Dynamics and Correlations in Exotic Nuclei

YITP, Kyoto, 9/20 – 10/28, 2011

Continuum Skyrme Hartree-Fock-Bogoliubov theory with Green's function method for exotic nuclei

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Peking University

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Outline

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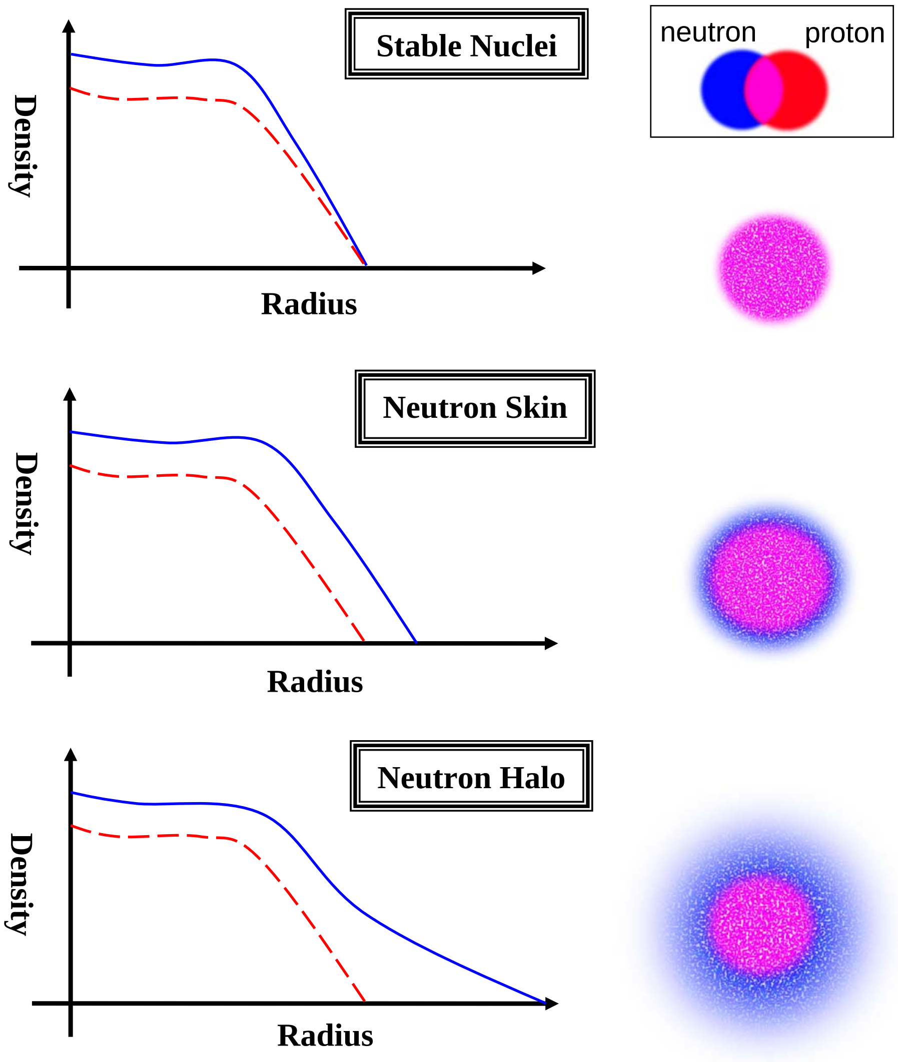
Introduction — *pair correlation in the weakly bound nuclei*

★ Exotic phenomena in nuclear physics



Tanihata PRL 55(1985)2676,
Hansen ARNPS 45(1995)591,
Jensen RMP 76(2004)215,...

Features:

- ✓ weakly bound
- ✓ large spacial density distribution
- ✓ sudden increase of the rms radius deviated from $A^{1/3}$ law

- ★ pair correlation
- ★ continuum effects

Our interests:
To investigate the pair correlation effects in the weakly bound neutron-rich nuclei

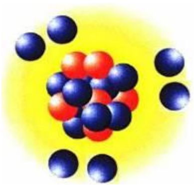
Introduction — *Pairing correlation effects to the weakly bound nuclei*

1-neutron halo: ^{11}Be, ^{19}C

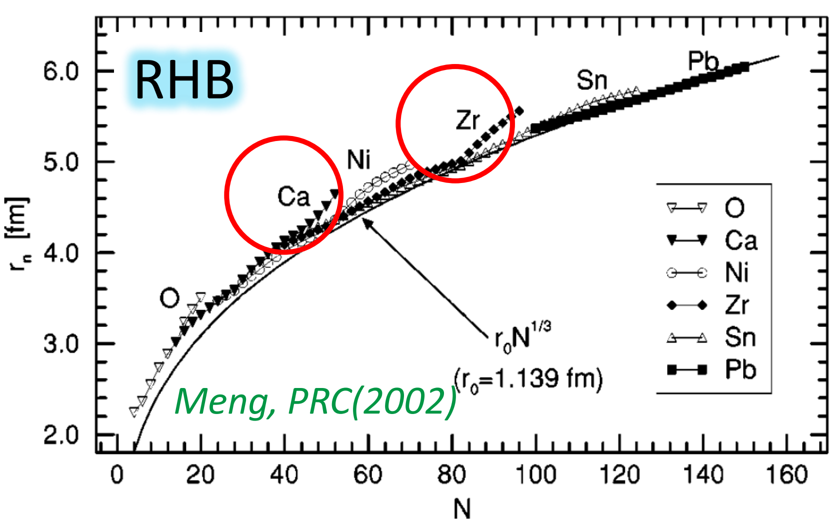
2-neutron halo: ^{6}He, ^{11}Li , ^{14}Be

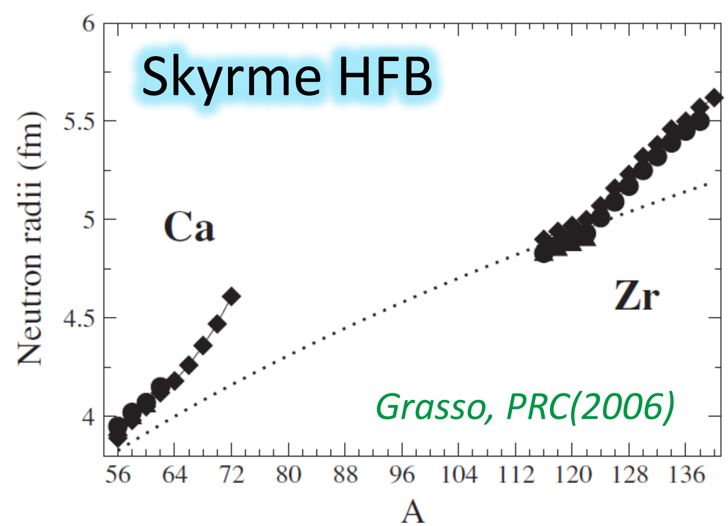
Exp: *Tanihata, PRL(1985)*
Jonson, PR(2004), …

The: *Meng, PRL(1996)* (RCHB)
Kuo, PRL(1997) (shell model)
Jensen, RMP(2004) (few body)
Tomaselli, PPNP(2007) (cluster),
…



neutron halo in heavier nuclei (*giant halo*): Ca, Zr





Meng, PRL(1998), PRC(2002), PPNP(2006); Sandulescu, PRC(2003); Zhang, SCS(2003); Geng, MPLA(2004); Long, PRC(2010)…

Grasso, PRC(2006); Terasaki, PRC(2006); Rotival, PRC(2009), …

Introduction — *Continuum contribution to the pair correlation*

★ **Weakly bound and unbound orbits with low angular momentum**

Since the centrifugal barrier is lower, the probability of the single-particle wave functions to stay inside the core nucleus decrease considerably as the binding energy becomes smaller.

- Decouple from the pair field ⇒ do not contribute to the pair correlation
  Hamamoto PRC(2003,2005,2006)
- Persistent contribution to the pair correlation
  Oba PRC(2009), Sagawa and Hagino, ...

⟹ **Different opinions!**

Example: p orbits in $N = 86$ isotones Sn∼Mo region ($Z = 50 \sim 42$), $\varepsilon = -0.5 \sim 0.7$ MeV

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Example: p orbits in $N = 86$ isotones Sn$\sim$Mo region ($Z = 50 \sim 42$), $\varepsilon = -0.5 \sim 0.7$ MeV

In order to clarify the pair correlation effects in the weakly bound nuclei

we should deal with the pair correlation and the continuum effect properly, which especially requires to describe the appropriate asymptotic behavior for density distribution in the coordinate space.

Introduction — *continuum HFB with Green's function method*

★ **Candidate theory for the description of exotic nuclei**

Hartree-Fock-Bogoliubov (HFB) theory: Bogoliubov quasiparticle ⇒ unified description of both the Hartree-Fock (HF) & pair correlation

- Non-relativistic: Skyrme *Dobaczewski NPA(1984)*
- Relativistic: covariant density functional theory

  Meng PRL(1996), Zhou PRC(2003,2010), Meng PPNP(2006), Long PRC(2010)

Discretized HFB: bad asymptotic behavior, no width for the q.p. resonance!

