Structure of halo nuclei and transfer reactions

F. Barranco and G. Potel

Sevilla University

R.A. Broglia

Milano University and INFN

The Niels Bohr Institute, Copenhagen

E. Vigezzi

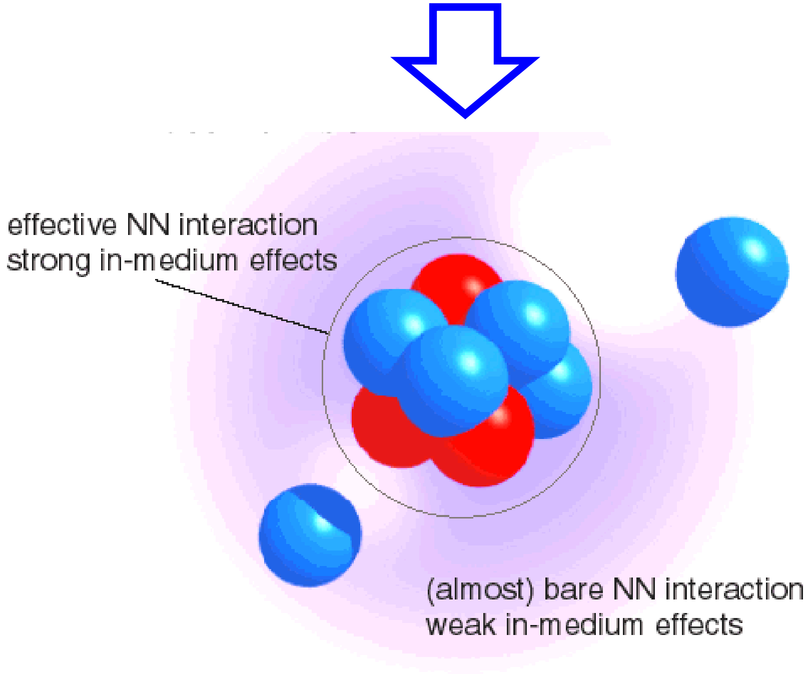
INFN Milano

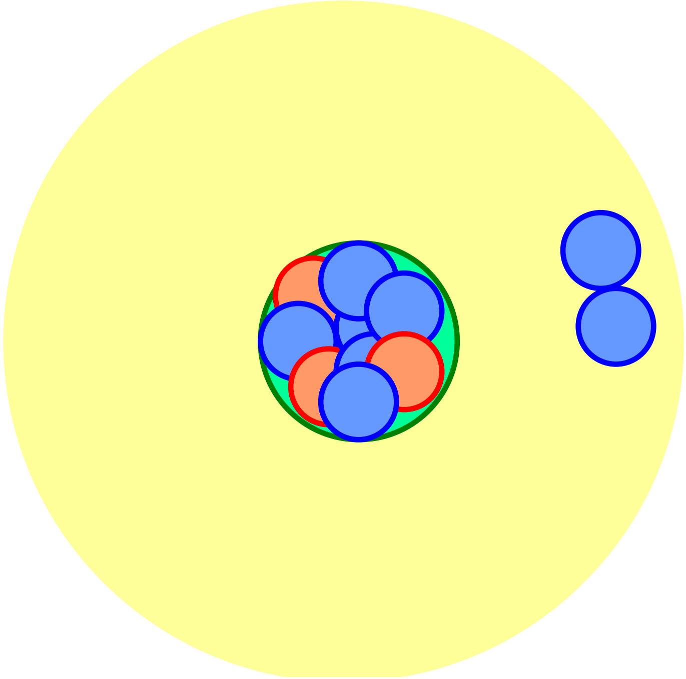
DCEN 2011, Kyoto

Outline

- Beyond the inert core approximation
- A dynamical model for one- (11Be, 10Li,..) and two-neutron halo nuclei (12Be, 11Li,…)
- Comparison with experiment: structure and reaction data

To what extent is this picture correct?



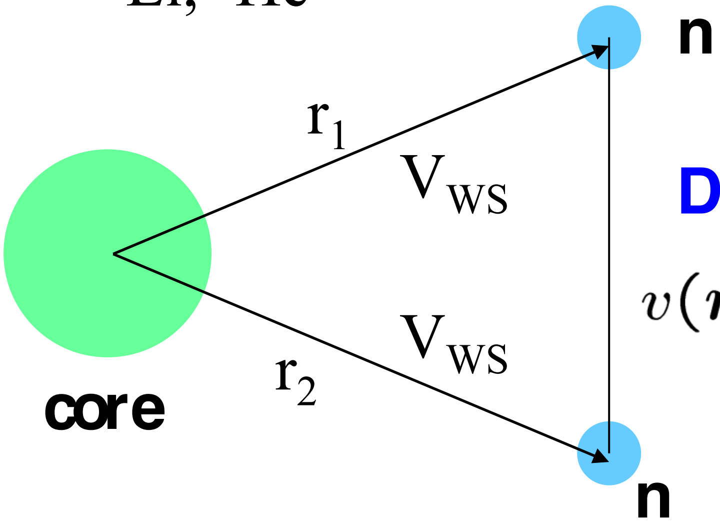


Talk by K. Hagino

Three-body model with density-dependent delta force

G.F. Bertsch and H. Esbensen,
Ann. of Phys. 209('91)327
H. Esbensen, G.F. Bertsch, K. Hencken,
Phys. Rev. C56('99)3054

^{11}Li, ^{6}He

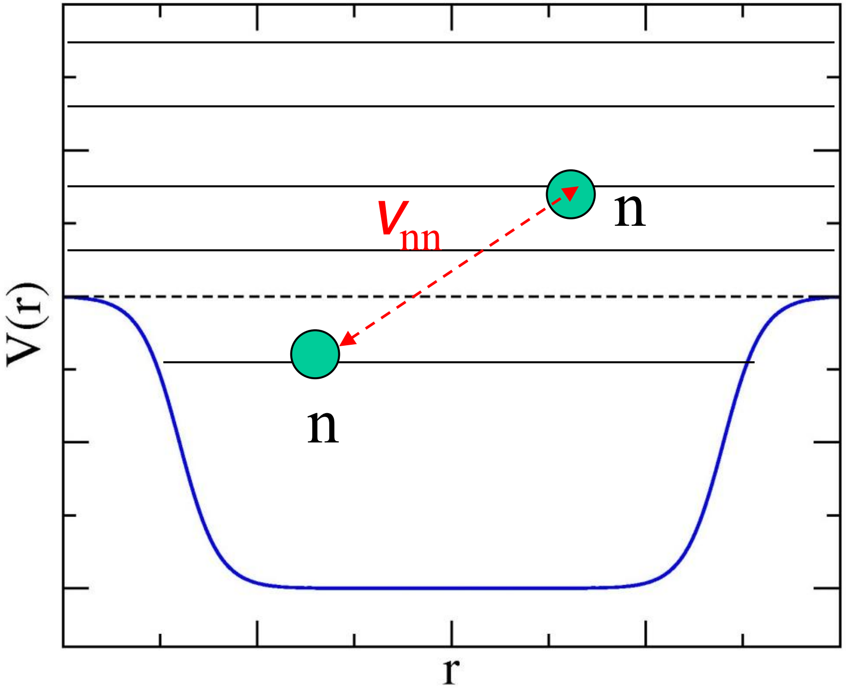


Density-dependent delta-force

$$\begin{aligned} v(\boldsymbol{r}_1, \boldsymbol{r}_2) &= v_0(1 + \alpha\rho(r)) \\ &\quad \times\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \end{aligned}$$

$$\boxed{H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}}$$

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$



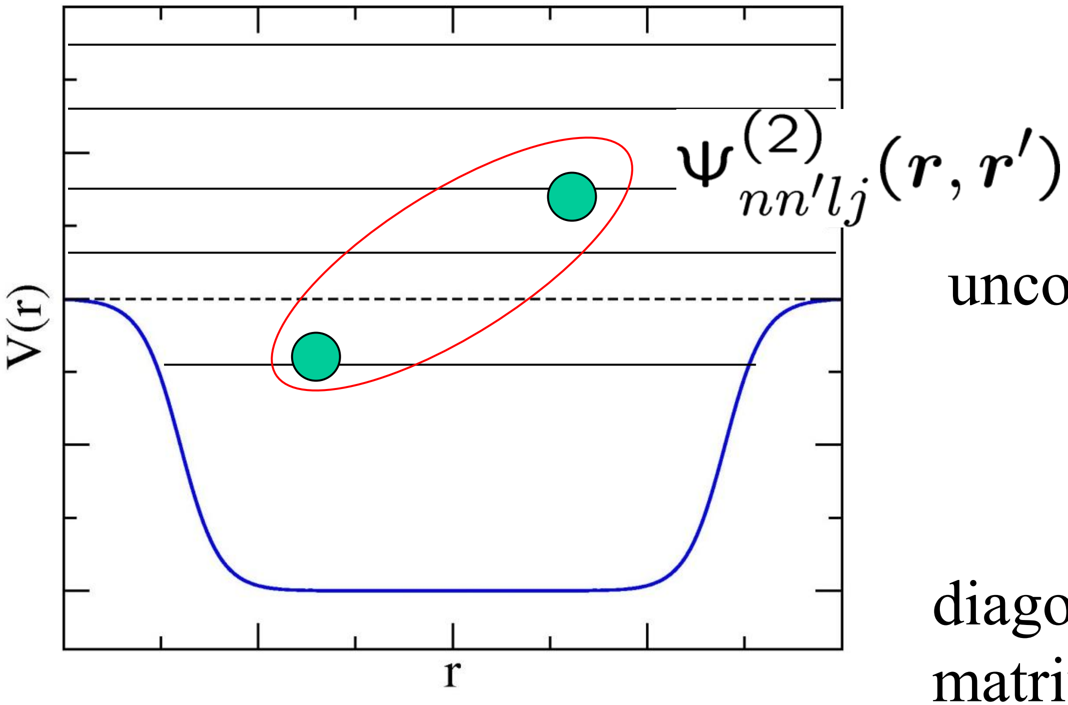
continuum states:
discretized in a large box

$$V_{nn}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\left(v_0 + \frac{v_\rho}{1 + \exp[(r_1 - R_\rho)/a_\rho]}\right)$$

- ✓ contact interaction
- ✓ v_0: free n-n
- ✓ density dependent term: medium many-body effects

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$

$$\boxed{\Psi_{gs}(\boldsymbol{r},\boldsymbol{r}') = \mathcal{A} \sum_{nn'lj} \alpha_{nn'lj}\, \Psi^{(2)}_{nn'lj}(\boldsymbol{r},\boldsymbol{r}')}$$



uncorrelated basis

⇓

diagonalization of Hamiltonian matrix

Good agreement with Faddeev calculations

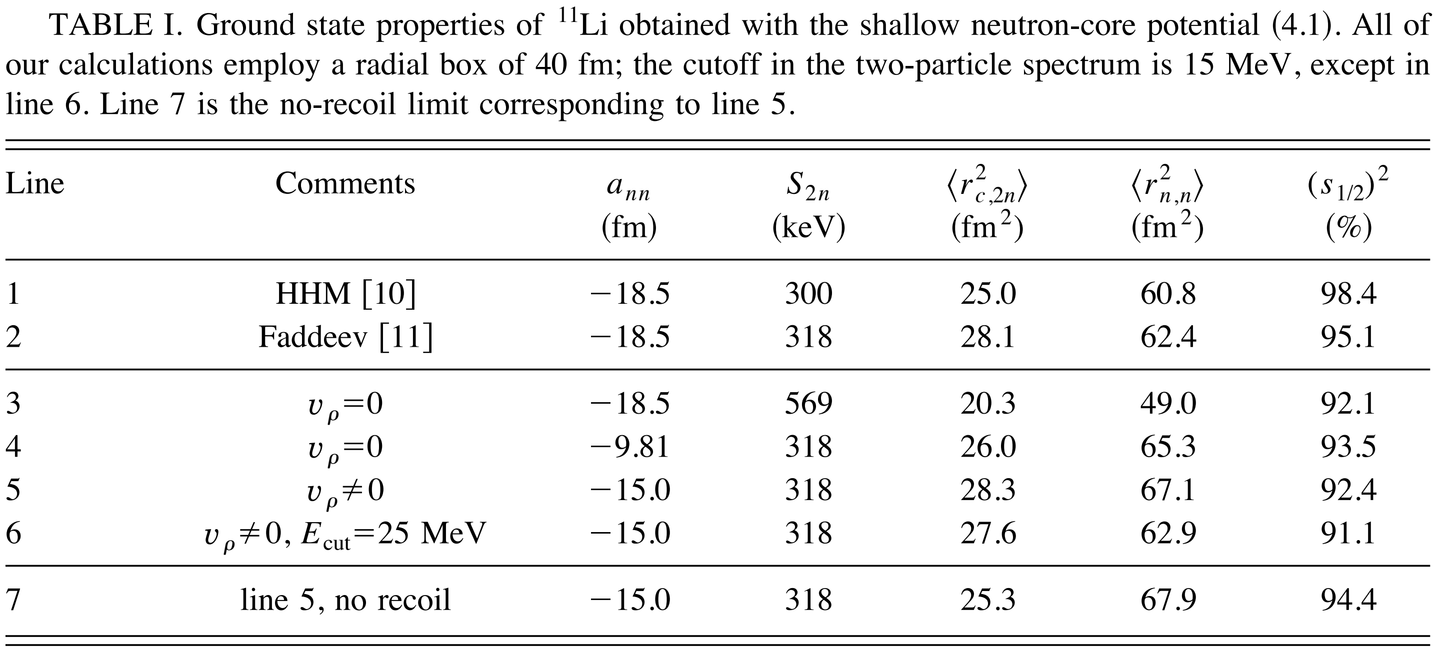


TABLE I. Ground state properties of ^{11}Li obtained with the shallow neutron-core potential (4.1). All of our calculations employ a radial box of 40 fm; the cutoff in the two-particle spectrum is 15 MeV, except in line 6. Line 7 is the no-recoil limit corresponding to line 5.

