Dynamics and Correlations in Exotic Nuclei Yukawa Institute for Theoretical Physics, Kyoto Univ. Sept. 23, 2011

# Real-time TDDFT for molecules and solids

K. Yabana



#### Collaborators:

Y. Shinohara T. Sugiyama Y. Kawashita T. Otobe J.-I. Iwata

K. Nobusada (QC) T. Nakatsukasa (NP) A. Rubio (CM) G.F. Bertsch (NP) Univ. Tsukuba Univ. Tsukuba Univ. Tsukuba JAEA, Kansai Univ. Tokyo

IMS RIKEN U. San Sebastian U. Washington



#### Nuclei Atoms, Molecules, Solids Composed of nucleons Electron many-body systems $10^{-10}$ m $10^{-15}$ m Size 1MeV 1eV Energy $10^{-17}$ s $10^{-23}$ s Time $10^9 eV$ Mass $5 \times 10^5 eV$ Interaction Nuclear force Coulomb force (Strong interaction) **Statistics** Fermion Fermion

# **Time-Dependent Density Functional Theory**

Successful for quantitative description of many-fermion dynamics

Nuclei (nucleon dynamics)

Atoms, Molecules, Solids (electron dynamics)

Linear response regime

- Giant resonances ( (Q)RPA )

Low-lying electronic excitaiton in moleculesOptical response of molecules and solids

#### Nonlinear regime, Initial value problem

- Heavy ion collision

- Laser science (Intense and ultra-short laser pulse) History: (TD)DFT in nuclear and electronic systems

#### **Nuclear Physics**

1970 Density matrix expansion Skyrme-HF calculation

1975 Continuum RPA (Shlomo-Bertsch)

1978 Real-time 3D calculation for fusion

1980 3D grid, high order finite difference

#### Electronic systems

1965 Hohenberg-Kohn Kohn-Sham

1980 Runge-Gross (extend HK theorem for TD)

1980 Continuum RPA (Zangwill-Soven)

1985- Atomic cluster physics

~1990 gradient correction  $\nabla \rho$ 

1994 3D grid, high order finite difference

1996 Real-time 3D calculation

Late 1990's ~ Development of Quantum chemistry method

### Continuum RPA

- Linearized TDDFT, spherical system, scattering boundary condition -

$$\delta\rho(\vec{r}) = \int d\vec{r} \, \Pi_0(\vec{r}, \vec{r}', \omega) \Biggl\{ \int d\vec{r}'' \frac{\delta h(\vec{r}')}{\delta \rho(\vec{r}'')} \delta\rho(\vec{r}'') + V(\vec{r}') \Biggr\}$$
$$\Pi_0(\vec{r}, \vec{r}', \omega) = \sum_j \phi_j^* G(\hbar\omega + \varepsilon_j) \phi_j + \phi_j G(-\hbar\omega + \varepsilon_j) \phi_j^*$$
$$G(\vec{r}, \vec{r}', E) = \langle \vec{r} | \frac{1}{E - h_0} | \vec{r}' \rangle = \sum_L \frac{u_L(r_<) v_L(r_>)}{rr'} \sum_M Y_{LM}^*(\hat{r}) Y_{LM}(\hat{r}') \Biggr\}$$





Fig. 9.1. Giant dipole resonance in <sup>16</sup>O. Dashed line: experimental; solid line, continuum RPA theory (Shlomo and Bertsch (1975)).

Shlomo, Bertsch 1975



Photoabsorption of

Giant Resonance in Metallic clusters (Mie plasmon)



$$\begin{cases} -\frac{\hbar^2}{2m}\vec{\nabla}^2 + \sum_a V_{ion}(\vec{r} - \vec{R}_a) + e^2 \int d\vec{r} \cdot \frac{n(\vec{r}, t)}{|\vec{r} - \vec{r}'|} + \mu_{xc}(n(\vec{r}, t)) + V_{ext}(\vec{r}, t) \\ & n(\vec{r}, t) = \sum_i \left| \psi_i(\vec{r}, t) \right|^2 \end{cases}$$