Green's function method — treat the continuum exactly

- Relativistic Random Phase Approximation (RPA)

  Shlomo NPA(1975), Daoutidis PRC(2009), Yang PRC(2010)
- Hartree-Fock-Bogoliubov (HFB)
  - Woods-Saxon HF and pair potential *Belyaev SJNP(1987)*
  - Axially deformed Woods-Saxon HF potential with selfconsistent pair potential (DDDI pairing interaction) *Oba PRC(2009)*
- HFB+RPA (QRPA) *Matsuo NPA(2001)*

My work: Y. Zhang, M. Matsuo, J. Meng, PRC **83**, 054301 (2011)

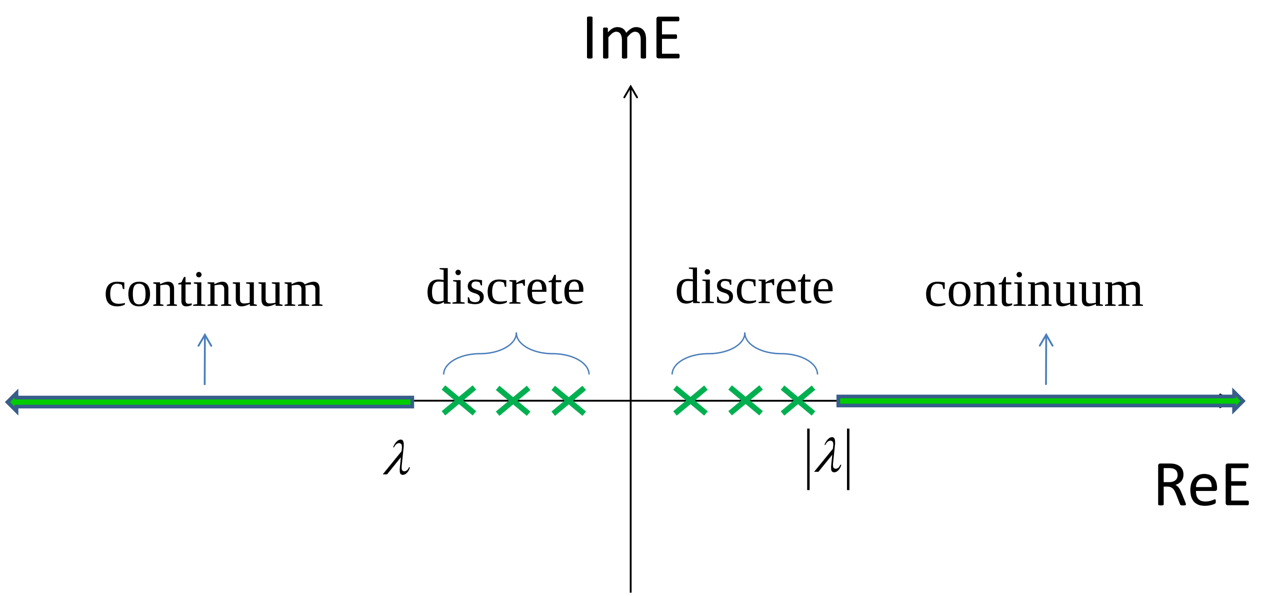
self-consistent continuum Skyrme HFB approach with Green's function ⇒ exotic nuclei

Outline

Hartree-Fock-Bogoliubov (HFB) theory

★ **HFB equation in the coordinate space**

$$\int d\mathbf{r}' \sum_{\sigma'} \begin{pmatrix} h(\mathbf{r}\sigma, \mathbf{r}'\sigma') - \lambda\delta(\mathbf{r}-\mathbf{r}')\delta_{\sigma\sigma'} & \tilde{h}(\mathbf{r}\sigma, \mathbf{r}'\sigma') \\ \tilde{h}^*(\mathbf{r}\tilde{\sigma}, \mathbf{r}'\tilde{\sigma}') & -h^*(\mathbf{r}\tilde{\sigma}, \mathbf{r}'\tilde{\sigma}') + \lambda\delta(\mathbf{r}-\mathbf{r}')\delta_{\sigma\sigma'} \end{pmatrix} \begin{pmatrix} \varphi_{1,i}(\mathbf{r}'\sigma') \\ \varphi_{2,i}(\mathbf{r}'\sigma') \end{pmatrix} = E_i \begin{pmatrix} \varphi_{1,i}(\mathbf{r}\sigma) \\ \varphi_{2,i}(\mathbf{r}\sigma) \end{pmatrix} \quad (1)$$



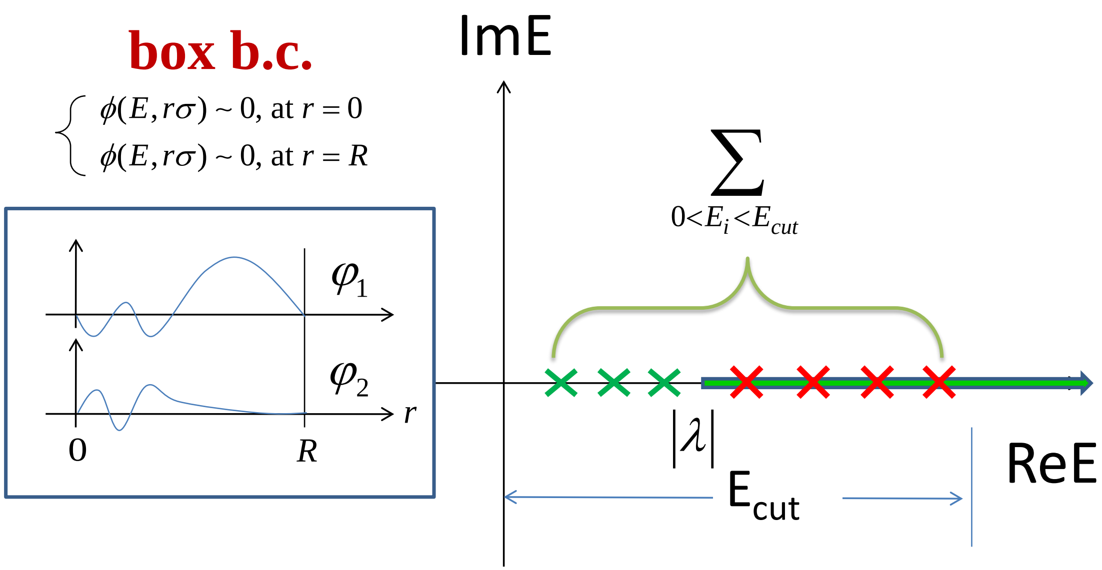
The particle and pair density matrices should be calculated as

$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \sum_{0<E_i<|\lambda|} \varphi_2(\mathbf{r}\sigma, E_i)\varphi_2^*(\mathbf{r}'\sigma', E_i) + \int_{|\lambda|}^{\infty} dn(E)\varphi_2(\mathbf{r}\sigma, E)\varphi_2^*(\mathbf{r}'\sigma', E), \quad (2a)$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = -\sum_{0<E_i<|\lambda|} \varphi_2(\mathbf{r}\sigma, E_i)\varphi_1^*(\mathbf{r}'\sigma', E_i) - \int_{|\lambda|}^{\infty} dn(E)\varphi_2(\mathbf{r}\sigma, E)\varphi_1^*(\mathbf{r}'\sigma', E). \quad (2b)$$

Discretized HFB approach

★ **Density matrices obtained by discretized HFB approach**



$$\rho(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \sum_{0<E_i<E_{\text{cut}}} \varphi_{2,i}(\mathbf{r}\sigma, E_i)\varphi^*_{2,i}(\mathbf{r}'\sigma', E_i) \quad (3a)$$

$$\tilde{\rho}(\mathbf{r}\sigma, \mathbf{r}'\sigma') = -\sum_{0<E_i<E_{\text{cut}}} \varphi_{2,i}(\mathbf{r}\sigma, E_i)\varphi^*_{1,i}(\mathbf{r}'\sigma', E_i) \quad (3b)$$

Problems:

- **BAD** asymptotic behavior for the density distribution
- **CANNOT** provide the width information for the quasiparticle resonance

Continuum HFB approach with Green's function

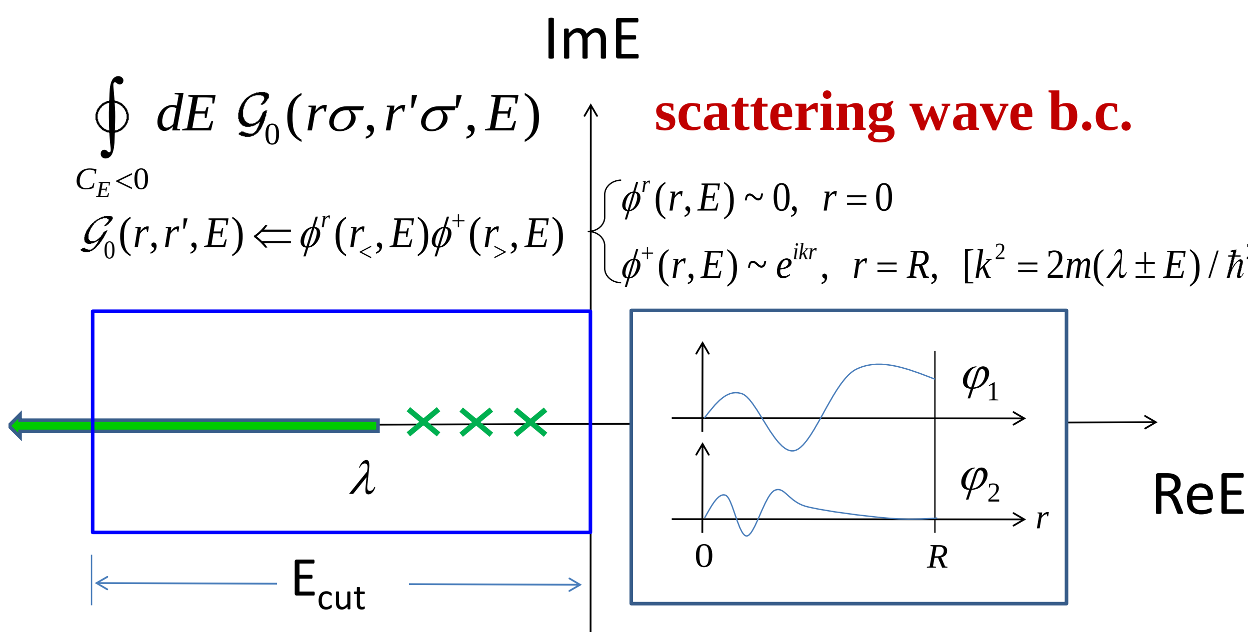
★ **Density matrices obtained by continuum HFB approach**