Line	Comments	a_{nn} (fm)	S_{2n} (keV)	$\langle r^2_{c,2n} \rangle$ (fm^2)	$\langle r^2_{n,n} \rangle$ (fm^2)	$(s_{1/2})^2$ (%)
1	HHM [10]	-18.5	300	25.0	60.8	98.4
2	Faddeev [11]	-18.5	318	28.1	62.4	95.1
3	$v_\rho = 0$	-18.5	569	20.3	49.0	92.1
4	$v_\rho = 0$	-9.81	318	26.0	65.3	93.5
5	$v_\rho \neq 0$	-15.0	318	28.3	67.1	92.4
6	$v_\rho \neq 0$, $E_{\text{cut}} = 25$ MeV	-15.0	318	27.6	62.9	91.1
7	line 5, no recoil	-15.0	318	25.3	67.9	94.4

H. Esbensen, G.F. Bertsch, K. Hencken, Phys. Rev. C 56 (1997) 3054

Relax some of the assumptions of Bertsch and Esbensen:

Inert core

Different potentials for s- and p- waves

Zero range interaction, with ad hoc density dependence

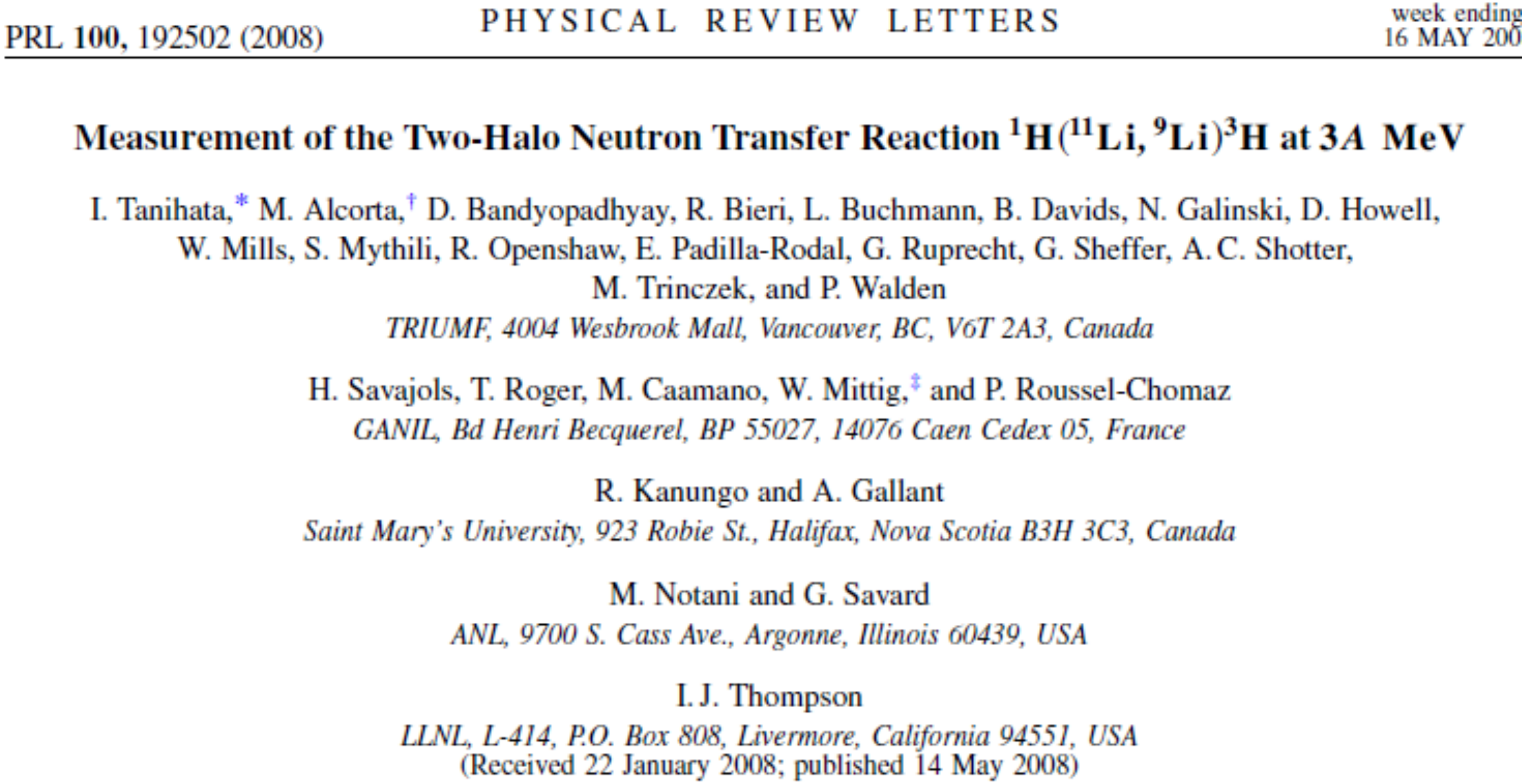
H. Esbensen, G.F. Bertsch, K. Hencken,
Phys. Rev. C 56 (1997) 3054

Low-lying collective modes of the core taken into account

Standard mean field potential

Bare N-N interaction (Argonne)

^{10}Li, ^{11}Li F. Barranco et al. EPJ A11 (2001) 385
^{11}Be, ^{12}Be G. Gori et al. PRC 69 (2004) 041302(R)



PRL **100,** 192502 (2008) PHYSICAL REVIEW LETTERS week ending
16 MAY 200

Measurement of the Two-Halo Neutron Transfer Reaction $^{1}\mathrm{H}(^{11}\mathrm{Li},{}^{9}\mathrm{Li})^{3}\mathrm{H}$ at $3A$ MeV

I. Tanihata,[*] M. Alcorta,[†] D. Bandyopadhyay, R. Bieri, L. Buchmann, B. Davids, N. Galinski, D. Howell,
W. Mills, S. Mythili, R. Openshaw, E. Padilla-Rodal, G. Ruprecht, G. Sheffer, A. C. Shotter,
M. Trinczek, and P. Walden
TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada

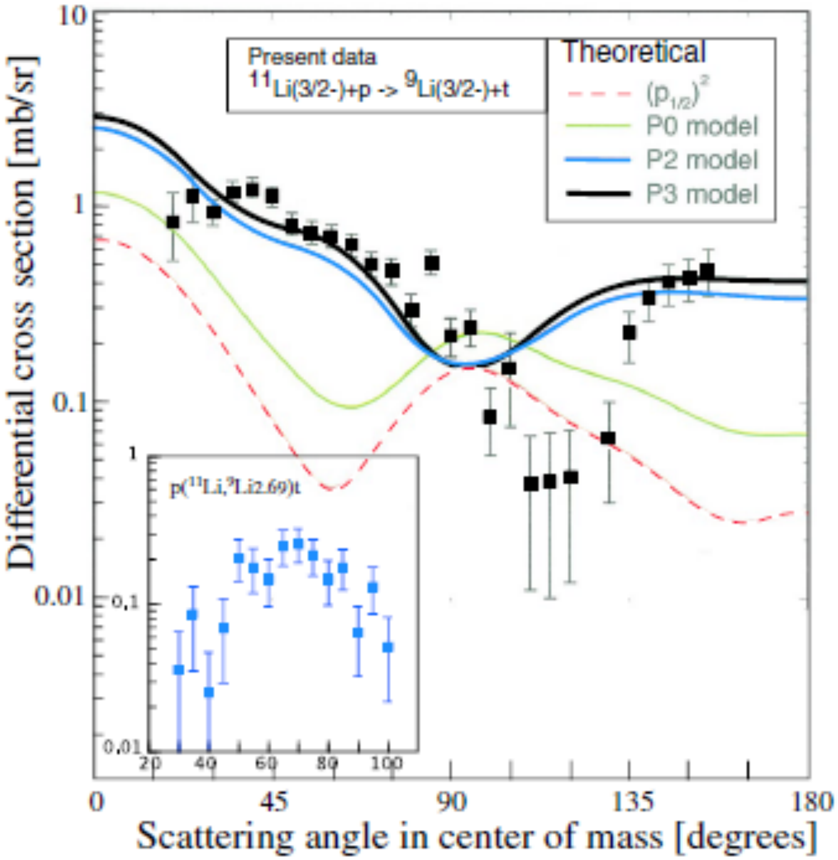
H. Savajols, T. Roger, M. Caamano, W. Mittig,[‡] and P. Roussel-Chomaz
GANIL, Bd Henri Becquerel, BP 55027, 14076 Caen Cedex 05, France

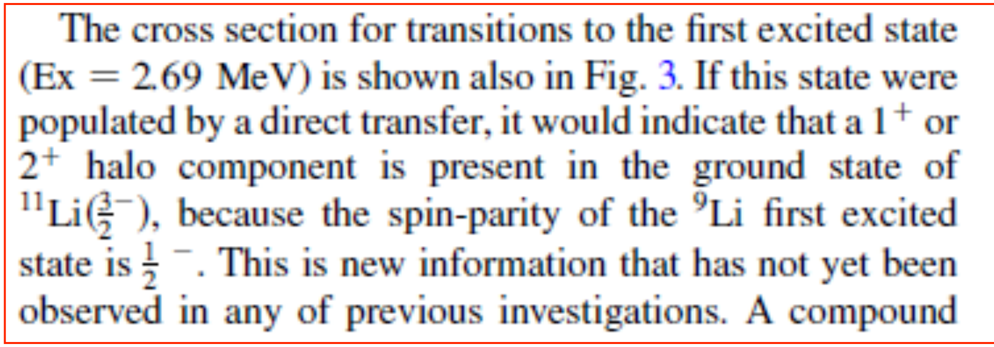
R. Kanungo and A. Gallant
Saint Mary's University, 923 Robie St., Halifax, Nova Scotia B3H 3C3, Canada

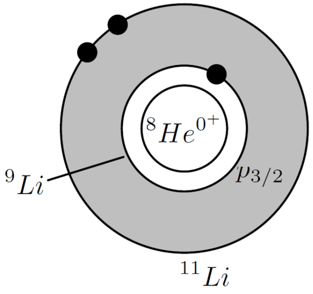
M. Notani and G. Savard
ANL, 9700 S. Cass Ave., Argonne, Illinois 60439, USA

I. J. Thompson
LLNL, L-414, P.O. Box 808, Livermore, California 94551, USA

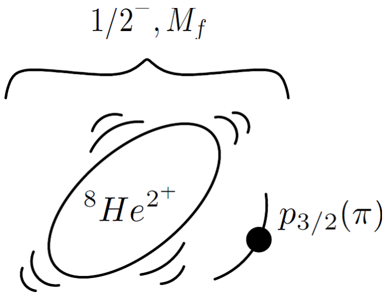
(Received 22 January 2008; published 14 May 2008)




The cross section for transitions to the first excited state (Ex = 2.69 MeV) is shown also in Fig. 3. If this state were populated by a direct transfer, it would indicate that a 1^+ or 2^+ halo component is present in the ground state of ^{11}Li($\frac{3}{2}^-$), because the spin-parity of the ^{9}Li first excited state is $\frac{1}{2}^-$. This is new information that has not yet been observed in any of previous investigations. A compound



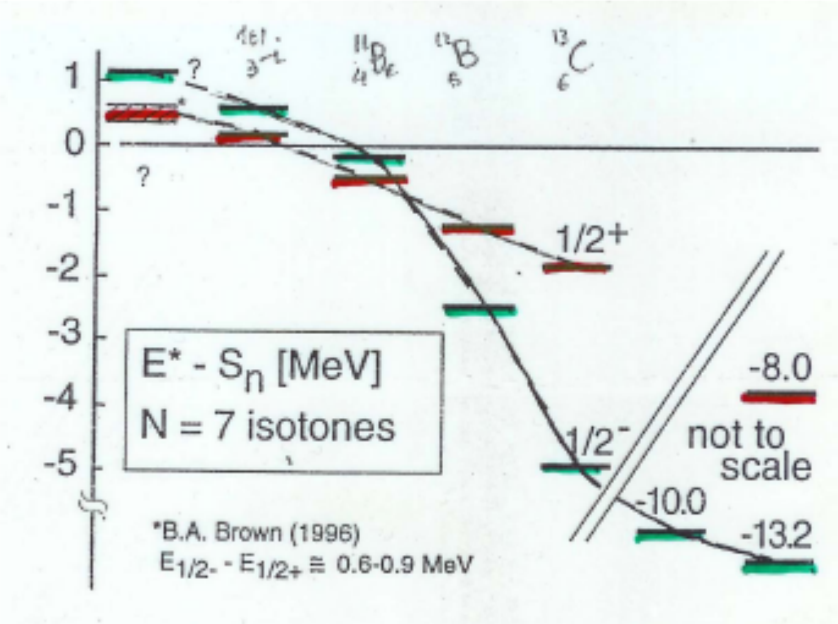
Schematic depiction of ^{11}Li



First excited state of ^{9}Li

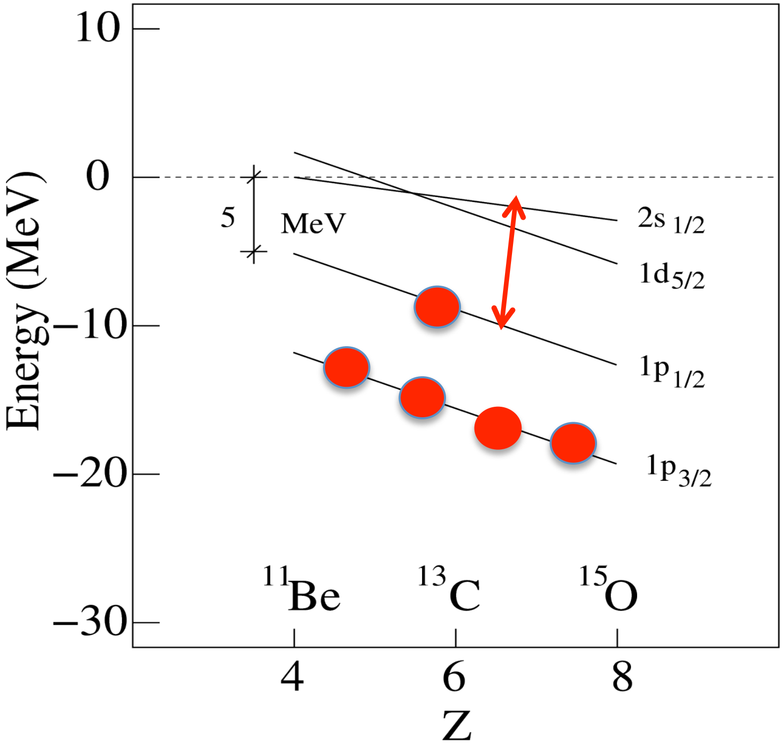
Parity inversion in N=7 isotones

Experimental systematics



Mean-field results

(Sagawa,Brown,Esbensen PLB 309(93)1)



