Extended RPA from time-dependent density-matrix theory

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Formulation

Extended RPA(ERPA): Equation-of-motion approach

Excitation operator

$$Q_\mu^+ = \sum (x_{\lambda\lambda'}^\mu : a_\lambda^+ a_{\lambda'} : + X_{\lambda_1\lambda_2\lambda_1'\lambda_2'}^\mu : a_{\lambda_1}^+ a_{\lambda_2}^+ a_{\lambda_2'} a_{\lambda_1'} :)$$

$$Q_\mu^+ \left| \Psi_0 \right\rangle = \left| \Psi_\mu \right\rangle$$

$$Q_\mu \left| \Psi_0 \right\rangle = 0$$

Equations of motion

$$\left\langle \Psi_0 \right| [[: a_{\alpha'}^+ a_\alpha :, H], Q_\mu^+] \left| \Psi_0 \right\rangle = \omega_\mu \left\langle \Psi_0 \right| [: a_{\alpha'}^+ a_\alpha :, Q_\mu^+] \left| \Psi_0 \right\rangle$$

$$\left\langle \Psi_0 \right| [[: a_{\alpha'}^+ a_{\beta'}^+ a_\beta a_\alpha :, H], Q_\mu^+] \left| \Psi_0 \right\rangle = \omega_\mu \left\langle \Psi_0 \right| [: a_{\alpha'}^+ a_{\beta'}^+ a_\beta a_\alpha :, Q_\mu^+] \left| \Psi_0 \right\rangle$$

ERPA equation

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} x^{\mu} \\ X^{\mu} \end{pmatrix} = \omega_{\mu}\begin{pmatrix} S_{11} & T_{12} \\ T_{21} & S_{22} \end{pmatrix}\begin{pmatrix} x^{\mu} \\ X^{\mu} \end{pmatrix}$$

$$A(\alpha\alpha':\lambda\lambda') = \langle\Psi_0|[[:a^+_{\alpha'}a_{\alpha}:,H],:a^+_{\lambda}a_{\lambda'}:]|\Psi_0\rangle$$

$$B(\alpha\alpha':\lambda_1\lambda_2\lambda_1'\lambda_2') = \langle\Psi_0|[[:a^+_{\alpha'}a_{\alpha}:,H],:a^+_{\lambda_1}a^+_{\lambda_2}a_{\lambda_2'}a_{\lambda_1'}:]|\Psi_0\rangle$$

$$C(\alpha\beta\alpha'\beta':\lambda\lambda') = \langle\Psi_0|[[:a^+_{\alpha'}a^+_{\beta'}a_{\beta}a_{\alpha}:,H],:a^+_{\lambda}a_{\lambda'}:]|\Psi_0\rangle$$

$$D(\alpha\beta\alpha'\beta':\lambda_1\lambda_2\lambda_1'\lambda_2') = \langle\Psi_0|[[:a^+_{\alpha'}a^+_{\beta'}a_{\beta}a_{\alpha}:,H],:a^+_{\lambda_1}a^+_{\lambda_2}a_{\lambda_2'}a_{\lambda_1'}:]|\Psi_0\rangle$$

$$S_{11}(\alpha\alpha':\lambda\lambda') = \langle\Psi_0|[:a^+_{\alpha'}a_{\alpha}:,:a^+_{\lambda}a_{\lambda'}:]|\Psi_0\rangle$$

$$T_{12}(\alpha\alpha':\lambda_1\lambda_2\lambda_1'\lambda_2') = \langle\Psi_0|[:a^+_{\alpha'}a_{\alpha}:,:a^+_{\lambda_1}a^+_{\lambda_2}a_{\lambda_2'}a_{\lambda_1'}:]|\Psi_0\rangle$$

$$T_{21}(\alpha\beta\alpha'\beta':\lambda\lambda') = \langle\Psi_0|[:a^+_{\alpha'}a^+_{\beta'}a_{\beta}a_{\alpha}:,:a^+_{\lambda}a_{\lambda'}:]|\Psi_0\rangle$$

$$S_{22}(\alpha\beta\alpha'\beta':\lambda_1\lambda_2\lambda_1'\lambda_2') = \langle\Psi_0|[:a^+_{\alpha'}a^+_{\beta'}a_{\beta}a_{\alpha}:,:a^+_{\lambda_1}a^+_{\lambda_2}a_{\lambda_2'}a_{\lambda_1'}:]|\Psi_0\rangle$$

$|\Psi_0\rangle$ is HF g.s.: Second RPA (SRPA)

We use the TDDM ground state as $|\Psi_0\rangle$

Time-dependent density-matrix theory (TDDM)

TDDM determines the time evolution of
ρ and C_2

$$\rho(11':t) = \left\langle \Phi(t) \,|\, a^+(1')a(1) \,|\, \Phi(t) \right\rangle$$

$$C_2(121'2':t) = \left\langle \Phi(t) \,|{:}\, a^+(1')a^+(2')a(2)a(1) \,{:}|\, \Phi(t) \right\rangle$$

Time derivatives of ρ and C_2

$$i\hbar \frac{\partial}{\partial t} \rho = \left\langle \Phi(t) \,|\, [a^+(1')a(1), H] \,|\, \Phi(t) \right\rangle = F_1(\rho, C_2)$$

$$i\hbar \frac{\partial}{\partial t} C_2 = F_2(\rho, C_2, C_3) \approx F_2(\rho, C_2)$$

BBGKY hierarchy is truncated

S. J. Wang & W. Cassing, Ann. Phys. 159, 328 (1985)

Ground state: A stationary solution of TDDM Eqs.

