Two-particle transfer and pairing correlations: interplay of reaction mechanism and structure properties

Andrea Vitturi

Dynamics and Correlations in Exotic Nuclei (DCEN2011)
Kyoto, 2011

How to use dynamics to study pairing correlations?

The main road is clearly provided by the study of those processes where a pair of particles in involved, e.g. transferred from/to another nucleus (two-particle transfer) or ejected onto the continuum (two-particle break-up).

Unfortunately, the situation is different, for example, from low-energy one-step Coulomb excitation, where the excitation probability is directly proportional to the B(Eλ) values. Here the reaction mechanism is much more complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious. The situation is actually more complicated even with respect to other processes (as inelastic nuclear excitation) that may need to be treated microscopically, but where the reaction mechanism is somehow well established.

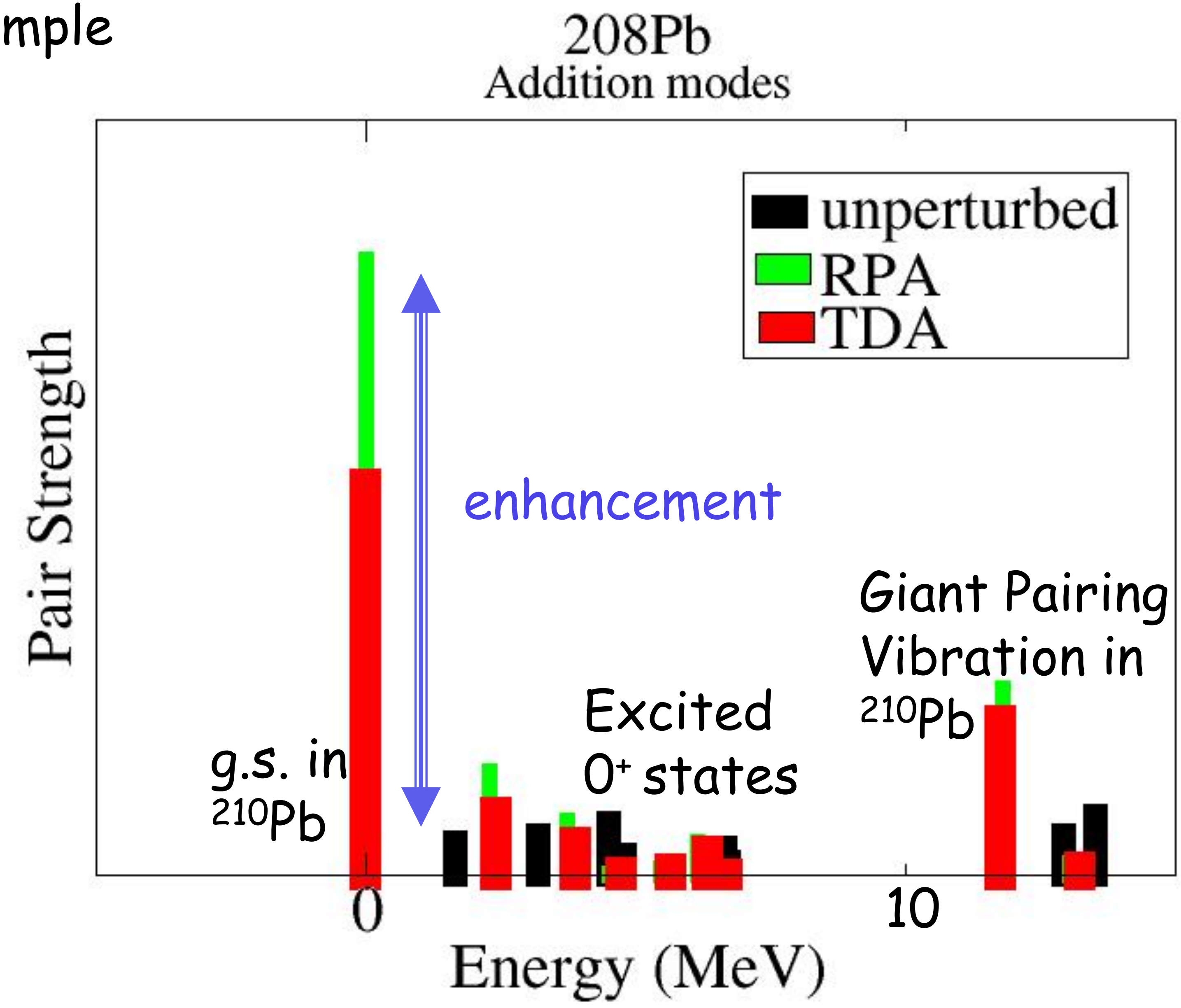
It is often assumed that the cross section for two-particle transfer just scale with T_0, the square of the matrix element of the pair creation (or removal) operator

$$P^{+} = \sum_j [a^{+}_j a^{+}_j]_{00}$$

For this reason the easiest way to define and measure the collectivity of pairing modes is to compare with single-particle pair transition densities and matrix elements to define some “pairing” single-particle units and therefore “pairing” enhancement factors.

Obs: We discuss here monopole pairing modes, i.e. 0+states

Example



enhancement

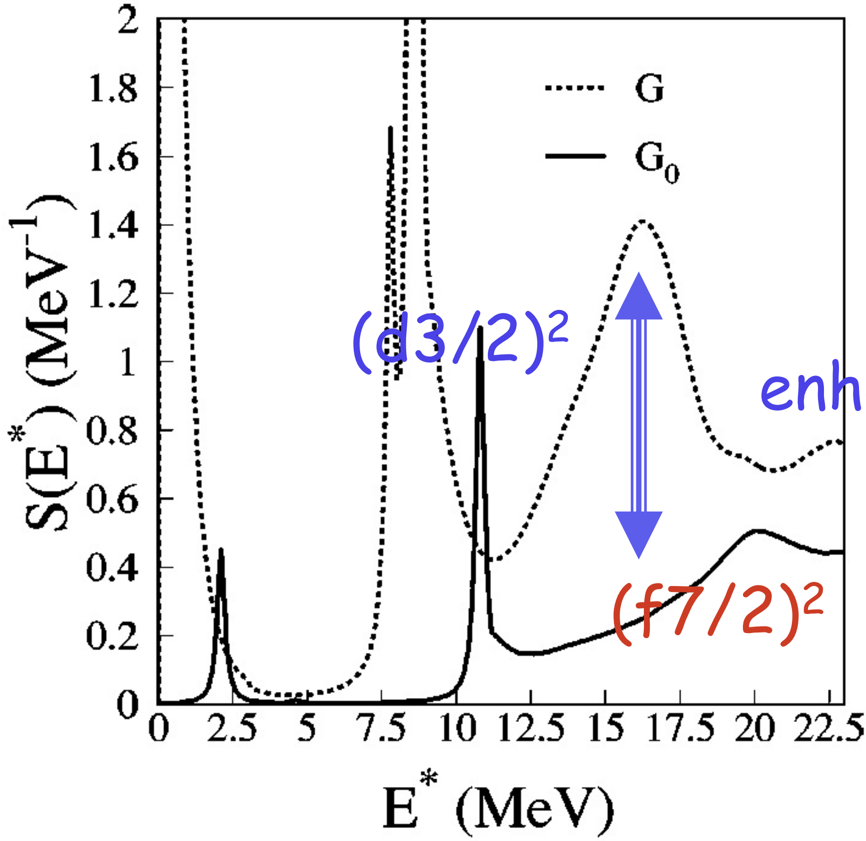
g.s. in ^{210}Pb

Excited 0^{+} states

Giant Pairing Vibration in ^{210}Pb

Pair strength function

^{22}O



$(d3/2)^2$

enhancement

$(f7/2)^2$

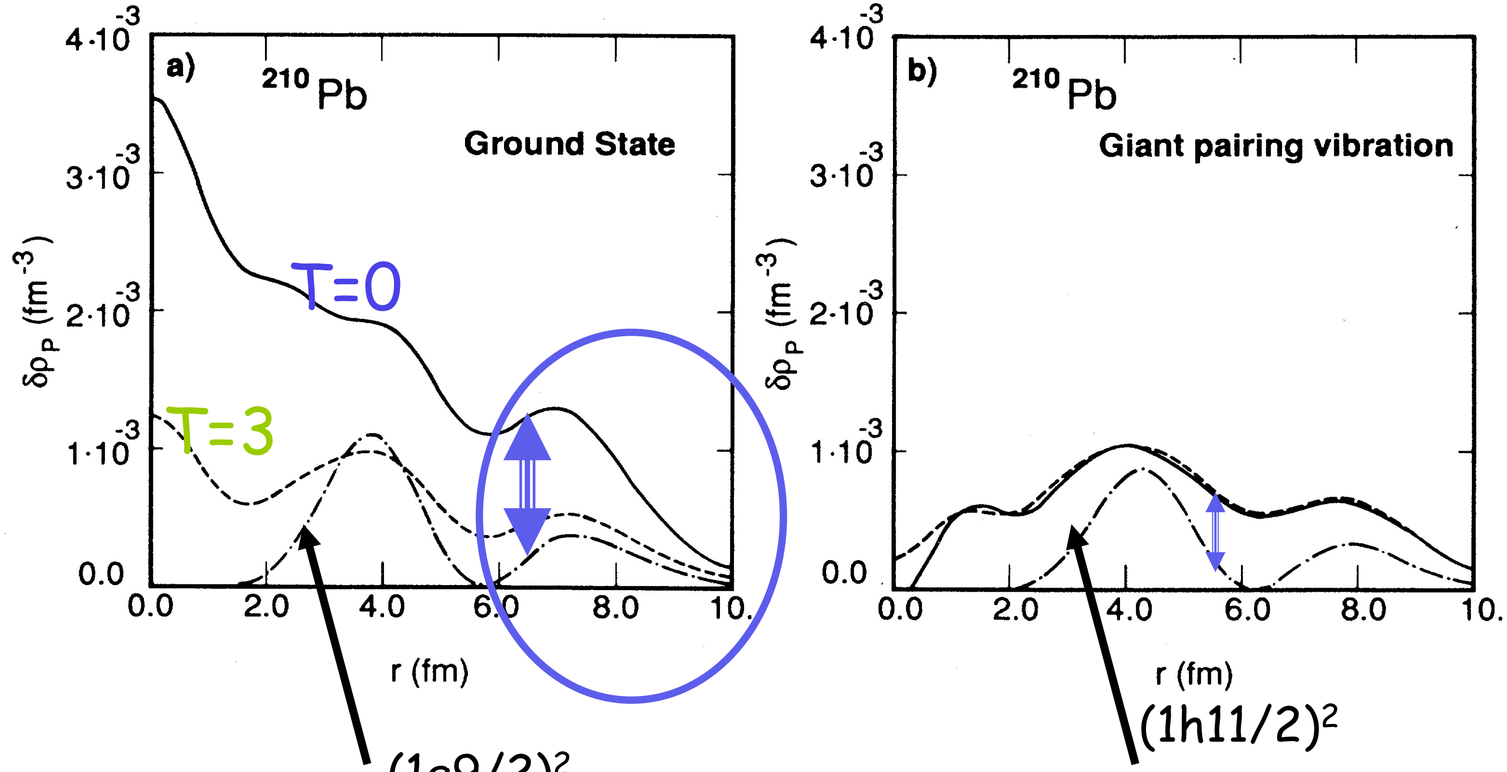
Khan, Sandulescu,Van Giai, Grasso

But the two-particle transfer process in not sensitive to just the pair matrix element. We have to look at the radial dependence, which is relevant for the reaction mechanism associated with pair transfer processes.

Comparison with pure single-particle configurations

pair transition density

$$\rho^{\nu}{}_{P}(r,r)=\kappa^{\nu}\,(r\sigma)=\langle 0|c(r\sigma)\overline{c}(r\sigma)|\nu\rangle$$

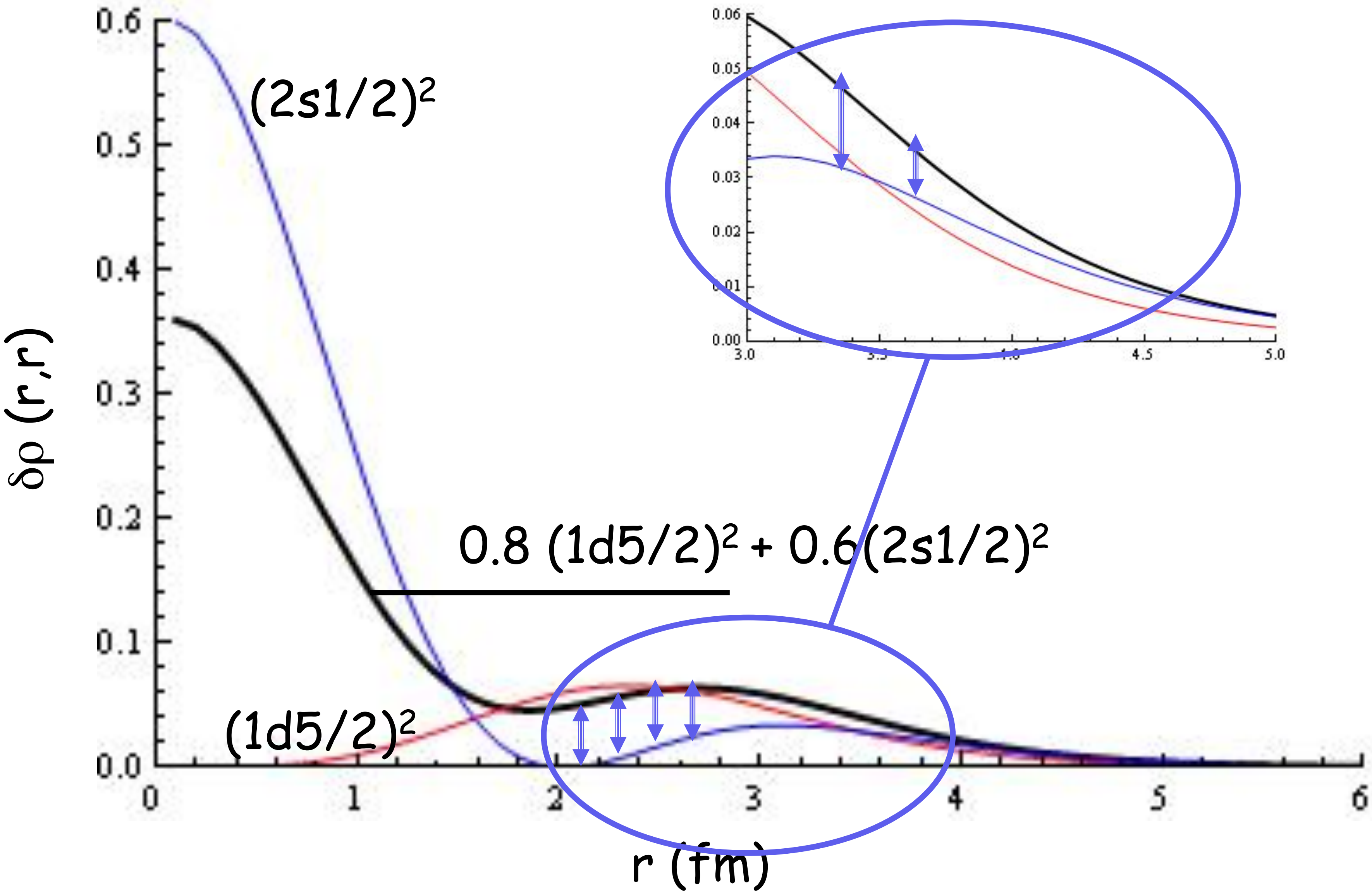


$(1g9/2)^2$

$(1h11/2)^2$

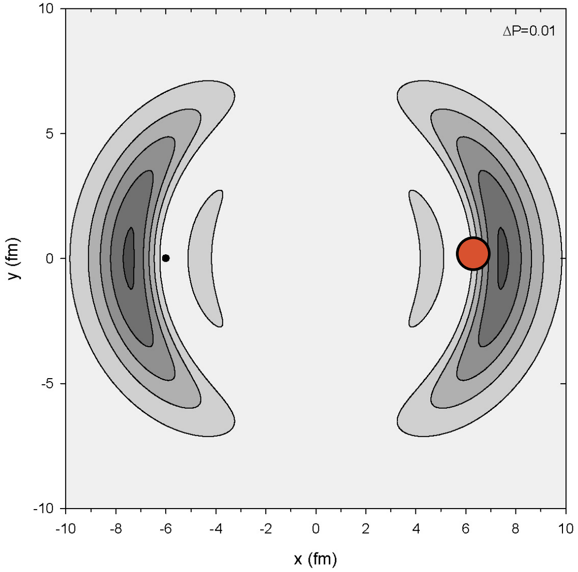
Lotti, Vitturi etal

^{18}O

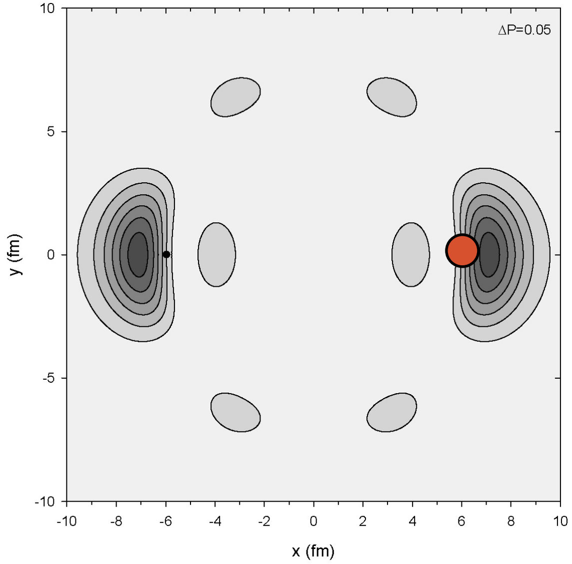

$(2s1/2)^2$
$0.8\,(1d5/2)^2 + 0.6(2s1/2)^2$
$(1d5/2)^2$
$\delta\rho\,(r,r)$
r (fm)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
0.06
0.05
0.04
0.03
0.02
0.01
0.00
3.0
3.5
4.0
4.5
5.0

^{206}Pb

$(3p1/2)^2$

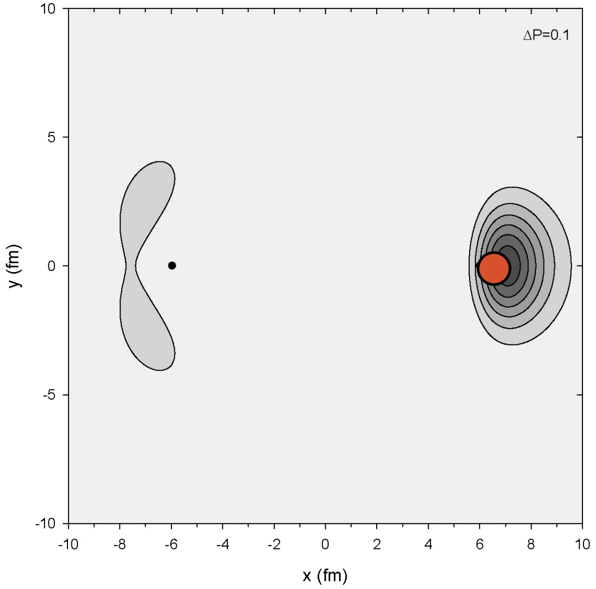


$(2f5/2)^2$



Correlated ground state

$|\Psi(r_1,r_2)|^2$ as a function of r_2, for fixed r_1



OBS: mixing of configurations with opposite parity

● position of particle 1

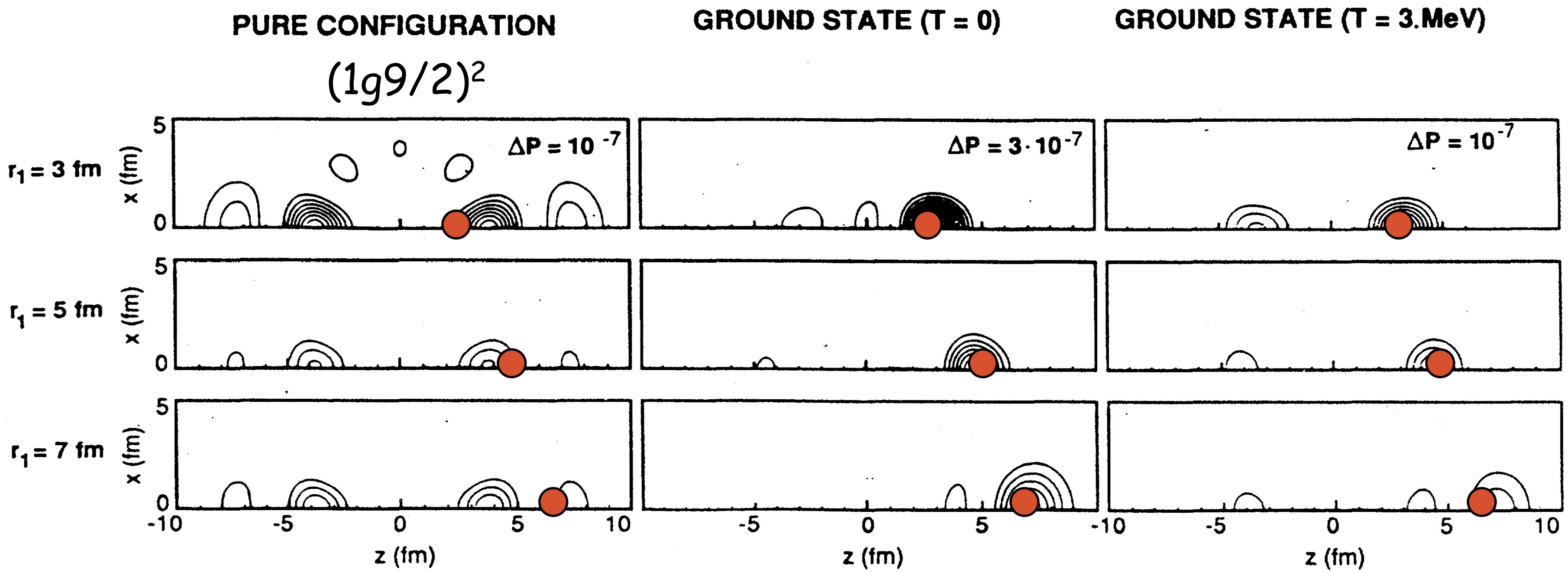
Lotti etal

particle-particle spatial correlations

$|\Psi(r_1,r_2)|^2$ as a function of r_2, for fixed r_1

Neutron addition mode: ground state of ^{210}Pb

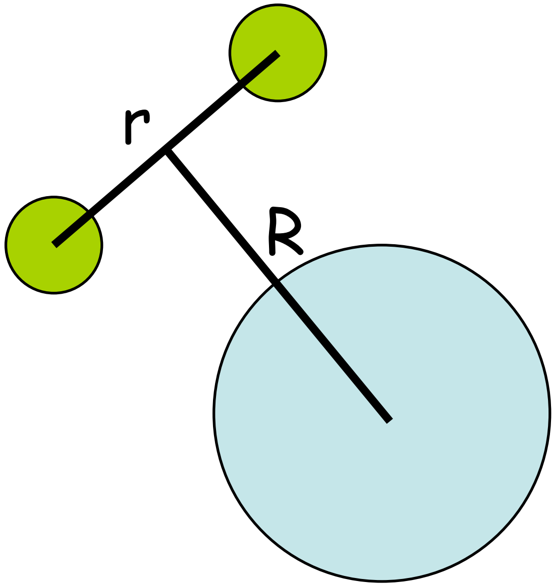
● position of particle 1



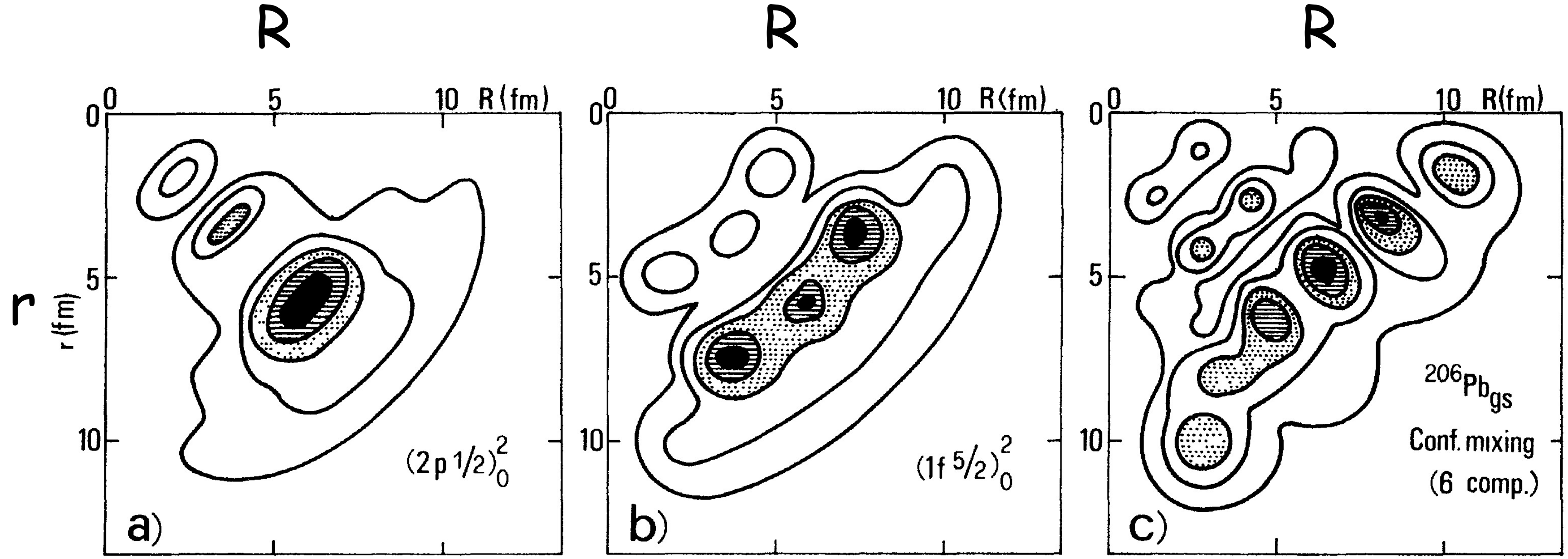
$(1g9/2)^2$

$\delta\rho_P(R,r)$

Catara etal, 1984

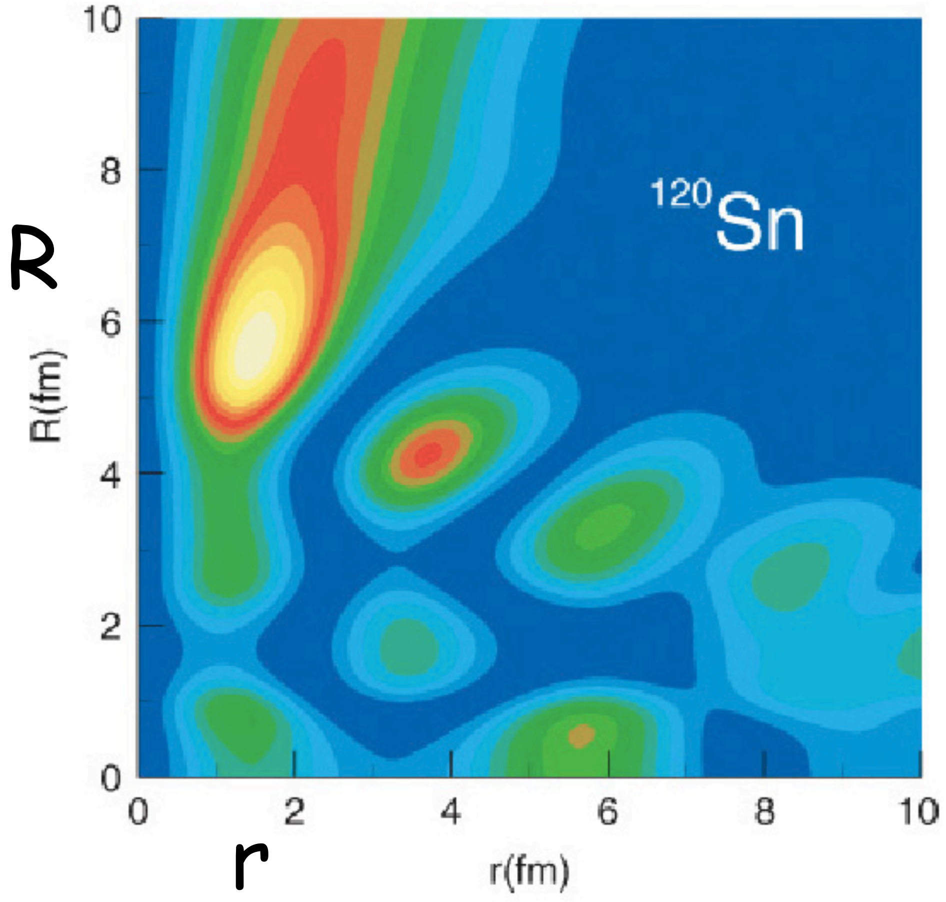


^{206}Pb



$(3p1/2)^2$ $(2f5/2)^2$ Correlated g.s.

