Recent development of CDCC

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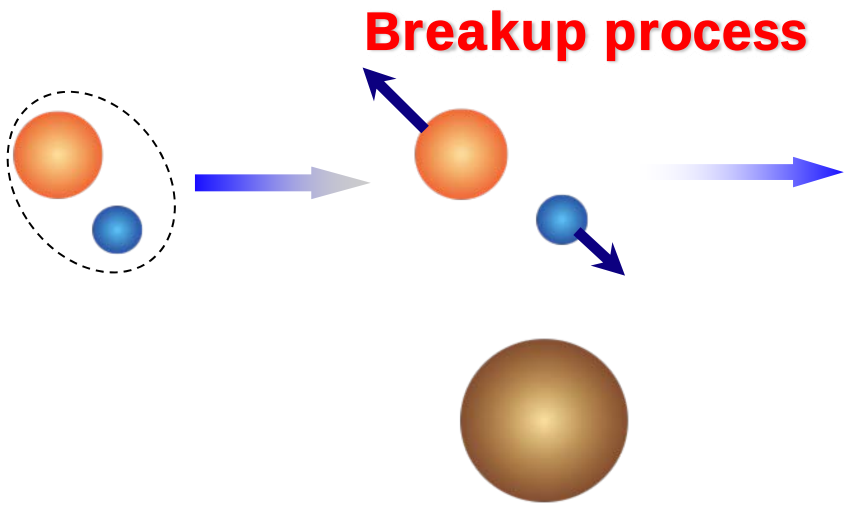
1. Foundation of CDCC ← Moro-san
2. Microscopic version of CDCC
3. Eikonal reaction theory (ERT) } Ogata-san, Minomo-kun
4. Four-body CDCC ← Matsumoto-san

Slides including the recent data are deleted.

CDCC

(The method of Continuum-Discretized Coupled Channels)



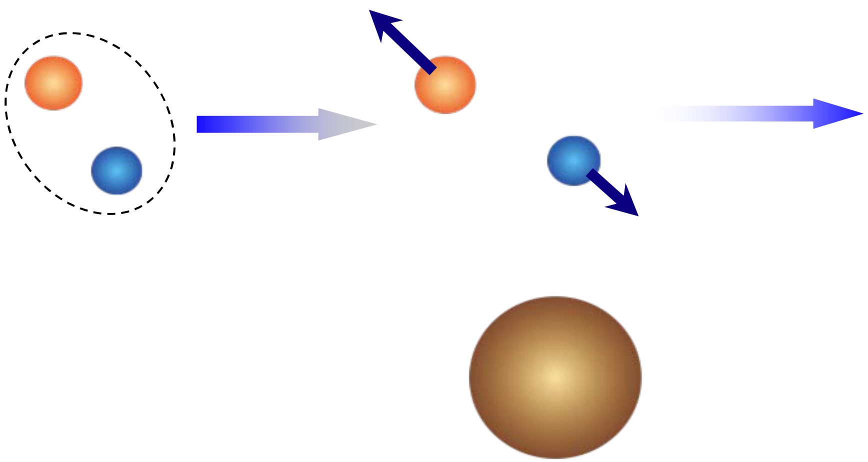
Review Papers

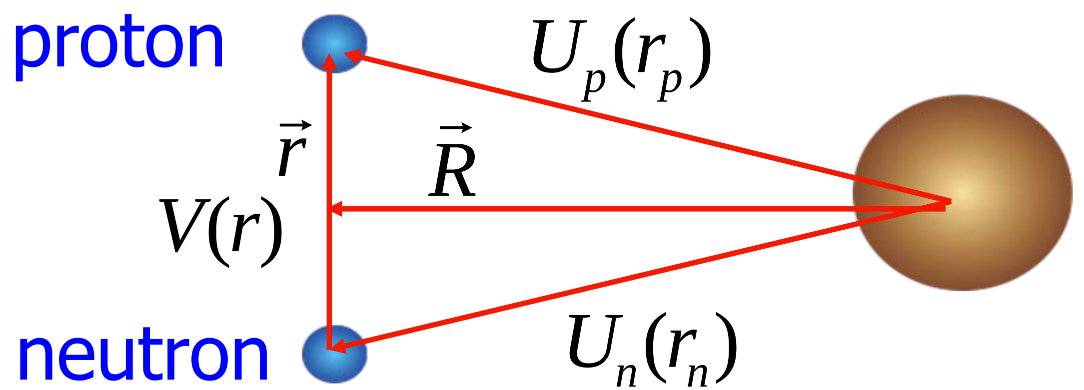
Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.89,1(1986)

Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. 154(1987),126.

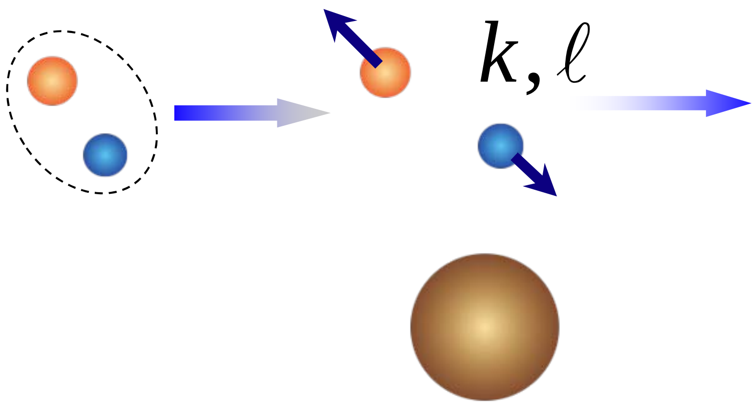
Deuteron scattering as a simple case

Deuteron scattering





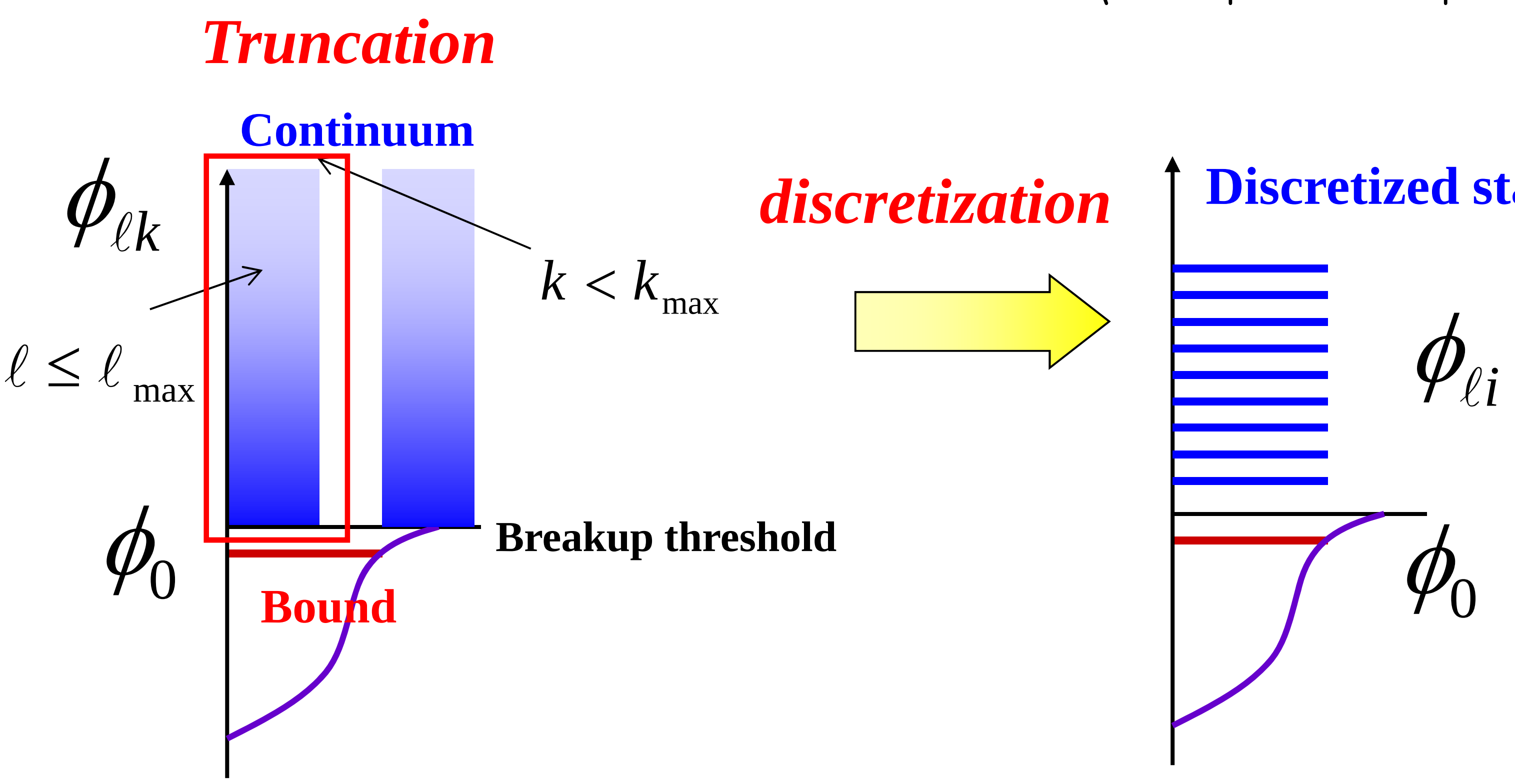
$$\left(E - K - V(r) - U_{\mathrm{p}}(r_p) - U_{\mathrm{n}}(r_n)\right)\psi = 0$$



$$\psi = \phi_0 \chi_0 + \sum_{\ell}^{\infty} \int_0^{\infty} dk \phi_{\ell k} \chi_{\ell k}$$

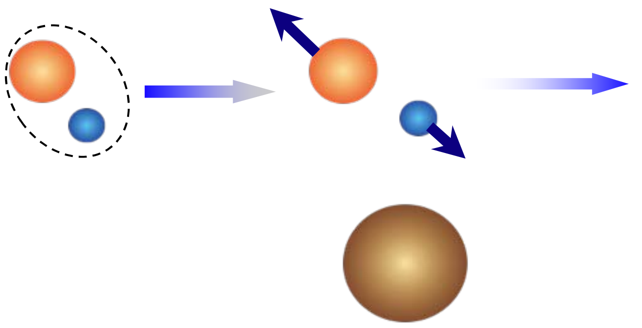
$$\psi = \phi_0 \chi_0 + \sum_{\ell}^{\ell_{\max}} \int_0^{k_{\max}} dk \phi_{\ell k} \chi_{\ell k} \cong \phi_0 \chi_0 + \sum_{\ell}^{\ell_{\max}} \sum_{i}^{i_{\max}} \phi_{\ell i} \chi_{\ell i}$$

CC equation: $\langle \phi_{\ell' i'} | H - E | \psi \rangle = 0$



CDCC

(The method of Continuum-Discretized Coupled Channels)



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Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.89,1(1986)

Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. 154(1987),126.