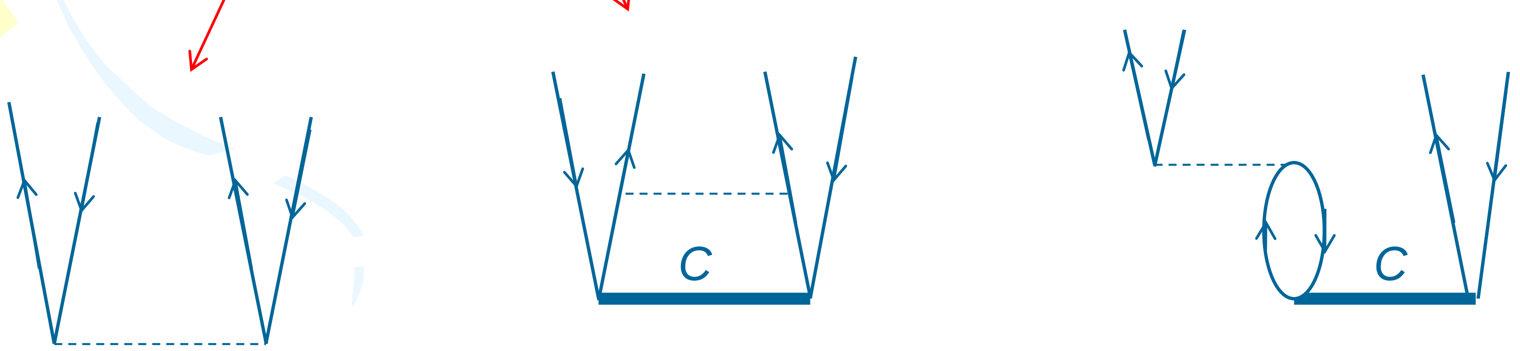
$$i\hbar \frac{d}{dt} n_{\alpha\alpha'} = F_{1\alpha\alpha'}(n, C) = 0$$

$$i\hbar \frac{d}{dt} C_{\alpha\beta\alpha'\beta'} = F_{2\alpha\beta\alpha'\beta'}(n, C) = 0$$

Expressions for F_1 and F_2

$$F_{1\alpha\alpha'}(n,C) = (\varepsilon_\alpha - \varepsilon_{\alpha'})n_{\alpha\alpha'} + \sum_{\lambda_1\lambda_2\lambda_3}\left\{ \langle \alpha\lambda_3 | v | \lambda_1\lambda_2 \rangle C_{\lambda_1\lambda_2\alpha'\lambda_3} - C_{\alpha\lambda_3\lambda_1\lambda_2} \langle \lambda_1\lambda_2 | v | \alpha'\lambda_3 \rangle \right\}$$

$$F_{2\alpha\beta\alpha'\beta'}(n,C) = (\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'})C_{\alpha\beta\alpha'\beta'} + B_{\alpha\beta\alpha'\beta'} + P_{\alpha\beta\alpha'\beta'} + H_{\alpha\beta\alpha'\beta'}$$



2p-2h excitations p-p and h-h correlations P-h correlations

Gradient method can be used

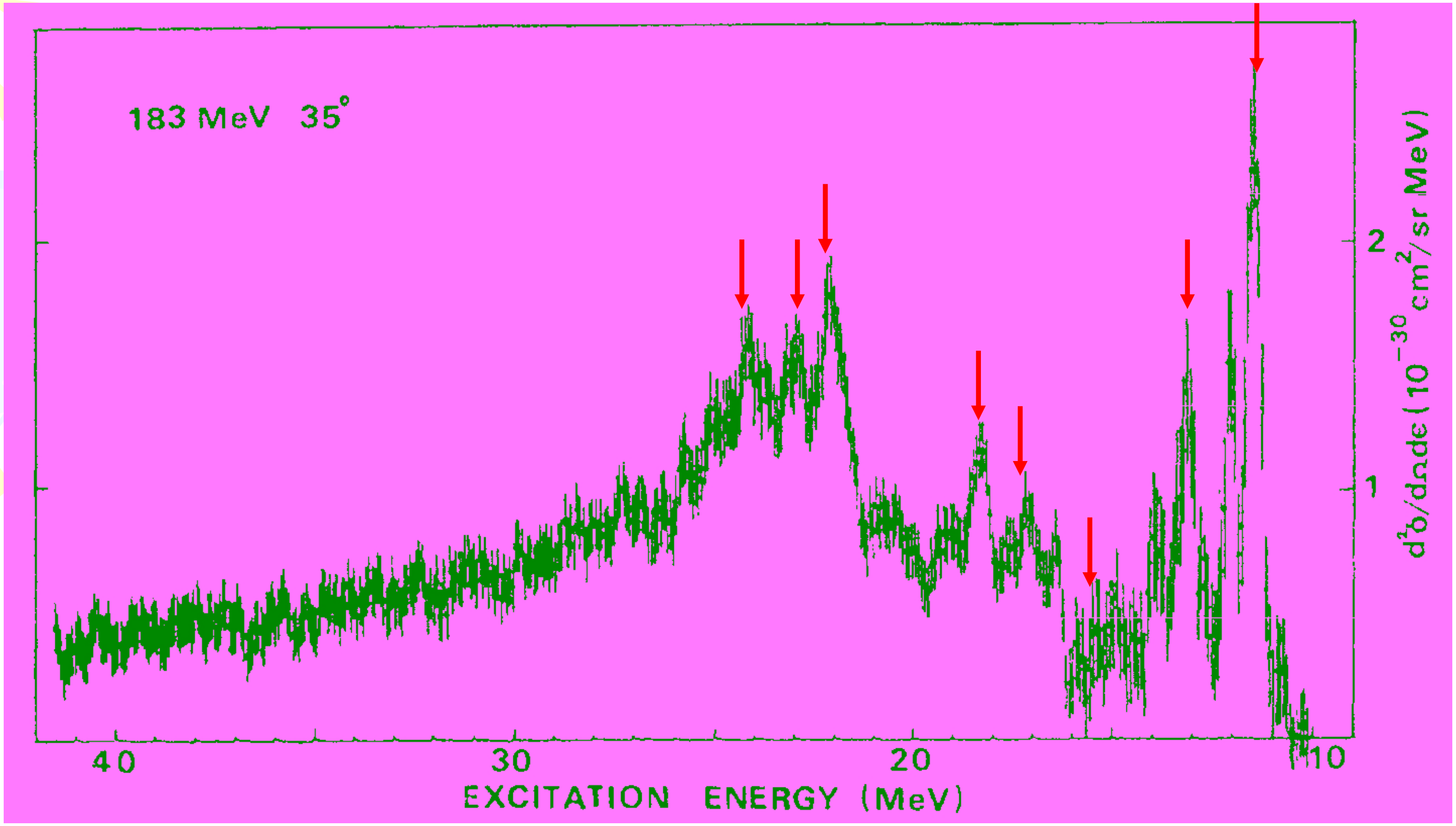
$$\begin{pmatrix} n(N+1) \\ C(N+1) \end{pmatrix} = \begin{pmatrix} n(N) \\ C(N) \end{pmatrix} - \begin{pmatrix} \frac{\delta F_1}{\delta n} & \frac{\delta F_1}{\delta C_2} \\ \frac{\delta F_2}{\delta n} & \frac{\delta F_2}{\delta C_2} \end{pmatrix}^{-1} \begin{pmatrix} F_1(N) \\ F_2(N) \end{pmatrix}$$