Admixture of $d_{5/2}$ x 2^+ configuration in the $1/2^+$ g.s. of ^{11}Be is about 20%

Calculated ground state

$$|1/2+\rangle = \sqrt{0.87}\,|s_{1/2}\rangle + \sqrt{0.13}\,|d_{5/2}\otimes 2+\rangle$$

Exp.:

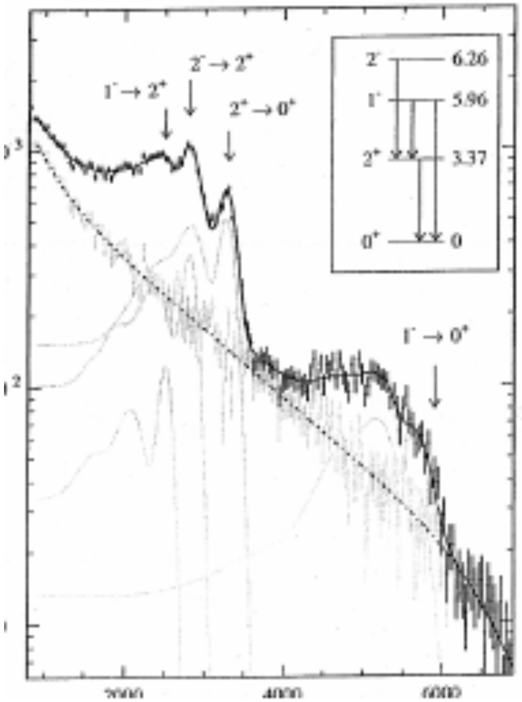
J.S. Winfield et al., Nucl.Phys. **A683** (2001) 48

$$|1/2+\rangle = \sqrt{0.84}\,|s_{1/2}\rangle + \sqrt{0.16}\,|d_{5/2}\otimes 2+\rangle$$

^{11}Be(p,d)^{10}Be in inverse kinematic detecting both the ground state and the 2+ excited state of ^{10}Be.

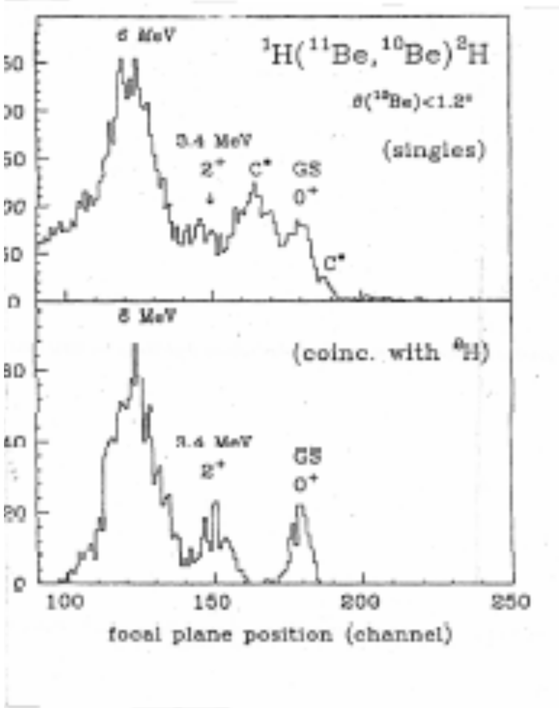
9Be(11Be,10Be+ γ) X

T. Aumann et al.
PRL 84(2000)35

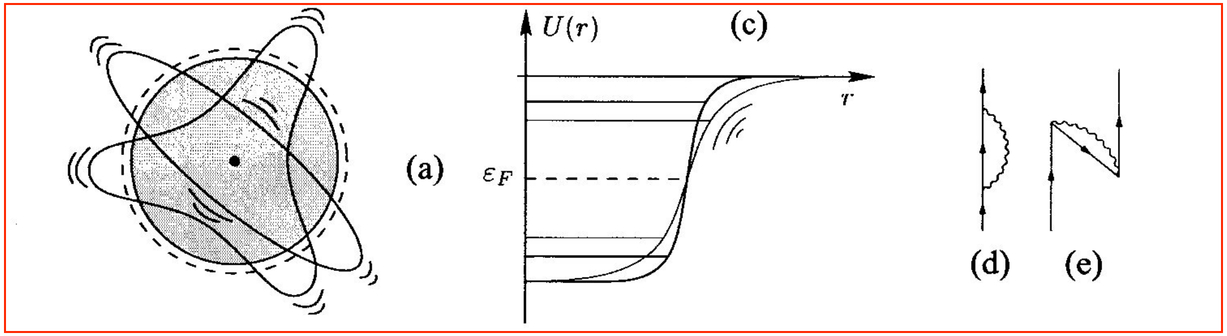


p(11Be,10Be)d

S. Fortier et al.
Phys. Lett.B461(1999)22

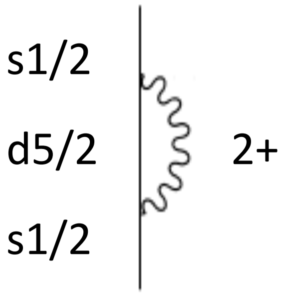


^{11}Be



H. Sagawa et al., PLB 309 (1993)1

Eshift = - 2.5 MeV

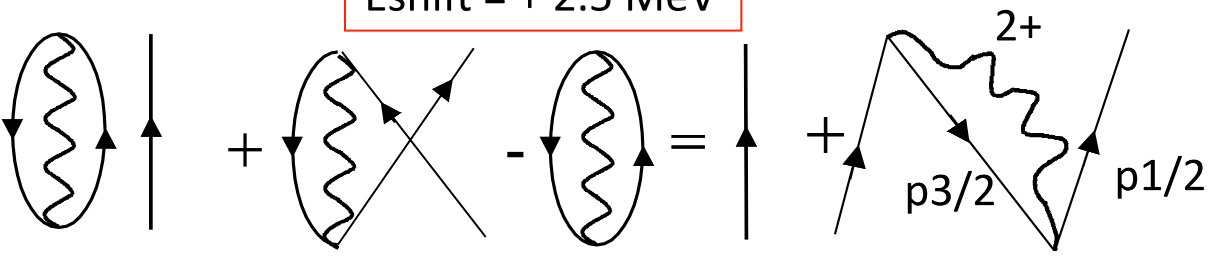


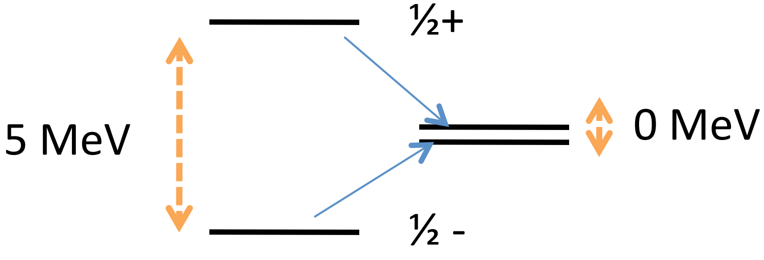
Self-energy

+

Pauli blocking of core ground state correlations

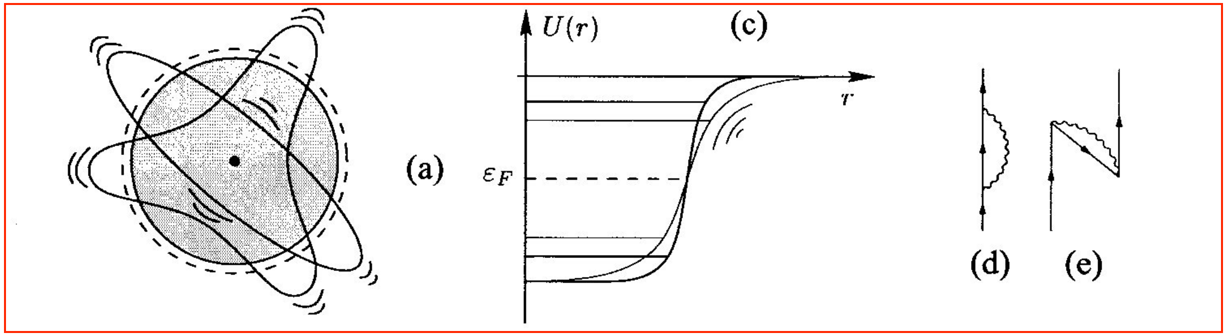
Eshift = + 2.5 MeV



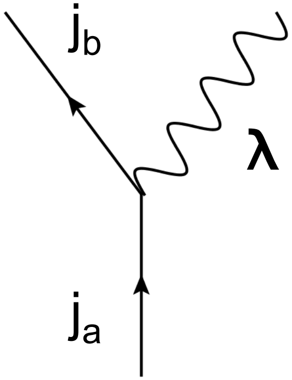


↓

Level inversion



Mean field potential

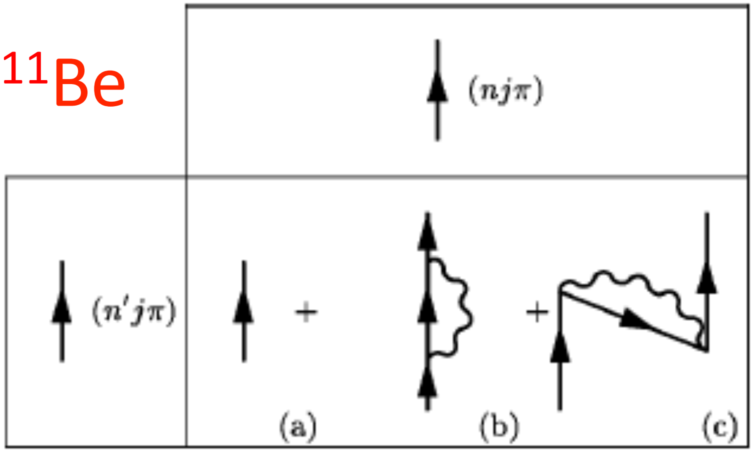


$$= \frac{1}{\sqrt{4\pi}} \langle j_a \lambda | j_b \rangle \beta_\lambda \left\langle j_a \left| \frac{\partial U}{\partial r} \right| j_b \right\rangle = h(a, b\lambda)$$

From B(EL) experimental value
in the core nucleus

Effective, energy-dependent matrix (Bloch-Horowitz)

^{11}Be



Main ingredients of our calculation

Fermionic degrees of freedom:

- s1/2, p1/2, d5/2 Wood-Saxon levels up to 150 MeV (discretized continuum) from a standard (Bohr-Mottelson) Woods-Saxon potential

Bosonic degrees of freedom:

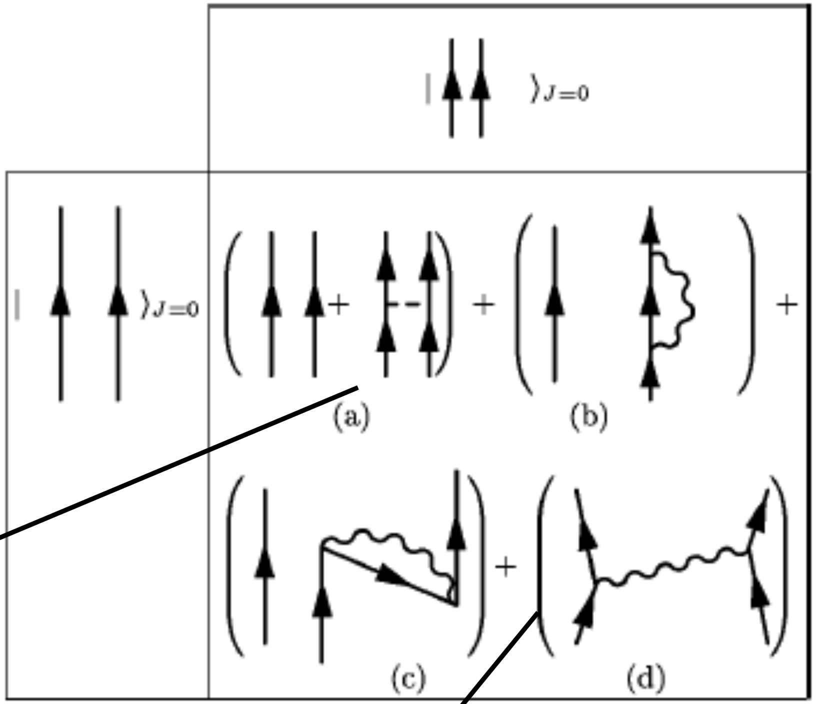
- 2+ and 3- QRPA solutions with energy up to 50 MeV; residual interaction: multipole-multipole separable with the coupling constant tuned to reproduce E(2+)=3.36 MeV and $0.6<\beta_2<0.7$

