Continuum particle-vibration coupling method in coordinate-space representation.

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Particle-Vibration formalism in coordinate space

Hartree-Fock

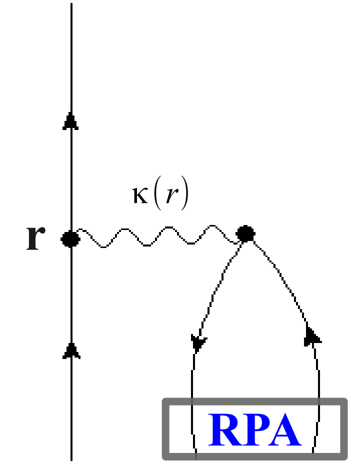
•Description of the single particle motion in a nucleus.

RPA

•Description of the vibration of the nucleus.

PVC Hamiltonian

$$\hat{H}_{PVC} = \int d\boldsymbol{r} \delta\hat{\rho}(\boldsymbol{r})\kappa(\boldsymbol{r}) \sum_{\sigma} \hat{\psi}^{\dagger}(\boldsymbol{r}\sigma)\hat{\psi}(\boldsymbol{r}\sigma)$$



•Wick's theorem

Self-energy function

$$\Sigma(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma'; \omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \kappa(\boldsymbol{r}) G(\boldsymbol{r}\sigma, \boldsymbol{r}'\sigma'; \omega - \omega') \kappa(\boldsymbol{r}') i R(\boldsymbol{r}, \boldsymbol{r}'; \omega')$$

Continuum HF Green's function

Causality!!

Continuum RPA

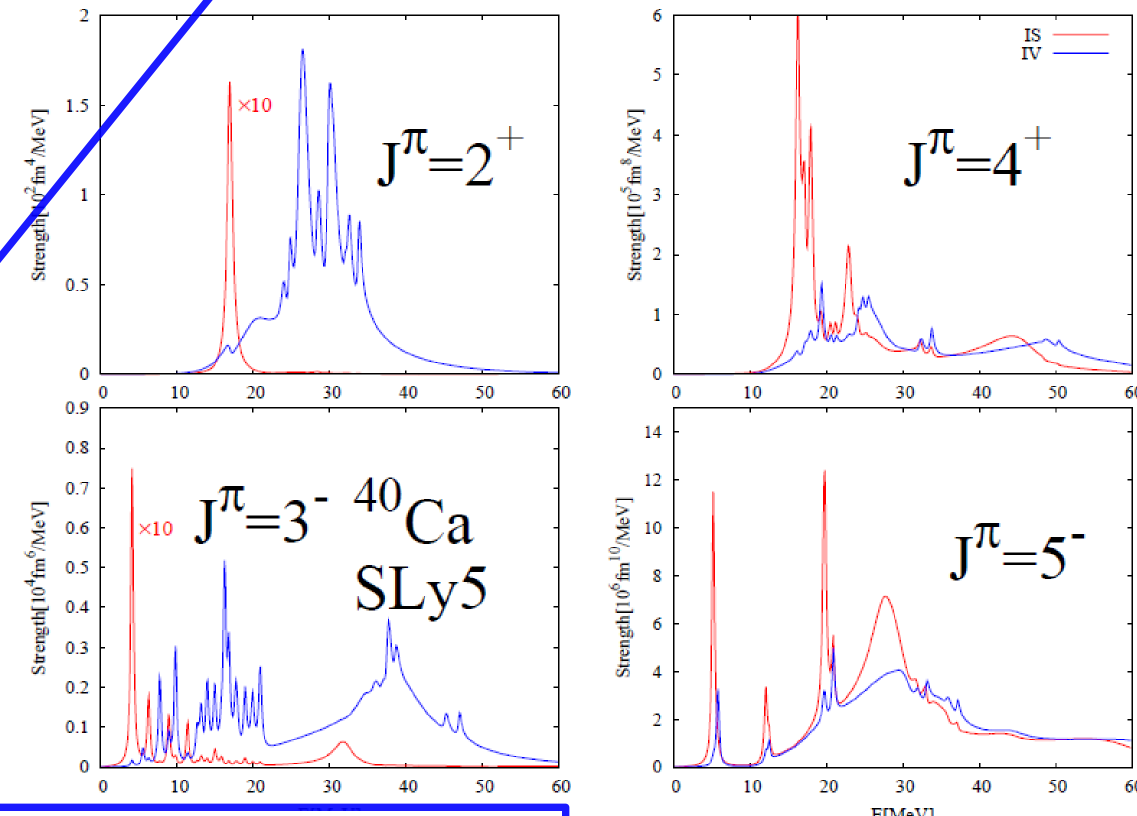
A. L. Fetter, J. D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill, 1971.

Self-energy function

$$\Sigma_{lj}(rr';\omega) = \sum_{l'j',L} \frac{|\langle lj||Y_L||l'j'\rangle|^2}{2j+1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0,l'j'}(rr';\omega-\omega') \frac{\kappa(r')}{r'^2} iR_L(rr';\omega')$$

Landau-Migdal approximation: k_F=1.33

Causal Skyrme continuum RPA response function



$$G_{0,lj}(rr';E) = \frac{1}{W(u,v)} u_{lj}(r_<;E) v_{lj}(r_>;E)$$

$$G^R_{0,lj}(rr',\omega) = G_{0,lj}(rr';\omega+i\eta)$$

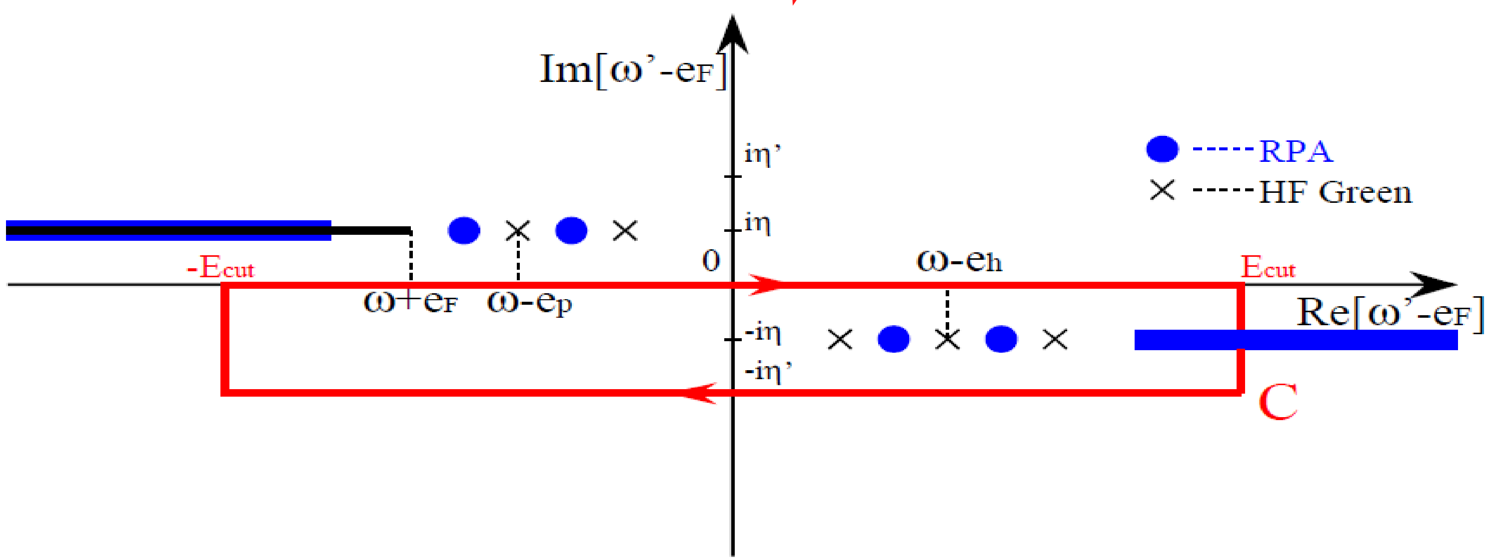
Causal Skyrme HF Green's function

$$G^C_{0,lj}(rr',\omega) = G_{0,lj}(rr';\omega+i\eta) + \sum_{n_h l_h j_h} \frac{2i\eta}{(\omega - e_{n_h l_h j_h})^2 + \eta^2} \phi_{n_h l_h j_h}(r) \phi_{n_h l_h j_h}(r')$$

Self-energy function

$$\Sigma_{lj}(rr';\omega) = \sum_{l'j',L} \frac{|\langle lj||Y_L||l'j'\rangle|^2}{2j+1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0,l'j'}(rr';\omega-\omega') \frac{\kappa(r')}{r'^2} iR_L(rr';\omega')$$

- Numerical contour integration on the complex energy plane.



