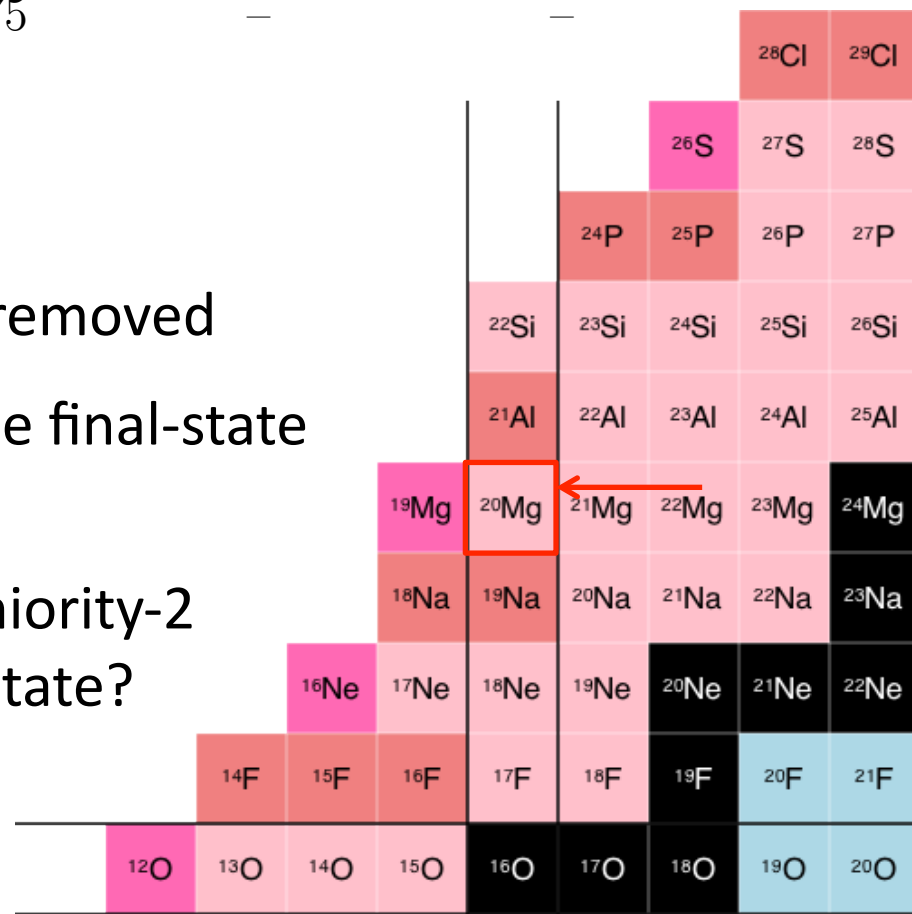
J_f^π	$[0d_{5/2}]^2$	$[0d_{3/2}]^2$	$[1s_{1/2}]^2$	$[0d_{3/2}][0d_{5/2}]$	$[0d_{5/2}][1s_{1/2}]$	$[0d_{3/2}][1s_{1/2}]$
0^+	0.8029	0.2546	0.3784	—	—	—
2^+	0.4566	0.1010	—	-0.1937	0.5244	-0.1983
4^+	-0.0153	—	—	-0.0175	—	—

$N=10 \rightarrow N=8$

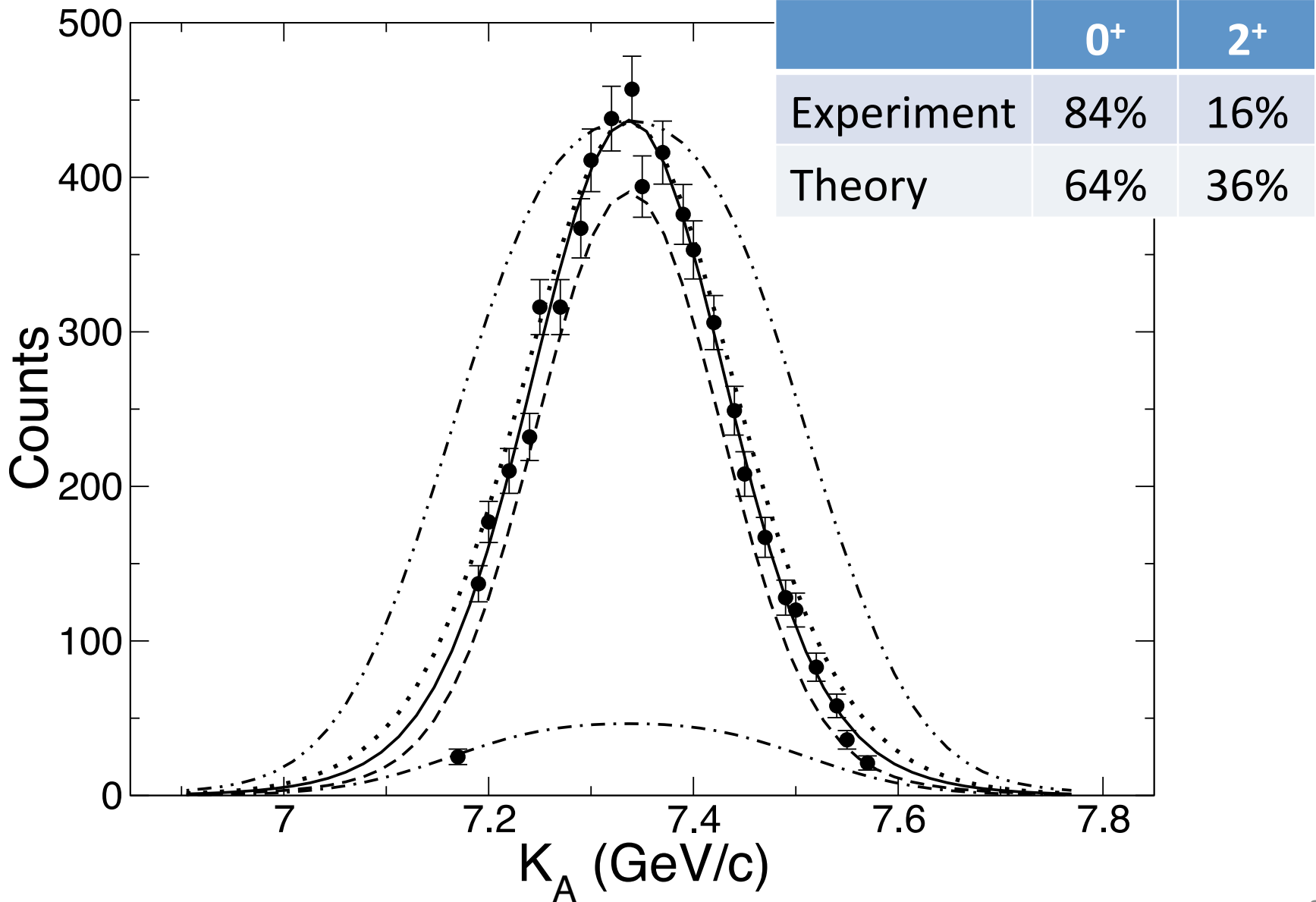
Last two neutrons in the sd-shell removed

Must necessarily be coupled to the final-state spin in the ^{22}Mg ground state

What could this tell use about seniority-2 components of the ^{22}Mg ground state?



Example 2: $^{22}\text{Mg}(-2n)$ momentum distributions



Spatial correlations

Joint position probability

Cross section in terms of joint position probability

$$\sigma_{\text{str}}^{(f)} = \int d\vec{b} \int d\vec{s}_1 \int d\vec{s}_2 \mathcal{P}_f(\vec{s}_1, \vec{s}_2) \mathcal{O}_{\text{str}}(c, 1, 2),$$

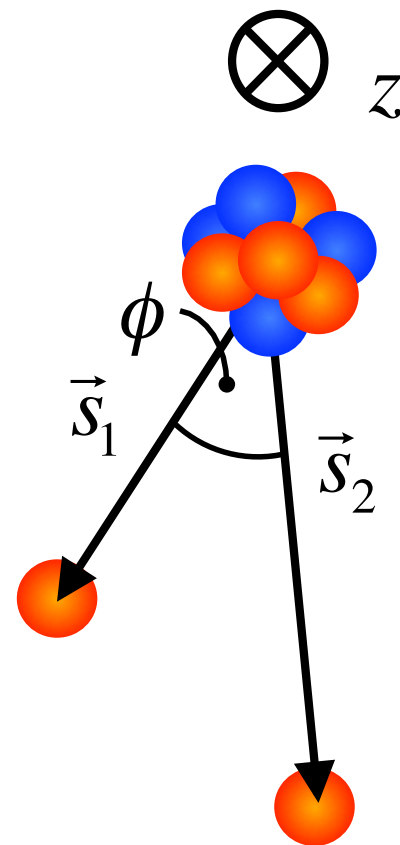
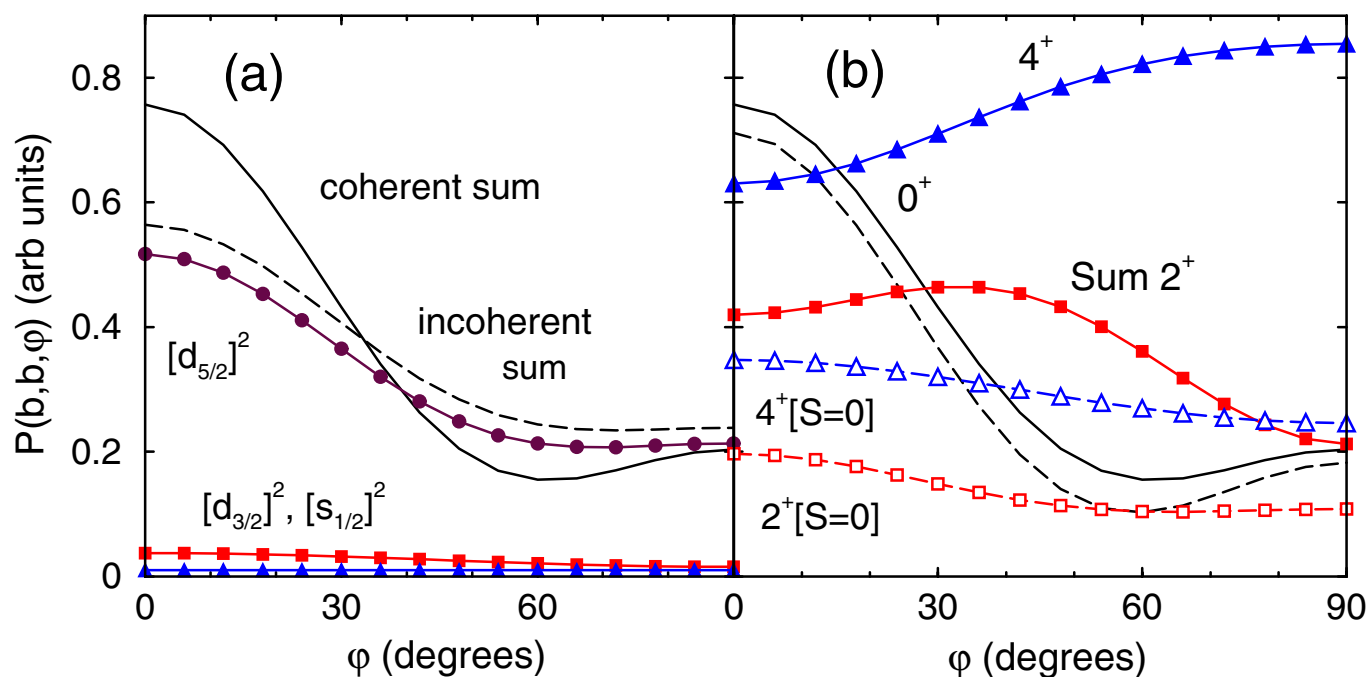
Joint position probability

$$\mathcal{P}_f(\vec{s}_1, \vec{s}_2) = \frac{1}{\hat{J}_i^2} \sum_{M_i M_f} \int dz_1 \int dz_2 \langle |\Psi_{J_i M_i}^{(F)}|^2 \rangle_{\text{sp}}.$$