N : Iteration step

M. Tohyama et al., Eur. Phys. J. A 21, 217(2004)

GQR in ^{16}O

Spectrum of electrons scattered from ^{16}O



A. Hotta et al., Phys. Rev. Lett. 33(1974)790

Calculation parameters

Effective interaction: Skyrme III

Single-particle states:

$$x^{\mu}_{\alpha\alpha'} : \varepsilon_{\alpha} \leq 40\mathrm{MeV}, \ell \leq 4\hbar, \quad R = 20\mathrm{fm}$$

$$n_{\alpha\alpha}, C_{\alpha\beta\alpha'\beta'}, X^{\mu}_{\alpha\beta\alpha'\beta'} : 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}$$

Residual interaction:

$$v(\vec{r}_1 - \vec{r}_2) = [t_0(1 + x_0 P_{\sigma}) + t_3 \rho / 2]\delta^3(\vec{r}_1 - \vec{r}_2) \times f$$

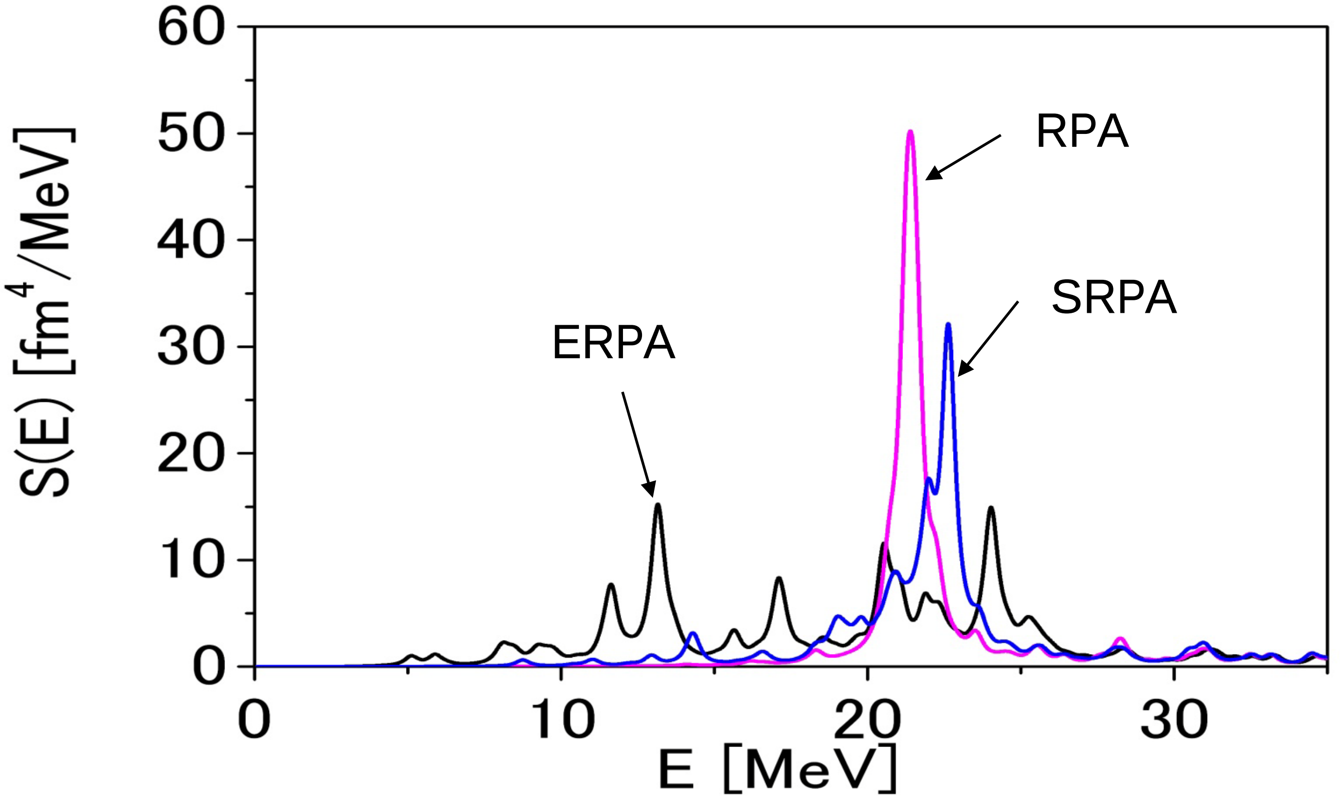
$$t_0 = -1128.75\,\mathrm{MeVfm}^3, x_0 = 0.45, t_3 = 14000\,\mathrm{MeVfm}^6$$