Cauchy's theorem

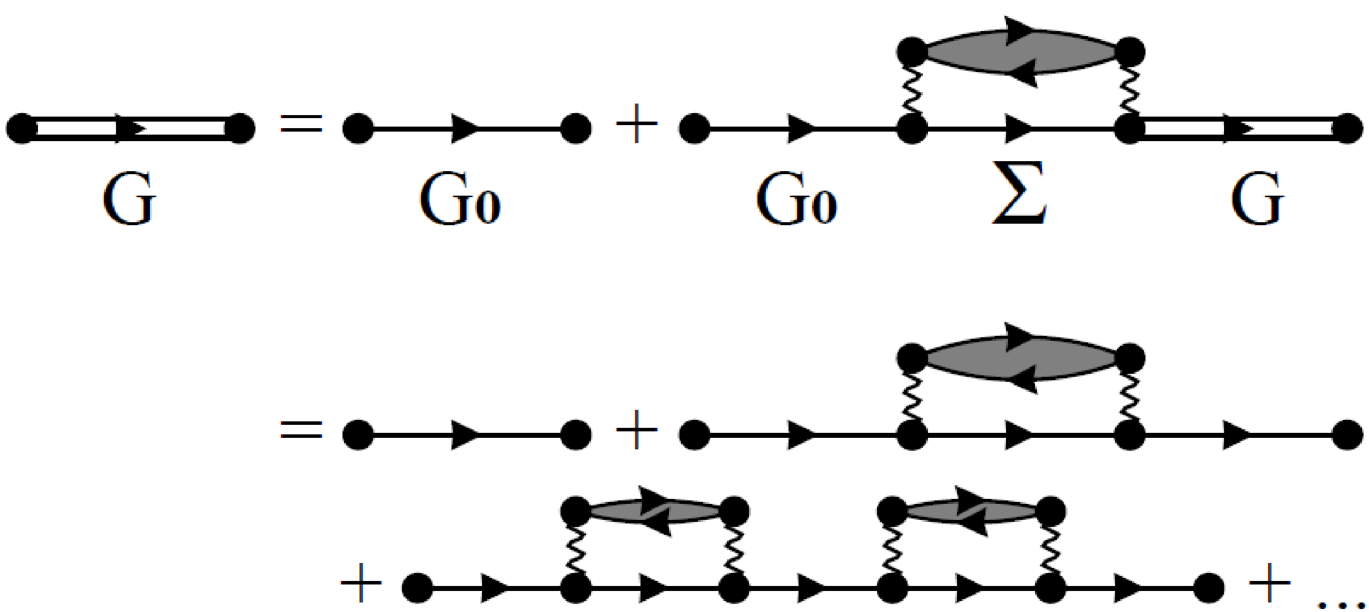
In case of the spectral representation...

$$\Sigma(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma';\omega) = \sum_{h,\nu} \frac{\phi_h(\boldsymbol{r}\sigma)\delta\rho_\nu^*(\boldsymbol{r})\kappa(\boldsymbol{r})\phi_h^*(\boldsymbol{r}'\sigma')\delta\rho_\nu(\boldsymbol{r}')\kappa(\boldsymbol{r}')}{\omega - e_h + E_\nu - i\eta} + \sum_{p,\nu} \frac{\phi_p(\boldsymbol{r}\sigma)\delta\rho_\nu(\boldsymbol{r})\kappa(\boldsymbol{r})\phi_p^*(\boldsymbol{r}'\sigma')\delta\rho_\nu^*(\boldsymbol{r}')\kappa(\boldsymbol{r}')}{\omega - e_p - E_\nu + i\eta}$$

Dyson equation

$$G_{lj}(rr';\omega) = G_{0,lj}(rr';\omega) + \iint dr_1 dr_2 \; G_{0,lj}(rr_1;\omega)\Sigma_{lj}(r_1 r_2;\omega) G_{lj}(r_2 r';\omega)$$

$$\Rightarrow \quad G(rr') = (1 - G_0\Sigma)^{-1} G_0(rr').$$



-- Level density

$$\rho_{0,lj}(\omega) = \sum_n \delta(\omega - \epsilon^{(0)}_{nlj}) \text{ for bound states.} (\omega < 0)$$

HF level density

$$\rho_{0,lj}(\omega) = \frac{\pm 1}{\pi} \int_0^R dr \mathrm{Im} G_{0,lj}(rr,\omega)$$

$$(\omega - \frac{p^2}{2m}) G_{Free} = 1.$$

HF level density*

$$\bar{\rho}_{0,lj}(\omega) = \frac{\pm 1}{\pi} \int_0^R dr \mathrm{Im}\left(G_{0,lj}(rr,\omega) - G_{Free,lj}(rr,\omega).\right)$$

$$\rho_{0,lj}(\omega) = \frac{1}{\pi} \frac{d\delta^{(0)}_{lj}}{d\omega}$$

S. Shlomo, Nucl.Phys.A539(1992),17

HF+PVC level density

$$\bar{\rho}_{lj}(\omega) = \frac{\pm 1}{\pi} \int dr \mathrm{Im}\left(G_{lj}(rr,\omega) - G_{Free,lj}(rr,\omega)\right)$$

Dyson equation

$$G_{lj}(rr';\omega) = G_{0,lj}(rr';\omega) + \iint dr_1 dr_2\, G_{0,lj}(rr_1;\omega)\Sigma_{lj}(r_1 r_2;\omega) G_{lj}(r_2 r';\omega)$$

---- $\rho_{Free,lj}(\omega) \propto \sqrt{\frac{2m}{\hbar^2}} \frac{R}{2\pi\sqrt{\omega}}$

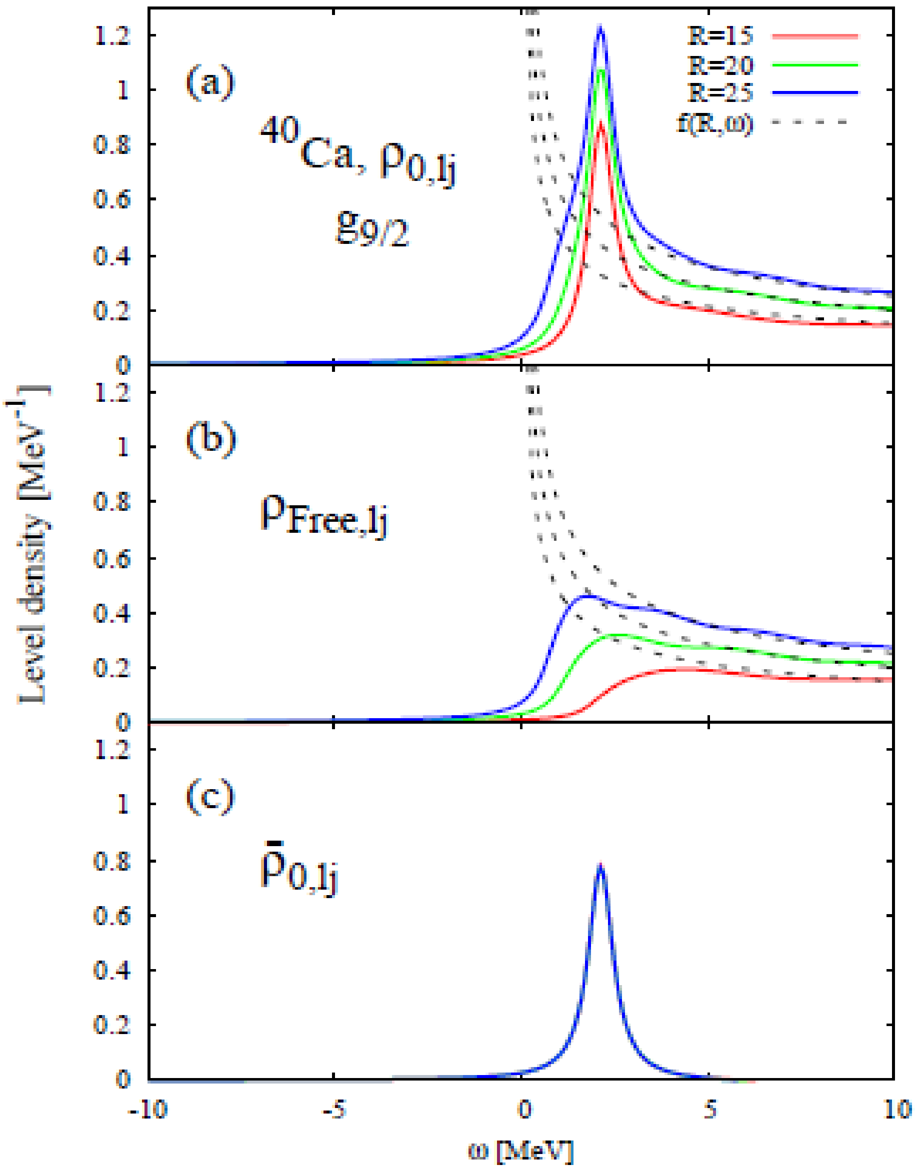
HF level density*

$$\bar{\rho}_{0,lj}(\omega) = \frac{\pm 1}{\pi} \int_0^R dr \mathrm{Im} \left(G_{0,lj}(rr,\omega) - G_{Free,lj}(rr,\omega) . \right)$$