Reaction probes JPP – two-nucleon overlap projected onto the impact parameter plane

Spatial correlations I

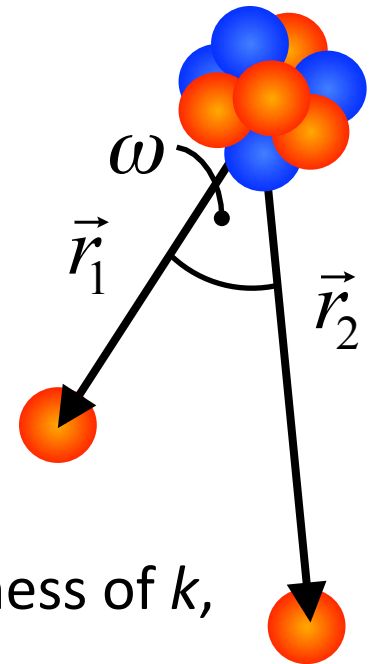
Joint position probability projected on to the impact parameter plane, displayed as the azimuthal angular separation of nucleons 1 and 2 for fixed b .



Spatial correlations II

Determines the probability for angular separation, reaction geometry determines how ω is probed

$$\Gamma_{\ell_1 \ell_2 \ell'_1 \ell'_2}^L(\omega) = (-1)^L \frac{\hat{\ell}_1 \hat{\ell}'_1 \hat{\ell}_2 \hat{\ell}'_2 \hat{L}^2}{(4\pi)^2} \sum_k W(\ell_1 \ell_2 \ell'_1 \ell'_2; Lk) \\ \times (-1)^k (\ell_1 0 \ell'_1 0 | k 0) (\ell_2 0 \ell'_2 0 | k 0) P_k(\cos \omega)$$



Evenness with respect to $\cos \omega = 0$ depends on evenness of k , itself dependent on evenness of l_1 and l'_1 (l_2 and l'_2)

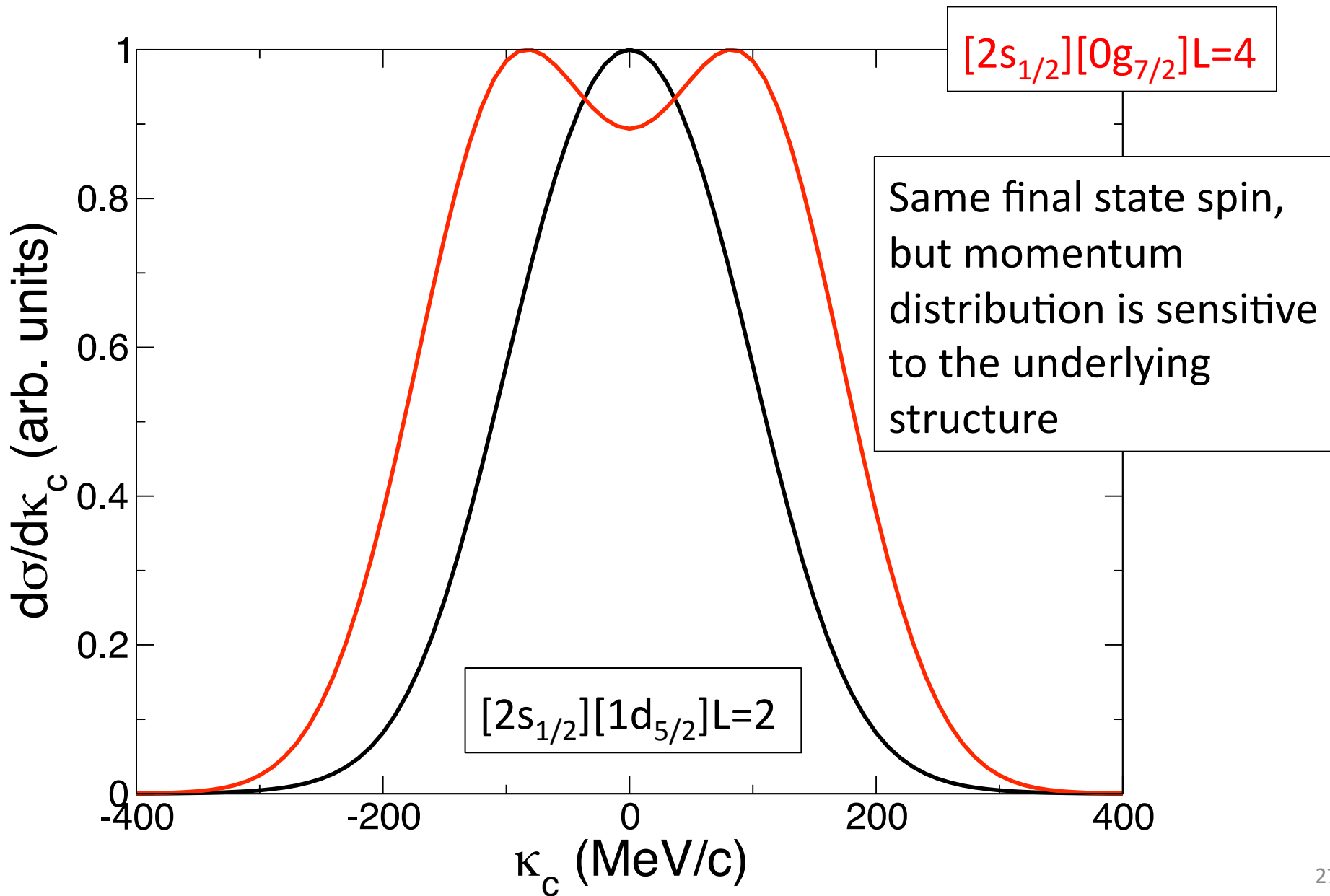
1. Structure sensitivity of momentum distributions?
2. Large basis calculations
3. Cross-shell excitations

Catara *et al.*, PRC 29, 1091 (1984)

Pinkston, PRC 29, 1123 (1984)

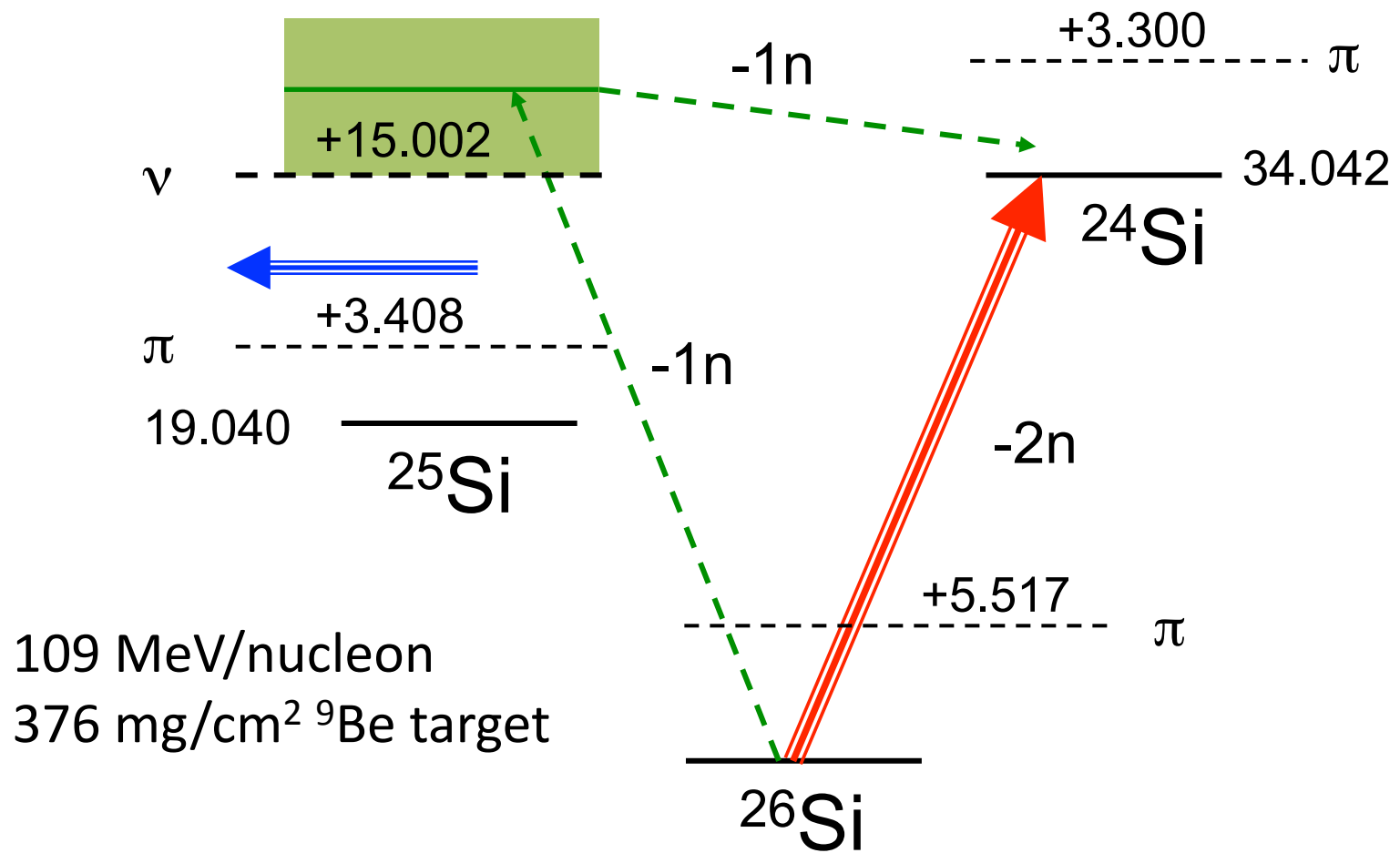
Tischler *et al.*, PRC 58, 2591 (1998)