Ground state of ^{16}O

Occupation probabilities

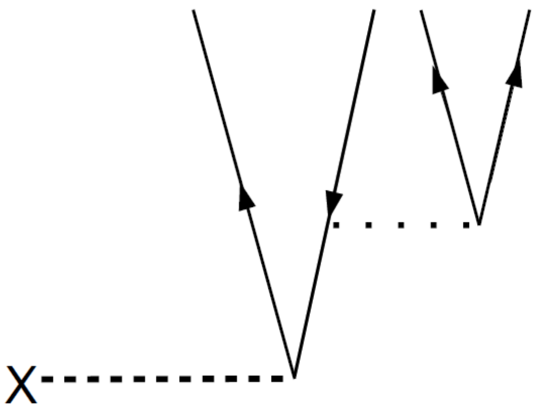
	Protons		Neutrons	
	ε_α(MeV)	n_α	ε_α(MeV)	n_α
$1p_{3/2}$	-18.3	0.90	-21.9	0.90
$1p_{1/2}$	-12.3	0.88	-15.7	0.88
$1d_{5/2}$	-3.8	0.10	-7.1	0.10
$2s_{1/2}$	1.0	0.02	-1.5	0.02

^{16}O: Strength function for r^2Y_{20}



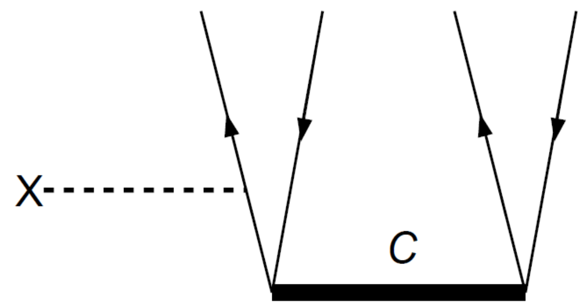
Damping processes

1p-1h ⟶ 2p-2h



X:External field

g. s. ⟶ 2p-2h



Included in SRPA and ERPA

Included only in ERPA

First excited states in tin isotopes

Excited states in tin isotopes

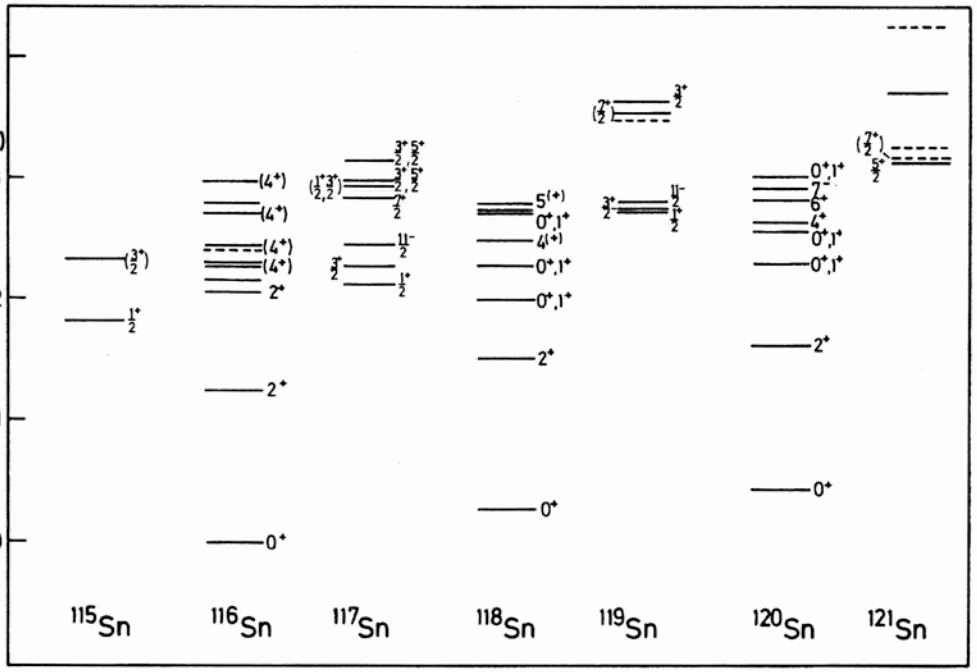
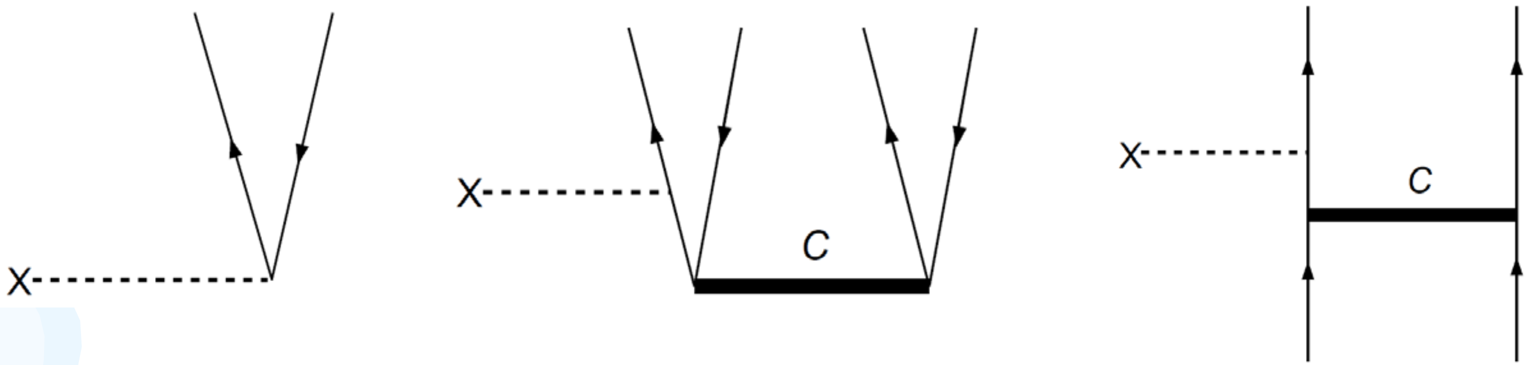