$$(E-\mathcal{H})\mathcal{G}_0(\mathbf{r}\sigma,\mathbf{r}'\sigma',E) = \delta(\mathbf{r}-\mathbf{r}')\delta_{\sigma\sigma'}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

$$\mathcal{G}_0 = \sum_i \frac{\phi_i(\mathbf{r}\sigma)\phi_i^\dagger(\mathbf{r}'\sigma')}{E-E_i} + \frac{\bar{\phi}_{\tilde{i}}(\mathbf{r}\sigma)\bar{\phi}_{\tilde{i}}^\dagger(\mathbf{r}'\sigma')}{E+E_i} \Longrightarrow$$

$$\rho(\mathbf{r}\sigma,\mathbf{r}'\sigma') = \frac{1}{2\pi i}\oint_{C_{E<0}} dE\ \mathcal{G}_0^{11}(\mathbf{r}\sigma,\mathbf{r}'\sigma',E),$$

$$\tilde{\rho}(\mathbf{r}\sigma,\mathbf{r}'\sigma') = \frac{1}{2\pi i}\oint_{C_{E<0}} dE\ \mathcal{G}_0^{12}(\mathbf{r}\sigma,\mathbf{r}'\sigma',E)$$



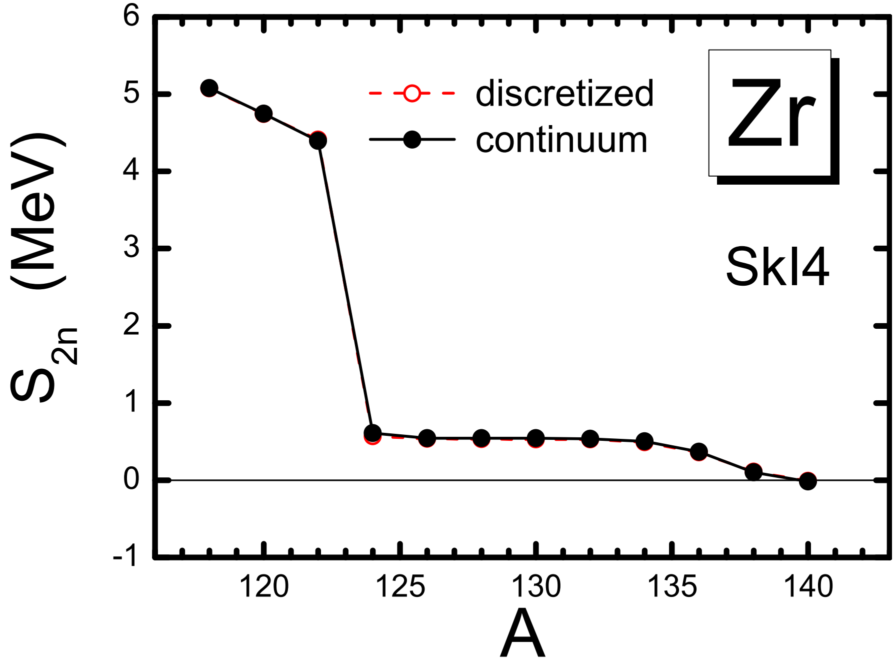
Advantages:

- **PROPER** asymptotic behavior for the density distribution
- **CAN** provide the width information for the quasiparticle resonance

Outline

Zirconium isotopes

★ **Giant Halo phenomenon**



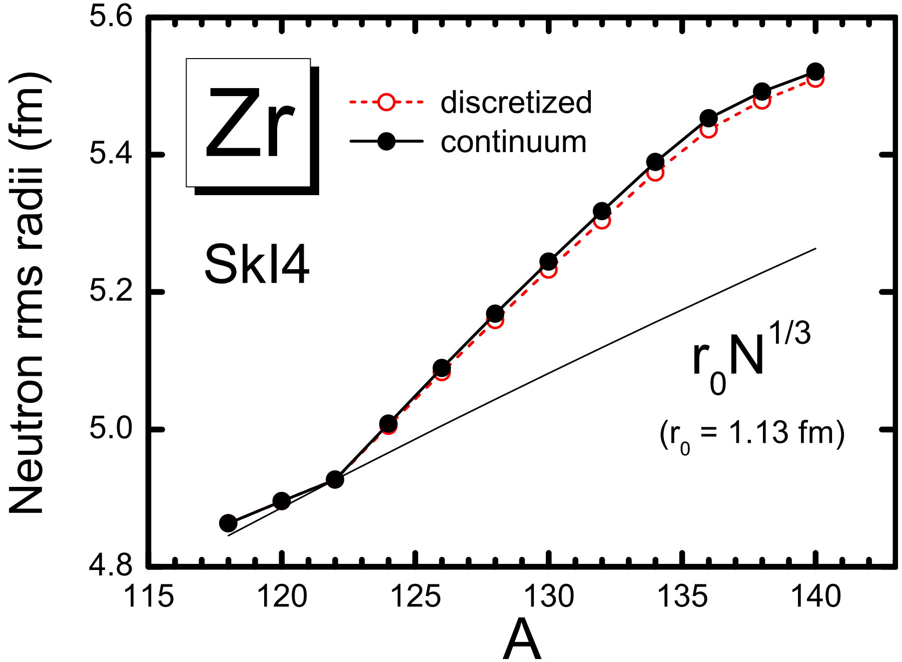


Figure: Two-neutron separation energies S_{2n} (left) and neutron rms radius (right) for Zr isotopes obtained by discretized and continuum Skyrme HFB approach with SkI4 parameter set.

Almost the same bulk information between the two.

Zirconium isotopes

★ **Densities by discretized and continuum HFB approach**

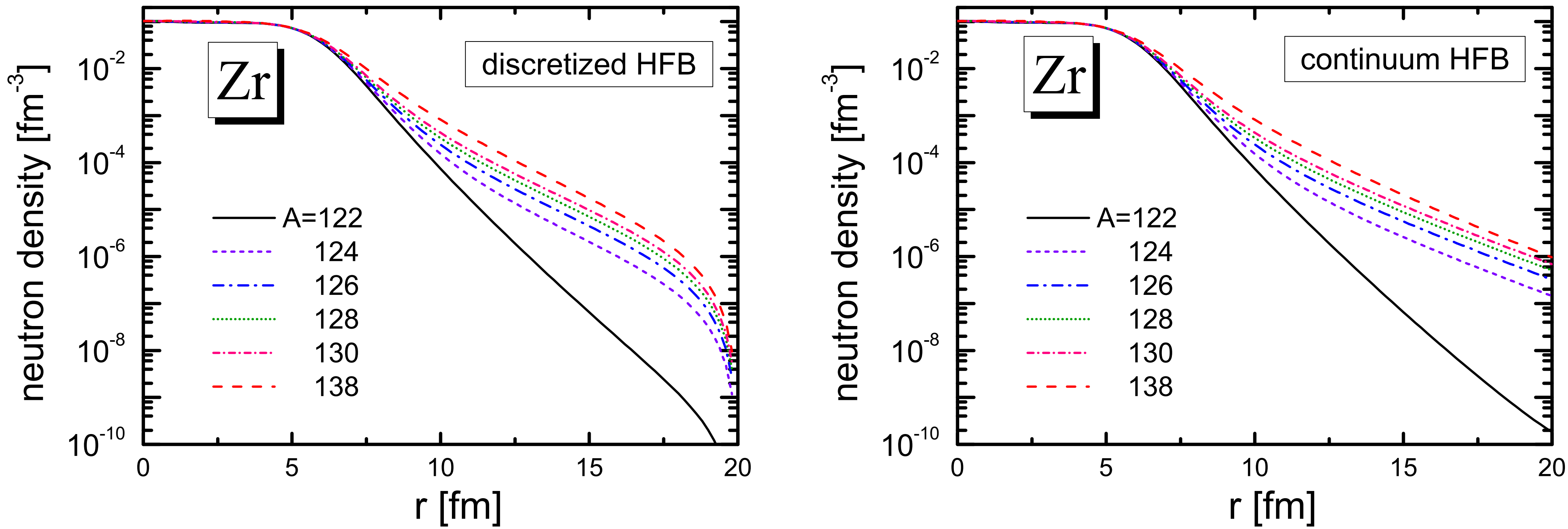


Figure: Neutron density for Zr isotopes obtained by discretized (left) and continuum (right) Skyrme HFB approach using Green's function method with SkI4 parameter set.