A dynamical description of two-neutron halos

^{11}Li
F. Barranco et al. EPJ A11 (2001) 385

^{12}Be
G. Gori et al. PRC 69 (2004) 041302(R)

Energy-dependent matrix



Bare interaction

Induced interaction

Theoretical calculation for ^{10}Li and ^{11}Li

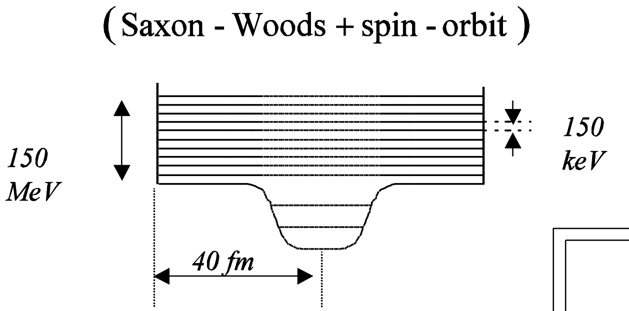
Low-lying dipole strength

↓

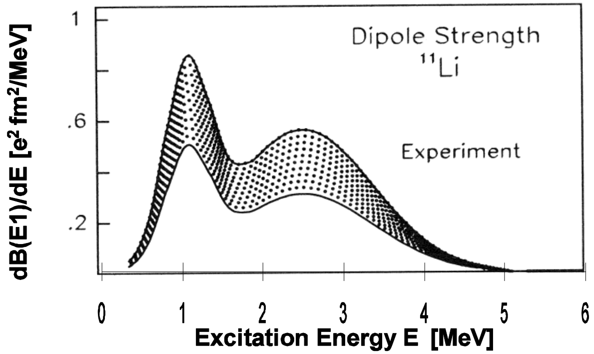
s-p mixing

Phenomenological input:
properties
of collective models

Predictions:
binding energy,
spectroscopic factors

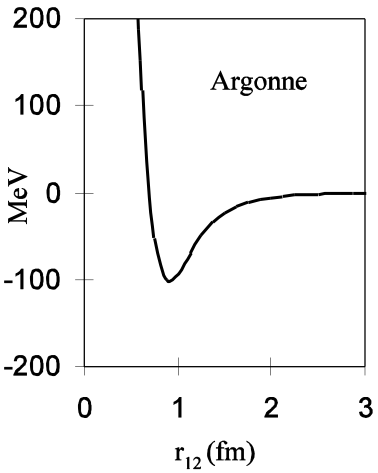


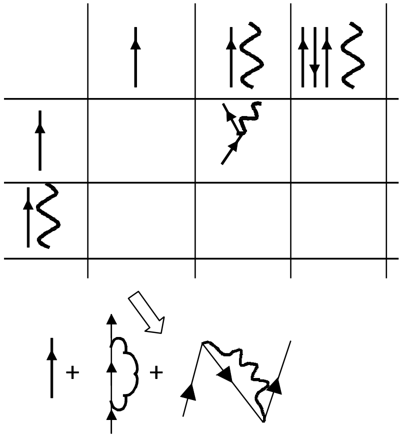
Vibrations

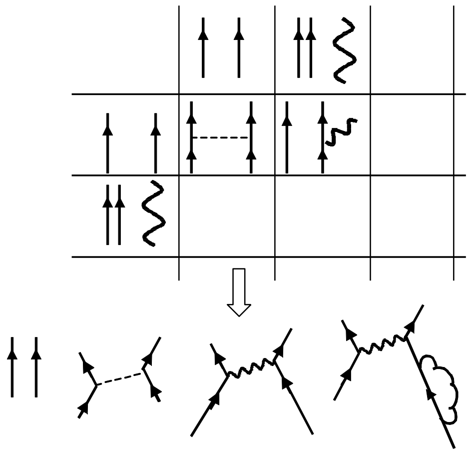


$B(E2)\uparrow = [5.2 \pm 0.6]10^{-3}e^2b^2 \quad (^{10}Be)$

Bare interaction







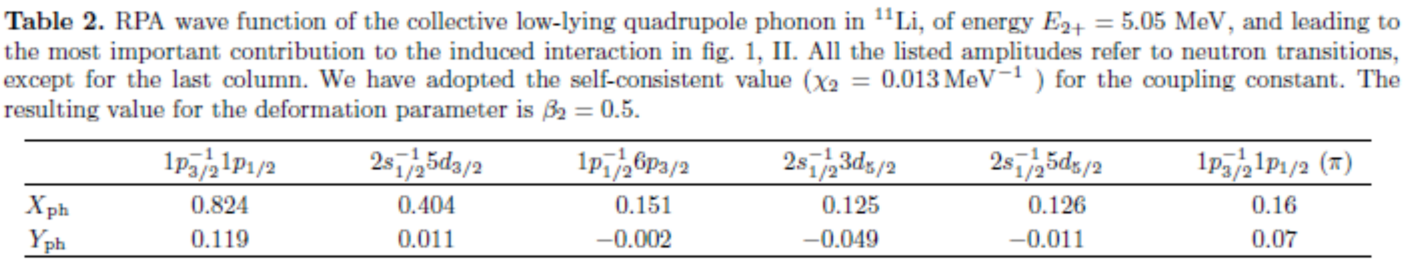
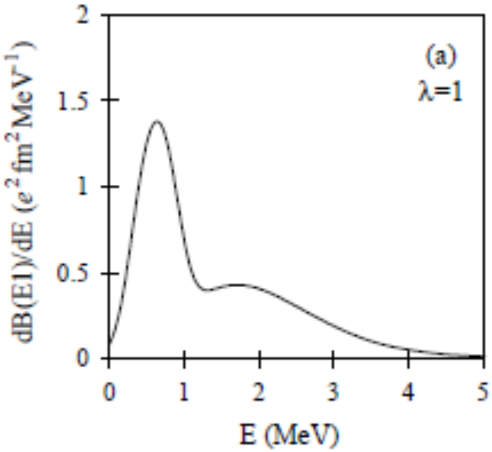


Table 2. RPA wave function of the collective low-lying quadrupole phonon in ^{11}Li, of energy $E_{2^+} = 5.05$ MeV, and leading to the most important contribution to the induced interaction in fig. 1, II. All the listed amplitudes refer to neutron transitions, except for the last column. We have adopted the self-consistent value ($\chi_2 = 0.013\,\mathrm{MeV}^{-1}$) for the coupling constant. The resulting value for the deformation parameter is $\beta_2 = 0.5$.

	$1p^{-1}_{3/2}1p_{1/2}$	$2s^{-1}_{1/2}5d_{3/2}$	$1p^{-1}_{1/2}6p_{3/2}$	$2s^{-1}_{1/2}3d_{5/2}$	$2s^{-1}_{1/2}5d_{5/2}$	$1p^{-1}_{3/2}1p_{1/2}$ (π)
X_{ph}	0.824	0.404	0.151	0.125	0.126	0.16
Y_{ph}	0.119	0.011	−0.002	−0.049	−0.011	0.07

B(E1) calculated with separable force; coupling constant tuned to reproduce experimental strength;
part of the strength comes from admixture of GDR



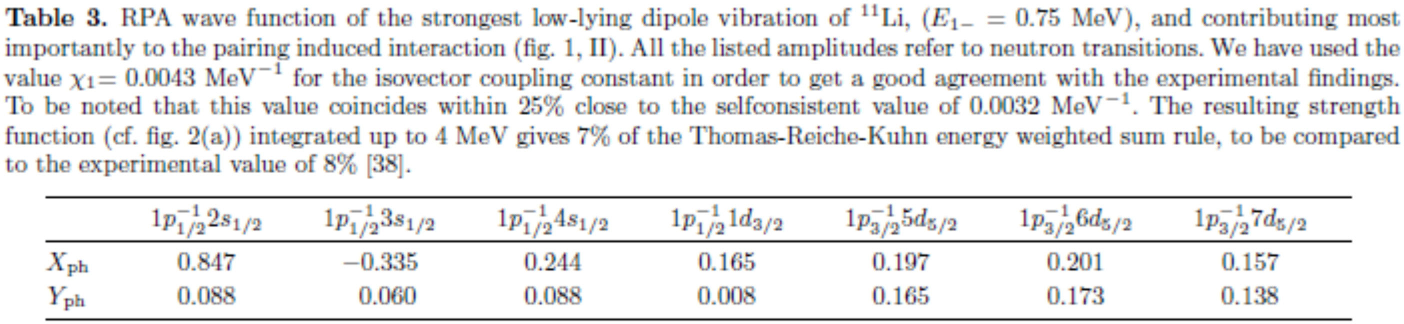
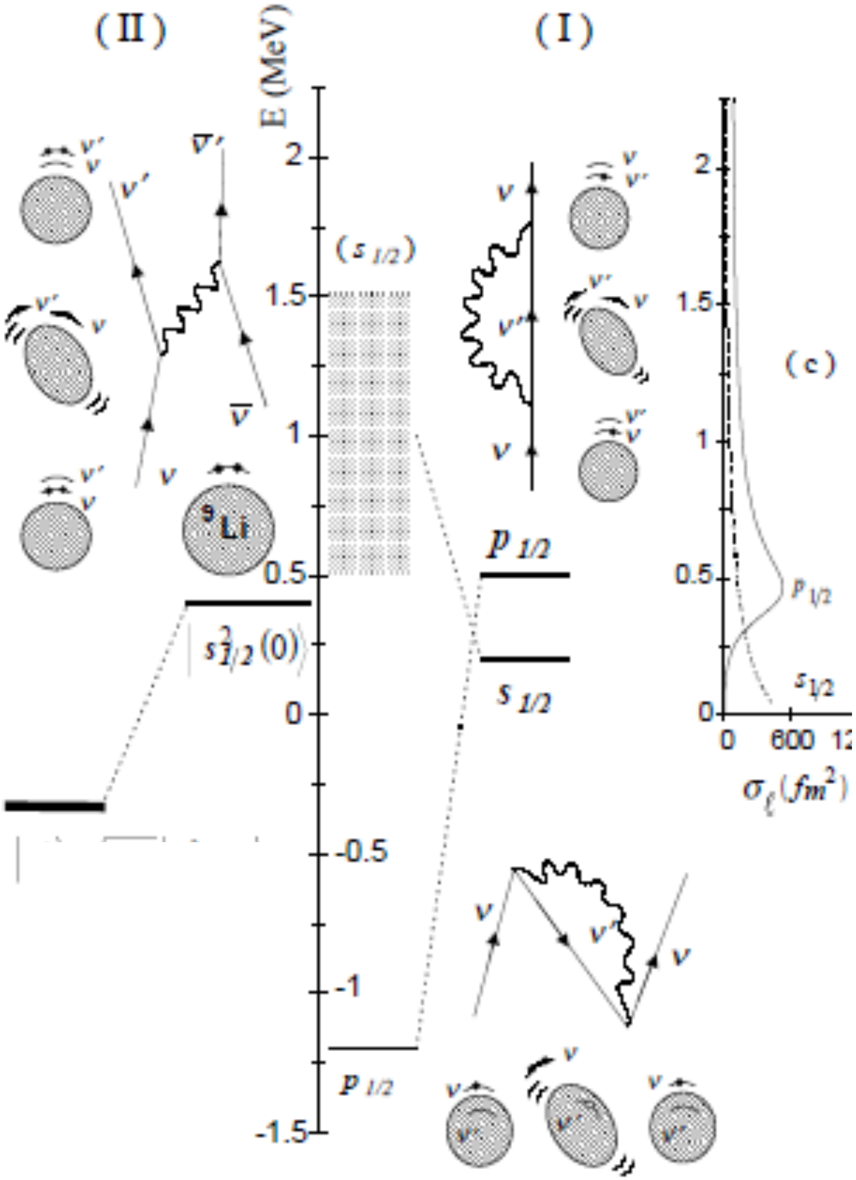


Table 3. RPA wave function of the strongest low-lying dipole vibration of ^{11}Li, ($E_{1^-} = 0.75$ MeV), and contributing most importantly to the pairing induced interaction (fig. 1, II). All the listed amplitudes refer to neutron transitions. We have used the value $\chi_1 = 0.0043$ MeV^{-1} for the isovector coupling constant in order to get a good agreement with the experimental findings. To be noted that this value coincides within 25% close to the selfconsistent value of 0.0032 MeV^{-1}. The resulting strength function (cf. fig. 2(a)) integrated up to 4 MeV gives 7% of the Thomas-Reiche-Kuhn energy weighted sum rule, to be compared to the experimental value of 8% [38].