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Pairing model: HFB+QRPA

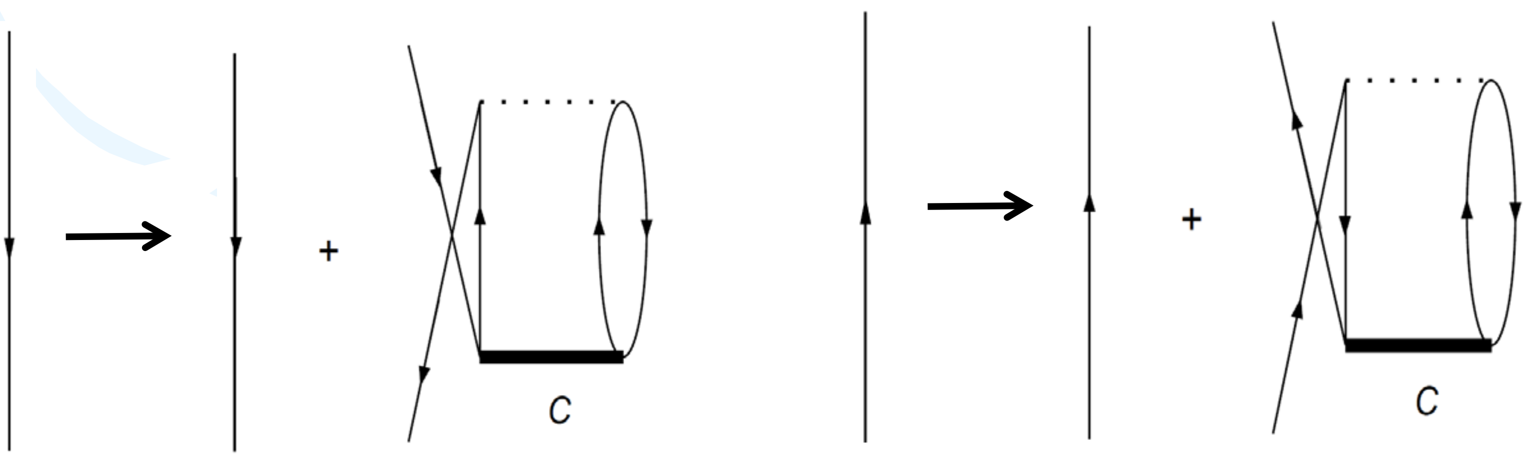
Effects of ground-state correlations in ERPA

Excitation processes



X : External field

Self-energies



Single-particle states:

$n_{\alpha\alpha}\ , C_{\alpha\beta\alpha'\beta'}\ , X^{\mu}_{\alpha\beta\alpha'\beta'} : 1g_{7/2}\ , 2d_{5/2}\ , 1h_{11/2}\ , 2d_{3/2}\ , 3s_{1/2}$ for neutrons

Residual interaction for matrices *B*, *C* and *D*:

$$v(\vec{r}_1 - \vec{r}_2) = [t_0 + t_3 \rho / 2] \delta^3 (\vec{r}_1 - \vec{r}_2) \times f'$$