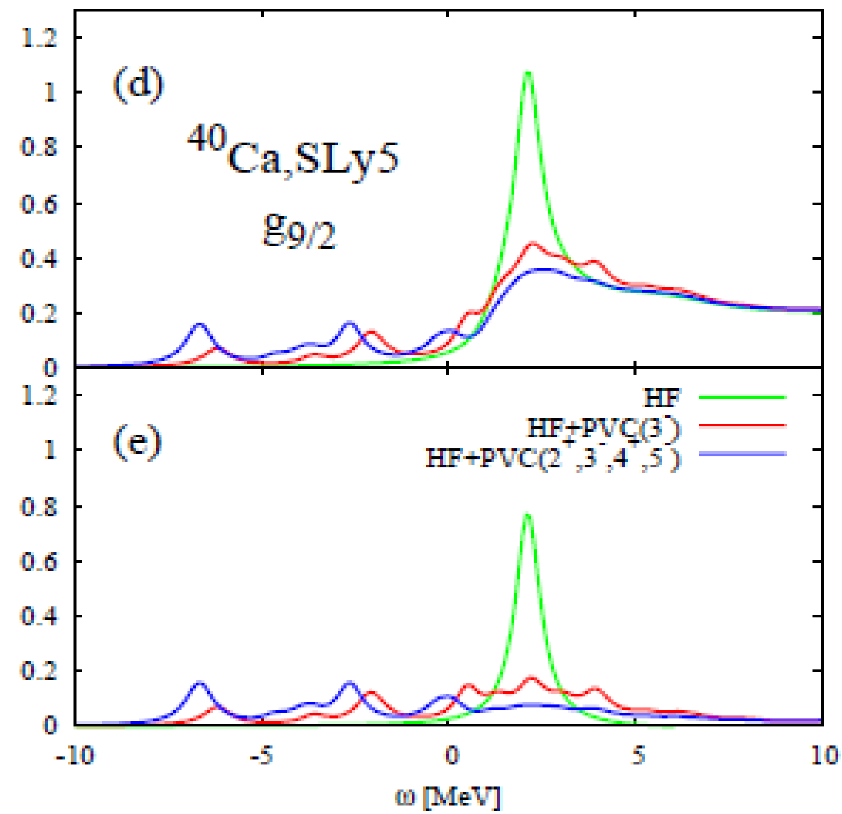
HF+PVC level density

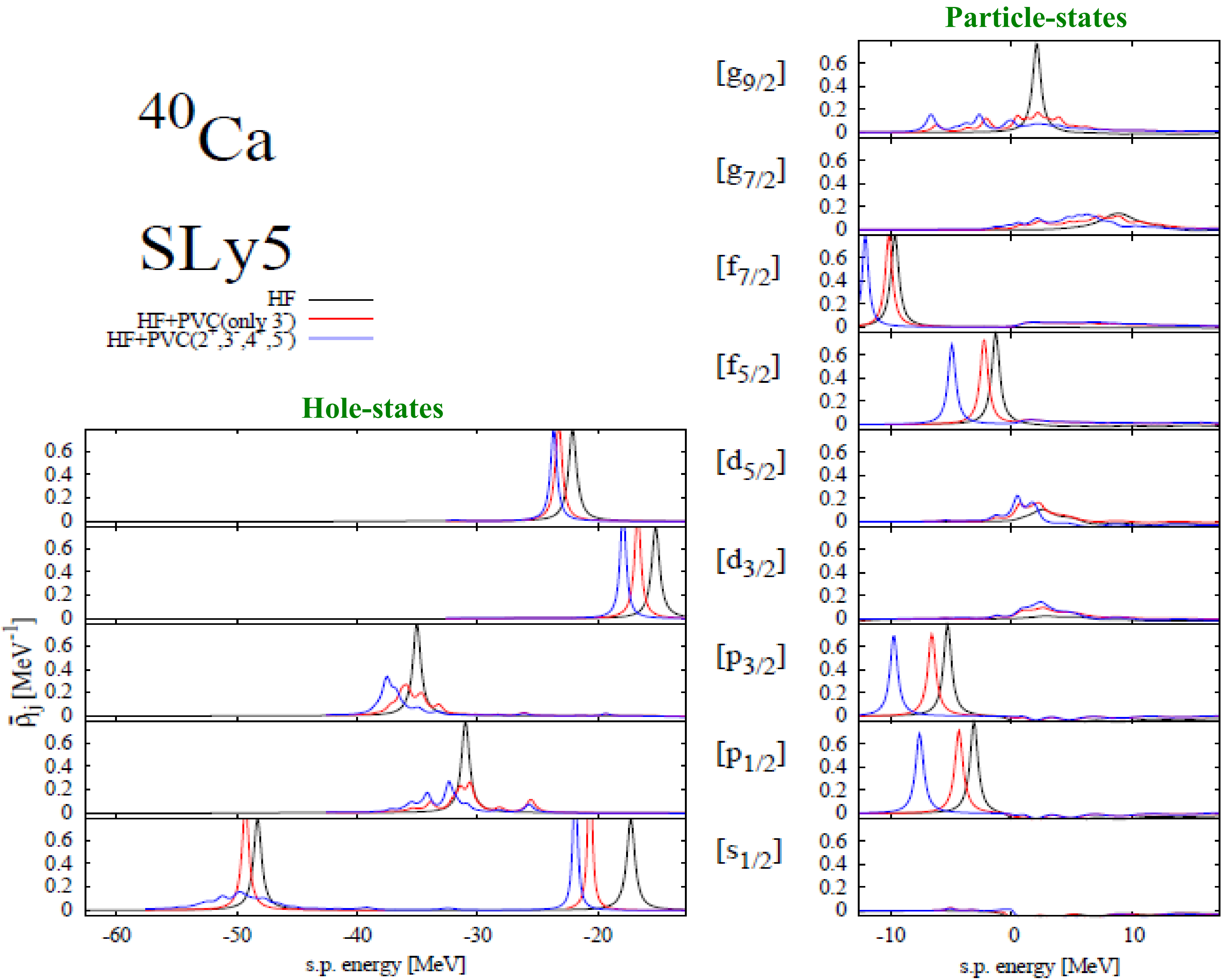
$$\bar{\rho}_{lj}(\omega) = \frac{\pm 1}{\pi} \int dr \mathrm{Im} \left(G_{lj}(rr,\omega) - G_{Free,lj}(rr,\omega) \right)$$