Proper asymptotic behavior for the continuum Skyrme HFB approach.

Zirconium isotopes — *quasiparticle resonances*

Occupation number density $n(E)$

$$\rho(\mathbf{r},E)=\frac{1}{\pi}\mathrm{Im}\sum_{\sigma}\mathcal{G}_0^{(11)}(\mathbf{r}\sigma,\mathbf{r}\sigma,-E-i\epsilon),\quad n(E)\equiv\int d\mathbf{r}\ \rho(\mathbf{r},E),\ \text{which satisfies}\ \langle N\rangle=\int_0^{E_{\rm cut}}dE\ n(E)$$

contributions from the q.p. states within unit q.p. energy to the particle number at certain energy E.

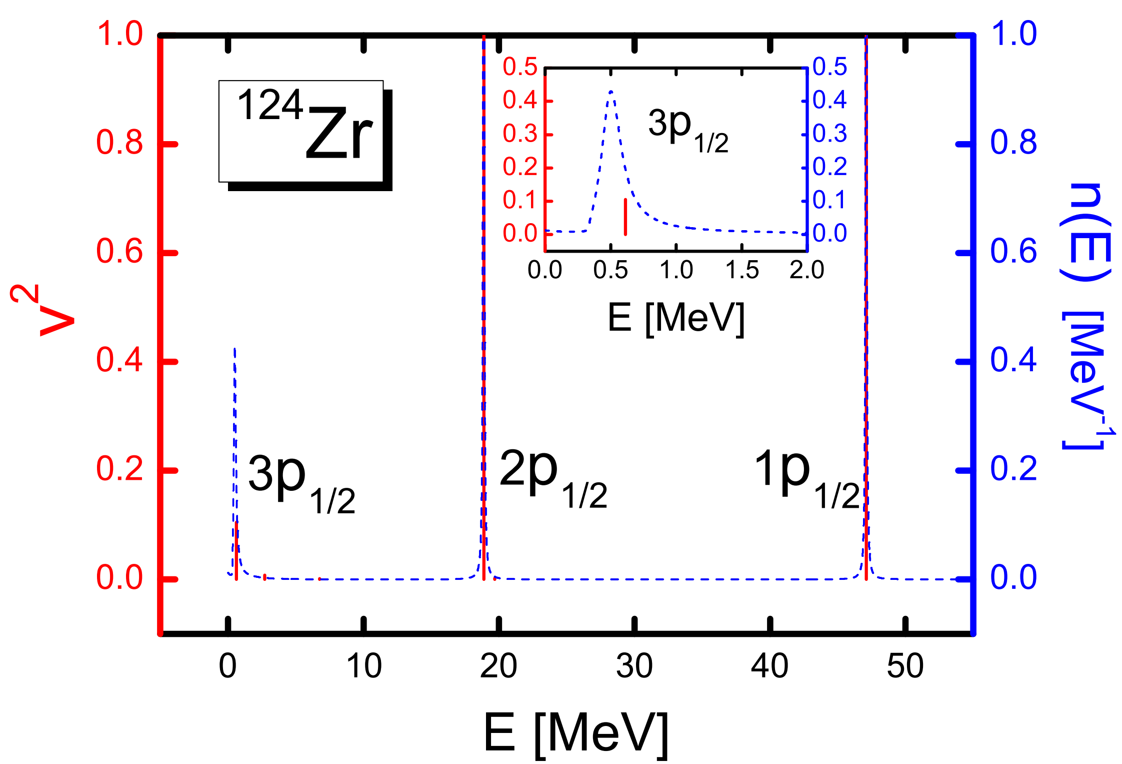


Figure: Occupation number density $n(E)$ of p states for ^{124}Zr obtained by the continuum Skyrme HFB approach and the occupation probability (v^2) obtained by the discretized Skyrme HFB approach.

Zirconium isotopes — *quasiparticle resonances*

★ Quasiparticle resonances and widths

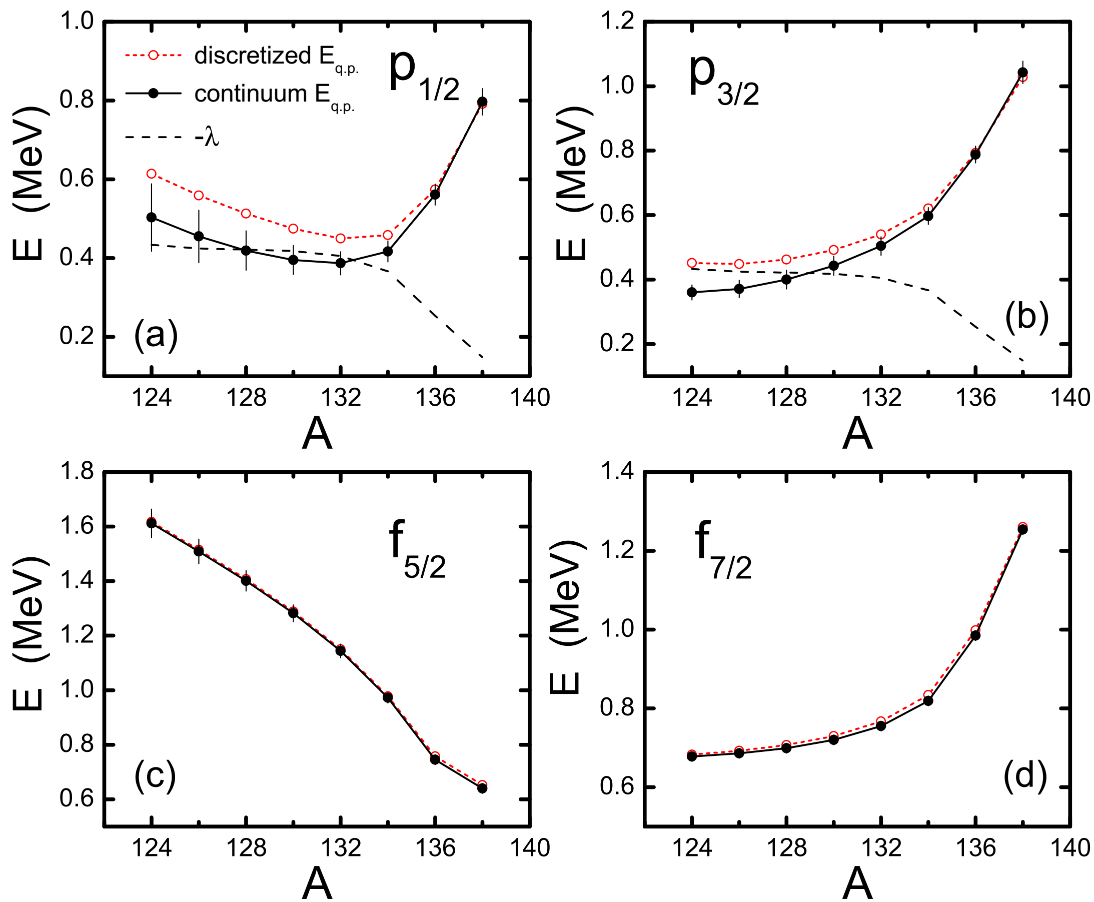
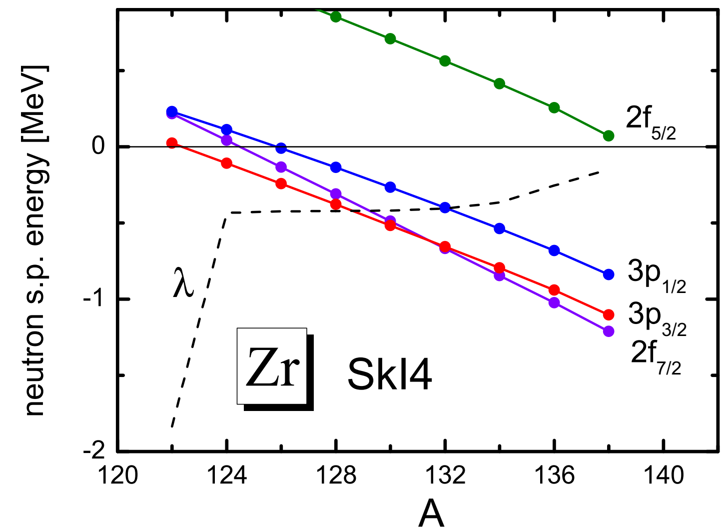


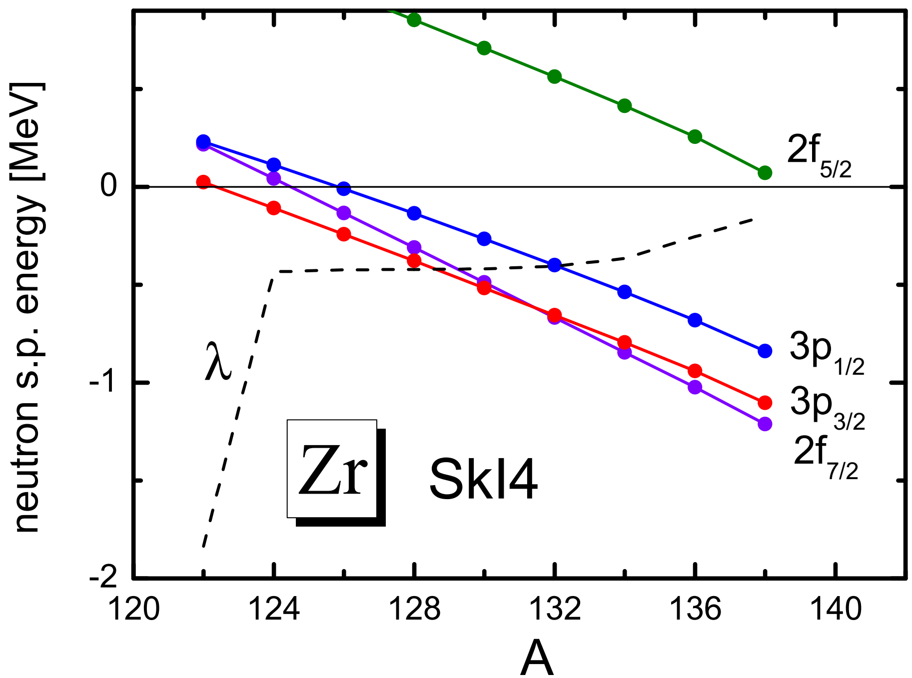
Figure: Quasiparticle resonance energy and width for *p* and *f* states around the fermi surface in Zr isotopes.