This is similar to a density-dependent pairing force

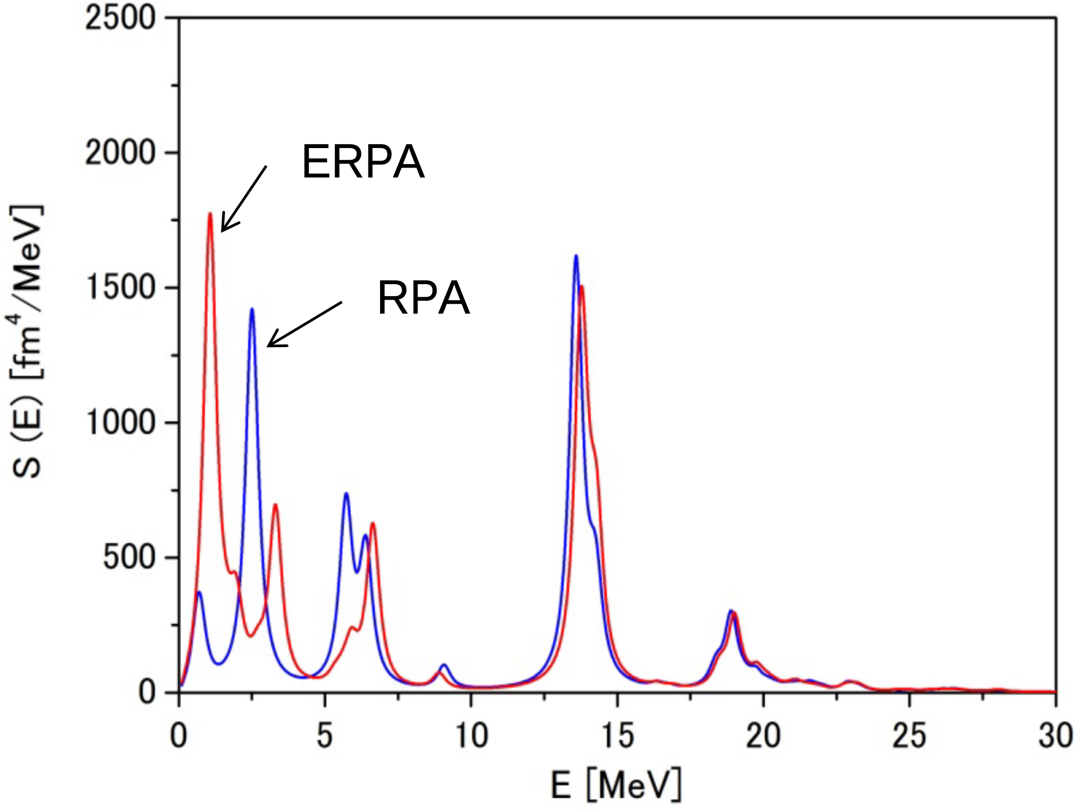
Ground state

Occupation probabilities in ^{114}Sn

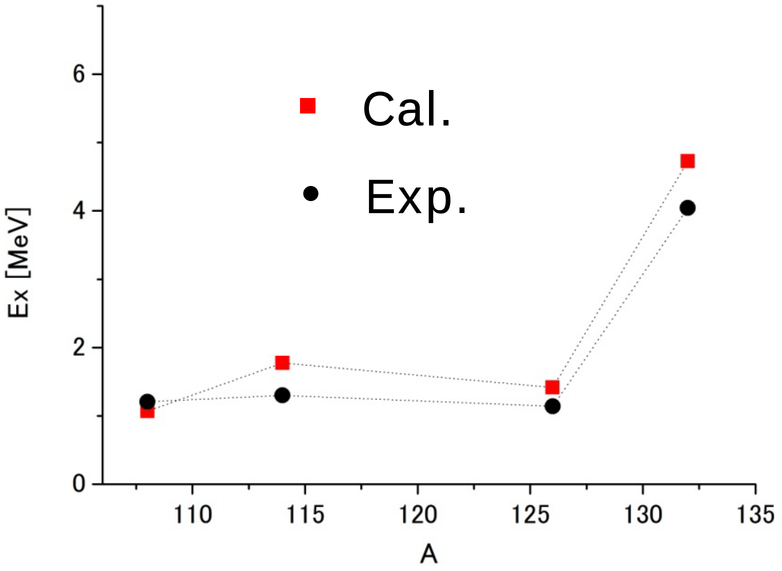
	ε_α [MeV]	$n_{\alpha\alpha}$
$1g_{7/2}$	-10.89	0.948
$2d_{5/2}$	-9.68	0.815
$1h_{11/2}$	-8.02	0.106
$3s_{1/2}$	-7.14	0.035
$2d_{3/2}$	-6.85	0.045

