Superfluidity in neutron-star crusts

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What is a neutron star?

Neutron stars are formed in the catastrophic gravitational collapse of massive stars at the endpoint of their evolution during type II supernova explosions.

Neutron stars are the **most compact stars** in the Universe:

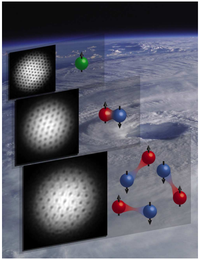
$M \sim 1 - 2M_\odot$
$R \sim 10$ km

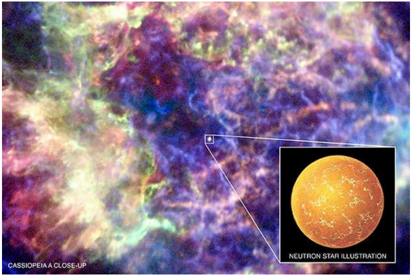
$\Rightarrow \bar{\rho} \sim 10^{14} - 10^{15}$ g.cm^{-3}

PSR J1846-0258 (Chandra)

Why studying superfluidity in neutron stars?

Neutron stars are by nature quantum systems: they contain highly degenerate matter which can therefore exhibit various phenomena observed in condensed matter physics like superfluidity.



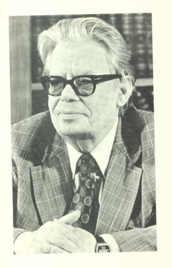


Cassiopeia A (NASA)

Superfluidity affects the evolution of neutron stars: pulsar glitches, pulsations, precession, cooling, magnetic field...

Nuclear superfluidity in neutron stars

The BCS theory was applied to nuclei by Bohr, Mottelson, Pines and Belyaev
Phys. Rev. 110, 936 (1958).
Mat.-Fys. Medd. K. Dan. Vid. Selsk. 31 , 1 (1959).

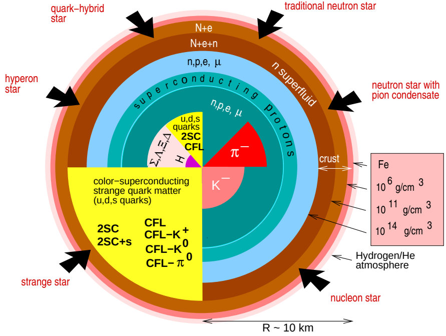


N.N. Bogoliubov, who developed a microscopic theory of superfluidity and superconductivity, was the first to explore its application to nuclear matter.
Dokl. Ak. nauk SSSR 119, 52 (1958).

Superfluidity in neutron stars was suggested long ago (before the actual discovery of neutron stars) by Migdal in 1959.
It was first studied by Ginzburg and Kirzhnits in 1964.
Ginzburg and Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006, (1964).

Superfluidity and superconductivity in neutron stars

In spite of their names, neutron stars are not only made of neutrons! As a consequence, they could contain **various kinds of superfluids and superconductors**.

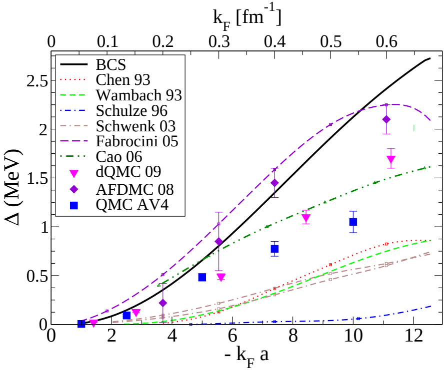


picture from F. Weber

Neutron stars are expected to contain at least a neutron superfluid in their crust.

Superfluidity in neutron-star crusts

Most microscopic calculations have been performed in **uniform neutron matter**.



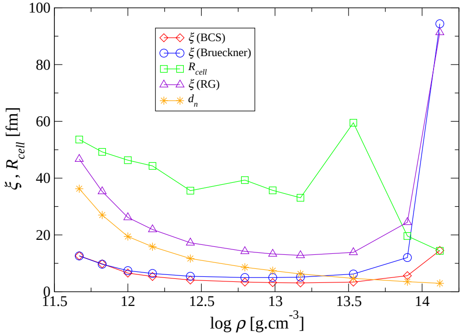
Microscopic calculations using different methods predict different density dependence of the $^1S_0$ pairing gaps.

Gezerlis & Carlson, Phys. Rev. C 81, 025803 (2010).

Is the neutron superfluid in the crust really uniform? What is the effect of the nuclei?

Effects of nuclear clusters on superfluidity?

The effects of the clusters cannot be ignored because the superfluid coherence length is smaller than the lattice spacing.



