Cluster-gas-like states and monopole excitations

T. Yamada

Cluster-gas-like states and monopole excitations

- **Isoscalar monopole excitations in light nuclei**
- **Cluster-gas-likes states: ^{12}C, ^{16}O, ^{11}B, ^{13}C**

(maybe skipped here) Funaki's talk

Outline of my talk

1. Introduction
2. IS monopole strengths in light nuclei
   Typical case : ^{16}O
   RPA and SRPA calculations, compared with Exp.
3. 4α OCM for 0+ states in ^{16}O
   (OCM=Orthogonality Condition Model)
3. OCM energy weighted sum rule of IS monopole transition
4. IS monopole strength function with 4α OCM,
   compared with Exp.
6. Emphasize two features of IS monopole excitations.
7. Dual nature of ^{16}O ground state.
8. Summary

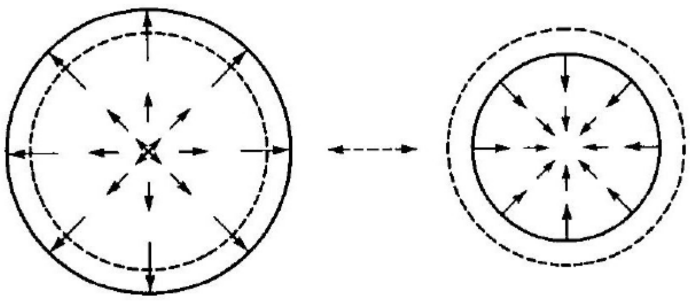
- **Isoscalar Monopole Excitation**

  ↔ density fluctuation

<u>Typical example</u>

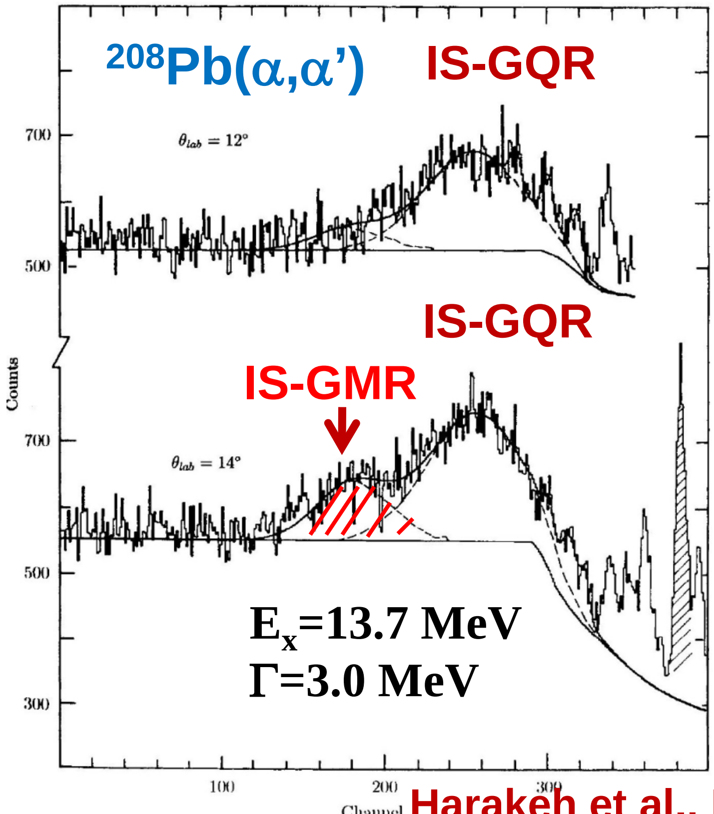
IS-GMR (heavy, medium-heavy nuclei)

exhaust EWSR ~ 100%



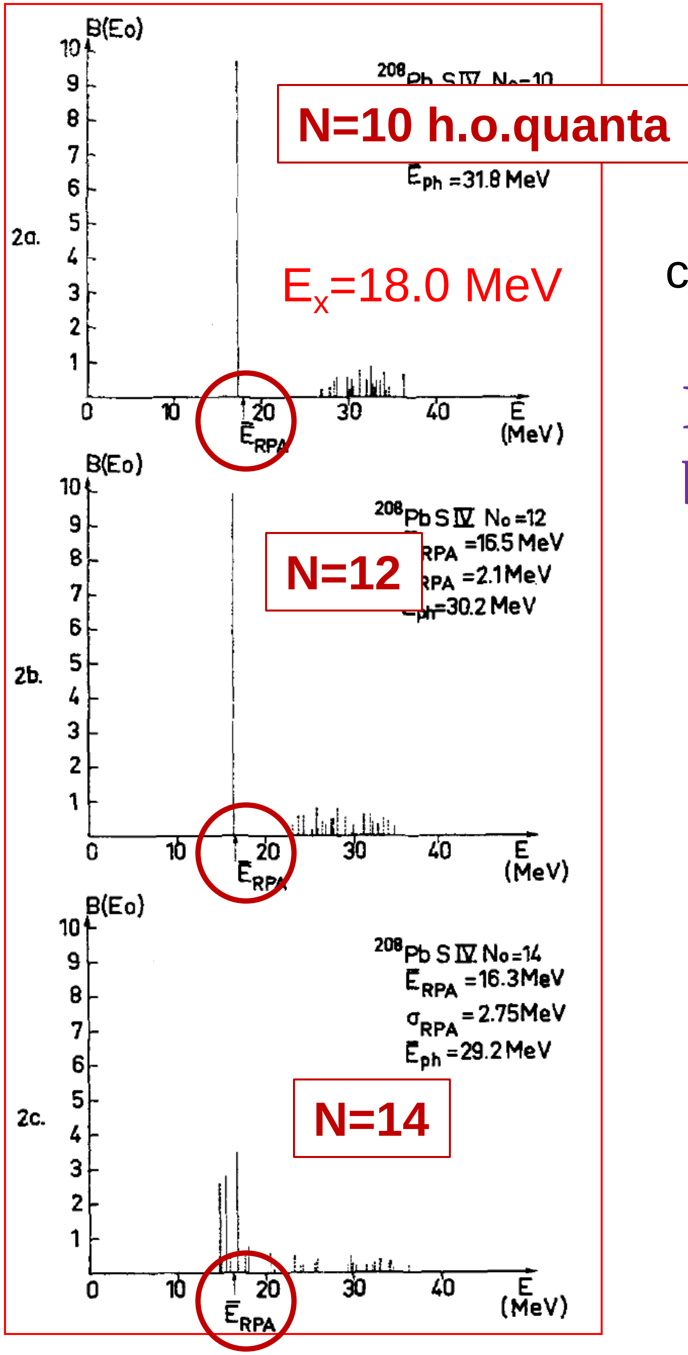
breathing mode

$$E_x \simeq 80/A^{-1/3} \quad [\mathrm{MeV}]$$



Harakeh et al., PRL(1977)

^{208}Pb: RPA



collective motion with coherent 1p-1h states

IS-GMR is well reproduced by RPA cal.

Blaizot, Gogny, Grammticos, NPA(1976)

- **Light Nuclei (*p*-, *sd*-shell,,,)**

Isoscalar monopole strengths are fragmented.

- **For example, ^{16}O, (^{12}C,^{11}B,^{13}C,^{24}Mg,…)**

IS-monopole response fun. of ^{16}O(α,α')

(i) discrete peaks at $E_x \leq 15$ MeV

~20% of EWSR

large M(E0) states ⇔ cluster states

(ii) three-bump structure : E=18, 23, 30 MeV

IS Monopole Strength Function of ^{16}O Exp. vs Cal.

^{16}O(α,α')

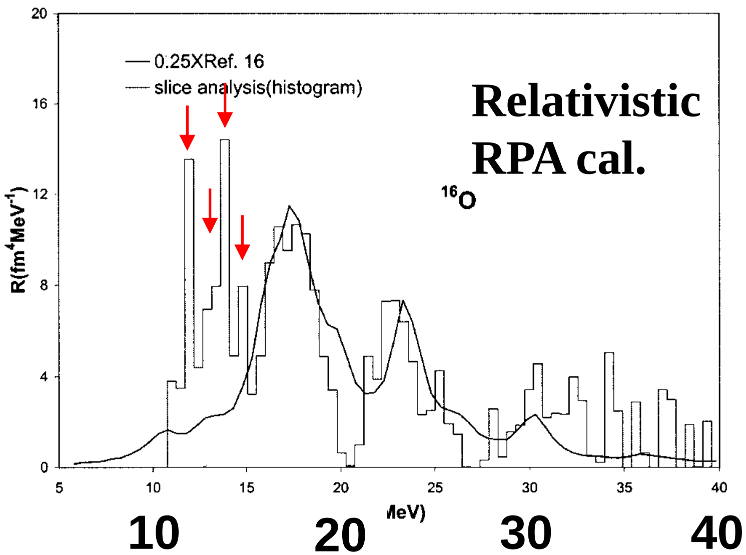


FIG. 7. ... histogram ... ength con- verted to monopole response fun ... shows the monopole response function from ... by 0.25 and shifted by 4.2 MeV.

Exp. condition: E_x > 10 MeV

Exp: histogram

Lui et al., PRC 64 (2001)

discrete peaks at Ex≤15 MeV

three bumps at 18, 23, 30 MeV

Cal: real line

Relativistic RPA

Ma et al., PRC 55 (1997)

Multiplied by 0.25

Shifted by 4.2 MeV

Not well reproduced by RRPA cal.

Non-rela. Mean-field calculations of IS-monopole strengths for ^{16}O

(1) RPA calculation: 1p-1h excitations

Blaizot et al., NPA265 (1976)

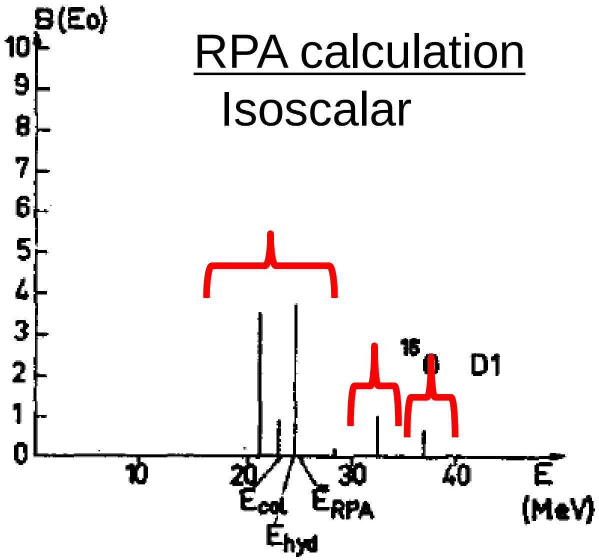
(2) Second-order RPA (SRPA) calculations:
: 1p1h + 2p2h

i) Drozdz et al., PR197(1980): D1-force

ii) Papakonstantino et al., PLB671(2009): UCOM

iii) Gambacurta et al., PRC81(2010) :
full SRPA calculation with Skyrme force

RPA calculation



Blaizot et al., NPA(1976)

Experiment

$^{16}O(\alpha,\alpha')$

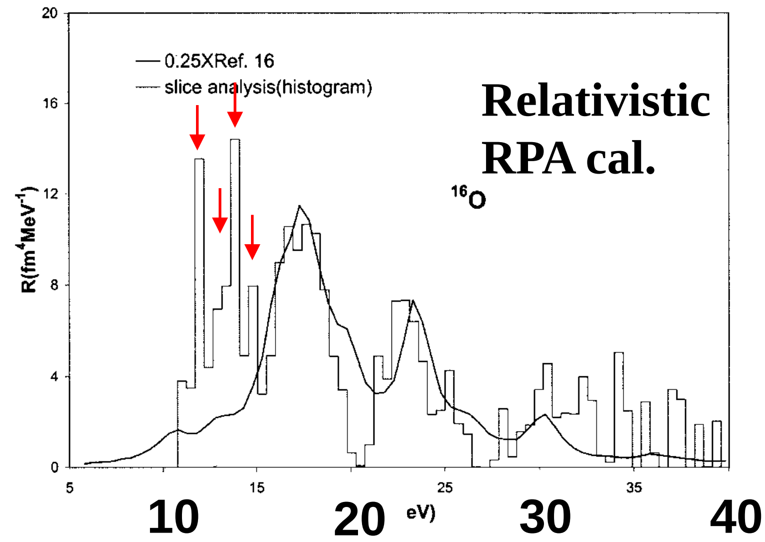


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the mo… shi…

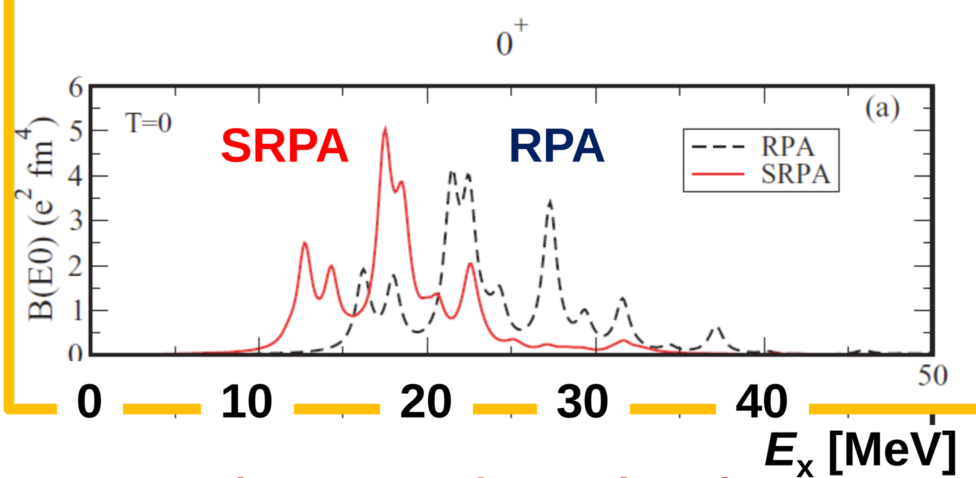
Exp. condition: E_x > 10 MeV

(1) Reproduction of 3-bump structure,
but the energy positions are by about 3-5 MeV higher than the data.