# Important residual interaction - Dynamical Screening Effect



## History: (TD)DFT in nuclear and electronic systems

#### **Nuclear Physics**

1970 Density matrix expansion Skyrme-HF calculation

1975 Continuum RPA (Shlomo-Bertsch)

1978 Real-time 3D calculation for fusion

1980 3D grid, high order finite difference

#### Electronic systems

1965 Hohenberg-Kohn Kohn-Sham

1980 Runge-Gross (extend HK theorem for TD)

1980 Continuum RPA (Zangwill-Soven)

1985- Atomic cluster physics

~1990 gradient correction  $\nabla \rho$ 

1994 3D grid, high order finite difference

1996 Real-time 3D calculation

Late 1990's ~ Development of Quantum chemistry method Nonlinear regime: Initial value problem

# Nuclear fusion reaction of ${}^{16}\text{O}{-}^{16}\text{O}$ Spatial grid: 30x28x16 ( $10^{-15}$ m), Time-step $4x10^2$ ( $10^{-22}$ s) H. Flocard, S.E. Koonin, M.S. Weiss, Phys. Rev. 17(1978)1682.



FIG. 2. Contour lines of the density integrated over the coordinate normal to the scattering plane for an  ${}^{16}O + {}^{16}O$  collision at  $E_{1ab} = 105$  MeV and incident angular momentum  $L = 13\hbar$ . The times t are given in units of  $10^{-22}$  sec.

## History: (TD)DFT in nuclear and electronic systems

#### **Nuclear Physics**

1970 Density matrix expansion Skyrme-HF calculation

1975 Continuum RPA (Shlomo-Bertsch)

1978 Real-time 3D calculation for fusion

1980 3D grid, high order finite difference

Electronic systems

1965 Hohenberg-Kohn Kohn-Sham

1980 Runge-Gross (extend HK theorem for TD)

1980 Continuum RPA (Zangwill-Soven)

1985- Atomic cluster physics

~1990 gradient correction  $\nabla \rho$ 

1994 3D grid, high order finite difference

1996 Real-time 3D calculation

Late 1990's ~ Development of Quantum chemistry method

## TDDFT in Web of Science



## History: (TD)DFT in nuclear and electronic systems

### **Nuclear Physics**

1970 Density matrix expansion Skyrme-HF calculation

1975 Continuum RPA (Shlomo-Bertsch)

1978 Real-time 3D calculation for fusion

1980 3D grid, high order finite difference

#### Electronic systems

1965 Hohenberg-Kohn Kohn-Sham

1980 Runge-Gross (extend HK theorem for TD)

1980 Continuum RPA (Zangwill-Soven)

1985- Atomic cluster physics

~1990 gradient correction  $\nabla \rho$ 

1994 3D grid, high order finite difference

1996 Real-time 3D calculation

Late 1990's ~ Development of Quantum chemistry method Linear polarizability from real-time TDDFT calculation

$$\begin{cases} -\frac{\hbar^{2}}{2m}\vec{\nabla}^{2} + \sum_{a}V_{ion}(\vec{r} - \vec{R}_{a}) + e^{2}\int d\vec{r} \cdot \frac{n(\vec{r}', t)}{|\vec{r} - \vec{r}'|} + \mu_{xc}(n(\vec{r}, t)) + V_{ext}(\vec{r}, t) \\ N(\vec{r}, t) = \sum_{i}\left|\psi_{i}(\vec{r}, t)\right|^{2} \end{cases}$$

Basic ideaK. Yabana, G.F. Bertsch, Phys. Rev. B54, 4484 (1996)<br/>K. Yabana et.al, phys.stat.sol.(b)243, 1121 (2006)Applied electric field: $V_{ext}(\vec{r},t) = eE(t)z$ Induced polarization: $p(t) = \int d\vec{r} z n(\vec{r},t) = \int dt' \alpha(t-t')E(t')$ Frequency dep. polarizability: $\alpha(\omega) = \int dt e^{i\omega t} \alpha(t) = \frac{\int dt e^{i\omega t} p(t)}{\int dt e^{i\omega t} E(t)}$ 

Simplest choice:  $E(t) \propto \delta(t)$  then,  $\alpha(\omega) \propto \int dt e^{i\omega t} p(t)$ 

Linear response in real-time: Hit the molecule and see response.