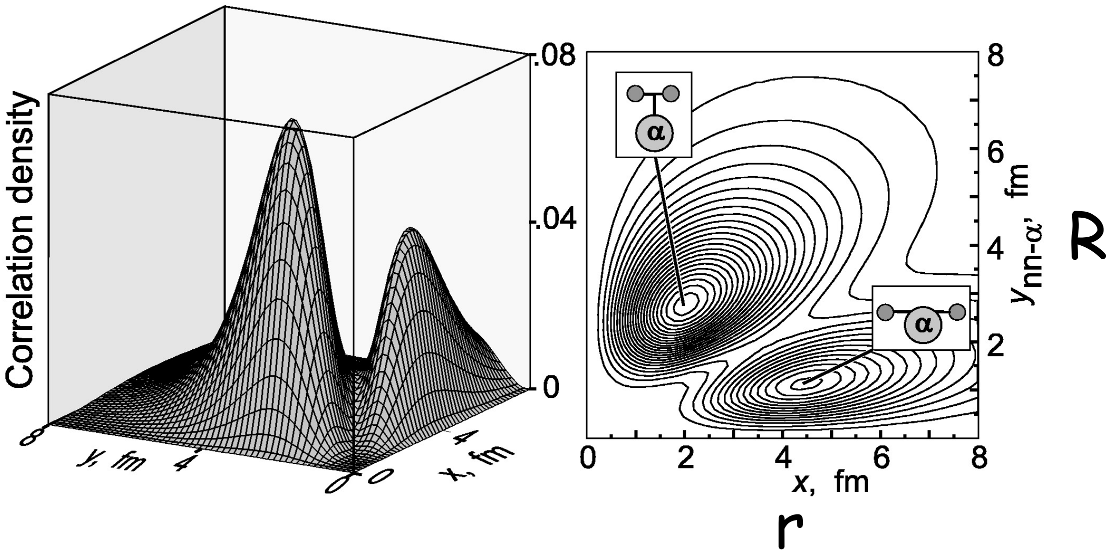
$\delta\rho_P(R,r)$



Pillet, Sandulescu, Schuck

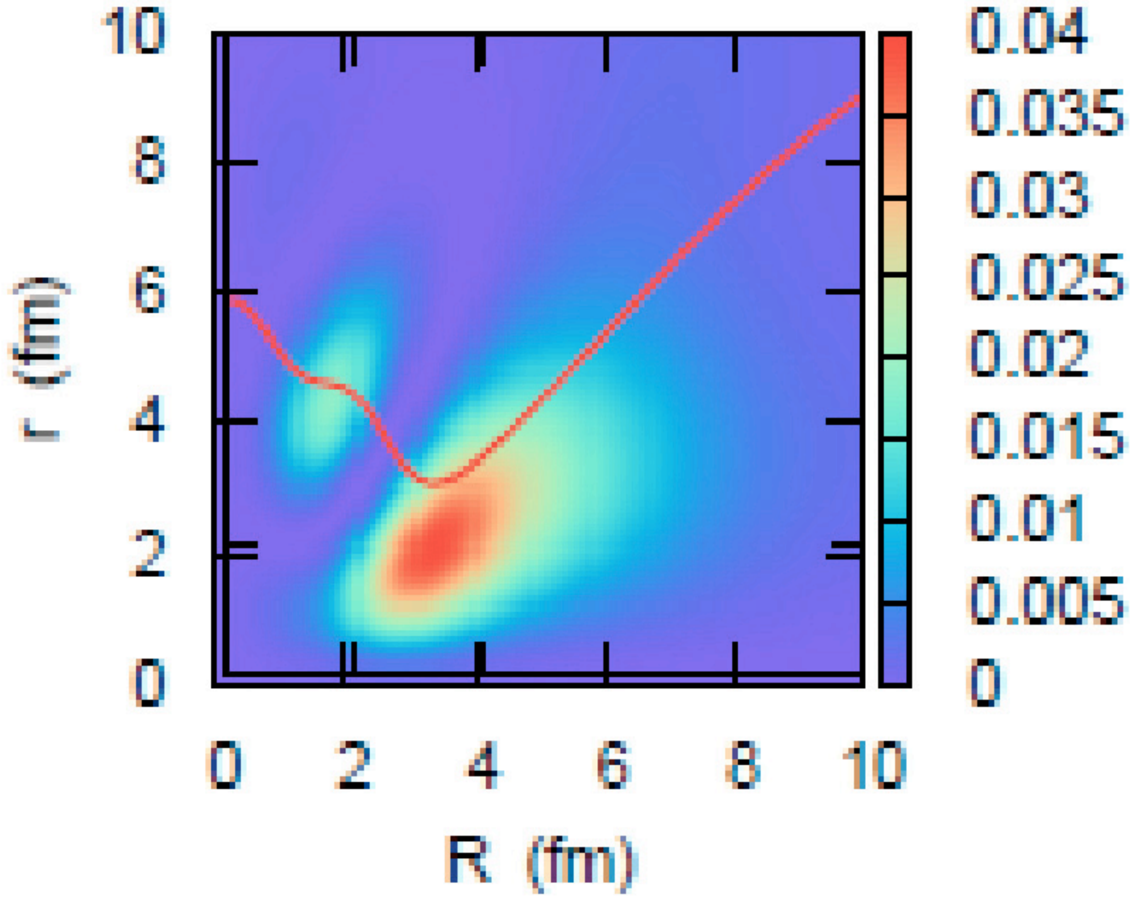
Interesting problem:
how is changed the picture as we move closer or even beyond the drip lines?

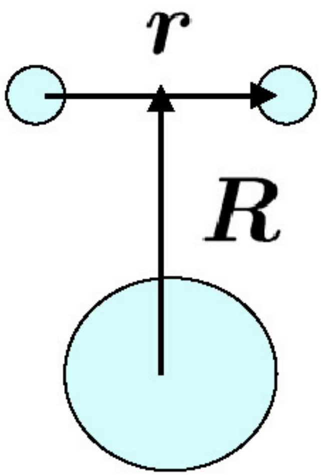
Example:
the case of ^{6}He



Oganessian, Zagrebaev, Vaagen, 1999

Other example: the case of ^{11}Li



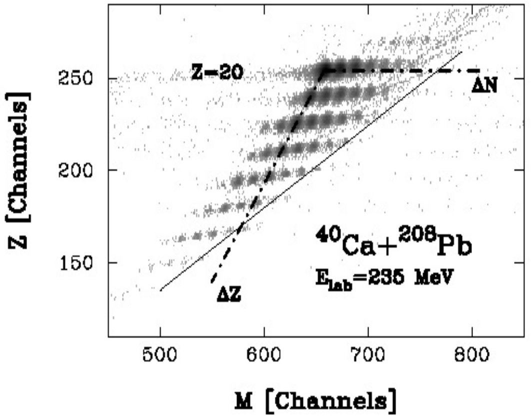


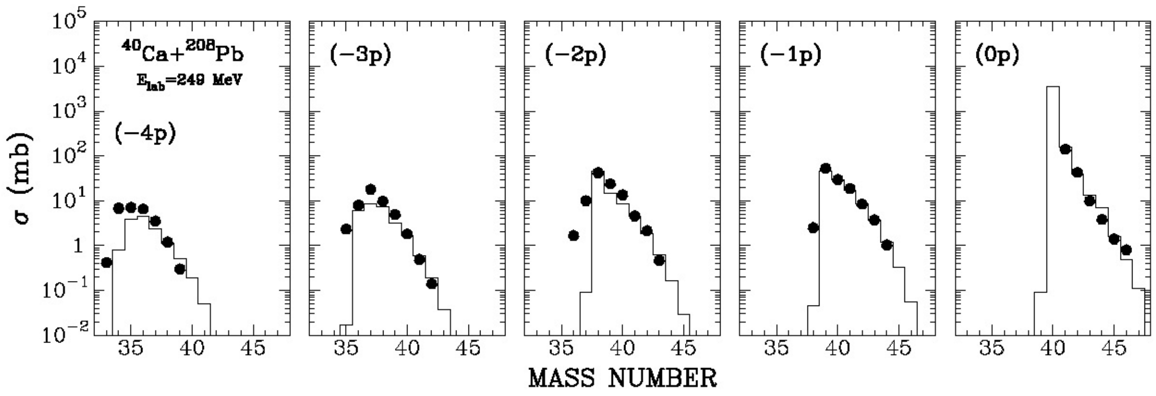
K.Hagino, H. Sagawa, and P. Schuck,
J. of Phys. G37(‘10) 064040.

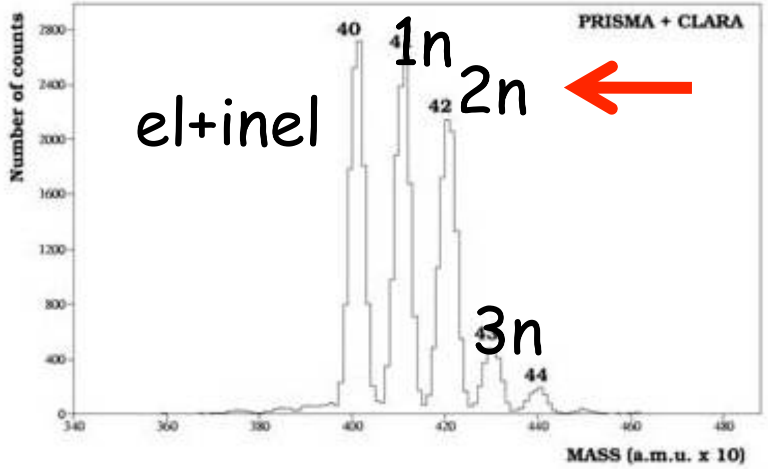
Two-particle transfer reactions

Example of multinucleon transfers at Legnaro

$^{40}Ca+^{208}Pb$







el+inel

1n

2n

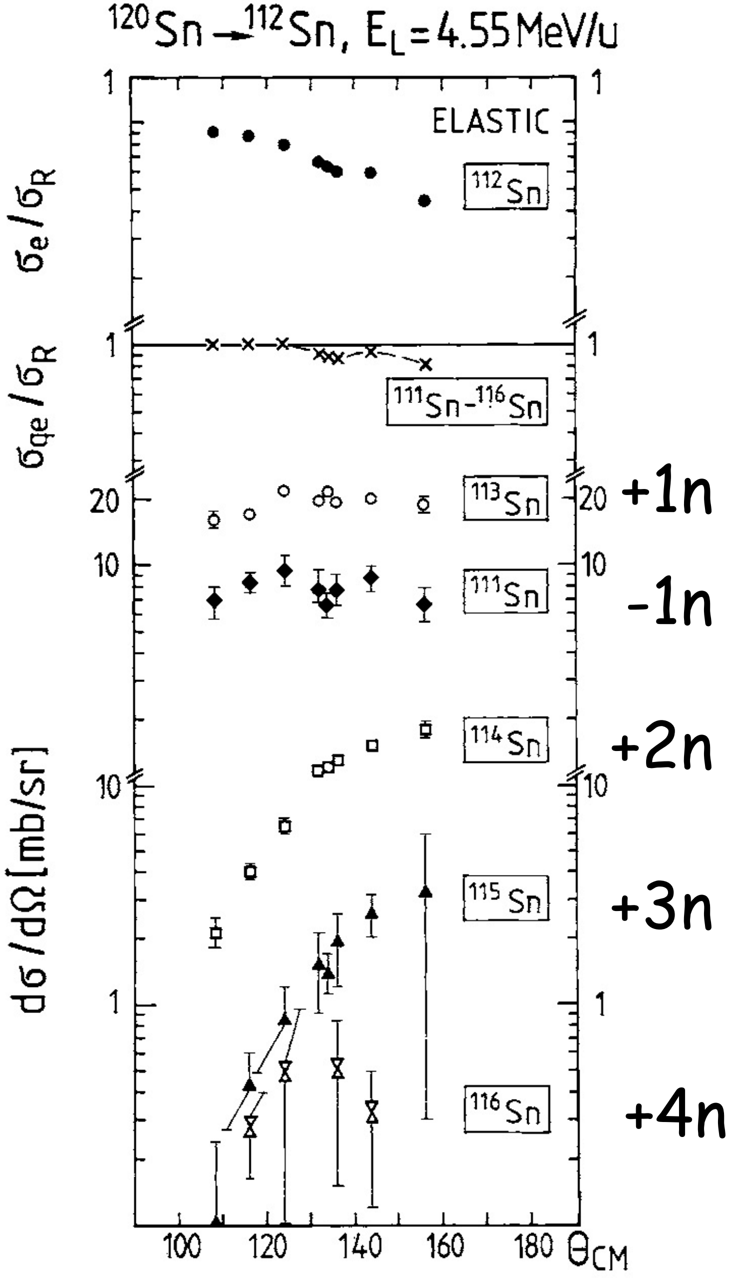
3n

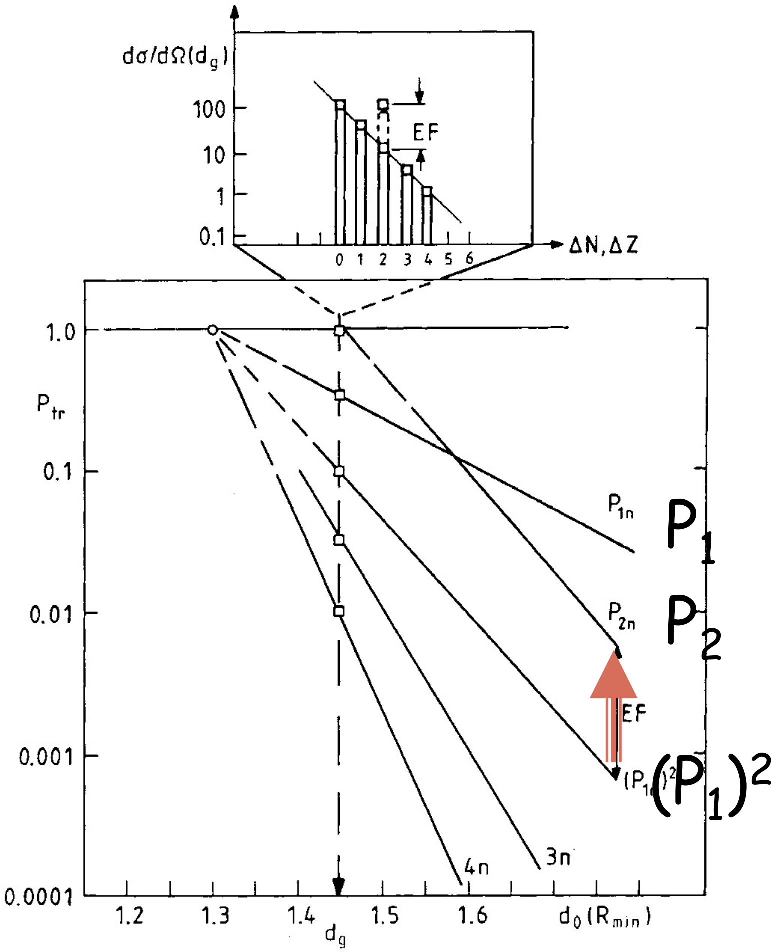
Neutron transfer channels

(odd-even transfer effect?)

The classical example:
Sn+Sn
(superfluid on superfluid)

Von Oertzen, Bohlen etal

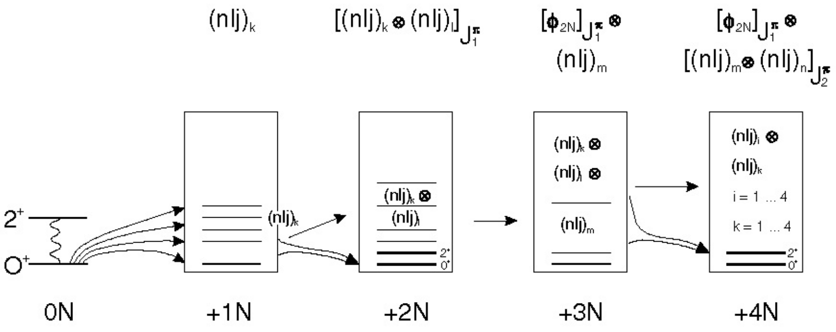


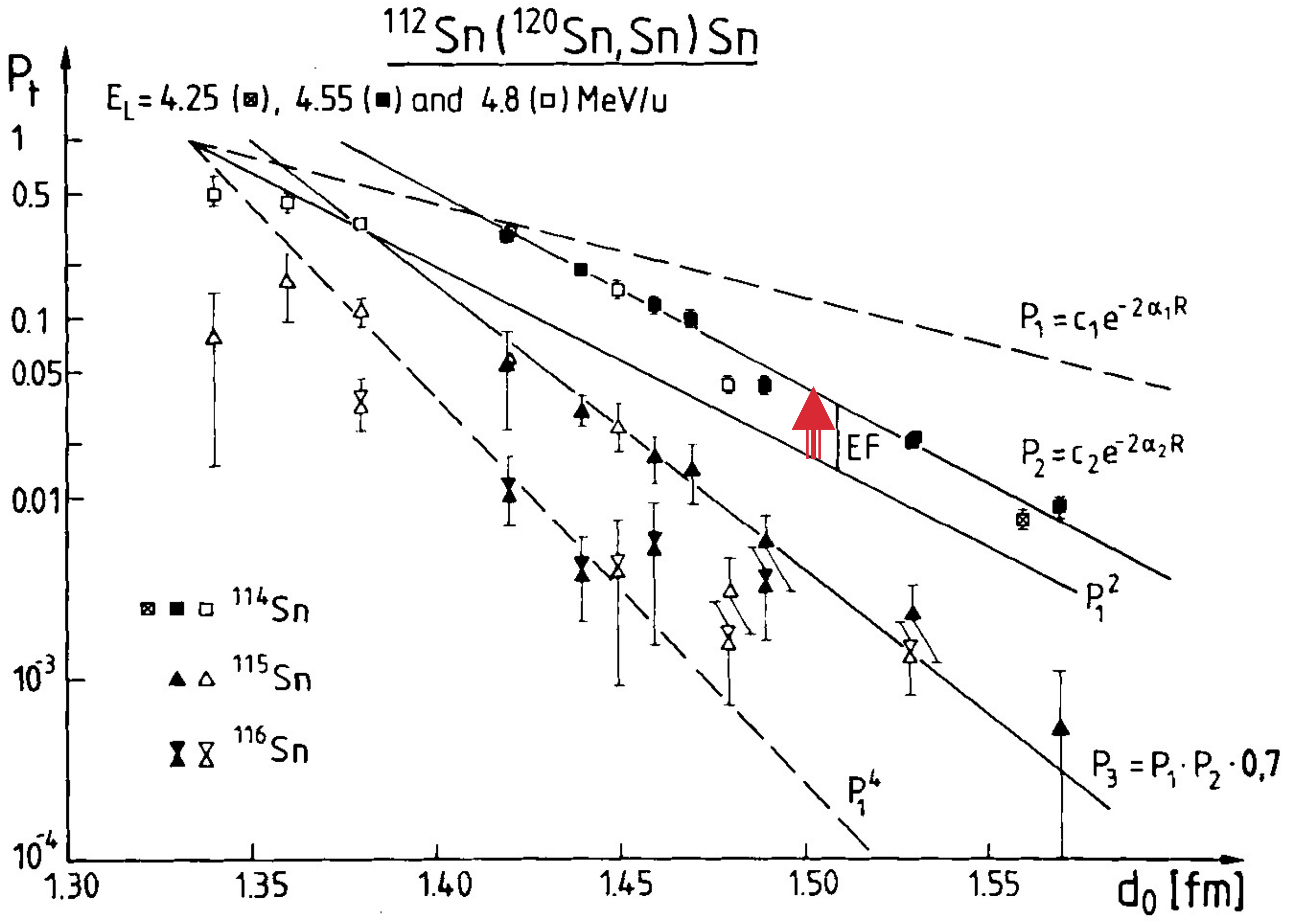


distance of closest approach

A way to define a pairing "enhancement" factor, by plotting transfer probabilities not as function of the scattering angle, but as function of the distance of closest approach of the corresponding classical trajectory

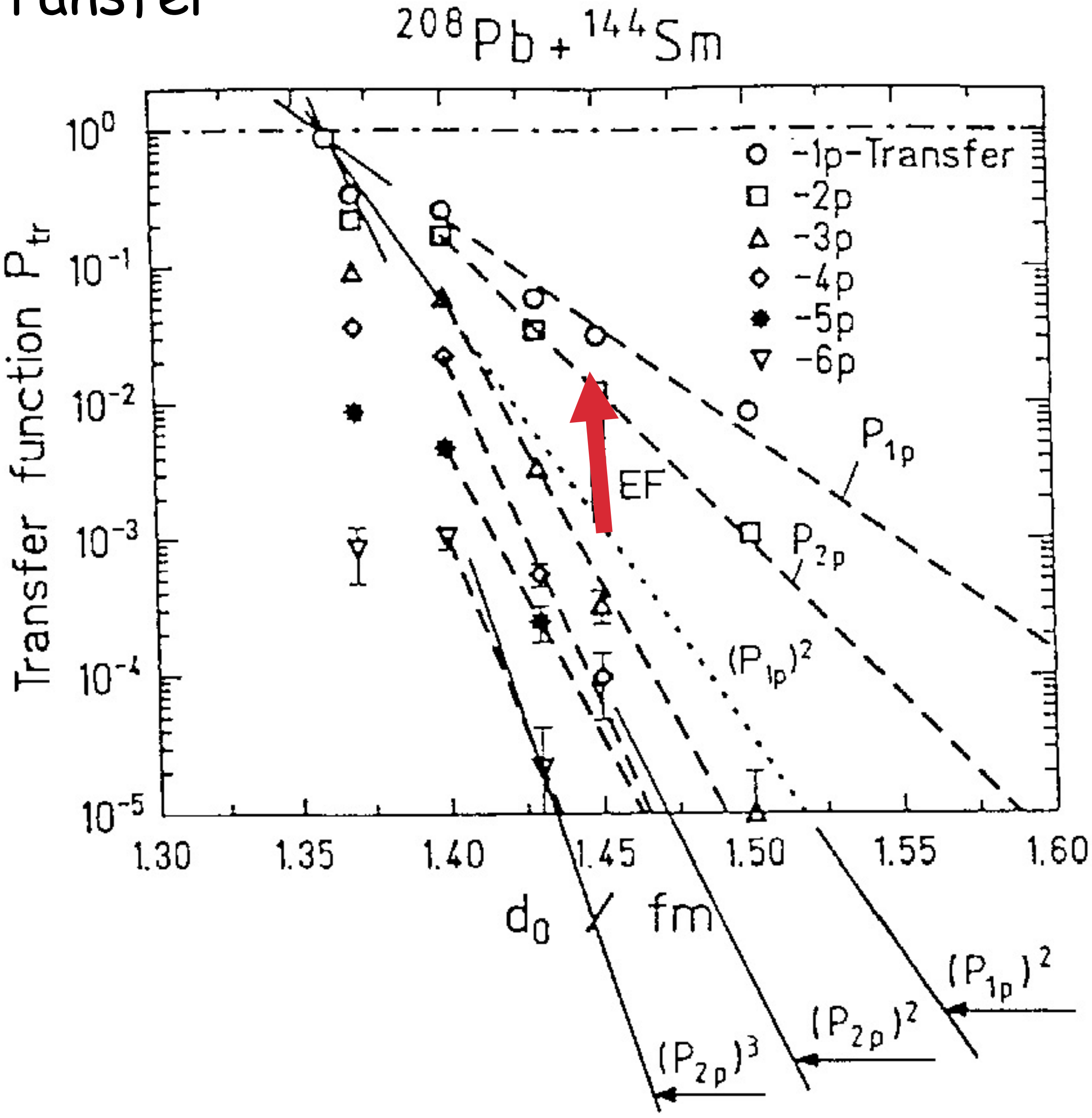
configurations in multi-nucleon transfer





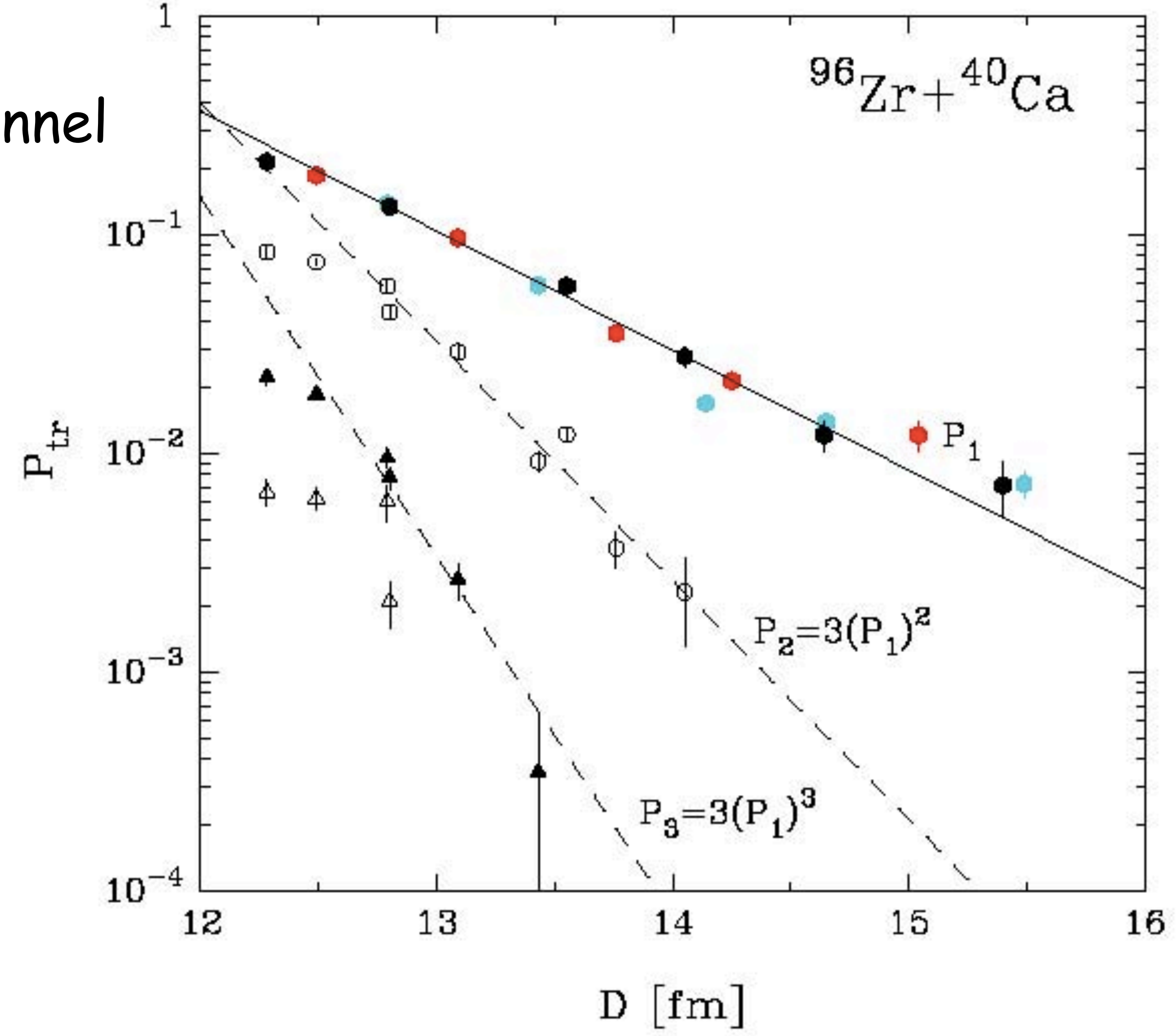
General problem: how separate the contribution of 0+ states?