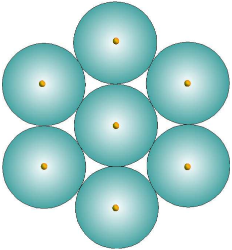
Pippard's definition

$$\xi = \frac{\hbar^2 k_{\mathrm{F}}}{\pi m_n \Delta}$$

based on the HF calculations of Negele&Vautherin

Superfluidity in neutron-star crusts with the Wigner-Seitz approximation

Superfluidity in neutron-star crusts has been already studied by several groups using the nuclear energy density functional theory in the W-S approximation.



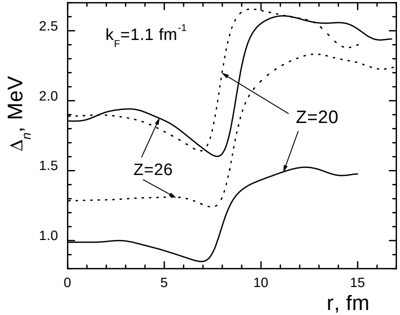
The effects of the clusters are found to be dramatic at densities $\gtrsim$ 0.03 nucleons per fm^3; in some cases the pairing gaps are almost completely suppressed

Baldo et al., Eur.Phys.J. A 32, 97(2007).

Limitations of the W-S approximation

Problems

- the results **depend very strongly on the boundary conditions** which are not unique
- the nucleon densities and pairing fields exhibit **spurious fluctuations** due to box-size effects



Spurious shell effects $\propto 1/R^2$ are very large in the bottom layers of the crust and are enhanced by the self-consistency of the calculations.

Baldo et al., Eur.Phys.J. A 32, 97(2007).

Nuclear band theory

Solution

The band theory treats consistently both the nuclear clusters and the unbound neutrons.
Chamel et al., Phys.Rev.C75(2007)055806.

"*I found to my delight that the wave differed from the plane wave of free electrons only by a periodic modulation.*" F. Bloch

$$\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} u_{\alpha\boldsymbol{k}}(\boldsymbol{r})$$

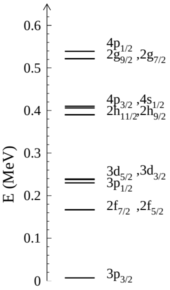
$$u_{\alpha\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{T}) = u_{\alpha\boldsymbol{k}}(\boldsymbol{r})$$

- $\alpha \rightarrow$ rotational symmetry around the lattice sites
- $\boldsymbol{k} \rightarrow$ translational symmetry of the crystal

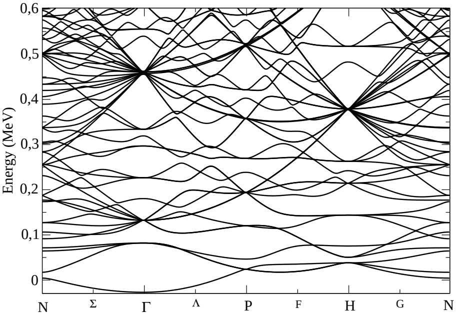
Neutron band structure

Body-centered cubic crystal of zirconium like clusters with
$N = 160$ (70 unbound) and $\bar{\rho} = 7 \times 10^{11}$ g.cm^{-3}

W-S approximation



nuclear band theory



Chamel et al, Phys.Rev.C75 (2007), 055806

Anisotropic multi-band neutron superfluidity

In the dense layers of the crust, spatial inhomogeneities are small and the Hartree-Fock-Bogoliubov equations can thus be decoupled into the BCS equations

$$\Delta_{\alpha\boldsymbol{k}} = -\frac{1}{2}\sum_{\beta}\sum_{\boldsymbol{k'}} \bar{v}^{\rm pair}_{\alpha\boldsymbol{k}\,\alpha-\boldsymbol{k}\,\beta\boldsymbol{k'}\,\beta-\boldsymbol{k'}} \frac{\Delta_{\beta\boldsymbol{k'}}}{E_{\beta\boldsymbol{k'}}} \tanh\frac{E_{\beta\boldsymbol{k'}}}{2T}$$

$$\bar{v}^{\rm pair}_{\alpha\boldsymbol{k}\,\alpha-\boldsymbol{k}\,\beta\boldsymbol{k'}\,\beta-\boldsymbol{k'}} = \int {\rm d}^3r\, v^{\pi}[\rho_n(\boldsymbol{r}),\rho_p(\boldsymbol{r})]\,|\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})|^2|\varphi_{\beta\boldsymbol{k'}}(\boldsymbol{r})|^2$$

$$E_{\alpha\boldsymbol{k}} = \sqrt{(\varepsilon_{\alpha\boldsymbol{k}}-\mu)^2+\Delta^2_{\alpha\boldsymbol{k}}}$$

$\varepsilon_{\alpha\boldsymbol{k}}$, μ and $\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})$ are obtained from **band structure calculations**.