(2) No reproduction of discrete peaks at $E_x \leq 15$ MeV

Experiment

SRPA (+RPA) calculation



Gambacurta et al., PRC(2010)

^{16}O(α,α')

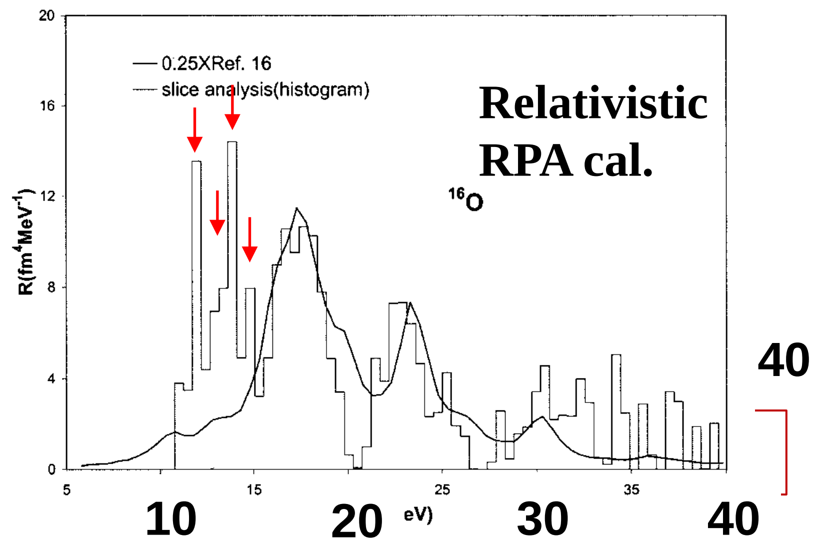


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the mo... shi...

Exp. condition: E_x > 10 MeV

(1) Gross structure at higher energy region (Ex > 18 MeV) , i.e. 3-bump structure, is reproduced by SRPA calculation.

(2) Discrete peaks at E_x≤15 MeV are not reproduced well.
In particular, the transition to 2nd 0+ state (E_x=6.1 MeV) is not seen in SRPA (+RPA) calculation.

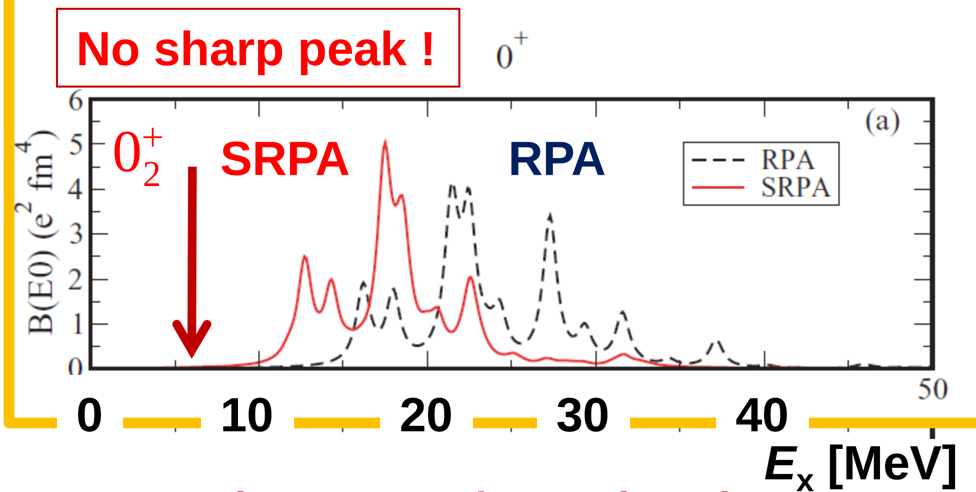
		Experiment				4α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

over 15%
of total EWSR

20%
of total EWSR

Experiment

SRPA (+RPA) calculation



Gambacurta et al., PRC(2010)

^{16}O(α,α')

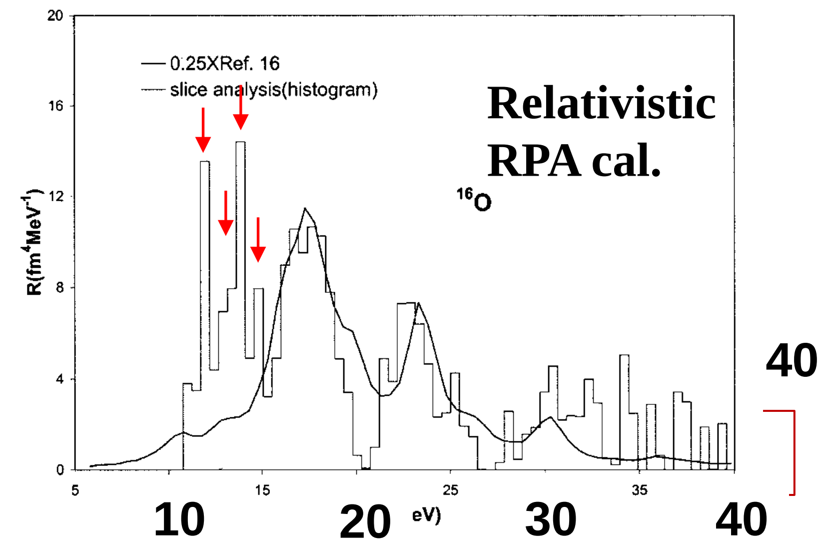


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the mo... shi...

Exp. condition: E_x > 10 MeV

(1) Gross structure at higher energy region (Ex > 18 MeV) , i.e. 3-bump structure, is reproduced by SRPA calculation.

(2) Discrete peaks at E_x≤15 MeV are not reproduced well.
In particular, the transition to 2nd 0+ state (E_x=6.1 MeV) is not seen in SRPA (+RPA) calculation.

RPA and SRPA calculations for ^{16}O

(1) Gross structure at higher energy region, i.e. 3-bump structure, is reproduced by RPA and SPRA calculations.

(2) Discrete peaks at $E_x \leq 15$ MeV are not reproduced well. In particular, the transition to 2^{nd} 0+ state (E_x=6.1 MeV) is not seen in SRPA and RPA calculations.

This peak should exit sharply at E_x=6.1 MeV because $\Gamma \leq 1$ eV.

Purposes of my talk

- What kind of states contribute to the discrete peaks?

- Recently, 4α OCM succeeded in describing the structure of the lowest six 0+ states up to 4α threshold ($E_x \approx 15$ MeV).

Funaki's talk

- We will study the IS monopole strength function with the 4α OCM.

Cluster-model analyses of ^{16}O

- **α+^{12}C OCM**

  Y. Suzuki, PTP55 (1976), 1751

- **α+^{12}C GCM**

  M. Libert-Heinemann, D. Bay et al., NPA339 (1980)

- **4α THSR wf** Not include α + ^{12}C configuration.

  Tohsaki, Horiuchi, Schuck, Roepke, PRL87 (2001)
  Funaki, Yamada et al., PRC82(2010)

- **4α OCM** 4α-gas, α + ^{12}C, shell-model-like configurations

  Funaki, Yamada et al., PRL101 (2008)

  Reproduction of lowest six 0+ states up to 4α threshold (15MeV)

$^{16}O = \alpha + ^{12}C$ cluster model

Y. Suzuki, PTP55 (1976), 1751

Even-parity

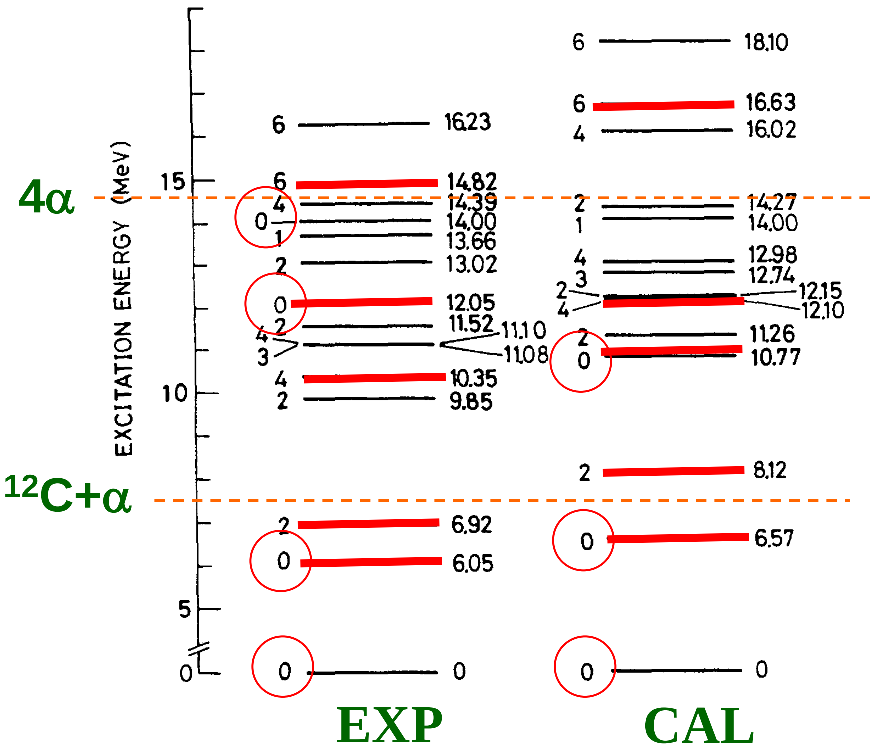


Fig. 2 (a). Energy levels of ^{16}O for the even-parity states [Ref. 30)].

Odd-parity

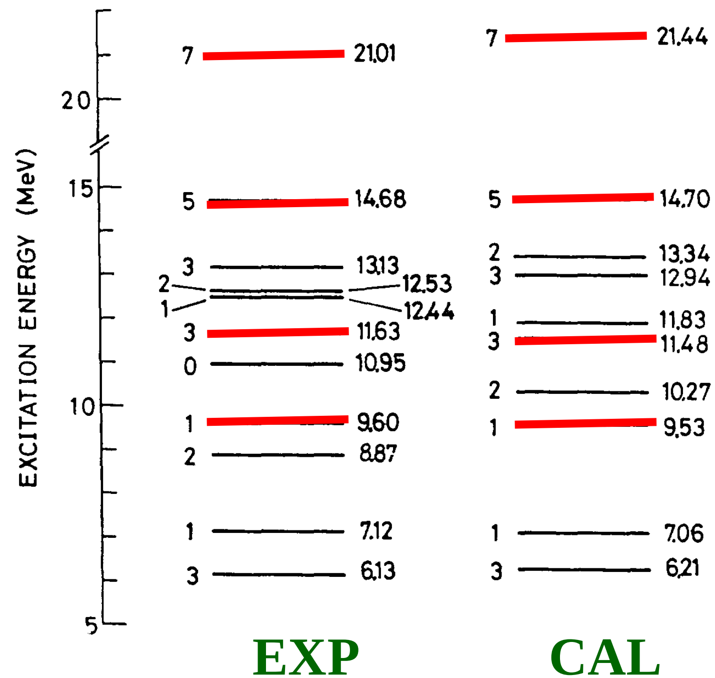
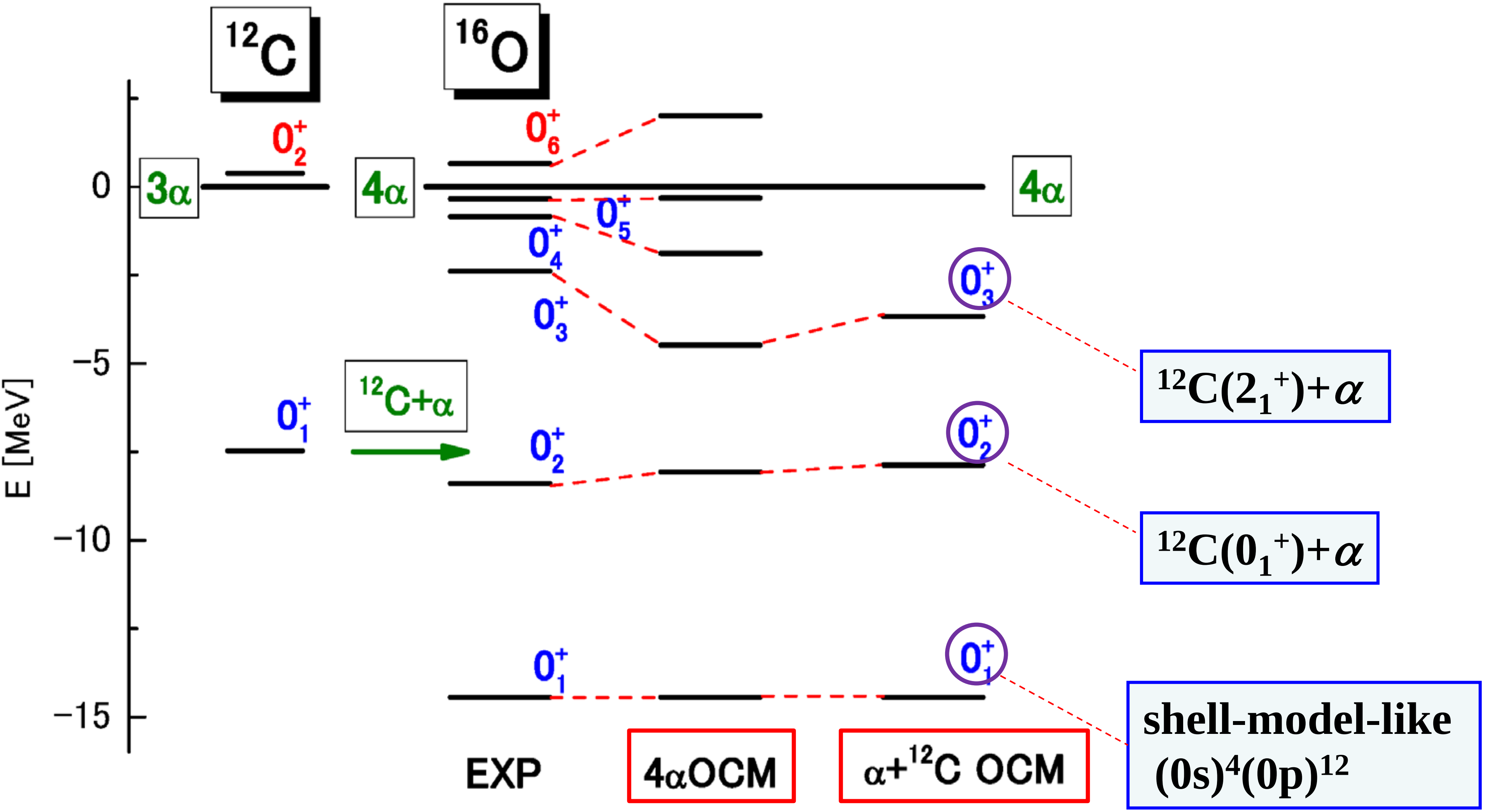