HF level density



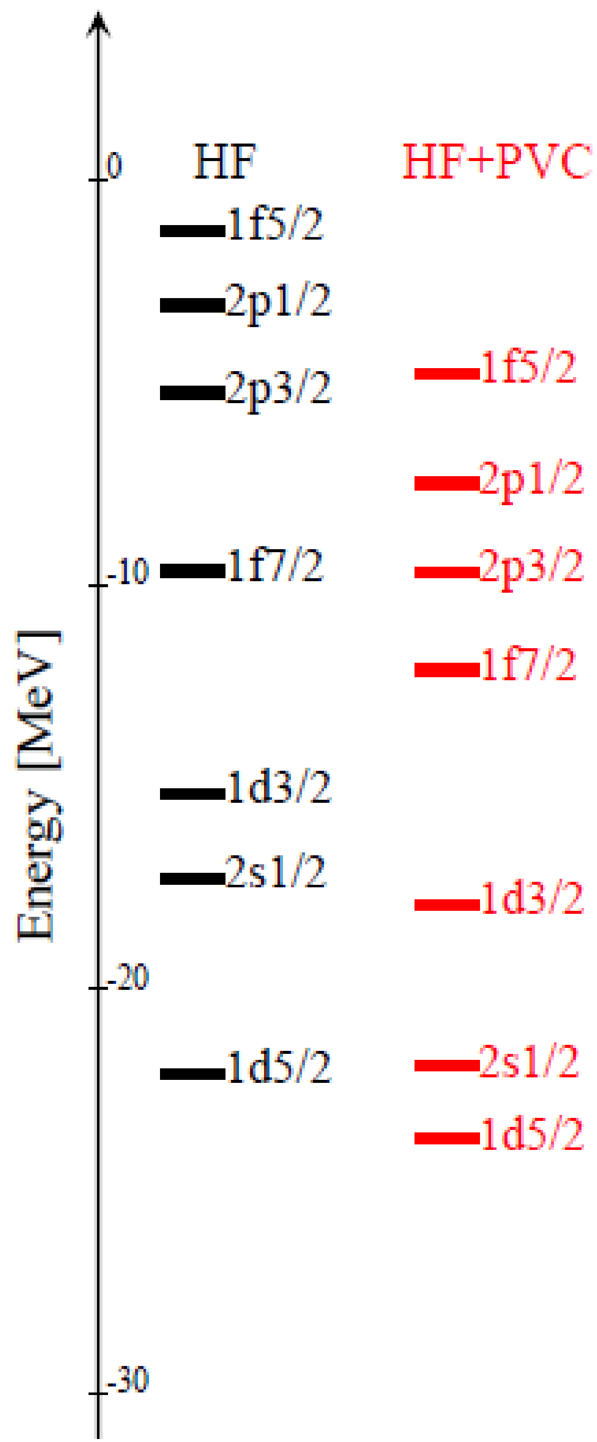
HF v.s. HF+PVC level density



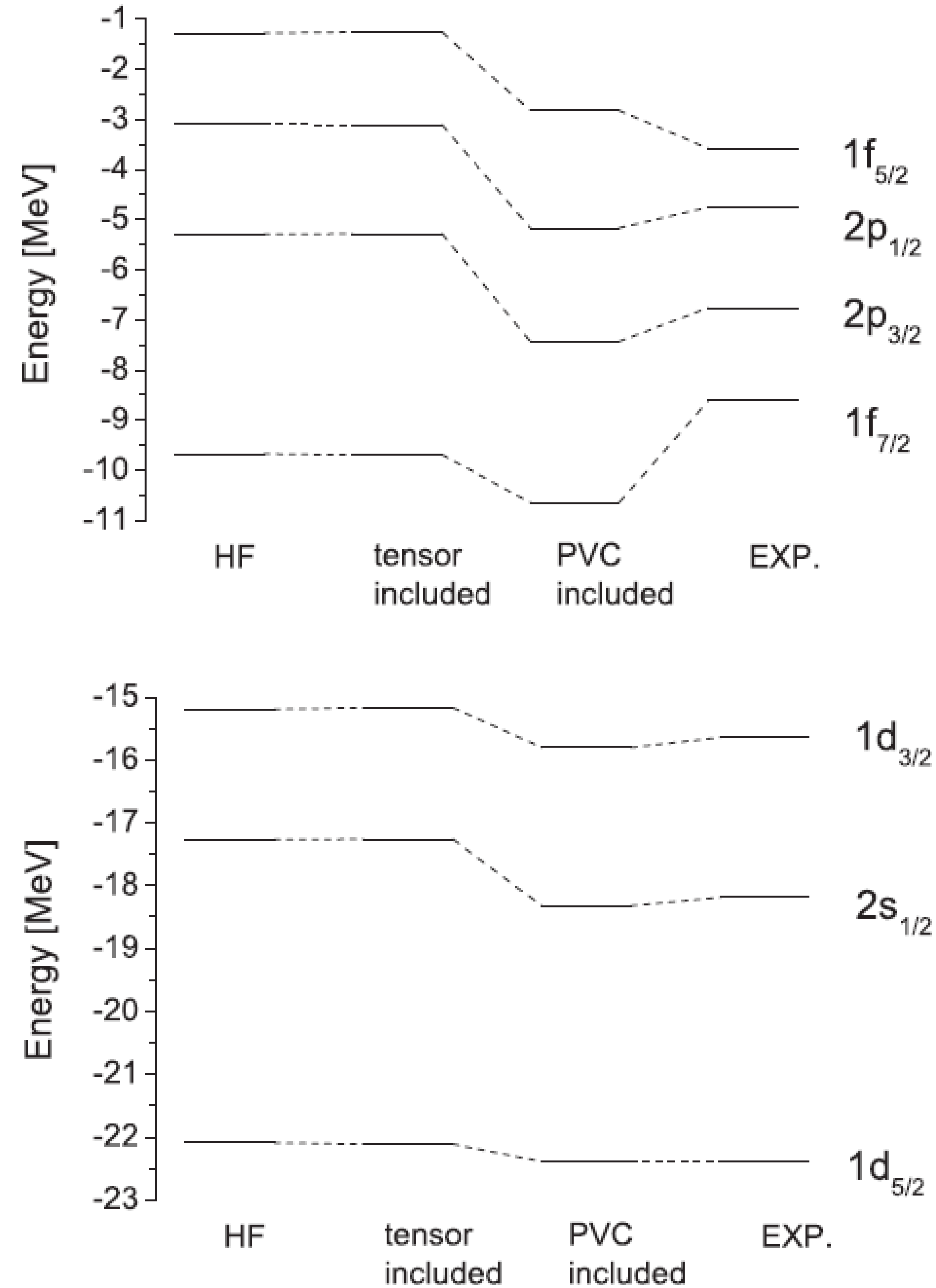


Particle-states
Hole-states
^{40}Ca
SLy5
HF
HF+PVC(only 3⁻)
HF+PVC(2⁺,3⁻,4⁺,5⁻)
[g9/2]
[g7/2]
[f7/2]
[f5/2]
[d5/2]
[d3/2]
[p3/2]
[p1/2]
[s1/2]
$\bar{\rho}_{lj}$ [MeV^{-1}]
s.p. energy [MeV]
s.p. energy [MeV]

Our results



Gianluca Colo, Hiroyuki Sagawa, and Pier Francesco Bortignon, PRC **82**, 064307 (2010)



Nuclear Data Sheets 107, 225 (2006)

Nuclear Data Sheets 94, 429 (2001)

Comparisons with the experimental data

^{40}Ca

SLy5

$E_x(^{39}\mathrm{Ca}) = e_{d3/2} - \omega$

$E_x(^{41}\mathrm{Ca}) = \omega - e_{f7/2}$

Possible configurations of the fragmentation for $d_{5/2}$

$2^+ \otimes 2s_{1/2}$, $2^+ \otimes 1d_{3/2}$

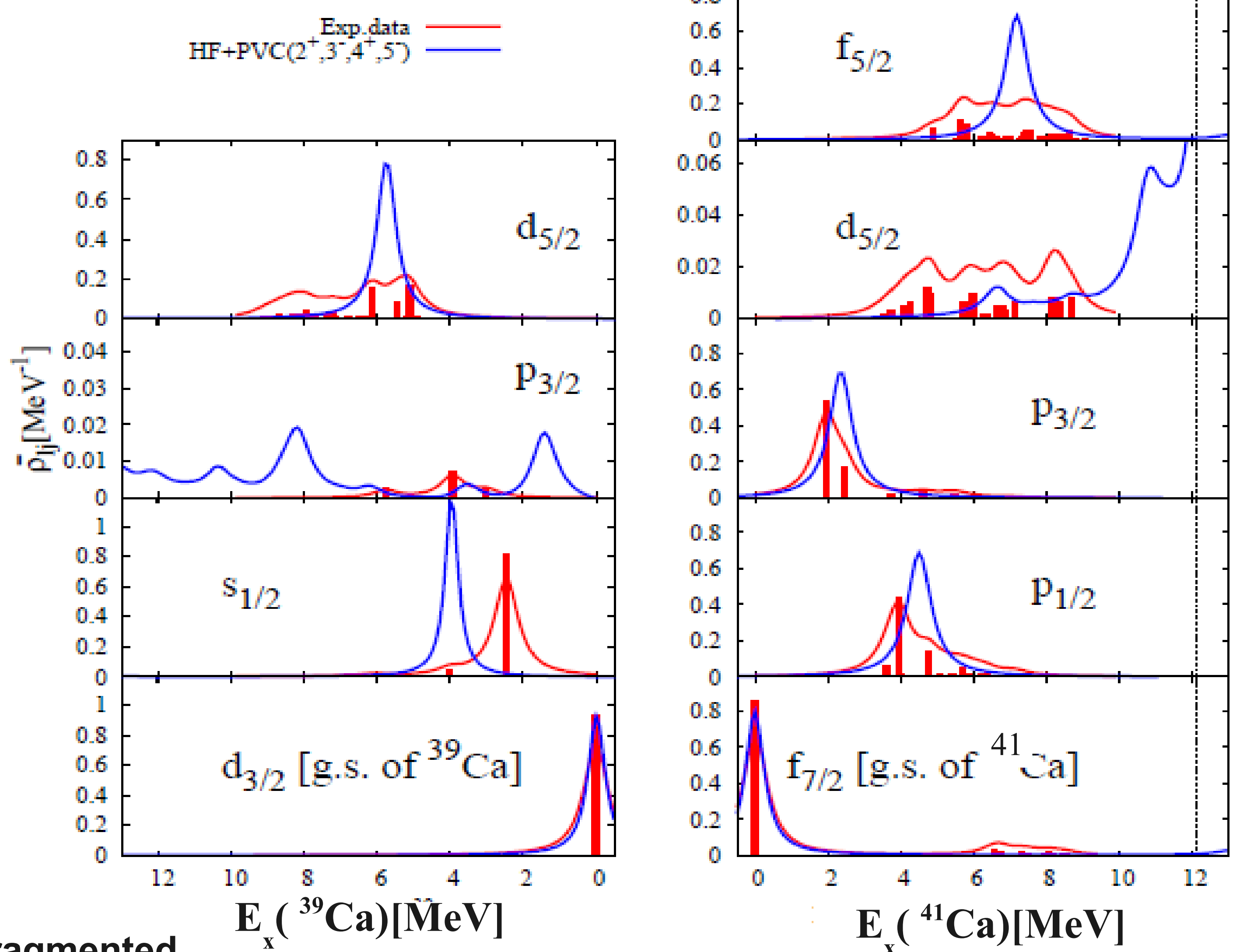
Theory(RPA):