Example: $^{208}\text{Pb}(-2p) \rightarrow ^{206}\text{Hg}(J_f=3^+)$



$^{26}\text{Si}(-2n) \rightarrow ^{24}\text{Si}$ separation thresholds

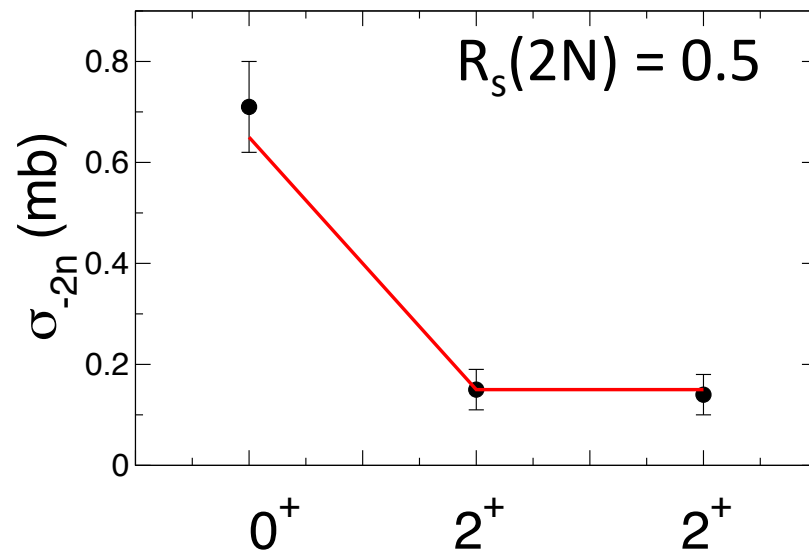
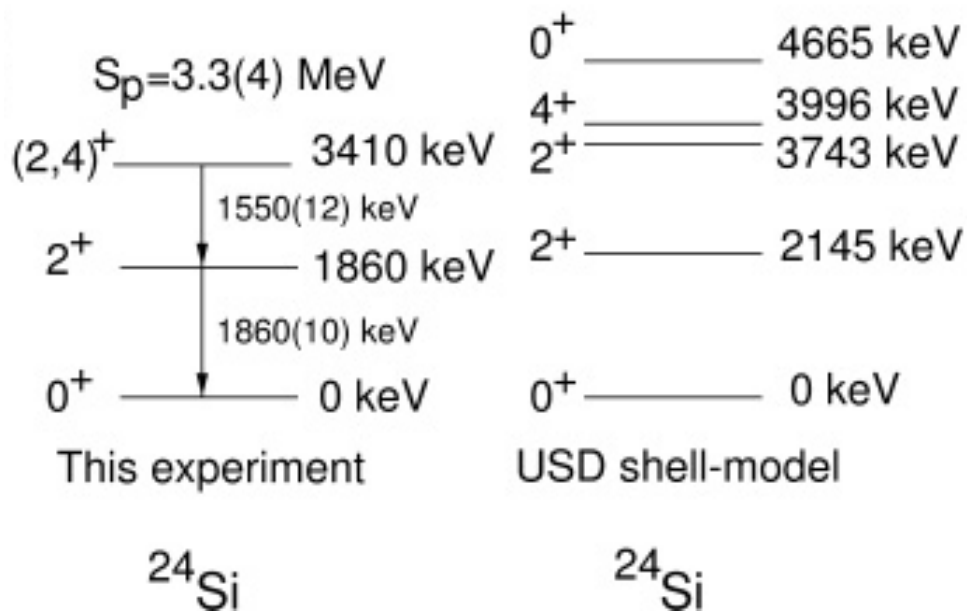
$^{26}\text{Si}(-2n)$



109 MeV/nucleon
376 mg/cm² ^9Be target

$^{26}\text{Si}(-2n)$: Cross section results

Results



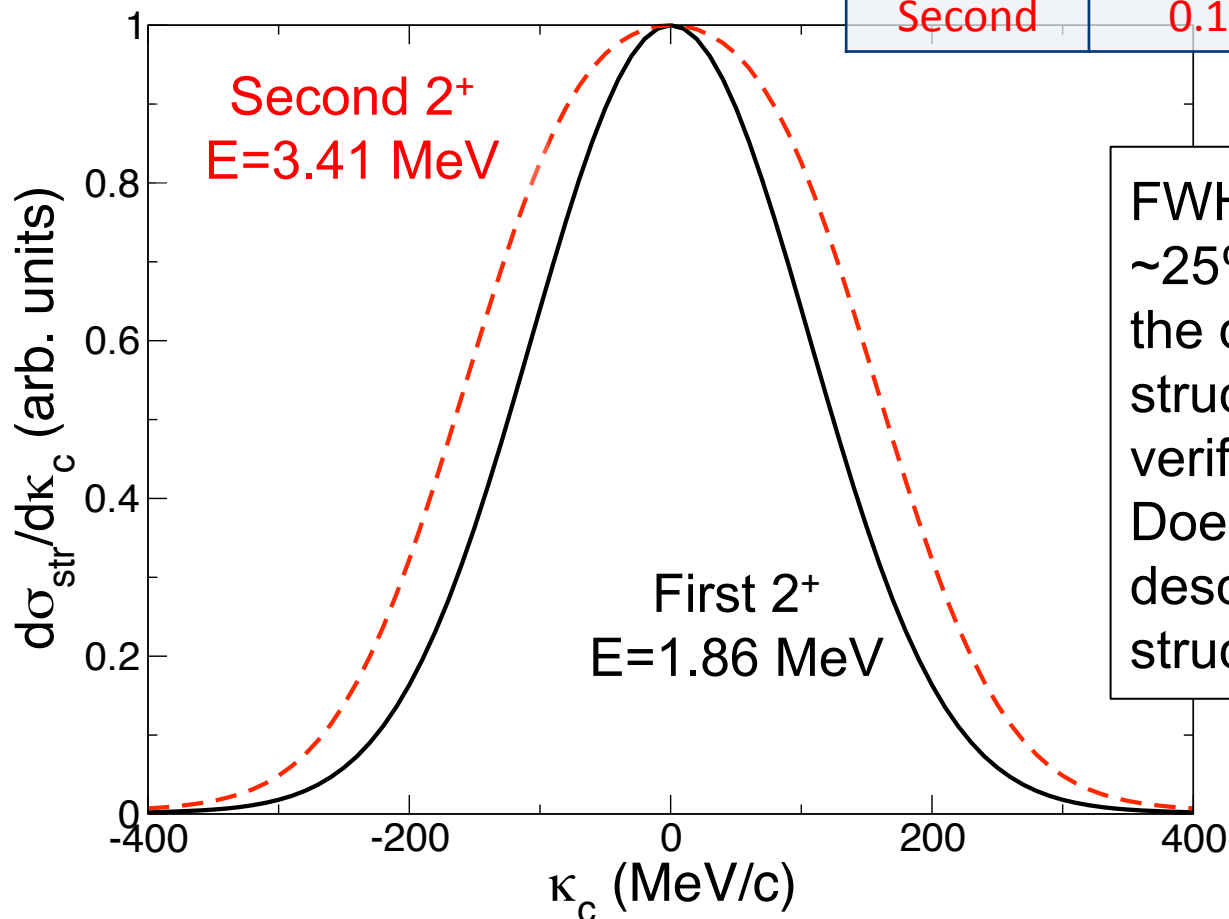
Shell model (sd-shell, USD) two-nucleon amplitudes

State	$[0d_{5/2}]^2$	$[0d_{5/2}, 0d_{3/2}]$	$[0d_{3/2}]^2$	$[1s_{1/2}, 0d_{3/2}]$	$[1s_{1/2}, 0d_{5/2}]$
2^+ (First)	-0.70074	0.43499	0.00594	-0.00188	-0.02781
2^+ (Second)	-0.38021	-0.12354	-0.12945	-0.15876	-0.58292
4^+ (First)	1.57469	0.41519	-	-	-

Structure Sensitivity

$^{26}\text{Si}(-2n)$

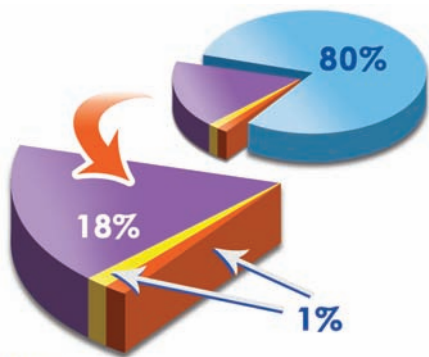
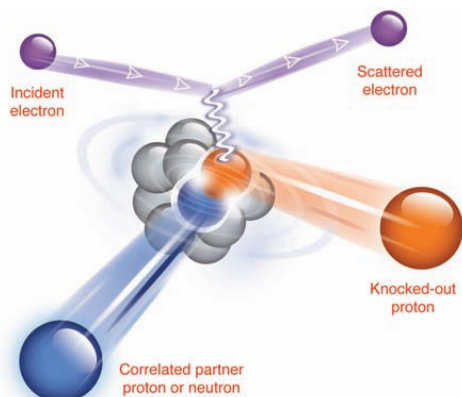
2^+ states	σ_{str} (mb)	σ_{-2n} (mb)	σ_{exp} (mb)
First	0.19	0.30	0.15(4)
Second	0.19	0.30	0.14(4)



FWHM of second state is ~25% larger, attributed to the different underlying structure – can this be verified by experiment? Does the SM accurately describe the underlying structure?

$N=Z$ nuclei: knockout of a proton and neutron

Motivation...



■ Single nucleons
■ n-p ■ n-n ■ p-p

Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C .