Chamel et al., Phys.Rev.C81,045804 (2010).

Validity of the decoupling approximation

The decoupling approximation means that

$$\int \mathrm{d}^3\boldsymbol{r}\, \varphi^*_{\alpha\boldsymbol{k}}(\boldsymbol{r})\Delta(\boldsymbol{r})\varphi_{\beta\boldsymbol{k}}(\boldsymbol{r}) \simeq \delta_{\alpha\beta} \int \mathrm{d}^3\boldsymbol{r}\, |\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})|^2 \Delta(\boldsymbol{r})$$

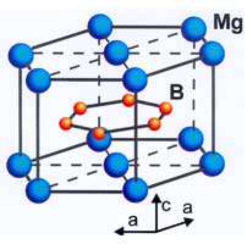
This approximation is justified whenever $\Delta(\boldsymbol{r})$ varies slowly as compared to $\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})$ for those states in the vicinity the Fermi level.

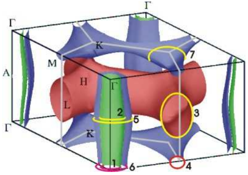
- bad for weakly bound nuclei (delocalized continuum states involved while $\Delta_q(\boldsymbol{r})$ drop to zero outside nuclei)
- good for strongly bound nuclei
- exact for uniform matter

$\Rightarrow$ reasonable for dense layers of neutron-star crusts

Analogy with terrestrial multi-band superconductors

Multi-band superconductors were first studied by Suhl et al. in 1959 but clear evidence were found only in 2001 with the discovery of MgB_2 (two-band superconductor)





In neutron-star crusts,

- the number of bands can be huge $\sim$ up to a thousand!
- both intra- and inter-band couplings must be taken into account

Neutron band structure calculation

1. The equilibrium structure of the inner crust is determined ignoring pairing using the Extended Thomas-Fermi (up to the 4th order)+Strutinsky Integral method with Skyrme functionals.

   Onsi et al., Phys.Rev.C77,065805 (2008).

Advantages of the ETFSI method

- **very fast approximation to the full Hartree-Fock method**
- avoids the difficulties related to boundary conditions but **include proton quantum shell effects** (neutron shell effects are negligibly small)

  Chamel et al.,Phys.Rev.C75(2007),055806.

2. The neutron band structure is then calculated using the ETFSI fields.

Brussels Skyrme functionals

Experimental data:

- 2149 measured atomic masses from the 2003 Atomic Mass Evaluation ($\sigma \lesssim 0.58$ MeV)
- charge radii from the 2003 AME ($\sigma \lesssim 0.03$ fm)
- compressibility $230 \leq K_v \leq 250$ MeV

Many-body calculations with realistic forces:

- isoscalar effective mass $M_s^*/M = 0.8$
- effective mass splitting $M_s^* > M_v^*$
- equation of state of symmetric and neutron matter
- ${}^1S_0$ pairing gaps of symmetric and neutron matter
- stability against spurious spin and spin-isospin instabilities

Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010).

With these constraints, these functionals are well-suited for describing neutron-star crusts.

Non-empirical pairing functional

The pairing functional is fully determined by microscopic $^1S_0$ pairing gaps $\Delta_q(\rho_n, \rho_p)$ in homogeneous matter

$$\mathcal{E}_{\rm pair}(\boldsymbol{r}) = \frac{1}{4}\sum_{q=n,p} v^{\pi q}[\rho_n(\boldsymbol{r}), \rho_p(\boldsymbol{r})]\tilde{\rho}_q(\boldsymbol{r})^2$$

$$v^{\pi q}[\rho_n, \rho_p] = -\frac{8\pi^2}{I_q(\rho_n, \rho_p)}\left(\frac{\hbar^2}{2M_q^*(\rho_n, \rho_p)}\right)^{3/2}$$

$$I_q = \sqrt{\mu_q}\left[2\log\left(\frac{2\mu_q}{\Delta_q}\right) + \Lambda\left(\frac{\varepsilon_\Lambda}{\mu_q}\right)\right]$$

