Short-range and tensor correlations in nuclei

Dynamics and Correlations in Exotic Nuclei (DCEN2011)
Mini workshop: Clusters and neutrons in weakly bound systems
Yukawa Institute of Theoretical Physics, Kyoto, Japan
October 3, 2011

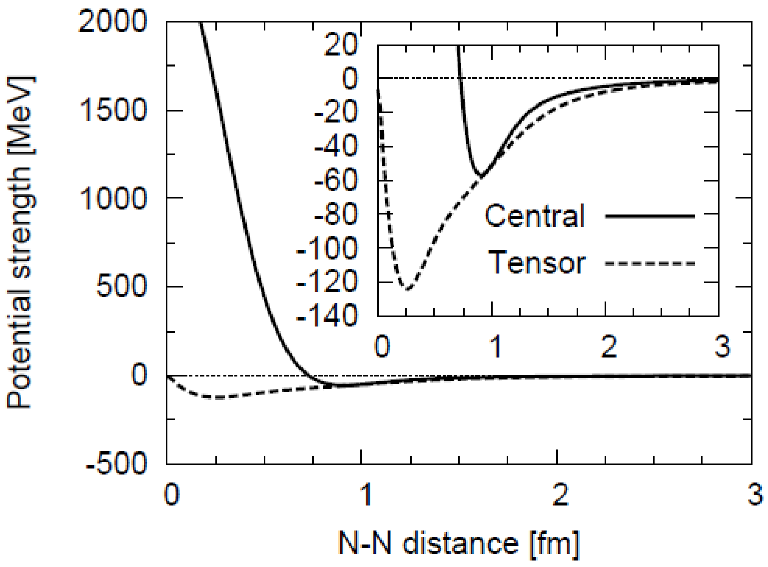
Wataru Horiuchi (RIKEN)

Collaborators:
Hans Feldmeier, Thomas Neff (GSI)
Yasuyuki Suzuki (Niigata, RIKEN)

Introduction

- "Realistic" nucleon-nucleon force
  - N-N phase shift
  - Deuteron properties
- Coordinate space
  - Short-range repulsion
  - Strong tensor component
- Momentum space
  - High momentum
  - Off diagonal matrix element

Argonne potential (V8')



Outline

- Purpose of this study
  - Short-range and tensor correlations
    - Low-momentum effective interaction
    - Measurement : (e, e’) experiment at JLab(R. Subedi, Science 320, 1476).
    - Saturation properties of nuclear matter
- Approach: Variational calculation with explicitly correlated basis
  - Correlated Gaussian and global vector
  - Stochastic variational method

    → “Exact” many-body states
- Results
  - One-body densities
  - Two-body densities for different spin-isospin channels
  - Comparison with the Unitary Correlation Operator Method (UCOM)
- Summary and outlook

Variational calculation for many-body systems

Hamiltonian

$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \left(\sum_{i<j<k}^{A} v_{ijk} \right)$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L} \cdot \boldsymbol{S}$$

Argonne V8' interaction: central, tensor, spin-orbit

Generalized eigenvalue problem

$$\Psi_{JM_J} = \sum_{i=1}^{K} c_i \Psi(\alpha_i)$$

$$\sum_{j=1}^{K} (H_{ij} - E B_{ij}) c_j = 0 \quad (i = 1, \ldots, K)$$

$$\begin{pmatrix} H_{ij} \\ B_{ij} \end{pmatrix} = \langle \Psi(\alpha_i) | \begin{pmatrix} H \\ 1 \end{pmatrix} | \Psi(\alpha_j) \rangle$$

Basis function

$$\Psi_{(LS)JM_J TM_T} = \mathcal{A} \left\{ \left[\psi_L^{\rm (space)} \psi_S^{\rm (spin)} \right]_{JM_J} \psi_{TM_T}^{\rm (isospin)} \right\}$$

$$\psi_{SM_S}^{\rm (spin)} = \left| [\cdots [[[\tfrac{1}{2} \tfrac{1}{2}] S_{12} \tfrac{1}{2}] S_{123}] \cdots] SM_S \right\rangle$$

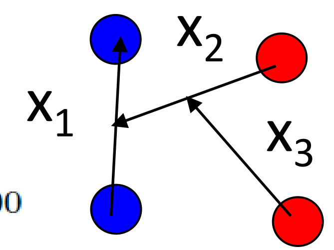
Explicitly correlated basis function

Correlated Gaussian

Explicit correlations among particles

$$\exp\left(-\frac{1}{2}ar^2\right) \to \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right) = \exp\left(-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right)$$

$$\exp\left(A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right) \sim \sum_n (\boldsymbol{x}_i\cdot\boldsymbol{x}_j)^n \sim \sum_{\ell=n,n-2,\dots} [\mathcal{Y}_\ell(\boldsymbol{x}_i)\mathcal{Y}_\ell(\boldsymbol{x}_j)]_{00}$$



Global vector

Rotational motion with total angular momentum L

$$r^l Y_{lm}(\hat{\boldsymbol{r}}) \equiv \mathcal{Y}_{lm}(\boldsymbol{r}) \to \mathcal{Y}_{LM_L}(\tilde{u}\boldsymbol{x}) = \mathcal{Y}_{LM_L}\Big(\sum_{i=1}^{A-1} u_i\boldsymbol{x}_i\Big)$$

$$\mathcal{Y}_{LM_L}(u_1\boldsymbol{x}_1+u_2\boldsymbol{x}_2) = \sum_{\ell=0}^{L}\sqrt{\frac{4\pi(2L+1)!}{(2\ell+1)!(2L-2\ell+1)!}}\; u_1^\ell u_2^{L-\ell}[\mathcal{Y}_\ell(\boldsymbol{x}_1)\mathcal{Y}_{L-\ell}(\boldsymbol{x}_2)]_{LM_L}$$