Bevatron fragmentation experiments from 1975 show very large np removal cross sections (^{12}C target)

Beam	pp	nn	np
^{12}C (2100 A MeV)	5.81(29)	4.11(22)	35.1(34)
^{12}C (1050 A MeV)	6.49(48)	4.44(25)	27.9(22)
^{12}C (250 A MeV)	5.88(970)	5.33(81)	47.5(24)
^{16}O (2100 A MeV)	4.71(31)	1.67(12)	41.8(33)

Simplest $[0p_{3/2}]^8$ structure suggests
 $\sigma_{\text{NN}}/\sigma_{\text{np}} \approx 6/16 \approx 2.7$ for $^{12}\text{C}(-2\text{N})$

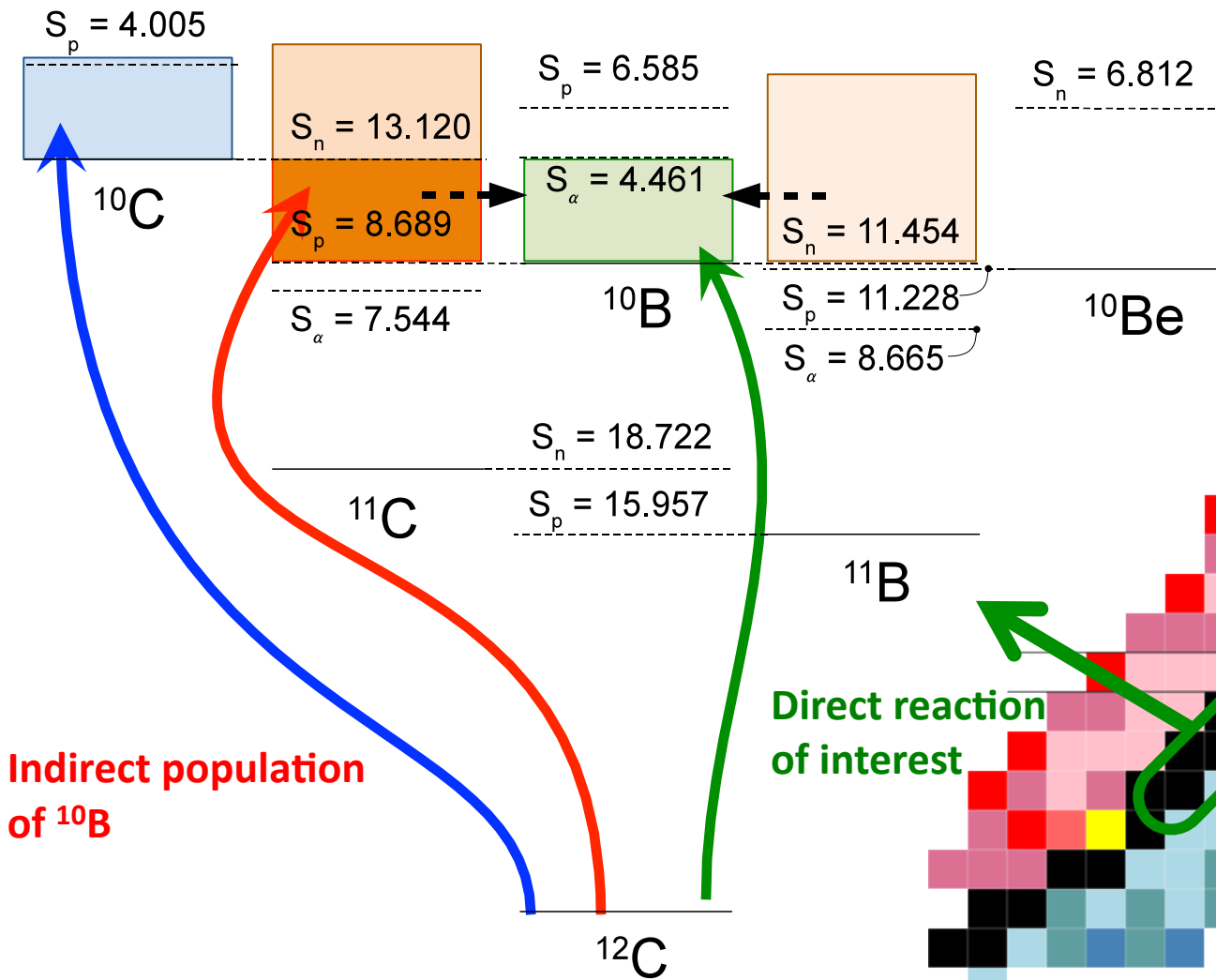
Lindstrom *et al.*, LBNL Report 3650 (1975)

Greiner *et al.*, PRL 35, 152 (1975)

Kidd *et al.*, 37, 2613 PRC (1988)

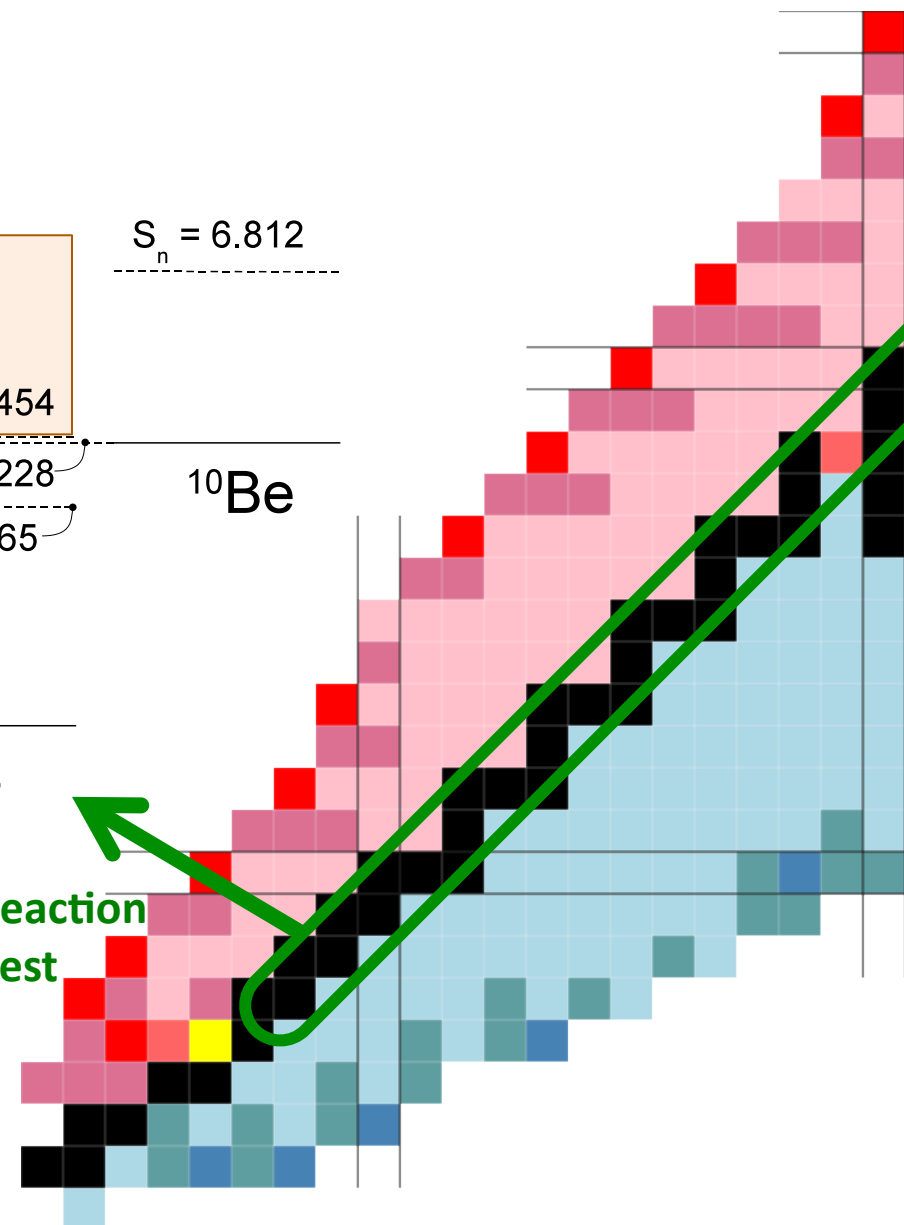
$^{12}\text{C}(-np)$: direct vs. indirect

Particle Separation Thresholds



Indirect population of ^{10}B

Direct reaction of interest



^{12}C beam LBL Bevatron results (1975)

Fragmentation of ^{16}O and ^{12}C projectiles studies at LBNL; beam energies 2.1 GeV/nucleon; cross sections and momentum distribution widths published (though averaged over targets)

Oxbash p-shell shell model structure input using WBP (and PJT interactions)

^{12}C projectile, WBP interaction

Residue	^{10}C		^{10}Be		^{10}B	
	exp.	theory	exp.	theory	exp.	theory
$\sigma_{-2\text{N}}$ (mb)	4.11±0.22	5.04	5.81±0.29	6.52	35.1±3.4	19.02
Width (MeV/c)*	121±6	120	129±4	127	134±3	132

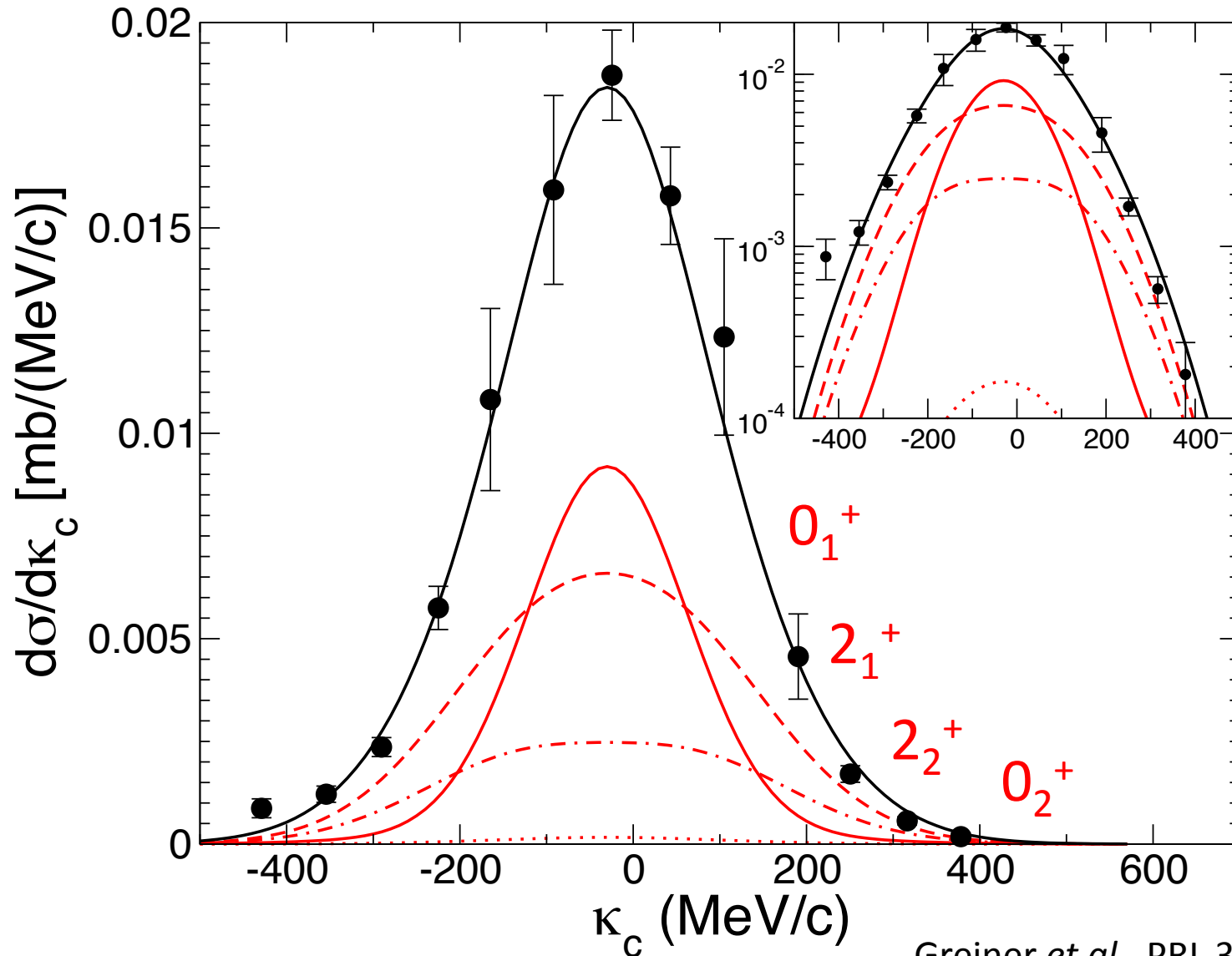
*Target averaged

Lindstrom *et al.*, LBNL Report 3650 (1975)