	$1p^{-1}_{1/2}2s_{1/2}$	$1p^{-1}_{1/2}3s_{1/2}$	$1p^{-1}_{1/2}4s_{1/2}$	$1p^{-1}_{1/2}1d_{3/2}$	$1p^{-1}_{3/2}5d_{5/2}$	$1p^{-1}_{3/2}6d_{5/2}$	$1p^{-1}_{3/2}7d_{5/2}$
X_{ph}	0.847	−0.335	0.244	0.165	0.197	0.201	0.157
Y_{ph}	0.088	0.060	0.088	0.008	0.165	0.173	0.138

Results for ^{10}Li and ^{11}Li



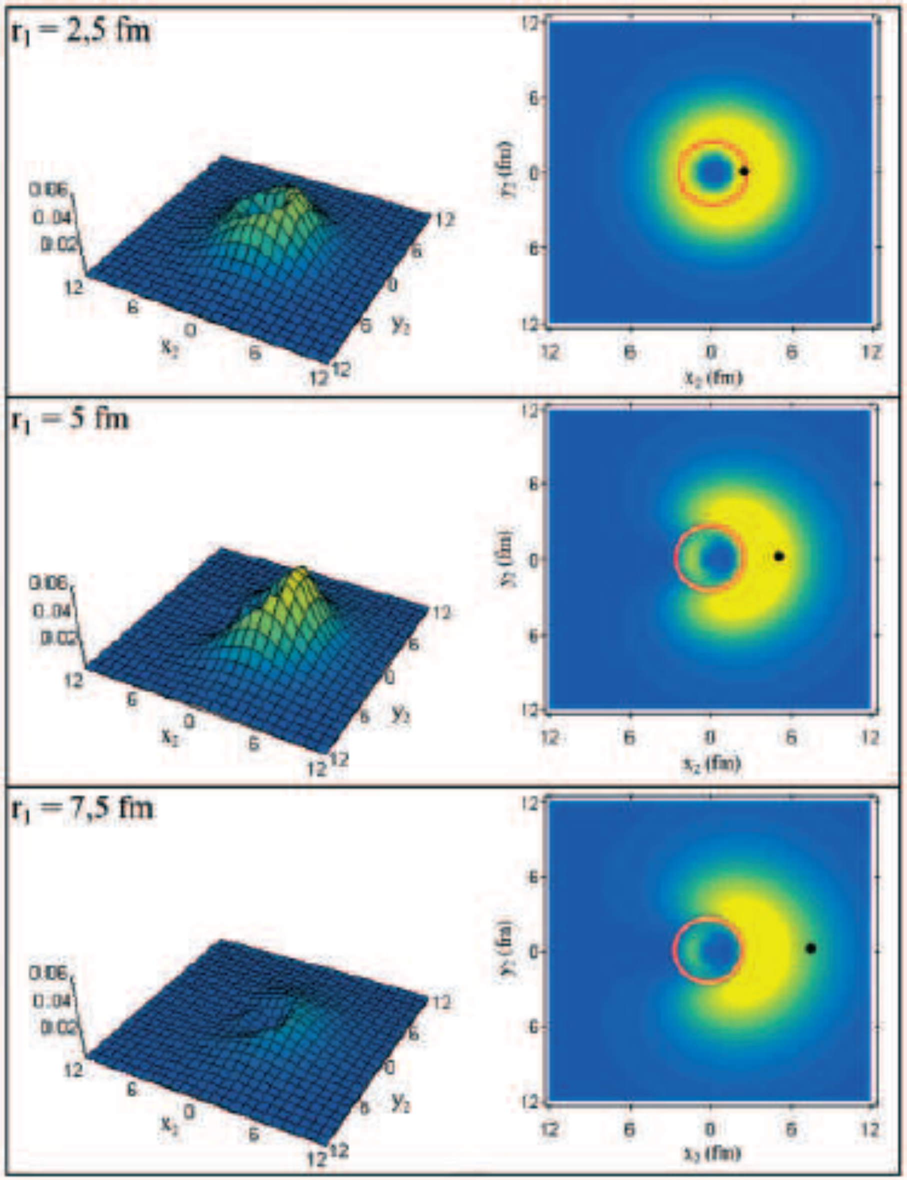
		Exp.	Theory	
			particle-vibration +Argonne	mean field
$^{10}_{3}$Li$_7$ (not bound)	s	0.1-0.2 MeV	0.2 MeV (virtual)	~ 1 MeV (virtual)
	p	0.5-0.6 MeV	0.5 MeV (res.)	-1.2 MeV (bound)
$^{11}_{3}$Li$_8$ (bound)	S_{2n}	0.369 MeV	0.33 MeV	2.4 MeV
	s^2,p^2	50% , 50%	41% , 59%	0% , 100%
	$\langle r^2\rangle^{1/2}$	3.55±0.1 fm	3.9 fm	
	$\Delta p_\perp$	48±10 MeV/c	55 MeV/c	

11Li correlated wave function

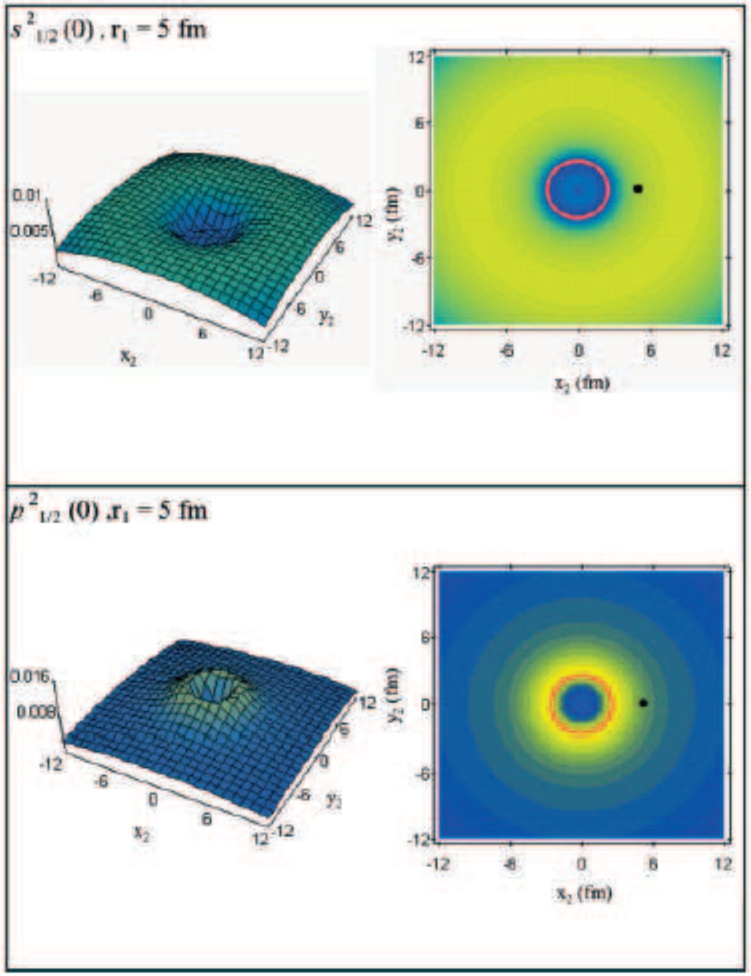
$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-;0\rangle + 0.1|(sd)_{2^+} \otimes 2^+;0\rangle$$

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

Correlated halo wavefunction



Uncorrelated



^{11}Li correlated wave function

The halo wavefunction is made out of components which are superposition of single-particle wavefunctions in the discretized continuum, leading to a bound state:

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

A part of the wavefunction is explicitly coupled to 1- and 2+ vibrations:

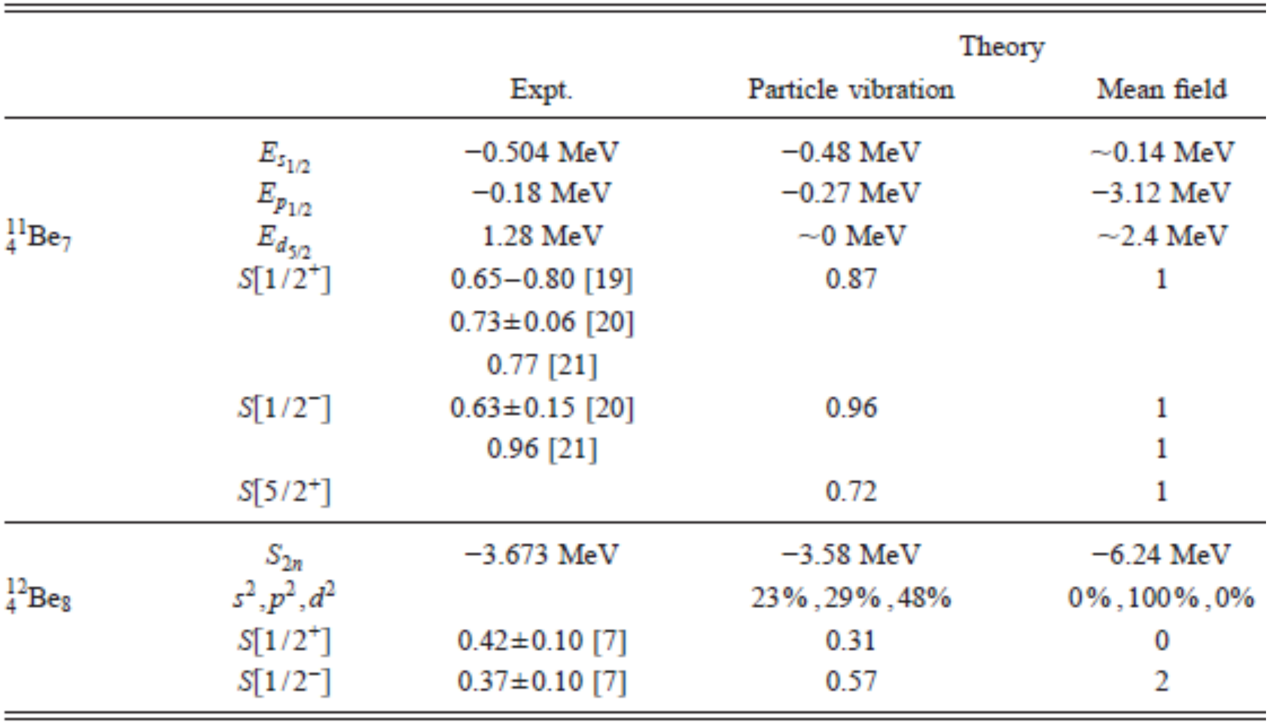
$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.1|(sd)_{2^+} \otimes 2^+; 0\rangle$$

Results for ^{11}Be,^{12}Be
Good agreement between theory and experiment concerning energies and spectroscopic factors

New result for S[1/2+]: $0.28^{+0.03}_{-0.07}$

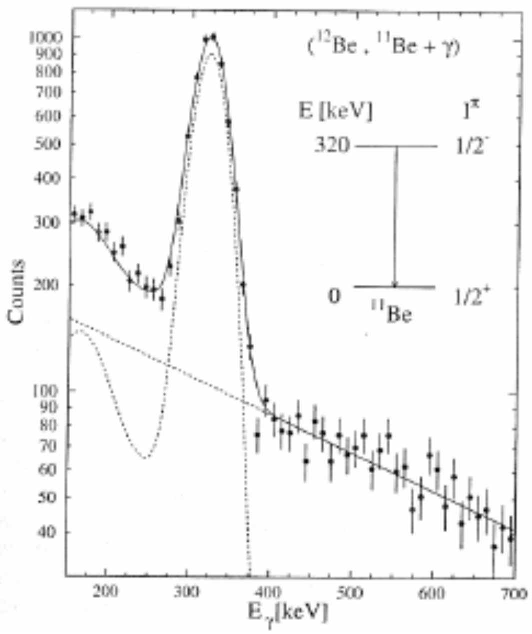
Kanungo et al.
PLB 682 (2010) 39

Spectroscopic factors from (12Be,11Be+γ) reaction to ½$^{+}$ and ½- final states:
S[1/2-]= 0.37±0.10   S[1/2+]= 0.42±0.10