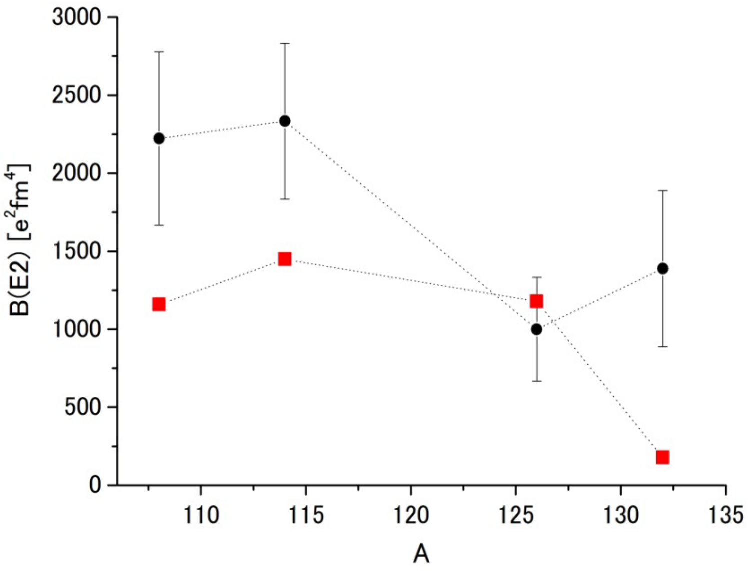
Quadrupole excitation in ^{108}Sn

Strength function for $r^2 Y_{20}$

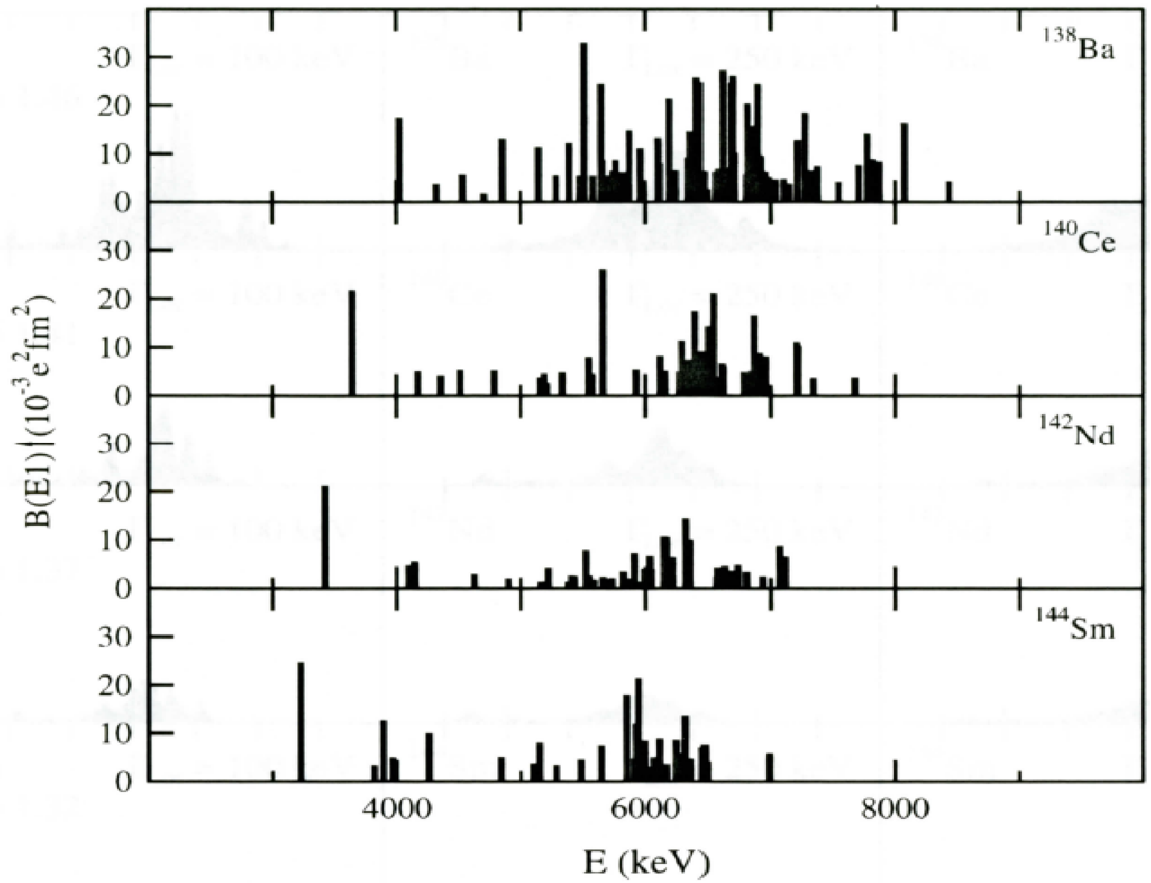


Excitation energy of 2_1^+ $B(E2)$ of 2_1^+



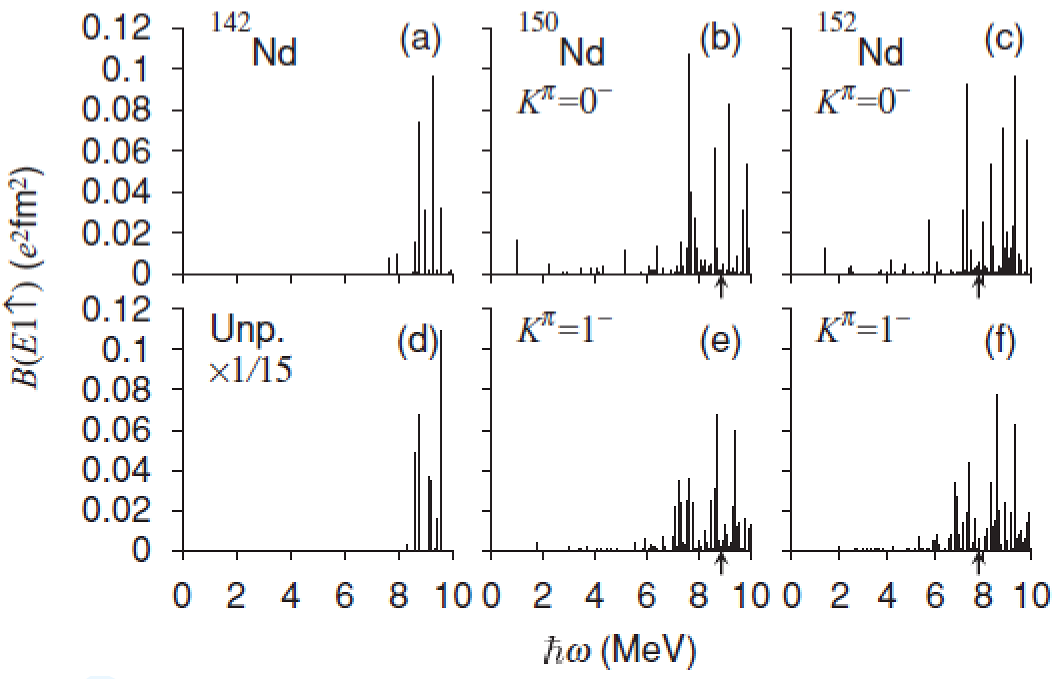


Pygmy dipole resonances (PDRs) in N=82 isotones



S. Volz et al., Nucl. Phys. A779 (2006)1

QRPA calculation



Yoshida & Nakatsukasa, Phys. Rev. C83, 021304(2011)

Single-particle states:

$$X^{\mu}_{\alpha\beta\alpha'\beta'}\,(\mathrm{p}):2p_{3/2},2p_{1/2},1g_{9/2},1g_{7/2},2d_{5/2},2d_{3/2},3s_{1/2},1h_{11/2},1h_{9/2}$$

$$X^{\mu}_{\alpha\beta\alpha'\beta'}\,(\mathrm{n}):2d_{5/2},2d_{3/2},1h_{11/2},1h_{9/2},2f_{7/2},1i_{13/2}$$

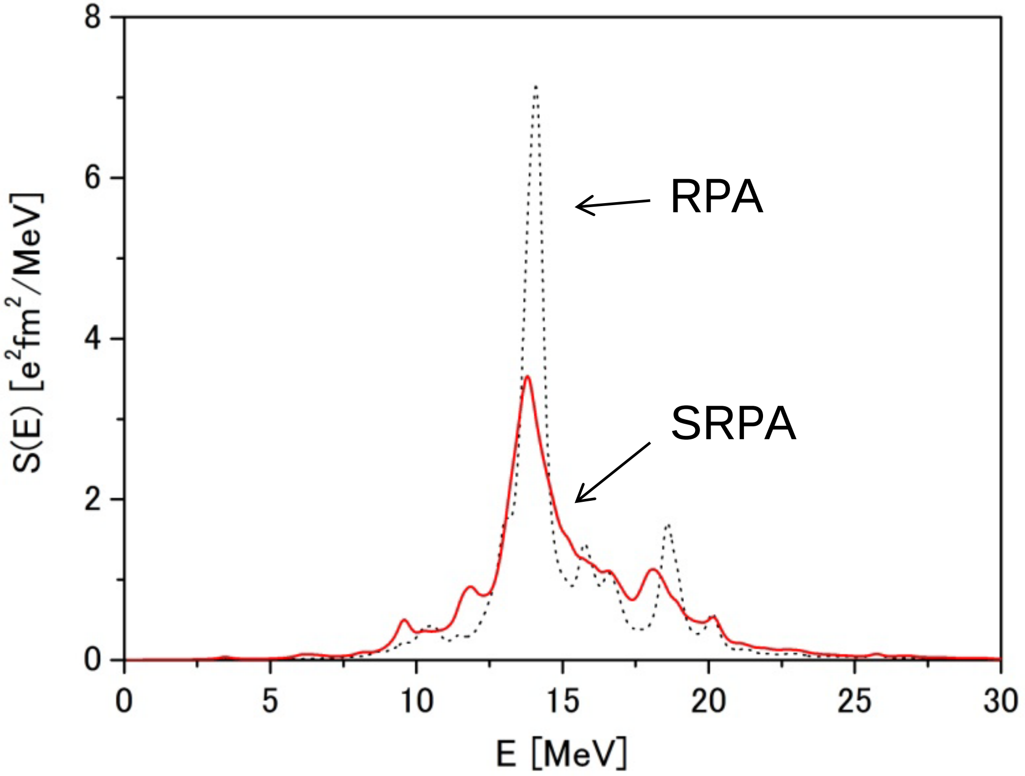
Residual interaction for matrix *D*:

$$v(\vec{r}_1-\vec{r}_2)=[t_0(1+x_0P_\sigma)+t_3\rho/2]\delta^3(\vec{r}_1-\vec{r}_2)+v_0\delta^3(\vec{r}_1-\vec{r}_2)$$

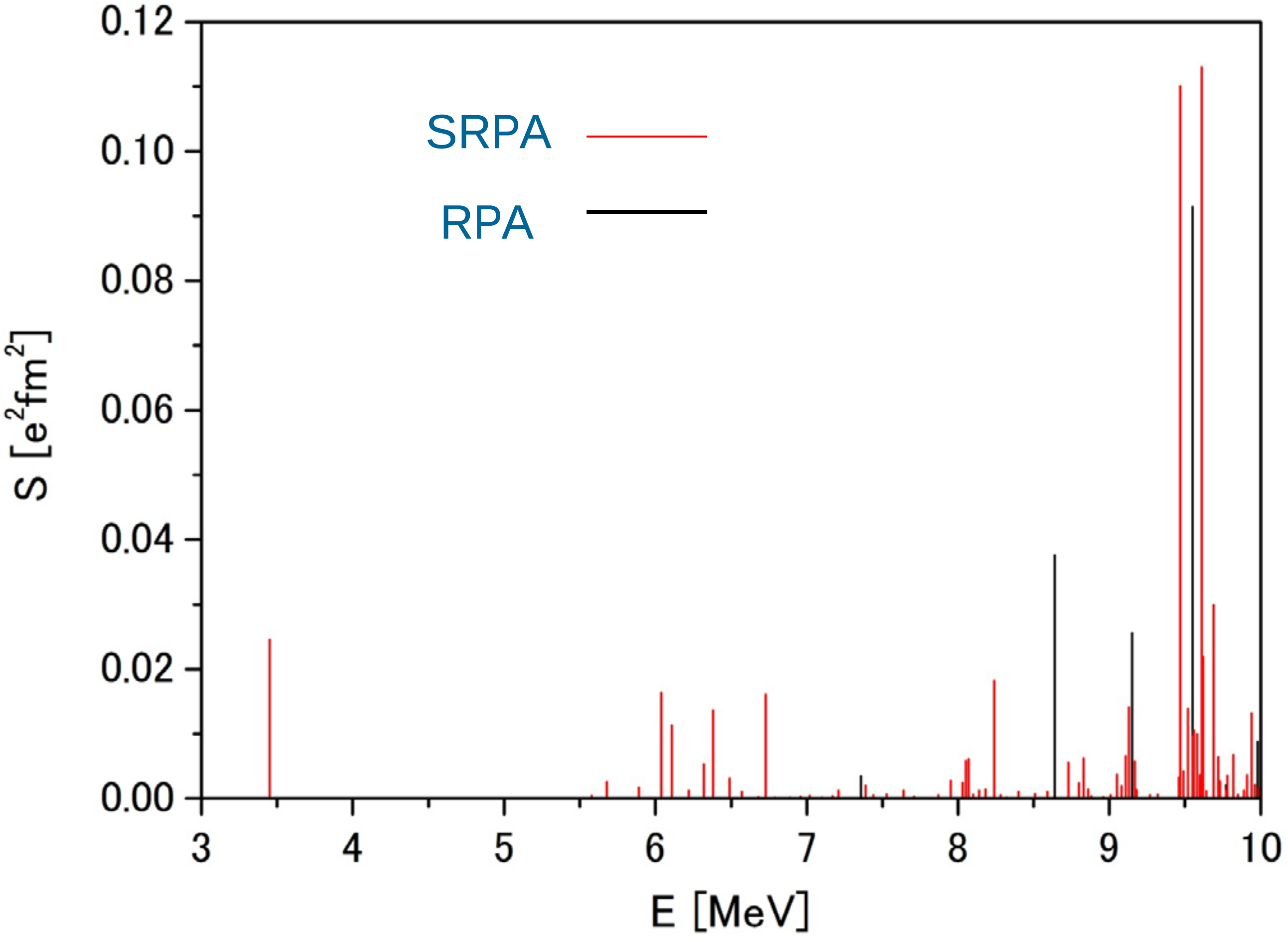
$$v_0=-570\,\mathrm{MeVfm}^3$$

Neglect of g.s. correlations $\longrightarrow$ SRPA calculation

Dipole response in ^{142}Nd



Dipole strength in ^{142}Nd



Dipole states below 8.5MeV (except for 1_1^-)

	$\bar{E}_x$ [MeV]		$\sum B(E1)\uparrow$ [e^2fm^2]	
Nucleus	SRPA	Exp.	SRPA	Exp.
^{138}Ba	7.57	6.49	0.360	0.681±0.119
^{140}Ce	7.12	6.28	0.369	0.308±0.059
^{142}Nd	6.99	6.07	0.366	0.184±0.031
^{144}Sm	6.60	5.69	0.345	0.208±0.035

Summary

- ERPA from TDDM was presented
  ERPA includes the effects of ground-state correlations
- Applications to
  GQR in ^{16}O
  2_1^+ states in tin isotopes
  PDRs in N=82 isotones
  show that ERPA works