Greiner *et al.*, PRL 35, 152 (1975)

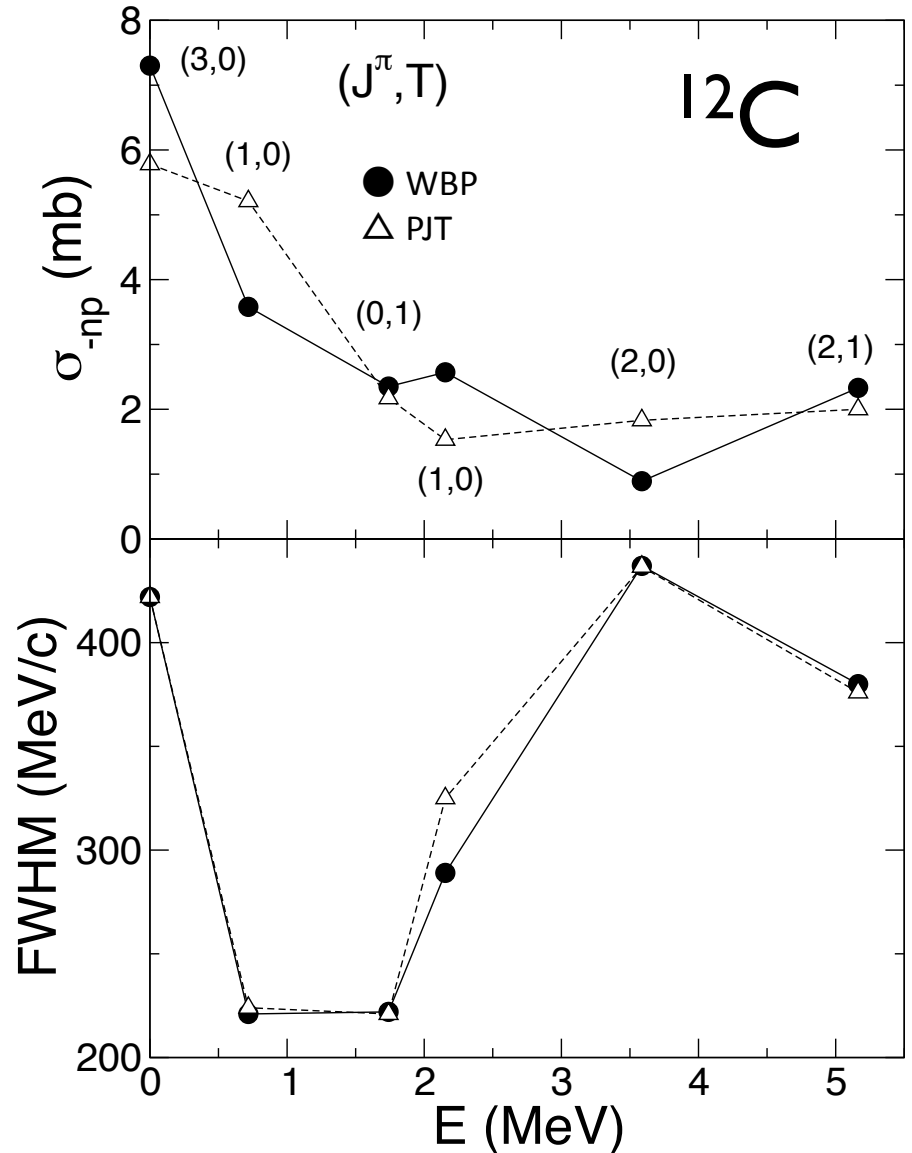
$^{12}\text{C}(-2p) \rightarrow ^{10}\text{Be}$ momentum distribution

^{10}Be final state inclusive



np-knockout: ^{12}C

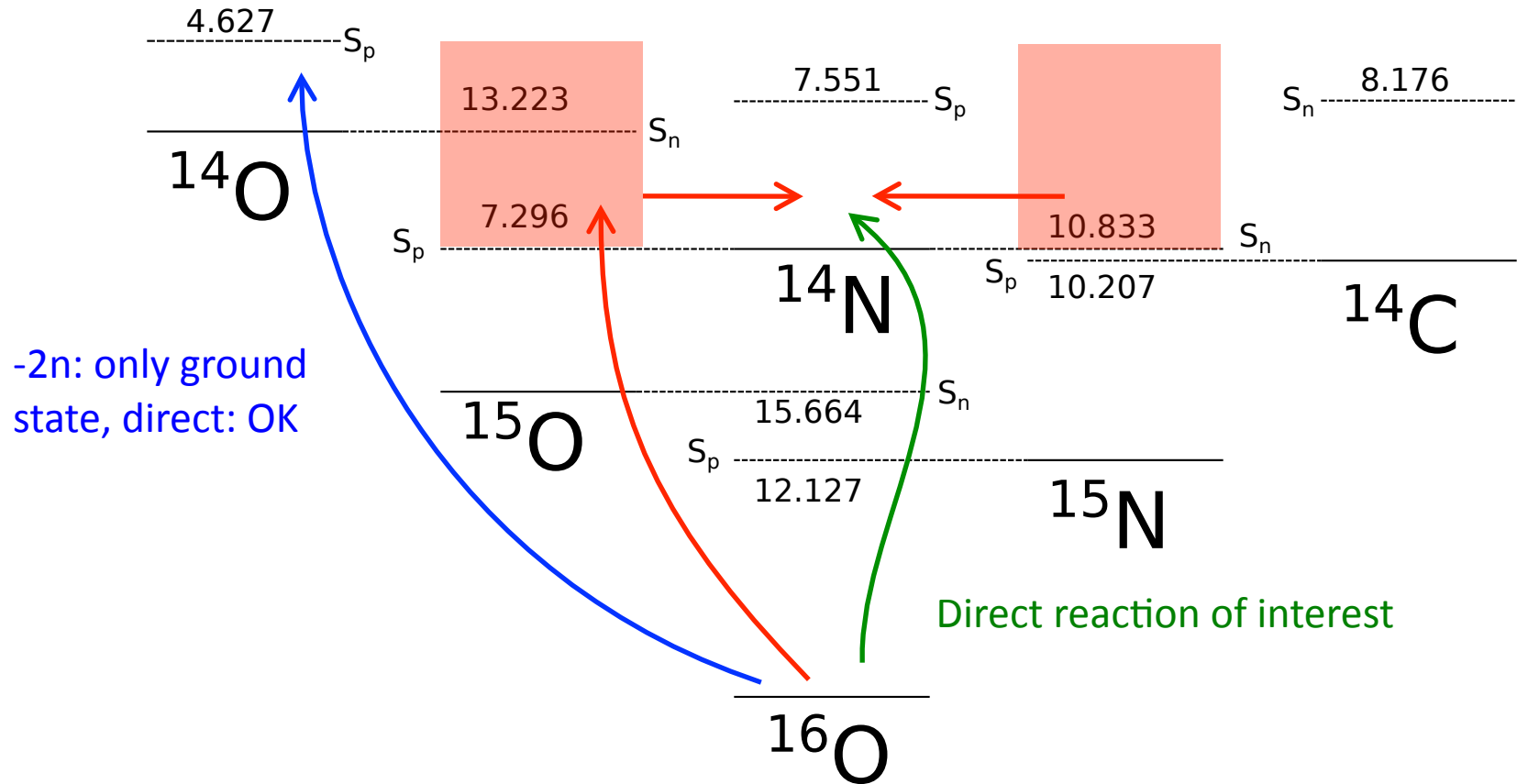
- Cross section underestimated: T=1 removal apparently well described (^{10}C , ^{10}Be), so deficiency with T=1 states?
- Some interaction sensitivity for T=0 states, weaker for T=1 states
- Distinct pattern of momentum distributions widths
 - Sensitivity to underlying structure
 - Indications of indirection removal
- Calculations using NCSM amplitudes are underway – can these large basis ($N_{\text{max}}=6$) account for the cross section deficit?



$^{16}\text{O}(-np)$: direct vs. indirect

Nucleon separation thresholds

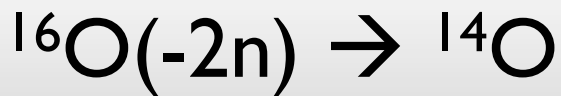
Possible indirect contributions to removal,
(though little predicted by SM)



Oxbash shell model input: spspdpf, wbt

Truncate to p-shell ($0 \hbar\omega$) (also $0+2 \hbar\omega$, $0+2+4 \hbar\omega$)

Harmonic oscillator wave functions used, $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$