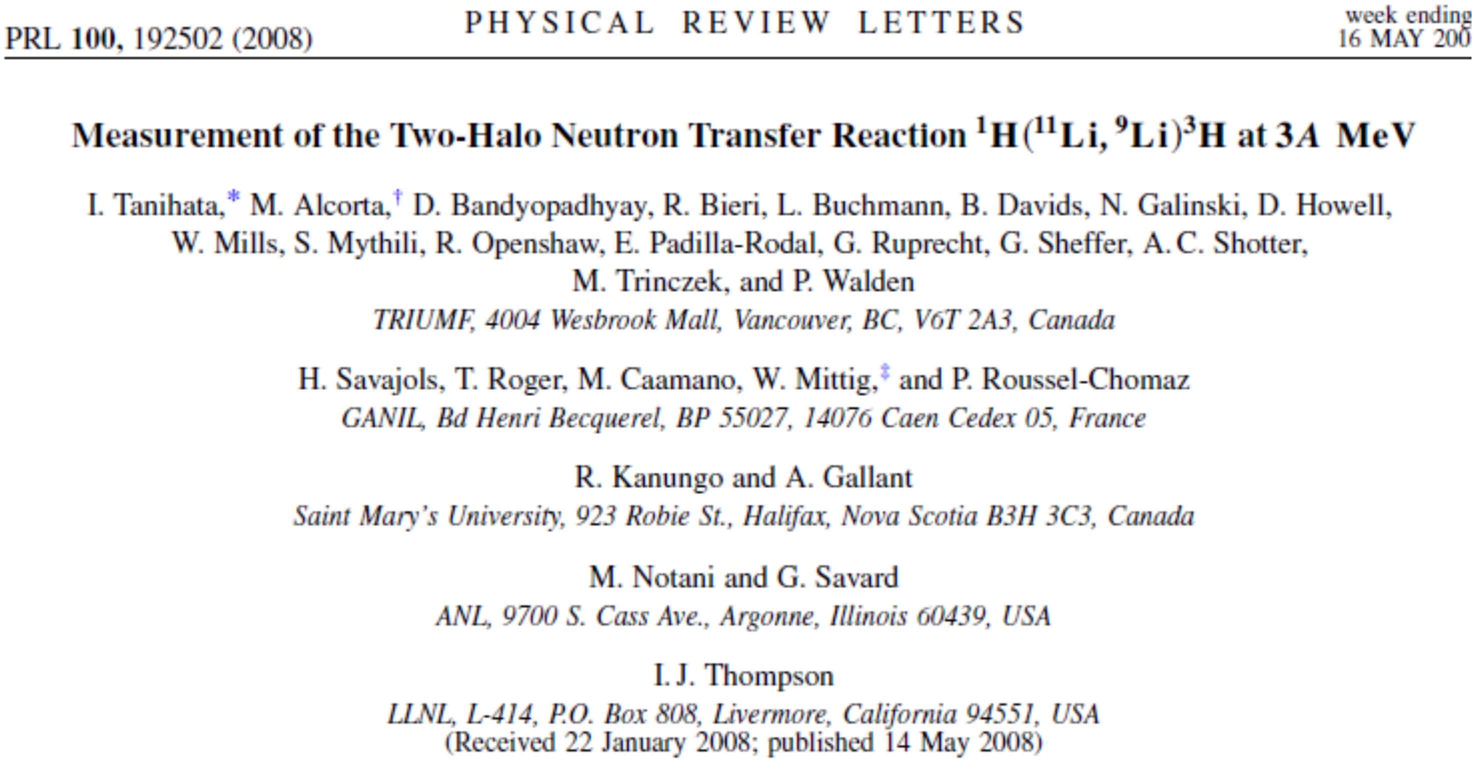


			Theory	
		Expt.	Particle vibration	Mean field
$^{11}_{4}$Be$_7$	$E_{s_{1/2}}$	−0.504 MeV	−0.48 MeV	~0.14 MeV
	$E_{p_{1/2}}$	−0.18 MeV	−0.27 MeV	−3.12 MeV
	$E_{d_{5/2}}$	1.28 MeV	~0 MeV	~2.4 MeV
	$S[1/2^+]$	0.65–0.80 [19]	0.87	1
		0.73±0.06 [20]		
		0.77 [21]		
	$S[1/2^-]$	0.63±0.15 [20]	0.96	1
		0.96 [21]		1
	$S[5/2^+]$		0.72	1
$^{12}_{4}$Be$_8$	S_{2n}	−3.673 MeV	−3.58 MeV	−6.24 MeV
	s^2,p^2,d^2		23%,29%,48%	0%,100%,0%
	$S[1/2^+]$	0.42±0.10 [7]	0.31	0
	$S[1/2^-]$	0.37±0.10 [7]	0.57	2

A. Navin et al.,
PRL 85(2000)266



Probing ^{11}Li halo-neutrons correlations via (p,t) reaction



PRL **100,** 192502 (2008) PHYSICAL REVIEW LETTERS week ending 16 MAY 200

Measurement of the Two-Halo Neutron Transfer Reaction ^{1}H(^{11}Li, ^{9}Li)^{3}H at 3*A* MeV

I. Tanihata,[*] M. Alcorta,[†] D. Bandyopadhyay, R. Bieri, L. Buchmann, B. Davids, N. Galinski, D. Howell,
W. Mills, S. Mythili, R. Openshaw, E. Padilla-Rodal, G. Ruprecht, G. Sheffer, A.C. Shotter,
M. Trinczek, and P. Walden
TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada

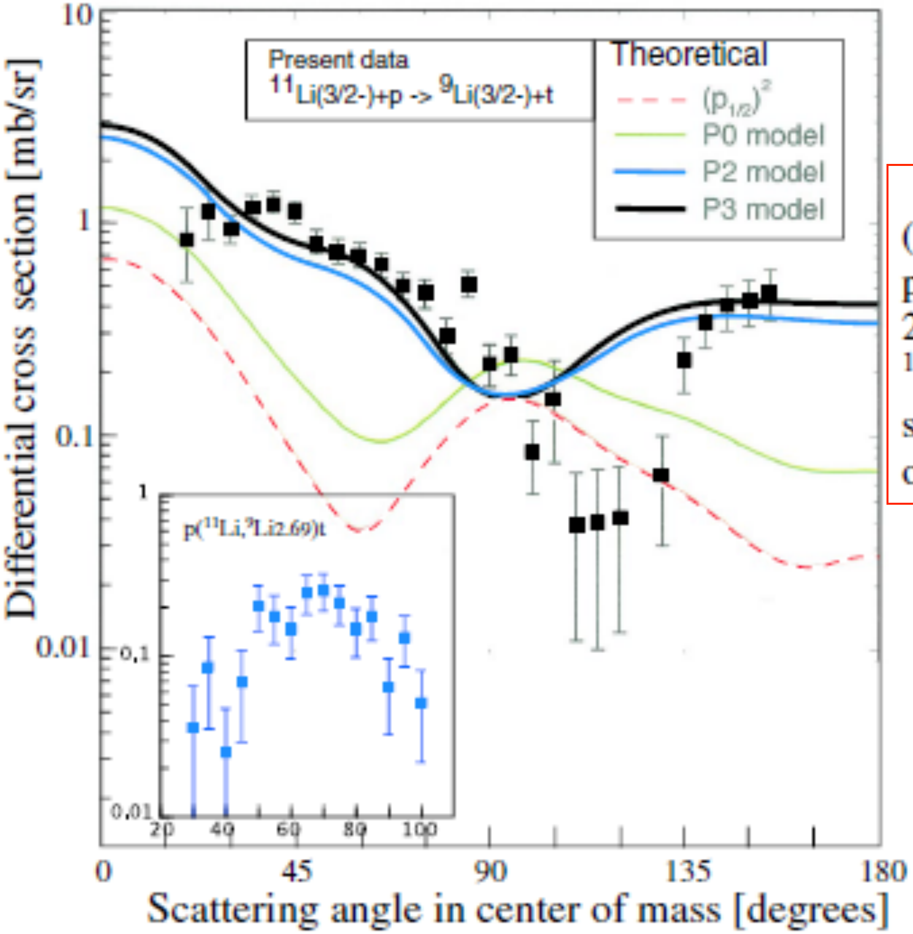
H. Savajols, T. Roger, M. Caamano, W. Mittig,[‡] and P. Roussel-Chomaz
GANIL, Bd Henri Becquerel, BP 55027, 14076 Caen Cedex 05, France

R. Kanungo and A. Gallant
Saint Mary's University, 923 Robie St., Halifax, Nova Scotia B3H 3C3, Canada

M. Notani and G. Savard
ANL, 9700 S. Cass Ave., Argonne, Illinois 60439, USA

I. J. Thompson
LLNL, L-414, P.O. Box 808, Livermore, California 94551, USA

(Received 22 January 2008; published 14 May 2008)



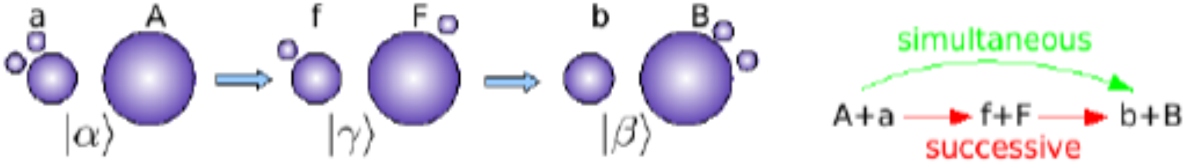
The cross section for transitions to the first excited state (Ex = 2.69 MeV) is shown also in Fig. 3. If this state were populated by a direct transfer, it would indicate that a 1^+ or 2^+ halo component is present in the ground state of $^{11}\mathrm{Li}(\frac{3}{2}^-)$, because the spin-parity of the $^9\mathrm{Li}$ first excited state is $\frac{1}{2}^-$. This is new information that has not yet been observed in any of previous investigations. A compound

TABLE I. Optical potential parameters used for the present calculations.

	V MeV	r_V fm	a_V fm	W MeV	W_D MeV	r_W fm	a_W fm	V_{so} MeV	r_{so} fm	a_{so} fm
$p + {}^{11}\mathrm{Li}$ [10]	54.06	1.17	0.75	2.37	16.87	1.32	0.82	6.2	1.01	0.75
$d + {}^{10}\mathrm{Li}$ [11]	85.8	1.17	0.76	1.117	11.863	1.325	0.731	0		
$t + {}^{9}\mathrm{Li}$ [12]	1.42	1.16	0.78	28.2	0	1.88	0.61	0		

Calculation of absolute two-nucleon transfer cross section by finite-range DWBA calculation

simultaneous and successive contributions



the initial and final channel wave functions are

$$
\begin{aligned}
|\alpha\rangle &= \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2)\phi_A(\xi_A)\chi_{aA}(\mathbf{r}_{aA}) \\
|\beta\rangle &= \phi_b(\xi_b)\phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2)\chi_{bB}(\mathbf{r}_{bB})
\end{aligned}
$$

very schematically, the first order (*simultaneous*) contribution is

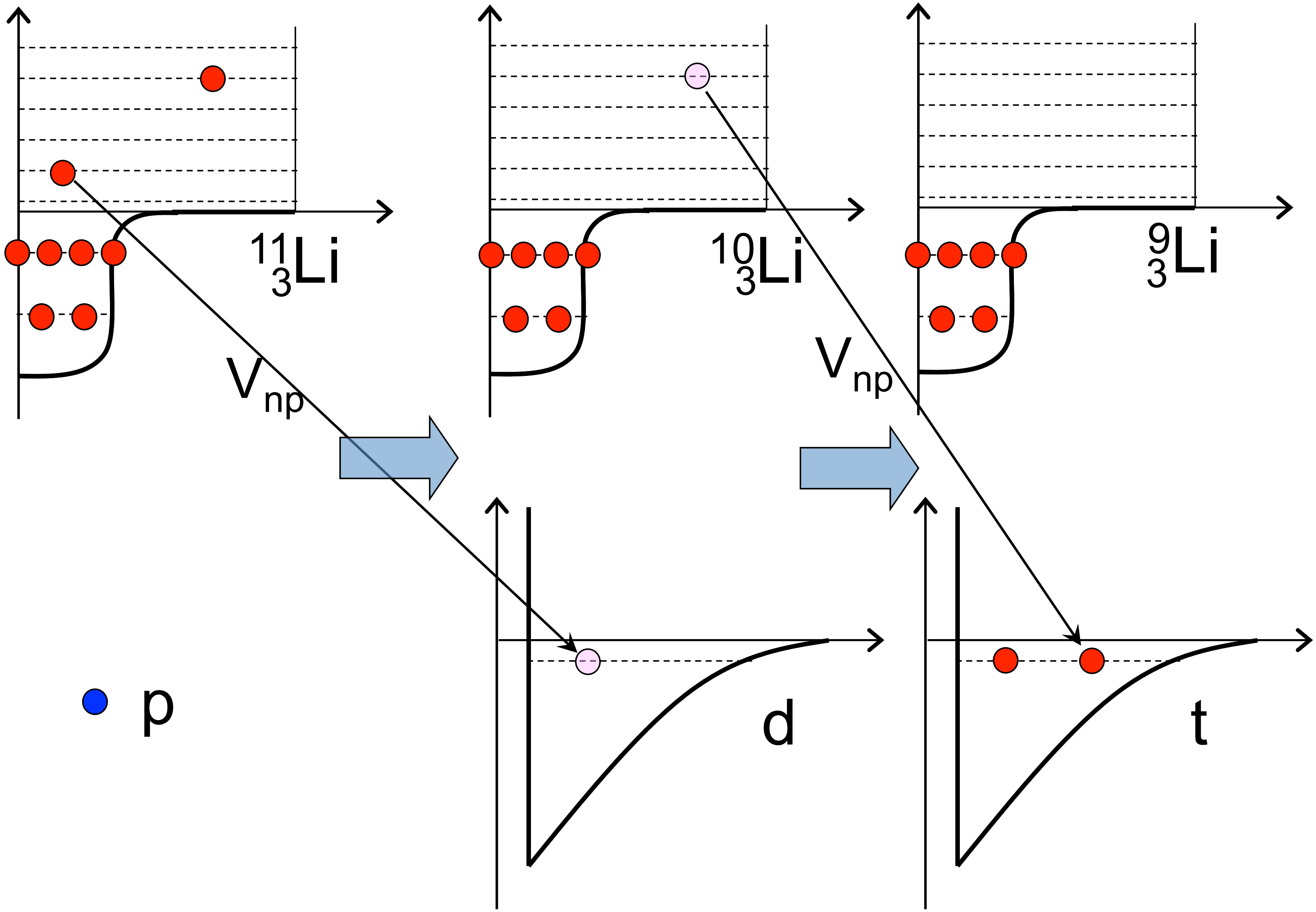
$$T^{(1)} = \langle\beta|V|\alpha\rangle,$$