Fig. 2 (b). Energy levels of ^{16}O for the odd-parity states [Ref. 30)].

$^{12}C+\alpha$: molecular states



Funaki et al., PRL101 (2008)

Suzuki, PTP55 (1976)

OCM (orthogonality condition model)

- An approximation of RGM (resonating group method)
- Relative motions among c.o.m. of clusters are exactly solved under an orthogonality condition arising from Pauli-Blocking effects

For example of *n*α system, S. Sato, Prog. Thor. Phys. 40 (1968)

Fermion w.f.: $\Phi^{(F)} = \mathcal{A}\left\{\prod_{i=1}^{N}\phi_{\alpha_i}^{\mathrm{int}}\chi^{rel}\right\}$, $(H-E)\Phi^{(F)}=0$, $\langle\Phi^{(F)}|\Phi^{(F)}\rangle=1$,

RGM eq.: $(\mathrm{H}-E\mathcal{N})\chi^{rel}=0$, $\langle\chi^{rel}|\mathcal{N}|\chi^{rel}\rangle=1$,

α - cluster w.f.: $\Phi^{(B)}=\sqrt{\mathcal{N}}\,\chi^{rel}$, $\left(\frac{1}{\sqrt{\mathcal{N}}}\mathrm{H}\frac{1}{\sqrt{\mathcal{N}}}-E\right)\Phi^{(B)}=0$, $\langle\Phi^{(B)}|\Phi^{(B)}\rangle=1$,

Approximation: $\frac{1}{\sqrt{\mathcal{N}}}H\frac{1}{\sqrt{\mathcal{N}}} \Rightarrow T+\sum_{i<j}V_{2\alpha}^{eff}(i,j)+\sum_{i<j<k}V_{3\alpha}^{eff}(i,j,k)=T+V^{eff}$

Orthogonality condition

OCM equation: $(T+V^{eff}-E)\Phi^{(B)}=0$ with $\langle u_F|\Phi^{(B)}\rangle=0$, $\langle\Phi^{(B)}|\Phi^{(B)}\rangle=1$

u_F : **Pauli forbidden states,** $\mathcal{N}\,u_F=0$ or $\mathrm{A}\left\{\prod_{i=1}^{N}\phi_{\alpha_i}^{\mathrm{int}}u_F\right\}=0$

$\Phi^{(B)}$: **Symmetrized w.f. with relative (Jacobi) coordinates**

Easy to formulate 2α+t OCM and 3α+n OCM based on GEM

Framework of 4α OCM

Total w.f. : internal w.f.s of α clusters × relative w.f.

$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$

$\phi(\alpha) : (0s)^4$

$$\langle u_F | \Psi(J^\pi) \rangle = 0$$

: Orthogonality condition arising from Paul-blocking effects:
i.e. orthogonal to u_F (Pauli-forbidden states)

Hamiltonian for $\Psi(J^\pi)$: 4-body Hamiltonian

$$H_{\mathrm{OCM}} = \sum_{i=1}^{4} T_i - T_{cm} + \sum_{i<j}^{4} \left[V_{2\alpha}^{(N)}(i,j) + V_{2\alpha}^{(\mathrm{Coul})}(i,j) \right]$$

$$+ \sum_{i<j<k}^{4} V_{3\alpha}(i,j,k) + V_{4\alpha}(1,2,3,4)$$

2α potential: phase shifts
Energy spectra of $^{12}C(0+,2+,4+,3-,1-)$
Energy of ^{16}O g.s.

How to solve 4-body problem with orthogonality condition

Combing Gaussian Expansion Method (GEM) and OCM

Gaussian Gaussian Gaussian

$$\Psi(J^\pi) = \sum_{\nu} f_c(\nu) \times \hat{S}\left[\left[\varphi_{\ell 1}(\xi_1, \nu_1)\varphi_{\ell 2}(\xi_2, \nu_2)\right]_{L12} \varphi_{\ell 3}(\xi_3, \nu_3)\right]_J$$

$$H_{\mathrm{OCM}}\Psi(J^\pi) = E\Psi(J^\pi)$$

$$\langle u_F | \Psi(J^\pi) \rangle = 0$$

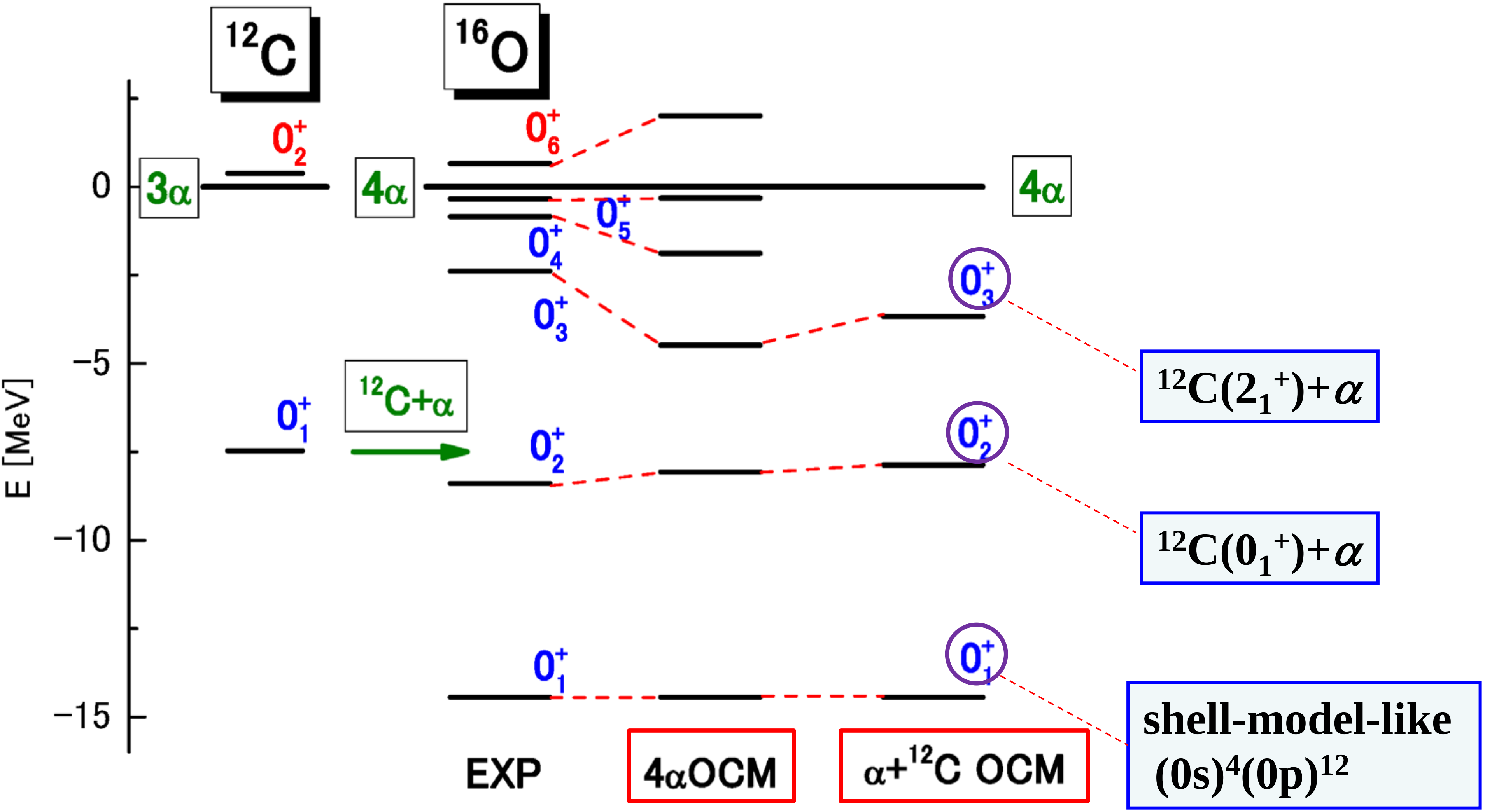
GEM (Gaussian Expansion Method):

Numerical precision is equivalent to Faddeev-Yakubousky eq. for ^{4}He = 4-body problem with realistic NN forces

Kamimura: PRA38(1988),
Hiyama, Kino, Kamimura: PPNP51(2003),
Kamada et al.: PRC64(2001)

GEM+OCM: Structure study of Light Hypernuclei

Hiyama, Yamada: PPNP63(2009)



Funaki et al.,
PRL101 (2008)

Suzuki,
PTP55 (1976)

		Experimental data				4α OCM		
	E_x [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
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0^+_6	15.1		no data	0.166		5.6	1.0	0.14

over 15%
of total EWSR

20%
of total EWSR

Single-particle IS-monopole strength

$$M(\mathrm{E0}) \sim \langle u_{\mathrm{f}} \,|\, r^2 \,|\, u_{\mathrm{i}} \rangle \sim (3/5) \times R^2 = 5.4\ \mathrm{fm}^2$$

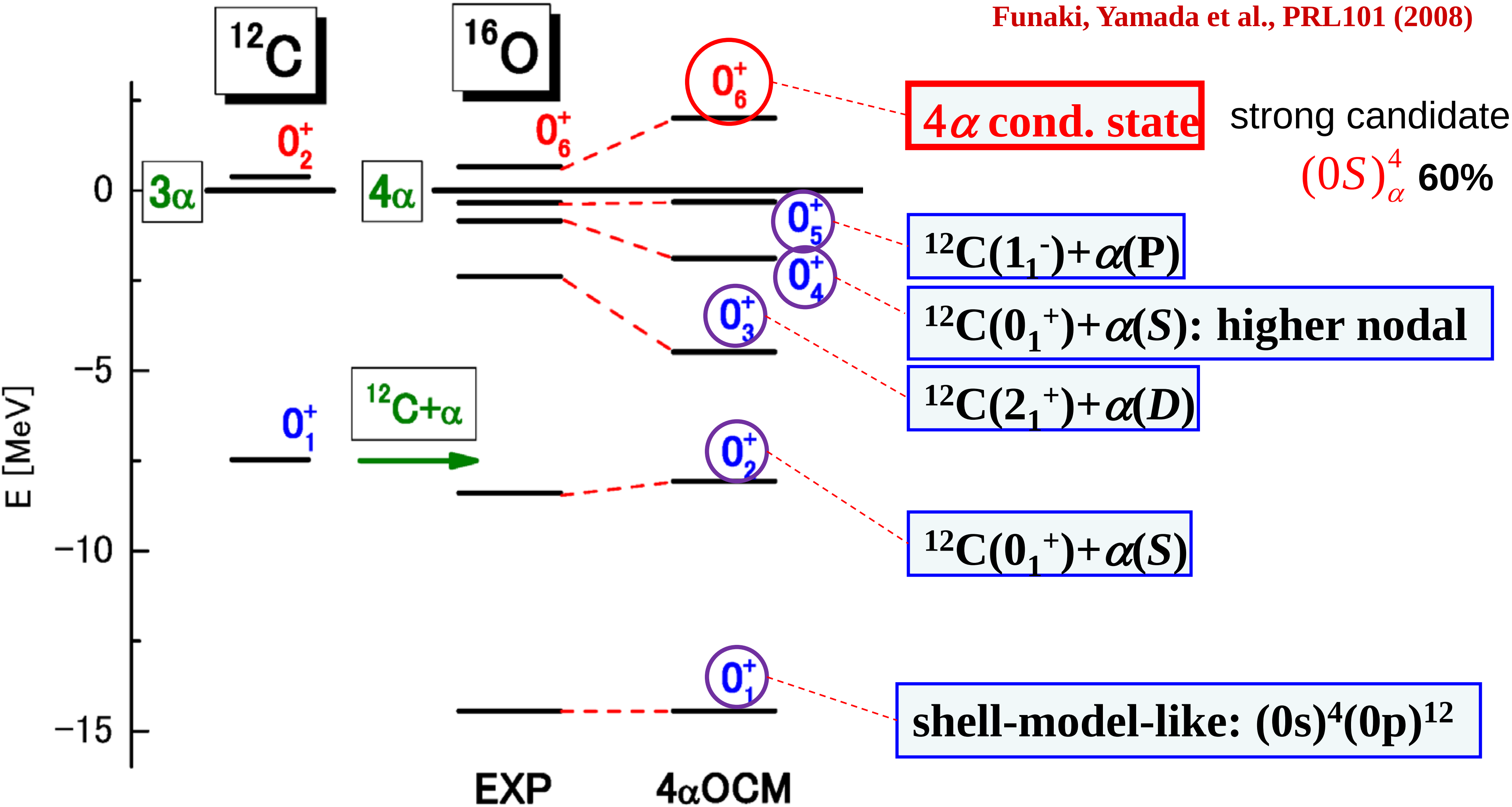
(using R = nuclear radius = 3 fm)