Global Vector Representation (GVR)

Parity $(-1)^{L_1+L_2}$

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Correlated basis approach

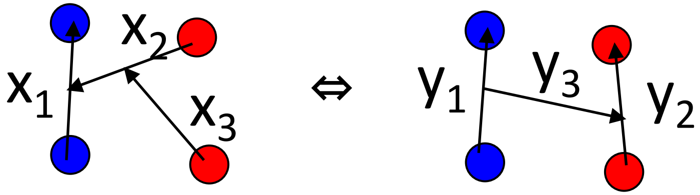
Double Global Vector Representation

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$F_{(L_1 L_2)LM}(u_1, u_2, A, \boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)\left[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})\right]_{LM}$$

- Formulation for N particle system
- Matrix elements can analytically be obtained
- Functional form does not change under any coordinate transformation

$$\boldsymbol{y} = T\boldsymbol{x} \implies \tilde{\boldsymbol{y}}B\boldsymbol{y} = \tilde{\boldsymbol{x}}\widetilde{T}BT\boldsymbol{x} \qquad \tilde{v}\boldsymbol{y} = \widetilde{\widetilde{T}v}\boldsymbol{x}$$



Stochastic variational method

K. Varga and Y. Suzuki, PRC52, 2885 (1995).

- Examine randomly generated basis and increase (or replace) the number of basis until convergence is reached

^{4}He energy agrees with the benchmark calculation

H. Kamada et al., PRC64, 044001 (2001)

How to extract correlated information

- Antisymmetrized many-body states Φ
  - Two-body: ^{2}H(d)
  - Three-body: ^{3}H(t), ^{3}He(h)
  - Four-body: ^{4}He(α), ^{4}He(0_2^+) (α*)

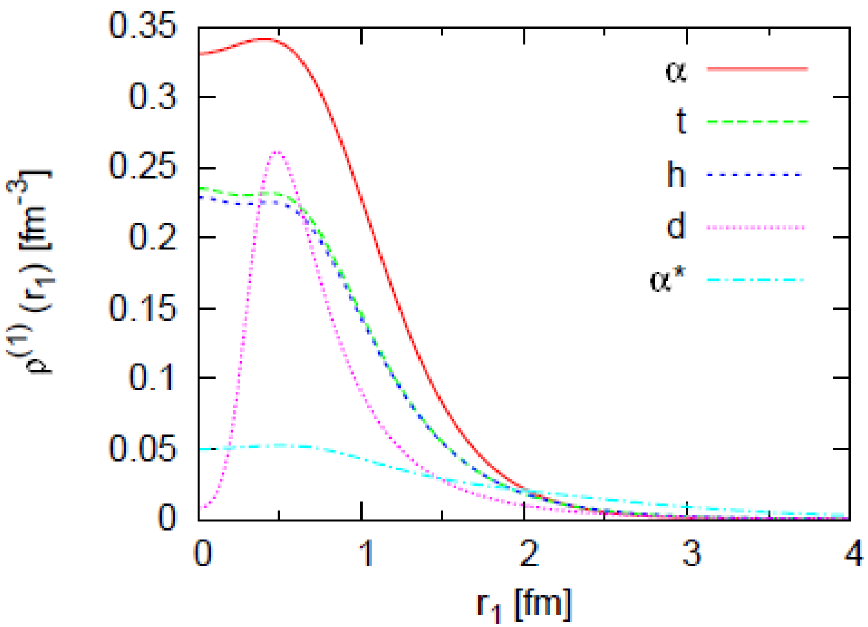
- *A*-body density: all information on correlation
  - Too much information
    - Position or momentum vectors: *A*
    - Spin-isospin possibilities: 4**A*
  - Two-body correlation

    -> integrate over A-2 particle degrees of freedom

One-body densities

Coordinate space

$$\rho^{(1)}(\boldsymbol{r}_1) = \langle \Phi | \sum_{i=1}^{A} \delta^3(\boldsymbol{r}_i - \boldsymbol{r}_1) | \Phi \rangle$$



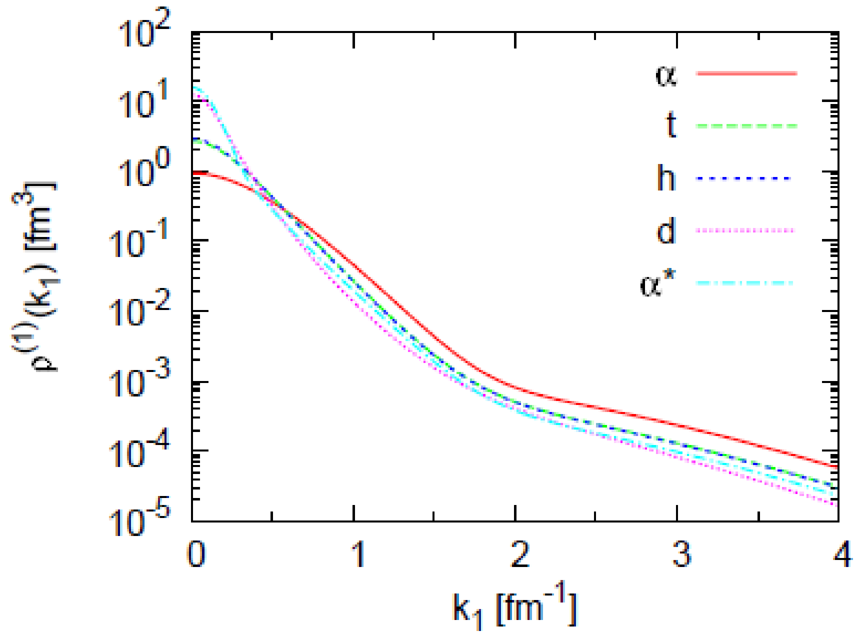
Strongly depends on the system

Dilute 3N+N structure in α*

W.H. and Y. Suzuki, PRC78, 034305 (2008)