Theoretical foundation

Austern, Yahiro and Kawai, PRL 63, 2649(1989)

Austern, Kawai and Yahiro, PRC 53, 394(1996)

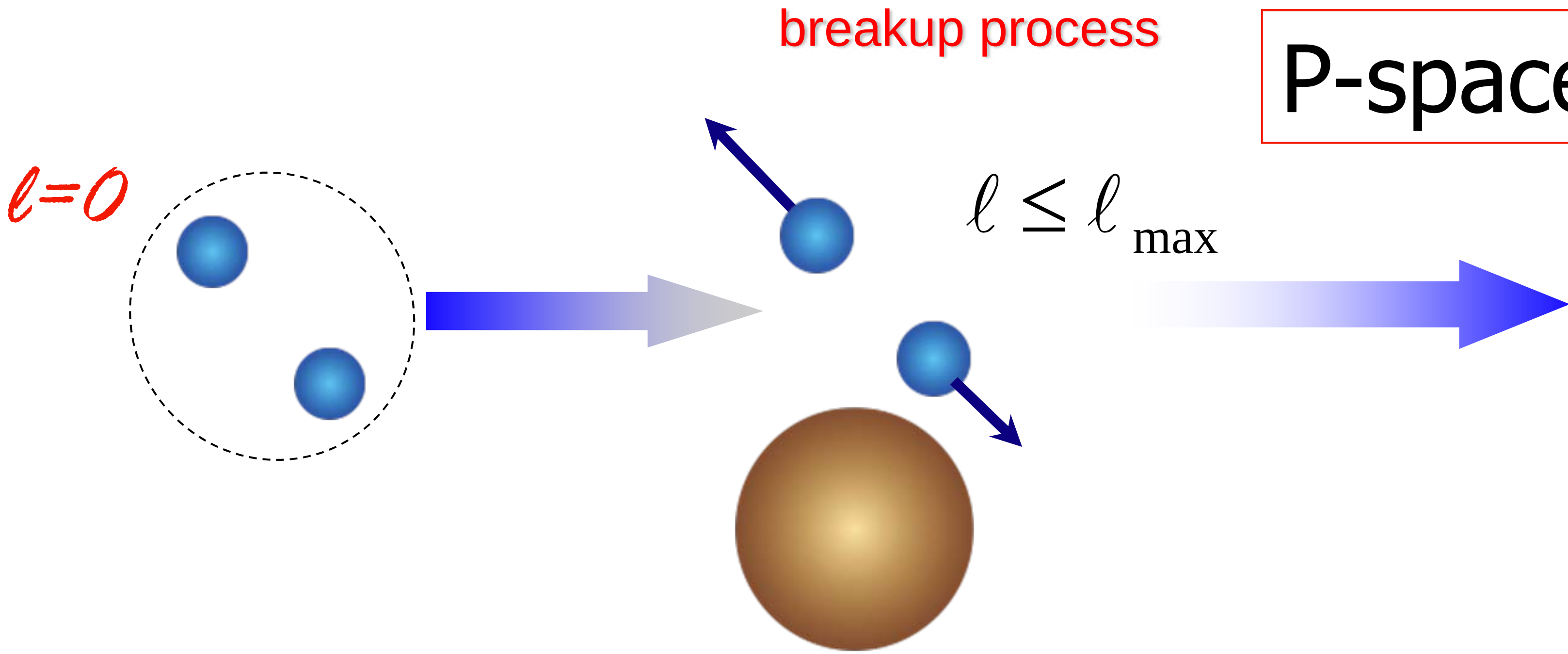
Numerical comparison between CDCC and Faddeev solutions

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.

1. Foundation of CDCC

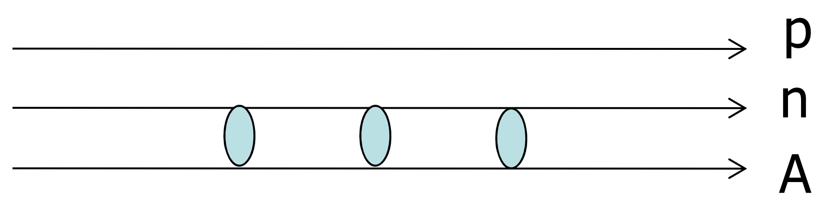
Austern,Yahiro,Kawai，PRL63，2649（1989)

P-space



CDCC-equation $(E - K - V - PU_{\mathrm{p}}P - PU_{\mathrm{n}}P)\psi = 0$

Disconnected diagram

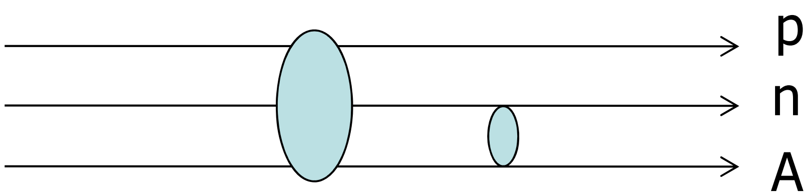


$$PU_{\mathrm{p}}P = P\ \exp[-r_{\mathrm{p}}^2]\ P = \int d\Omega_r\ \exp[-(\boldsymbol{R} - \boldsymbol{r}/2)^2] = \exp[-R^2 - r^2/4]\ j_0(Rr)$$

$$(E - K - V(r) - U_p - U_n)\psi = 0$$

Faddeev decomposition

$$\psi = \psi_d + \psi_p + \psi_n$$



Faddeev equations

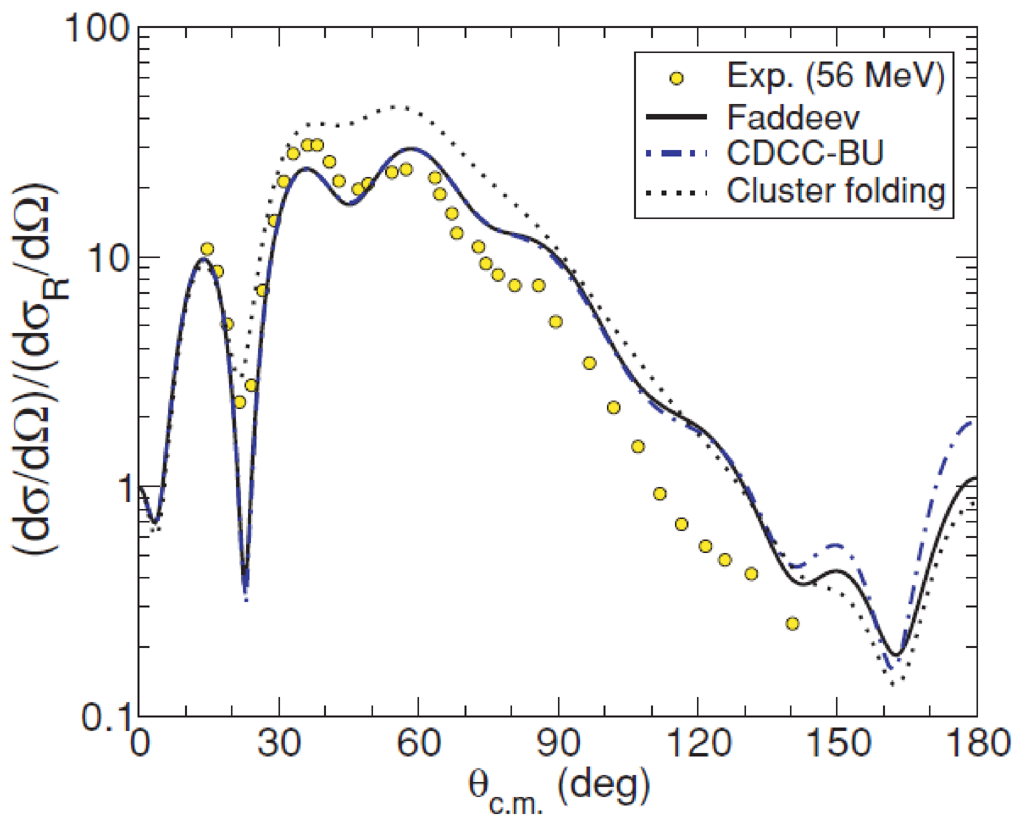
$$[E - K - V \qquad]\psi_d = 0$$

$$[E - K - U_p]\psi_p = (U_p \qquad)\psi_d + U_p\psi_n$$

$$[E - K - U_n]\psi_n = (U_n \qquad)\psi_d + U_n\psi_p$$

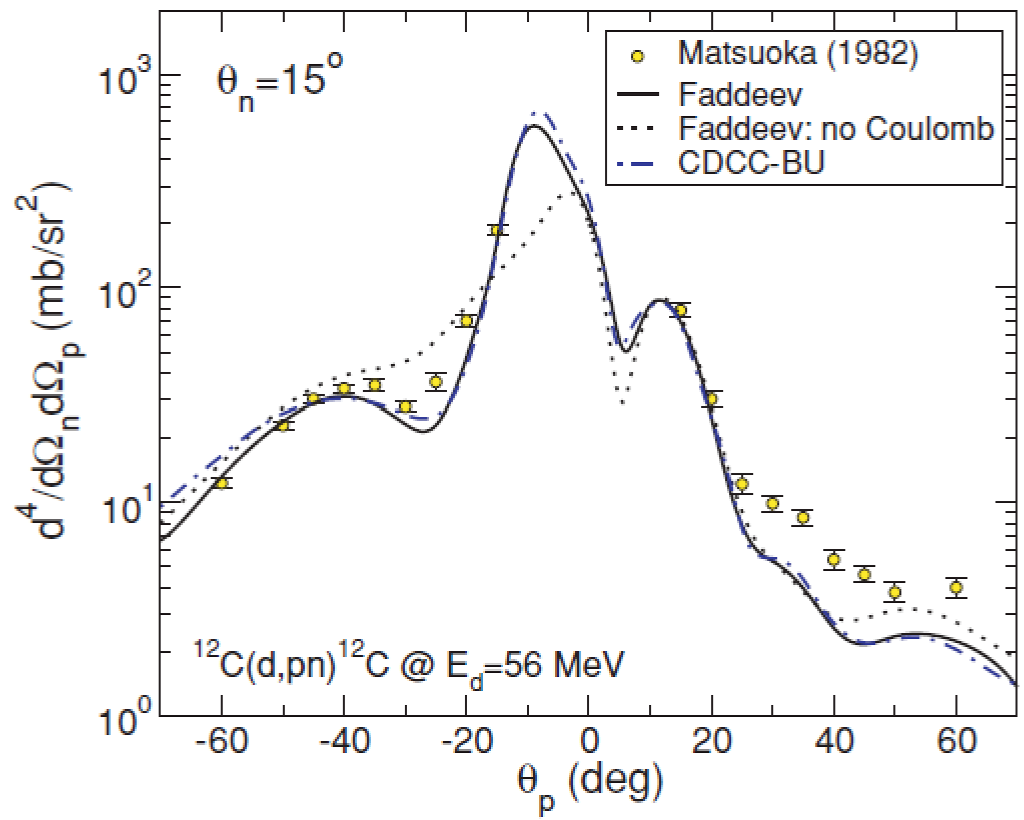
Comparison between CDCC and Faddeev solutions

A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76 (2007), 064602.



d+^{12}C at 56 MeV
Elastic scattering

$^{12}C(d,pn)$ at 56 MeV

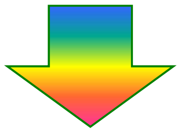


What is CDCC?