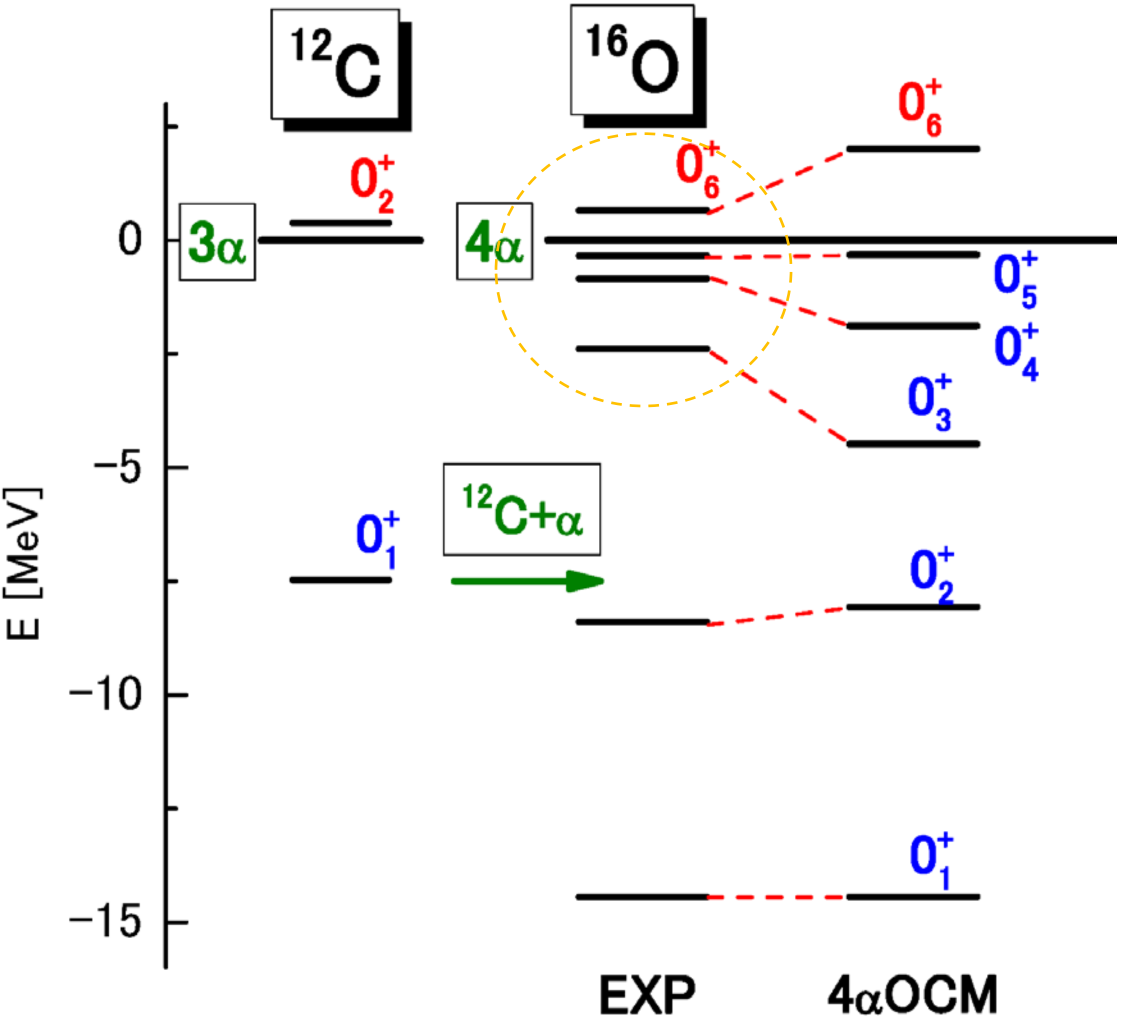
Uniform-density approximation for $u_f(r)$ and $u_i(r)$

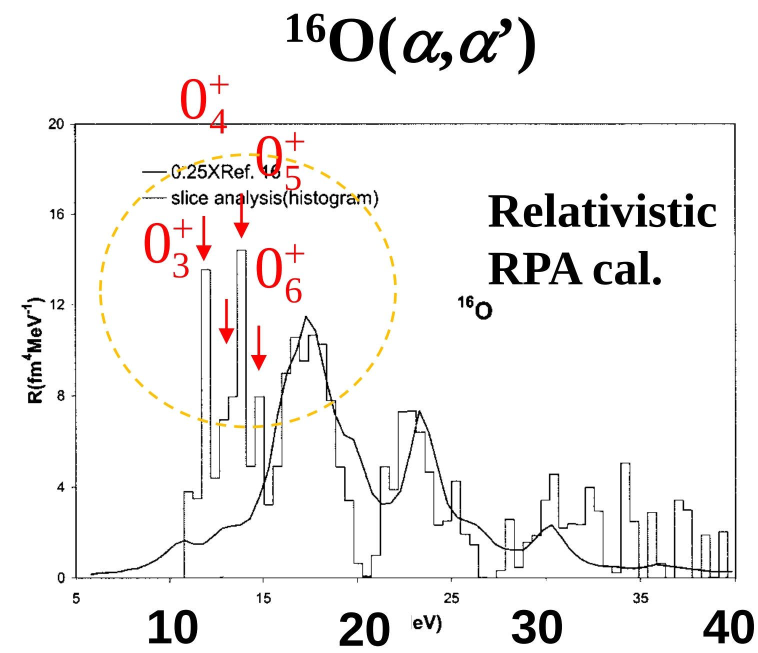
$$u(r) = (3/R^3)^{1/2} \quad \text{for} \quad 0 \le r \le R$$
$$u(r) = 0 \quad \text{for} \quad R < r$$

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)







Exp. condition: E_x > 10 MeV

shifted by 4.2 MeV.

Exp: histogram

Lui et al., PRC 64 (2001)

Cal: real line

Ma et al., PRC 55 (1997)

Multiplied by 0.25

Shifted by 4.2 MeV

IS monopole strength function of ^{16}O

Monopole Strength Function

$$S(E) = \sum_n \delta(E - E_n) |\langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle|^2, \quad \mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$R(E) = \langle 0_1^+ | \frac{\mathcal{O}^\dagger \mathcal{O}}{E - H + i\epsilon} | 0_1^+ \rangle,$$ $\left| \mathbf{0}_{\mathbf{n}}^+ \right\rangle$: **resonance state with E_n - iΓ_n/2**

$$S(E) = -\frac{1}{\pi} \mathrm{Im}[R(E)]$$
$$= \frac{1}{\pi} \sum_n \frac{\Gamma_n / 2}{(E - E_n)^2 + (\Gamma_n / 2)^2} \left| M(0_n^+ - 0_1^+) \right|^2$$

IS-monopole m.e.: $M(0_n^+ - 0_1^+) = \langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle$

Energy weighted sum rule (total-EWSR): **rms radius of ^{16}O**

$$\sum_n (E_n - E_1) \left| M(0_n^+ - 0_1^+) \right|^2 = \frac{2\hbar^2}{m} \times 16 \times R^2,$$

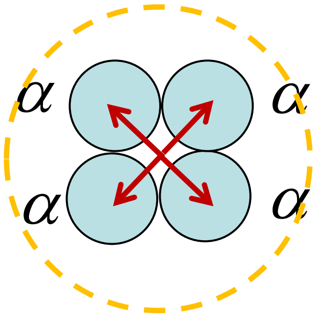
$$R = \sqrt{\frac{1}{16} \left\langle 0_1^+ \left| \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\rm cm})^2 \right| 0_1^+ \right\rangle}$$

Energy weighted sum rule within 4α OCM (OCM-EWSR)

4α OCM and OCM-EWSR

Total w.f. : internal w.f.s of α clusters × relative w.f

$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$



IS monopole matrix element

$$M^{\mathrm{OCM}}(0_n^+ - 0_1^+) = \langle \tilde{\Psi}(0_n^+) | \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}})^2 | \tilde{\Psi}(0_1^+) \rangle,$$

Interesting characters of IS monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$= \sum_{k=1}^{4}\sum_{i=1}^{4} (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 + \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2$$

internal part of each α-cluster

relative parts acting on relative motions of 4α's with respect to c.o.m. of ^{16}O

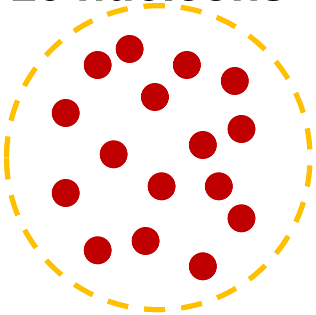
$$= \sum_{i=1}^{4} (\mathbf{r}_i - \mathbf{R}_{\alpha})^2 + \sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{^{12}\mathrm{C}})^2 + 3(\mathbf{R}_{\alpha} - \mathbf{R}_{^{12}\mathrm{C}})^2$$

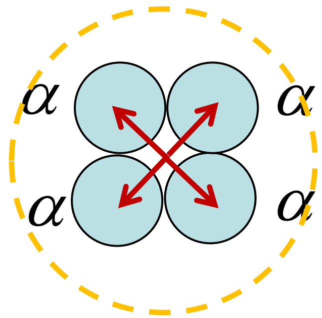
internal part of α

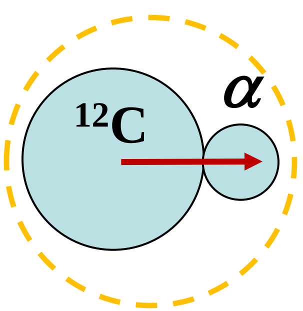
internal part of ^{12}C

relative part acting on relative motion of α and ^{12}C

16 nucleons





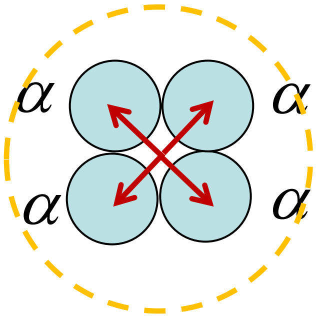


Decompositions into Internal part and relative part play an important role in understanding monopole excitations of ^{16}O.

4α OCM and OCM-EWSR

Total w.f. : internal w.f.s of α clusters × relative w.f

$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$

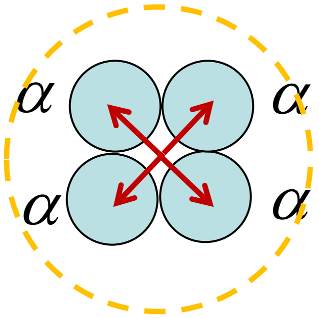


IS monopole matrix element

$$M^{\mathrm{OCM}}(0_n^+ - 0_1^+) = \langle \tilde{\Psi}(0_n^+) | \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}})^2 | \tilde{\Psi}(0_1^+) \rangle,$$

4α OCM and OCM-EWSR

Total w.f. : internal w.f.s of α clusters × relative w.f



$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$

IS monopole matrix element

$$M^{\rm OCM}(0_n^+ - 0_1^+) = \langle \tilde{\Psi}(0_n^+) | \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2 | \tilde{\Psi}(0_1^+) \rangle,$$

$$= \langle \Psi(0_n^+) | \sum_{k=1}^{4} 4(\boldsymbol{R}_{\alpha_k} - \boldsymbol{R}_{\rm cm})^2 | \Psi(0_1^+) \rangle + 16 \times R(\alpha)^2 \delta_{n1},$$

rms radius of α

16 nucleons

relative part

internal part of α clusters

where we used

$$R(\alpha) = \sqrt{\langle \phi(\alpha) | \frac{1}{4} \sum_{i=1}^{4} (\boldsymbol{r}_i - \boldsymbol{R}_\alpha)^2 | \phi(\alpha) \rangle}$$

$$\langle \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) | \sum_{k=1}^{4} \sum_{i=1}^{4} (\boldsymbol{r}_{i+4(k-1)} - \boldsymbol{R}_{\alpha_k})^2 | \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \rangle = 16 \times R(\alpha)^2,$$

4α OCM and OCM-EWSR

Total w.f. : internal w.f.s of α clusters × relative w.f.