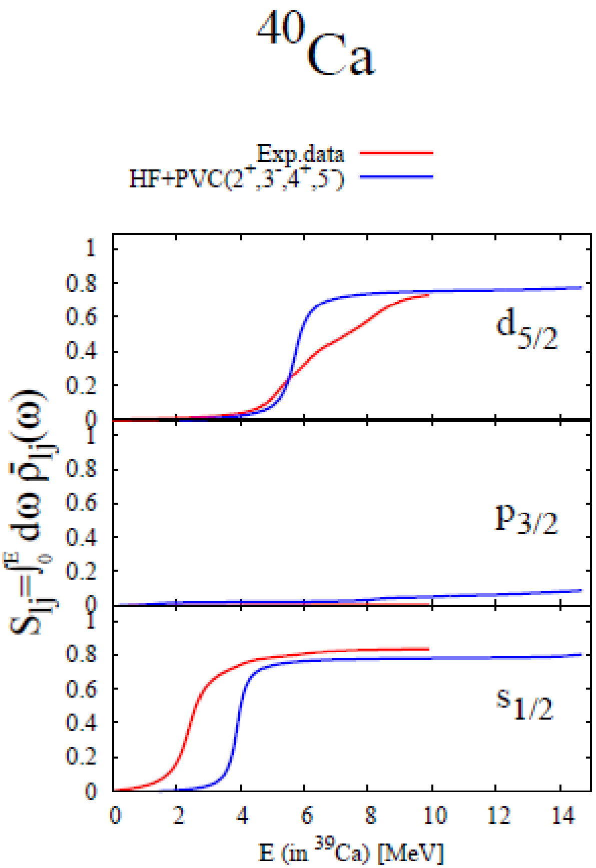
•No low-lying 2+

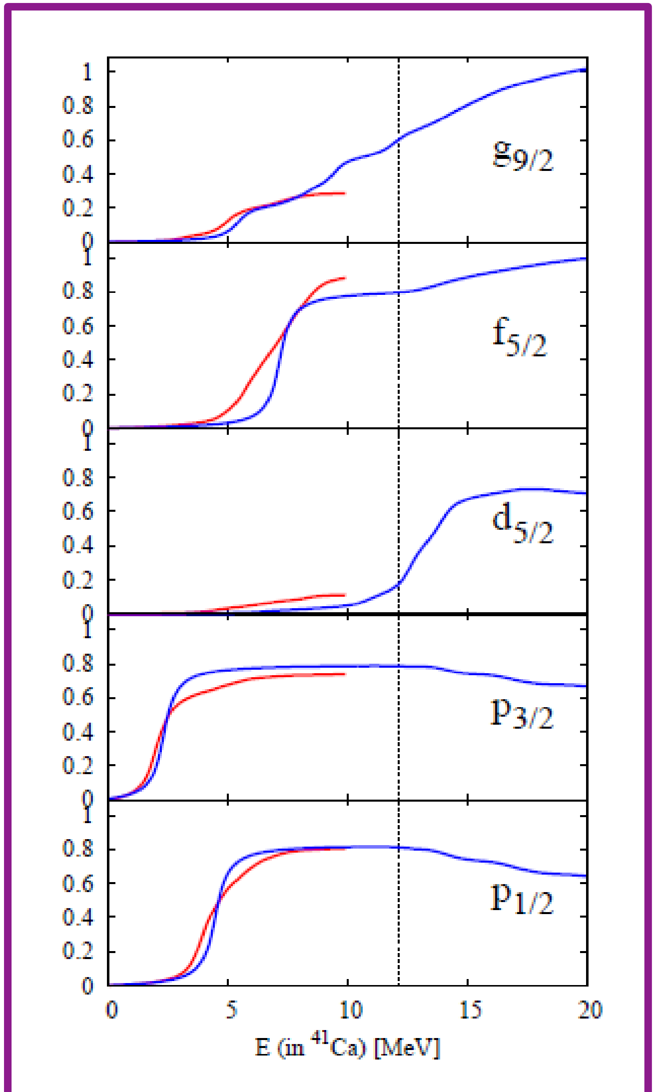
•Sharp ISGQR

Experiment:

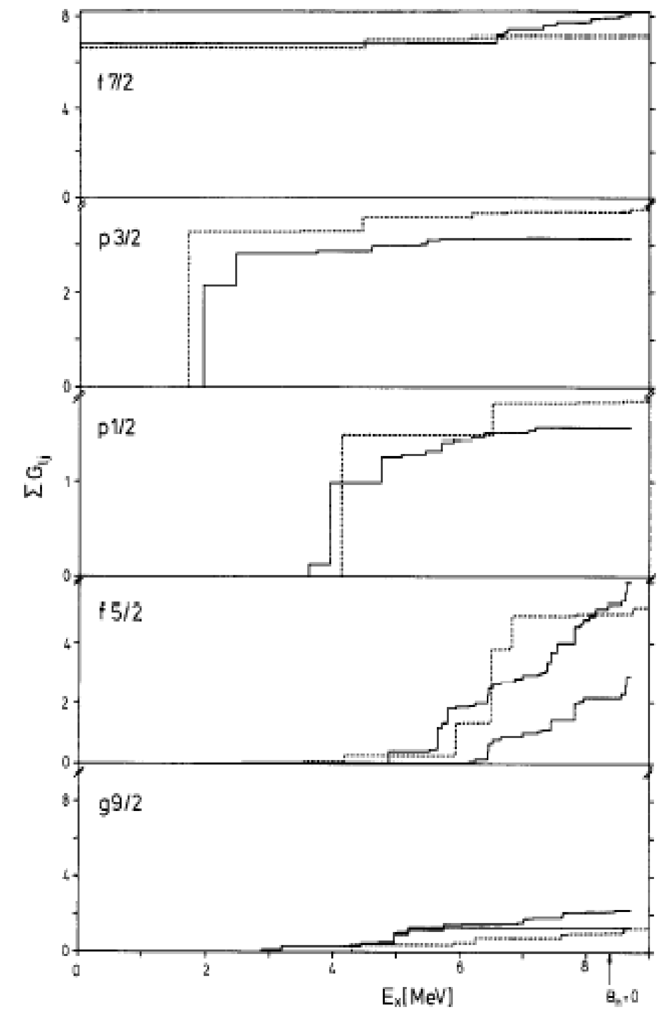
• ISGQR is very much fragmented







F.J. ECKLE', et.al.
Nuclear Physics A506 (1990) 159-195



$$S_{lj}=\int_0^E d\omega\ \bar{\rho}_{lj}(\omega)$$

Summary

- We investigated a new particle-vibration coupling method which treats the continuum properly.
  - *Key points:*
    - *Causal continuum HF Green's function and continuum RPA response function.*
    - *Contour integration on the complex energy plane.*
    - *Dyson equation in coordinate space representation.*

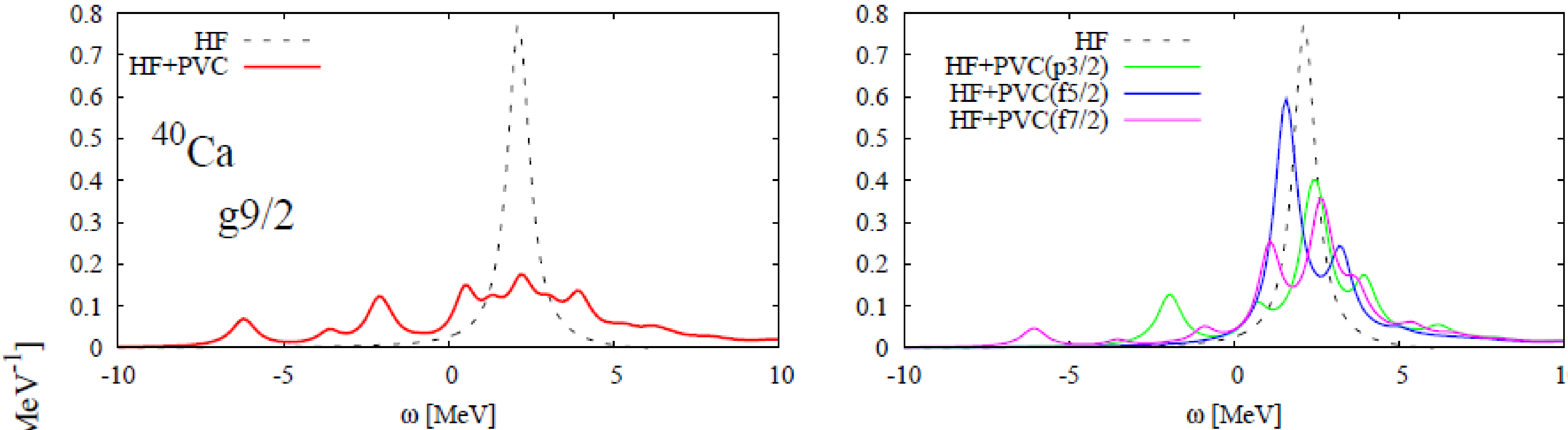
- Our formulation is based on the microscopic effective mean field theory; the Skyrme HF and Random-phase approx.(RPA).