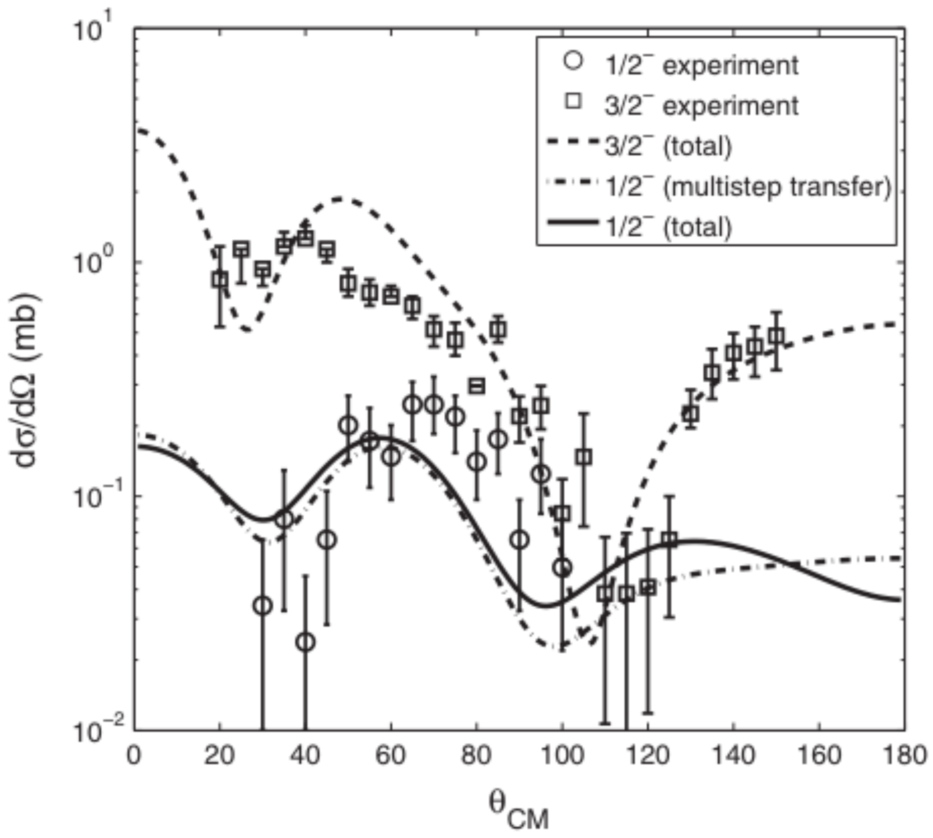
while the second order contribution can be separated in a *successive* and a *non-orthogonality* term

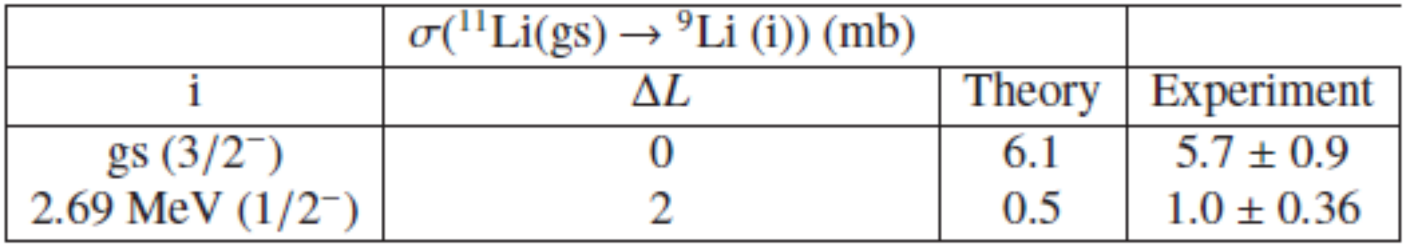
$$
\begin{aligned}
T^{(2)} &= T^{(2)}_{succ} + T^{(2)}_{NO} \\
&= \sum_{\gamma}\langle\beta|V|\gamma\rangle G\langle\gamma|V|\alpha\rangle - \sum_{\gamma}\langle\beta|\gamma\rangle\langle\gamma|V|\alpha\rangle.
\end{aligned}
$$

B.F. Bayman and J. Chen, Phys. Rev. C 26 (1982) 150
M. Igarashi, K. Kubo and K. Yagi, Phys. Rep. 199 (1991) 1
G. Potel et al., arXiv: 0906.4298

$$\sum_{n_1,n_2} a_{n_1,n_2}[\psi_{n_1}(r_1)\psi_{n_2}(r_2)]_{00}$$





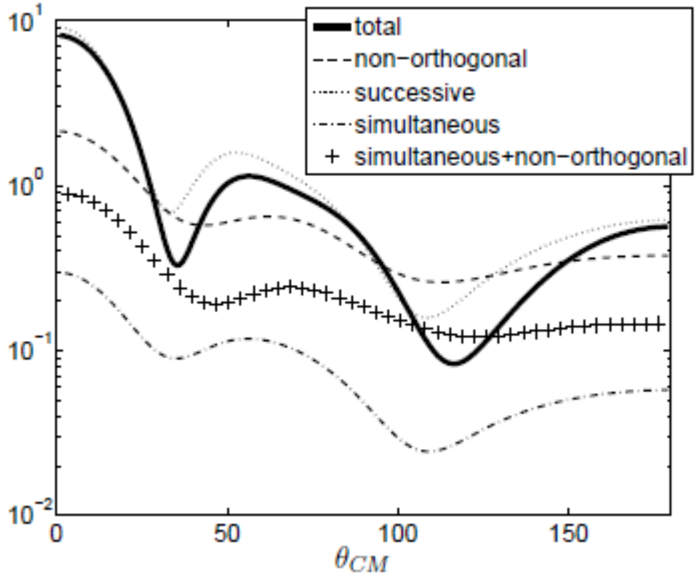


	$\sigma(^{11}\text{Li(gs)} \rightarrow {}^{9}\text{Li (i)})$ (mb)		
i	ΔL	Theory	Experiment
gs ($3/2^-$) 2.69 MeV ($1/2^-$)	0 2	6.1 0.5	5.7 ± 0.9 1.0 ± 0.36

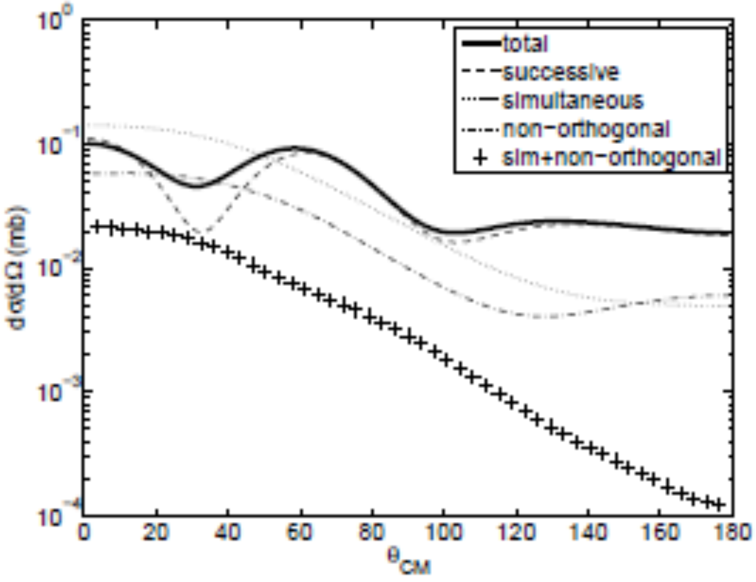
G. Potel et al., PRL 105 (2010) 172502

Decomposition into successive and simultaneous contributions

3/2- ground state

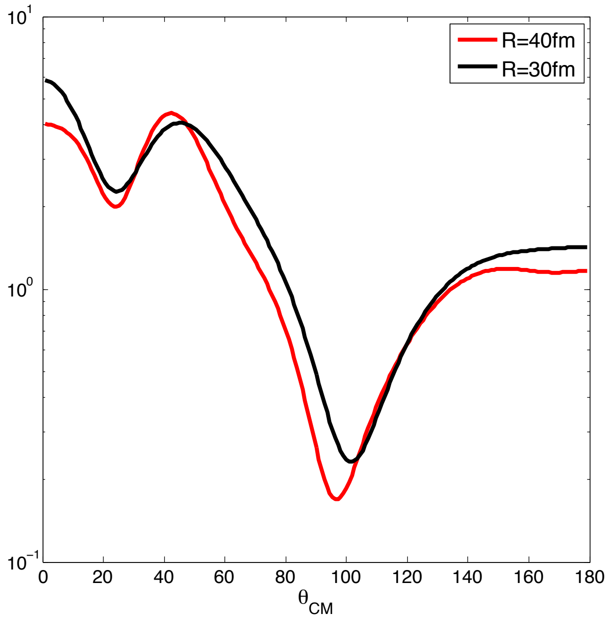


1/2- excited state

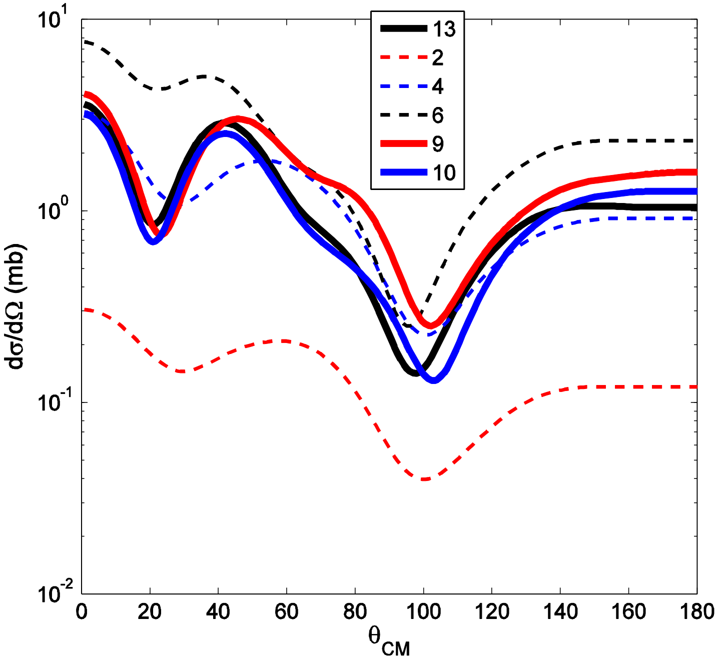


Convergence of the calculation

With box radius



With number of intermediate states

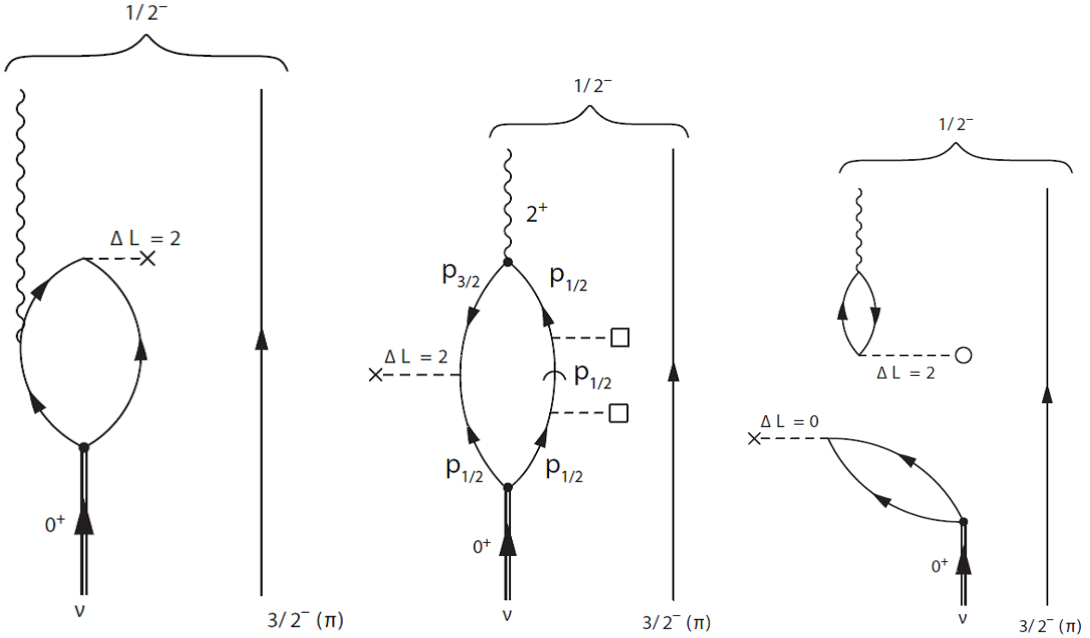


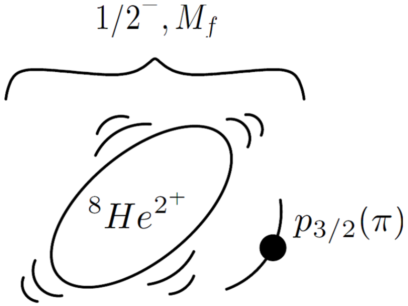
Channels c leading to the first $1/2^-$ excited state of ^{9}Li

$c = 1$: Transfer of the two halo neutrons

$c = 2$: Transfer of a $p_{1/2}$ halo neutron and a $p_{3/2}$ core neutron

$c = 3$: Transfer to the ground state + inelastic excitation





$$P^{(1)} = 1.3 \times 10^{-3}$$

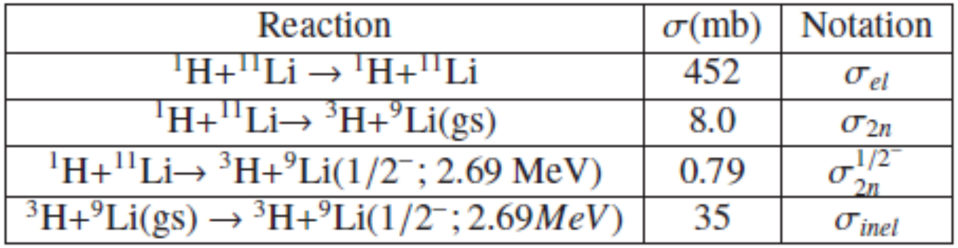
$$P^{(2)} = 4.6 \times 10^{-5}$$

$$P^{(3)} = 2.6 \times 10^{-6}$$

$\sigma_c = \frac{\pi}{k^2} \sum_l (2l+1)|S_l^{(c)}|^2$, $P^{(c)} = \sum_l |S_l^{(c)}|^2$ $(c = 1, 2, 3)$.