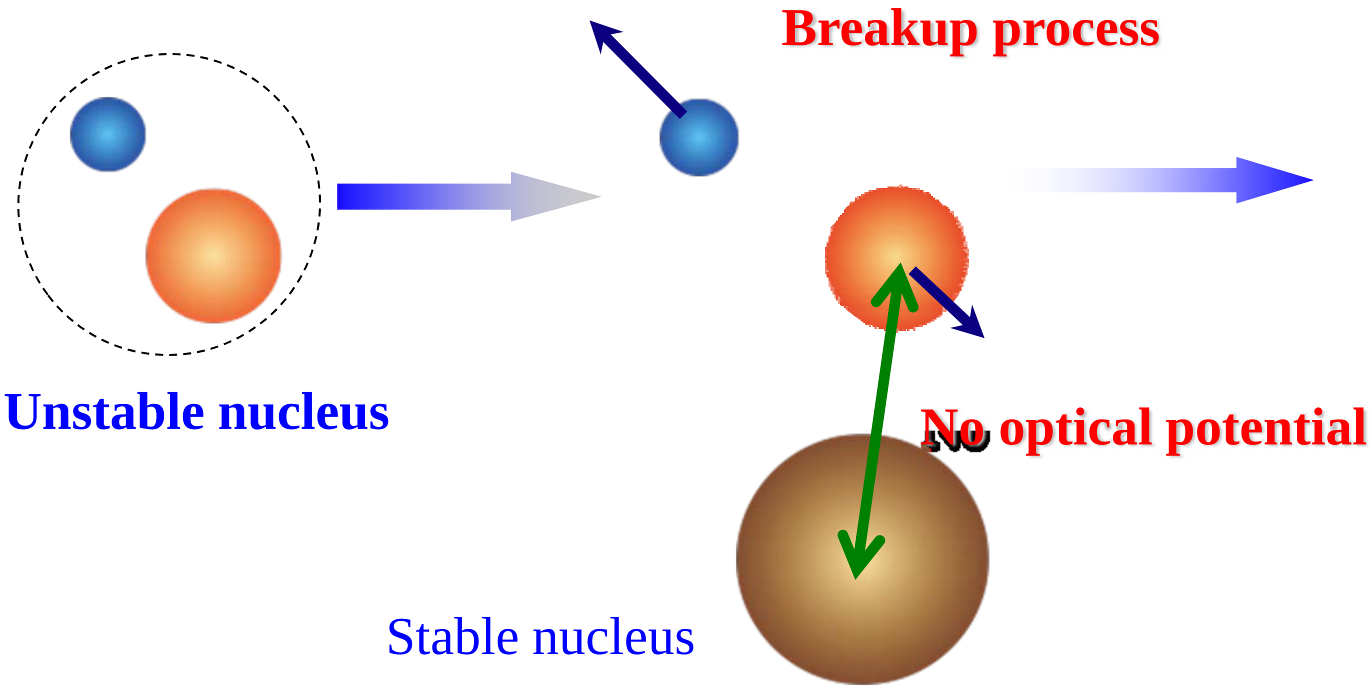
- The primary approximation of CDCC is the model-space approximation.
- The discretization of k-continuum is the secondary approximation.

2. Microscopic version of CDCC

Scattering of unstable nuclei



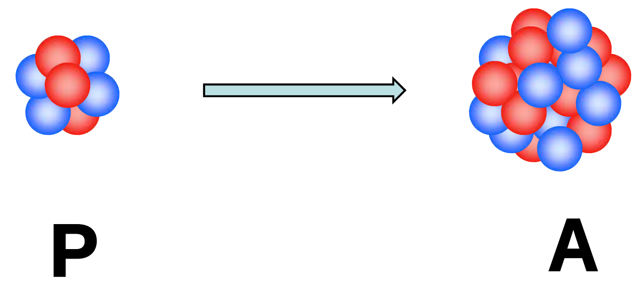
Microscopic reaction theory

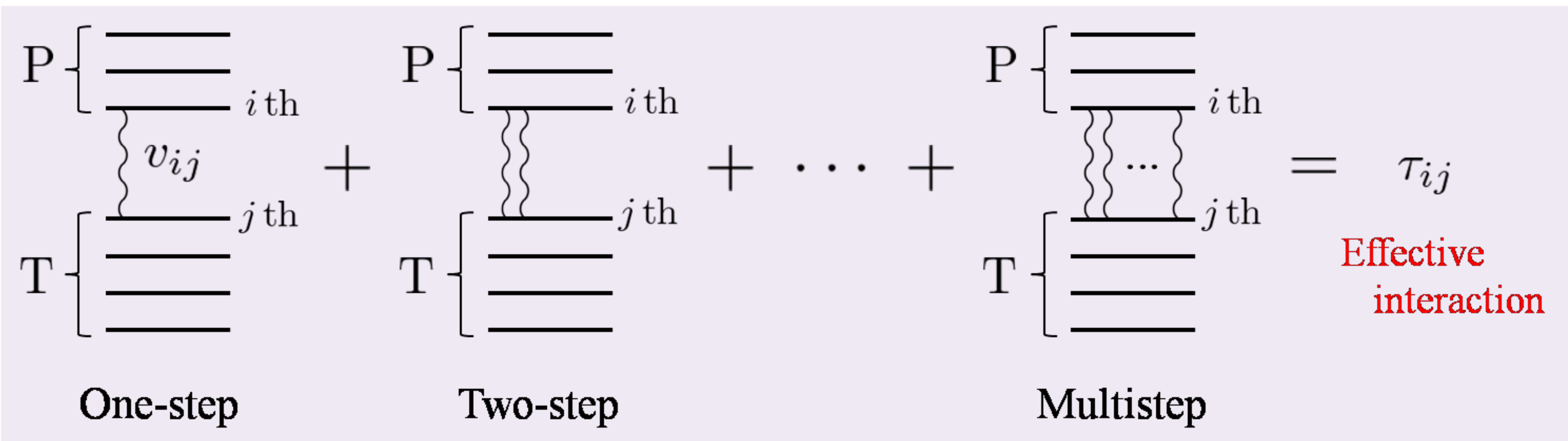


Many-body Schrödinger equation with realistic NN interaction

Realistic NN interaction

$$\left(K + h_P + h_A + \boxed{\sum_{i\in P,\, j\in A} v_{ij}} - E\right)\Psi = 0$$

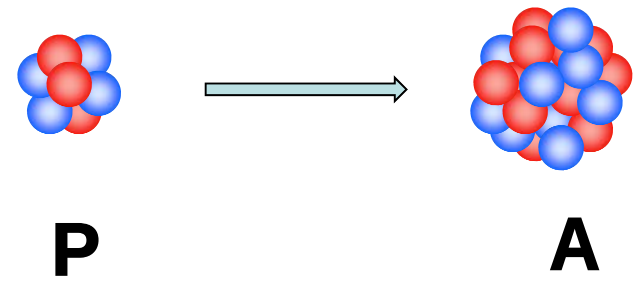




Schrödinger equation with resummation

M. Yahiro, K. Minomo, K. Ogata, M. Kawai, Prog.Theor.Phys.120:767-783,2008.

$$\left(K + h_P + h_A + \frac{Y-1}{Y}\boxed{\sum_{i\in P,\, j\in A}\tau_{ij}} - E\right)\Psi = 0$$



$$Y = A_P A_T \Rightarrow 12\times 12 = 144$$ for $^{12}C+^{12}C$ scattering

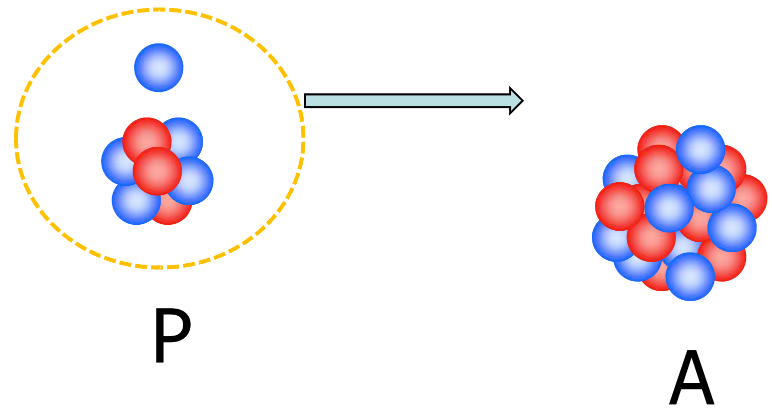
$$\left(K + h_P + h_A + \boxed{\sum_{i\in P,\, j\in A}\tau_{ij}} - E\right)\Psi = 0$$

Effective NN interaction = G-matrix interaction

2.1. Double-folding model

Folding potential

$$U_{opt} = \langle \Phi_P \Phi_A | \sum_{i \in P, j \in A} g_{ij} | \Phi_P \Phi_A \rangle$$



G-matrix: K. Amos et al.,
(Melbourne group)
Adv. Nucl. Phys. Vol.25 (2000) 275

Microscopic calculation such as
HF and AMD
with Gogny D1S interaction.

Bonn-B NN interaction
+ phenomenological imaginary potential.

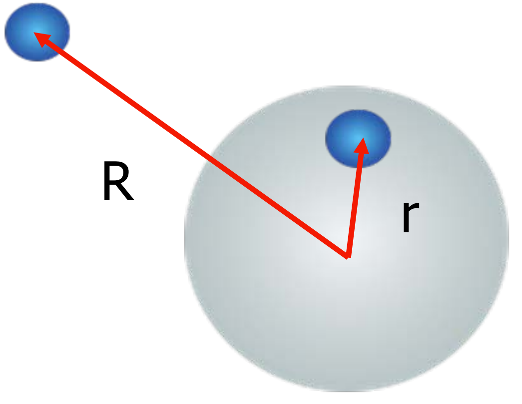
Proton scattering as a simple case

$$g_{0j} = g(r_{0j})(1 + P_{EX})$$

Non-local

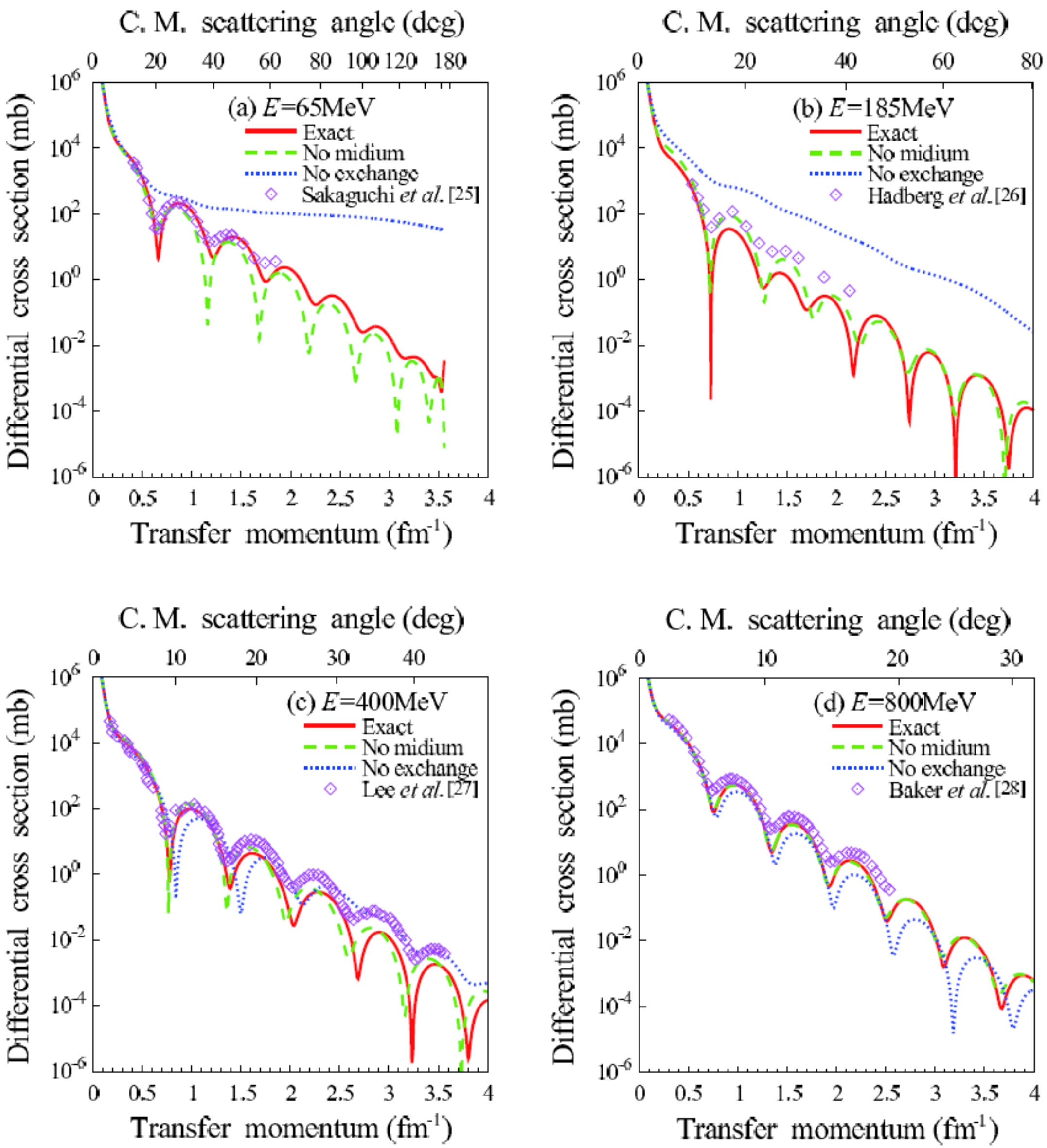
$$\left[-\frac{\hbar^2}{2\mu}\nabla_R^2 + U^{\mathrm{DR}}(\mathbf{R}) + V_{\mathrm{c}}(R)\,\delta_{-1/2}^{\nu_1} - E\right]\chi_{\mathbf{K},\nu_1}(\mathbf{R}) = \int U^{\mathrm{EX}}(\mathbf{R},\mathbf{r})\chi_{\mathbf{K},\nu_1}(\mathbf{r})\,d\mathbf{r}$$