- We showed the level density as our numerical results with SLy5 in ^{40}Ca (^{208}Pb, ^{24}O). We obtained the consistent energy-shift with the previous PVC calculations(Gianluca's results) in single-particle energies of ^{40}Ca.

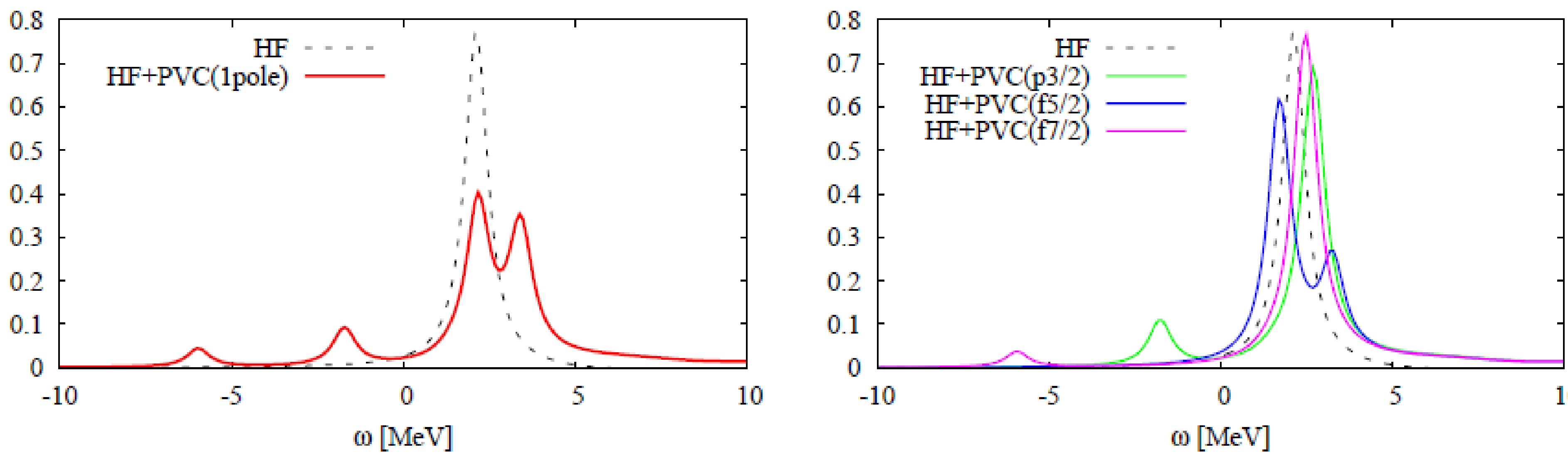
- We compared the level density defined by the Green's function with the experimental data. Our results are overall agreement with the experimental data in ^{39}Ca and ^{41}Ca.
  - Some difference($d_{5/2}$, $f_{7/2}$) maybe due to the disagreement between the RPA quadrupole phonon and the experimental data.
    → The improvement of RPA is needed?

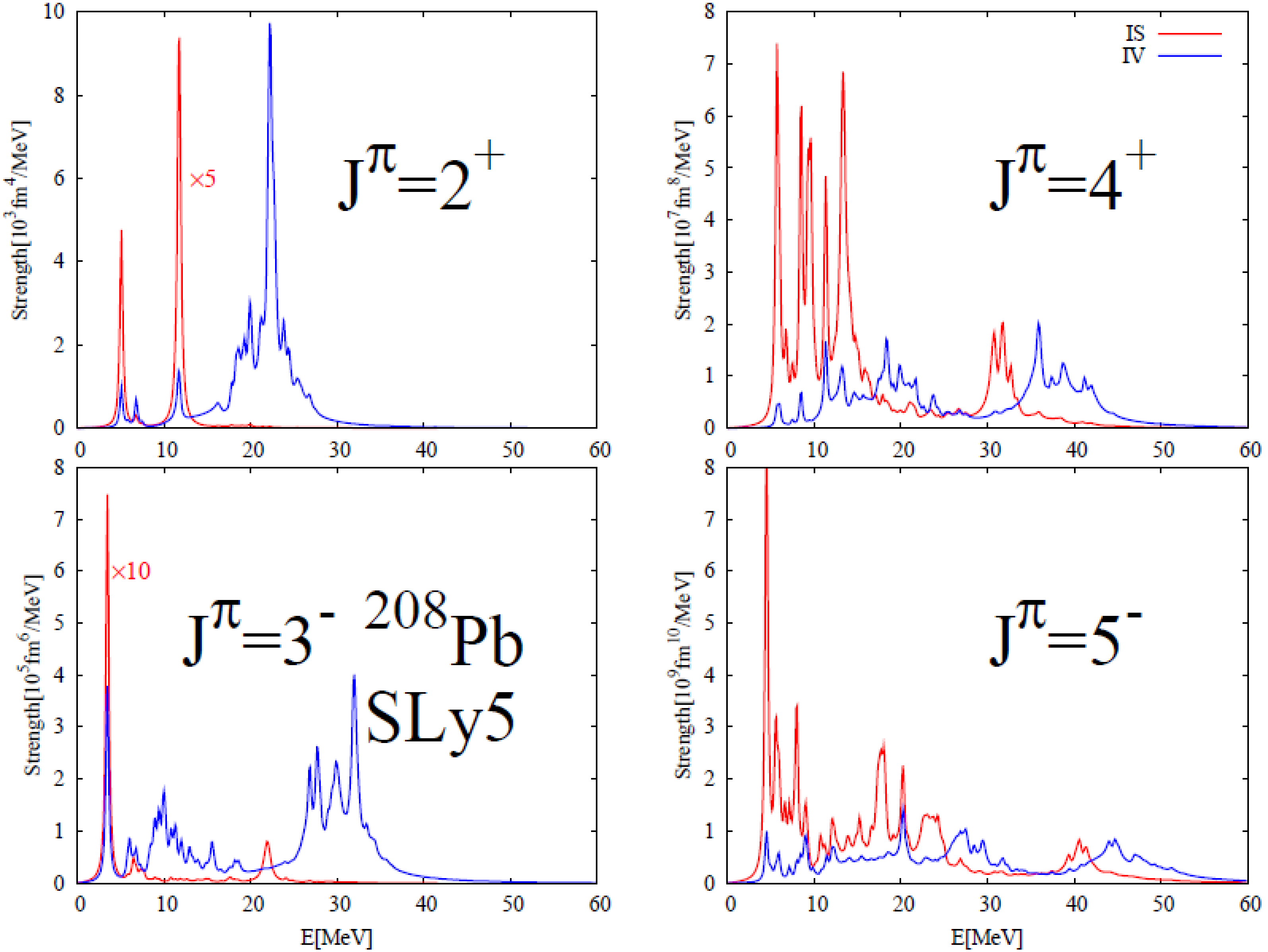
$$\Sigma_{lj}(rr';\omega) = \sum_{l'j',L} \frac{|\langle lj||Y_L||l'j'\rangle|^2}{2j+1} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0,l'j'}(rr';\omega-\omega') \frac{\kappa(r')}{r'^2} iR_L(rr';\omega')$$