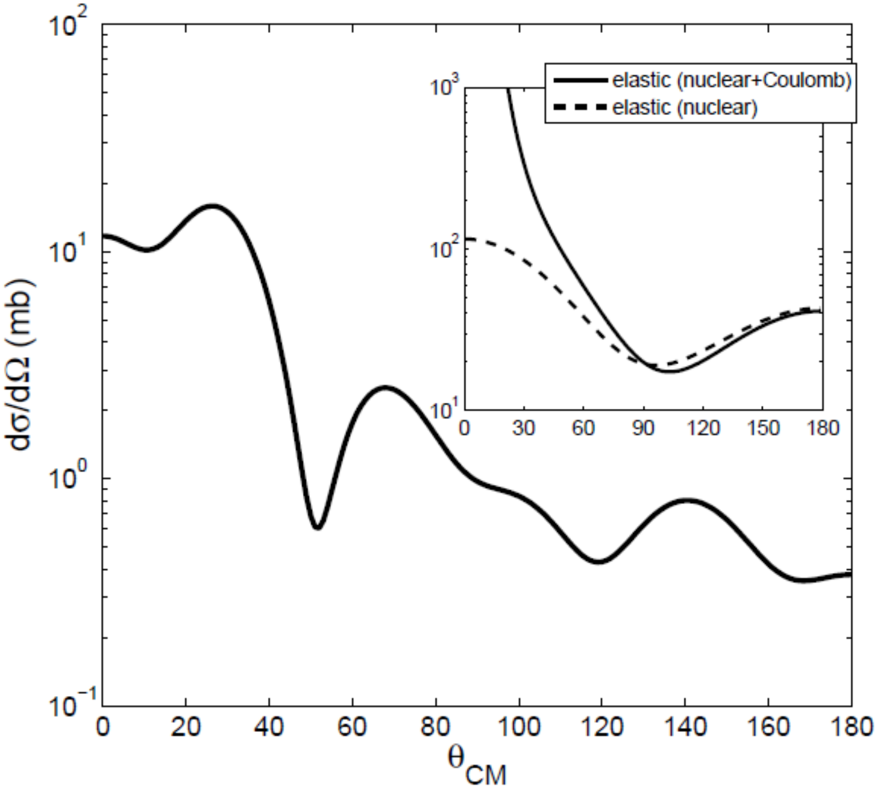
Small probabilities ⇒ use of second order perturbation theory.

Two-step effects : how important are they?

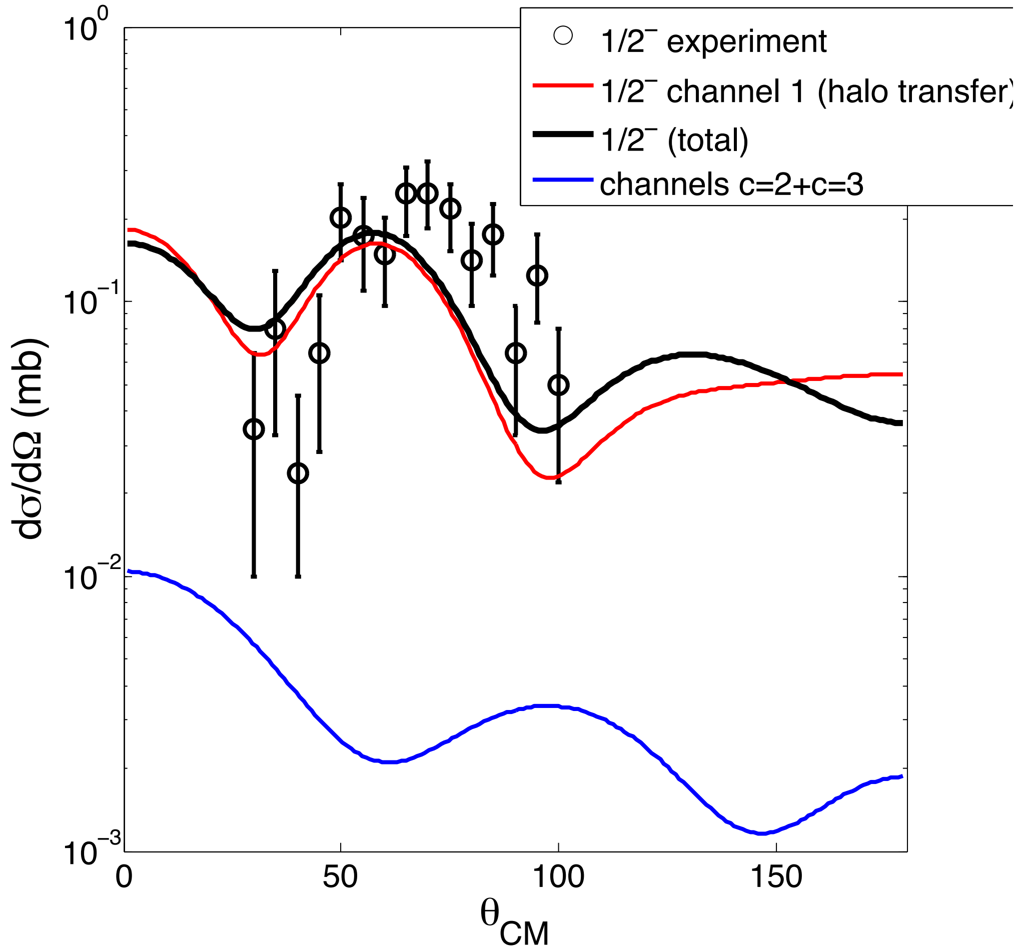


Reaction	σ(mb)	Notation
$^{1}H+^{11}Li \rightarrow ^{1}H+^{11}Li$	452	σ_{el}
$^{1}H+^{11}Li \rightarrow ^{3}H+^{9}Li(gs)$	8.0	σ_{2n}
$^{1}H+^{11}Li \rightarrow ^{3}H+^{9}Li(1/2^{-}; 2.69\ \mathrm{MeV})$	0.79	$\sigma_{2n}^{1/2^{-}}$
$^{3}H+^{9}Li(gs) \rightarrow ^{3}H+^{9}Li(1/2^{-}; 2.69 MeV)$	35	σ_{inel}

Excitation of ½- state following transfer



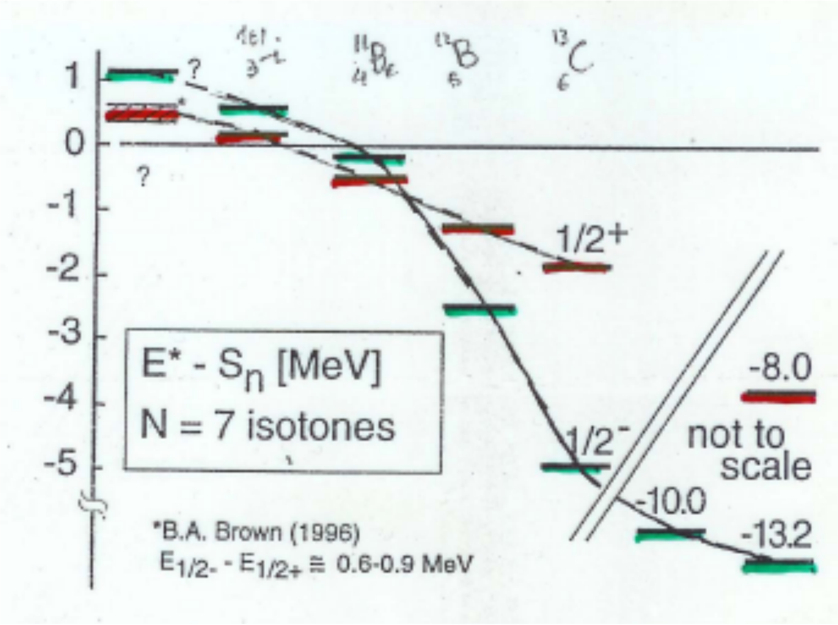
E. Vigezzi et al.,
J. Phys. G Conf. Ser. 312 (2011) 092061



10^0
10^{-1}
10^{-2}
10^{-3}
dσ/dΩ (mb)
0
50
100
150
θ_{CM}
○ 1/2⁻ experiment
1/2⁻ channel 1 (halo transfer
1/2⁻ (total)
channels c=2+c=3

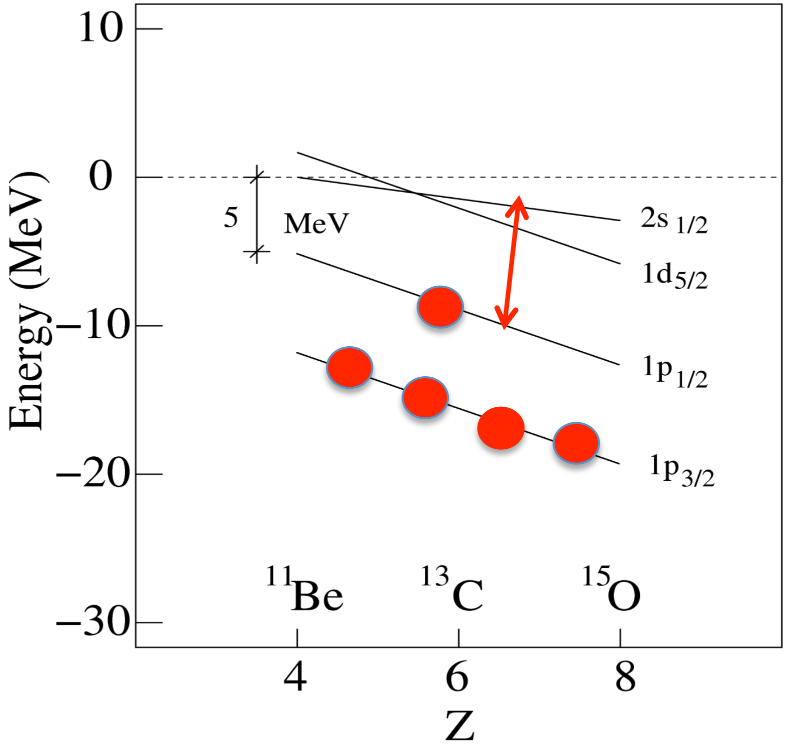
Parity inversion in N=7 isotones

Experimental systematics



Mean-field results

(Sagawa,Brown,Esbensen PLB 309(93)1)

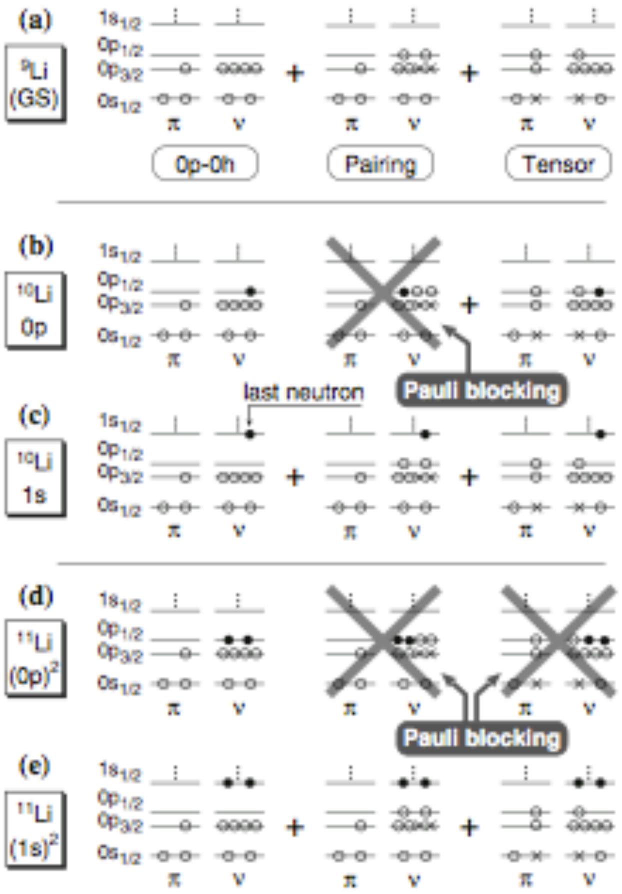


Comparison with the model by Ikeda, Myo et al.

K. Ikeda et al,
Lect. Notes in Physics 818 (2010)

and essentially all the theoretical works of 11Li had to accept that the 1s1/2 single particle state is brought down to the 0p1/2 state without knowing its reason …

The theoretical challenge on the halo structure is therefore summarized as follows. There are many indications that the *s*-wave component is very large in the ground state wave function. Hence, we have to find a mechanism to bring down the $s_{1/2}$ orbit with the amount to wash out the $N = 8$ magic structure.



$$
\begin{aligned}
|^{9}Li\rangle = {} & C_1 |(s_{1/2})^2_\pi (s_{1/2})^2_\nu (p_{3/2})_\pi (p_{3/2})^4_\nu\rangle_{J=3/2} \\
& + C_2 |(s_{1/2})^2_\pi (s_{1/2})^2_\nu (p_{3/2})_\pi (p_{3/2})^2_{\nu J=0} (p_{1/2})^2_{\nu J=0}\rangle_{J=3/2} \\
& + C_3 |[(s_{1/2})_\pi (s_{1/2})_\nu]_{J=1} (p_{3/2})_\pi (p_{3/2})^4_\nu [(p_{1/2})_\pi (p_{1/2})_\nu]_{J=1}\rangle_{J=3/2} \\
& + \cdots
\end{aligned}
$$

$p_{1/2}$ orbit is pushed up by pairing correlations and tensor force. Only 3/2- configurations are included: coupling to core vibrations (1/2-) is not considered. Binding energy is given as input. 50%(s^2)-50%(p^2) wavefunction is obtained

CONCLUSION:

According to a dynamical model of the halo nucleus 11Li,
a key role is played by the coupling of the valence nucleons
with the vibrations of the system.
The structure model has been tested with a detailed reaction
calculation, comparing with data obtained in a recent (t,p) experiment.
Theoretical and experimental cross section are in reasonable agreement.

Many open issues, among them:
Optical potentials
The role of the tensor force