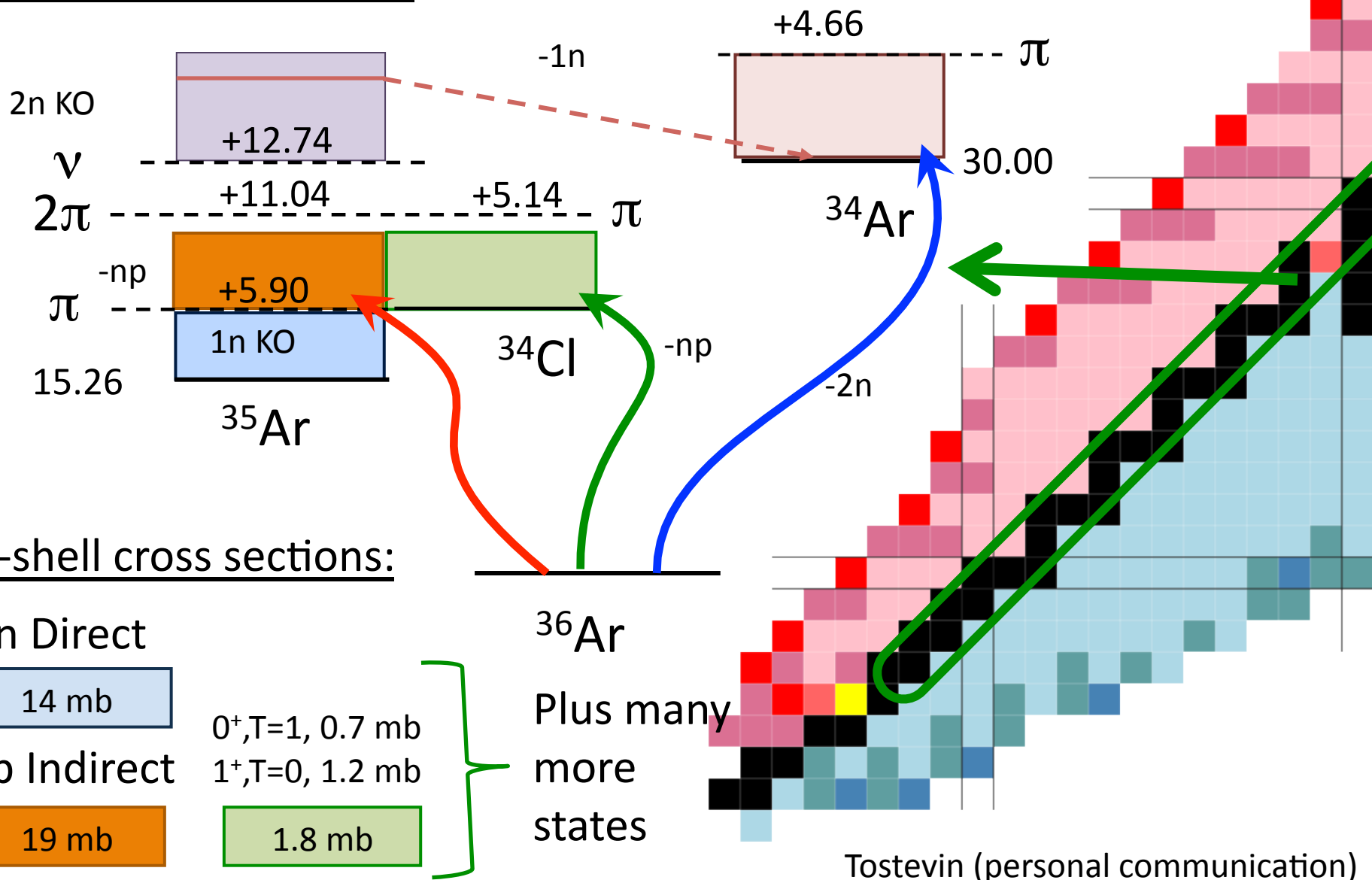
Preliminary Results

^{14}O	Experiment	$0\hbar\omega$ (WBT)	$2\hbar\omega$ (WBT)	$4\hbar\omega$ (WBT)
σ (mb)	1.67 ± 0.12	1.36	1.36	1.39
Width (MeV/c)	99 ± 6	99.6	96.5	94.4
$\Sigma(\text{TNA})^2$	-	1.00	0.90	0.83

- Good agreement for (very narrow) momentum distribution
- $0\hbar\omega$ theory underestimates experiment in contrast to exotic sd-shell cases and $^{12}\text{C}(-2N)$
 - Sizes of core and radial wave functions?
 - Centre of mass corrections to TNA?
- How important are cross-shell excitations?
 - Overlap smaller, cross-shell components enhance spatial correlations, maintaining cross section

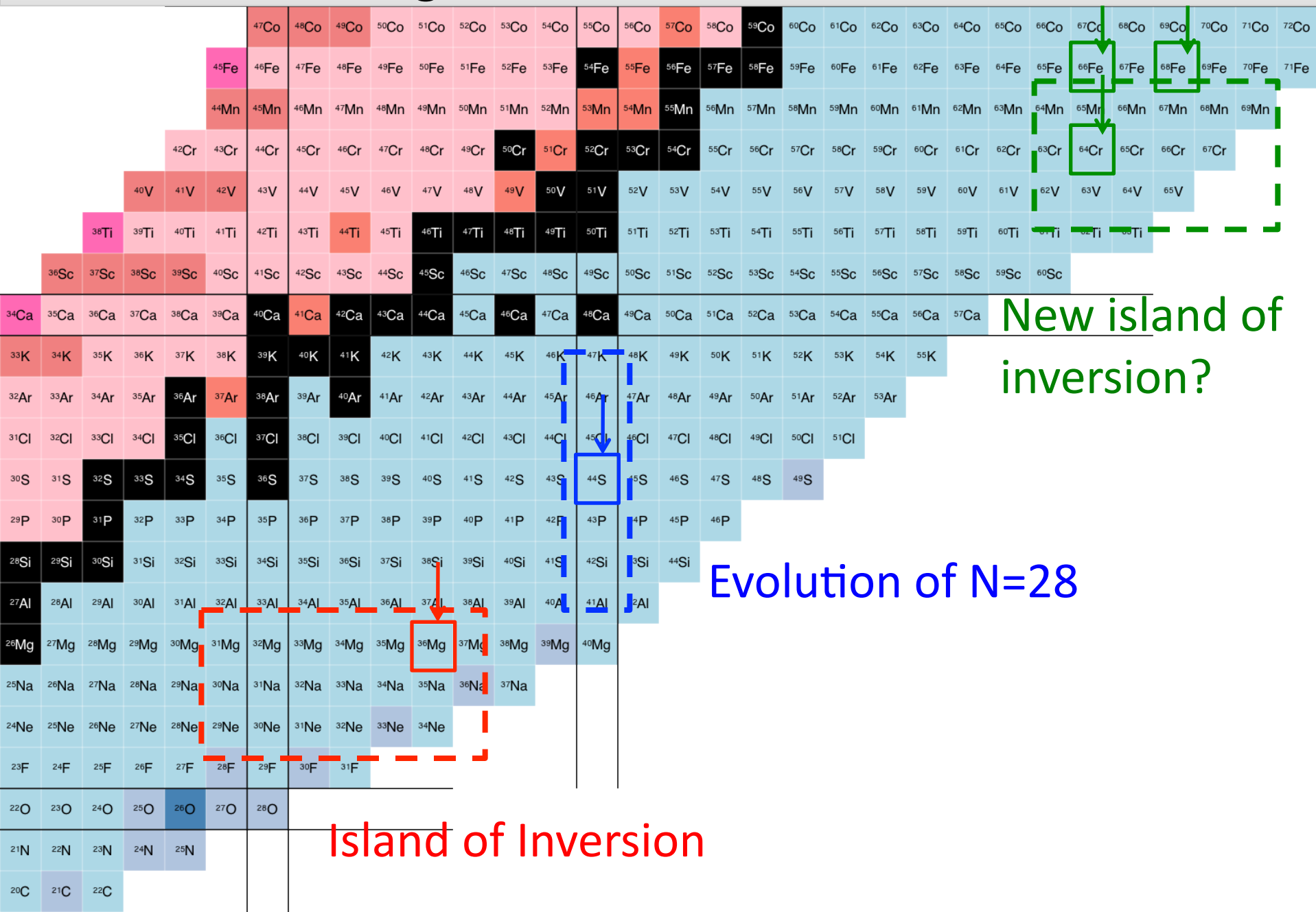
Along the $N=Z$ line?

$^{36}\text{Ar}(-2N)$ thresholds



Rapid structural change: applications and
further work

Structure changes in exotic nuclei

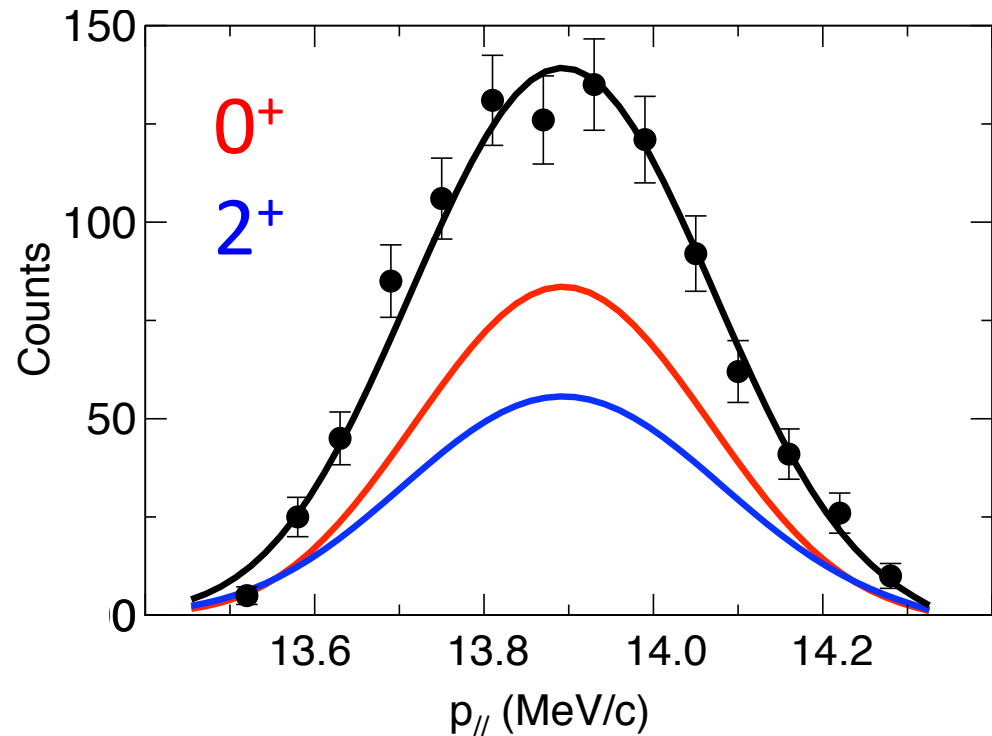
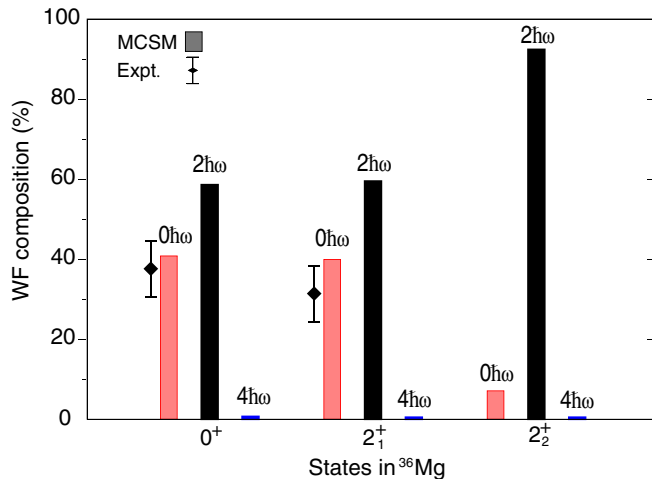


Island of Inversion: $^{38}\text{Si}(-2p) \rightarrow ^{36}\text{Mg}$

$^{38}\text{Si}(-2p) \rightarrow ^{36}\text{Mg}$

Ratio of experimental to pure 0 $\hbar\omega$ theoretical cross sections gives indications of fraction of 2 $\hbar\omega$ components, agreeing with MCSM calculations