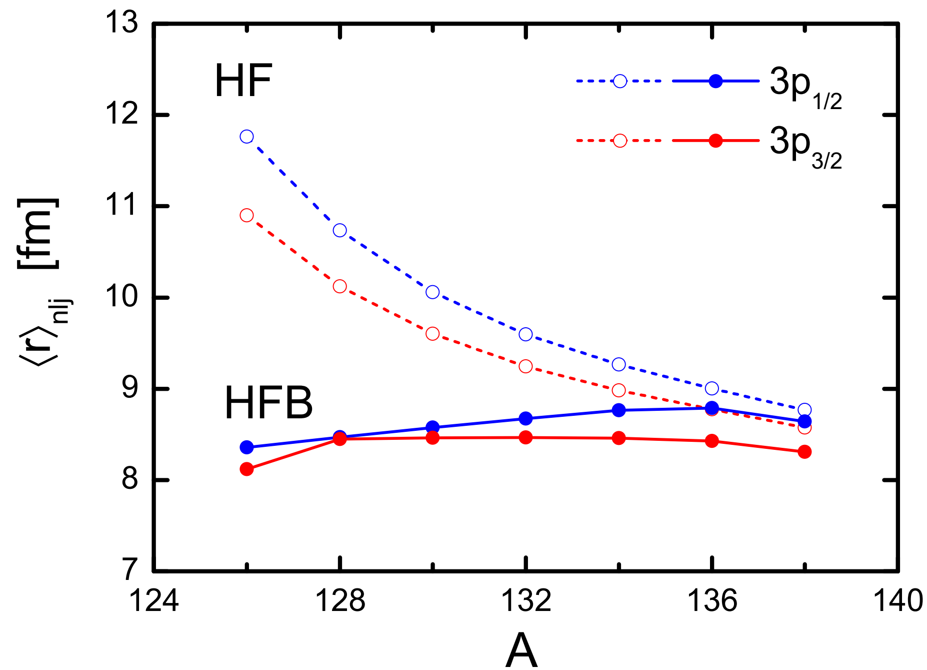


*E is roughly the distance between the s.p. energy and the fermi energy $E = \sqrt{(\varepsilon - \lambda)^2 + \Delta^2}$.

Zirconium isotopes — *role of the pair correlation*

★ **The rms radii of the weakly bound orbits near the fermi energy**



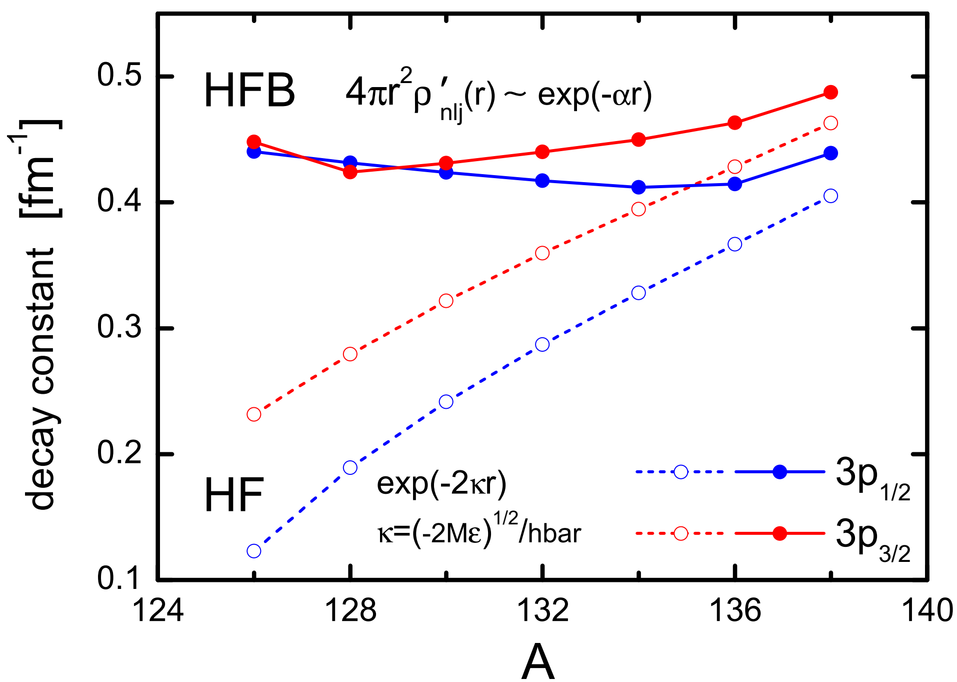


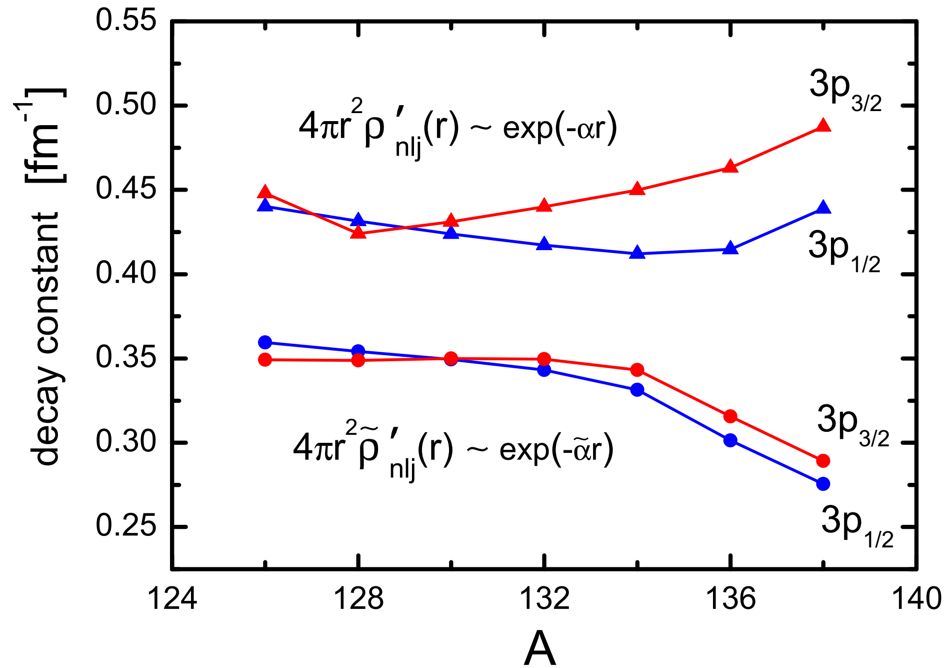
HF: $\langle r\rangle_{nlj} = \left(\frac{\int 4\pi r^4 \phi_{nlj}^2(r)dr}{\int 4\pi r^2 \phi_{nlj}^2(r)dr}\right)^{1/2}$, where $\phi_{nlj}(r)$ is the single-particle wave function.

HFB: $\langle r\rangle_{nlj} = \left(\frac{\int 4\pi r^4 \rho'_{nlj}(r)dr}{\int 4\pi r^2 \rho'_{nlj}(r)dr}\right)^{1/2}$, where $\rho'_{nlj}(r) = \int_0^{2\text{ MeV}} dE \rho_{lj}(r)(r,E)$.