$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$

$\phi(\alpha):\ (0s)^4$

EWSR of IS monopole transition in 4α OCM: OCM-EWSR

$$\sum_n (E_n - E_1)\left|M^{\rm OCM}(0_n^+ - 0_1^+)\right|^2 = \frac{1}{2}\langle \Psi(0_1^+)|[\mathcal{O}_{\rm OCM},[\mathcal{H},\mathcal{O}_{\rm OCM}]]|\Psi(0_1^+)\rangle,$$

$$O_{\rm OCM} = \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2,$$

4α OCM and OCM-EWSR

Total w.f. : internal w.f.s of α clusters × relative w.f.

$$\tilde{\Psi}(J^\pi) = \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \times \Psi(J^\pi).$$

$\phi(\alpha):\ (0s)^4$

EWSR of IS monopole transition in 4α OCM: OCM-EWSR

$$\sum_n (E_n - E_1)\left|M^{\rm OCM}(0_n^+ - 0_1^+)\right|^2 = \frac{1}{2}\langle\Psi(0_1^+)|[\mathcal{O}_{\rm OCM},[\mathcal{H},\mathcal{O}_{\rm OCM}]]|\Psi(0_1^+)\rangle,$$

$$O_{\rm OCM} = \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2,$$

$$= \frac{2\hbar^2}{m}\langle\Psi(0_1^+)|\sum_{k=1}^{4} 4(\boldsymbol{R}_{\alpha_k} - \boldsymbol{R}_{\rm cm})^2|\Psi(0_1^+)\rangle,$$

$$= \frac{2\hbar^2}{m} \times 16 \times (R^2 - R(\alpha)^2)$$

← OCM-EWSR

where $R = \sqrt{\frac{1}{16}\left\langle 0_1^+ \left| \sum_{i=1}^{16}(\mathbf{r}_i - \mathbf{R}_{\rm cm})^2 \right| 0_1^+ \right\rangle}$ **: r.m.s radius of ^{16}O**

$R(\alpha)$**: r.m.s radius of α-particle**

4α OCM and OCM-EWSR

Ratio of OCM-EWSR to total EWSR:

$$\frac{\text{OCM-EWSR}}{\text{total EWSR}} = 1 - \left(\frac{R(\alpha)}{R}\right)^2 = 1 - \left(\frac{1.47}{2.58}\right)^2 = 0.68.$$

4α OCM shares over 60% of the total EWSR value.

(c.f. $\alpha+^{12}C$ OCM : 31%)

This is one of important reasons that 4α OCM works rather well in reproducing IS monopole transitions at low-energy region of ^{16}O.

OCM-EWSR: $$\sum_n (E_n - E_1)\left|M^{\rm OCM}(0_n^+ - 0_1^+)\right|^2 = \frac{2\hbar^2}{m} \times 16 \times \left[R^2 - R(\alpha)^2\right]$$

total EWSR: $$\sum_n (E_n - E_1)\left|M(0_n^+ - 0_1^+)\right|^2 = \frac{2\hbar^2}{m} \times 16 \times R^2$$

Yamada et al.

IS monopole strength function $S(E)$ within 4α OCM framework

Monopole Strength Function with 4α OCM

$$S(E) = \sum_n \delta(E - E_n) |\langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle|^2, \quad \mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$R(E) = \langle 0_1^+ | \frac{\mathcal{O}^\dagger \mathcal{O}}{E - H + i\epsilon} | 0_1^+ \rangle,$$

$|\mathbf{0}_{\mathrm{n}}^+\rangle$**: resonance state with** E_{n} **-** $\mathrm{i}\Gamma_{\mathrm{n}}/2$

$$S(E) = -\frac{1}{\pi} \mathrm{Im}[R(E)]$$

$$= \frac{1}{\pi} \sum_n \frac{\Gamma_n / 2}{(E - E_n)^2 + (\Gamma_n / 2)^2} \left| M(0_n^+ - 0_1^+) \right|^2$$

$M(0_n^+ - 0_1^+) = \langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle$ **: calculated by 4α OCM**

$$\Gamma_{\mathrm{n}} = \sqrt{\Gamma_{\mathrm{n}}(\mathrm{OCM})^2 + (\mathrm{exp.\ resolution})^2}$$

50 keV

$\mathbf{E_n}$ **: experimental energy of *n*-th 0+ state**

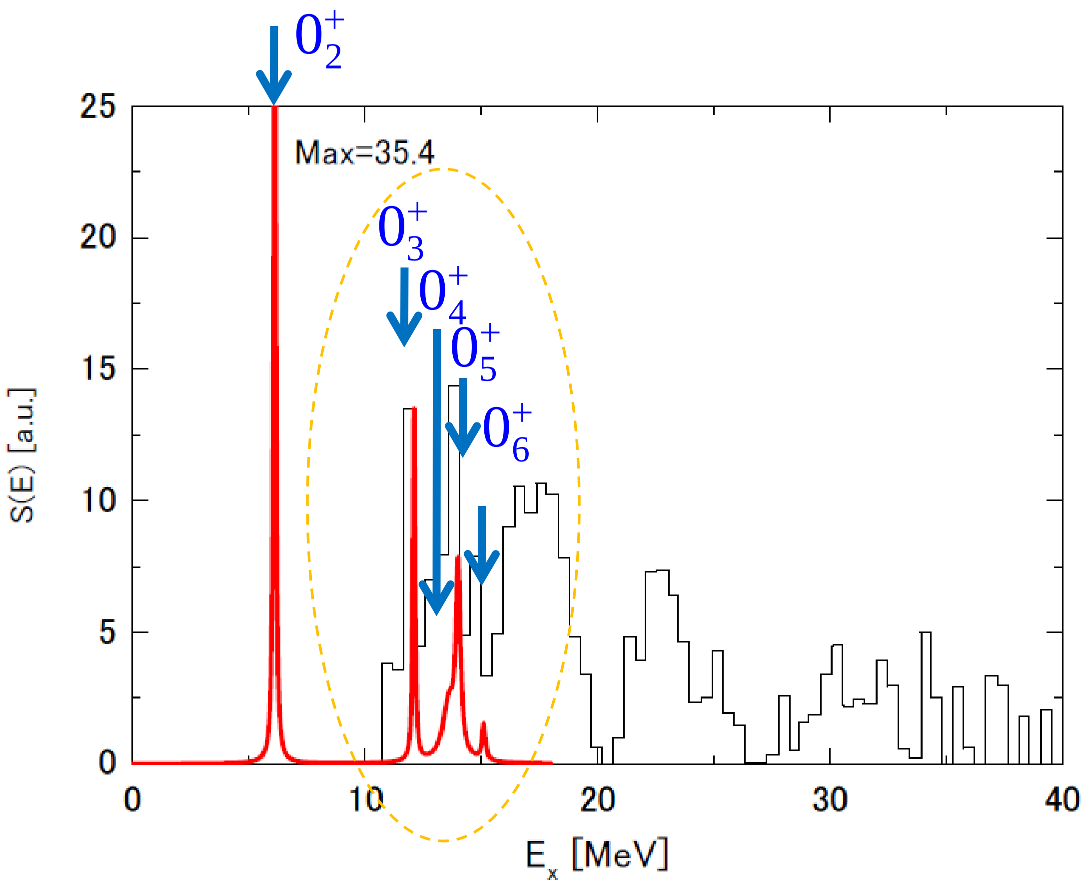
		Experiment				4α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

over 15%
of total EWSR

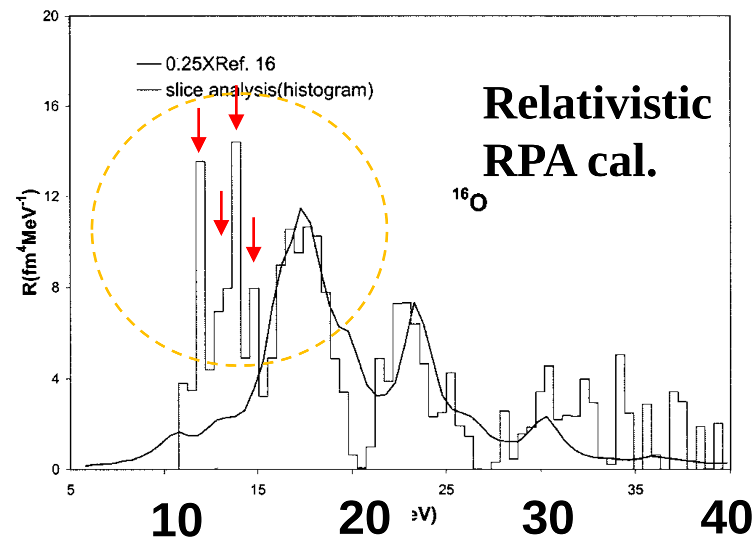
20%
of total EWSR

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



^{16}O(α,α')



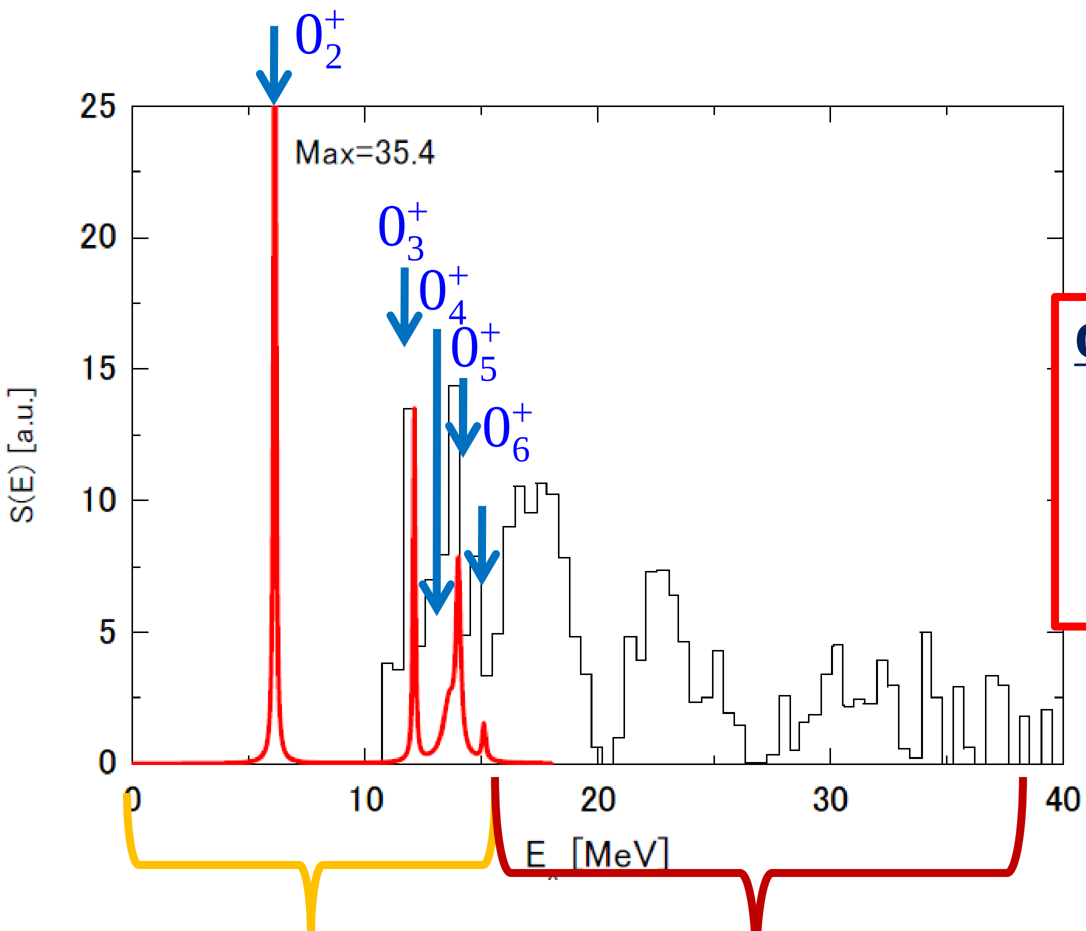
Exp. condition: E_x > 10 MeV

shifted by 4.2 MeV.