 $V_{ext}(\vec{r},t) \propto \delta(t) z$ 



Ethylene ( $C_2H_4$ ) molecule



#### Oscillator strength distribution from real-time TDDFT

K. Yabana, Y. Kawashita, T. Nakatsukasa, J.-I. Iwata, Charged Particle and Photon Interactions with Matter: Recent Advances, Applications, and Interfaces Chapter 4, Taylor & Francis, 2010.



$$\begin{cases} -\frac{\hbar^2}{2m}\vec{\nabla}^2 + \sum_a V_{ion}(\vec{r} - \vec{R}_a) + e^2 \int d\vec{r} \cdot \frac{n(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} + \mu_{xc}(n(\vec{r})) \\ \rightarrow 0 \quad (r \to \infty) \qquad \rightarrow -\frac{e^2}{r} \quad (r \to \infty) \end{cases}$$
  
LDA cannot describe correct asymptotic behavior (self-interaction problem)

+e

 $\rightarrow -\frac{e^2}{r}$   $(r \rightarrow \infty)$ 

Nonlocal Fock potential has correct form

Here we employ van Leeuwen – Baerends potential (LB94)

$$v_{xc}^{\sigma}(\vec{r}) = -\beta n_{\sigma}^{1/3}(\vec{r}) \frac{x_{\sigma}^{2}}{1+3\beta x_{\sigma} \sinh^{-1}(x_{\sigma})} \rightarrow -\frac{1}{r} \quad (r \rightarrow \infty) \qquad x_{\sigma} = \frac{|\nabla n_{\sigma}|}{n_{\sigma}^{4/3}}$$

Asymptotically correct behavior at large distance. HOMO energy = - IP

 $\ln e^{-\alpha r} \propto r$ 

#### Linear response in crystalline solid



For periodic Hamiltonian, we may apply Bloch's theorem

$$\psi_{nk}\left(\vec{r}+\vec{R}\right)=e^{i\vec{k}\vec{R}}\psi_{nk}\left(\vec{r}\right), \qquad h\left(\vec{r}+\vec{R}\right)=h(\vec{r})$$

Linear potential eE(t)z violates periodicity of the Hamiltonian.

We may recover periodicity by gauge transformation, employing vector potential  $\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t} \qquad \phi = eE(t)z \Leftrightarrow \vec{A} = \hat{z}e\int_{}^{t} dt' E(t')$   $i\hbar \frac{\partial}{\partial t}\psi(t) = \left[\frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}(t)\right)^{2} - e\phi(\vec{r}, t)\right]\psi(t)$ 

## Equation for vector potential: Dynamics of induced polarization



Bertsch, Iwata, Rubio, Yabana, Phys. Rev. B62(2000)7998.

### Electron dynamics in bulk silicon under intense laser pulse





#### Response to weak-field: dielectric function within TDDFT

Bertsch, Iwata, Rubio, Yabana, Phys. Rev. B62(2000)7998.



# Dielectric function of Si in TDDFT (Adiabatic LDA)



#### Quantitatively not sufficient

- Too small direct bandgap
- Lack of excitonic structure







G. Onida, L. Reining, A. Rubio, Rev. Mod. Phys. 74(2002)601.



arXiv:1107.0199 (July 1, 2011)

S. Sharma, J.K. Dewhurst, A. Sanna, E.K.U. Gross, Bootstrap approx. for the exchange-correlation kernel of time-dependent density functional theory

## TDDFT in nonlinear regime: Intense and Ultrashort Laser Pulse



Intense laser pulse on atoms and molecules induces nonlinear electron dynamics

Rescattering phenomena

- Ultrashort X-ray
- Atto-second science
- Molecular orbital tomography

- ...

Ethylene  $(C_2H_4)$  molecule



## Coulomb explosion: N<sub>2</sub> molecule under intense laser pulse

I=3.35x10<sup>15</sup>W/cm<sup>2</sup>, 27fs

N2 15 Ô -1 10 -2 -3 -4 -5 -6 N -7 -8 -10-15 -15 -10 -5 5 10 15 0 y,

"N2\_dens\_iter001.out" using 1:2:(log(\$3))

Y. Kawashita, Ph.D thesis

#### As the laser intensity increases,



time (au)

# Behavior around breakdown (1x10<sup>15</sup> W/cm<sup>2</sup>, 3.1eV, 40fs)

Initial stage < 15fs, dielectric screening  $\varepsilon(0) \approx 5.7$ 

#### Substantial excitation, 15-20fs

- phase difference between  $E_{ext}(t)$  and  $E_{tot}(t)$
- rapid increase of excited electron number and energy transfer
  - $\Rightarrow$  Dielectric breakdown