Zirconium isotopes — *role of the pair correlation*

★ **The decay constant of the weakly bound orbits near the fermi energy**





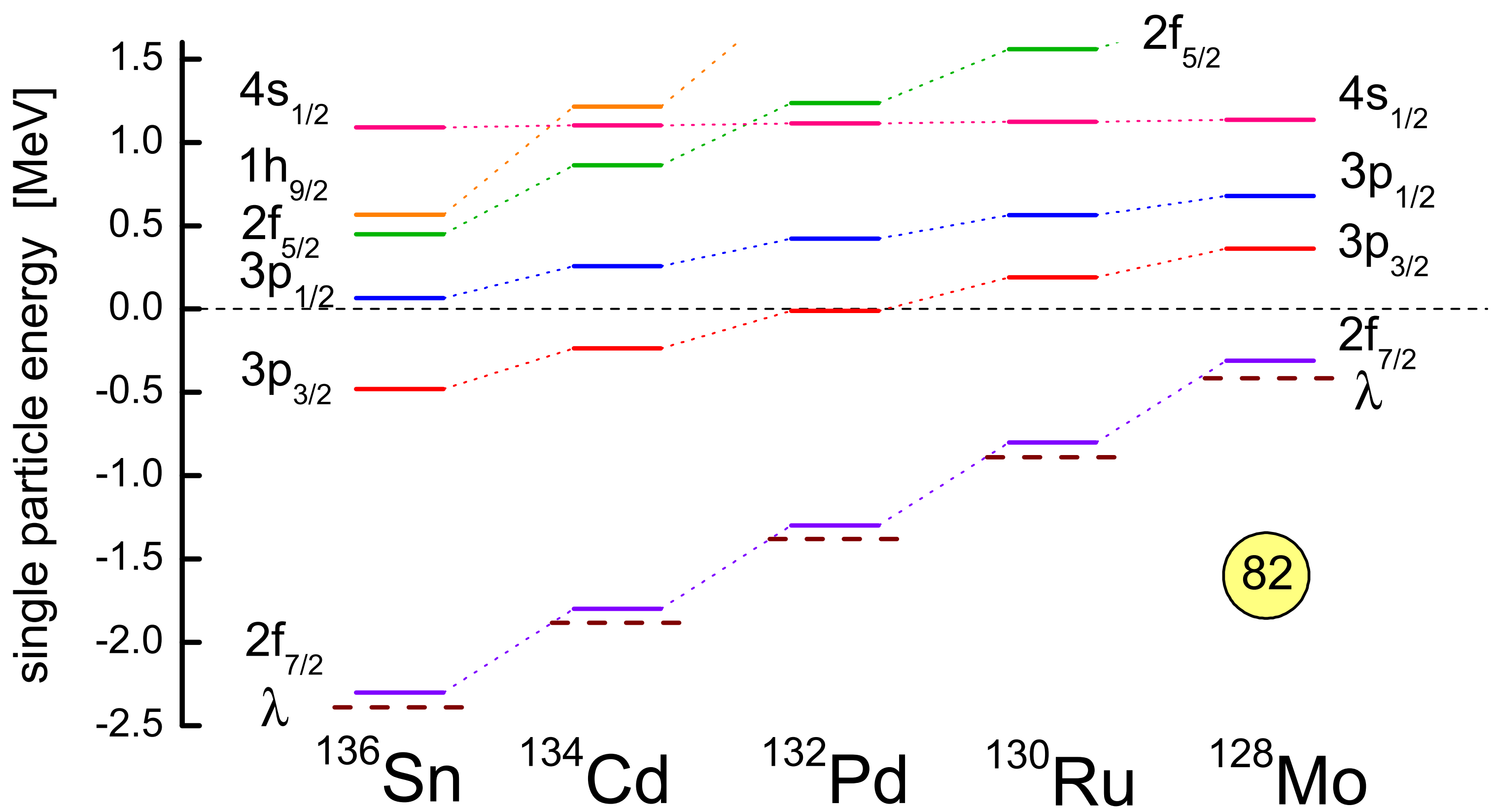
HF: $\phi_{njl} \sim \frac{\exp(-\kappa r)}{r}$, where $\kappa = \sqrt{-2m\varepsilon}/\hbar$

HFB: $\rho'_{nlj}(r) \sim \frac{\exp(-\alpha r)}{r^2}$, $\quad \tilde{\rho}'_{nlj}(r) \sim \frac{\exp(-\tilde{\alpha} r)}{r^2}$, where α and $\tilde{\alpha}$ are extracted from the numerical result of the density by fitting the asymptotic behavior to the exponential decay form.

Outline

$N = 86$ isotones — *continuum contribution to the pair correlation*

★ **Neutron single-particle spectrum for $N = 86$ isotones**

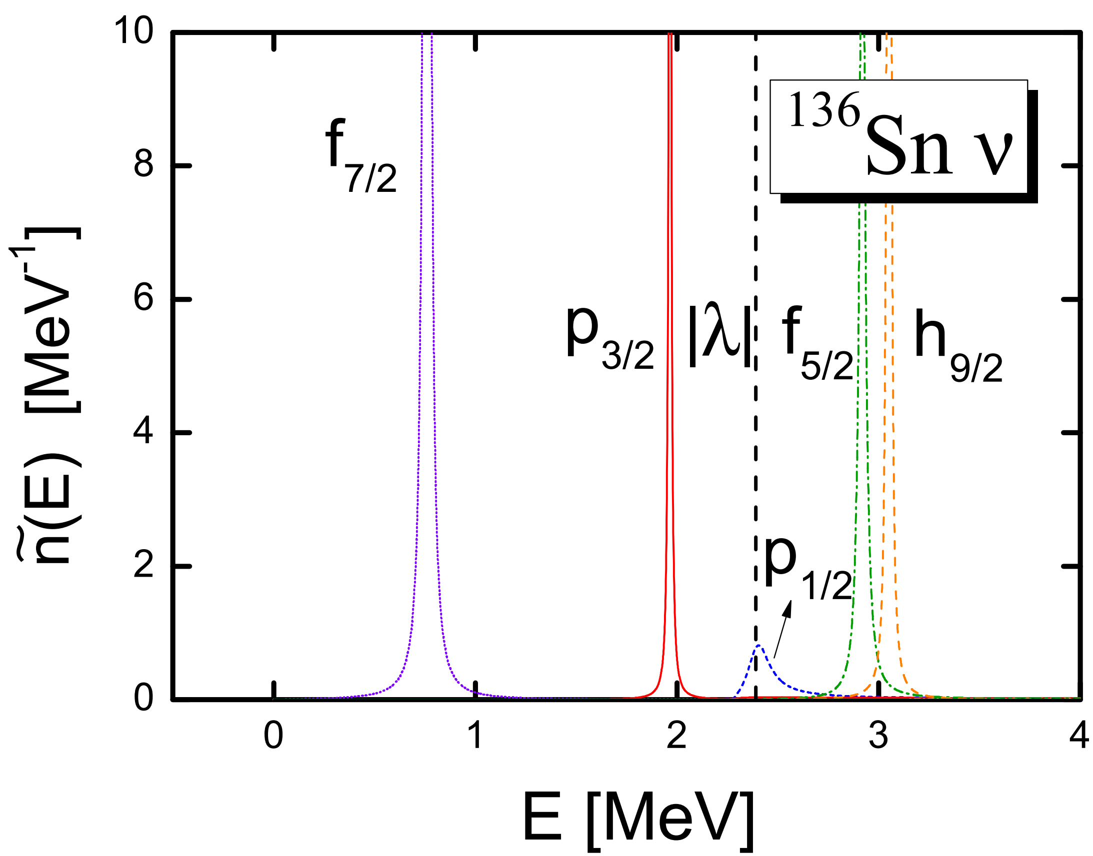


$N = 86$ isotones — *continuum contribution to the pair correlation*

Pair number density $\tilde{n}(E)$

$$\tilde{\rho}(\mathbf{r}, E) = \frac{1}{\pi}\mathrm{Im}\sum_{\sigma}\mathcal{G}_0^{(12)}(\mathbf{r}\sigma, \mathbf{r}\sigma, -E - i\epsilon),$$

$\tilde{\rho}(r) \equiv \int dE\ \tilde{\rho}(\mathbf{r}, E) \Rightarrow$ pair density; $\tilde{n}(E) \equiv \int d\mathbf{r}\ \tilde{\rho}(\mathbf{r}, E) \Rightarrow$ pair number density



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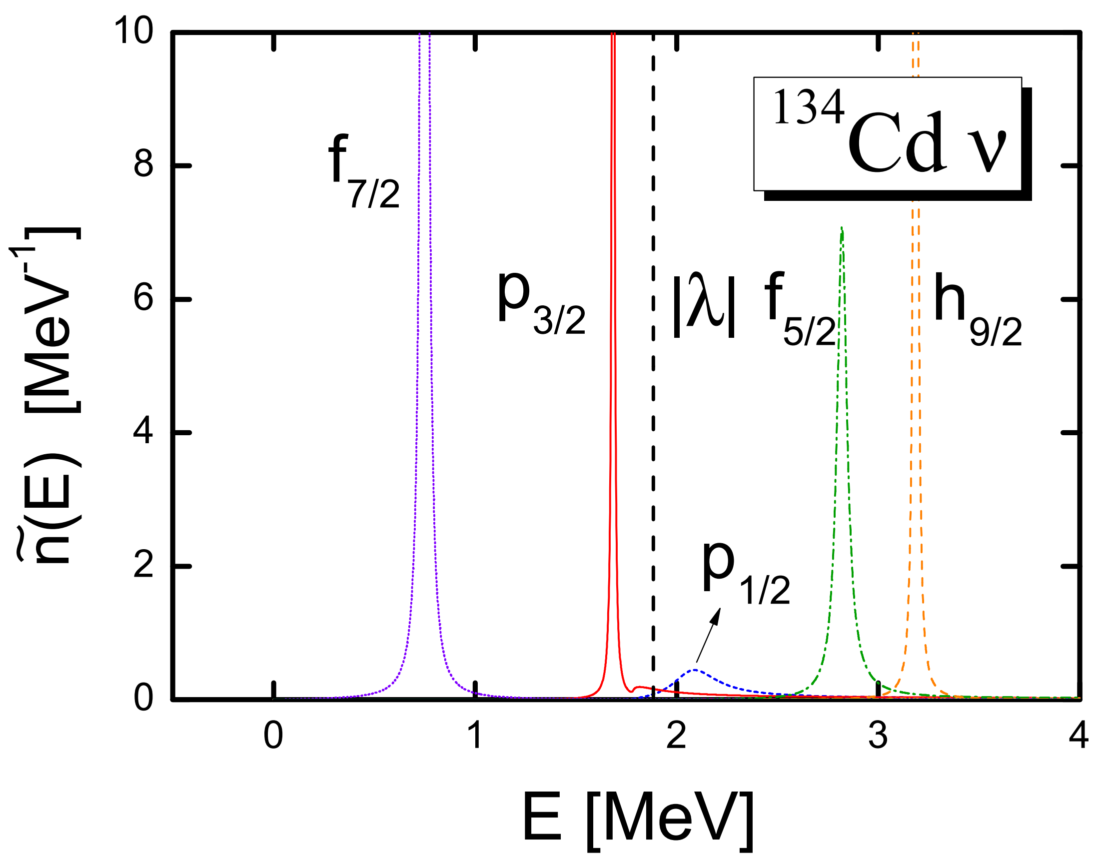