Proton transfer



Corradi etal, LNL

-2n channel
(^{42}Ca)



Reaction mechanism and models for two-particle transfer processes

Large number of different approaches, ranging from macroscopic to semi-microscopic and to fully microscopic. They all try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles).

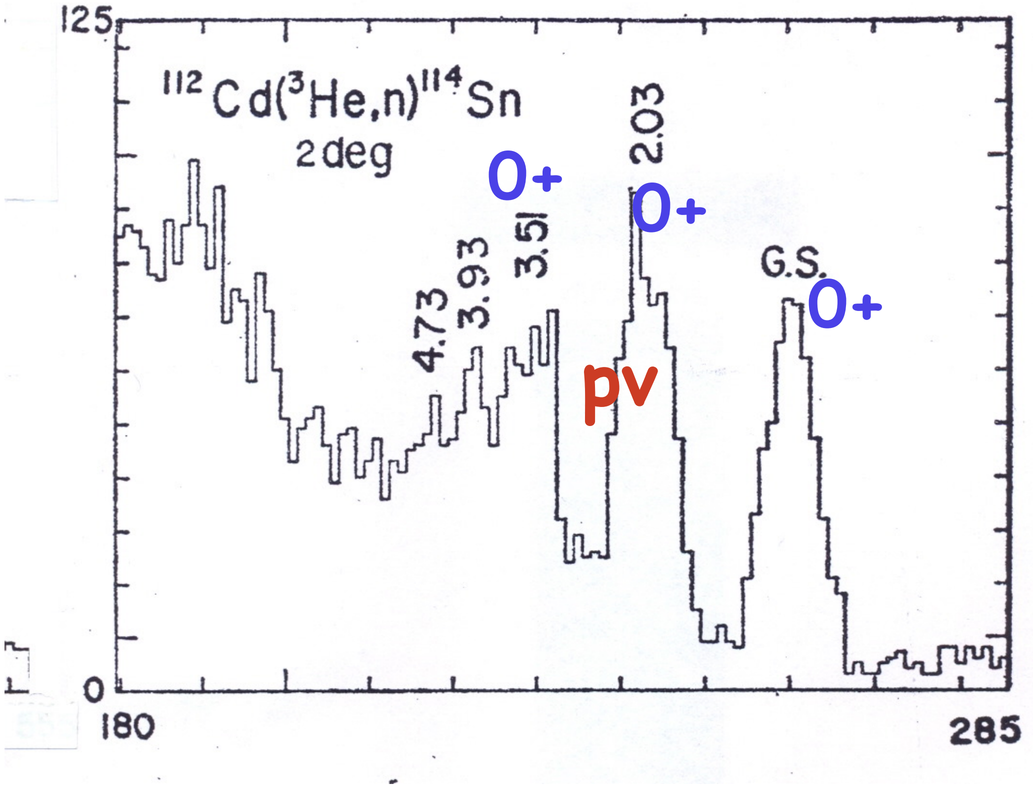
Aside from the precise description of the reaction mechanism (and therefore from the absolute values of the cross sections), some points are more or less well established

- Angular distributions
- Role of other multipole states
- Q-value effect

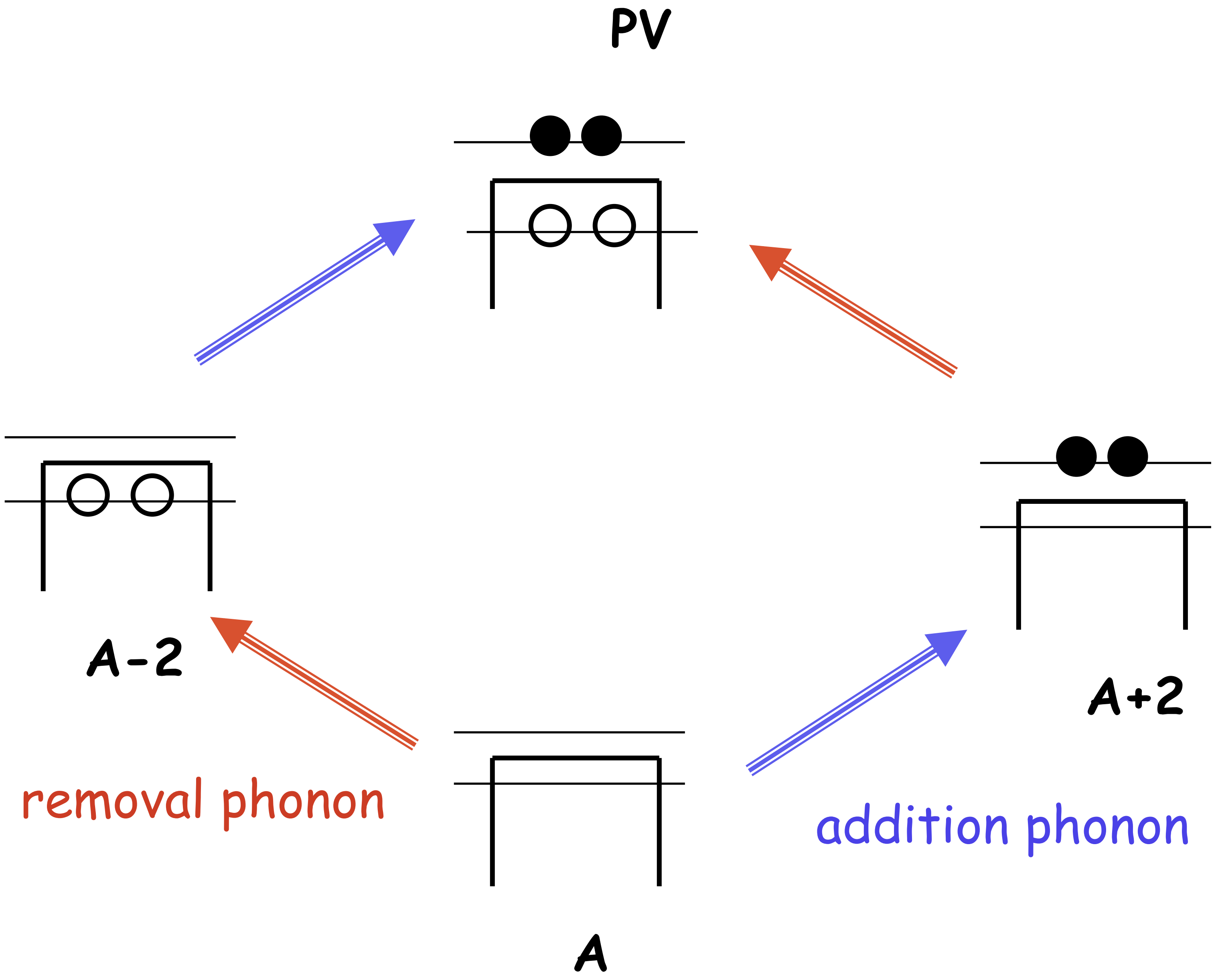
Angular distribution

With light ions at forward angles one excites selectively 0+ states

The excited states in 114Sn are of proton character at Z=50 closed shell



Pair transfer intensities



PV

A-2

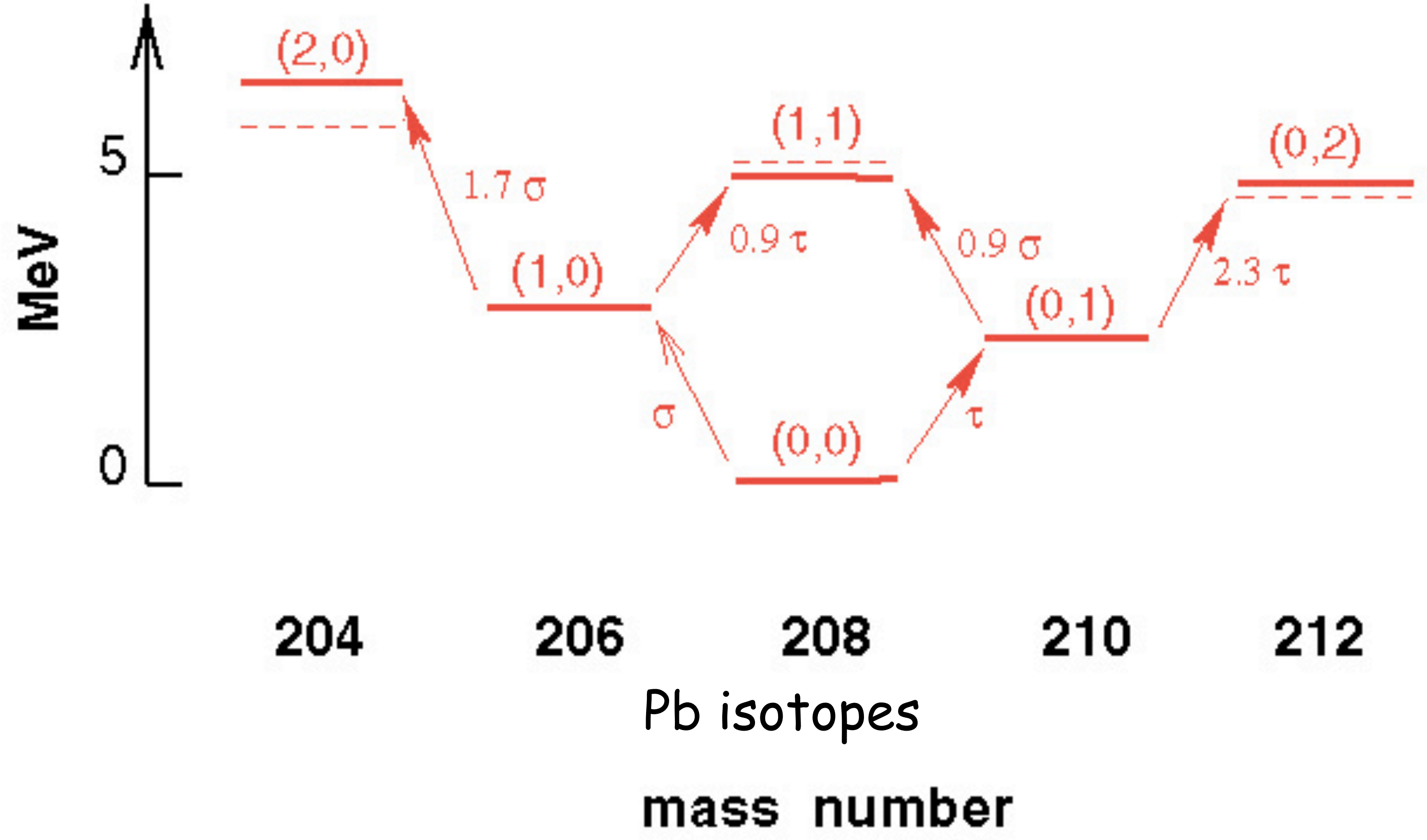
A+2

A

removal phonon

addition phonon

Vibrational pairing spectrum around closed shell: neutron case around ^{208}Pb

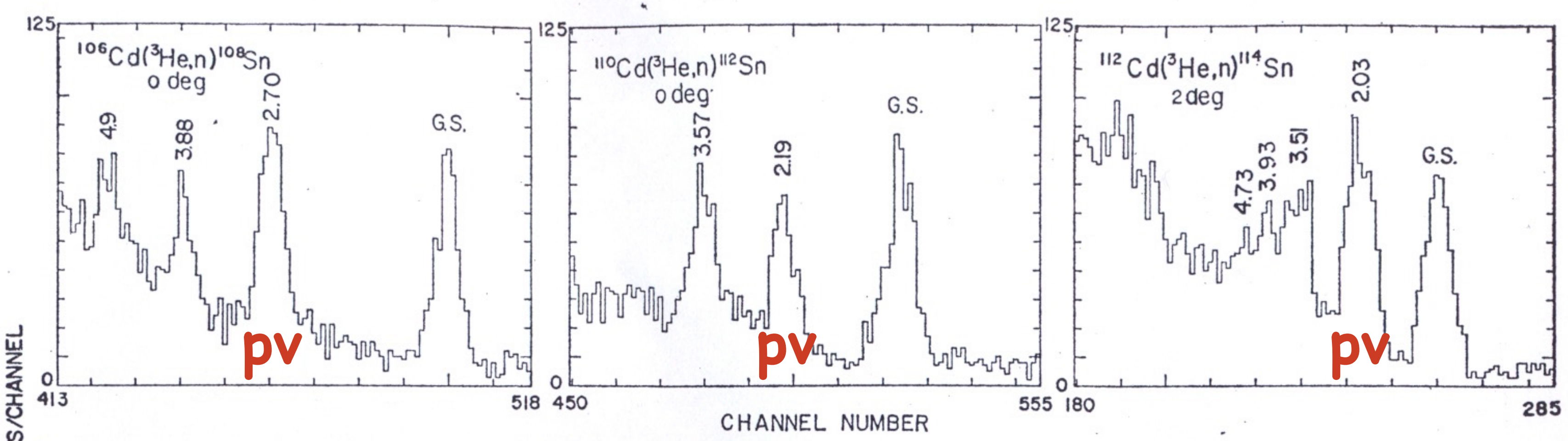


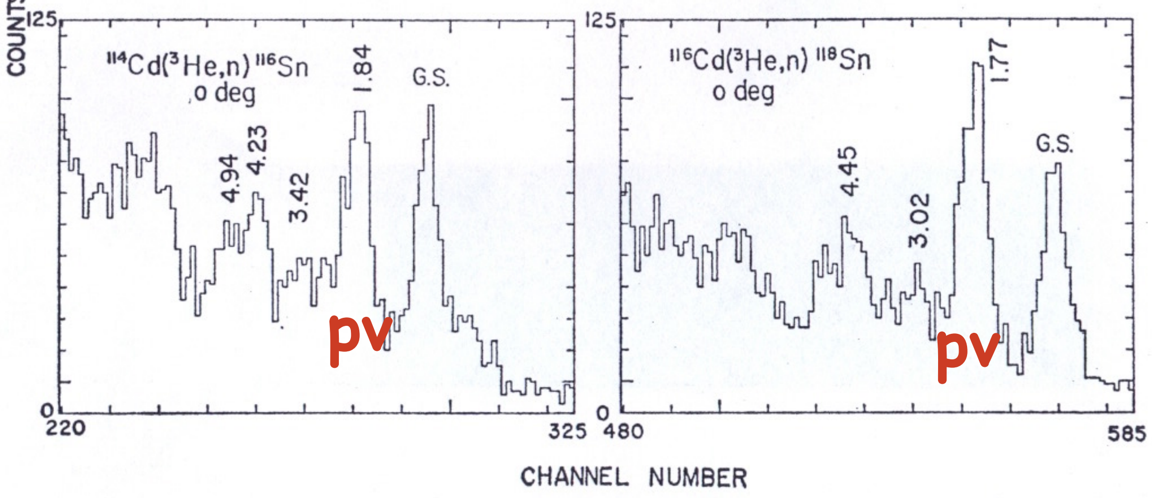
Pb isotopes

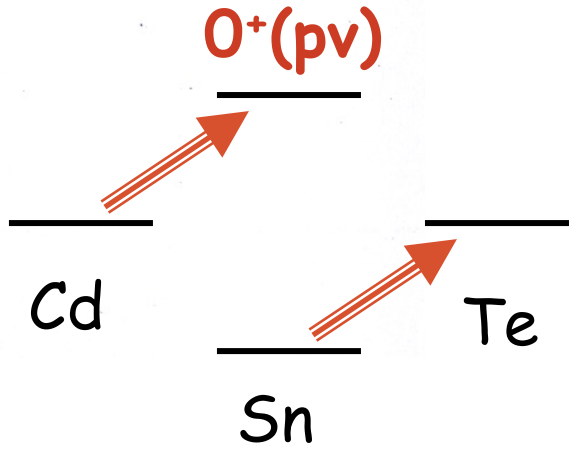
Bohr and Mottelson

Proton pairing vibration at Z=50 closed shell

He^3,n reactions



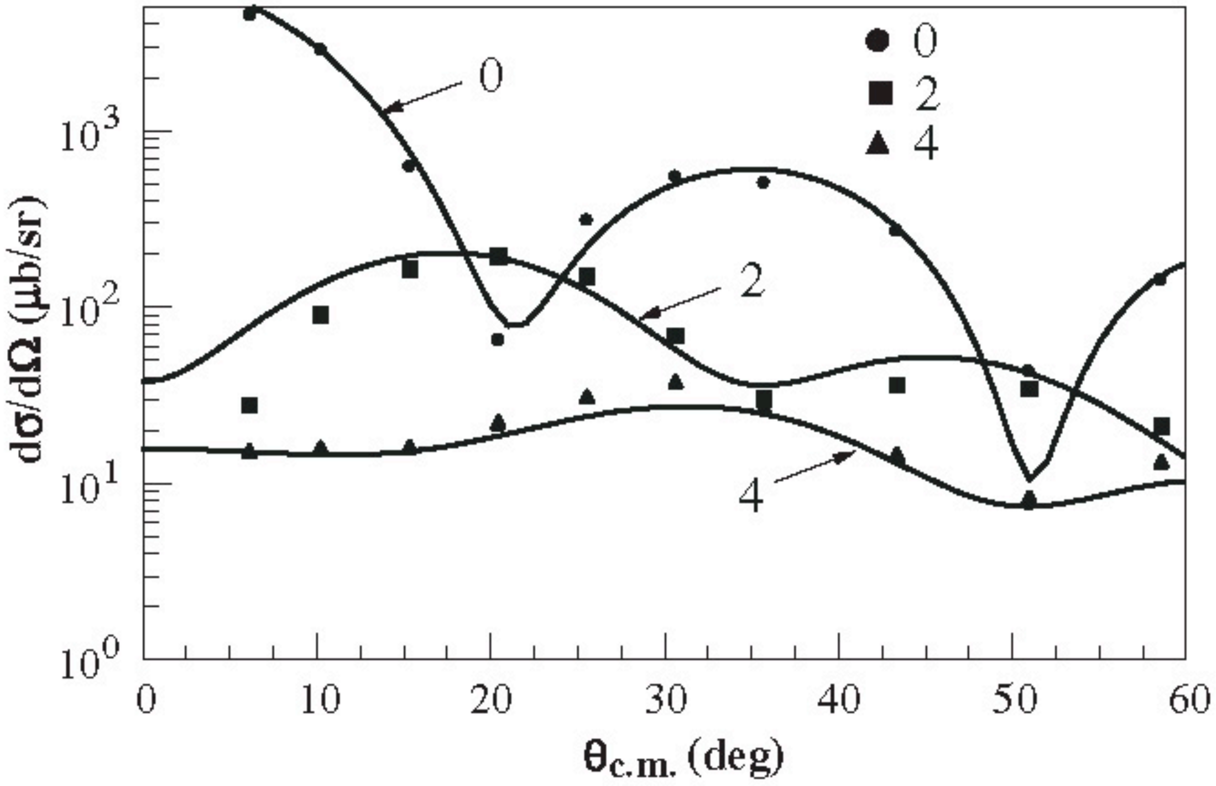




$^{112}Sn(p,t)^{110}Sn$

Lowest 0+,2+,4+ states

Guazzoni etal

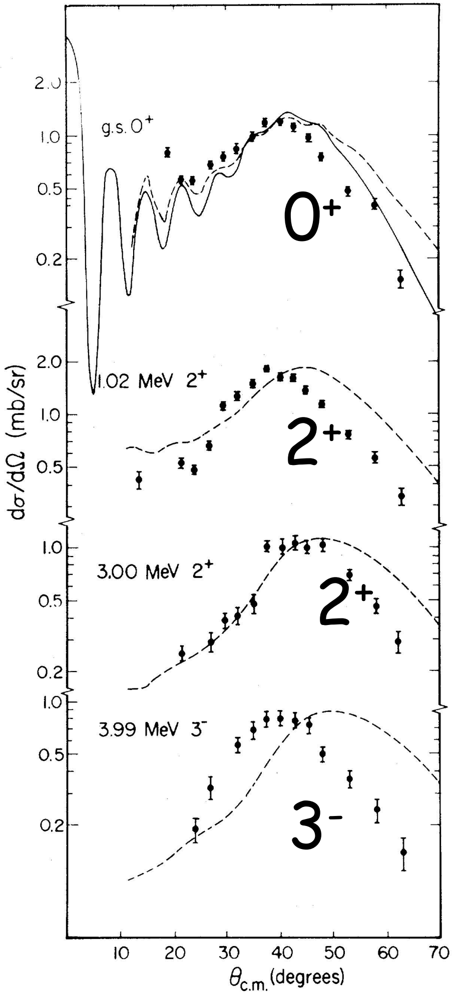


Obs: Cross section to 0+ state order of magnitude larger at 0° degrees

Angular distribution

Situation different for heavy-ions induced pair transfer processes: angular distributions are always peaked around the grazing angle, independently of the multipolarity