#### Metallic response, > 25 fs

 no further increase of excited electron number and energy transfer

Note: plasma frequency for 0.4/atom

$$\omega_p = \left(\frac{4\pi n_{ex}}{m\varepsilon(0)}\right) \approx 4\text{eV}$$

close to frequency of laser pulse, 3.1eV



Energy transfer from laser pulse to diamond



Two photon curve (green) Analytic theory by Keldysh (1965) (red)

## Interaction of Intense and ultrashort laser pulse with solids

We know the basic equation, but...

$$i\hbar\frac{\partial}{\partial t}\psi_{i} = \frac{1}{2m}\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}\right)^{2}\psi_{i} - e\phi\psi_{i} + \frac{\delta E_{xc}}{\delta n}\psi_{i} \qquad n = \sum_{i}\left|\psi_{i}\right|^{2}$$
$$\frac{1}{c^{2}}\frac{\partial^{2}\vec{A}}{\partial t^{2}} - \vec{\nabla}^{2}\vec{A} = \frac{4\pi}{c}\vec{j} \qquad \vec{\nabla}^{2}\phi = -4\pi\{en_{ion} - en_{e}\}$$

Light propagation in matter described by Maxwell equation

$$E(\vec{r},t), \quad B(\vec{r},t)$$



Electron dynamics described by time-dep. Kohn-Sham eq.  $\psi_i(\vec{r},t)$ 



For weak electromagnetic wave, we may apply perturbation theory for electron dynamics to obtain dielectric function  $\varepsilon(\omega)$ .

Then, Schroedinger and Maxwell equations decouple.

## Interaction of Intense and ultrashort laser pulse with solids

We know the basic equation, but...

$$i\hbar\frac{\partial}{\partial t}\psi_{i} = \frac{1}{2m}\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}\right)^{2}\psi_{i} - e\phi\psi_{i} + \frac{\delta E_{xc}}{\delta n}\psi_{i} \qquad n = \sum_{i}|\psi_{i}|^{2}$$
$$\frac{1}{c^{2}}\frac{\partial^{2}\vec{A}}{\partial t^{2}} - \vec{\nabla}^{2}\vec{A} = \frac{4\pi}{c}\vec{j} \qquad \vec{\nabla}^{2}\phi = -4\pi\{en_{ion} - en_{e}\}$$

Light propagation in matter described by Maxwell equation

$$E(\vec{r},t), \quad B(\vec{r},t)$$



Electron dynamics described by time-dep. Kohn-Sham eq.  $\psi_i(\vec{r},t)$ 



# wave length [µm]

Electron dynamics [nm]

For intense electromagnetic field,  $D \neq \varepsilon(\omega)E$ . We must solve "coupled Maxwell + Schroedinger eq". We also note that there are two different spatial scales, "multi-scale problem" Coupled Maxwell + TDDFT multi-scale simulation

- 1D propagation of laser pulse incident normally on Si surface -



Laser pulse on Si : Maxwell-TDDFT multi-scale calculation

Weak pulse, linear response regime

 $I = 10^{10} W/cm^2$ 



Laser pulse on Si : Maxwell-TDDFT multi-scale calculation

Intense pulse, nonlinear regime

I=5 x  $10^{12}$ W/cm<sup>2</sup>



# Computational aspects



k-points :  $8^3$  (reduced by symmetry)

Macroscopic (Maxwell) spatial grid: 256

Time step (commom) = 16,000

1024 Cores, 15 hours @ ISSP, Univ. Tokyo

#### Formation of electron-hole plasma at the surface of Si (under progress)



#### Related measurement:

## "Generation of dense electron-hole plasma in silicon"

K. Sokoowski-Tinten, D. von der Linde, Phys. Rev. B61, 2643 (2000)



## Summary

#### Real-space, real-time TDDFT calculation

useful for linear and nonlinear dynamics of condensed many-fermion systems, isolated and periodic systems.

#### Linear response regime

- accurate description for oscillator strength distribution

#### Nonlinear electron dynamics in ultrashort and ultraintense laser field

- interaction of intense and ultrashort laser pulse with matter
- propagation of light: Maxwell + TDDFT multi-scale simulation

Future problems

- Collision effect is important,
  - how to incorporate in systems with gap; Kadanoff-Baym eq?