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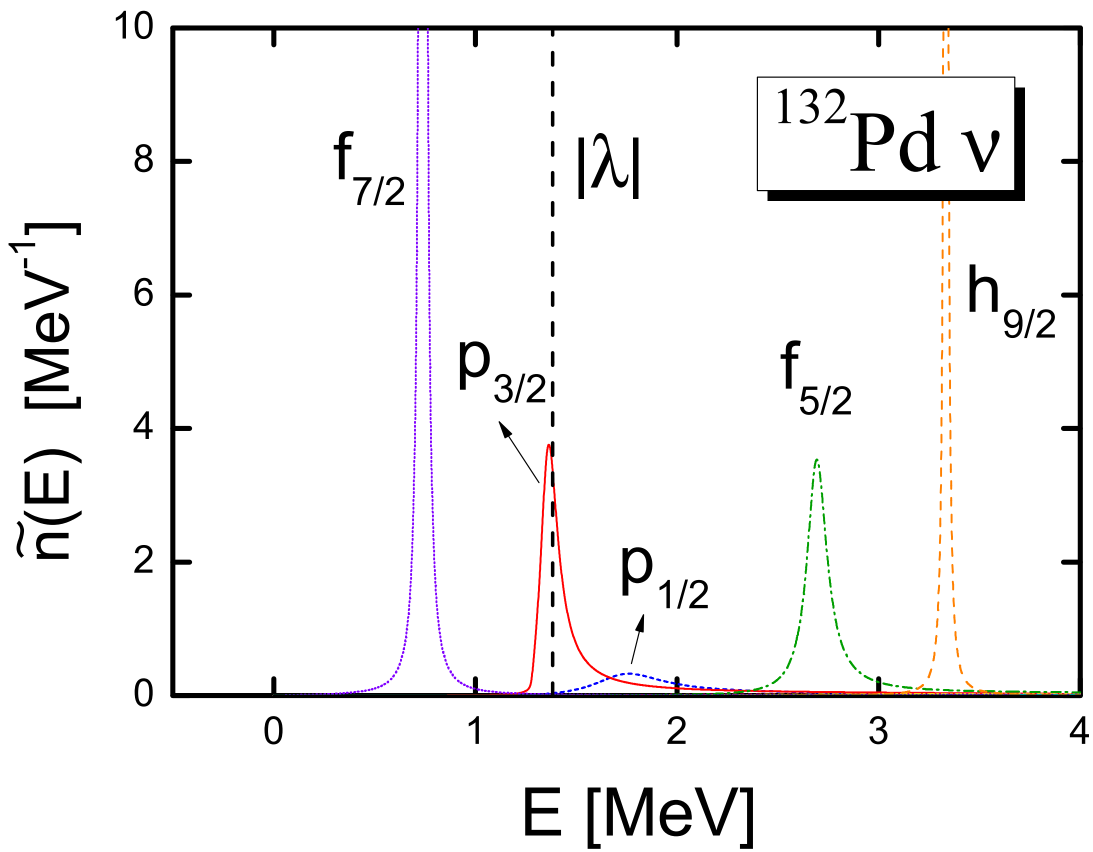

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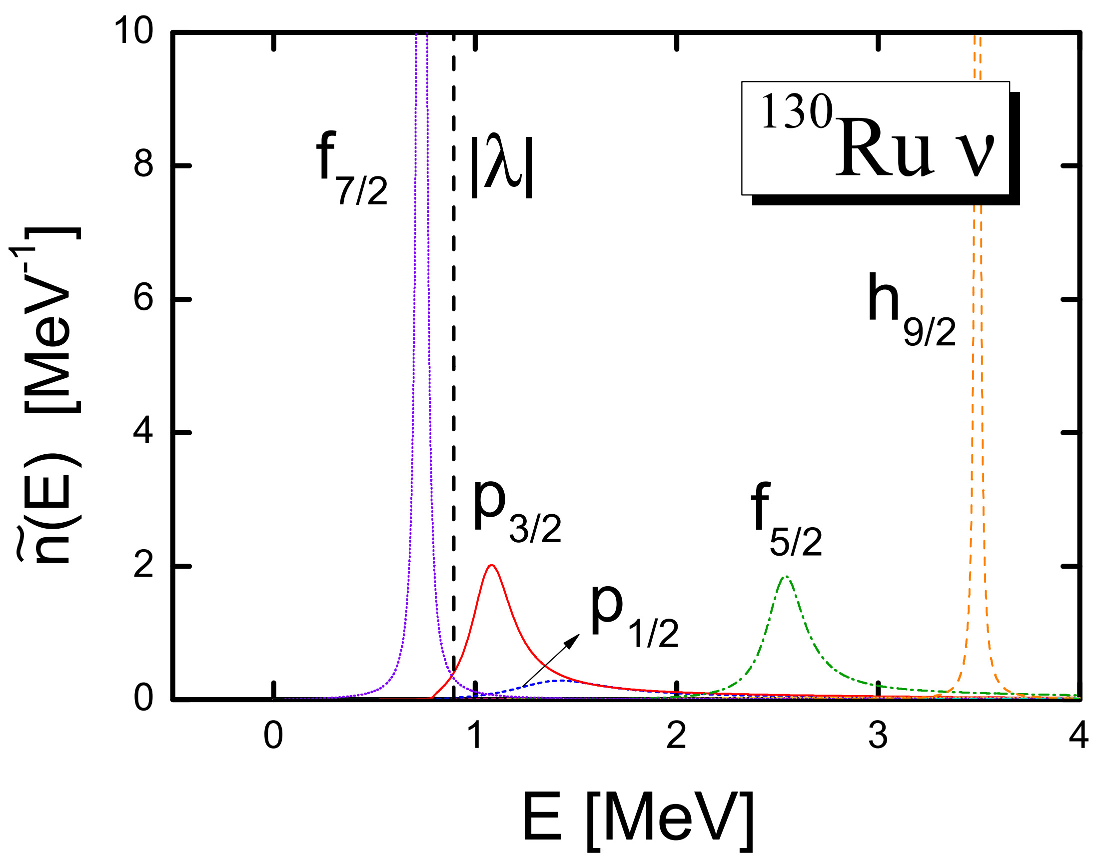

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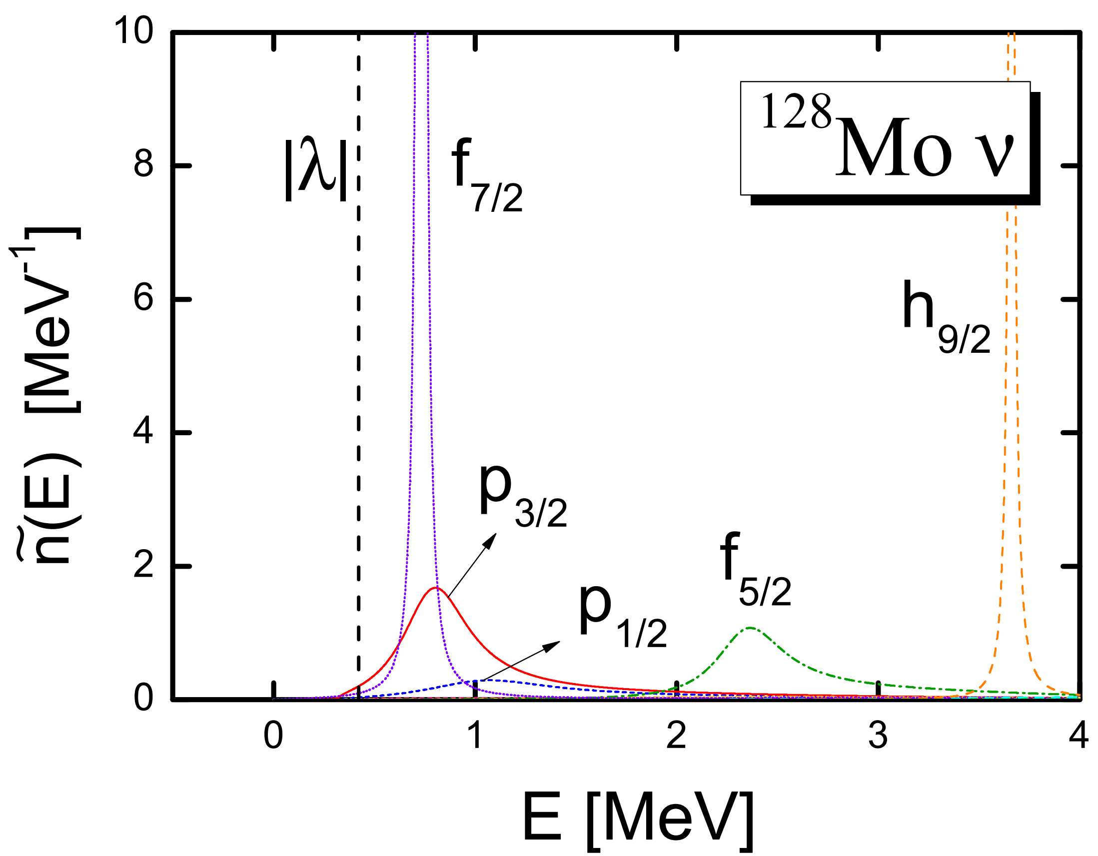