Exp. (mb)	0 $\hbar\omega$ Theory (mb)
0.10 \pm 0.01	0.28



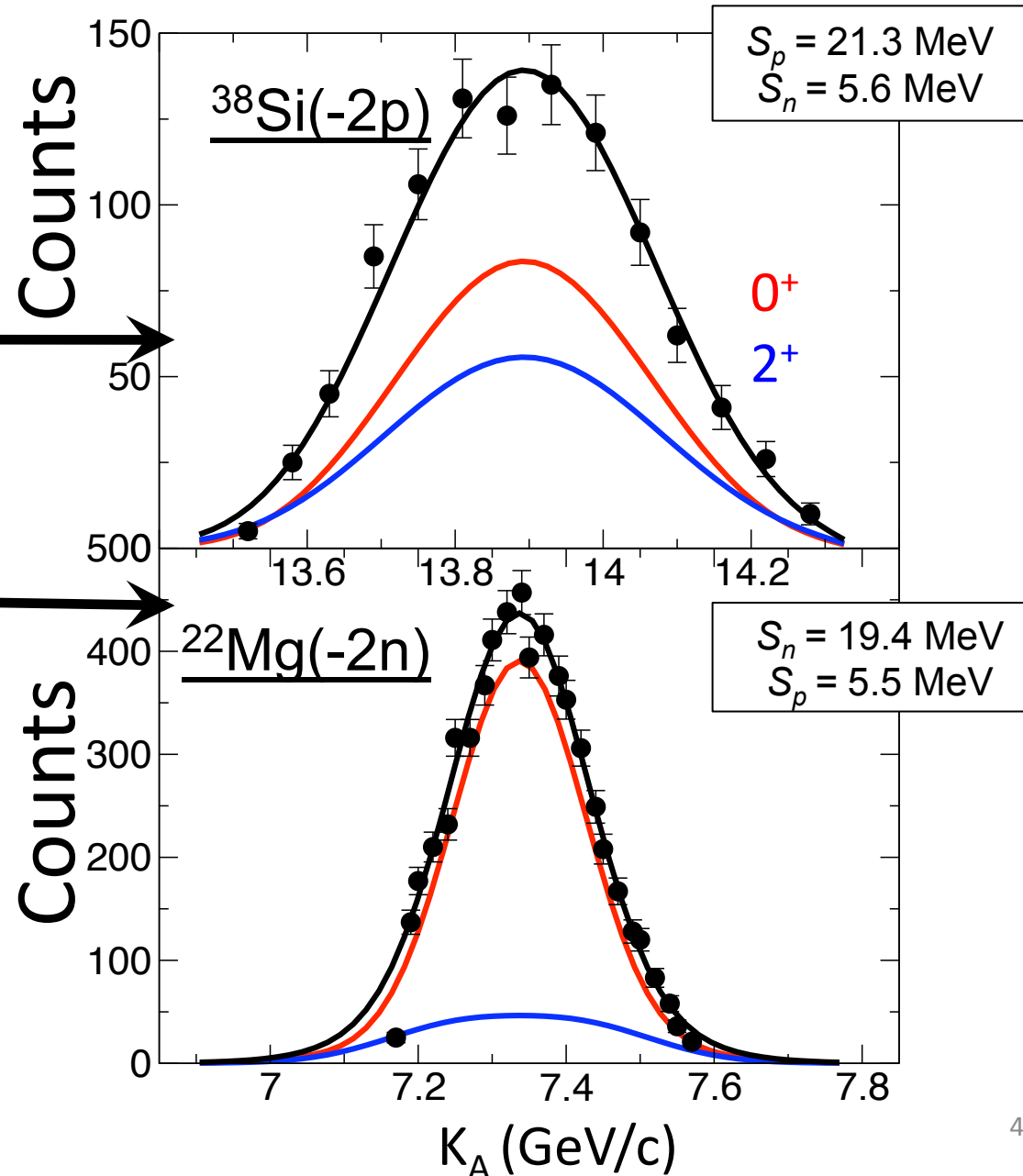
Gade et al., PRC 77, 044306 (2008)

Spherical model does not track structural changes

$^{22}\text{Mg}(-2n)$ vs. $^{38}\text{Si}(-2p)$

Target: ^9Be 376 mg/cm²
Energy: 83 A MeV
Beam: $\Delta K = 1.66\%$
Target: $\Delta K = 0.29$ GeV/c

Target: ^9Be 188 mg/cm²
Energy: 75.1 MeV
Beam: $\Delta K = 0.5\%$



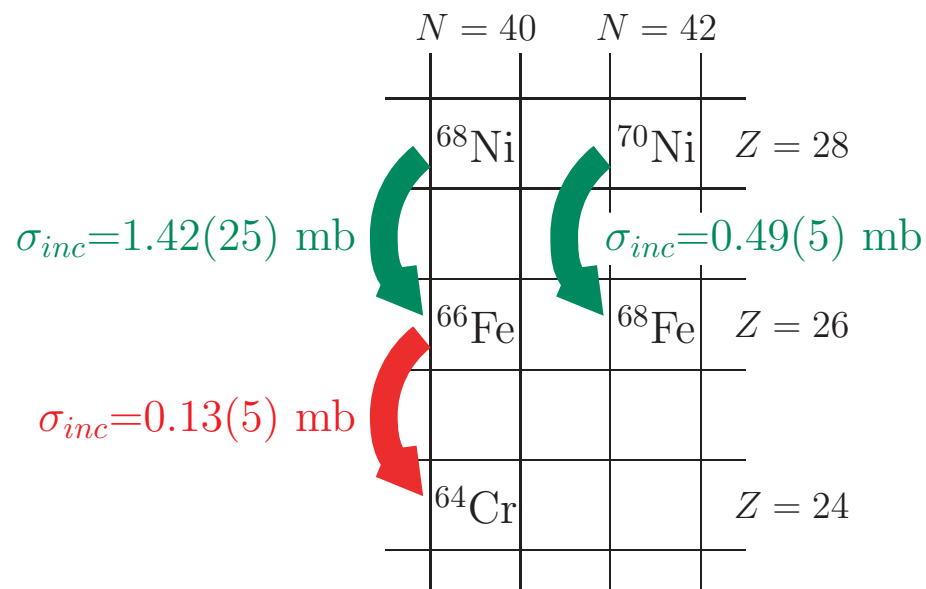
Gade *et al.*, PRC 76, 024317 (2007)

Gade *et al.*, PRL 99, 072502 (2007)

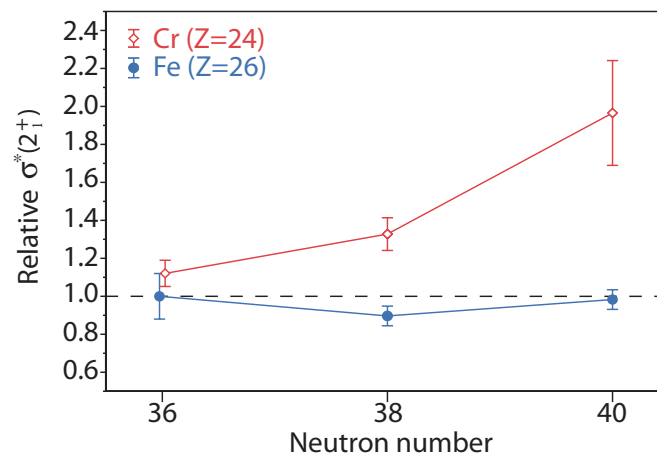
New island of inversion? $N=40$

Two-proton removal

- Cross section for $^{66}\text{Fe}(-2p)$ strongly suppressed relative to spherical theory and $^{68}\text{Ni}(-2p)$
- Symptomatic of reduced structure overlap
 - Rapid structure changes
 - Onset of deformation
- Inelastic scattering measurements indicate increasing deformation



Inelastic scattering



$N=28$ shell closure: $^{46}\text{Ar}(-2p) \rightarrow ^{44}\text{S}$

$^{46}\text{Ar}(-2p)$

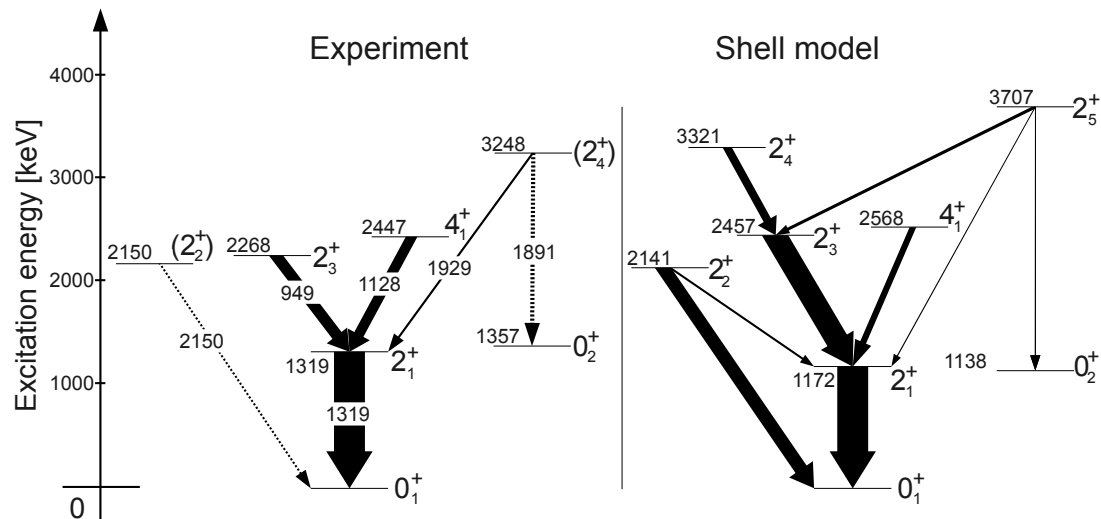
Structure down the
 $N=28$ shell closure