Schroedinger equation for proton scattering



The proton scattering from ^{90}Zr

K. Minomo, K. Ogata, M. Kohno, Y.R. Shimizu, M. Yahiro, J.Phys.G37:085011,2010.



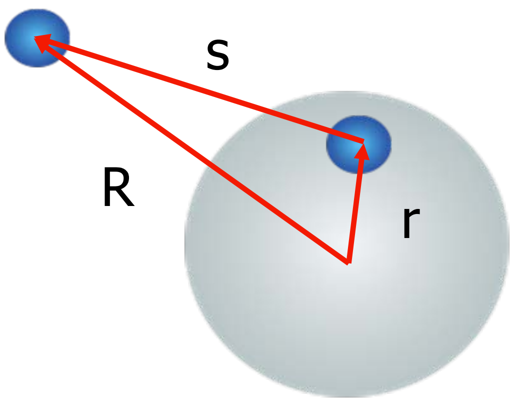
The Brieva-Rook localization

Nucl. Phys. A291,317

$$\int U_{EX}(R,R')\chi(R')dR' \approx \int U_{EX}(R,R')e^{i\vec{k}\cdot(\vec{R}'-\vec{R})}dR'\chi(R)$$

$$\downarrow$$

$$U_{\rm BR}^{\rm EX}(R) = \sum_{\nu_2,T_z} \int \rho_{\nu_2}^{\rm LFG}(\mathbf{R},\mathbf{r}) g_{T_z}^{\rm EX}(s;\rho_{\nu_2}(r_g)) j_0(K(R)s) d\mathbf{s}.$$

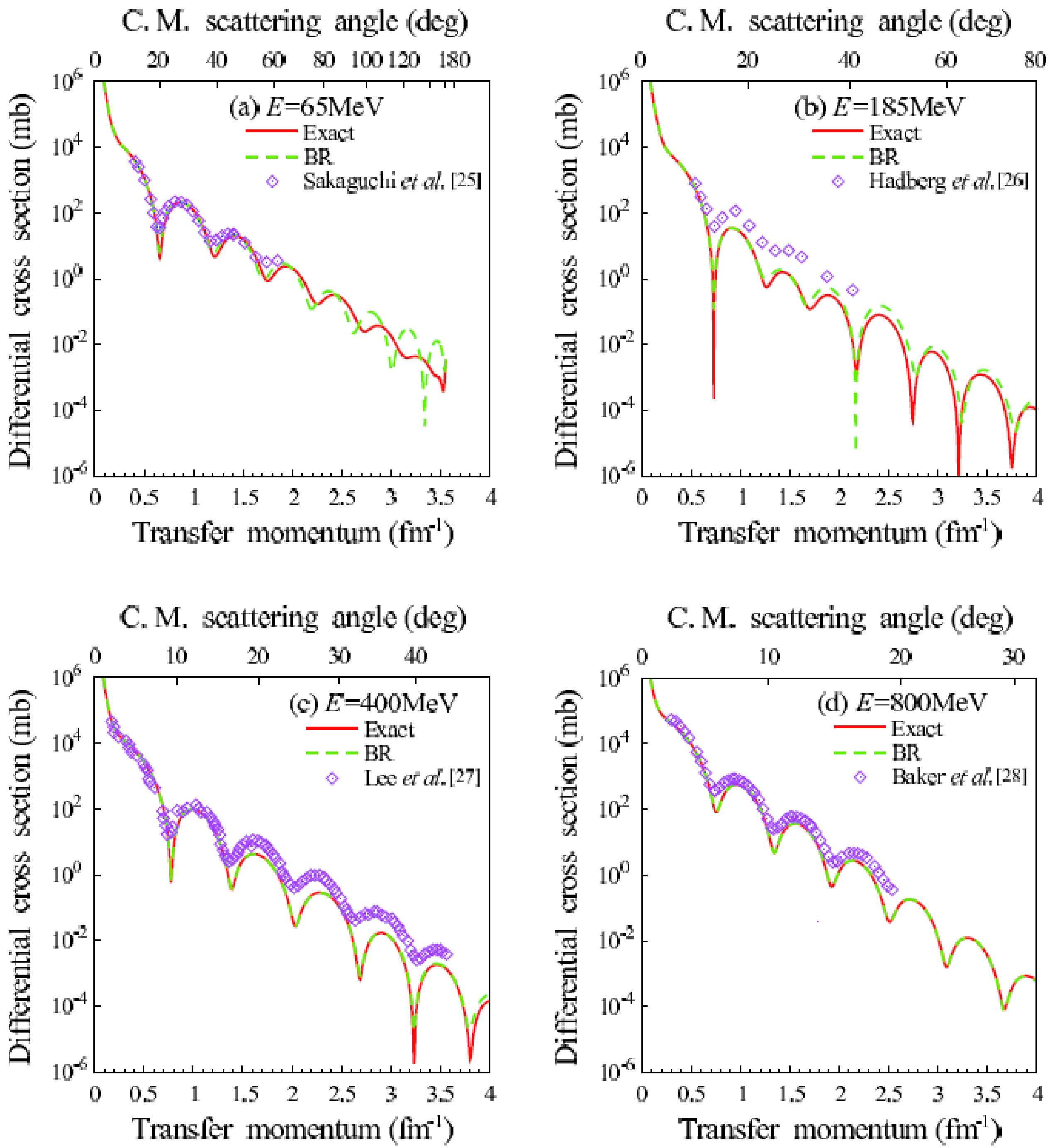


The mixed density

$$\rho_{\nu_2}(\mathbf{R},\mathbf{r}) = \sum_{nljj_z} \int \varphi^*_{\nu_2;nljj_z}(\mathbf{r},\xi)\, \varphi_{\nu_2;nljj_z}(\mathbf{R},\xi)\, d\xi,$$

The validity is tested in

K. Minomo, K. Ogata, M. Kohno, Y.R. Shimizu, M. Yahiro, J.Phys.G37:085011,2010.



C. M. scattering angle (deg)
Differential cross section (mb)
Transfer momentum (fm^{-1})
(a) E=65MeV
Exact
BR
Sakaguchi *et al.* [25]
(b) E=185MeV
Exact
BR
Hadberg *et al.* [26]
(c) E=400MeV
Exact
BR
Lee *et al.* [27]
(d) E=800MeV
Exact
BR
Baker *et al.* [28]

2.2. Application of double folding model to total reaction cross section

For stable nuclei

ρ ⇦ Electron scattering

For unstable nuclei

ρ ⇦ Spherical-HF or AMD

Theoretical Framework of AMD

N-body Hamiltonian : **Gogny D1S effective interaction**

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{T}_g + \sum_{i<j}^{A} \hat{v}_{nn}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_C(r_{ij}), \quad \hat{v}_{nn} : \text{GognyD1S}$$

Variational Wave Function : Parity projection, Gaussian wave packets

$$\Psi^{\pi} = \frac{1 + \pi \hat{P}_x}{2} \Psi_{\text{int}},$$

$$\Psi_{\text{int}} = \mathcal{A}\left\{\varphi_1(\boldsymbol{r}_1), \varphi_1(\boldsymbol{r}_2), ..., \varphi_A(\boldsymbol{r}_A)\right\}, \quad \varphi_i(\boldsymbol{r}_j) = \phi_i(\boldsymbol{r}_j)\chi_i\eta_i,$$

Variational Parameters: Width and centroids of Gaussian, nucleon spins

$$\phi_i(\boldsymbol{r}_j) = \left(\frac{2^3 \det M}{\pi}\right)^{1/2} \exp\left\{-(\boldsymbol{r}_i - \boldsymbol{Z}_j)\boldsymbol{M}(\boldsymbol{r}_i - \boldsymbol{Z}_j)\right\}$$

$$\chi_i = \boldsymbol{\alpha_i}\chi_{\uparrow} + \boldsymbol{\beta_i}\chi_{\downarrow}, \quad \eta_i = \text{proton or neutron}$$

$\boldsymbol{Z}_j$: Centroid of Gaussian wave packets (complex valued 3D vector)

M : Deformation and radius of Gaussian wave packets (real valued 3x3 matrix)

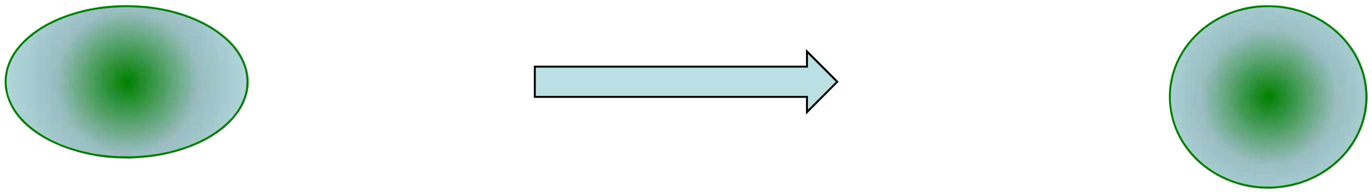
$\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i$: Direction of nucleon's spin (complex valued spinor)

Theoretical Framework of AMD

***J-projection*: Full 3D projection (non axial intrinsic state)**

$$\Psi^{J\pm}_{MK} = \int d\Omega D^{J*}_{MK}(\Omega)\hat{R}(\Omega)\Psi^{\pm}_{\text{int}},$$