$^{48}Ca(^{18}O,^{16}O_{gs})^{50}Ca(\lambda)$

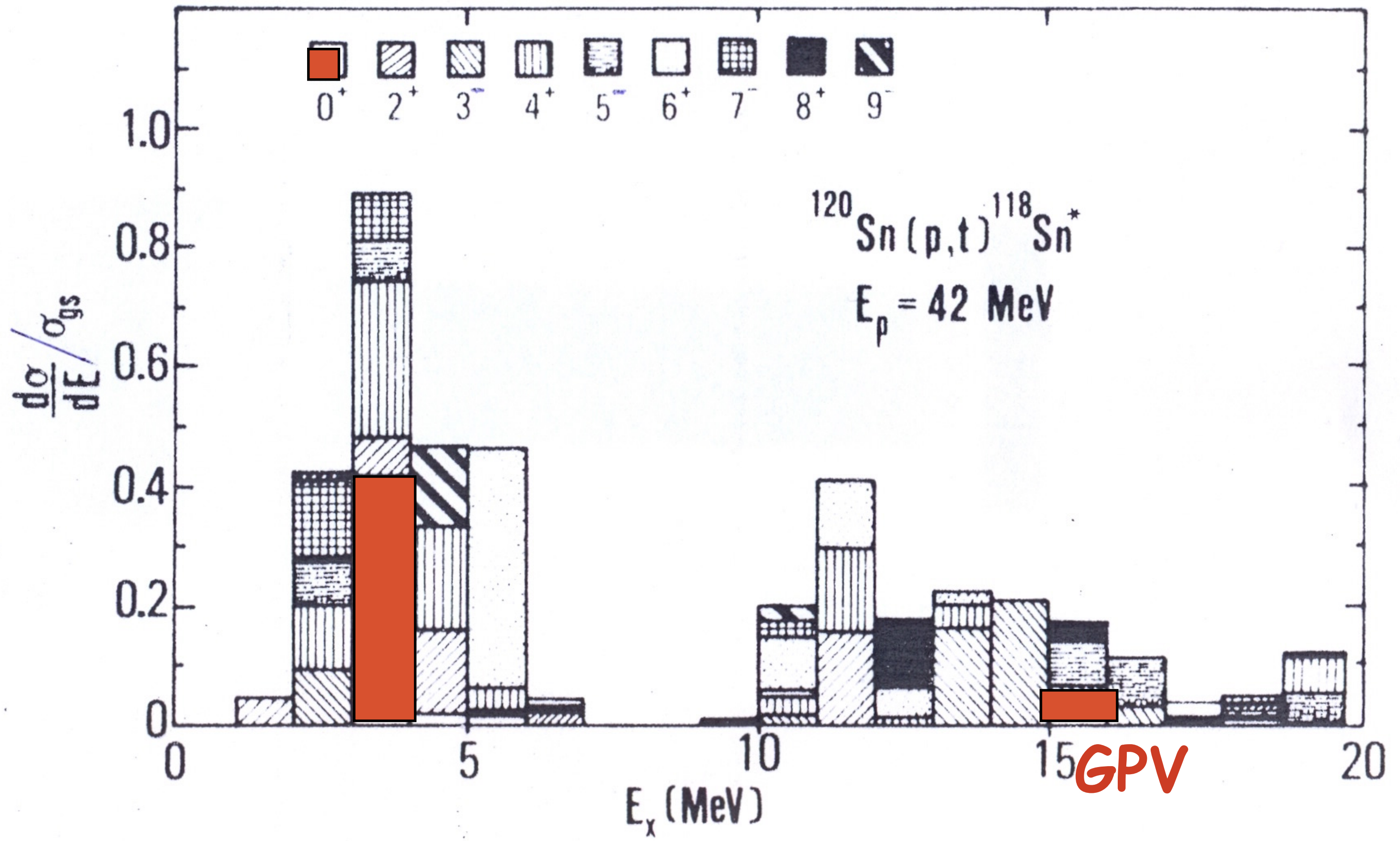


Higher multipolarities

Far from the very forward angles the pairing vibrational states are overwhelmed by states with other multipolarities

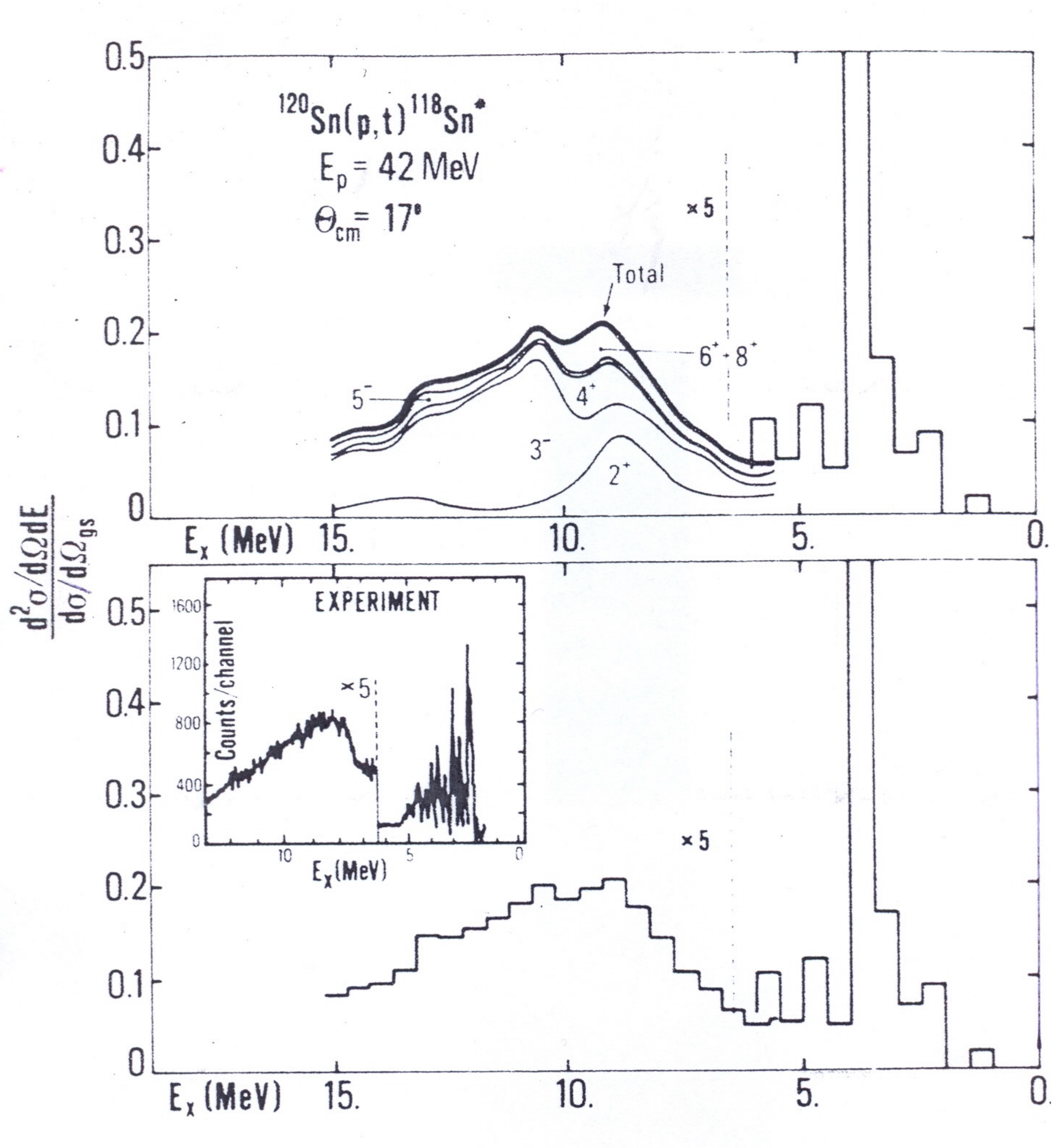
Example:
predicted total cross sections in
^{120}Sn(p,t)^{118}Sn* reaction

Searching the Giant Pairing Vibration …….



Bortignon and AV

Bump at 10 MeV does not come from GPV, but from incoherent sum of different multipolarities



Q-value effect

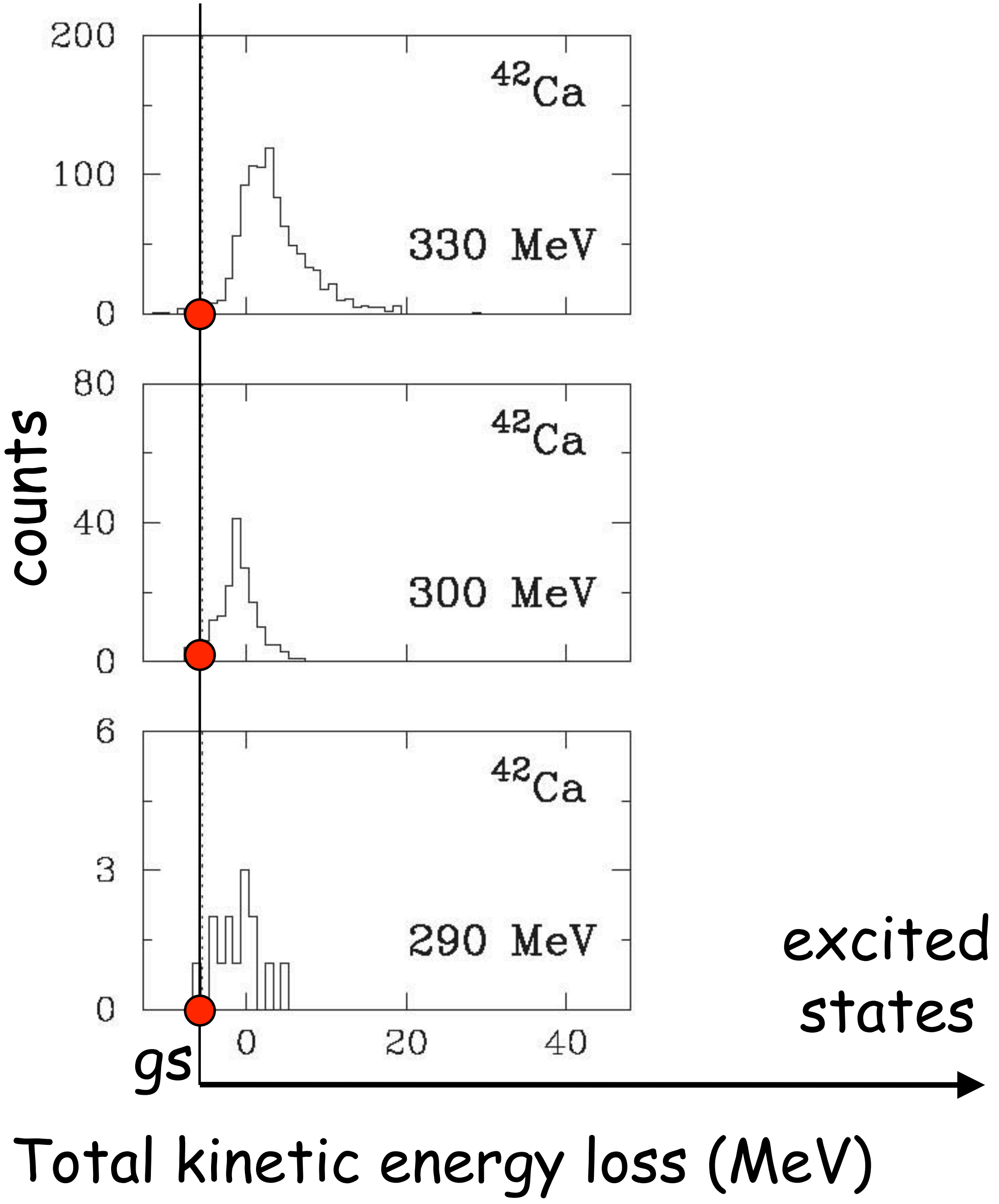
Keeping fixed any other parameter, the probability for populating a definite final channel depends on the Q-value of the reaction. The dependence (in first approximation a gaussian distribution centered in the optimum Q-value) is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

The optimum Q-value depends on the angular momentum transfer and on the charge of the transferred particles. In the specific case of L=0 two-neutron transfer, the optimal Q-value is zero.

Experimental evidence

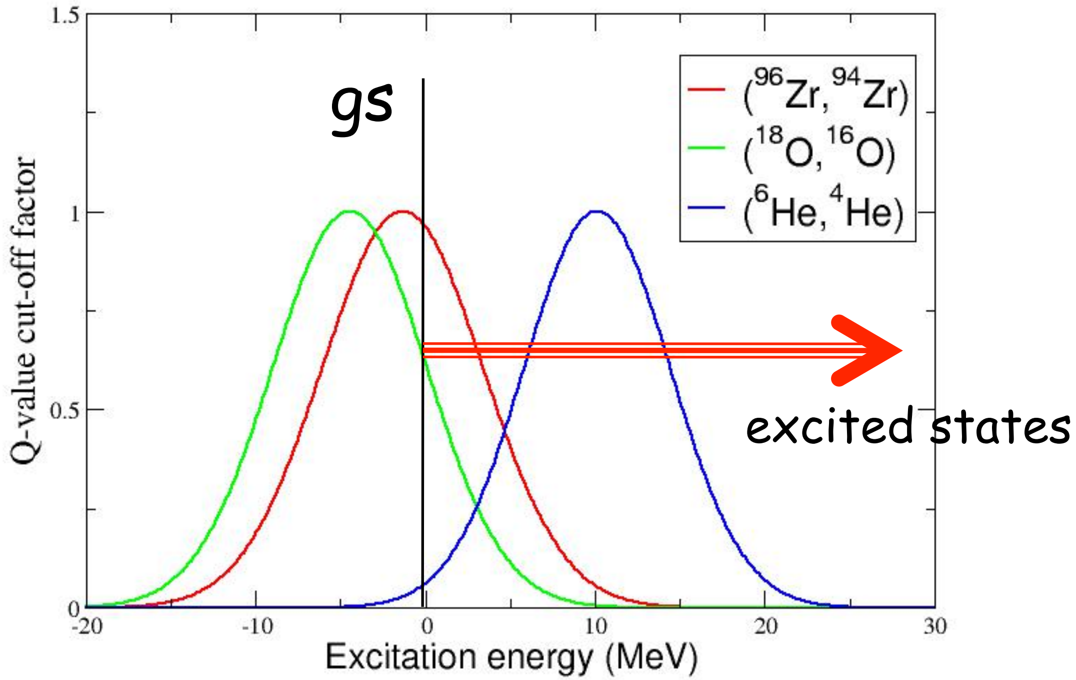
$^{96}Zr+^{40}Ca$

Selecting final
^{42}Ca mass partition



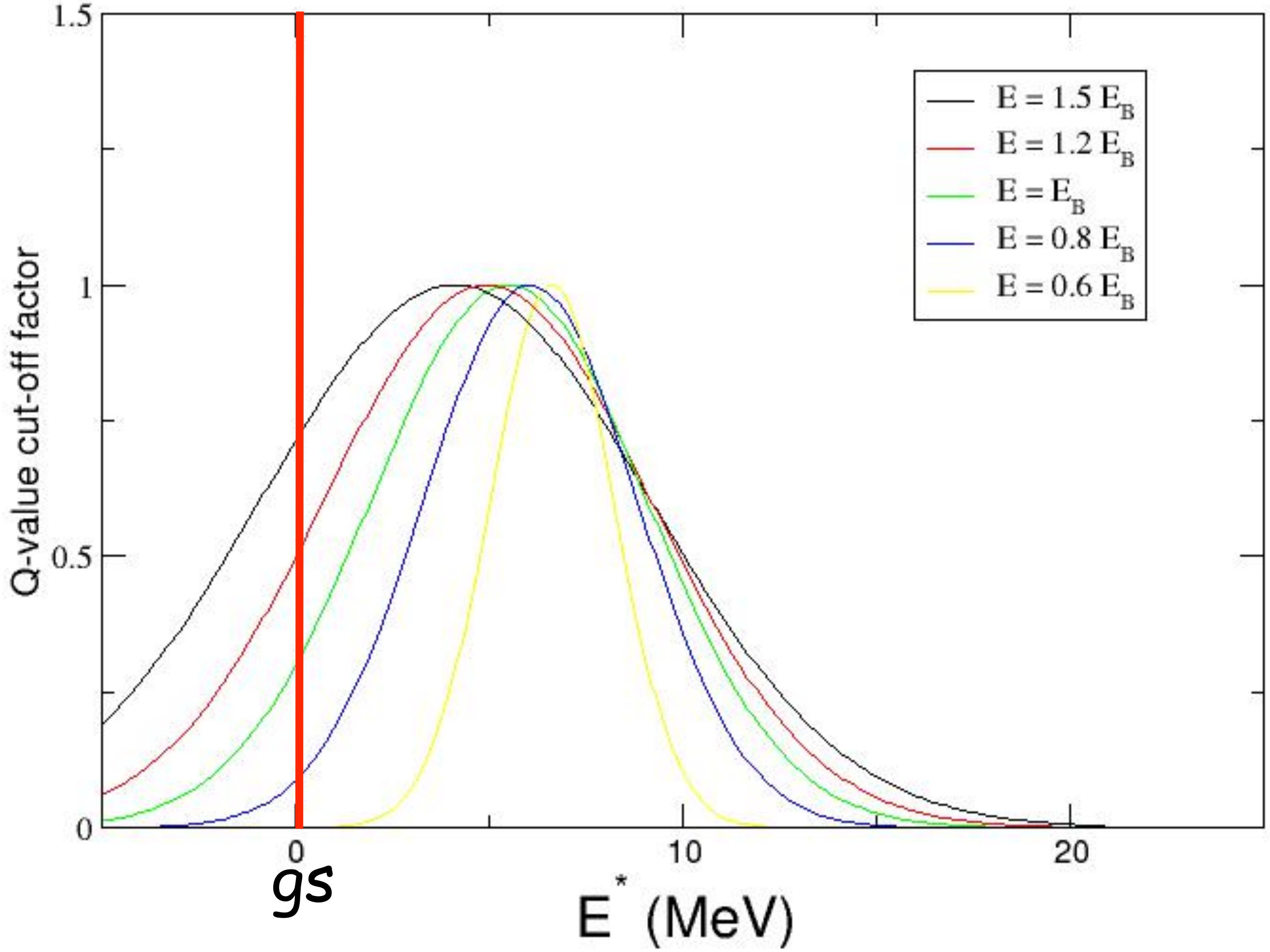
Playing with different combinations of projectile/target (having different Q_{gg}-value) one can favour different energy windows

Example: Target ^{208}Pb Final ^{210}Pb (at bombarding energy $E_{cm} = 1.2\ E_{barrier}$)



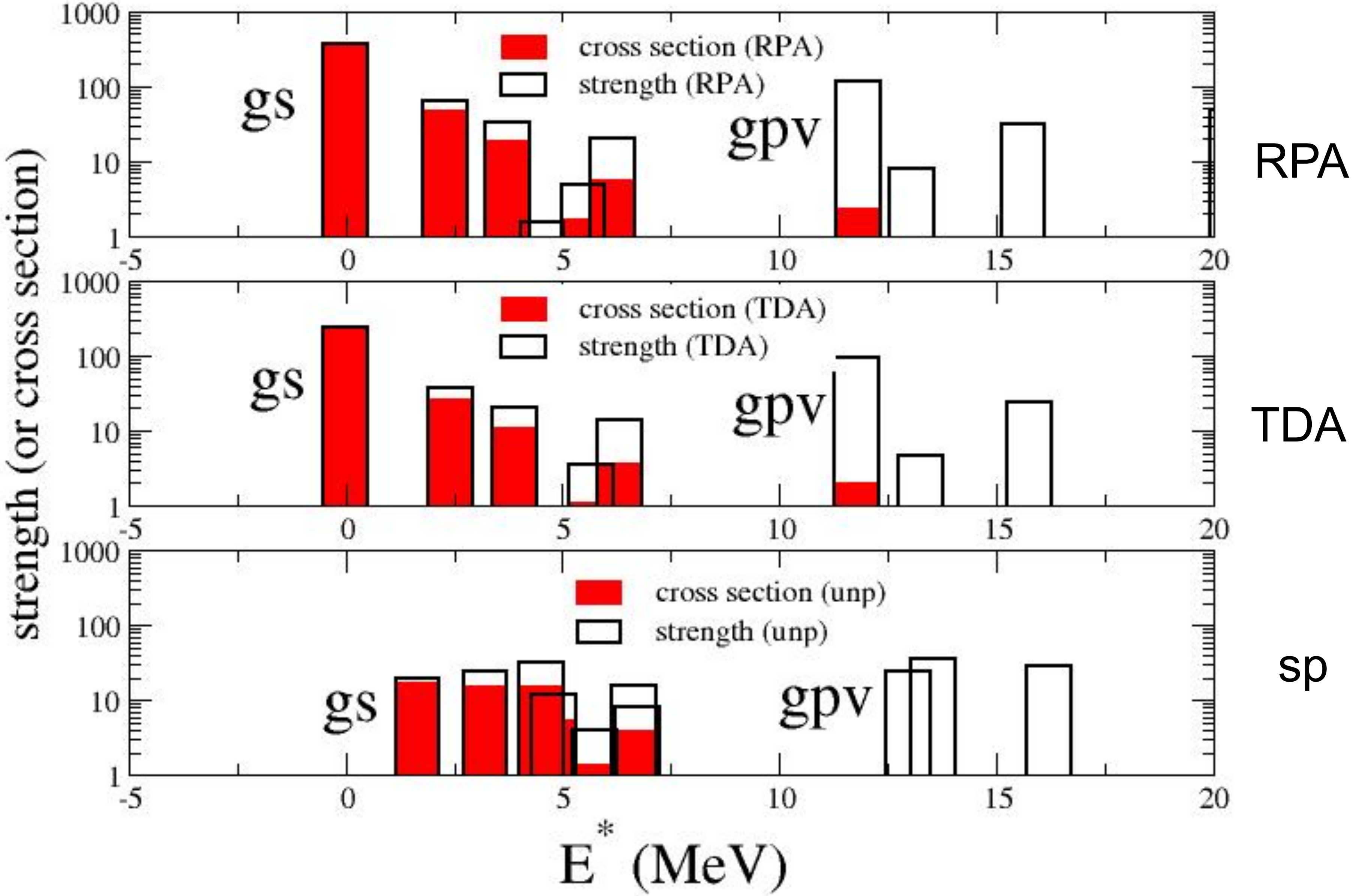
The width of the Q-value window increases
with the bombarding energy

$^{40}Ca(^{96}Zr,^{94}Zr)^{42}Ca$

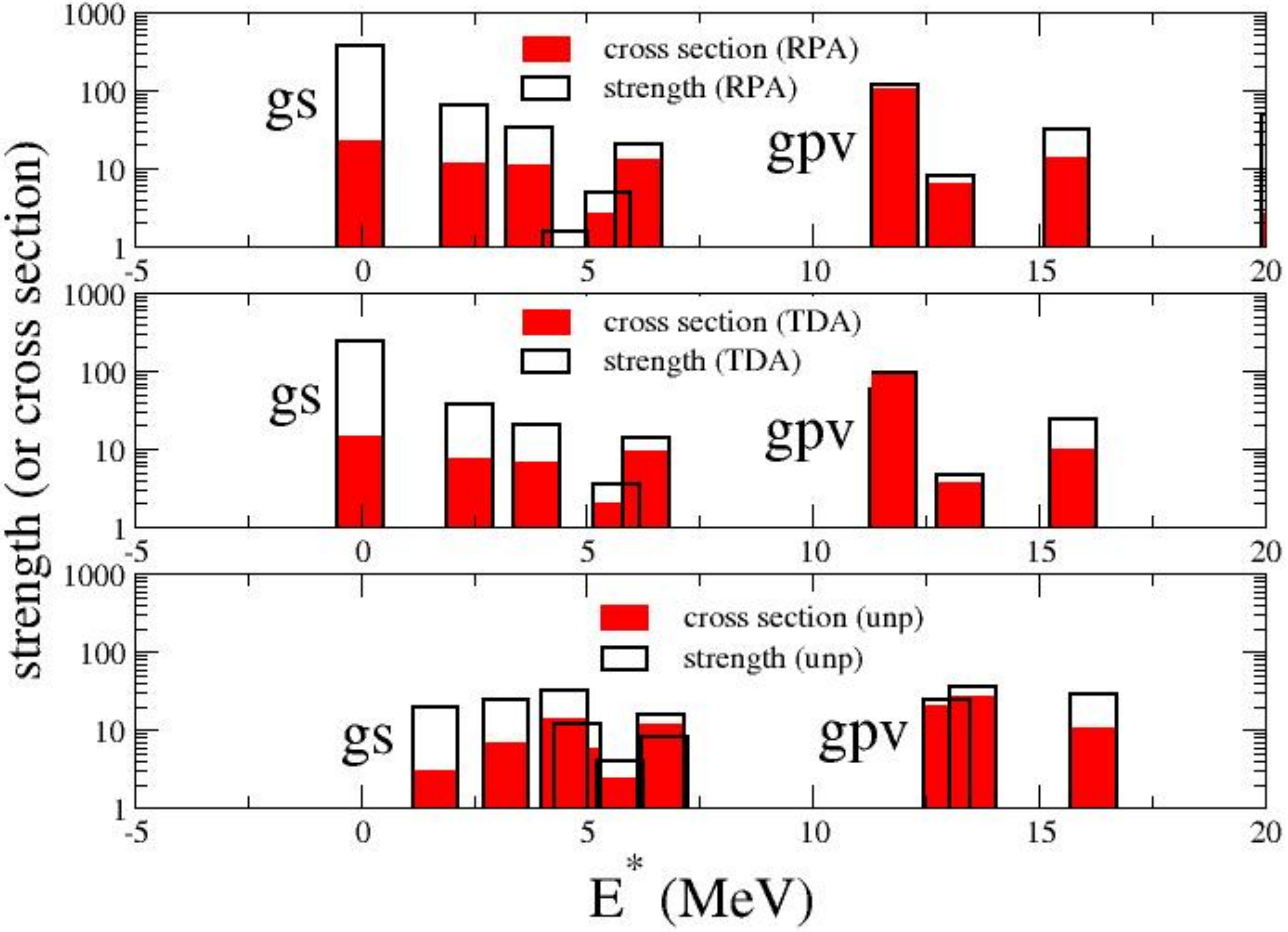


The pairing strength is therefore modulated by the Q-value cut-off to yield the final two-particle cross section

^{208}Pb(^{18}O,^{16}O)^{210}Pb (0^+ states)



^{208}Pb(^{6}He,^{4}He)^{210}Pb (0^{+} states)



RPA

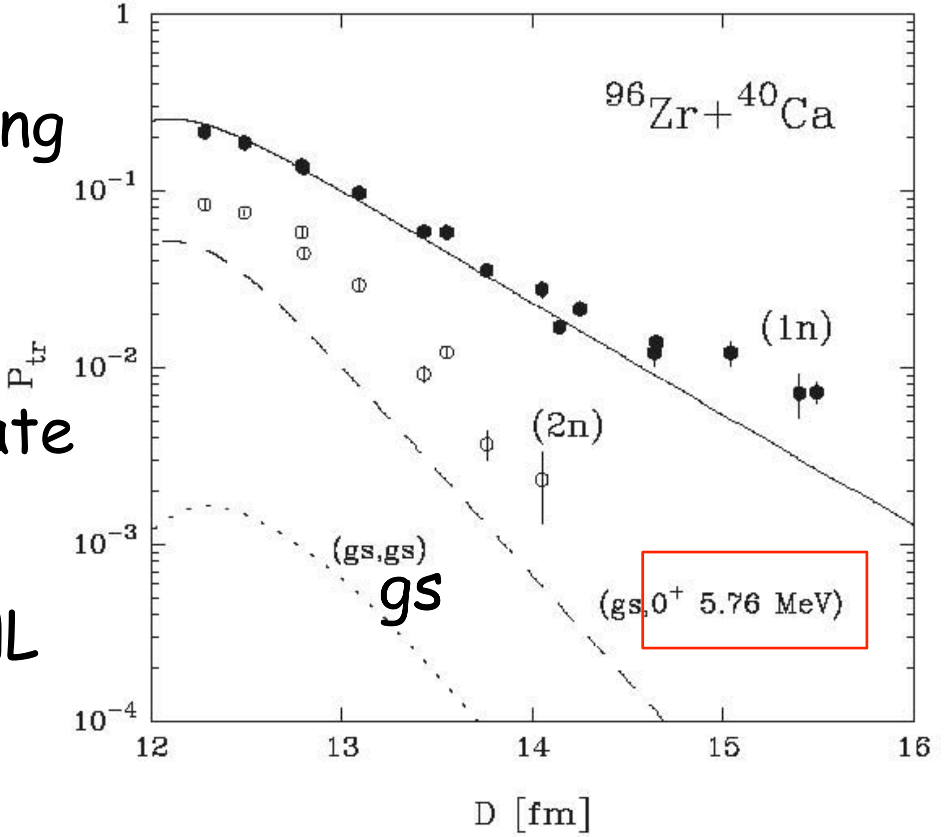
TDA

sp

As a result, the correlated states may be populated in a much weaker way than uncorrelated states

Example: $^{96}Zr+^{40}Ca$, leading to ^{42}Ca
In this case is favored the excitation of an uncorrelated 0+ state at about 6 MeV

Corradi, Pollarolo etal, LNL



Models for two-particle transfer reactions

Example (t,p)

Quantal

DWBA: one step di-neutron transfer

Microscopic construction of the di-neutron transfer form factor
(Glendenning or Bayman-Kallio methods)

Options:
zero range : only relative cross-sections
or
finite range : absolute cross sections
(but needs the use of proper triton wf)

Macroscopic approach

Complete parallelism with inelastic excitation of collective surface modes (vibrations and rotations in gauge space)

Reaction mechanism: one step transfer produced by a new generalized pair field

$$F(r) = \beta_P \, dU/dA = \beta_P \, (R/3A) \, dU/dr$$

Where the "deformation" parameter β_P is the pair -transfer matrix element and contains all the microscopy of the approach

Very simple, appropriate for situations with many other coupled open channels
Problem: recoil? Relative cross sections?