$0_1^+ \nu[2p2h]$

$2_1^+ \nu[2p2h]$

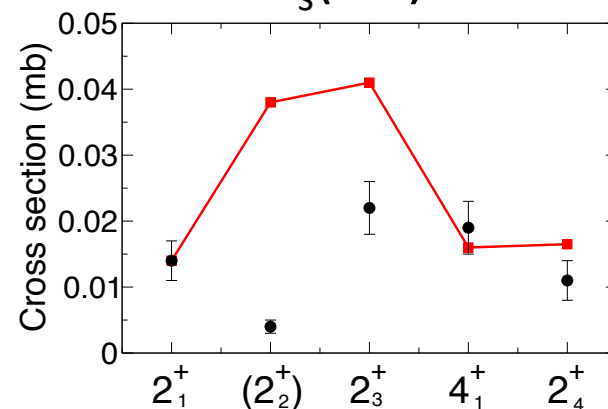
$0_2^+ \nu[0p0h]$

$4_1^+ \nu[1p1h]$



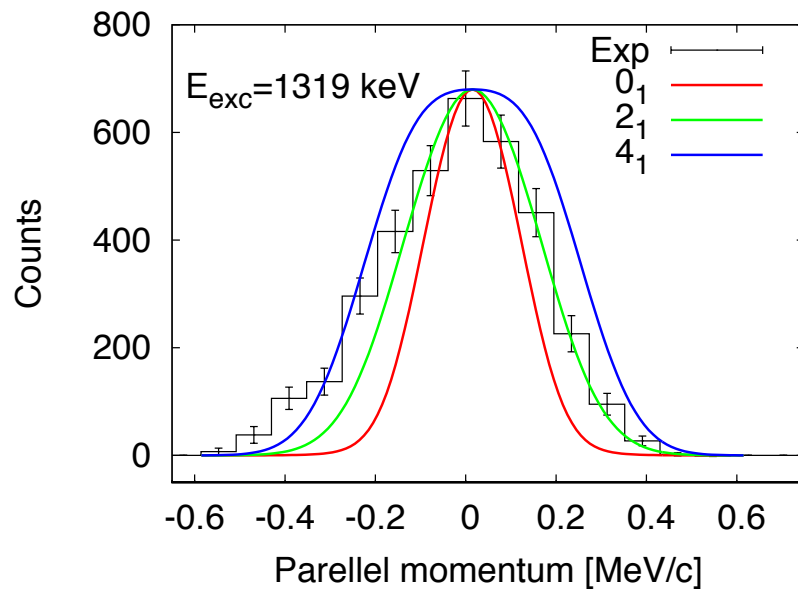
E_{level} (keV)	J^π	E_γ (keV)	J^π_{final}	σ (mb)	σ_{theory} (mb)
0	0^+				0.334
1319(7)	2_1^+	1319(7)	0_1^+	0.014(3)	0.028
1357(15)	0_2^+				0.163
2150(11)*	(2_2^+)	2150(11)	0_1^{+*}	0.004(1)	0.076
2268(8)	2_3^+	949(5)	2_1^+	0.022(4)	0.082
2447(9)	4_1^+	1128(6)	2_1^+	0.019(4)	0.032
3248(12)	(2_4^+)	1891(10)	0_2^{+*}	0.011(3)	0.033
		1929(7)	2_1^+		

$R_S(2N) = 0.5$



$N=28$ shell closure: $^{46}\text{Ar}(-2p) \rightarrow ^{44}\text{S}$

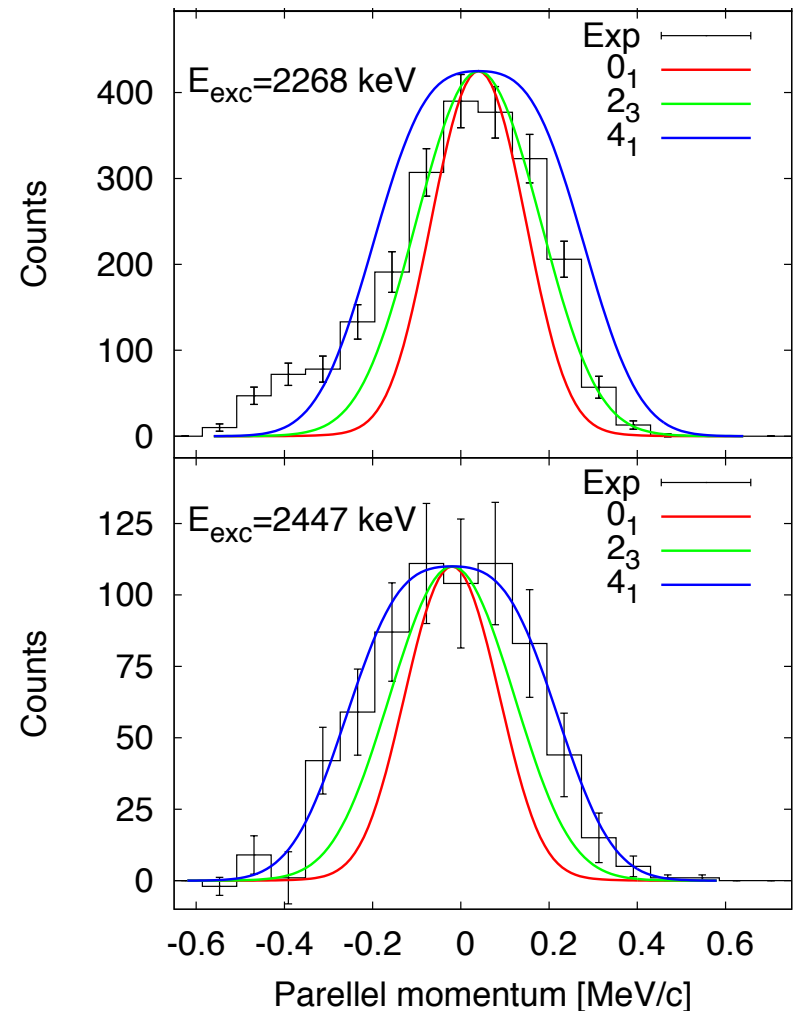
Residue momentum distributions



YKIS talk by Dario Vretenar:

$$\beta(^{34}\text{Ar}) = -0.19$$

$$\beta(^{44}\text{S}) = 0.34$$

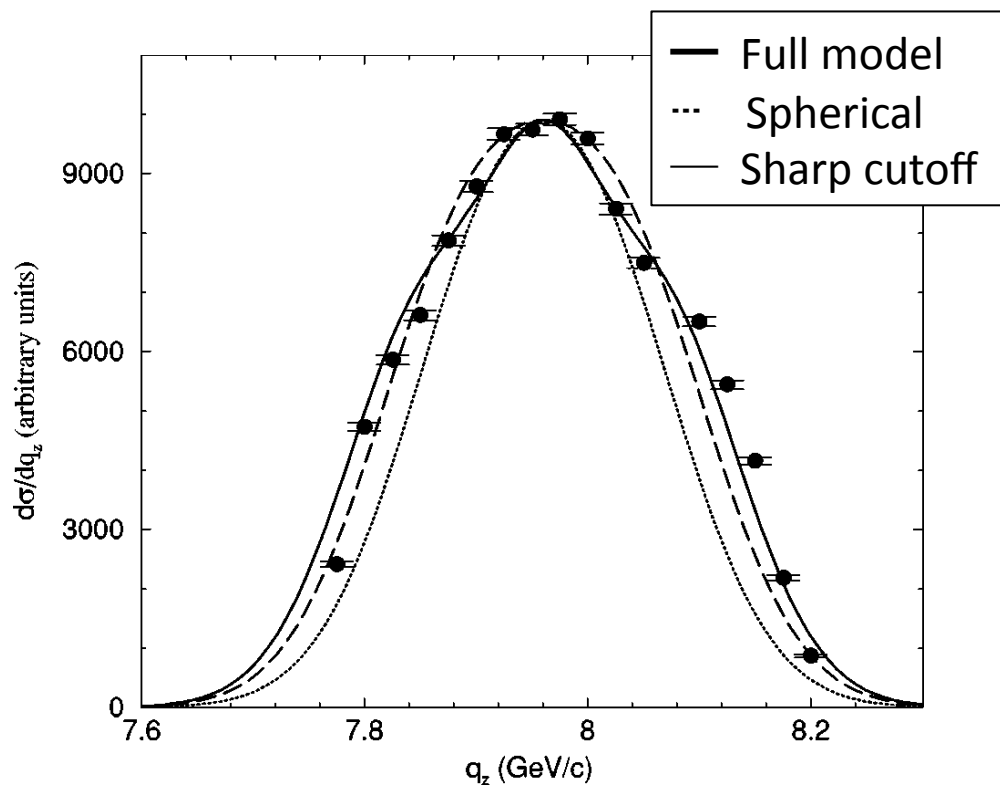


BUT ^{44}S and other $N=28$ isotones are strongly deformed

Deformation

$^{25}\text{Al}(-1p)$ [202]5/2, $\delta=0.34$

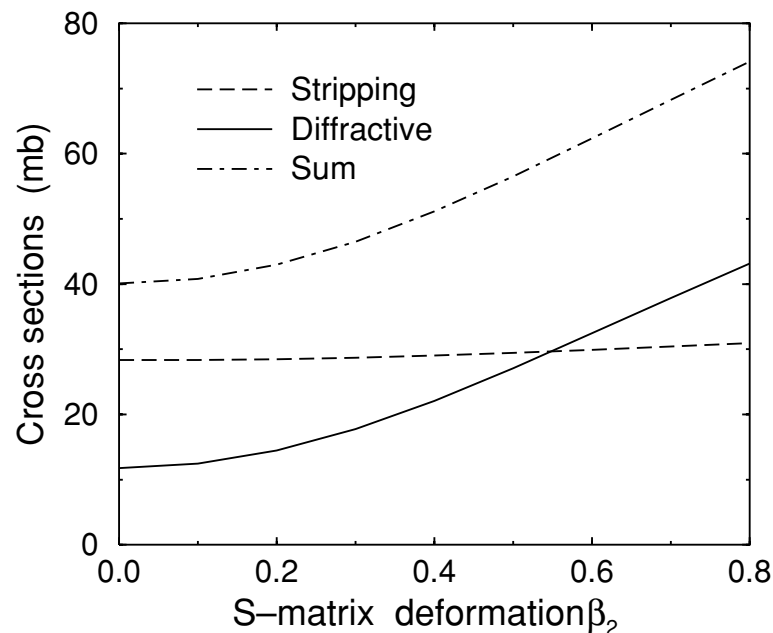
Dynamic core excitation
required for momentum
distribution shoulders



Sakharuk and Zelavinsky, PRC 61, 014609 (1999)

$^{17}\text{C}(-1n)$

Stripping and diffractive
cross sections as a
function of deformation



Batham *et al.*, PRC 71 064608 (2005)

Conclusions

Conclusions

- Two-nucleon removal offers an efficient route to detailed spectroscopic information on low-lying states in highly exotic nuclei
- Momentum distributions offer detailed tests of structure models
 - NSCL proposal on $^{26}\text{Si}(-2n)$
- Odd-odd systems more complicated (mixed I), but may yet exhibit structure sensitivity
- KO from $N=Z$ nuclei are intriguing: new final-state exclusive measurements are required to provide robust tests in stable nuclei
 - RIKEN proposal on $^{12}\text{C}(-np)$
- Rapid structural changes:
 - New (deformed) structure input (PSM, BCS+Nilsson)
 - Deformed reaction dynamics descriptions
 - Dynamic core excitation

Acknowledgements

J. A. Tostevin, P. H. Regan, Zs. Podolyak,
S. J. Steer



D. Bazin, B. A. Brown, A. Gade



J. Lee



P. Navratil



UK STFC Grants EP/D003628 and ST/F012012

UK EPSRC Grant EP/P503892/1