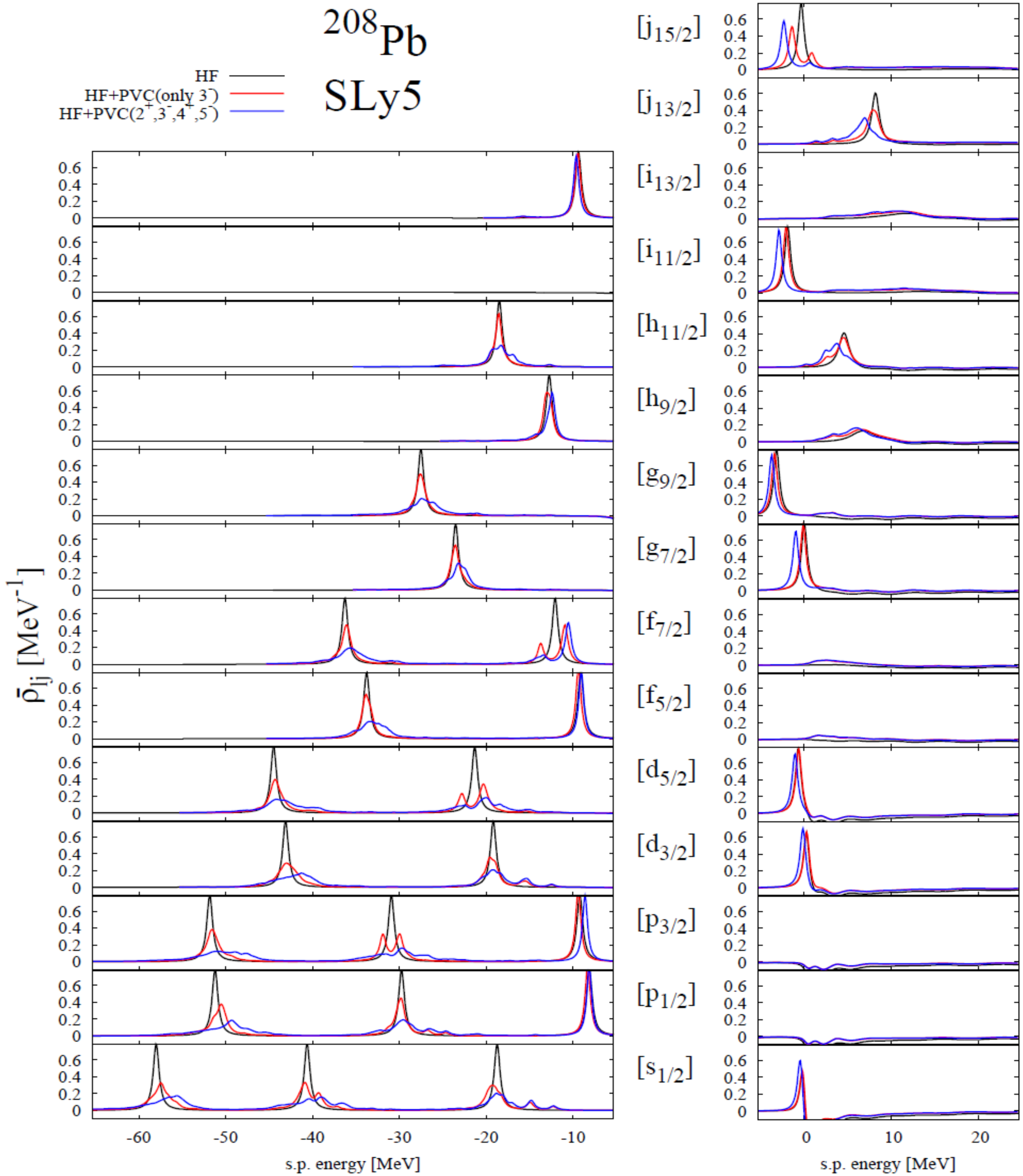
[3⁻, Full]



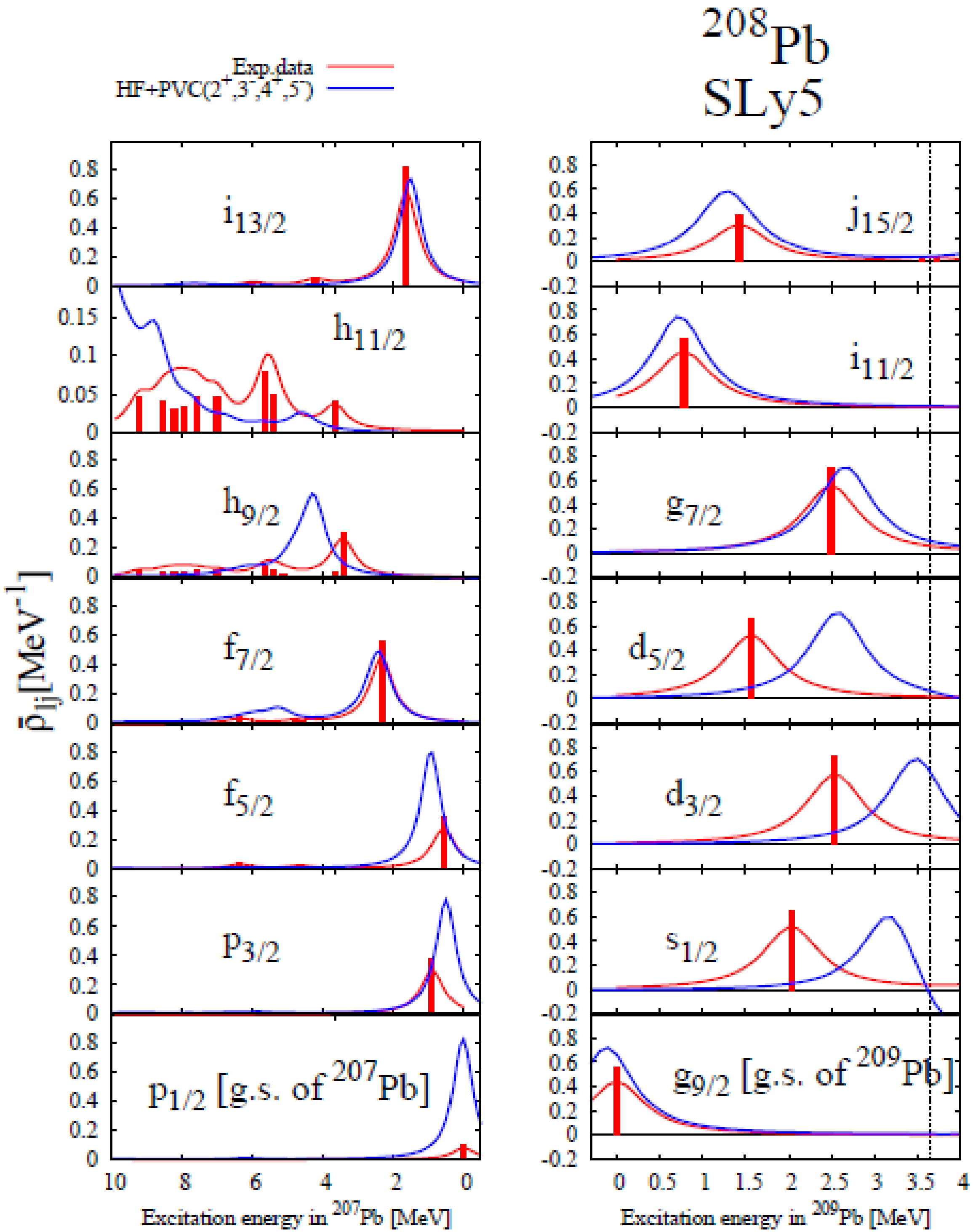
[3⁻, 1 pole]