Semi-microscopic approach

Reaction mechanism: one-step di-neutron (cluster) transfer

Microscopy: Formfactor obtained by double-folding the microscopic pair densities of initial and final states with some nucleon-nucleon interaction

or

Simple folding of microscopic pair density in the target with the one-body mean field of the projectile

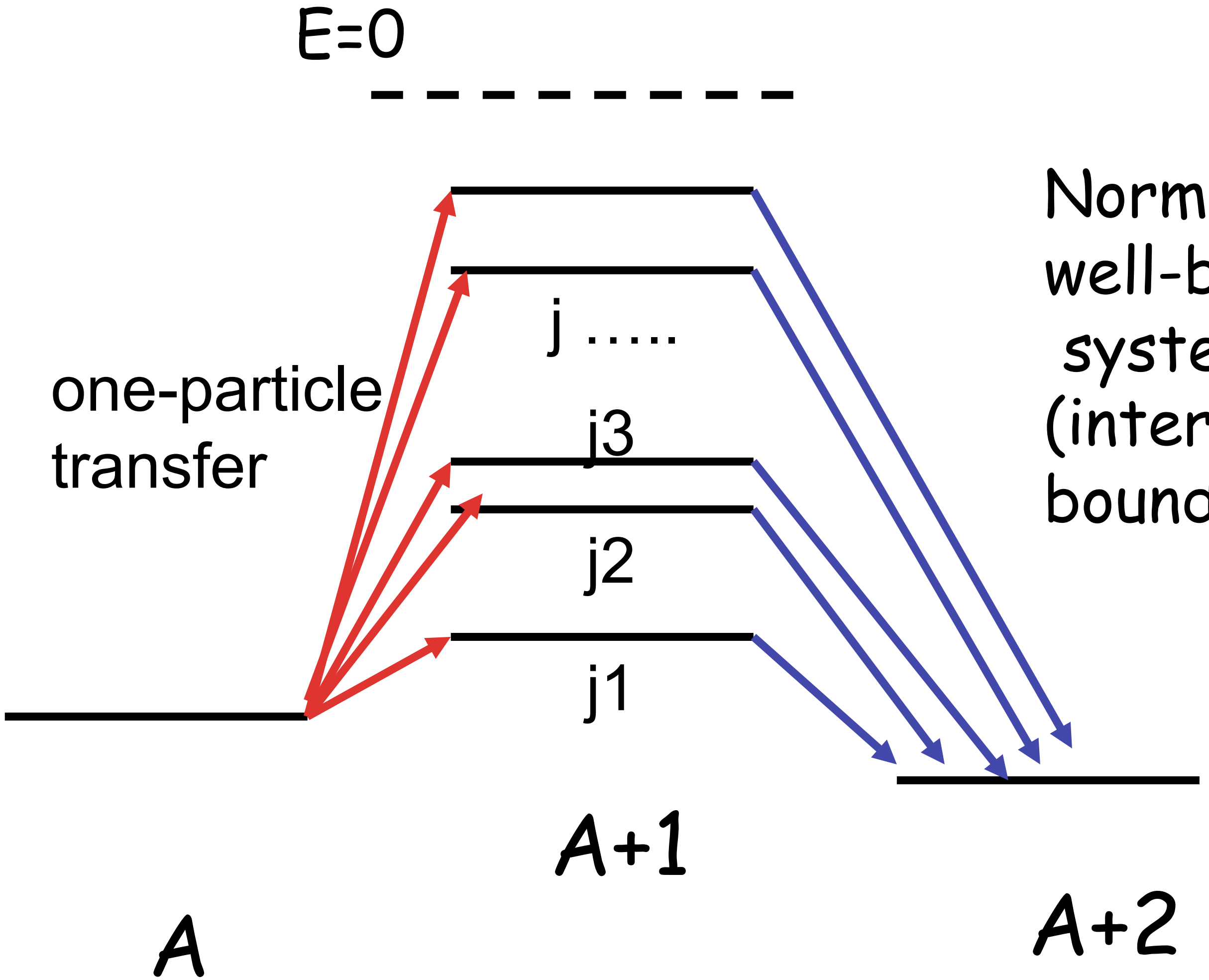
Fully microscopic approach ☺ (cf. talks by Vigezzi and Potel)

Reaction mechanism: Sequential two-step process (each step transfers one particle)

Microscopy: Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A+1) nuclei, due to the correlations in initial and final wave functions

Building blocks: single-particle formfactors and wf's

Problems: quantal calculations rather complex (taking into account full recoil), semiclassical more feasible (but approximate treatment of recoil)

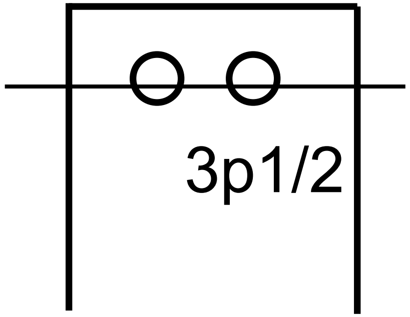


Normal
well-bound
systems
(intermediate
bound states)

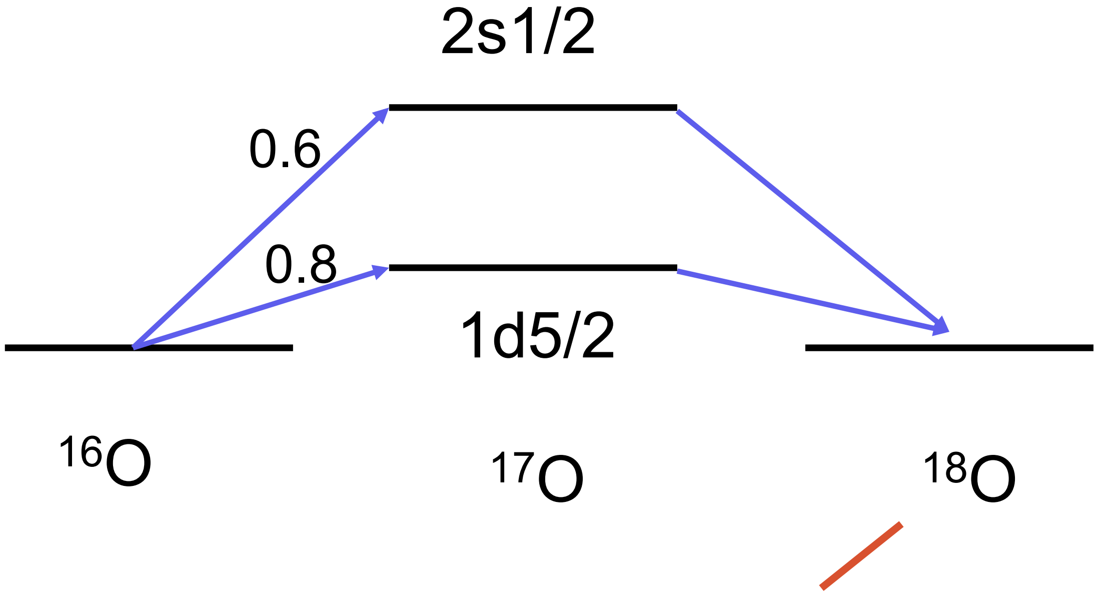
Example $|A=2> = \Sigma_i X_i [a_i^+ a_i^+]_0 |A>$

Example

$^{208}Pb(^{16}O,^{18}O)^{206}Pb$

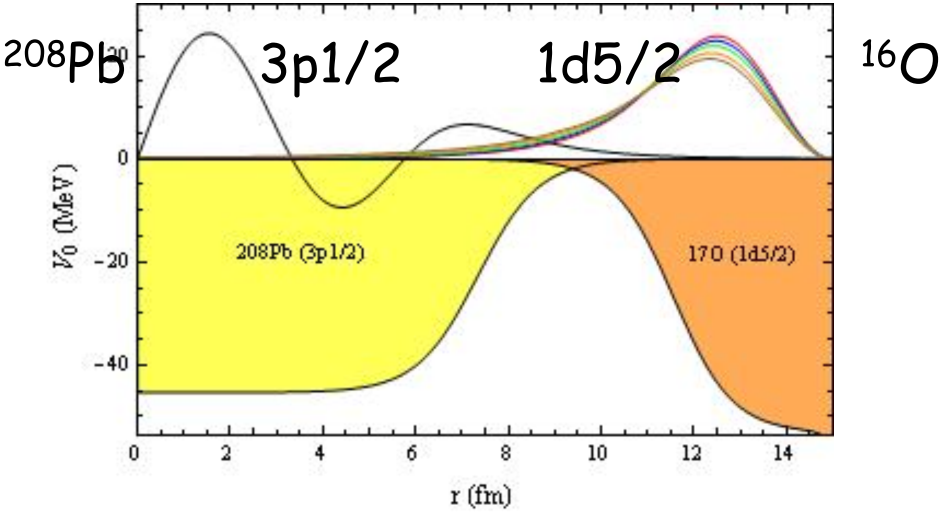


^{208}Pb



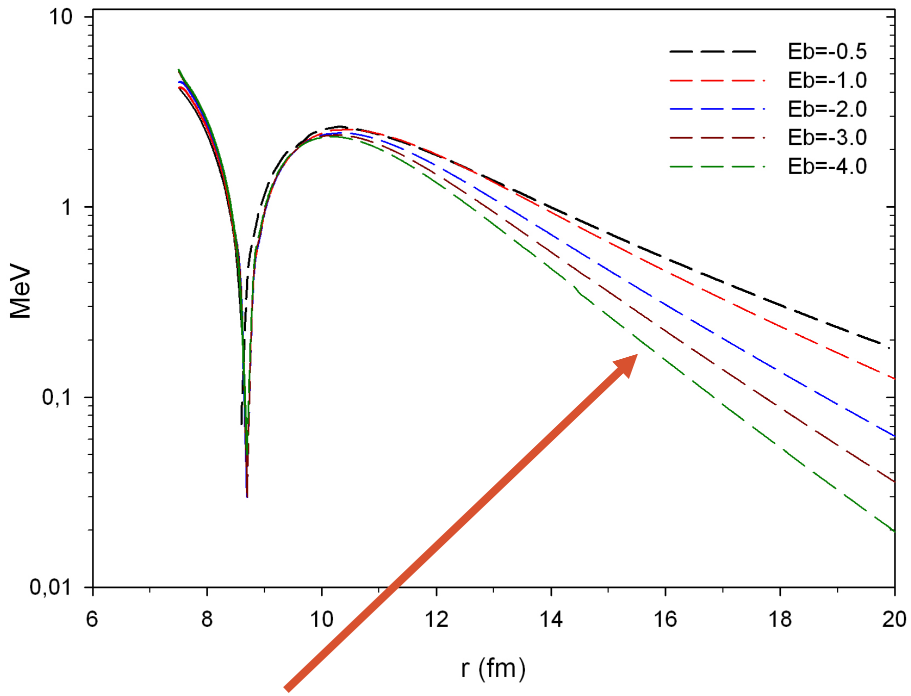
$0.8\ (1d5/2)^2 + 0.6(2s1/2)^2$

Basic blocks: single particle formfactors



3p1/2 (in Pb) ⇒ 1d5/2 (in O)

$\lambda = 3$



$$F_\lambda(r) = \int \phi_{in} V \phi_{fin} ds$$

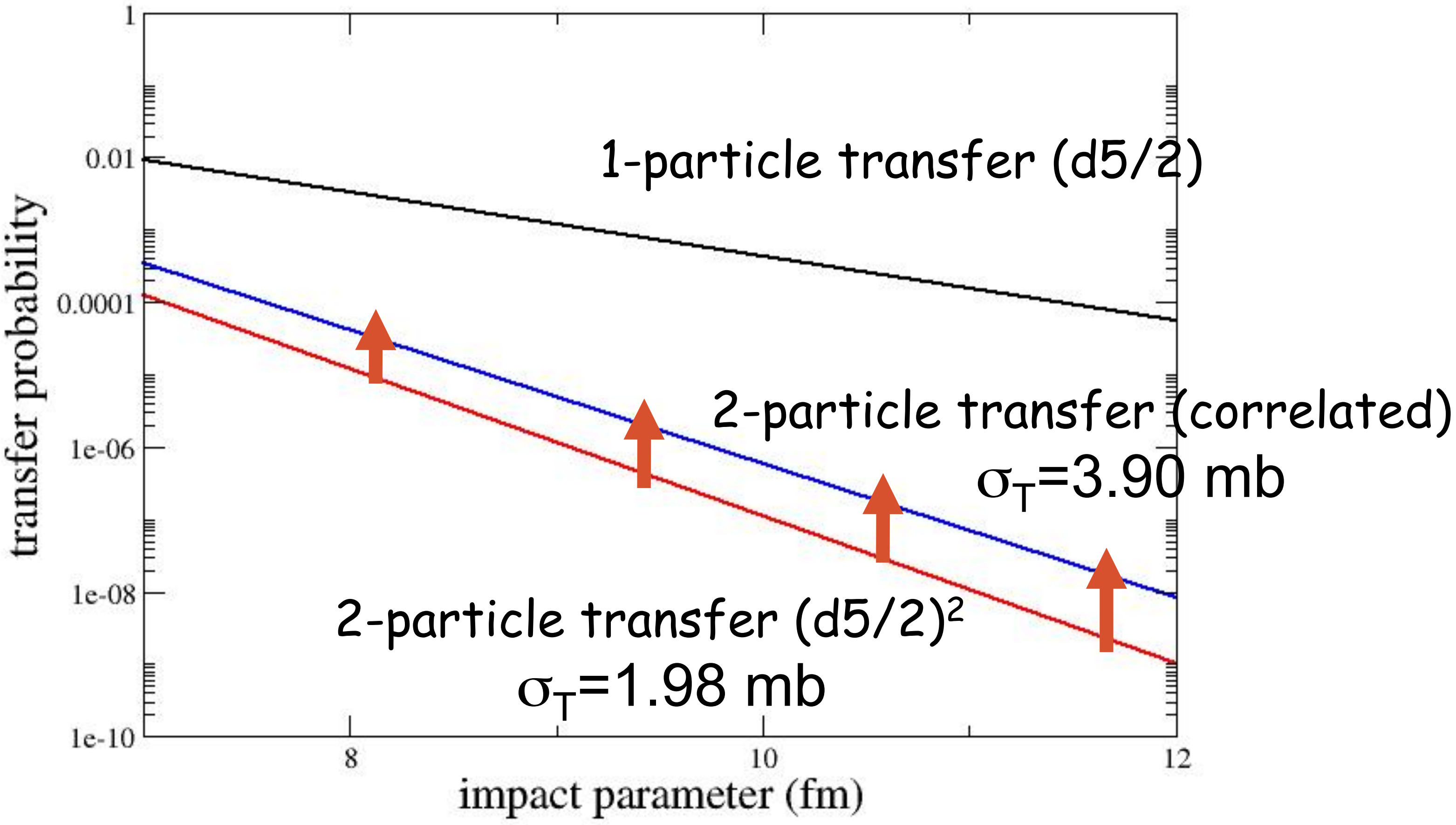
dependence on the binding energy

All microscopy and nuclear structure information are contained in the two-particle transfer amplitudes (from correlated initial and final wave functions), which give the weight of each two-step path, and in the single particle transfer formfactors, which need single particle wavefunctions in target and projectile

Obs: Basic idea: dominance of mean field, which provides the framework for defining the single -particle content of the correlated wave functions

Example of calculation

$^{208}\mathrm{Pb}(^{16}\mathrm{O},^{17,18}\mathrm{O})^{207,206}\mathrm{Pb}$

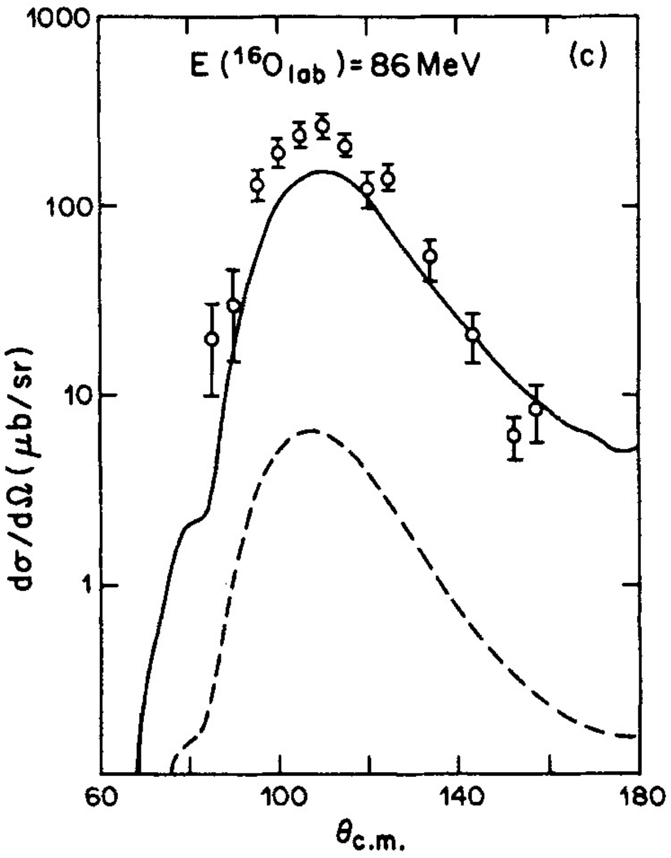


Obs: to get cross sections one needs optical potentials

^{208}Pb(^{16}O,^{18}O)^{206}Pb *gs*

—— two-step

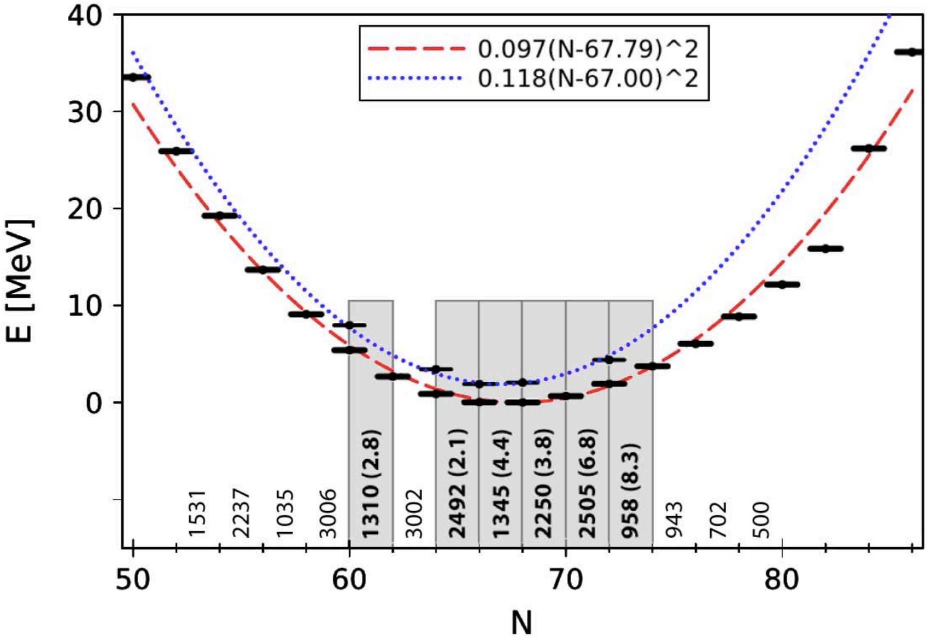
- - - - one-step



Maglione, Pollarolo, Vitturi, Broglia, Winther

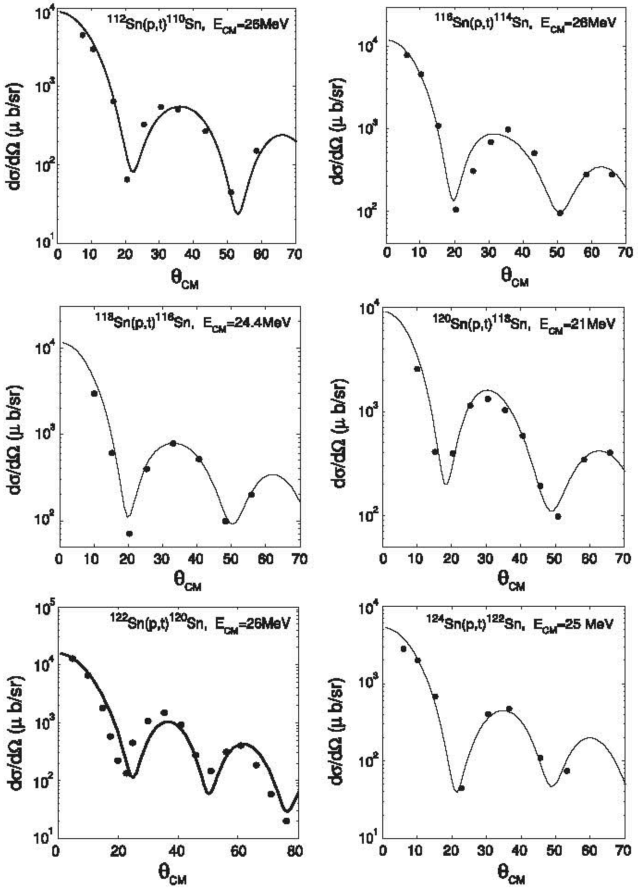
Other recent calculations in the talks by Enrico Vigezzi and Gregory Potel

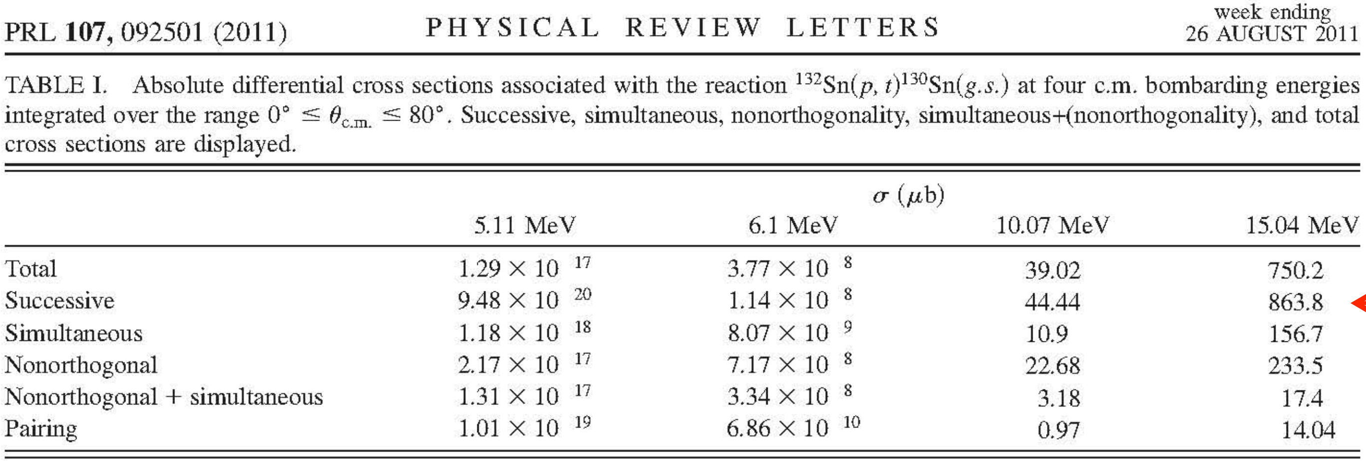
Example: (p,t) reactions on Sn isotopes
(typical example of pairing rotational band)



Vigezzi etal

Importance of different two-particle trasfer mechanisms (dependence on the bombarding energy)





PRL **107,** 092501 (2011) PHYSICAL REVIEW LETTERS week ending 26 AUGUST 2011

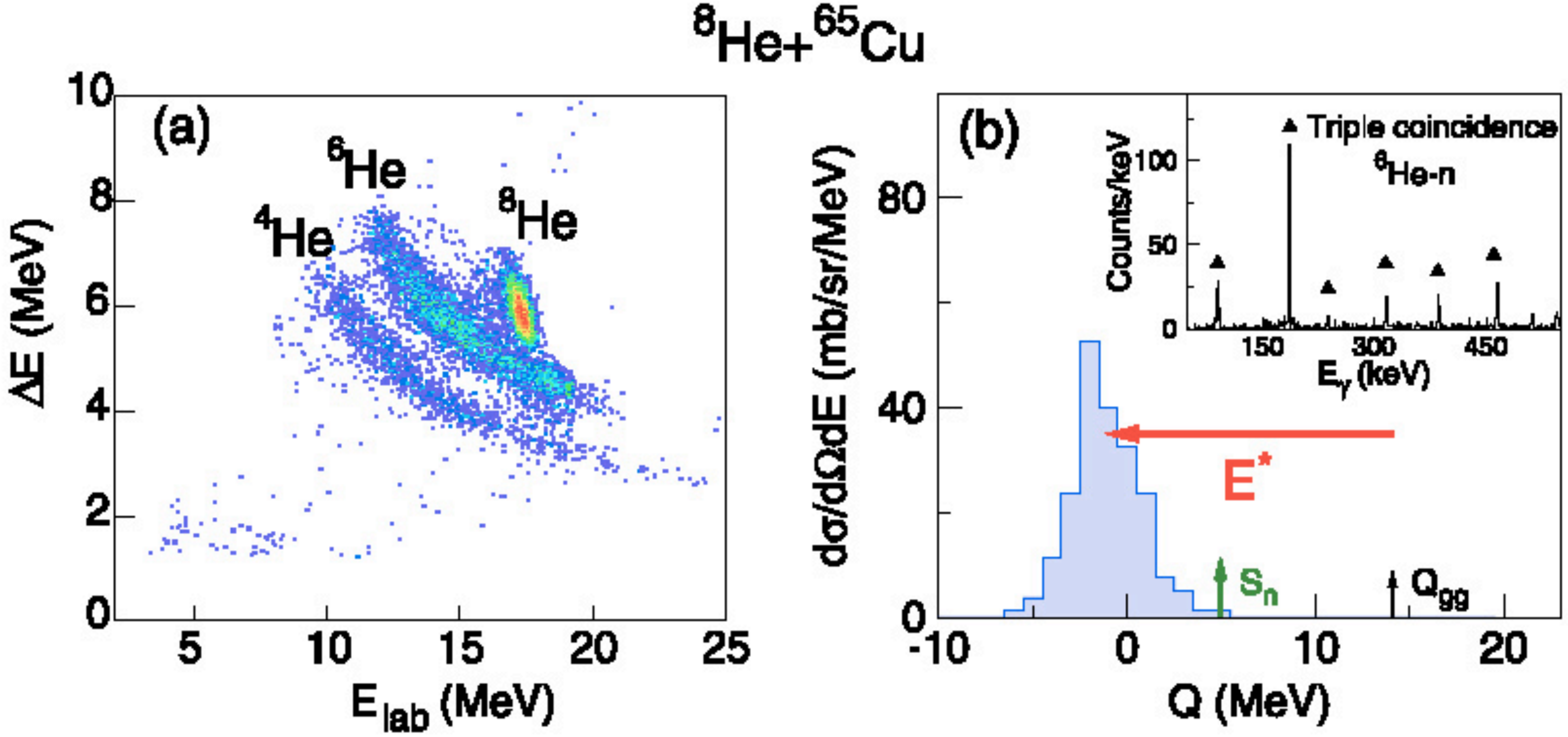
TABLE I. Absolute differential cross sections associated with the reaction ^{132}Sn$(p, t)^{130}$Sn$(g.s.)$ at four c.m. bombarding energies integrated over the range $0° \leq \theta_{\text{c.m.}} \leq 80°$. Successive, simultaneous, nonorthogonality, simultaneous+(nonorthogonality), and total cross sections are displayed.