Momentum space

$$\rho^{(1)}(\boldsymbol{k}_1) = \langle \Phi | \sum_{i=1}^{A} \delta^3(\boldsymbol{k}_i - \boldsymbol{k}_1) | \Phi \rangle$$



Size of the system

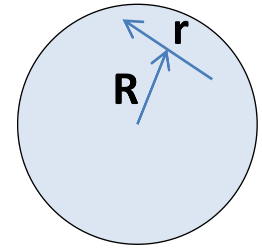
High momentum component

Two-body densities

Two-body density

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_1)\delta^3(\boldsymbol{r}_j-\boldsymbol{r}_2)\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

Spin (isospin) projector



Two-body density in relative coordinate

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

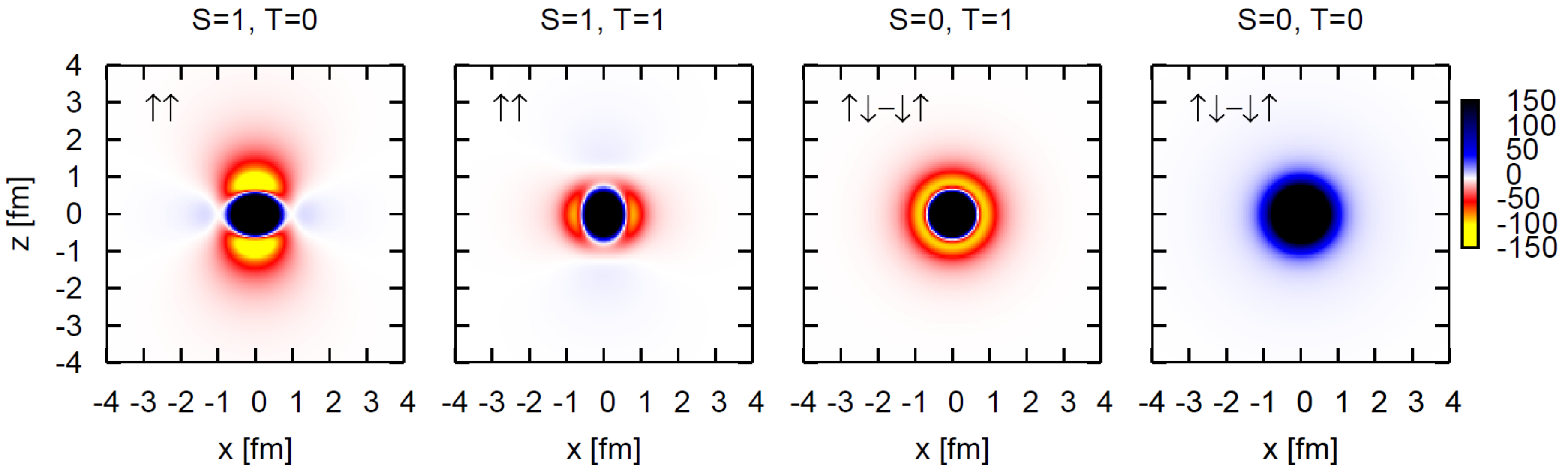
$$\begin{aligned}\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r})&=\int d\boldsymbol{R}\,\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R})\\&=\langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle\end{aligned}$$

Momentum space

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{k},\boldsymbol{K})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\delta^3(\boldsymbol{k}_i+\boldsymbol{k}_j-\boldsymbol{K})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{k})=\langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

Potential plot (Argonne V8')