It is likely to exist discrete peaks on a small bump at E_x < 15MeV
This small bump may come from the contribution from continuum states of α+^{12}C

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O

(1) α-clustering degree of freedom

(2) mean-field-type one

$(0s)^4(0p)^{12}$: SU(3)(00)= $^{12}C+\alpha$:

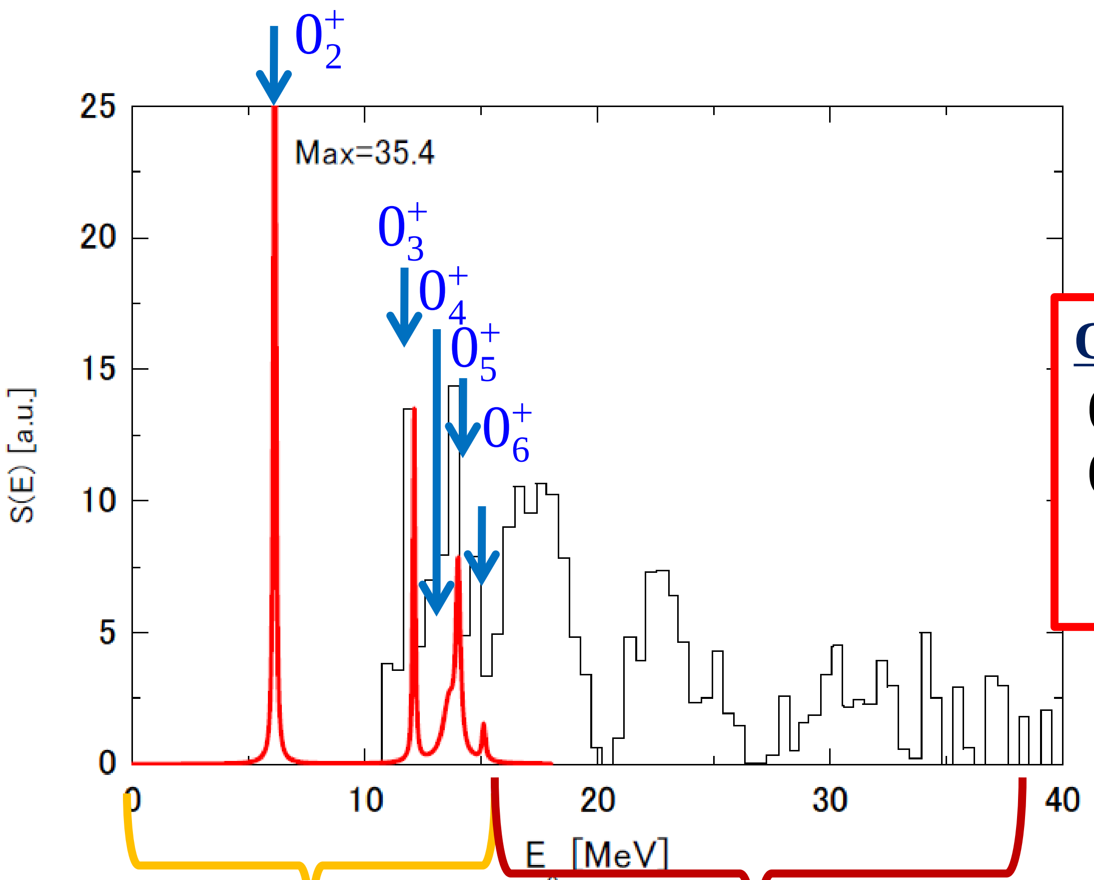
Bayman-Bohr theorem

Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O

(1) α-clustering degree of freedom

(2) mean-field-type one

$(0s)^4(0p)^{12}$: SU(3)(00)= ^{12}C+α :

Bayman-Bohr theorem

next discuss!

Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

Dual nature of ground state of ^{16}O

mean-field character and α-clustering character

Ground state of ^{16}O

(λ,μ)

Dominance of doubly-closed-shell structure: $(0s)^4(0p)^{12}$ **= SU(3)(0,0)**

Cluster-model calculations: 4α OCM, 4α THSR, α+^{12}C OCM,...
Mean-field calculations : RPA, QRPA, RRPA,......

Supported by no-core shell model calculations:
Dytrych et al., PRL98 (2007)

Bayman & Bohr, NPA9 (1958/59)

Bayman-Bohr theorem : **SU(3)[f]($\lambda\mu$) is equivalent to "a cluster-model wf"**

Doubly-closed-shell w.f, $(0s)^4(0p)^{12}$, is mathematically equivalent to a single α-cluster w.f.

This means that the ground state w.f. of ^{16}O originally has an α-clustering degree of freedom together with mean-filed-type degree of free dom.

We call dual nature of g.s.

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm}) = \left(\frac{32\nu}{\pi}\right)^{3/4} \exp(-16\nu \mathbf{R}_{\rm cm}^2)$$

$$\frac{1}{\sqrt{16!}} \det\left|(0s)^4(0p)^{12}\right| \times \left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1}$$ **: closed shell**

$$= N_0 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{40}(\xi_3, 3\nu)\phi_{L=0}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

relative wf (S-wave)

$$= N_2 \sqrt{\frac{12!4!}{16!}} A\left\{\left[u_{42}(\xi_3, 3\nu)\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0} \phi(\alpha)\right\}$$

relative wf (D-wave)

α-degree of freedom

➔G.S. has mean-field-type and α-cluster degrees of freedom.

We call dual nature of g.s.

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

$$\frac{1}{\sqrt{16!}}\det\left|(0s)^4(0p)^{12}\right|\times\left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1}$$: closed shell

c.o.m. w.f. of ^{16}O

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm})=\left(\frac{32\nu}{\pi}\right)^{3/4}\exp(-16\nu\mathbf{R}_{\rm cm}^2)$$

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relative wf (S-wave)

$$=N_2\sqrt{\frac{12!4!}{16!}}A\left\{\left[u_{42}(\xi_3,3\nu)\phi_{L=2}(^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}$$

relative wf (D-wave)

α-degree of freedom

➔G.S. has mean-field-type and α-cluster degrees of freedom.

Excitation of mean-field-type degree of freedom in g.s
➔ 1p1h states (3-bump structure)

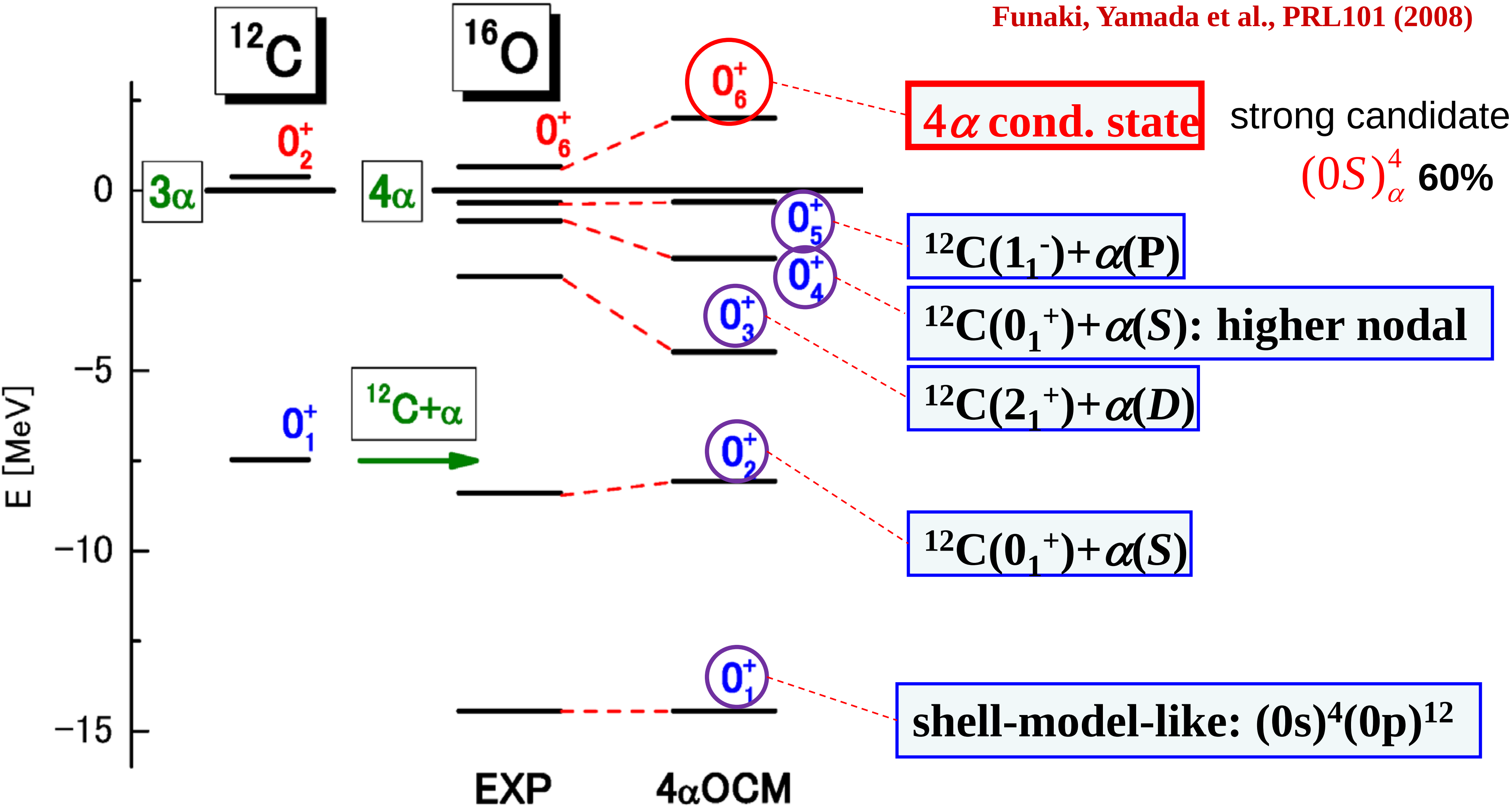
Excitation of α-cluster degree of freedom in g.s
➔ α+^{12}C cluster states: 2nd 0+, 3rd 0+

IS monopole operator

$$\mathcal{O}=\sum_{i=1}^{16}(\boldsymbol{r}_i-\boldsymbol{R}_{\rm cm})^2=\underbrace{\sum_{i=1}^{4}(\mathbf{r}_i-\mathbf{R}_\alpha)^2+\sum_{i=5}^{16}(\mathbf{r}_i-\mathbf{R}_{12C})^2}_{\text{internal parts}}+\underbrace{3(\mathbf{R}_\alpha-\mathbf{R}_{12C})^2}_{\text{relative part}}$$

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Monopole excitation to $0^+_{2,3}$ and 0^+_6 state

$0^+_{2,3}$ states: $^{12}C(0_1^+,2_1^+)+\alpha$ main configuration

Bayman-Bohr theorem:

$(0s)^4(0p)^{12}$ has an α + $^{12}C(0_1^+,2_1^+)$ degree of freedom

relative motion (α-^{12}C) is excited by IS monopole operator

0^+_6 state: **4α-gas like structure** main configuration

Bayman-Bohr theorem:

$(0s)^4(0p)^{12}$ has a 4α degree of freedom

relative motion among 4α is excited by IS monopole operator

Bayman-Bohr theorem

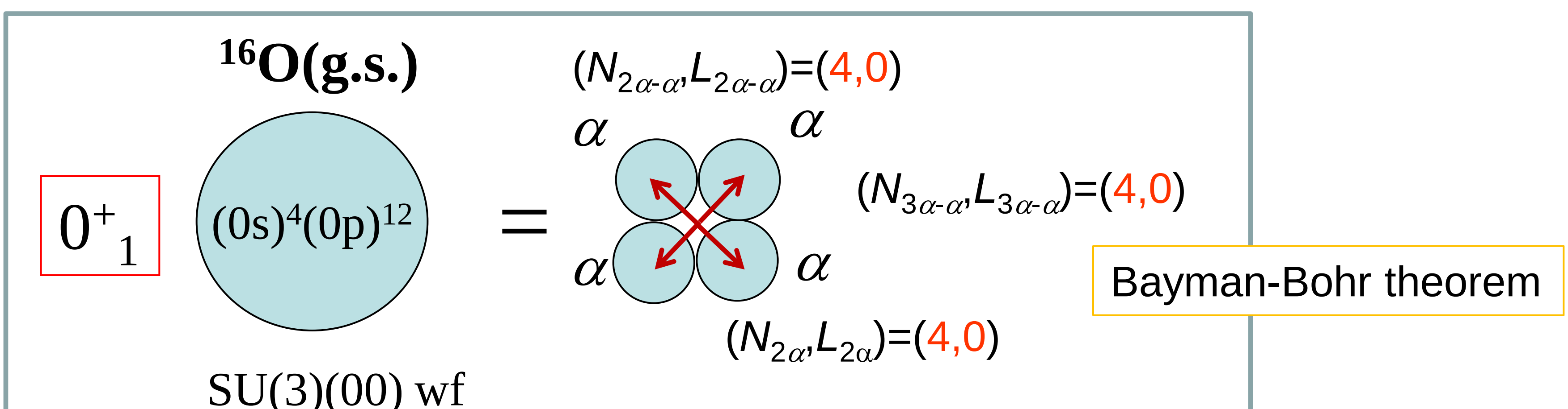
$$\frac{1}{\sqrt{16!}}\det|(0s)^4(0p)^{12}| \times [\phi_{\rm cm}(\boldsymbol{R}_{\rm cm})]^{-1}$$

: closed shell

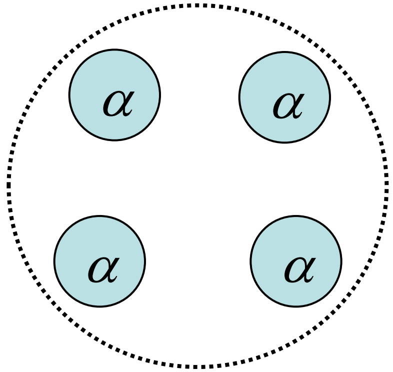
$$= \hat{N}_0\sqrt{\frac{4!4!4!4!}{16!}}\mathcal{A}\Big\{ \Big[u_{40}(\boldsymbol{\xi}_3, 3\nu)\Big[u_{40}(\boldsymbol{\xi}_2, \frac{8}{3}\nu)u_{40}(\boldsymbol{\xi}_1, 2\nu)\Big]_{L=0}\Big]_{J=0} \times\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\Big\}$$

4α-cluster wf

➔ **G.S. has a 4α-cluster degree of freedom.**



4α gas-like or $\alpha+{}^{12}$C(Hoyle) $\approx$ 0^+_6 state



4α -gas-like

M(E0)=1.0 fm^2 by 4α OCM

Monopole transition

$$\mathcal{O} = \sum_{i=1}^{16} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$: monopole operator

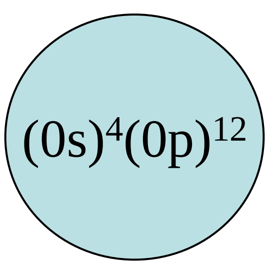
$$= \sum_{k=1}^{4}\sum_{i=1}^{4} (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 + \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2$$

internal part | **relative part**

coherent excitation

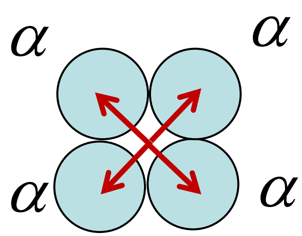
^{16}O(g.s.)