	σ (μb)			
	5.11 MeV	6.1 MeV	10.07 MeV	15.04 MeV
Total	1.29×10^{17}	3.77×10^{8}	39.02	750.2
Successive	9.48×10^{20}	1.14×10^{8}	44.44	863.8
Simultaneous	1.18×10^{18}	8.07×10^{9}	10.9	156.7
Nonorthogonal	2.17×10^{17}	7.17×10^{8}	22.68	233.5
Nonorthogonal + simultaneous	1.31×10^{17}	3.34×10^{8}	3.18	17.4
Pairing	1.01×10^{19}	6.86×10^{10}	0.97	14.04

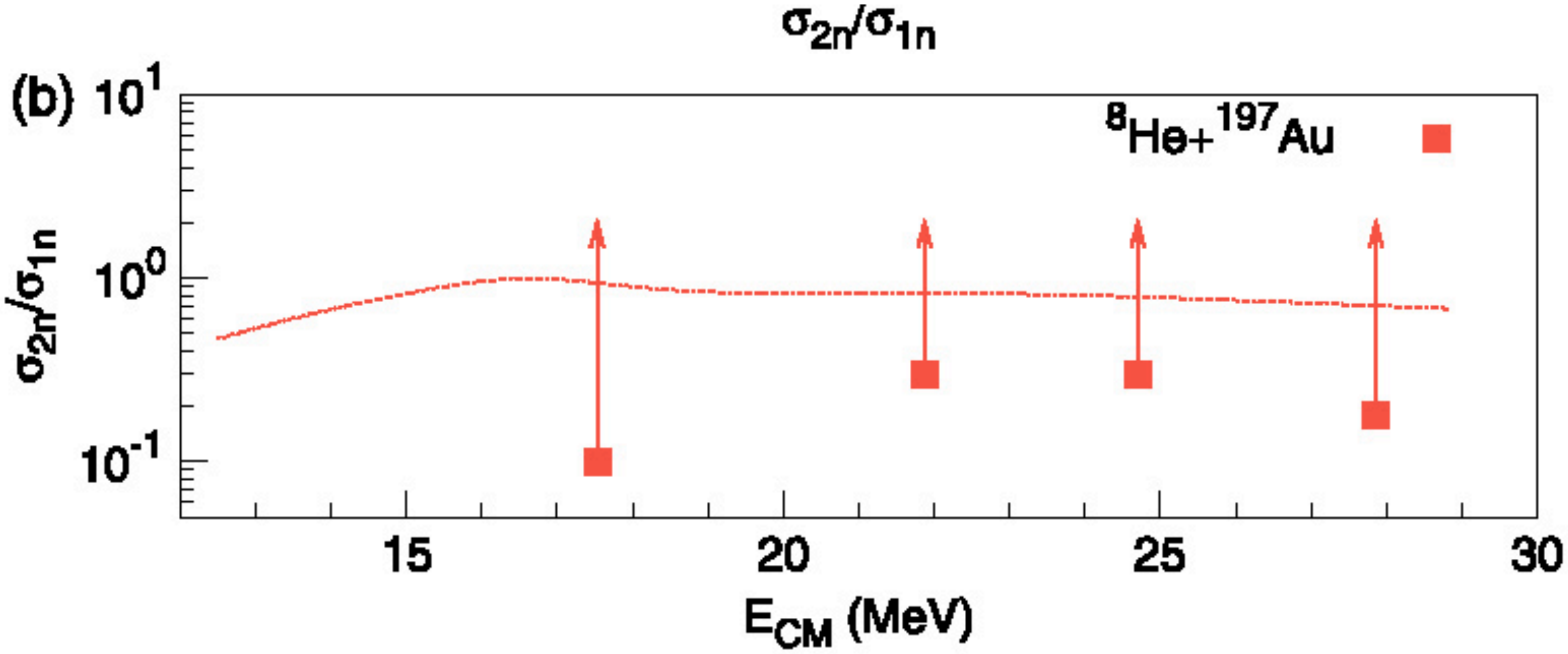
Basic problem:

how is changed the picture as we move closer or even beyond the drip lines?

Data from GANIL, Navin etal, 2011



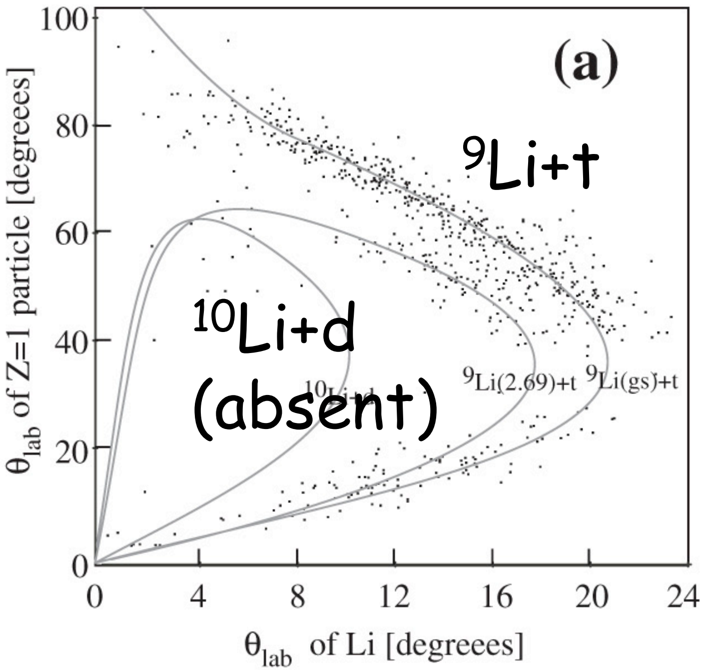
Extremely difficult to extract the fundamental $\sigma2/\sigma1$ ratio



^{11}Li+p -> ^{9}Li+t

Data from ISAC-2, TRIUMF

Isao Tanihata etal



^{9}Li+t

^{10}Li+d (absent)

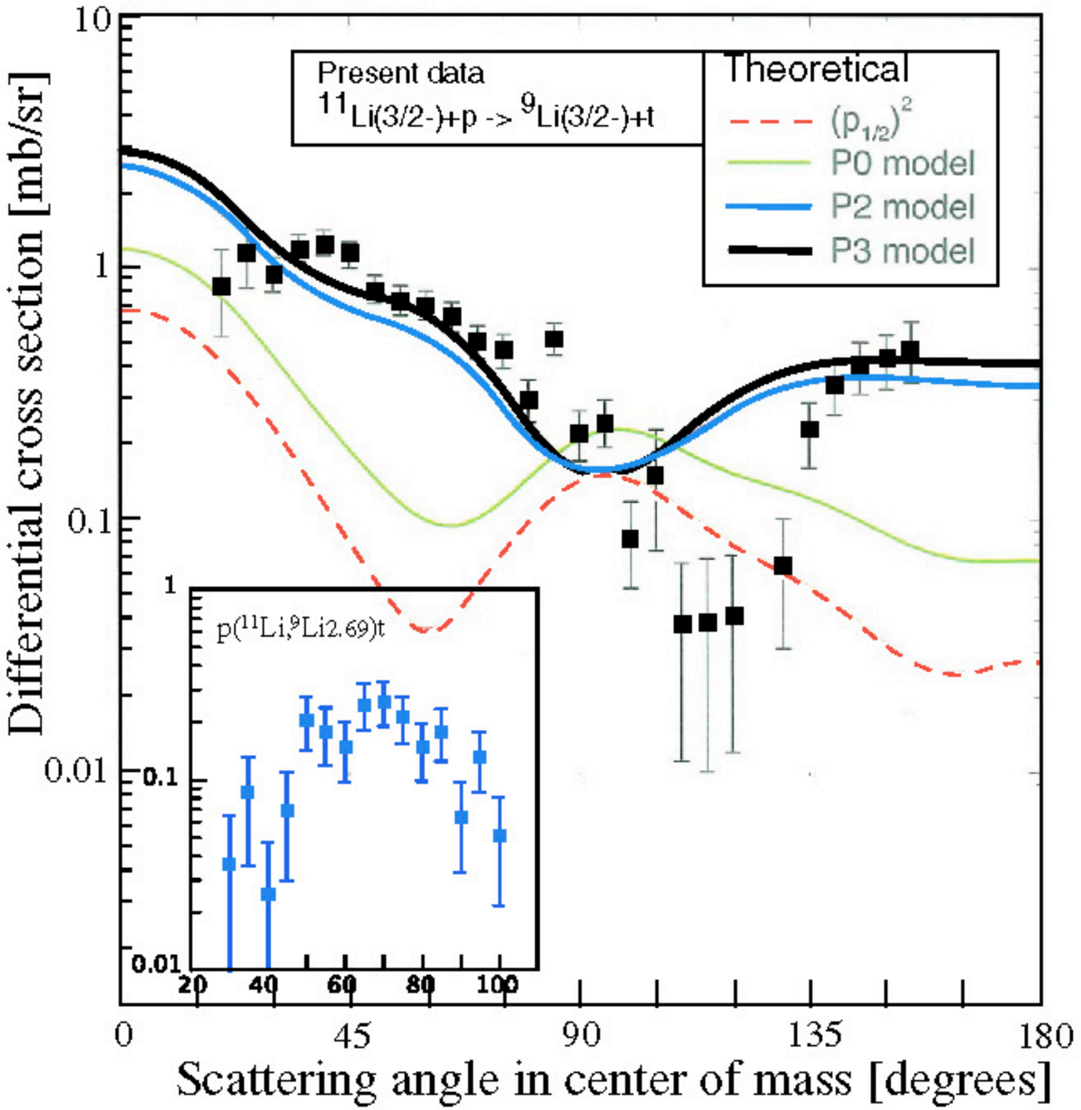
Tanihata, Thompson

Sensitivity to the pairing function in ^{11}Li

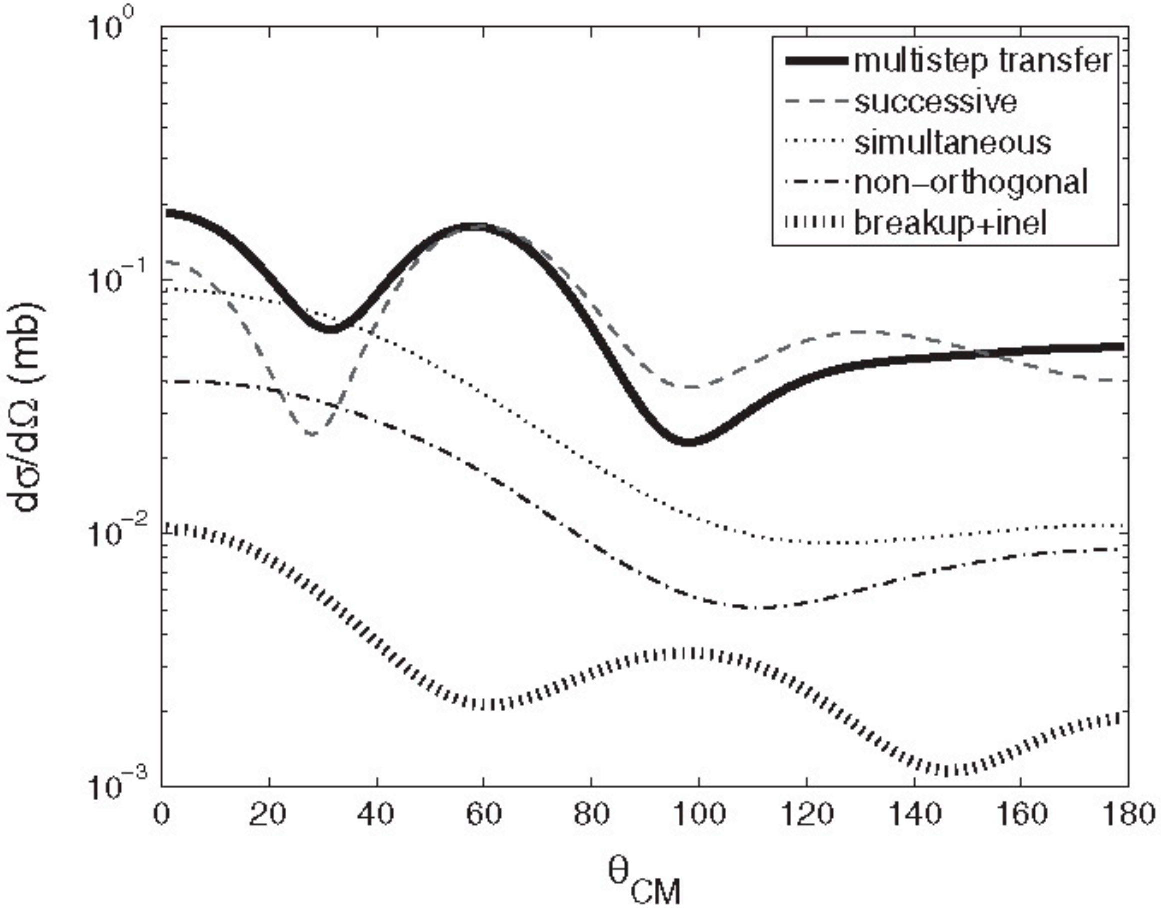
P0: 3% of $(s_{1/2})^2$

P2: 31% of $(s_{1/2})^2$

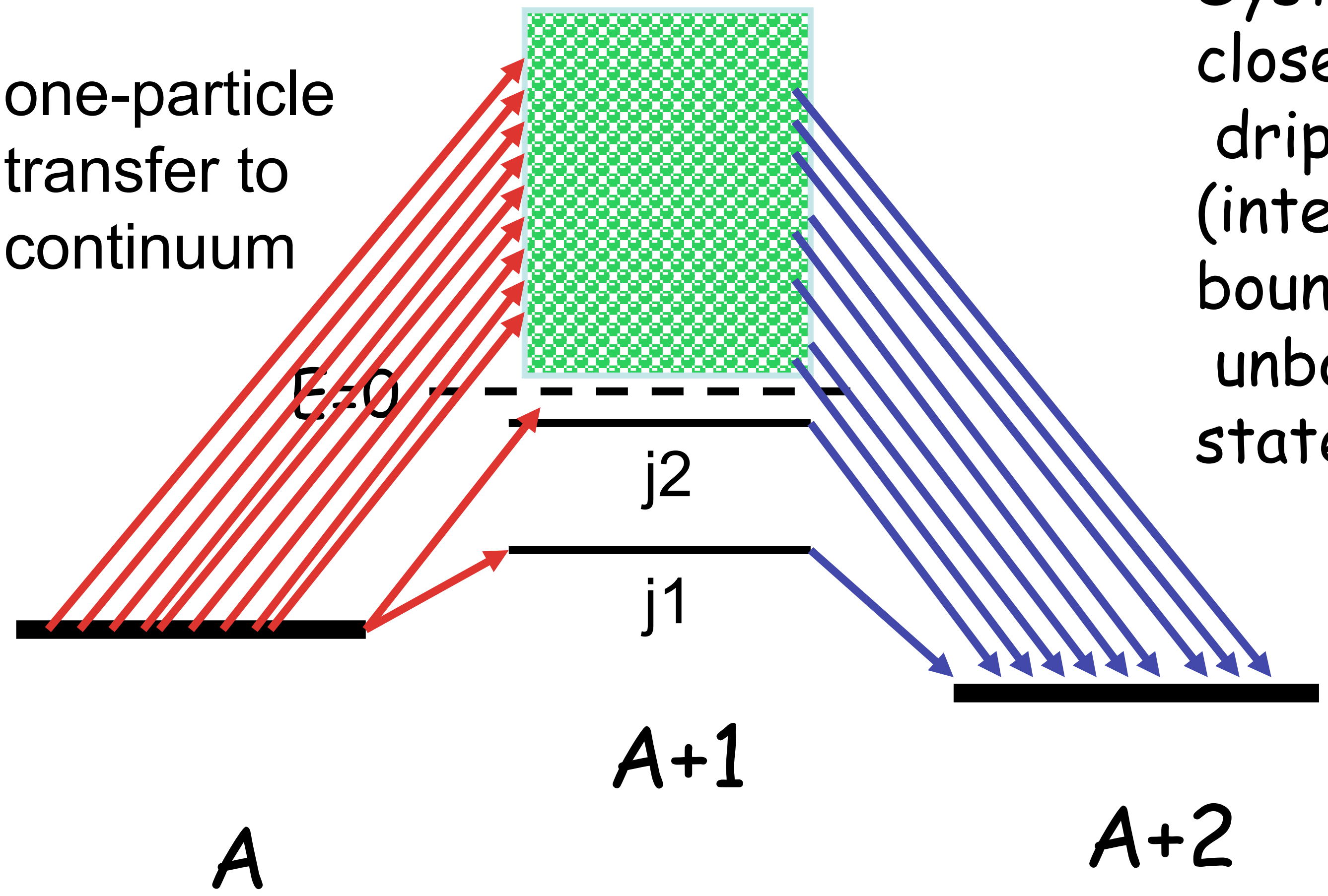
P3: 45% of $(s_{1/2})^2$



$^{11}Li+p \rightarrow ^{9}Li+t$



Potel etal, PRL, 2010

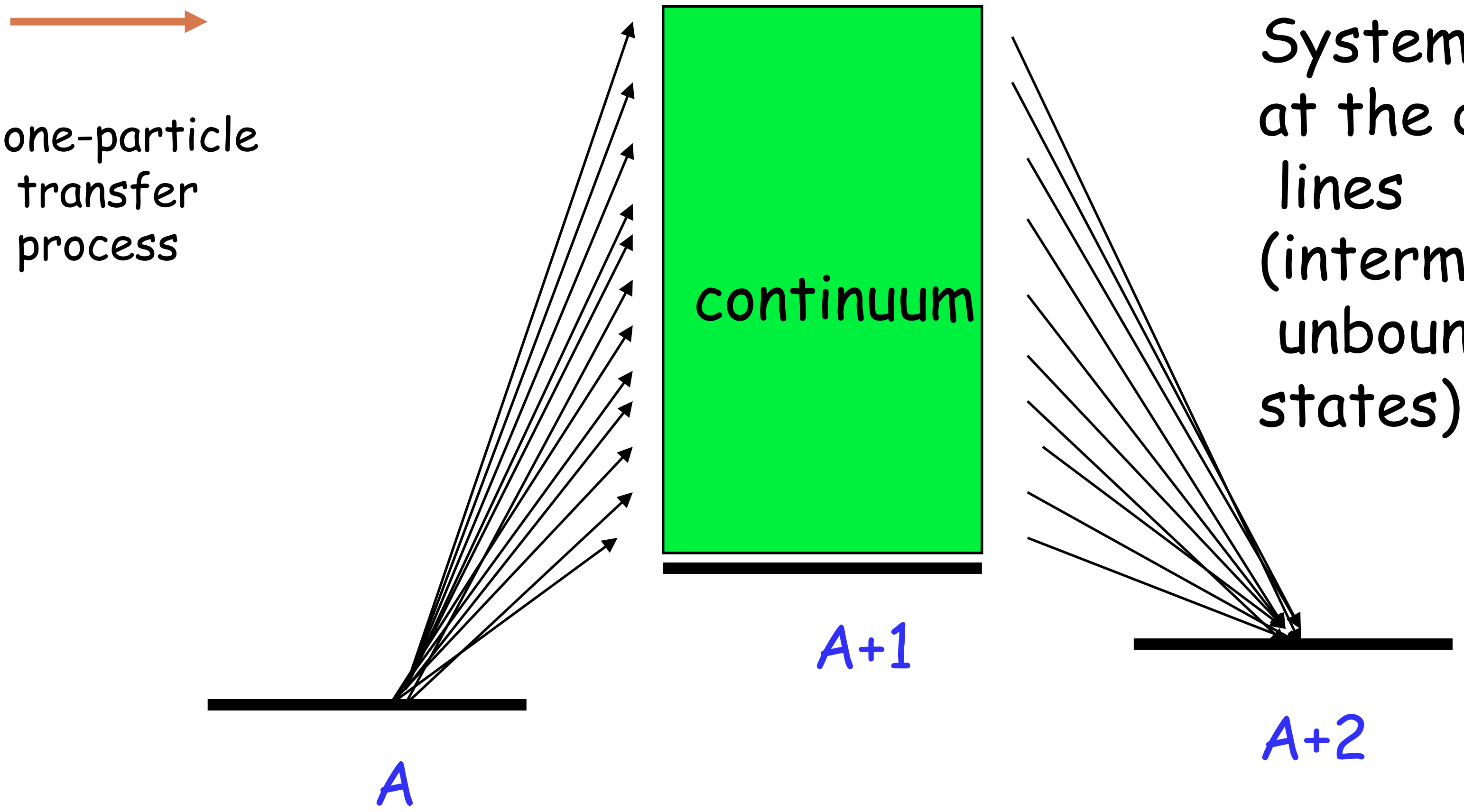


one-particle
transfer to
continuum

Systems
closer to the
drip lines
(intermediate
bound and
unbound
states)

Example

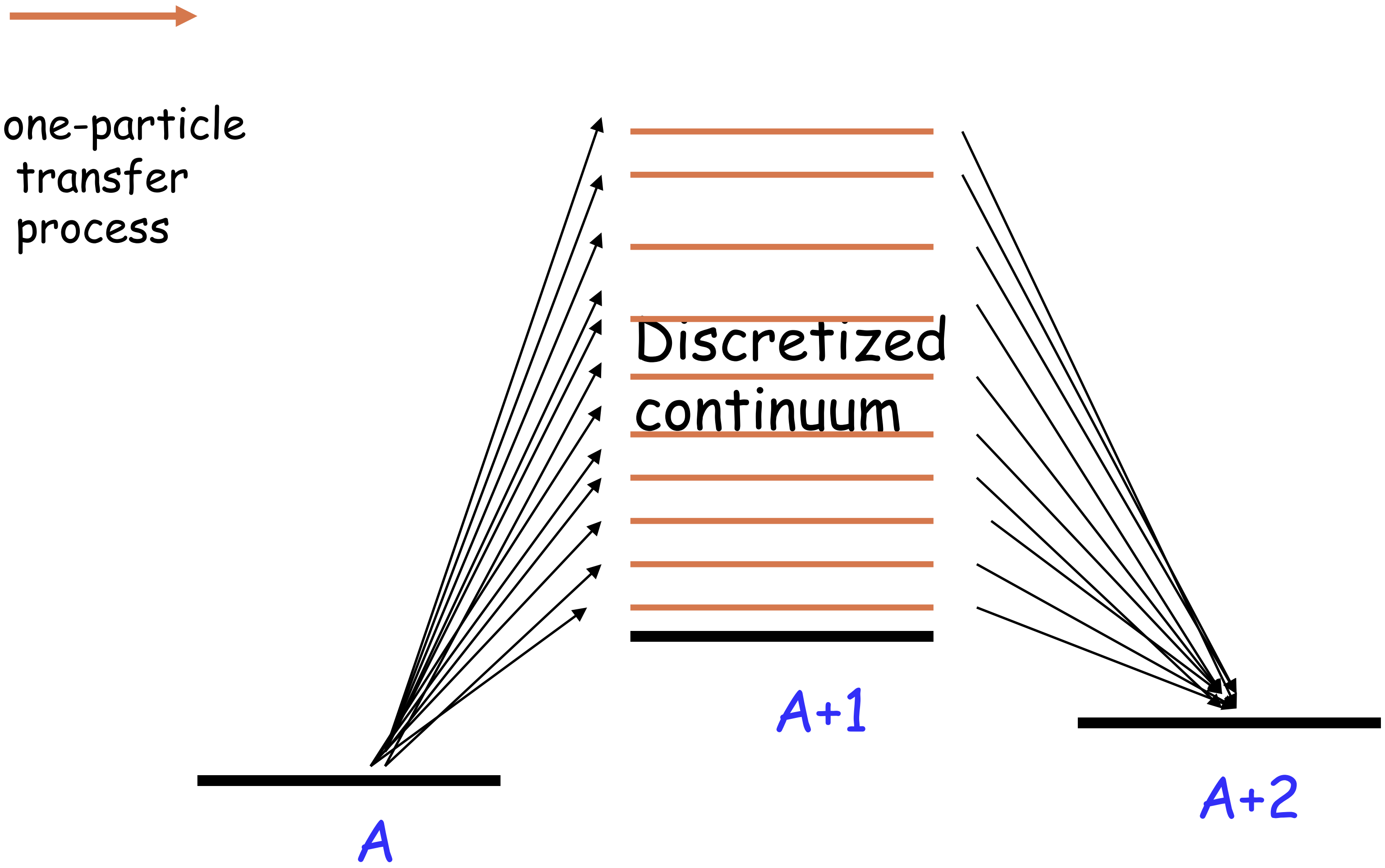
$$|A=2\rangle = \{ \Sigma_i X_i [a_i^+ a_i^+]_0 + \int dE\, X(E)\, [a^+(E) a^+(E)]_0 \} |A\rangle$$



Systems at the drip lines (intermediate unbound states)

$$|A=2> = \int dE\, X(E)\, [a^{+}(E)a^{+}(E)]_0\, |A>$$

Two-particle trasfer will proceed mainly by constructive interference of successive transfers through the (unbound) continuum intermediate states



The integration over the continuum intermediate states can becomes feasible by continuum discretization: but how many paths should we include? Thousands or few, for example only the resonant states?