Static deformation effect:
enlarge R and enhance σ_R



in the intrinsic frame

in the space-fixed frame

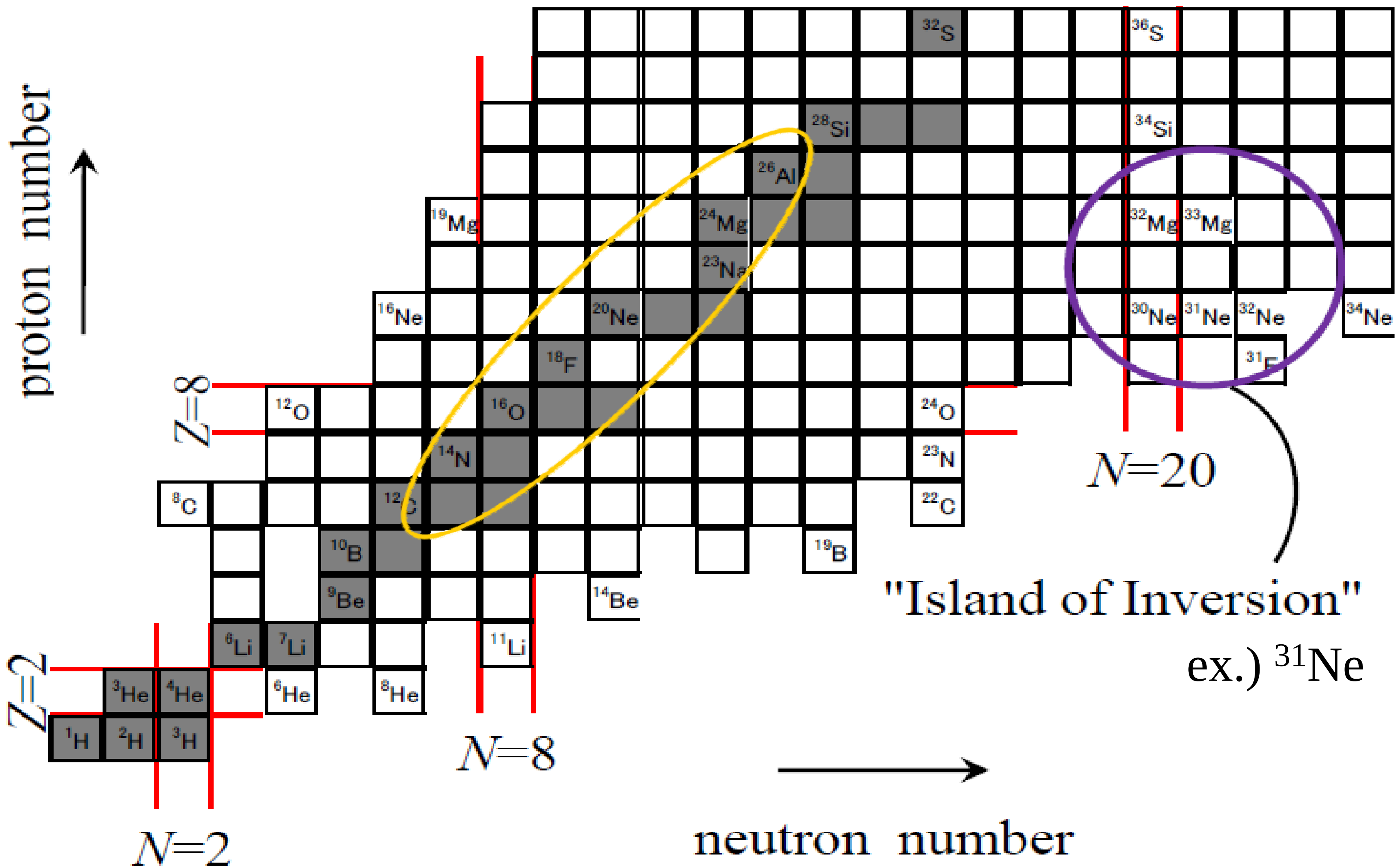
***GCM*:** Generator Coordinate is the quadrupole deformation β

$$\Psi^{J\pm}_{\alpha} = \sum_{iK} c_{ik}\Psi^{J\pm}_{MK}(\beta_i), \quad \sum_{jK'} H_{iKjK'}c_{jK',\alpha} = E_\alpha \sum_{jK'} N_{iKjK'}c_{jK',\alpha},$$

Hill-Wheeler equation

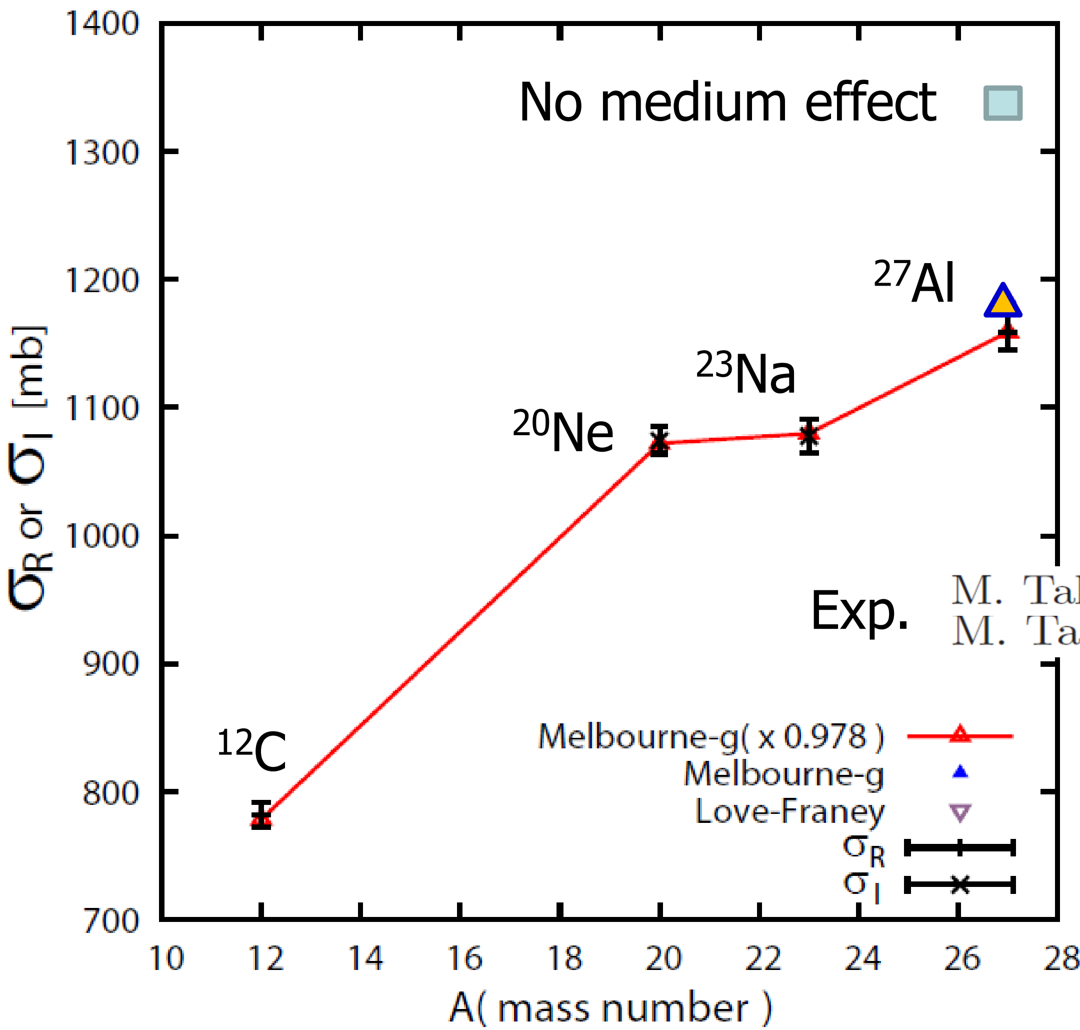
$$H_{iKjK'} = \langle\Psi^{J\pm}_{MK}(\beta_i)|\hat{H}|\Psi^{J\pm}_{MK'}(\beta_j)\rangle, \quad N_{iKjK'} = \langle\Psi^{J\pm}_{MK}(\beta_i)|\Psi^{J\pm}_{MK'}(\beta_j)\rangle$$

Nuclei near the neutron drip line



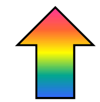
Reaction cross section

^{12}C scattering at 240 MeV/nucleon



M. Takechi, *et al.*, Phys. Rev. C **79**, 061601(R) (2009).
M. Takechi *et al.*, Nucl. Phys. **A834**, 412c (2010).

Projectile and target densities



Electron scattering

Further analysis for ^{31}Ne

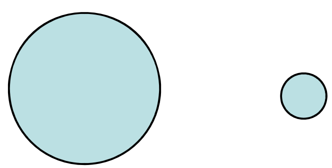
The discrepancy ← { Breakup effect 10 mb; Tail correction to AMD density }

Breakup effect
10 mb

Tail correction to
AMD density

↓

RGM+AMD

^{30}Ne  n

$$\Psi(^{31}\mathrm{Ne};3/2_1^-) = \sum_{nJ\pi} \mathcal{A}\left\{\chi_{nl}(r)Y_{lm}(\hat{\boldsymbol{r}})\Psi(^{30}\mathrm{Ne};J_n^\pi)\phi_n\right\},$$

Density profile of 31Ne

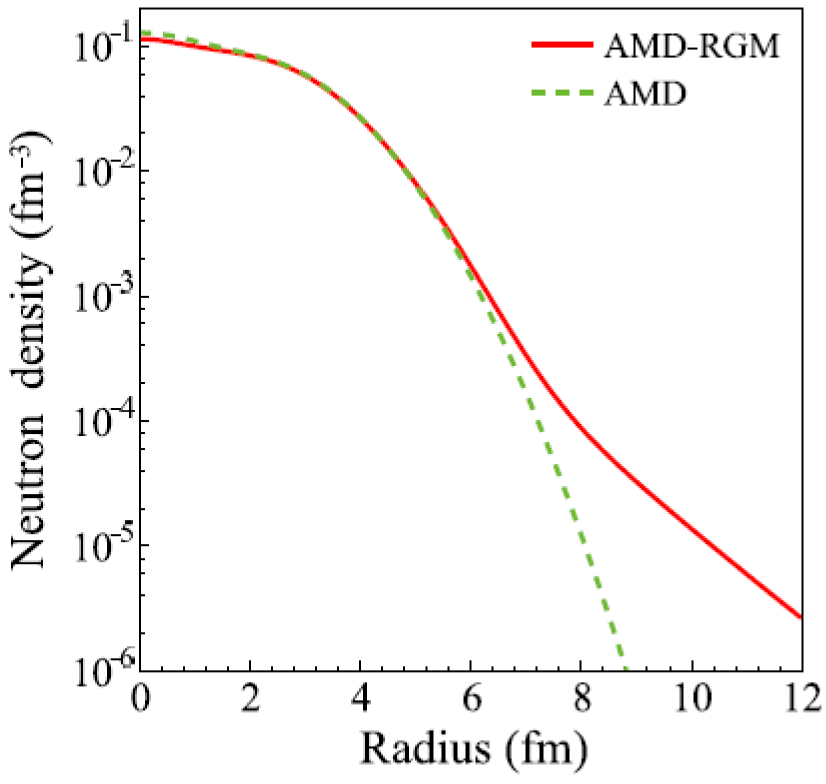
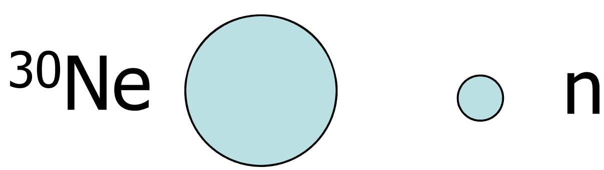


TABLE II: Configurations of the ground state of ^{31}Ne obtained by AMD-RGM and AMD.

