

Resonances in hadron physics

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YITP workshop:

Resonances and non-Hermitian systems in quantum mechanics

December 11–13, 2012

Contents

1. Introduction

Strong interactions, experiments, quarks, exotics, ...

2. T-matrices and poles

From simple to realistic examples

3. Topics of discussions

Quark *intrinsic* or Hadronic *composite (dynamical)*

Extraction of T-matrices from exp. (Kamano)

Compositeness of hadronic molecules (Hyodo)

Geometric aspects of poles of two level model (Nawa)

Lambda(1405) and complex matrix elements (Sekihara)

1. Introduction

1. Introduction

What Physics does ...

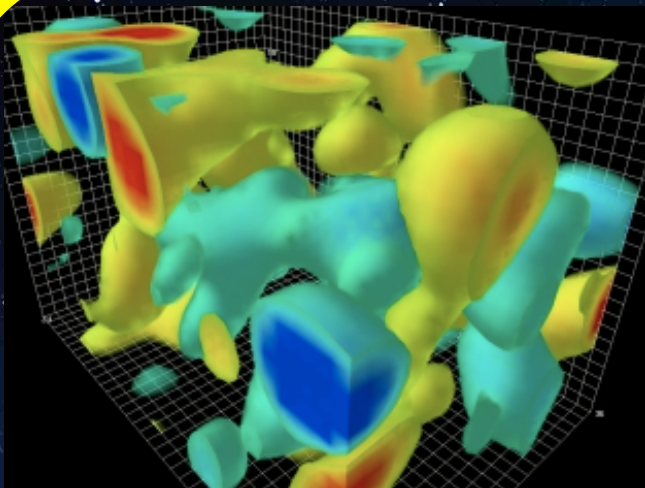
Disintegration Find fewer constituents and simple laws
Pursue the frontier

Reconstruction Reconstruct the world from constituents
Explain the diversity and complexity

Resonances have composite nature

Matter world is not straightforward

Multi-hierarchies (steps)/ not continuous



Vacuum fluctuating by the strong int

Nucleons from quarks

Atomic nucleus

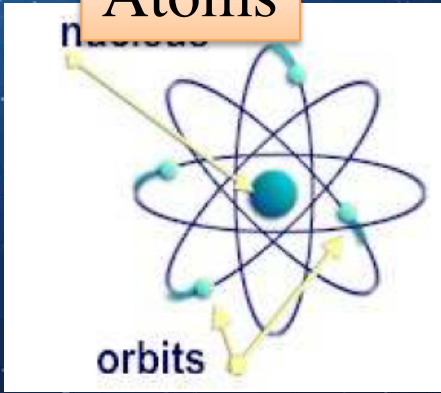
Subatomic world Hadron physics



Molecules



Atoms



Daily

Dec. 11-13, 2012

YITP workshop - Resonances

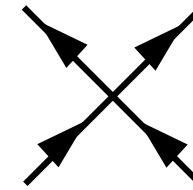
原子

To show the hierarchies of matter

Example: Nambu model (NJL)

Y. Nambu and G. Jona-Lasinio,
PR122, 345, 1961; PR124, 246, 1961

$$L_{NJL} = \bar{q}i\partial q + G \left[\underline{(\bar{q}q)^2} + \underline{(\bar{q}i\gamma_5 \vec{\tau} q)^2} \right]$$



SU(2) x SU(2) chiral symmetry

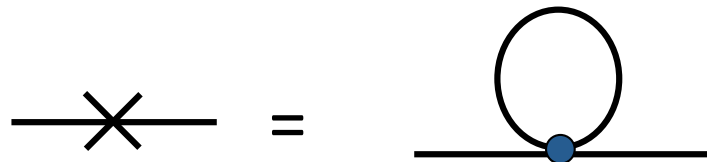
$$q \rightarrow \exp(i\vec{\tau} \cdot \vec{a}\gamma_5)q \quad \Rightarrow \quad \bar{q}q \leftrightarrow \bar{q}i\gamma_5 \vec{\tau} q$$

Two features:

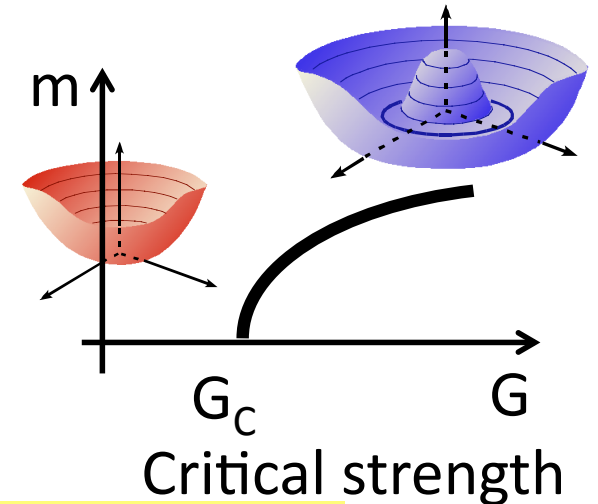
- (1) Mass generation
- (2) Massless pion

$$L_{NJL} = \bar{q}i\partial q + G \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau} q)^2 \right]$$