L=Even
Most attractive

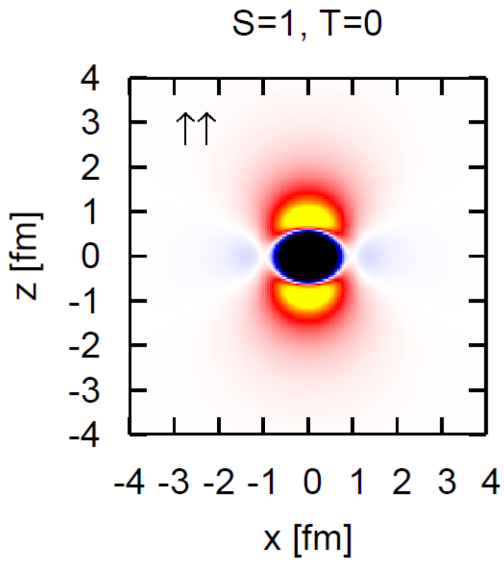
L=1

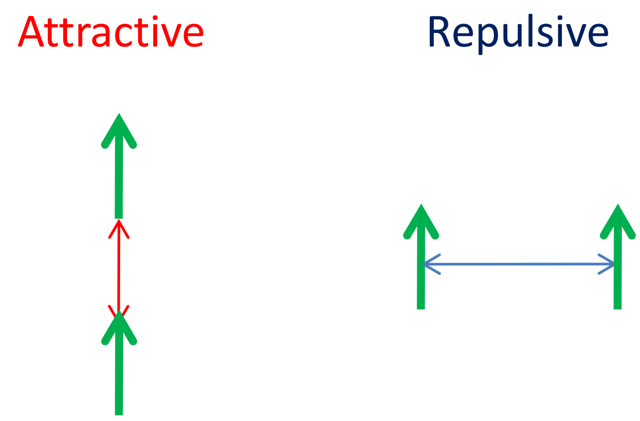
L=Even
Second attractive

Pair numbers in ST channel

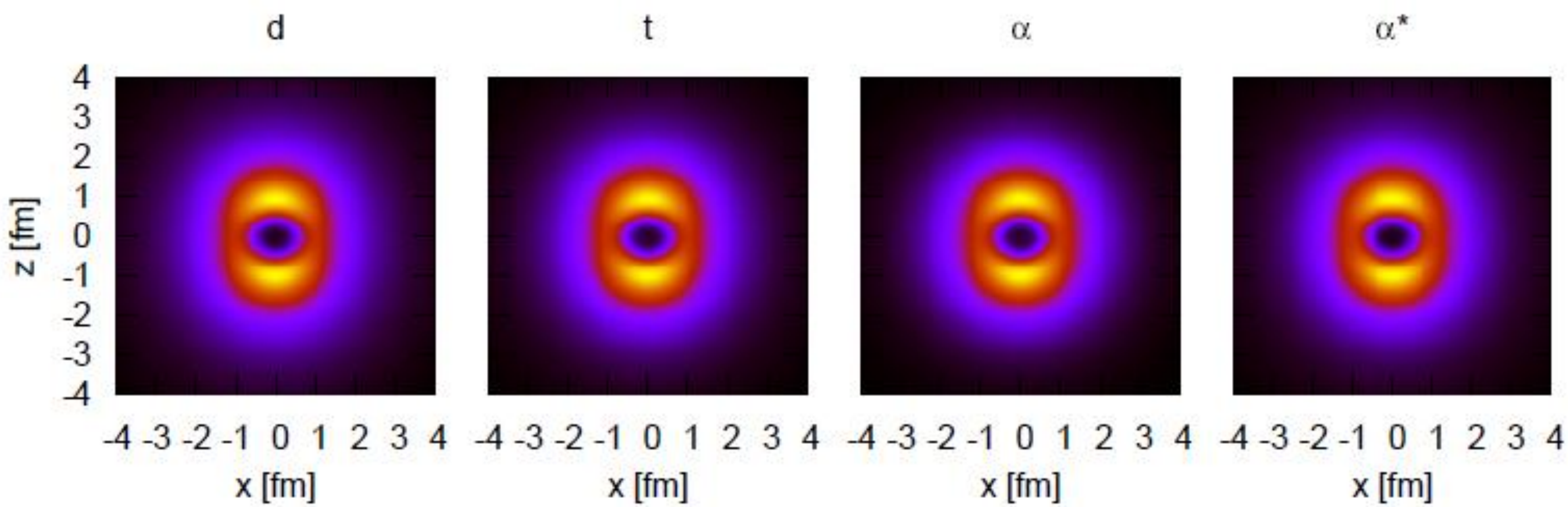
state\\(ST)	(10)	(01)	(11)	(00)
d	1	–	–	–
t	1.490	1.361	0.1385	0.009866
h	1.489	1.361	0.1394	0.01131
α	2.992	2.572	0.4282	0.008214
α^*	2.966	2.714	0.2862	0.03449

Two-body density (SM_S=11, T=0)