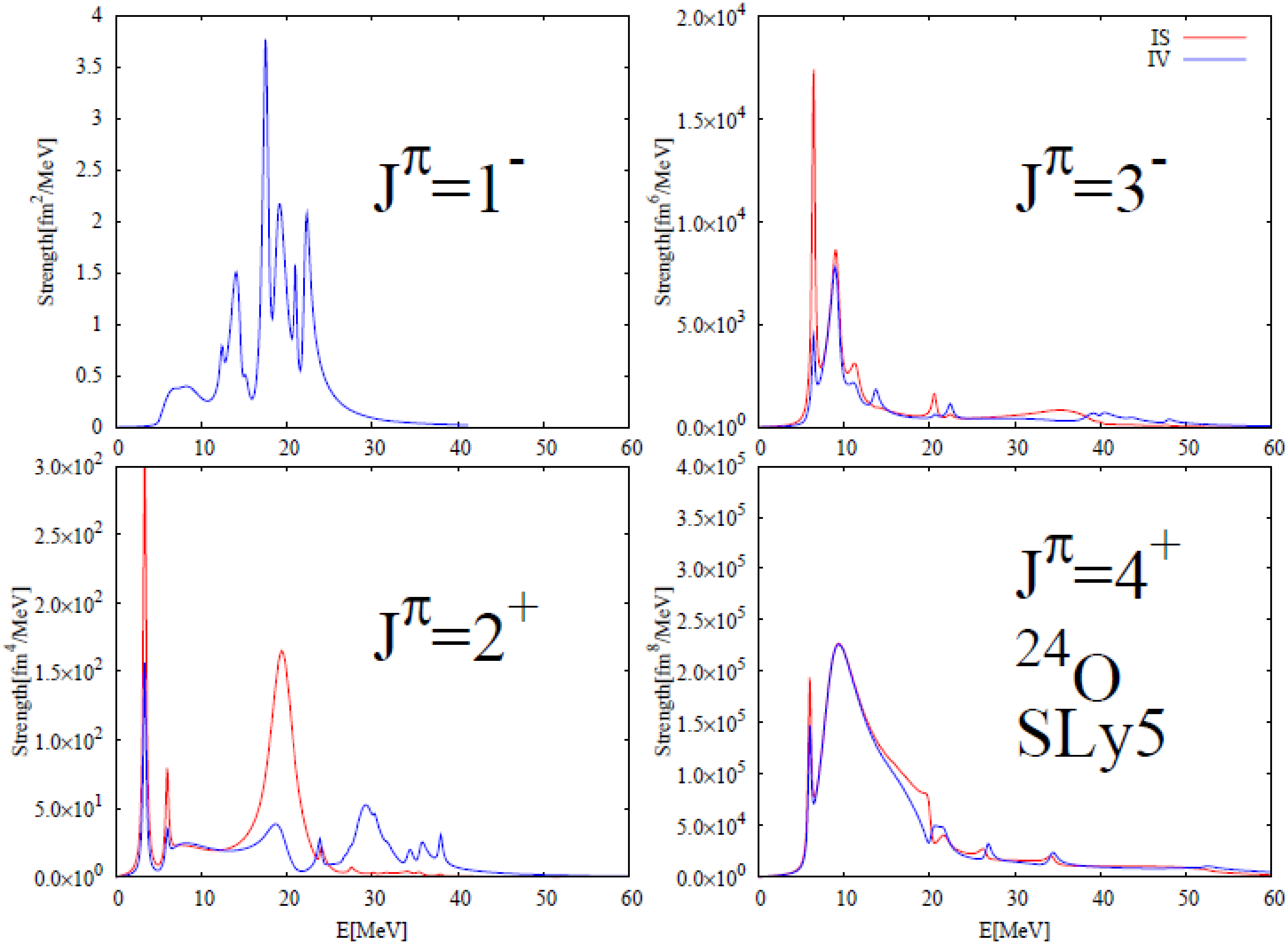

Strength[10^3fm^4/MeV]
×5
$J^\pi=2^+$
IS
IV
Strength[10^7fm^8/MeV]
$J^\pi=4^+$
Strength[10^5fm^6/MeV]
×10
$J^\pi=3^-$ ^{208}Pb
SLy5
Strength[10^9fm^{10}/MeV]
$J^\pi=5^-$
E[MeV]
E[MeV]



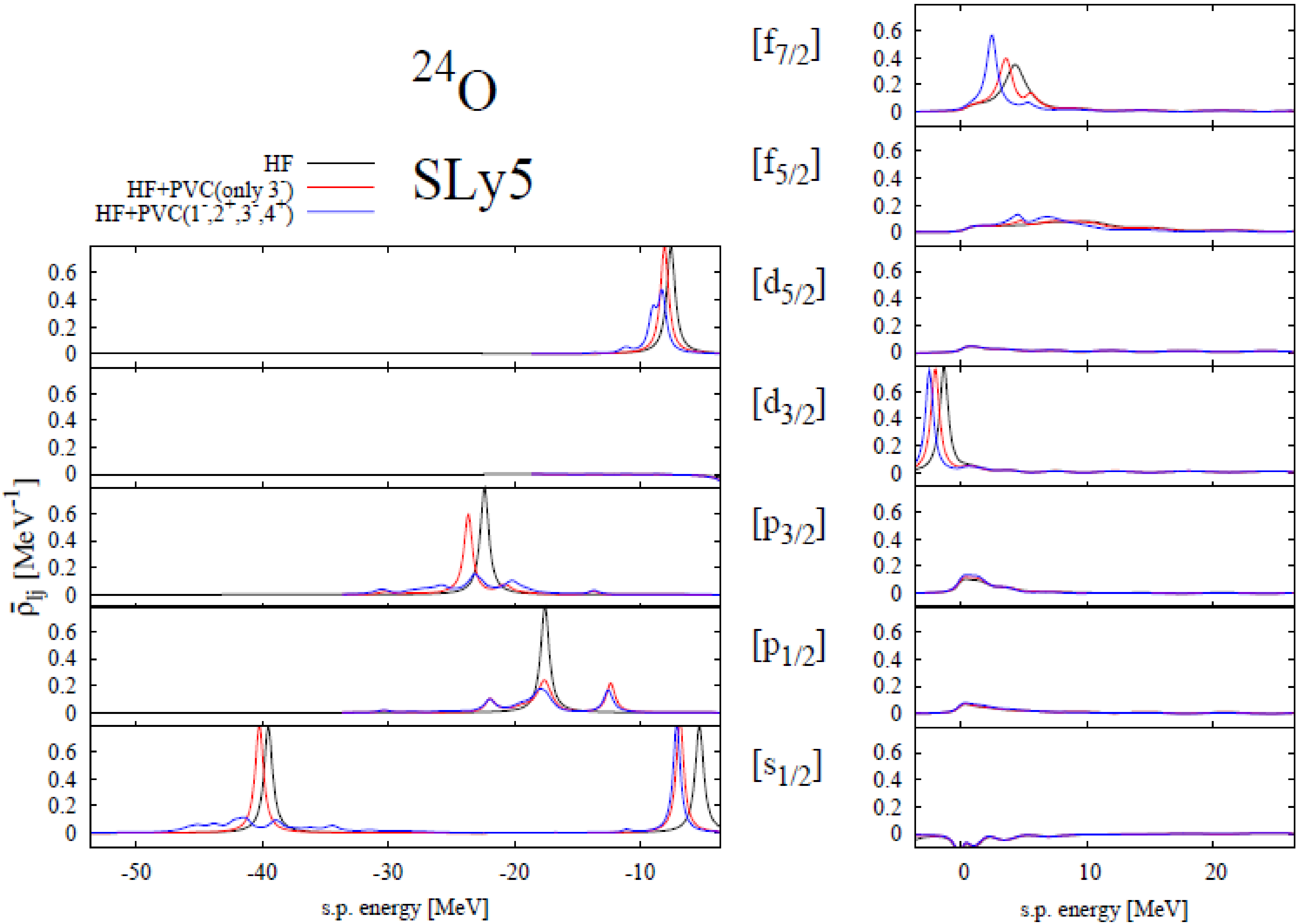
^{208}Pb
SLy5
HF
HF+PVC(only 3$^-$)
HF+PVC(2$^+$,3$^-$,4$^+$,5$^-$)
$\bar{\rho}_{lj}$ [MeV^{-1}]
[$j_{15/2}$]
[$j_{13/2}$]
[$i_{13/2}$]
[$i_{11/2}$]
[$h_{11/2}$]
[$h_{9/2}$]
[$g_{9/2}$]
[$g_{7/2}$]
[$f_{7/2}$]
[$f_{5/2}$]
[$d_{5/2}$]
[$d_{3/2}$]
[$p_{3/2}$]
[$p_{1/2}$]
[$s_{1/2}$]
s.p. energy [MeV]
s.p. energy [MeV]



^{208}Pb
SLy5
Exp.data
HF+PVC(2$^+$,3$^-$,4$^+$,5$^-$)
$\bar{\rho}_{lj}$[MeV^{-1}]
$i_{13/2}$
$h_{11/2}$
$h_{9/2}$
$f_{7/2}$
$f_{5/2}$
$p_{3/2}$
$p_{1/2}$ [g.s. of ^{207}Pb]
$j_{15/2}$
$i_{11/2}$
$g_{7/2}$
$d_{5/2}$
$d_{3/2}$
$s_{1/2}$
$g_{9/2}$ [g.s. of ^{209}Pb]
Excitation energy in ^{207}Pb [MeV]
Excitation energy in ^{209}Pb [MeV]



$J^{\pi}=1^{-}$
$J^{\pi}=3^{-}$
$J^{\pi}=2^{+}$
$J^{\pi}=4^{+}$
^{24}O
SLy5
IS
IV
Strength[fm^2/MeV]
Strength[fm^6/MeV]
Strength[fm^4/MeV]
Strength[fm^8/MeV]
E[MeV]



^{24}O
SLy5
HF
HF+PVC(only 3$^-$)
HF+PVC(1$^-$,2$^+$,3$^-$,4$^+$)
[$f_{7/2}$]
[$f_{5/2}$]
[$d_{5/2}$]
[$d_{3/2}$]
[$p_{3/2}$]
[$p_{1/2}$]
[$s_{1/2}$]
$\bar{\rho}_{lj}$ [MeV^{-1}]
s.p. energy [MeV]
s.p. energy [MeV]