0^+_1



=

$(N_{2\alpha\text{-}\alpha}, L_{2\alpha\text{-}\alpha})=(4,0)$



$(N_{3\alpha\text{-}\alpha}, L_{3\alpha\text{-}\alpha})=(4,0)$

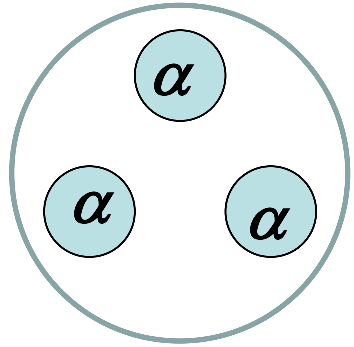
$(N_{2\alpha}, L_{2\alpha})=(4,0)$

SU(3)(00) wf

Bayman-Bohr theorem

^{12}C

3α gas-like = 0^+_2 (Hoyle) state



$$\mathcal{O} = \sum_{i=1}^{12} (\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm})^2$$

$$= \sum_{k=1}^{3} \sum_{i=1}^{4} (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2$$ internal part

$$+ \sum_{k=1}^{3} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2$$ relative part

portion of EWSR

$M(E0) = 5.4 \pm 0.2\ \text{fm}^2\ (16\%)$

Monopole transition

coherent excitation

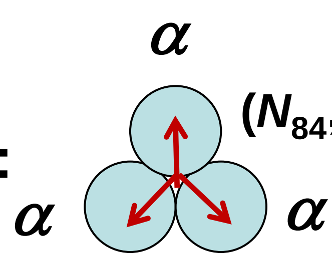
^{12}C(g.s.)

0^+_1

$(0s)^4(0p)^8$

SU(3)(04) wf

=



$(N_{84}, L_{84}) = (4,0)$

$(N_{44}, L_{44}) = (4,0)$

Yamada et al., PTP120 (2008)

Bayman-Bohr theorem

Dominance of SU(3)(04) : no-core shell model by Dytrych et al., PRL98 (2007)

^{12}C

Bayman-Bohr theorem

$$\phi_{J=0}(^{12}\mathrm{C}) = \left| (0s)^4 (0p)^{12}; (\lambda,\mu)=(0,4), J^{\pi}=0^+ \right\rangle_{\mathrm{internal}}$$ **: SU(3) wf**

$$= N_0 \sqrt{\frac{4!4!4!}{12!}} A\left\{ \left[u_{40}(\xi_2, \frac{8}{3}\nu) u_{40}(\xi_1, 2\nu) \right]_{J=0} \phi(\alpha)\phi(\alpha)\phi(\alpha) \right\}$$

3α-cluster wf

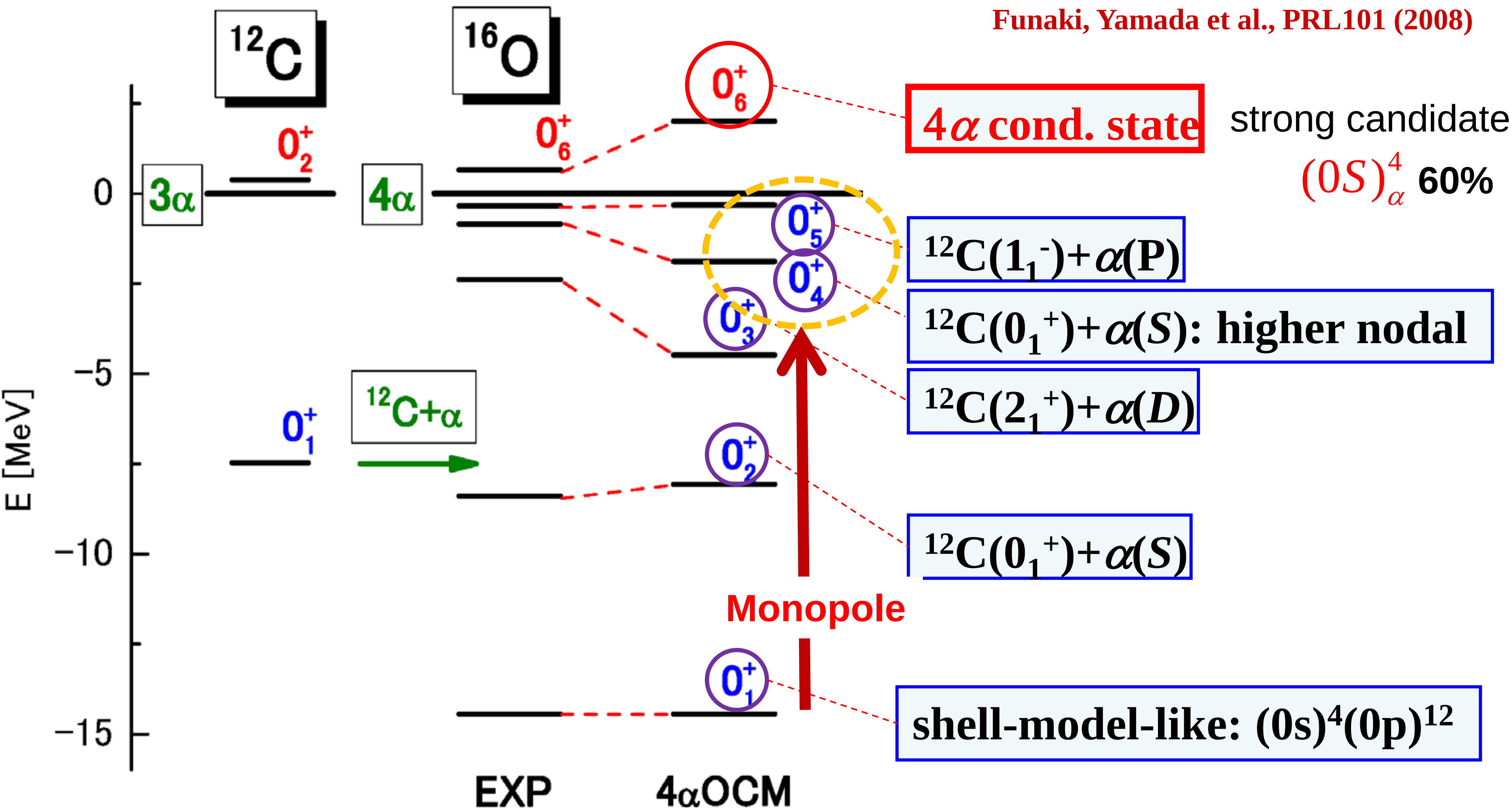
Dominance of SU(3)(04) in ^{12}C(g.s): confirmed by no-core shell model Dytrych et al., PRL98 (2007)

Yamada et al., PTP120 (2008)

This means that the ground state w.f. of ^{12}C originally has an α-clustering degree of freedom together with single-particle degree of free dom.

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Monopole excitation to 0^+_5 and 0^+_4 state

0^+_5 state: ^{12}C(1_1^-)+α(P) main configuration

Bayman-Bohr theorem:

$(0s)^4(0p)^{12}$ has no configuration of ^{12}C(1-)+α

Why this state is excited?

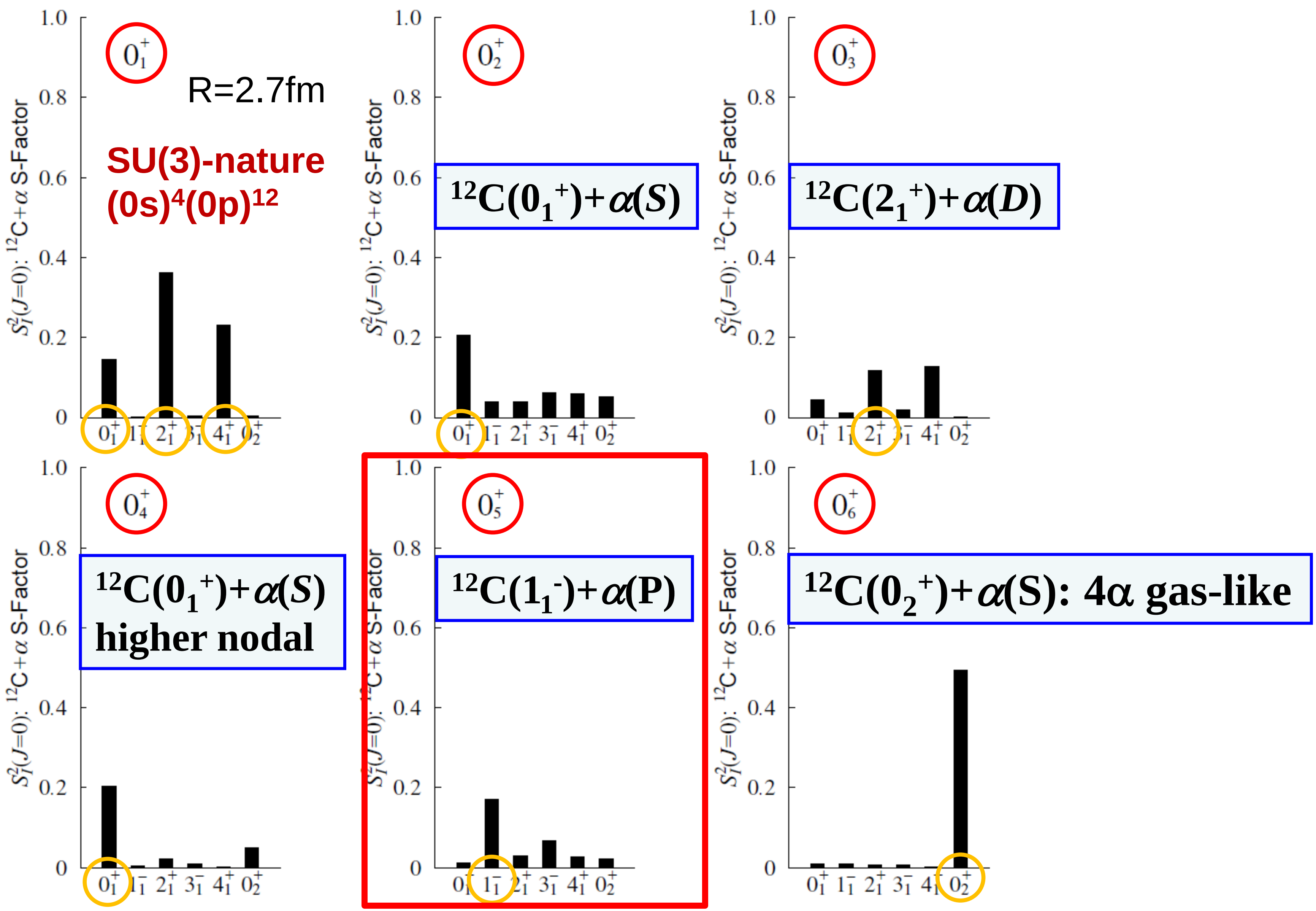
Coupling with ^{12}C(0+,2+)+α and ^{12}C(Hoyle)+α configuration