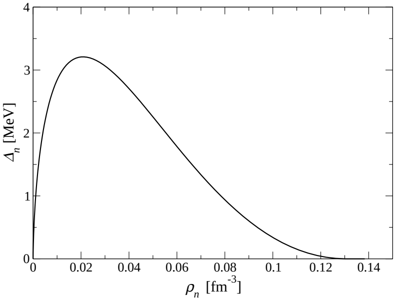
$$\Lambda(x) = \log(16x) + 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right) - 4$$

s.p. energy cutoff $\varepsilon_\Lambda = 16$ MeV above the Fermi level

Chamel, Phys. Rev. C 82, 014313 (2010)

Choice of the pairing gap

We take the ${}^1S_0$ BCS pairing gaps obtained with realistic NN potentials



Pairing gap obtained with Argonne V_{14} potential

- $\Delta(\rho)$ essentially independent of the NN potential
- $\Delta(\rho)$ completely determined by experimental ${}^1S_0$ NN phase shifts

Dean&Hjorth-Jensen,Rev.Mod.Phys.75(2003)607.

Neutron pairing gaps

ρ_n^f is the density of unbound neutrons
$\Delta_{\rm u}$ is the gap in neutron matter at density ρ_n^f
$\bar{\Delta}_{\rm u}$ is the gap in neutron matter at density ρ_n

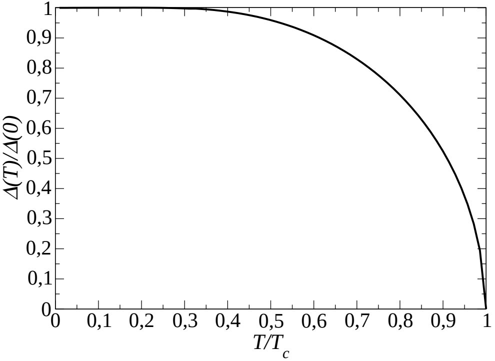
$\bar{\rho}$ [fm^{-3}]	Z	A	ρ_n^f [fm^{-3}]	$\Delta_{\rm F}$ [MeV]	$\Delta_{\rm u}$ [MeV]	$\bar{\Delta}_{\rm u}$ [MeV]
0.07	40	1258	0.060	1.44	1.79	1.43
0.065	40	1264	0.056	1.65	1.99	1.65
0.06	40	1260	0.051	1.86	2.20	1.87
0.055	40	1294	0.047	2.08	2.40	2.10
0.05	40	1304	0.043	2.29	2.59	2.33

- The presence of clusters reduces the gaps but much less than predicted by previous calculations
- Both bound and unbound neutrons contribute to the gap.

Chamel et al., Phys.Rev.C81,045804 (2010).

Average neutron pairing gap vs temperature

Example at $\bar{\rho} = 0.06\ \mathrm{fm}^{-3}$



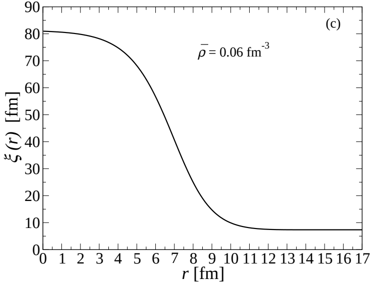
- $\Delta_{\alpha\boldsymbol{k}}(T)/\Delta_{\alpha\boldsymbol{k}}(0)$ is a universal function of T
- The critical temperature is approximately given by the usual BCS relation $T_{\mathrm{c}} \simeq 0.567\Delta_{\mathrm{F}}$

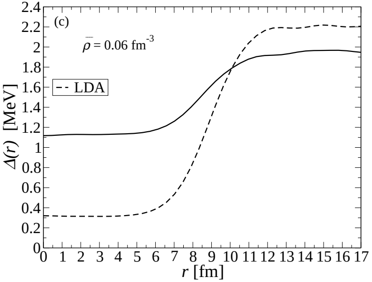
Pairing field and local density approximation

The effects of inhomogeneities on neutron superfluidity can be directly seen in the pairing field

$$\Delta_n(\boldsymbol{r}) = -\frac{1}{2} v^{\pi n}[\rho_n(\boldsymbol{r}), \rho_p(\boldsymbol{r})] \sum_{\alpha,\boldsymbol{k}}^{\Lambda} |\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})|^2 \frac{\Delta_{\alpha\boldsymbol{k}}}{E_{\alpha\boldsymbol{k}}}$$

Example: $\bar{\rho} = 0.06$ fm^{-3} and $T = 0$





Chamel et al., Phys.Rev.C81(2010)045804.