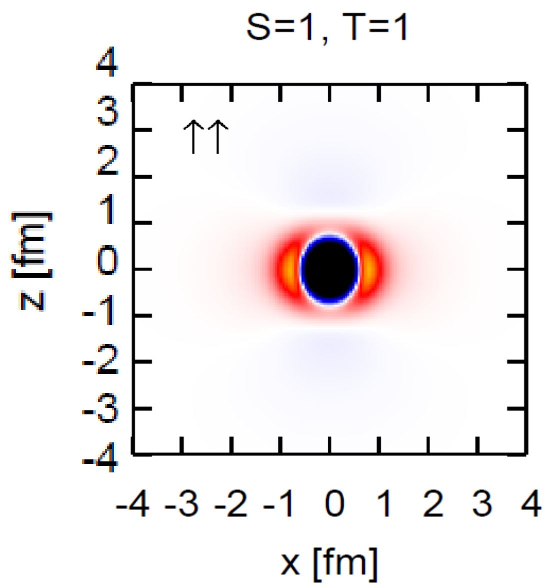


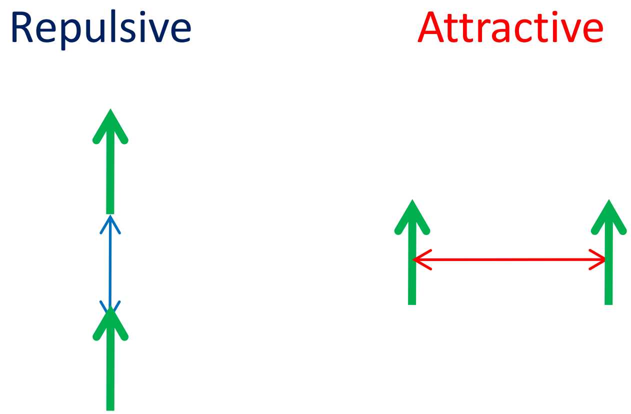


$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

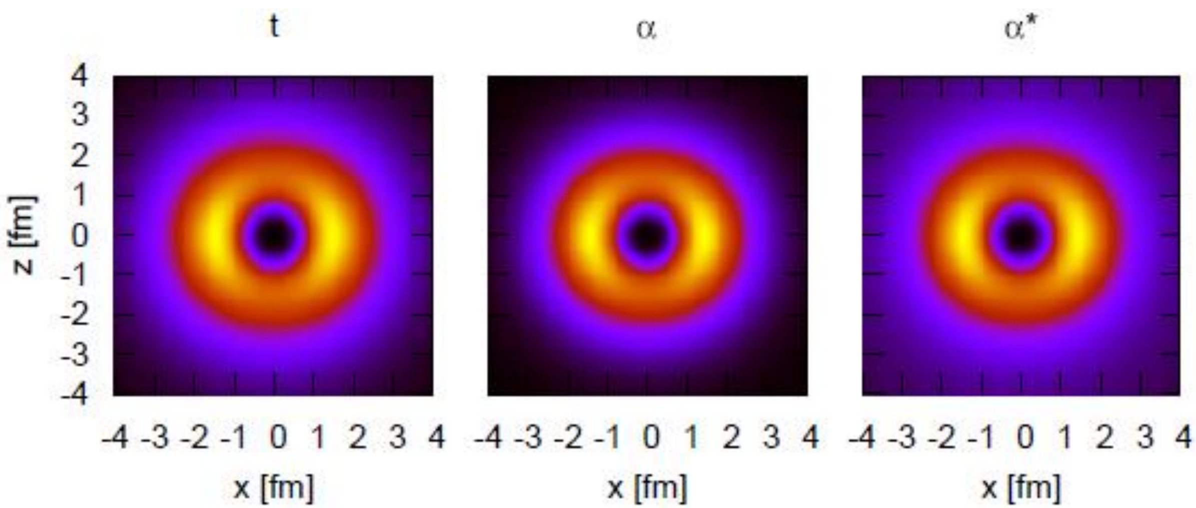


Two-body density (SM_S=11, T=1)

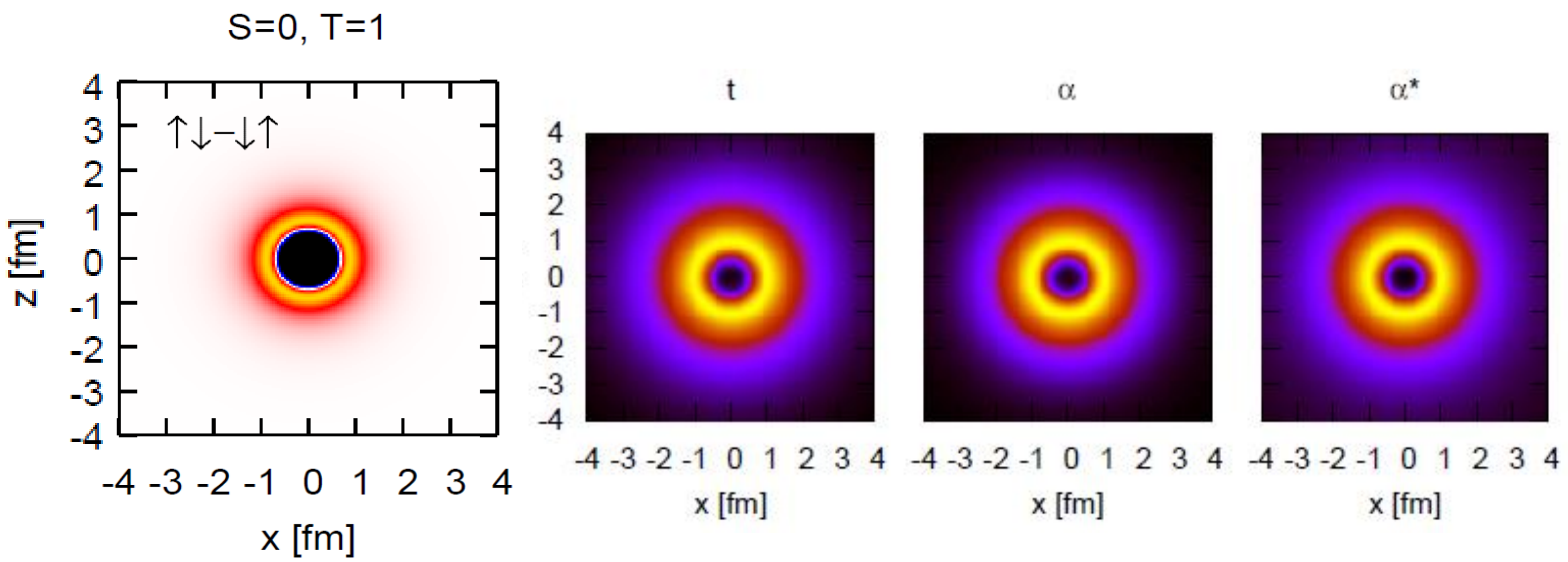




$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$



Two-body density (S=0, T=1)



S=0, T=0 channel -> small components in w. f.

One-to-one correspondence to the potential for all ST channels

Relevance to density functional theory

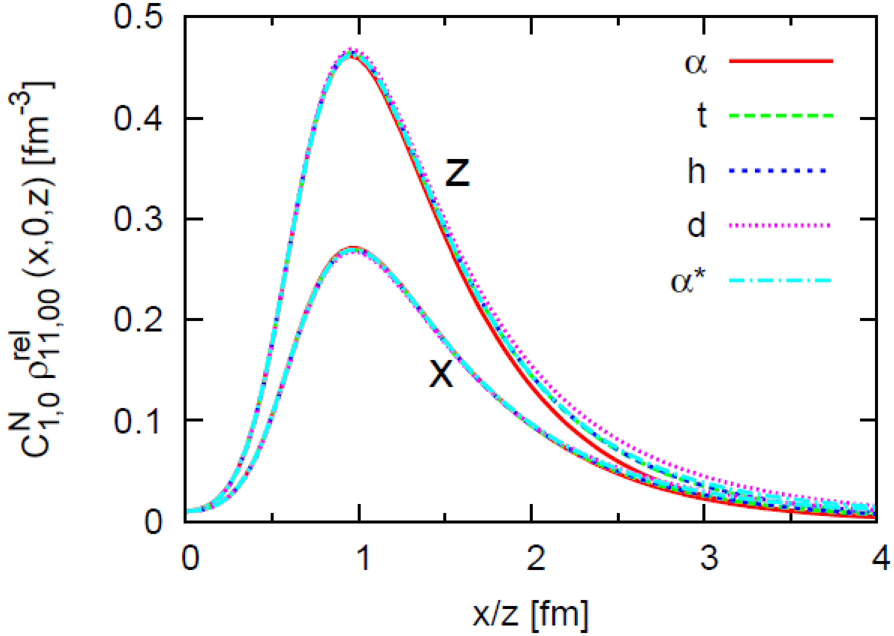
Y. Suzuki, W.H., Nucl. Phys. A 818, 188 (2009).