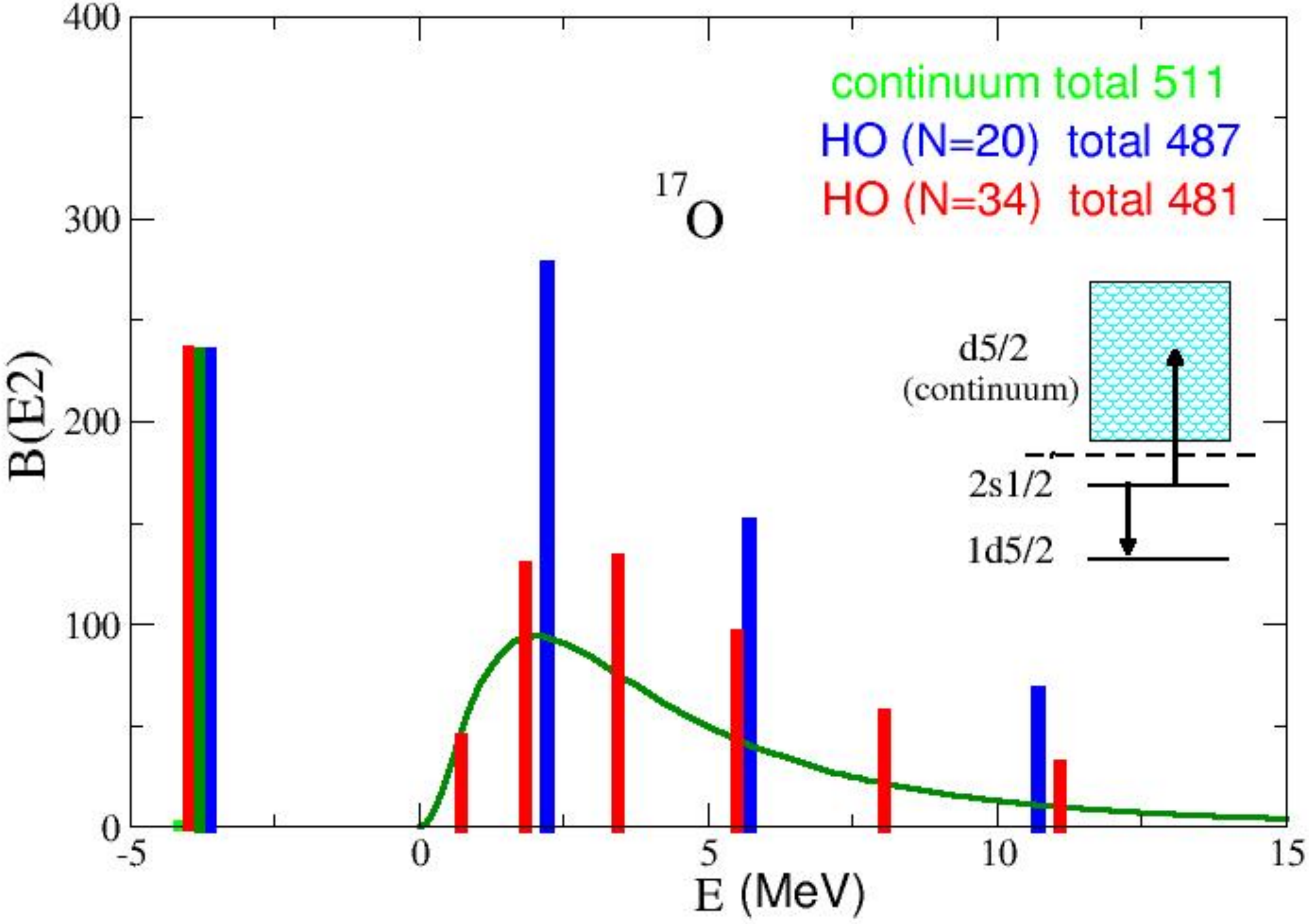
For weakly-bound systems at the drip lines it is mandatory to include in the models the positive energy part of the spectrum. If one wants to still use the same machinary used with bound states, the most popular approach is the discretization of the continuum. But the discretization MUST go in parallel in a consistent way both in the structure and in reaction parts.

All discretization procedures are equivalent as long as a full complete basis is used. In practice all procedudes contain a number of parameters and criteria, that make not all procedures equally applicable in practical calculations. Computational constraints may in fact become a severe problem.

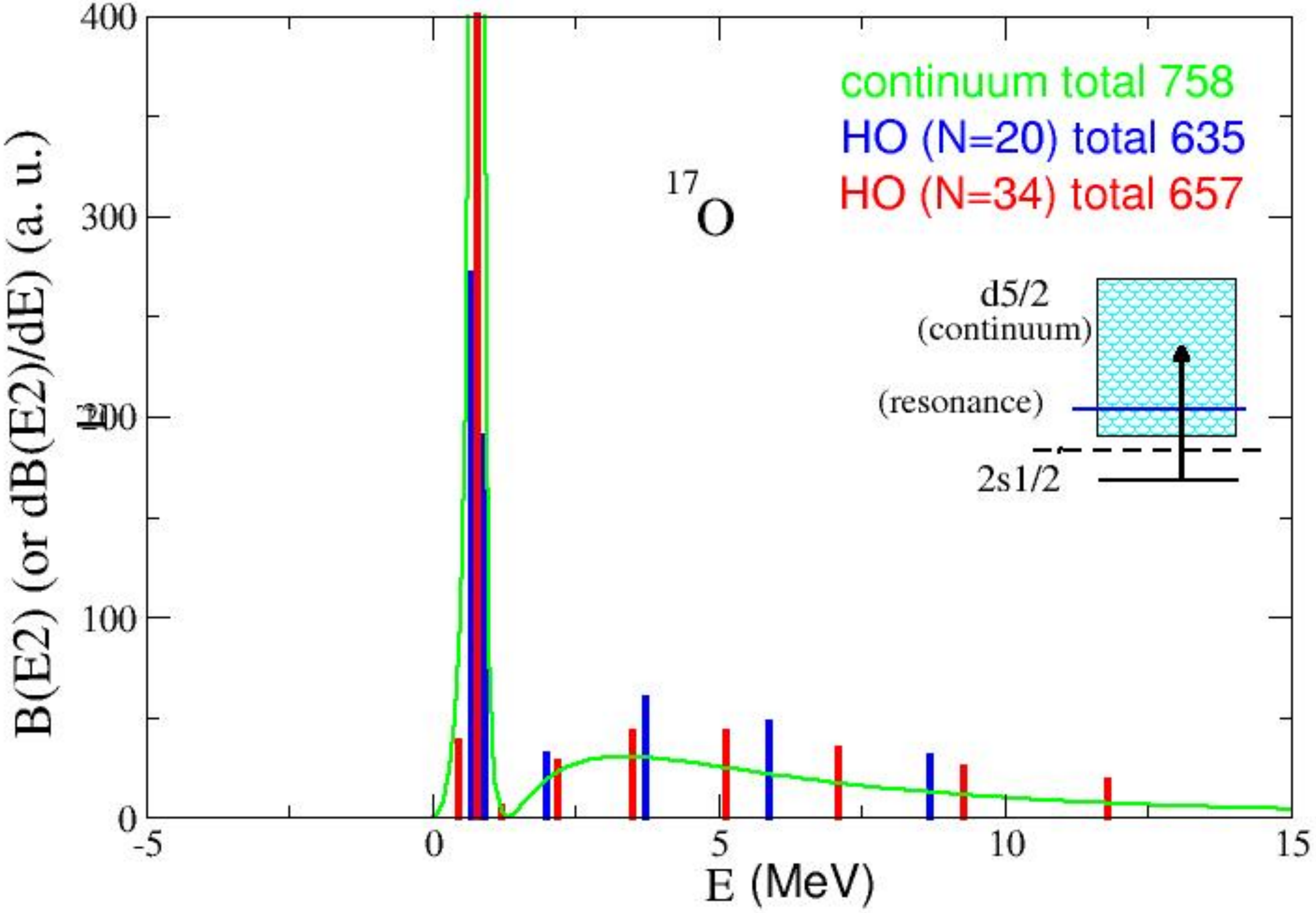
As possibilities we can consider

- diagonalization in a basis given by HO wave functions
- impose boundary conditions in a BOX
- the case of discretized wave functions with scattering boundary conditions (CDCC)
- Gamow states (complex energies)

Case of non-resonant continuum
(Woods-Saxon single-particle potential in a HO basis)

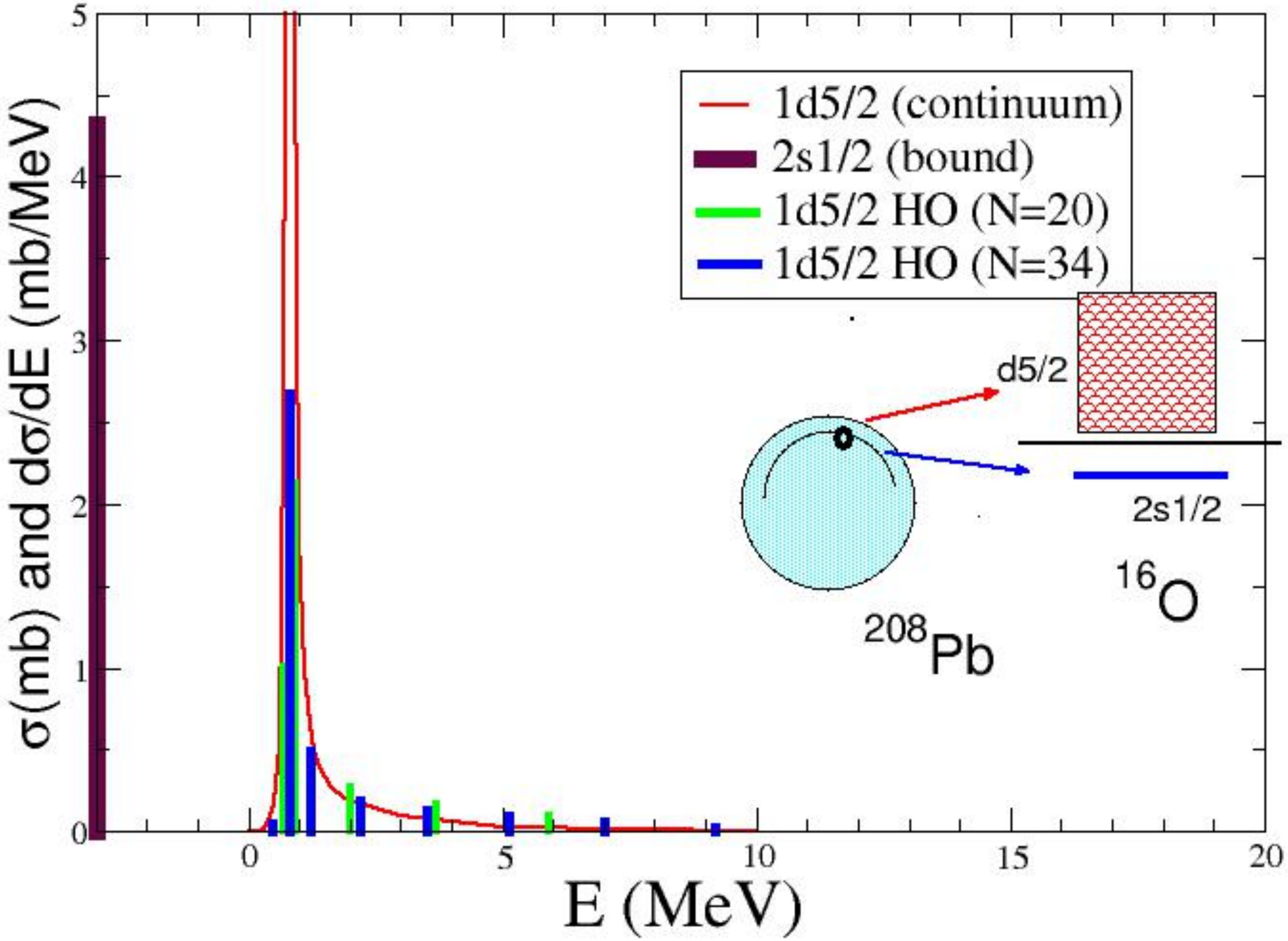


Case of resonant + non-resonant continuum



One-particle transfer (in DWBA)
Case of resonant + non-resonant continuum

$^{208}\mathrm{Pb}(^{16}\mathrm{O},{}^{17}\mathrm{O}^{*})^{207}\mathrm{Pb}(\mathrm{gs},(3p1/2)^{-1})$



Moving from the case of just one particle in the continuum to cases with more particles in the continuum

Simple test case in structure

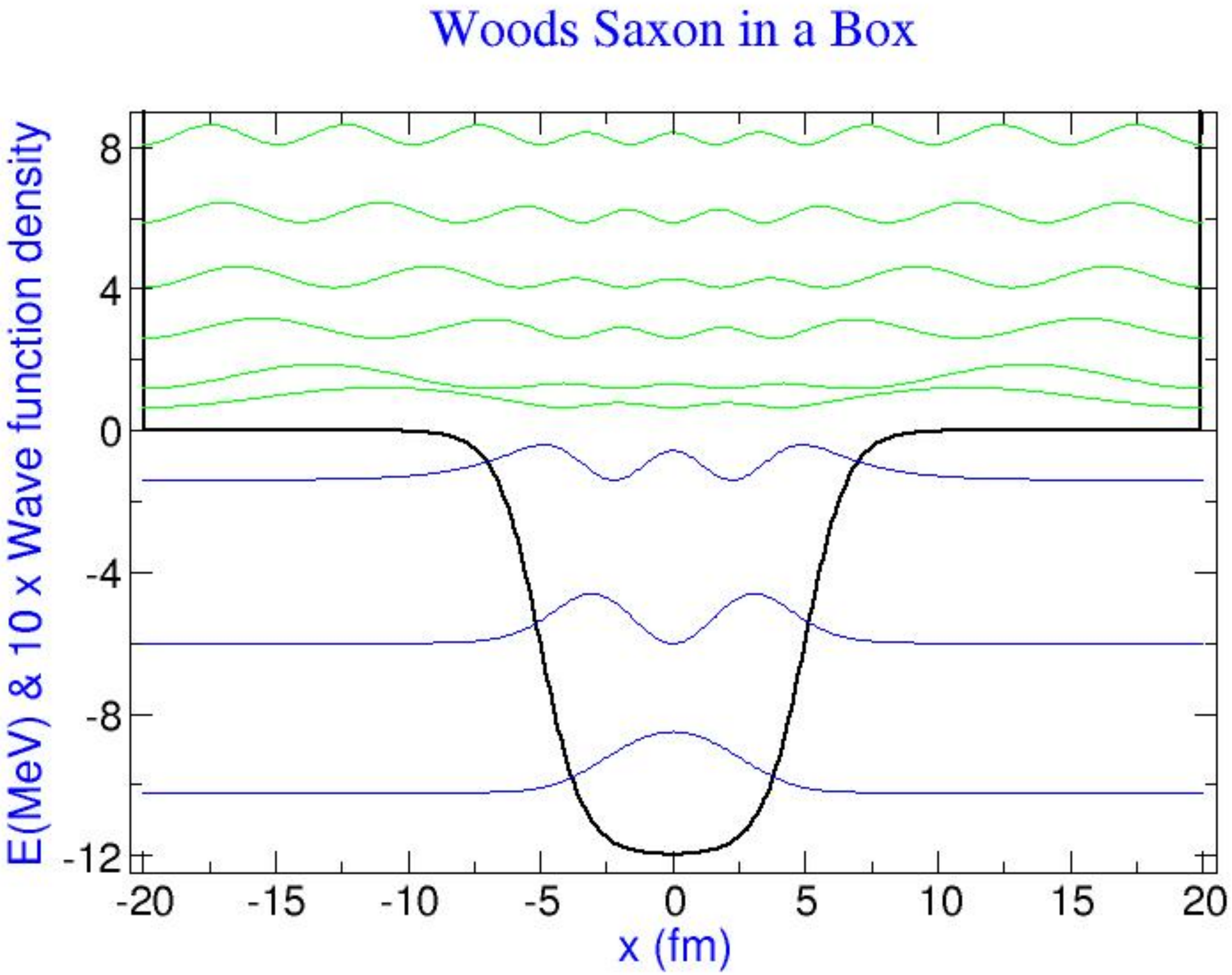
Two valence particles, moving in a one-dimensional Woods-Saxon potential V_0, interacting via a residual density-dependent short-range attractive interaction. Modelling a drip-line system, one can choose the Fermi surface in such a way that there are no available bound states, and the two unperturbed particles must be in the continuum. The residual interaction

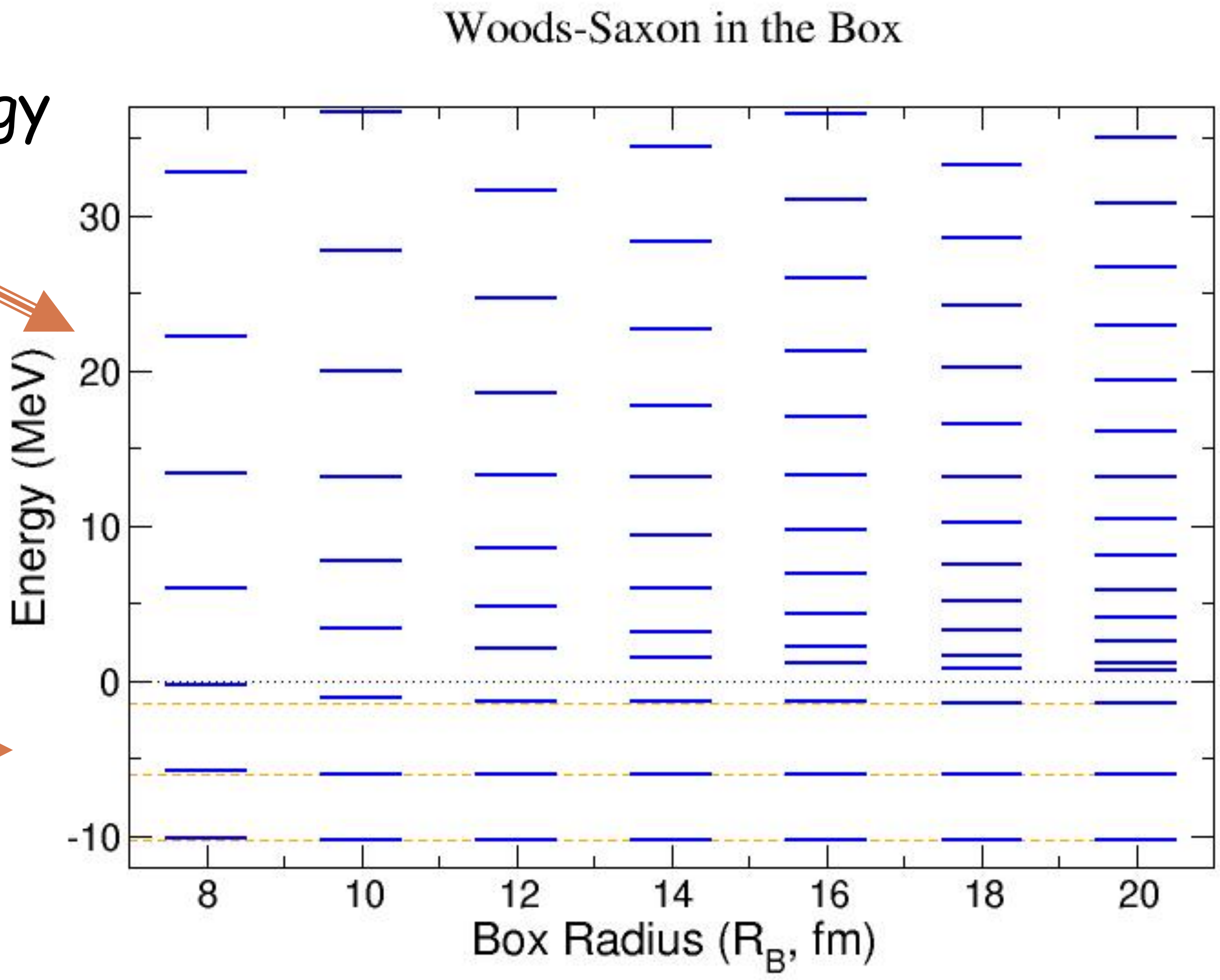
$$V(x_1,x_2) = V_0\,\delta(x_1-x_2)\,\rho((x_1+x_2)/2)/\rho_0$$

can be chosen in such a way that the final correlated wave function is however bound. Such a system is normally called "Borromean"

Diagonalization in a box

WS single-particle states obtained imposing boundary conditions at a box (R=20 fm)

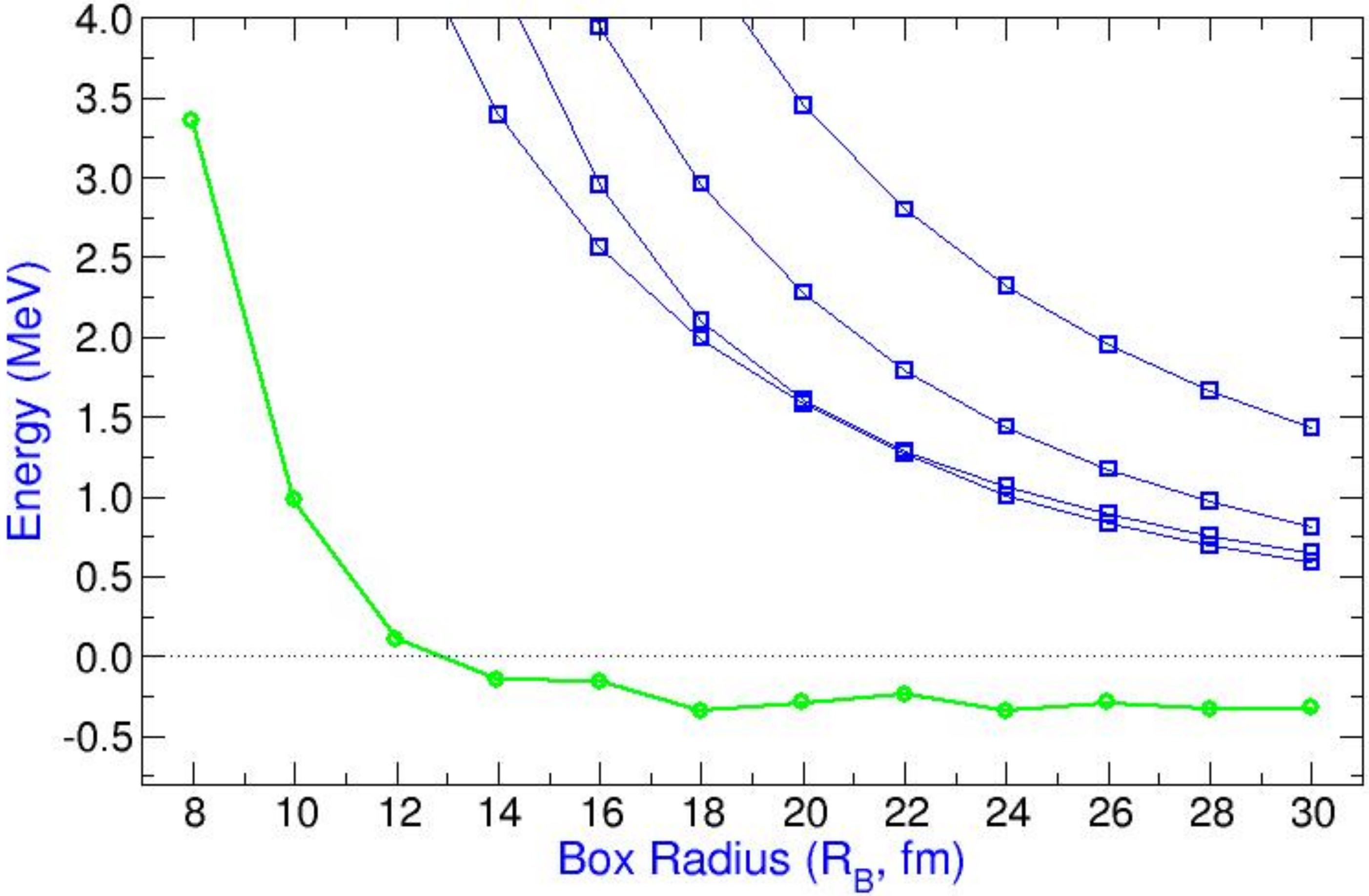




positive energy states

bound states

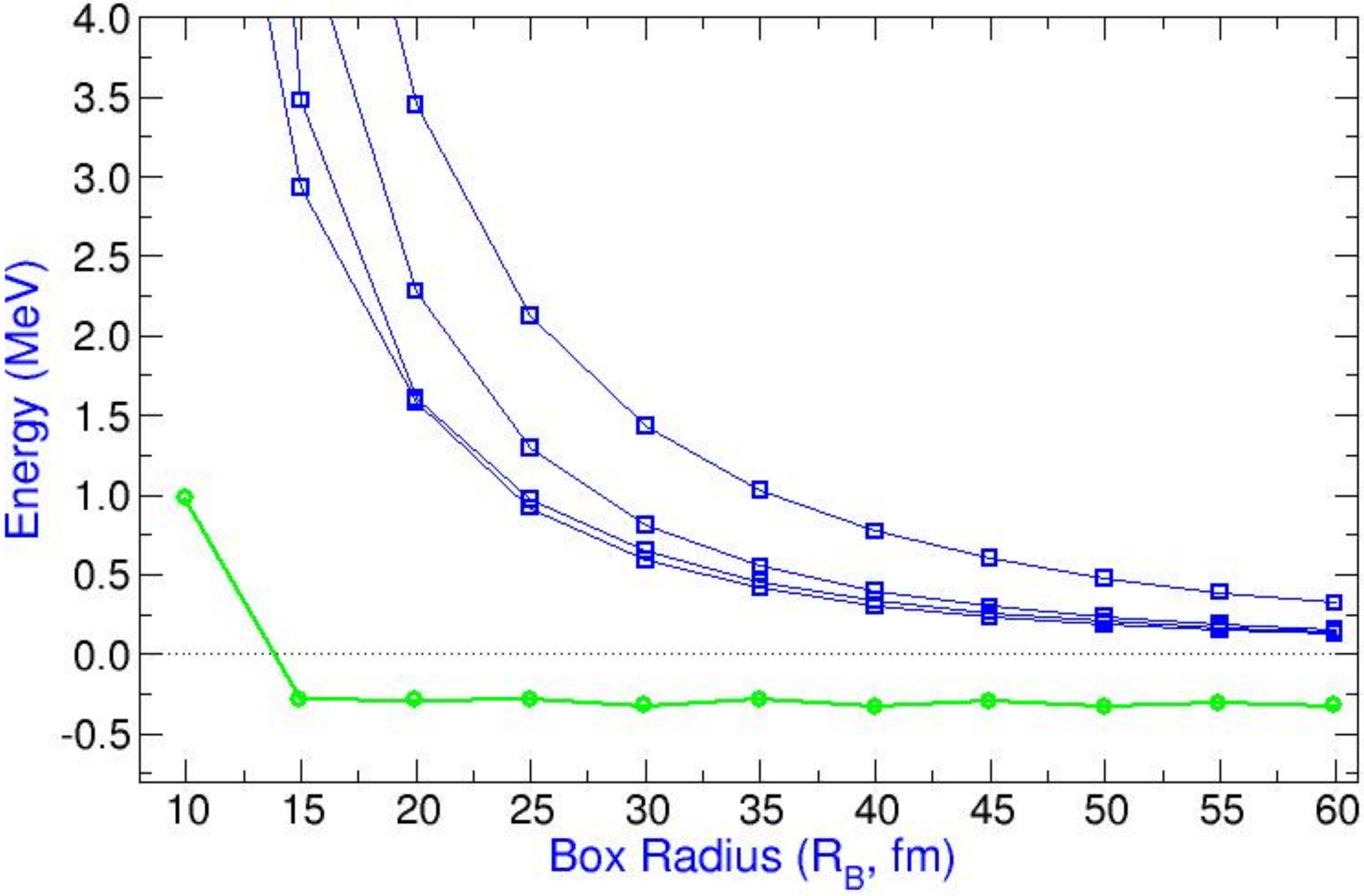
Correlated energy of the two-particle system (as a function of the box radius)



unphysical two
-particle states
(basis dependent)