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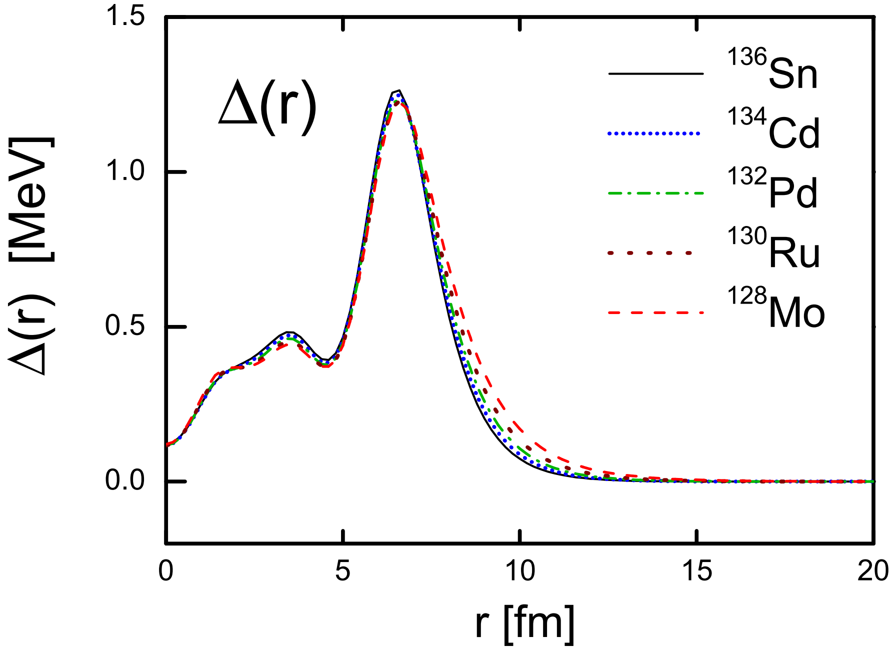

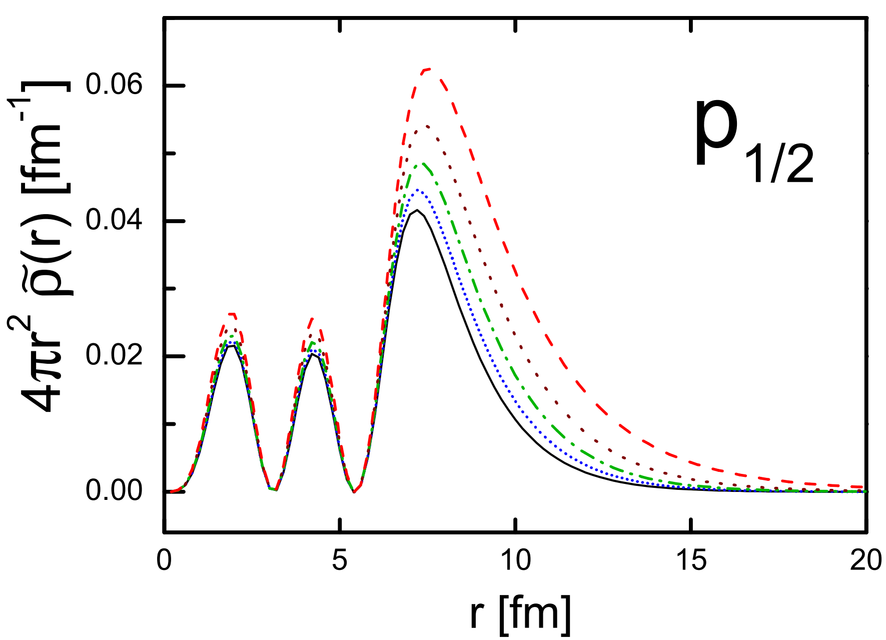

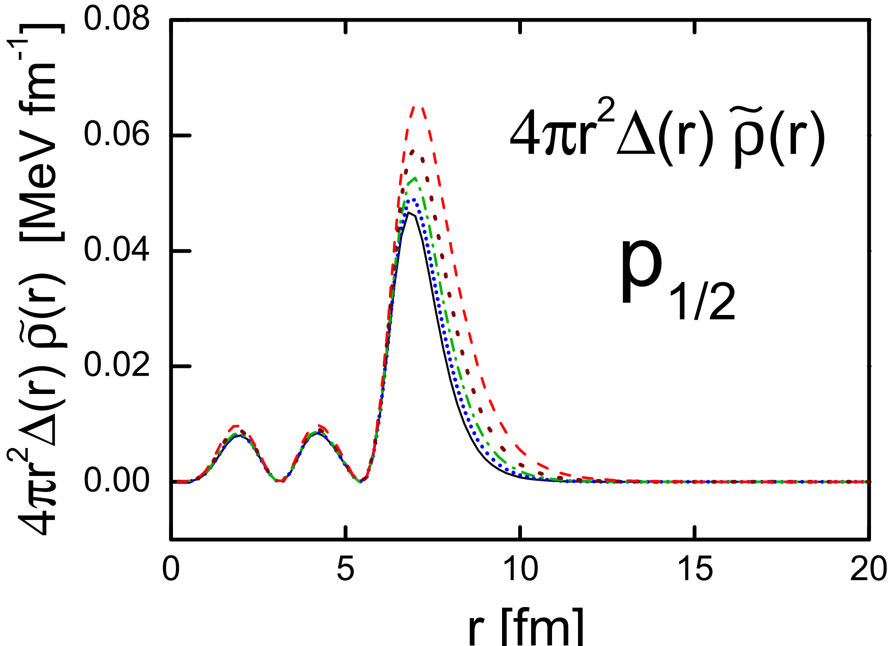

$N = 86$ isotones — *continuum contribution to the pair correlation*

★ **Pairing contribution within $0 \sim 4$ MeV for surface pairing**

		^{136}Sn	^{134}Cd	^{132}Pd	^{130}Ru	^{128}Mo
$\vert\lambda\vert$		2.390	1.884	1.383	0.894	0.421
$\langle\tilde{N}\rangle^{\text{tot.}}$		16.875	17.083	17.458	18.111	19.245
$E_{\text{pair}}^{\text{tot.}}$		-6.212	-6.162	-6.173	-6.280	-6.527
$\overline{\Delta}_{uv}^{\text{tot.}}$		0.736	0.721	0.707	0.694	0.678
	$\langle\tilde{N}\rangle$	0.528	0.591	0.679	0.813	1.040
$p_{3/2}$	E_{pair}	-0.156	-0.171	-0.193	-0.225	-0.275
	$\overline{\Delta}_{uv}$	0.591	0.579	0.568	0.554	0.529
	$\langle\tilde{N}\rangle$	0.179	0.199	0.226	0.268	0.339
$p_{1/2}$	E_{pair}	-0.051	-0.055	-0.061	-0.070	-0.085
	$\overline{\Delta}_{uv}$	0.570	0.553	0.540	0.522	0.501

✓ tot: $\langle\tilde{N}\rangle$, E_{pair} increase

✓ p states:

- $\langle\tilde{N}\rangle$: $+100\%$
- E_{pair} : $+70\%$
- $\overline{\Delta}$: -20%

The weakly bound and the continuum low-l orbits persist in contributing to the pair correlation!

Outline

Summary

Pairing correlation is important in the neutron-rich nuclei. In order to clarify the pair correlation effects in the weakly bound nuclei, we should deal with the pair correlation and the continuum effect properly.

- *Self-consistent continuum Skyrme Hartree-Fock-Bogoliubov approach with Green's function method in the coordinate space is established.*
  - $\surd$ proper asymptotic behavior for the density distribution
  - $\surd$ can provide the width information for quasiparticle resonance
- *Pairing correlation effects in the Zr giant halo nuclei*
  - $\star$ make the weakly bound orbits more bound
  - $\star$ make the pair density more extended in the neutron-rich isotopes as $\lambda \to 0$
- *Continuum contribution to the pair correlation ($N = 86$)*

  The weakly bound or unbound p orbits persist in the contribution to the pair correlation even when they form very broad quasiparticle resonances, and when they are located above the potential barrier.
- *Perspectives*

  link with exp. (phase-shift to obtain physical width), deformed case, relativistic case, ...

Thank you !