Universality of short-range correlations
Two-body density in coordinate space

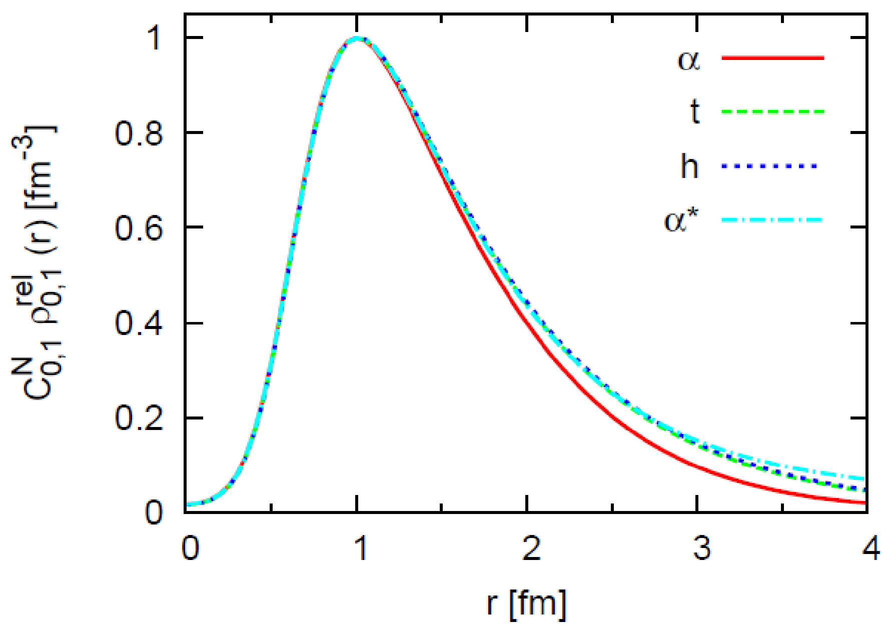
Density cut along z- and x-direction

Normalized at 1 fm of z axis

S=1 M_S=1, T=0



S=0, T=1



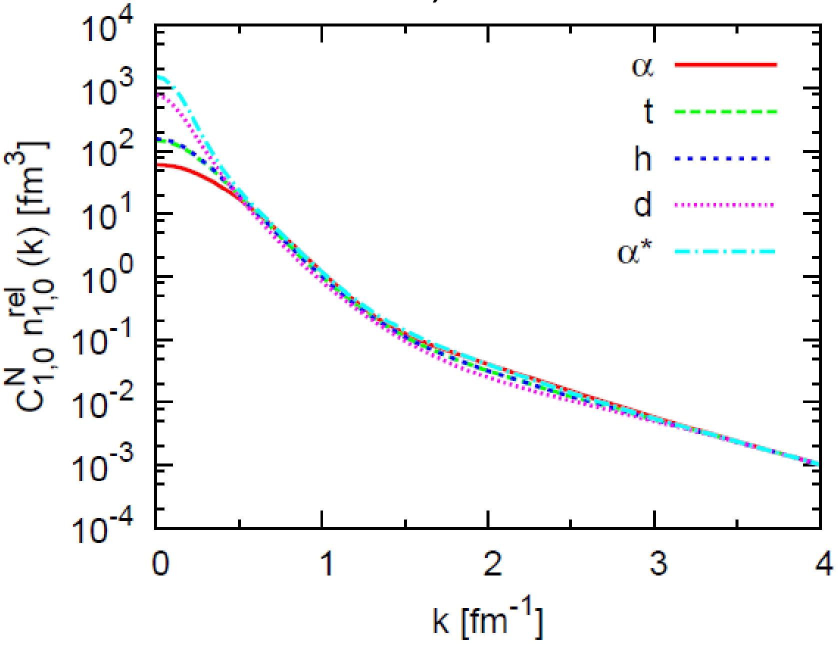
Universal behavior at short distances

Universality of short-range correlation Two-body density in momentum space

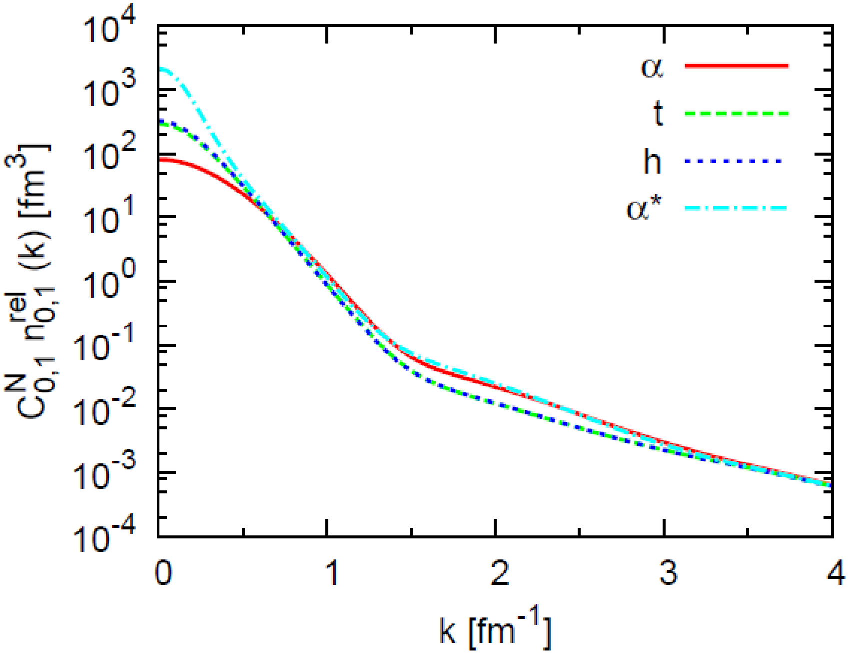
Two-body densities in momentum space

The same normalization factors are used as these in coordinate space.