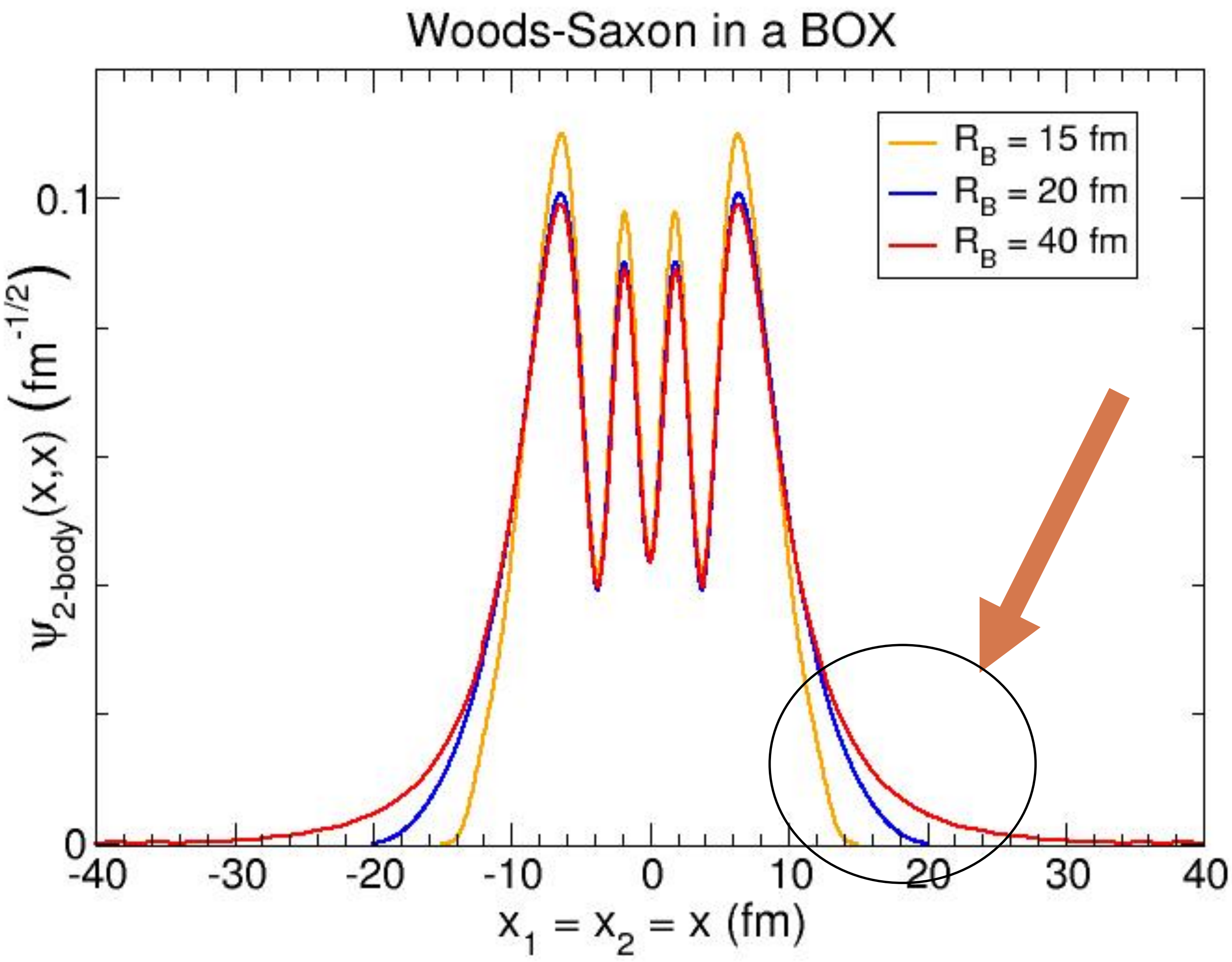
physical two-particle
bound state

The value of the binding energy is converging (with some oscillations) to the final value

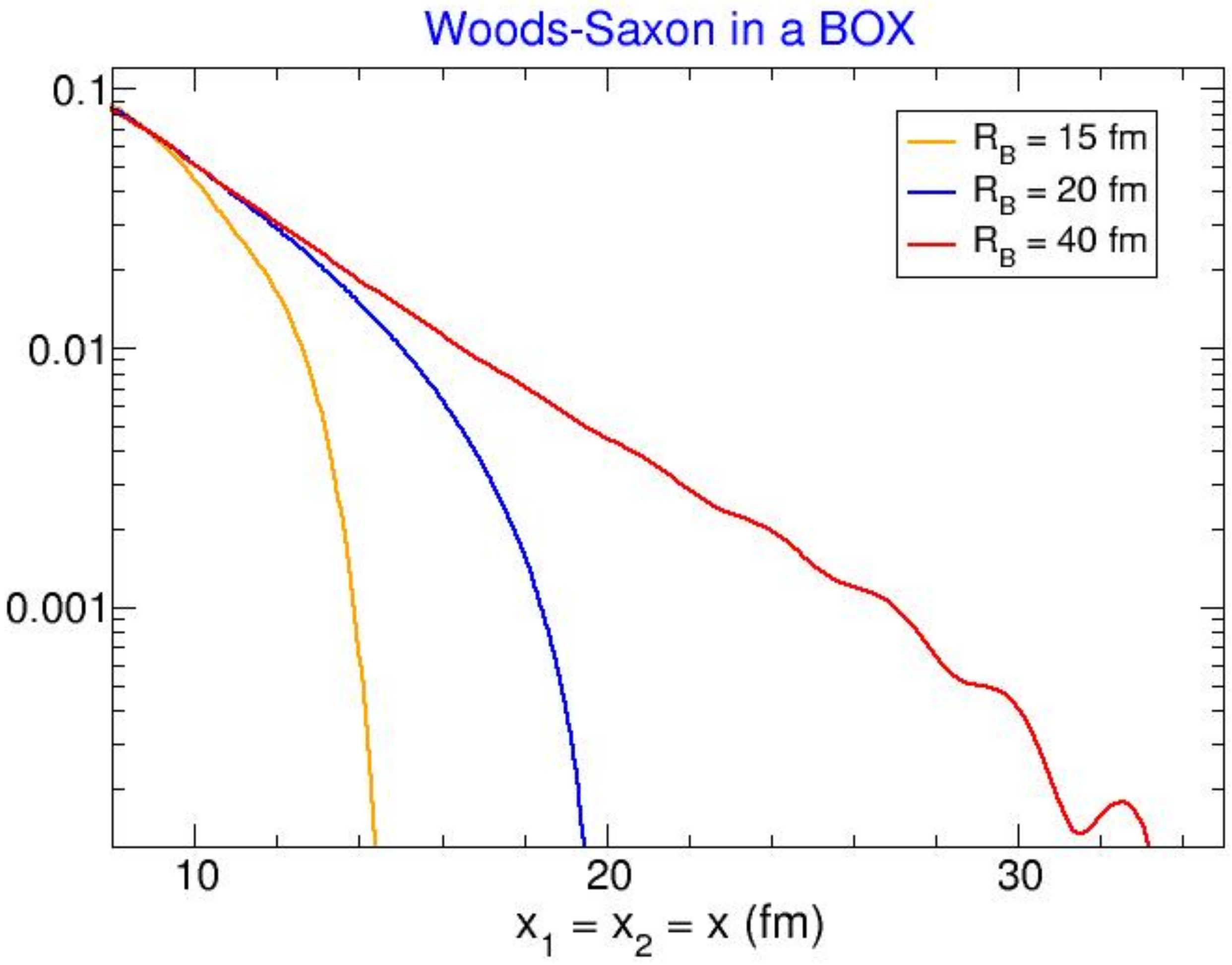


Energy already practically correct with a box
of 15 fm, but what about the wave function?
In particular, how does it behave in the tail?

Radial dependence $\delta\rho(x,x)$

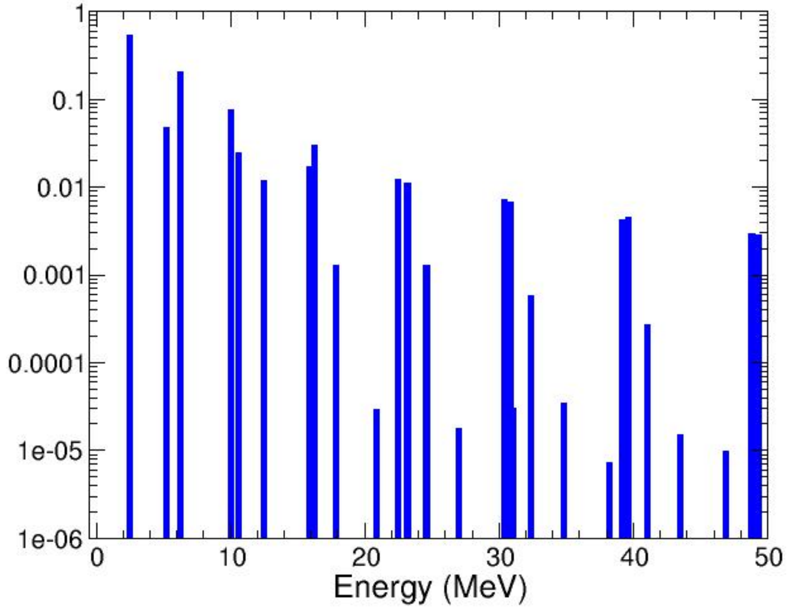


Logaritmic scale



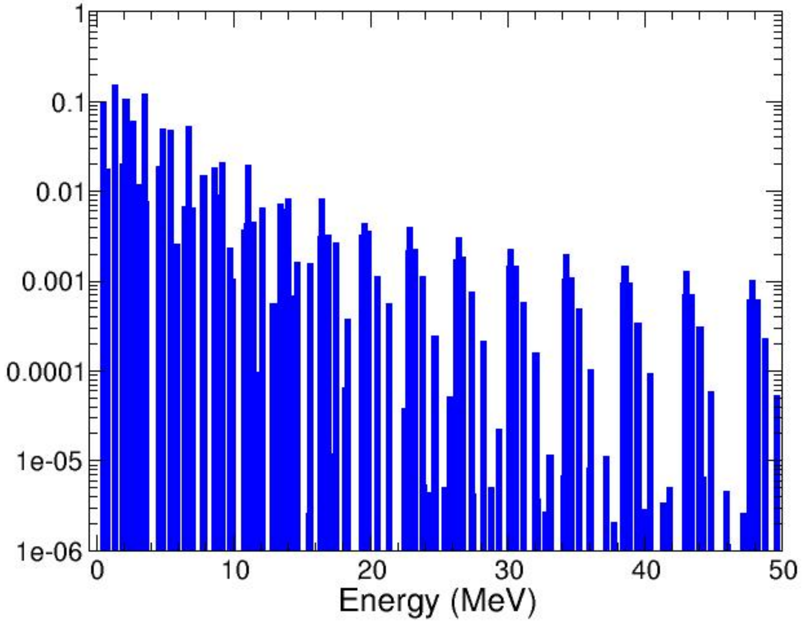
Correlated two-particle wave-function expanded over discretized two-particle positive energy states

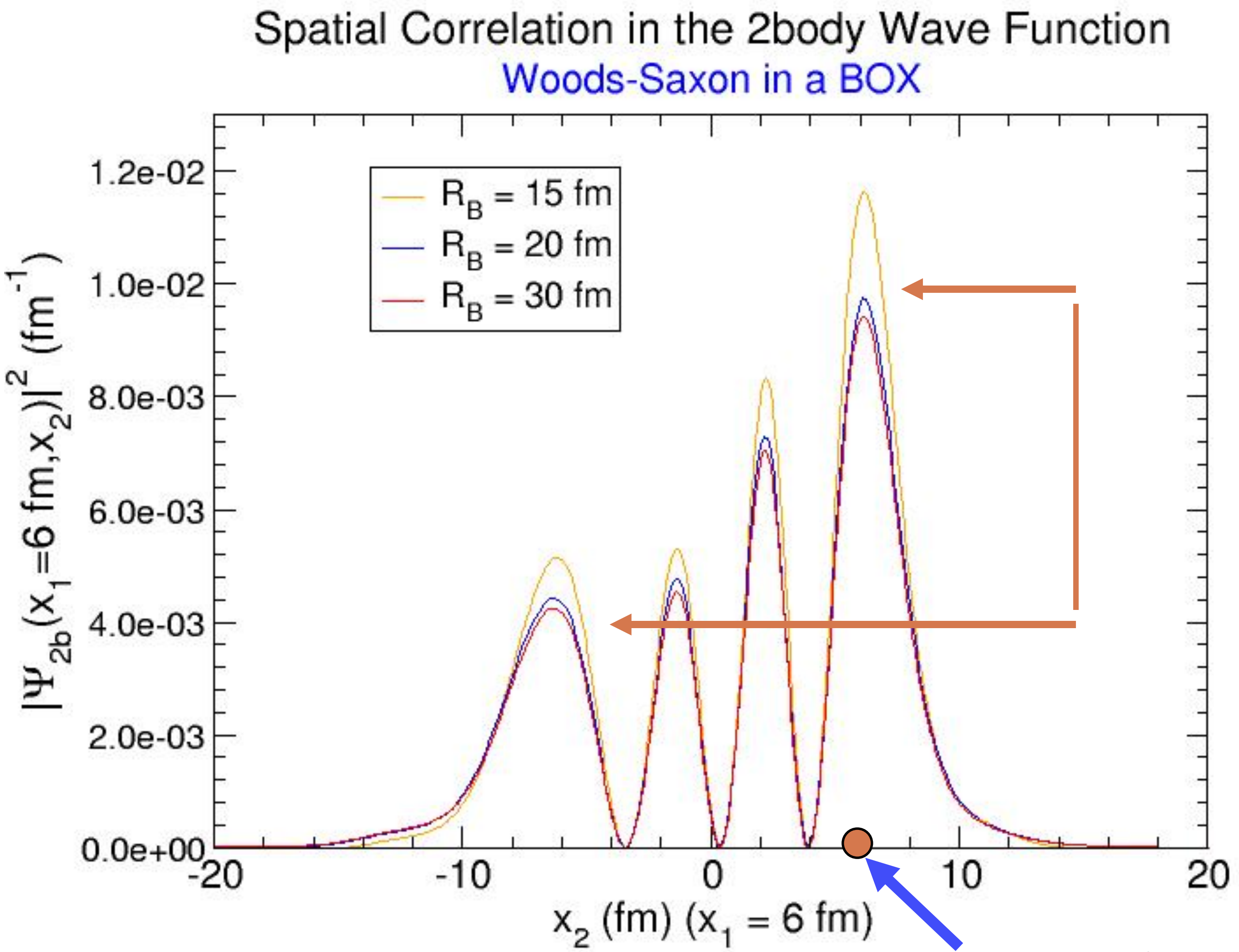
R=15fm



OBS Enormous number of components

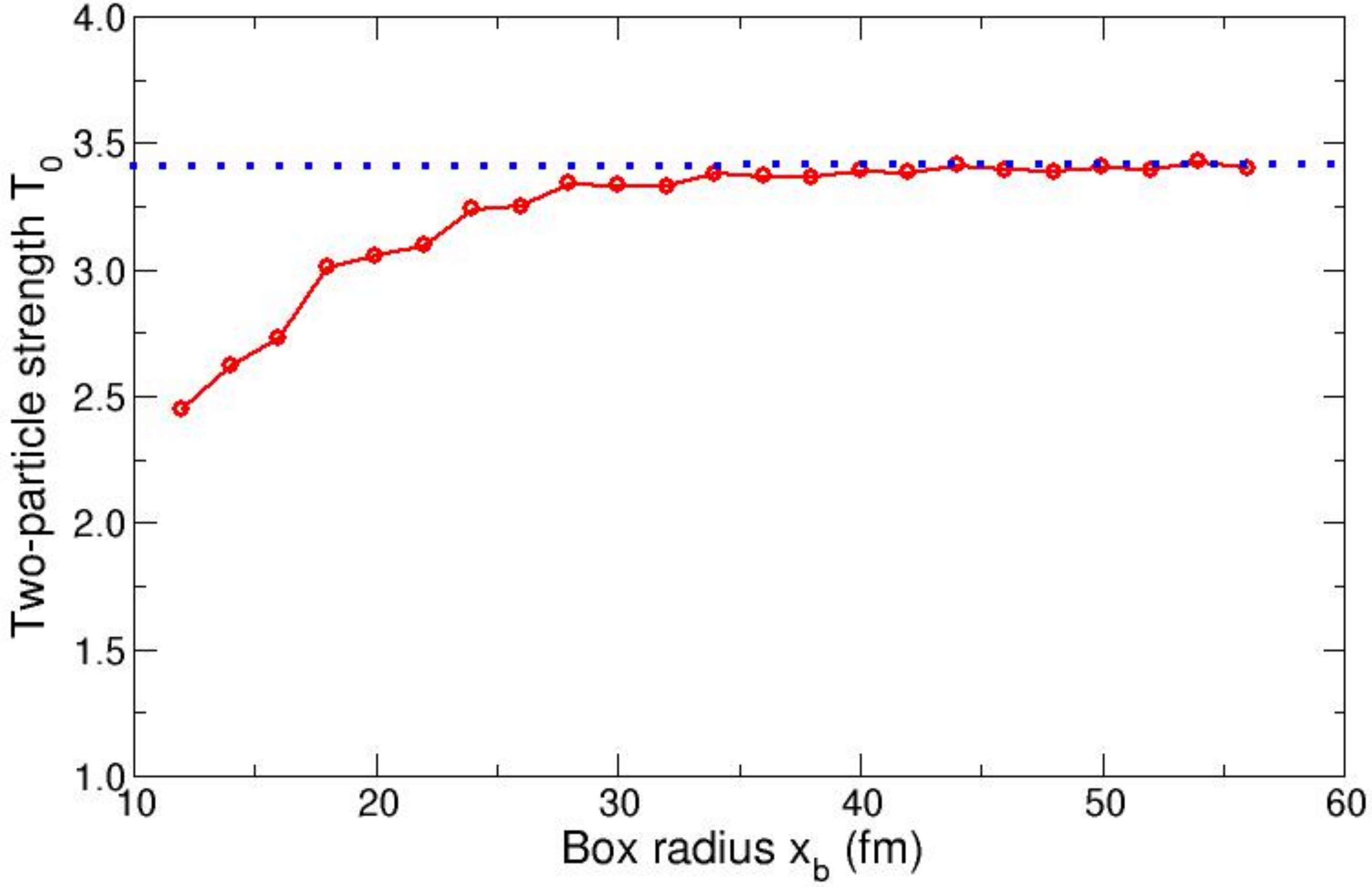
R=40fm




Spatial Correlation in the 2body Wave Function
Woods-Saxon in a BOX
R_B = 15 fm
R_B = 20 fm
R_B = 30 fm
$|\Psi_{2b}(x_1=6\ \mathrm{fm},x_2)|^2\ (\mathrm{fm}^{-1})$
1.2e-02
1.0e-02
8.0e-03
6.0e-03
4.0e-03
2.0e-03
0.0e+00
-20
-10
0
10
20
x_2 (fm) (x_1 = 6 fm)

particle 1

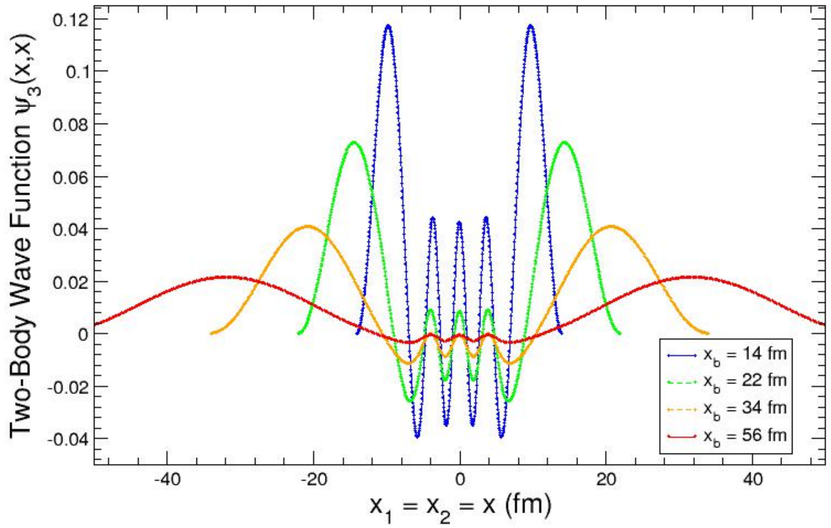
Pairing matrix element to the ground state (two-particle creation matrix element)

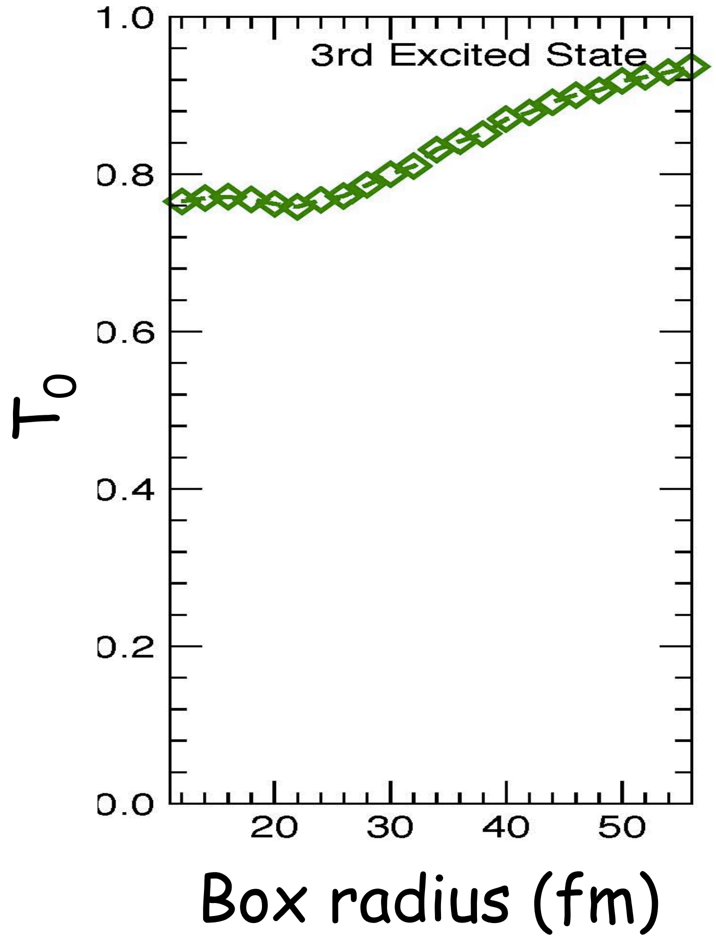


Pair strength to excited states

Example: Pair matrix element to a generic "discretized" state in the continuum

Two-body wave function





Similar results have been obtained by the
diagonalization in a harmonic oscillator basis
(with similar results, now as a function of the number of
HO shells). Correct behavior on the tail and converged
two-particle matrix element require very large number of
Shells (even more than 100 ….)

Conclusions:

There is a non-trivial connection between pairing correlations and two-particle transfer cross-sections.

The connection is even more complicated in the case of weakly-bound systems due to the role of continuum states, which are not easily incorporated in standard reaction frameworks.