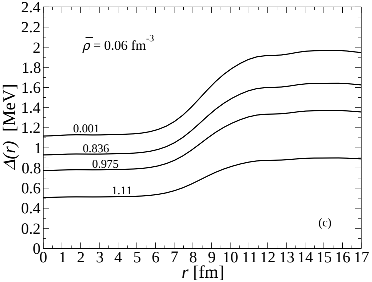
Pairing field at finite temperature

At $T > 0$, the neutron pairing field is given by

$$\Delta_n(\boldsymbol{r}) = -\frac{1}{2} v^{\pi n}[\rho_n(\boldsymbol{r}), \rho_p(\boldsymbol{r})] \sum_{\alpha,\boldsymbol{k}}^{\Lambda} |\varphi_{\alpha\boldsymbol{k}}(\boldsymbol{r})|^2 \frac{\Delta_{\alpha\boldsymbol{k}}}{E_{\alpha\boldsymbol{k}}} \tanh \frac{E_{\alpha\boldsymbol{k}}}{2T}$$

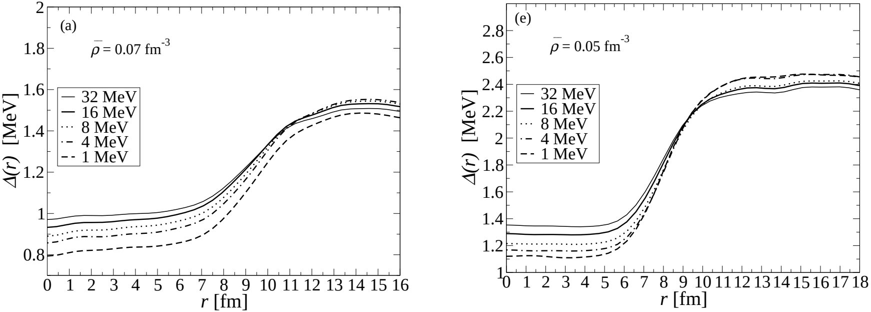
Example: $\bar{\rho} = 0.06$ fm^{-3}

The superfluid becomes more and more homogeneous as T approaches T_c



Chamel et al., Phys.Rev.C81(2010)045804.

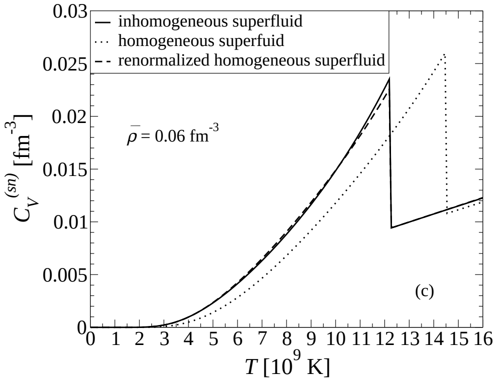
Impact of the pairing cutoff



$\bar{\rho}$ [fm^{-3}]	$\Delta_{F0}(16)$ [MeV]	$\Delta_{F0}(8)$	$\Delta_{F0}(4)$	$\Delta_{F0}(2)$	$\Delta_{F0}(1)$
0.070	1.39	1.38	1.37	1.36	1.29
0.050	2.27	2.25	2.27	2.26	2.24

Pairing gaps (hence also critical temperatures) are very weakly dependent on the pairing cutoff.

Impact on thermodynamic quantities : specific heat



- Band structure effects are small. This remains true for non-superfluid neutrons.
  Chamel et al, Phys. Rev. C 79, 012801(R) (2009)
- The renormalization of T_c comes from the density dependence of the pairing strength.

Dynamical response of the neutron superfluid in neutron-star crusts

The crust has a strong impact on the neutron superfluid hydrodynamics.

Pethick, Chamel, Reddy, Prog.Theor.Phys.Sup.186(2010)9.

$\bar{\rho}$ (fm^{-3})	ρ_n^f/ρ_n (%)	ρ_n^c/ρ_n^f (%)
0.0003	20.0	82.6
0.001	68.6	27.3
0.005	86.4	17.5
0.01	88.9	15.5
0.02	90.3	7.37
0.03	91.4	7.33
0.04	88.8	10.6
0.05	91.4	30.0
0.06	91.5	45.9
0.08	104	64.8

The density ρ_n^c of "conduction" neutrons (i.e. superfluid neutron density) can be much smaller than the density ρ_n^f of unbound neutrons!

Summary

1. The EDF theory allows for a consistent treatment of superfluid neutrons in neutron-star crusts.
2. The Brussels Skyrme functionals are constrained by experiments and N-body calculations:
   - they give an excellent fit to essentially all nuclear mass data ($\sigma \simeq 0.58$ MeV)
   - they reproduce various properties of homogeneous nuclear matter (EoS, effective masses, pairing gaps, *etc*)
3. Using the band theory of solids, we have shown that the crust affects both the static and the dynamic properties of the neutron superfluid.