S=1, T=0



S=0, T=1



Universal behavior at high momenta.

Comparison with UCOM

Unitary Correlation Operator Method (UCOM)

Short-range correlations
-> central and tensor correlation functions

Unitary transformation $|\Psi\rangle = \hat{C}\,|\Phi\rangle$

Effective Hamiltonian $\hat{H}_{\mathrm{UCOM}} = \hat{C}^{\dagger}\hat{H}\hat{C}$ "Correlated" Hamiltonian

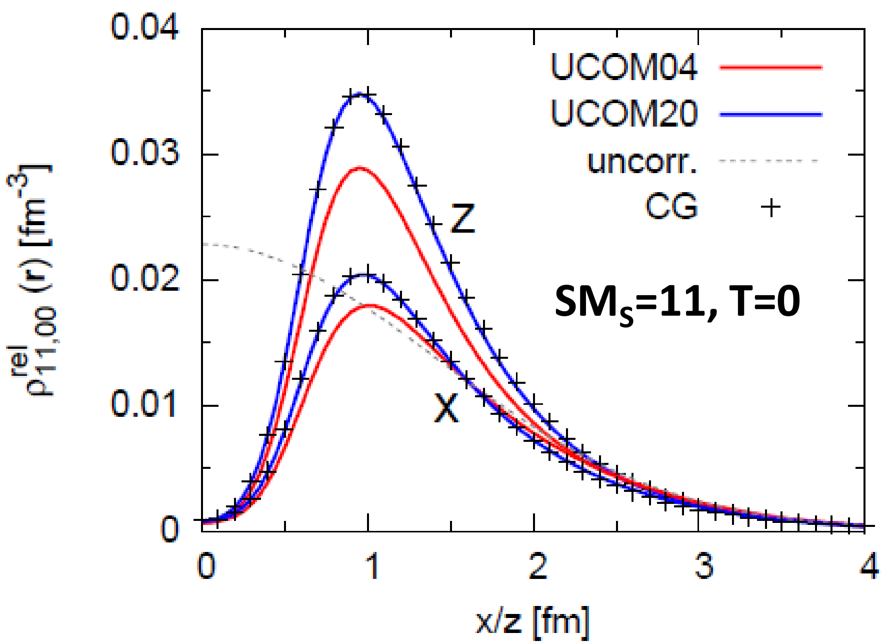
$\hat{H}_{\mathrm{UCOM}}\,|\Phi\rangle = E\,|\Phi\rangle$ Trial w.f. can be simple

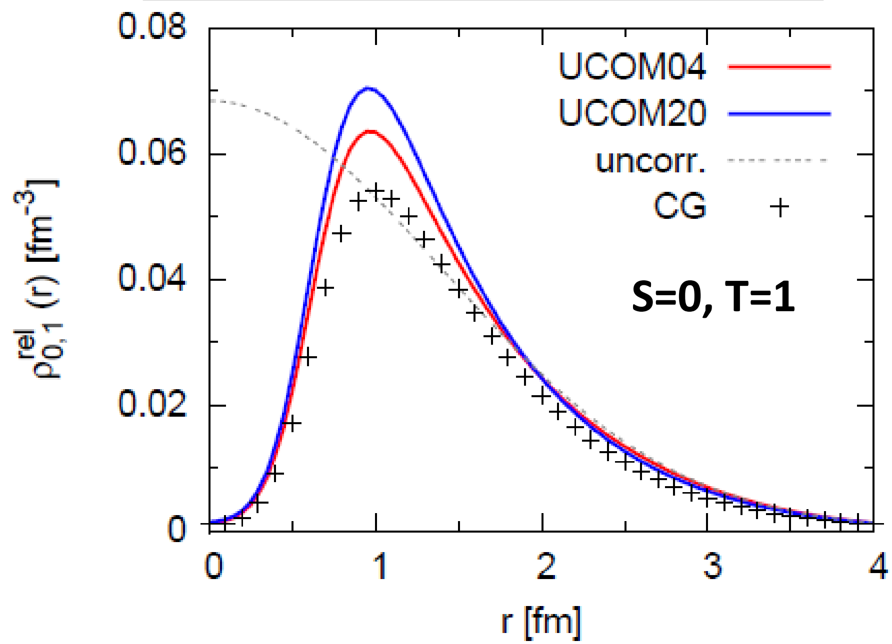
Simple trial w.f. $|\Phi\rangle = |(0s)^4\rangle$

Correlation functions: SRG evolved AV8'