	Amplitude	
	AMD-RGM	AMD
$^{30}\mathrm{Ne}(0^+) \otimes 1p_{3/2}$	56 %	37 %
$^{30}\mathrm{Ne}(2^+) \otimes 1p_{3/2}$	24 %	41 %
$^{30}\mathrm{Ne}(2^+) \otimes 0f_{7/2}$	9 %	12 %
$^{30}\mathrm{Ne}(1^-) \otimes 1s_{1/2}$	5 %	5 %
other components	6 %	5 %

Core excitation



Deformed Woods-Saxon model

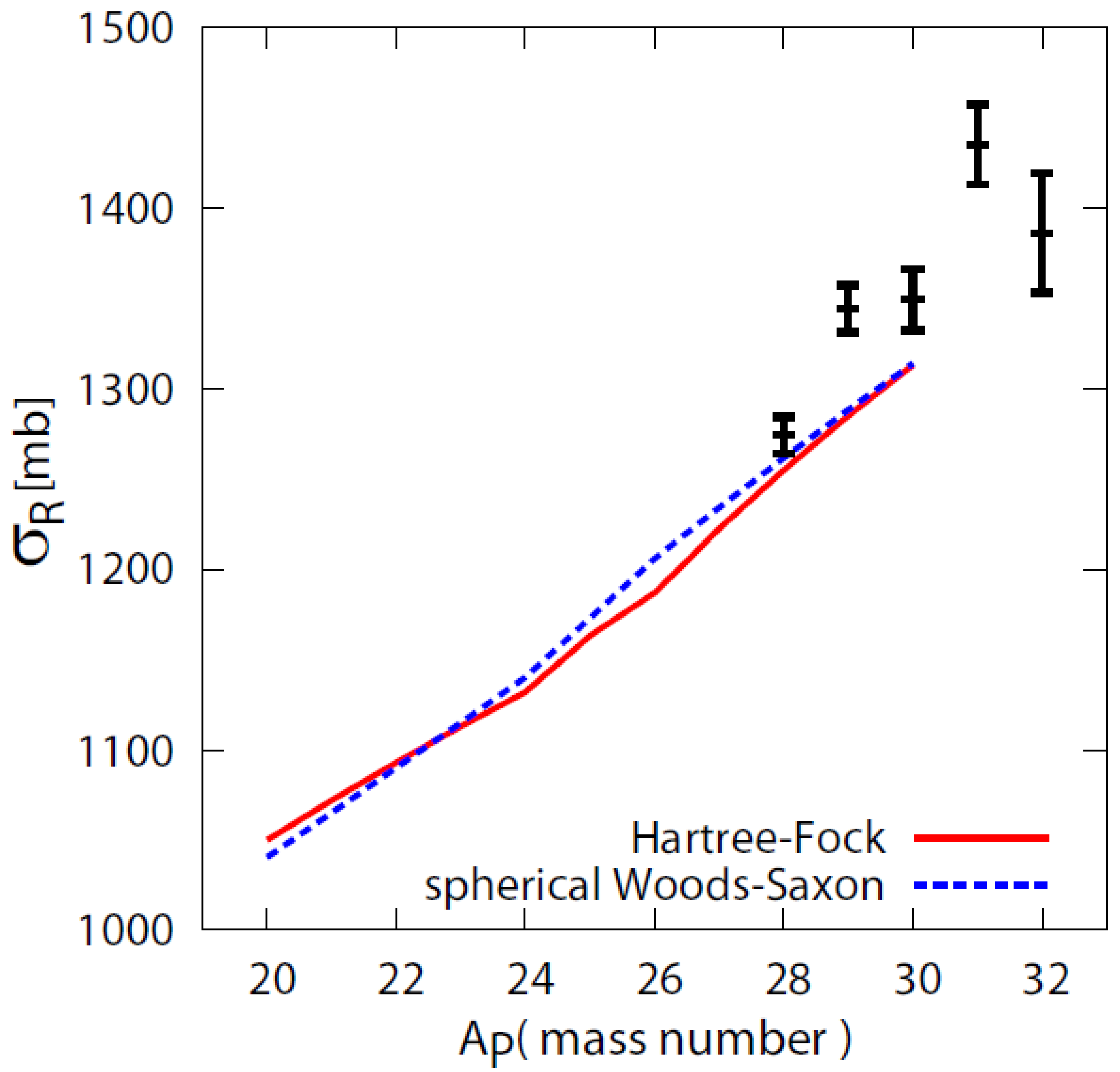
Wyss parametrization

Diffuseness,
depth,
Spherical radius ← The spectroscopic properties of high-spin states from light to heavy deformed stable nuclei, and also the RMS radii.

Deformation ← AMD calculation

Ramon Wyss parametrization

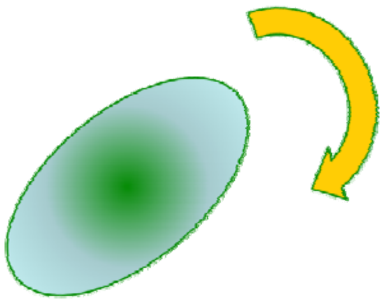
V_0 [MeV]	κ_c	κ_{so}	R_{0c} [fm]	R_{0so} [fm]	a [fm]	λ_{so}
-53.70	0.630	0.25461	$1.193A^{1/3} + 0.25$	$0.969 \times R_{0c}$	0.680	26.847



1500
1400
1300
1200
1100
1000
σ_R [mb]
20
22
24
26
28
30
32
Ap(mass number)
Hartree-Fock
spherical Woods-Saxon

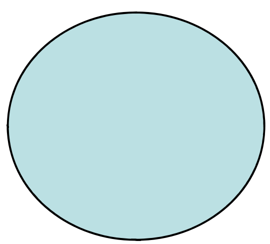
Adiabatic rotational motion (dynamical rotation effect)

Ω (Euler angles)



Adiabatic and eikonal approx.

$$\sigma_{\rm R} = \int d\boldsymbol{b}(1 - |S|^2)$$



$$S = \int \frac{d\Omega}{8\pi^2} \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz U(\boldsymbol{R},\Omega)\right]$$

$$U = U_0(R) + \Delta U(R,\Omega)$$

spherical Non-spherical

$$S = S_0 + S_0 \int \frac{d\Omega}{8\pi^2} \left(\frac{\delta^2}{2} + \cdots\right)$$

$$\begin{aligned} S_0 &= \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz U_0(R)\right], \\ \delta &= -\frac{i}{\hbar v}\int_{-\infty}^{\infty} dz \Delta U(\boldsymbol{R}, \Omega). \end{aligned}$$

Total reaction cross sections for Ne isotopes at low and high incident energies

1. **Low Energy (33-60 MeV/nucleon)**
   Exp. Data at GANIL

2. **High Energy (950 MeV/nucleon)**
   Exp. Data at GSI

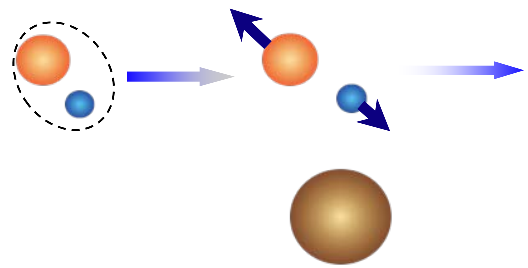
3. Eikonal reaction theory (ERT)

An extension of CDCC to inclusive reactions such as one-nucleon removal reaction. ⇐ Minomo-kun

Yahiro, Ogata and Minomo, Prog. Theor. Phys. 126(2011), 167-176.
Hashimoto, Yahiro, Ogata, Minomo and Chiba, Phys. Rev. C83(2011), 054617.

One nucleon removal reaction

$$\sigma_{-n} = \sigma_{\mathrm{bu}} + \sigma_{\mathrm{str}}$$



	σ_{bu}	σ_{str}
CDCC	○	×
Glauber	△	△
	× (Coulomb elastic breakup)	

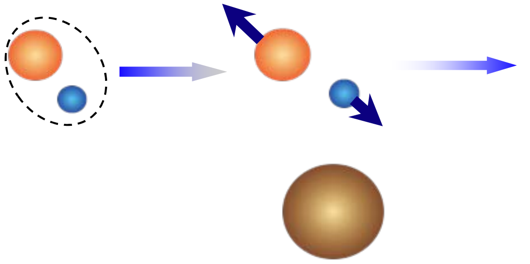
⇐ Eikonal reaction theory (ERT) ⇐ Minomo-kun

DEA, CCE ⇐ Baye-san

One nucleon removal reaction

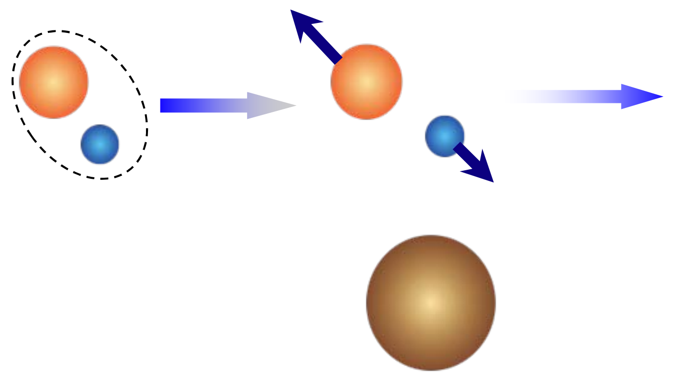
$$\sigma_{-n} = \sigma_{\mathrm{bu}} + \sigma_{\mathrm{str}}$$

CDCC ERT



This framework is applicable for the scattering system with strong Coulomb interaction.

Eikonal Reaction Theory



$$(E - \frac{\hbar^2}{2\mu}\Delta - \hat{h} - V_n - V_c)\psi = 0$$

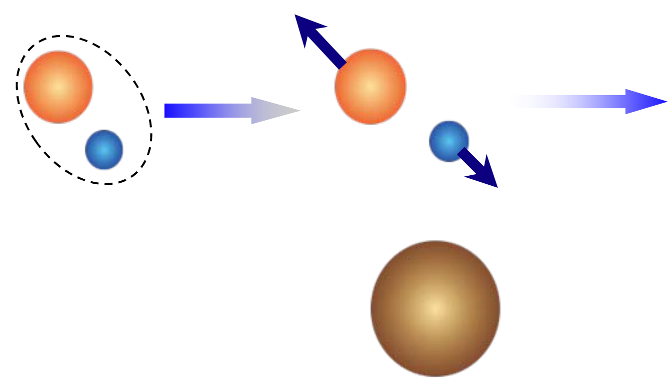
Eikonal assumption

$$\psi = e^{i\hat{k}z}\chi(r) \qquad \hat{k} = \frac{1}{\hbar}\sqrt{2\mu(E-\hat{h})}$$

Formal solution

$$S = \exp\left[-i\frac{1}{\hbar v}P\int_{-\infty}^{\infty}dz\,O^{-1}(V_n+V_c)O\right]$$

$$O = e^{i\hat{k}z}$$



Path ordering

$$P(V(z_1)V(z_2)) = \theta(z_1 - z_2)V(z_1)V(z_2) + \theta(z_2 - z_1)V(z_2)V(z_1)$$

$$\exp[-iP\int dzV(z)]$$

$$= 1 - i\int dzV(z) + \frac{(-i)^2}{2}\iint dz_1 dz_2 P(V(z_1)V(z_2)) + \cdots$$

$$(-i)^2 \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{z_1} dz_2 V(z_1)V(z_2)$$