Coherent contribution from these configurations

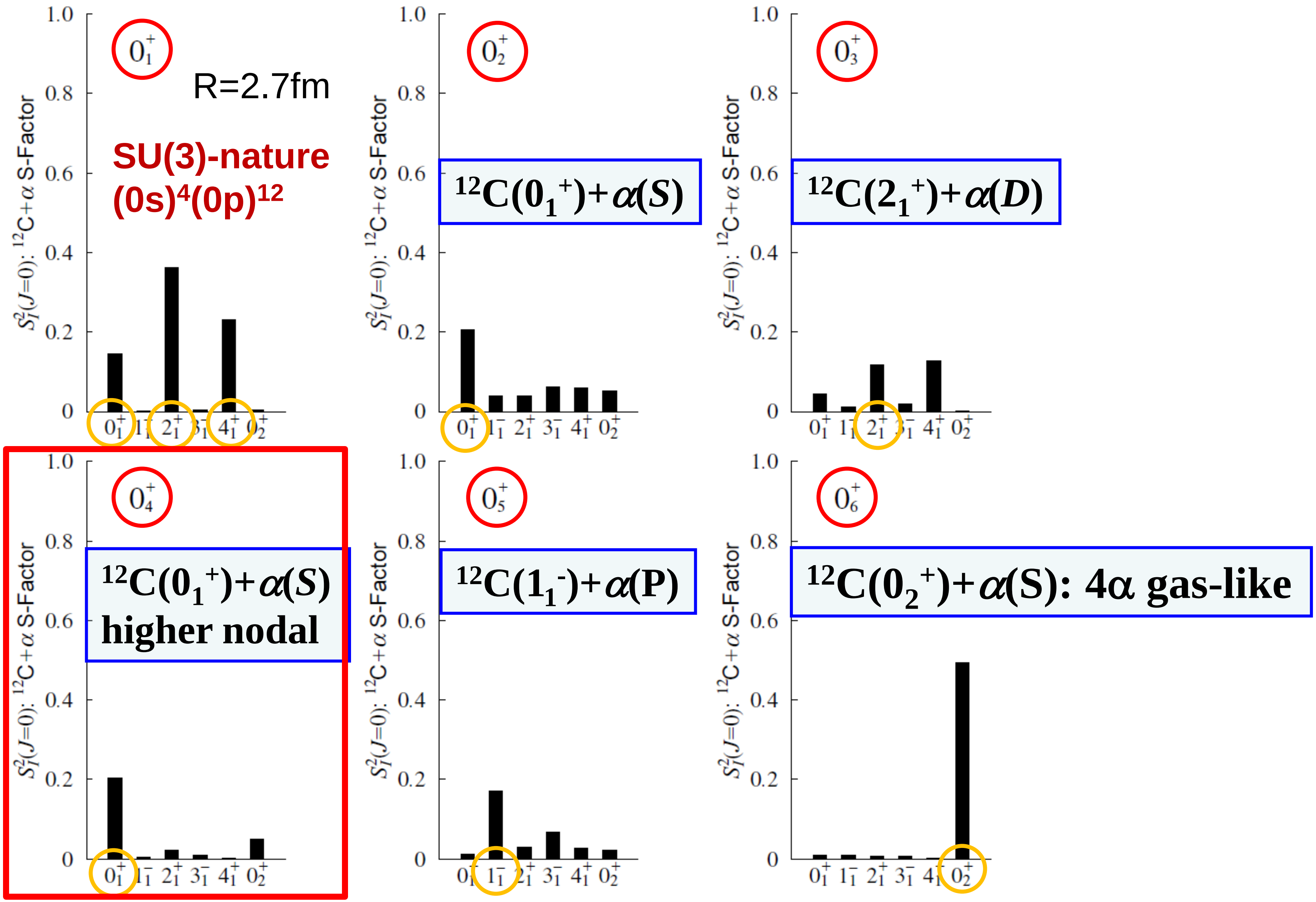
0^+_4 state: ^{12}C(0_1^+)+α(S): higher nodal

Coherent contributions from
^{12}C(0+,2+)+α and ^{12}C(Hoyle)+α configurations

S^2-factors of α+^{12}C(L^π) channels in 0^+ states of ^{16}O

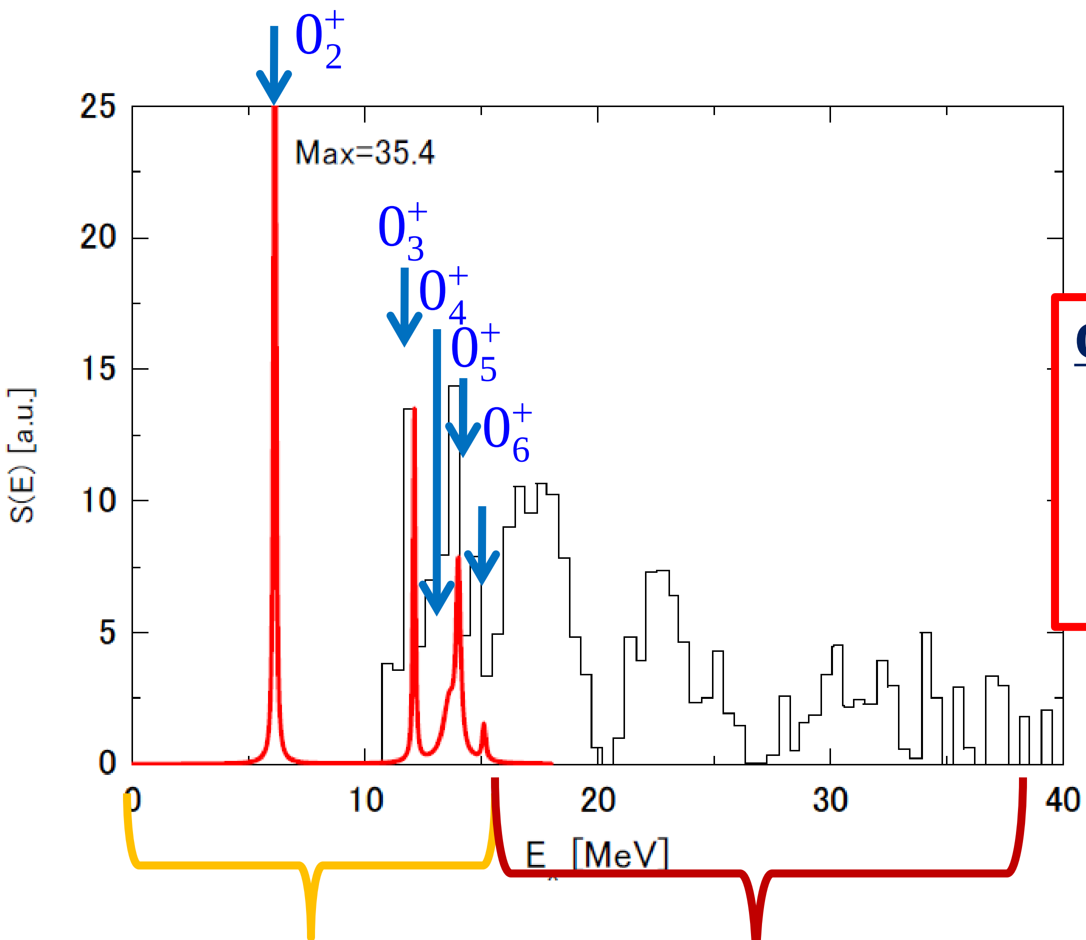


S^2-factors of $\alpha+^{12}C(L^\pi)$ channels in 0^+ states of ^{16}O



Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O

(1) α-clustering degree of freedom

(2) mean-field-type one

$(0s)^4(0p)^{12}$: SU(3)(00)= ^{12}C+α :

Bayman-Bohr theorem

Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

Dual nature of the ground states in ^{12}C and ^{16}O : common to all N=Z=even light nuclei

^{20}Ne, ^{24}Mg, ^{32}S, ….., ^{44}Ti,….

^{20}Ne

$$\Phi_J(^{20}\mathrm{Ne}) = |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\mathrm{internal}}$$ **: SU(3) wf**

$$= N_J\sqrt{\frac{4!16!}{20!}}\mathcal{A}\left\{\underline{u_{8J}(\boldsymbol{r}_{\alpha-^{16}\mathrm{O}})}\phi(\alpha)\phi(^{16}\mathrm{O})\right\}$$ **: cluster wf**

relative wf (J-wave)

Excitation of mean-field degree of freedom

➔ $K^\pi = 2^-$ band : $5p1h$ state

Excitation of α-cluster degree of freedom

➔ **α+^{16}O cluster states of $K^\pi = 0^+_4, 0^-$ bands**

$K^\pi = 0^+_4$ band : higher nodal states

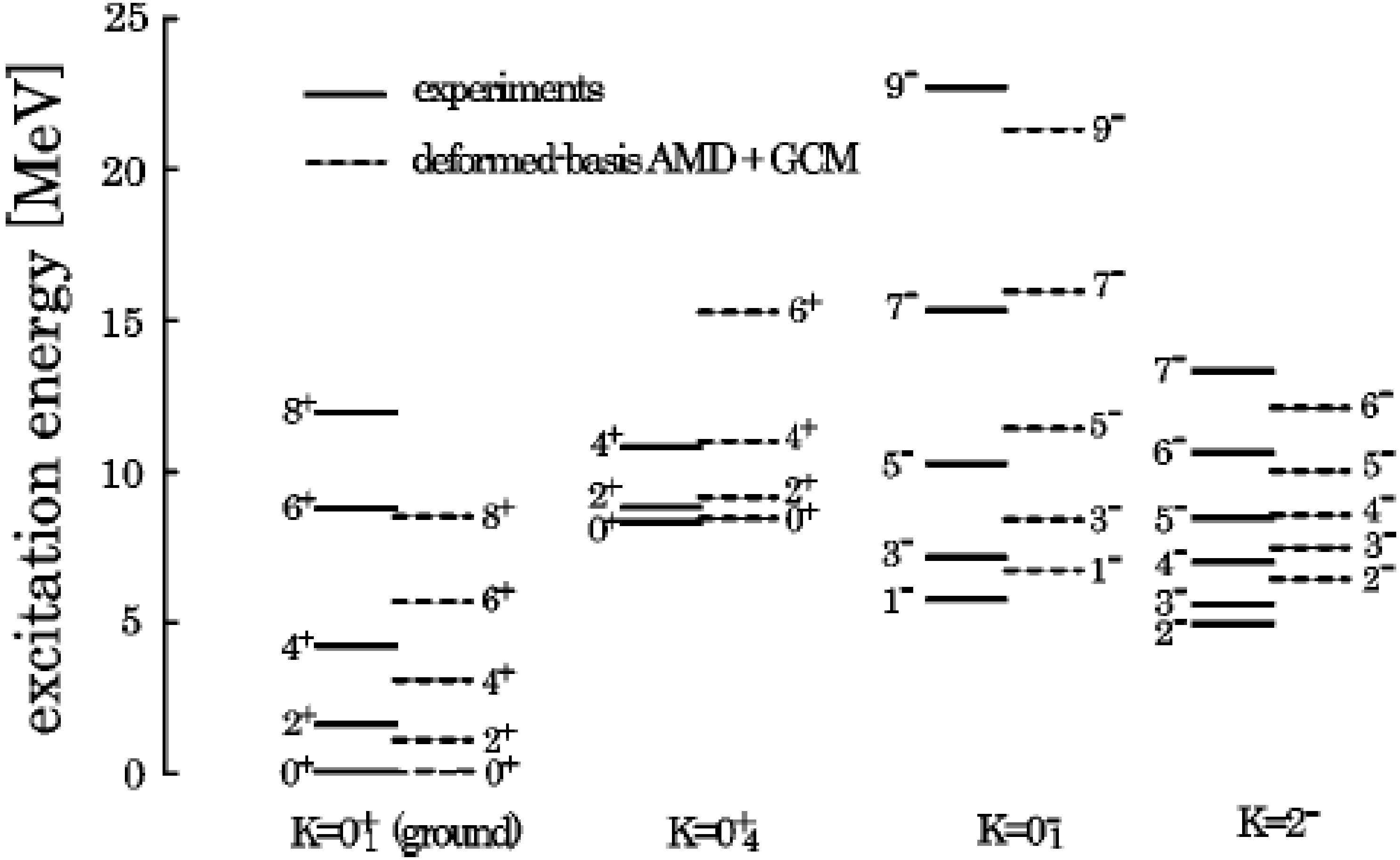
α+^{16}O comp. = 80% for low spins: Q=10 quanta

$K^\pi = 0^-$ band : parity − doublet states

Almost pure α+^{16}O structures for low spins: Q=9 quanta

^{20}Ne

AMD+GCM



M. Kimura, PRC69, 044319 (2004)

Two features of IS monopole excitation

Cluster states at low energy,
Mean-field-type states at higher energy

These features will persist in other light nuclei.

We predict that increasing mass number, the two features will gradually be vanishing.

Because,

effect of spin-orbit forces becomes stronger:

some SU(3) symmetries are mixed in g.s.

➔ Goodness of nuclear SU(3) symmetry: gradually disappearing.

This means that dual nature of g.s is corroding with increasing mass number.

➔ Two features of IS monopole excitation will be vanishing.

Eventually only 1p1h-type collective motions are strongly excited.

Hope: Systematic experiments!!

Summary

- **α-condensation in ^{12}C, ^{16}O, heavier 4n nuclei.**
- **Hoyle-analog states in ^{11}B, ^{13}C**
- **IS monopole tran. : useful to search for cluster states**

  ↔ B(E2): nuclear deformation (Rainwater)
- **IS monopole excitations have two features: ^{16}O (typical)**

  (i) α-cluster type: discrete peaks at $E_x \leq 15$ MeV

  (ii) mean-field type: 3-bump structure (18,23,30 MeV)
- **The origin: Dual nature of the ground state of ^{16}O.**

  G.S. has mean-field and α-cluster degrees of freedom

  : $(0s)^4(0p)^{12}$ = SU(3)(00) = α + ^{12}C w.f. by Bayman-Bohr theorem
- **Dual nature is common to all N=Z=even light nuclei**
- **Two features of IS monopole excitations will persist in other nuclei. Hope experiments.**