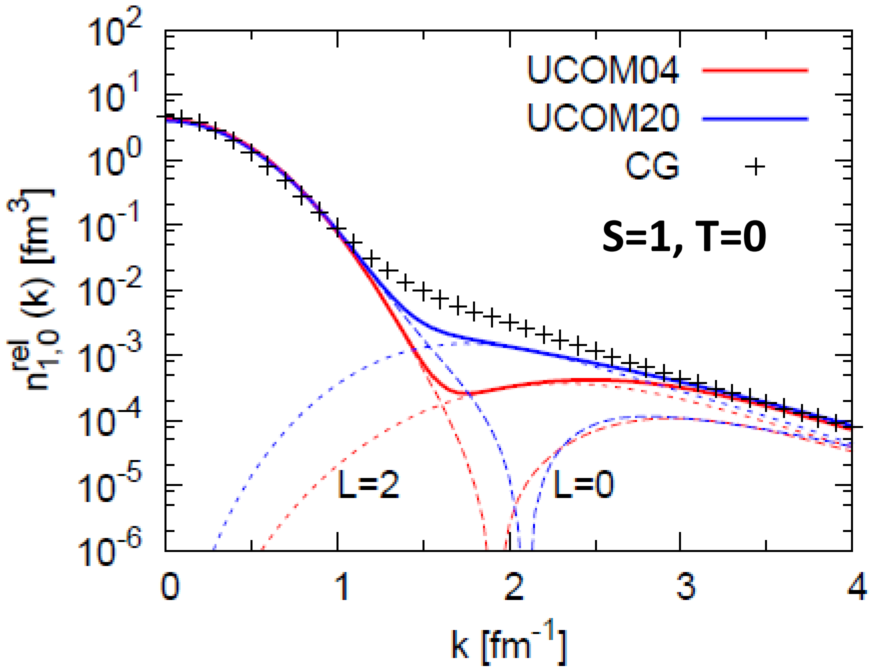
R. Roth, T. Neff, H. Feldmeier, Prog. Part. Nucl. Phys. 65, 50 (2010).

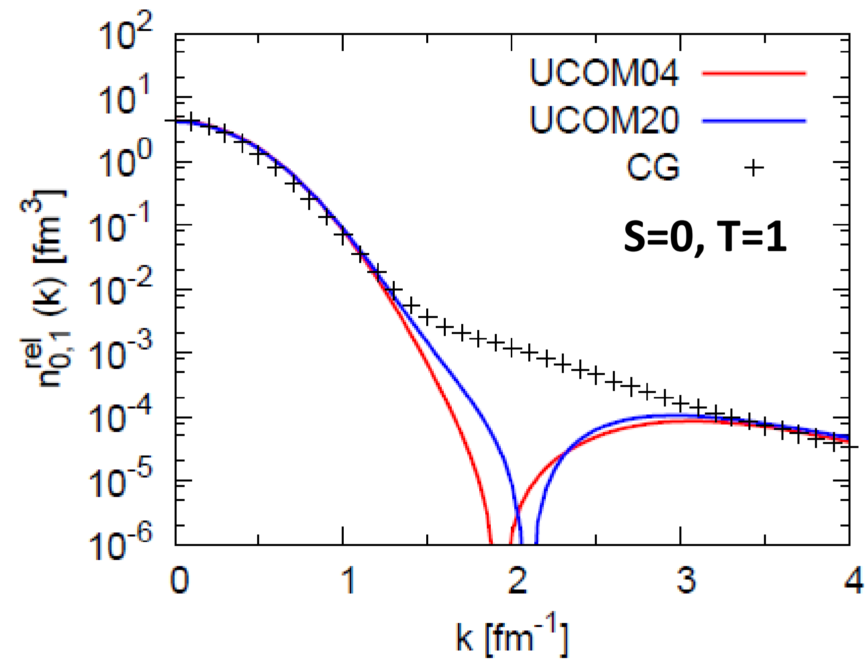
UCOM04(soft): -18.50 MeV
UCOM20(very soft): -25.10 MeV





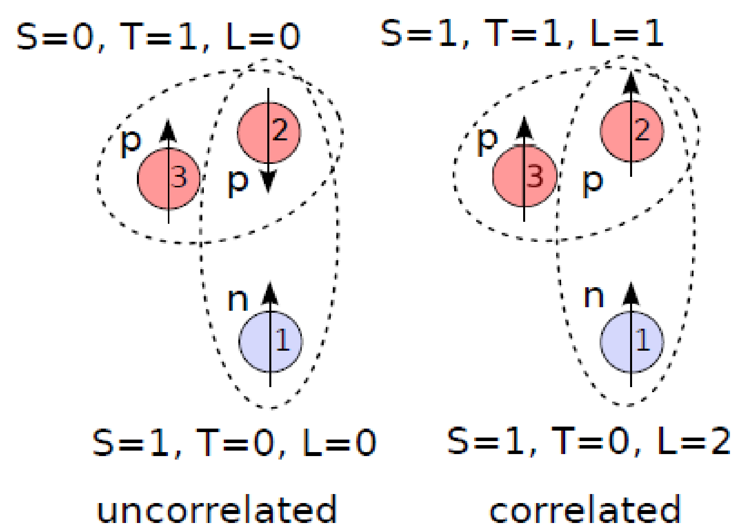
Comparison with UCOM





Lack of the three-body correlations

	$(0s)^4$	correlated
(ST)=(10) pair:	3.0	2.992
(00) pair:	0.0	0.008
(ST)=(01) pair:	3.0	2.572
(11) pair:	0.0	0.438



The deviation can be restored by a more elaborated trial wave function (NCSM, in progress).

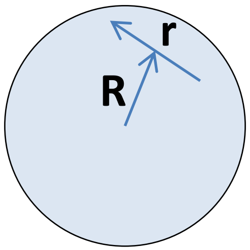
Summary and outlook

- Highly correlated many-body states (d, t, h, α, α*)
  - *Ab initio* type calculation with the Argonne V8' interaction
    - Correlated Gaussian with global vectors
    - Stochastic variational method
  - One-body densities
    - Quite different : strongly depends on the system
- Two-body densities
  - One-to-one correspondence between two-body potential and density
  - Universality at short distances (< 1 fm) and high momenta (> 3 fm^{-1})
- Comparison with the UCOM
  - Success of low-momentum interaction
  - Three-body correlation
    - Too simple trial wave function (-> no-core shell model, etc)
    - Correlation operator determined in two-body level

Accepted for publication in Phys. Rev. C (2011.9.20), arXiv:1107.4956.

- Outlook
  - Two-body density with two variables

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,|\Phi\rangle$$



  - More particle systems (A>4)