Eikonal decomposition of S

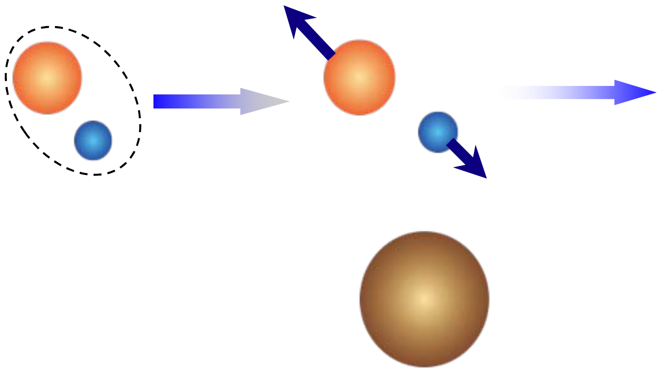
Yahiro, Ogata and Minomo, Prog. Theor. Phys. 126(2011), 167-176.

$$S = \exp\left[-i\frac{1}{\hbar v} P \int_{-\infty}^{\infty} dz O^{-1}(V_n + V_c) O \right]$$

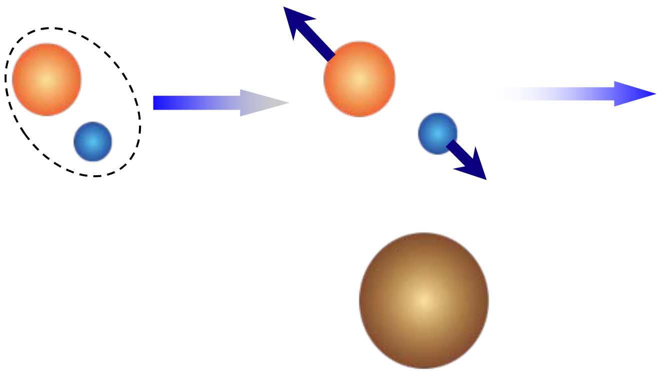
$$\approx \exp\left[-i\frac{1}{\hbar v} \int_{-\infty}^{\infty} dz V_n \right] \exp\left[-i\frac{1}{\hbar v} P \int_{-\infty}^{\infty} dz O^{-1} V_c O \right]$$

$$= S_n S_c$$

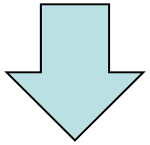
$$O^{-1} V_n O \approx V_n e^{i(k_i - k_f)a}$$



How to get S_c



$$(E - \frac{\hbar^2}{2\mu}\Delta - \hat{h} - V_n - V_c)\psi = 0$$



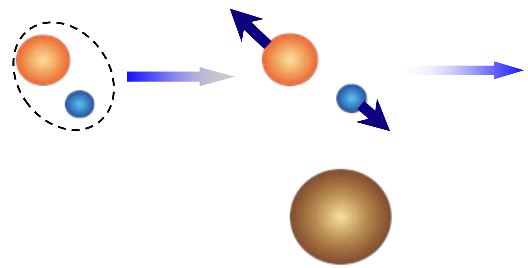
$$(E - \frac{\hbar^2}{2\mu}\Delta - \hat{h} - V_c)\psi_c = 0$$

Eikonal CDCC

K.Ogata, Hashimoto, Iseri, Kamimura, and Yahiro, PRC73, 024605 (2006).

One nucleon removal reaction

$$\sigma_{-n} = \sigma_{\mathrm{bu}} + \sigma_{\mathrm{str}}$$



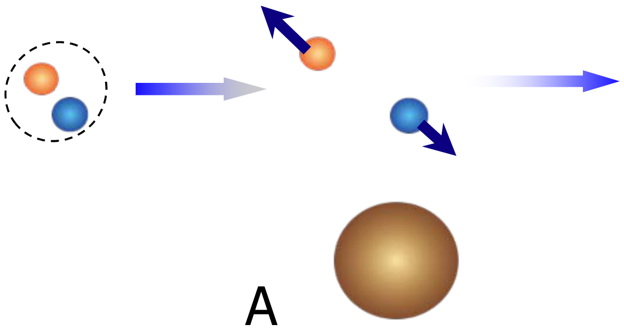
CDCC ERT

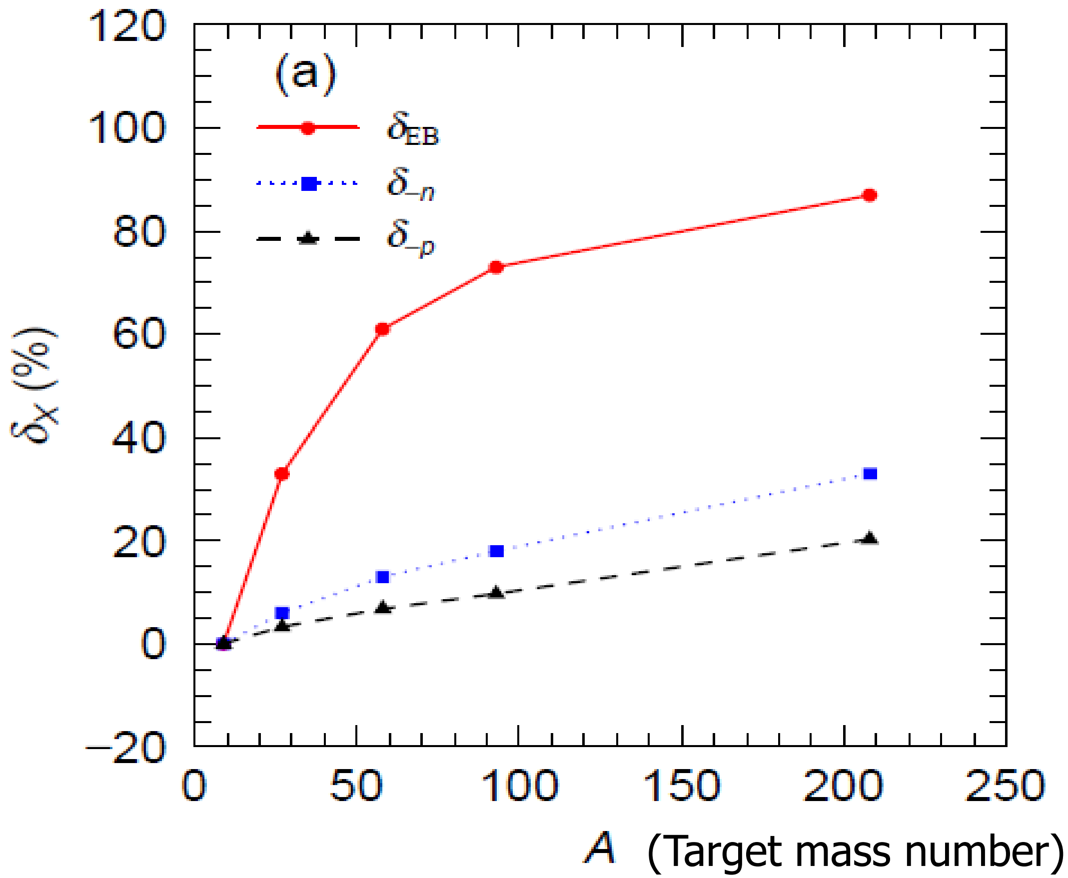
$$\sigma_{\mathrm{str}} = \int d^2\boldsymbol{b} \langle \varphi_0 | |S_{\mathrm{c}}|^2 (1 - |S_{\mathrm{n}}|^2) | \varphi_0 \rangle$$

Accuracy of the Glauber model

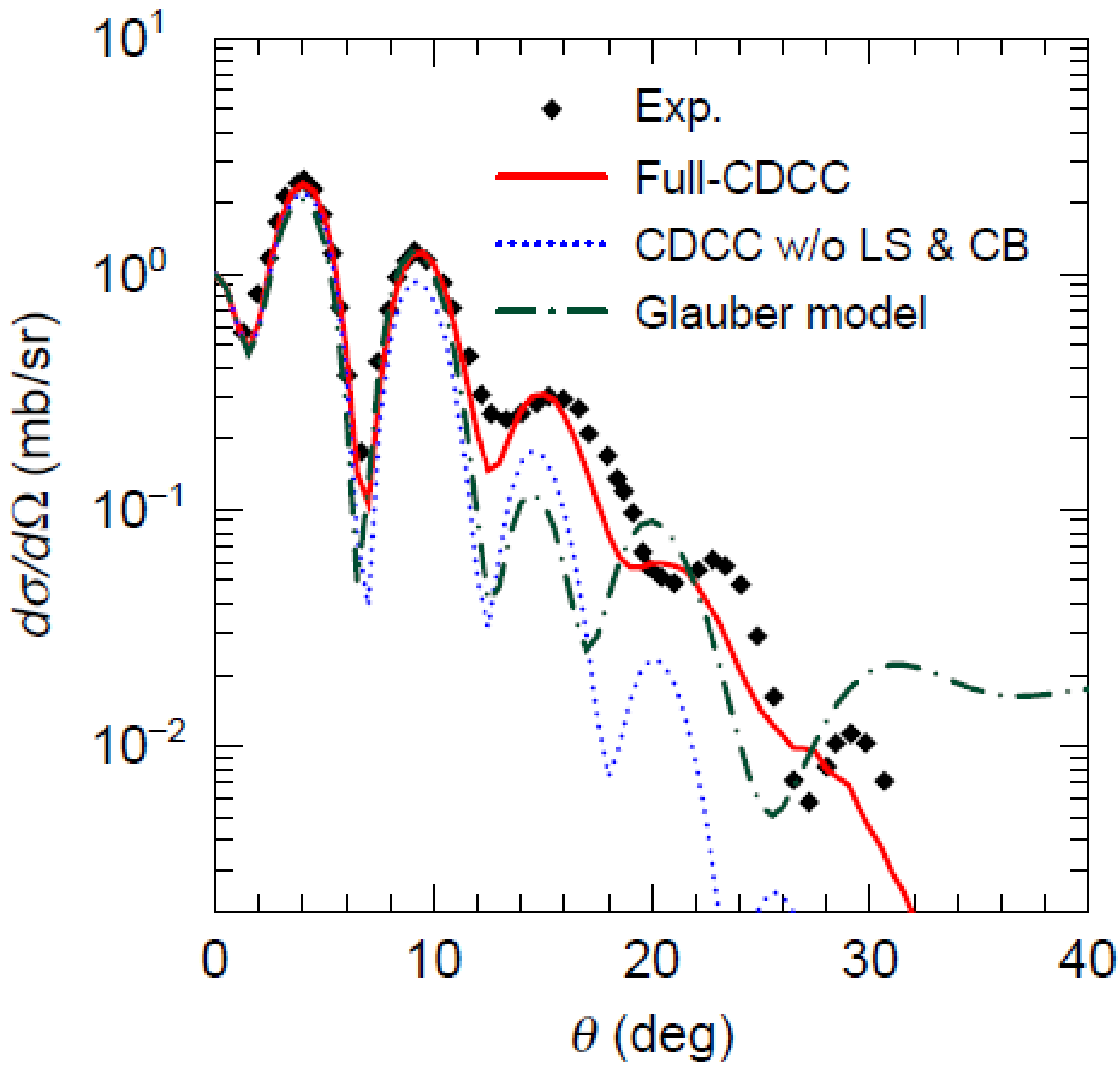
Hashimoto, Yahiro, Ogata, Minomo and Chiba, Phys. Rev. C83(2011), 054617.

Deuteron scattering from several targets at 200 MeV/nucleon

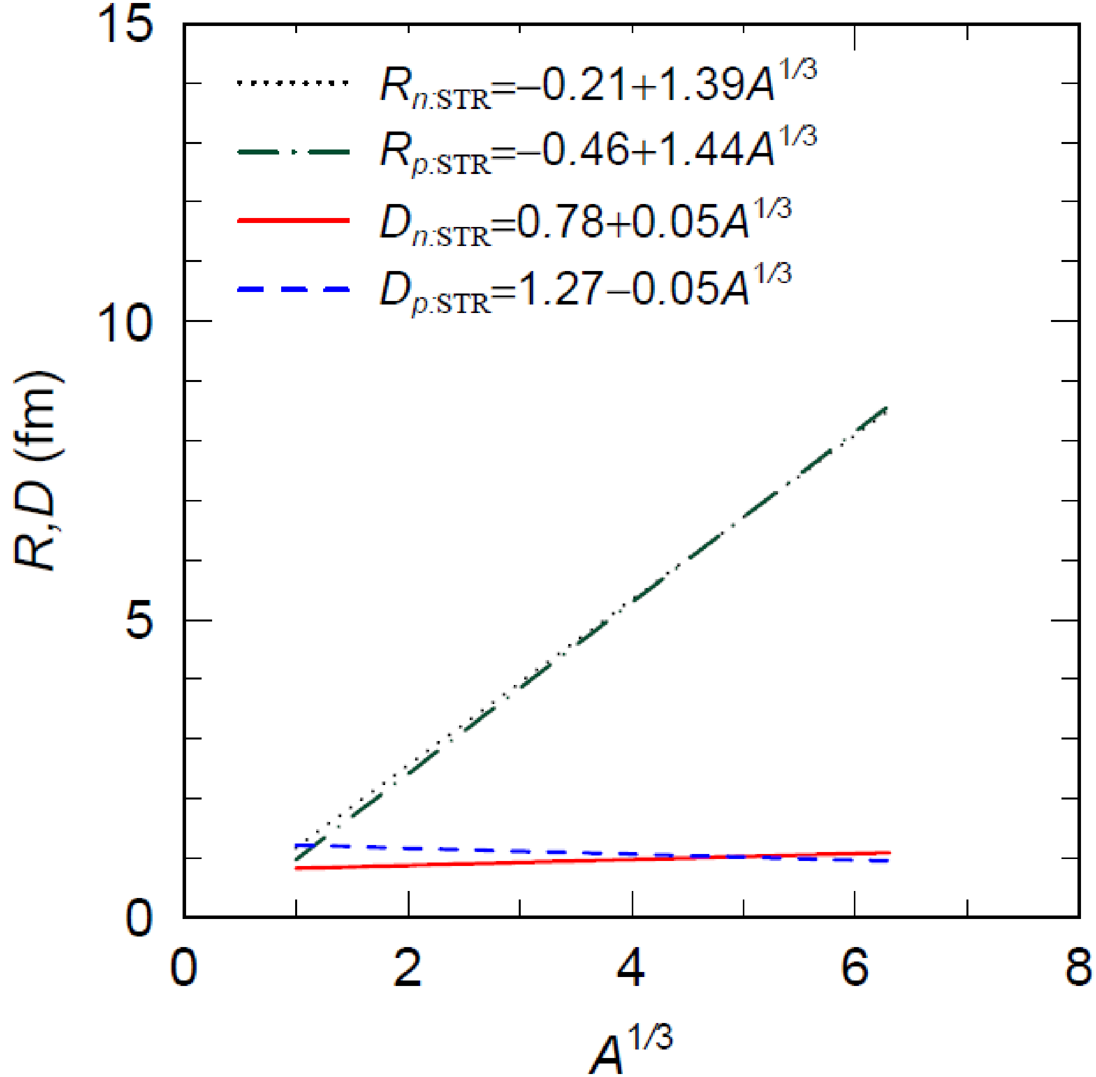


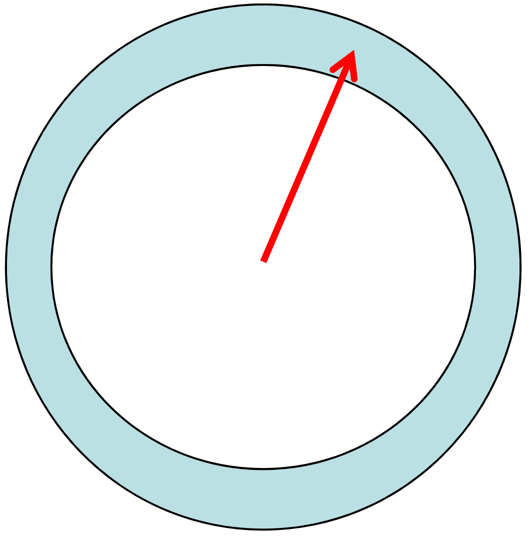


d+^{58}Ni elastic scattering at 400 MeV



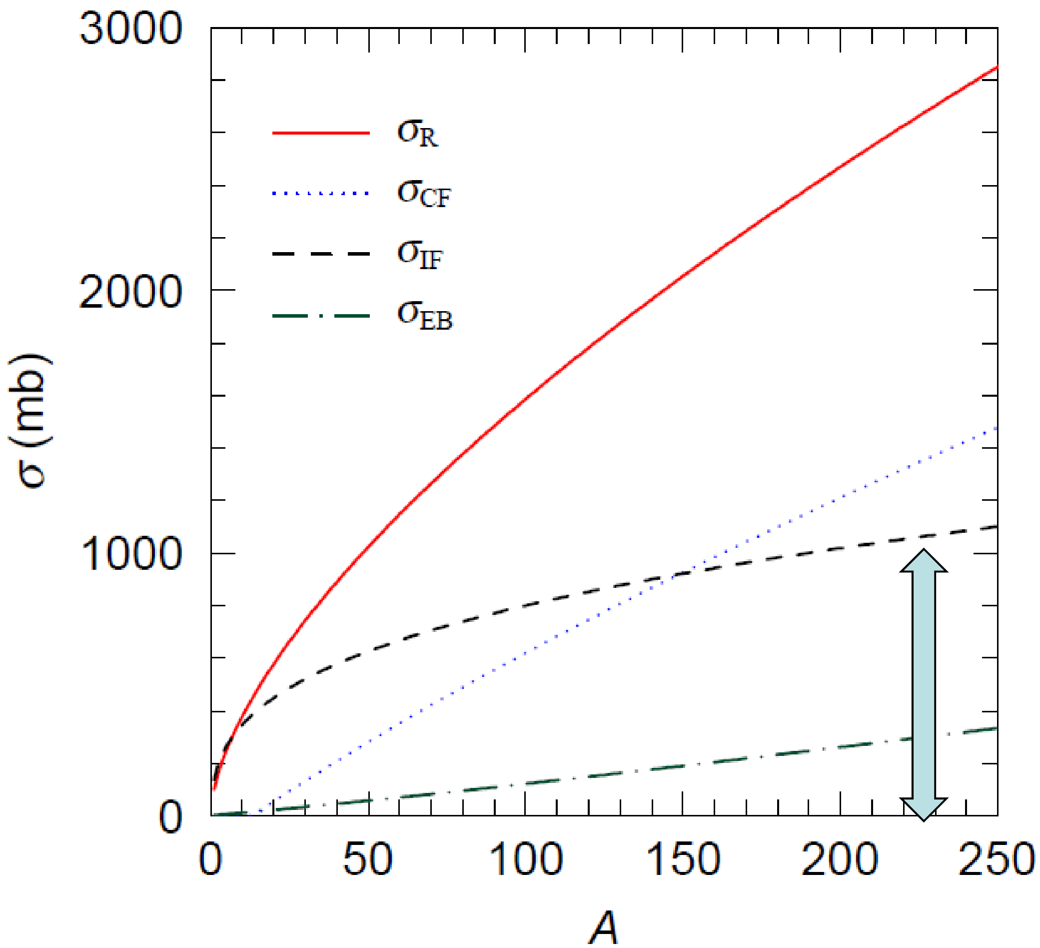
A-dependence of stripping cross section

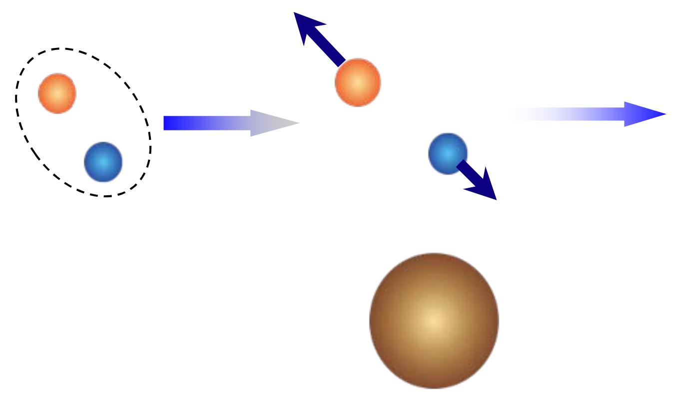




A-dependence of reaction cross section

$$\sigma_R = \sigma_{Elastic-breakup} + \sigma_{incomplete-fusion} + \sigma_{complete-fusion}$$





incomplete fusion

4. Four-body CDCC → Matsumoto-san

It is the method for four-body breakup.

Matsumoto, Kato and Yahiro, Phys.Rev.C82:051602,2010.
M. Rodriguez-Gallardo, J. M. Arias, J. Gomez-Camacho, A. M. Moro, I. J. Thompson, and J. A. Tostevin, Phys. Rev. C 80, 051601(R) (2009).

Summary

- A goal of nuclear physics is to construct the microscopic reaction theory. CDCC+AMD is a candidate for the theory. The theory can predict physics of unstable nuclei before the measurement.
- This method was applied for the reaction cross section of the scattering of Ne isotopes from C target at 250 MeV/nucleon. The static deformation effect is important, but the projectile breakup effect is small. ^{31}Ne is a halo nucleus with large deformation.
- We proposed ERT to treat inclusive reactions. ERT can treat Coulomb breakup properly.

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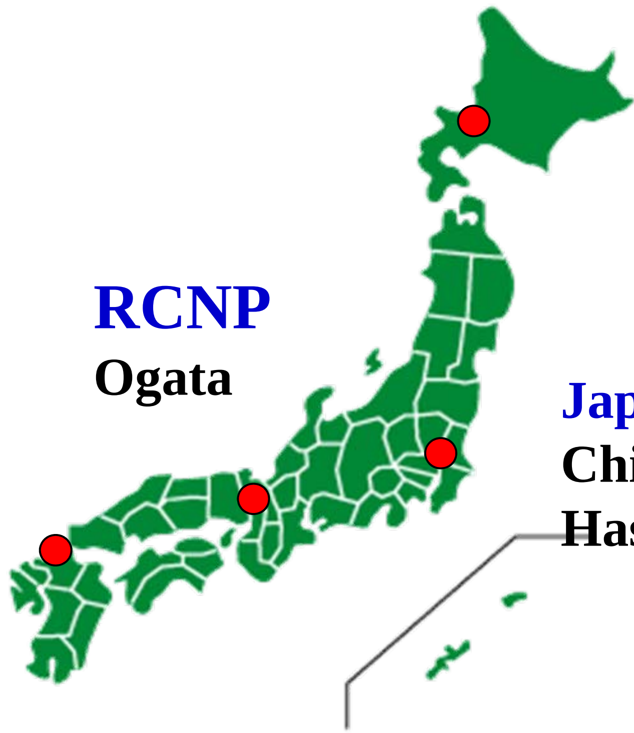

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