- **One particle: gap equation for m**

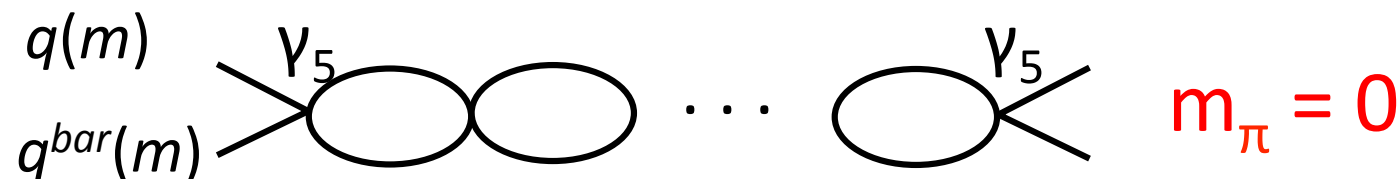


$$m = G \langle \bar{q}q \rangle = G \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m}{\sqrt{k^2 + m^2}}$$



Finite m for sufficiently strong G (or Λ) \rightarrow SSB

- **Pseudoscalar (γ_5) and scalar (1) channels:**
Massless pion and massive sigma



Bosonize the NJL \rightarrow Sigma model

$$L_{NJL} = \bar{q}i\partial q + G \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau} q)^2 \right]$$

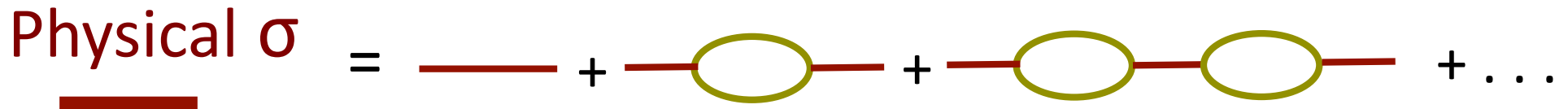
\Rightarrow

$$\mathcal{L}_\sigma = \frac{1}{2} \left((\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2 \right) - \frac{\mu^2}{2} (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + \dots$$



Quark intrinsic/bare/static

In the σ model



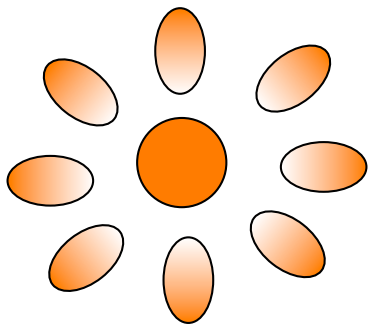
Composite, molecule/dressed/dynamical

Strong interaction
in comparison with EM force

Lagrangian

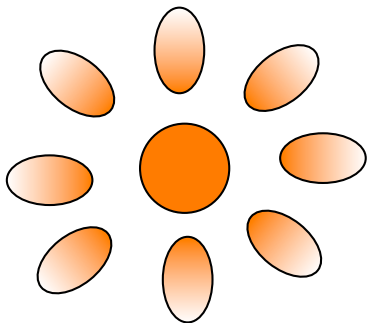
$$U(1): L_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} (i\partial - eA) \psi$$

$$SU(3): L_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} (i\partial - eA^a \lambda^a) \psi$$



Screening
Chargeless photons

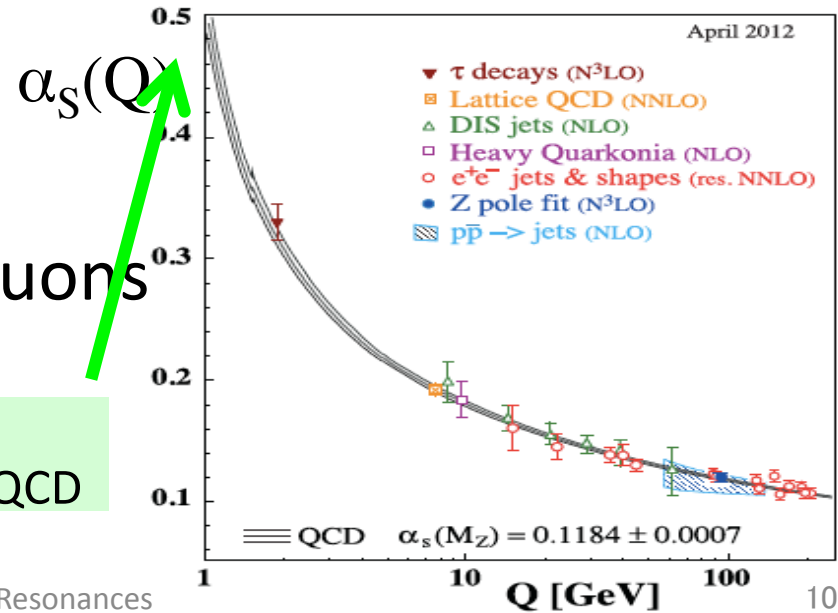
$$\alpha = \frac{1}{137} \sim const.$$



Anti-screening
(color) charged gluons

α_s diverges at Λ_{QCD}

PDG 2012



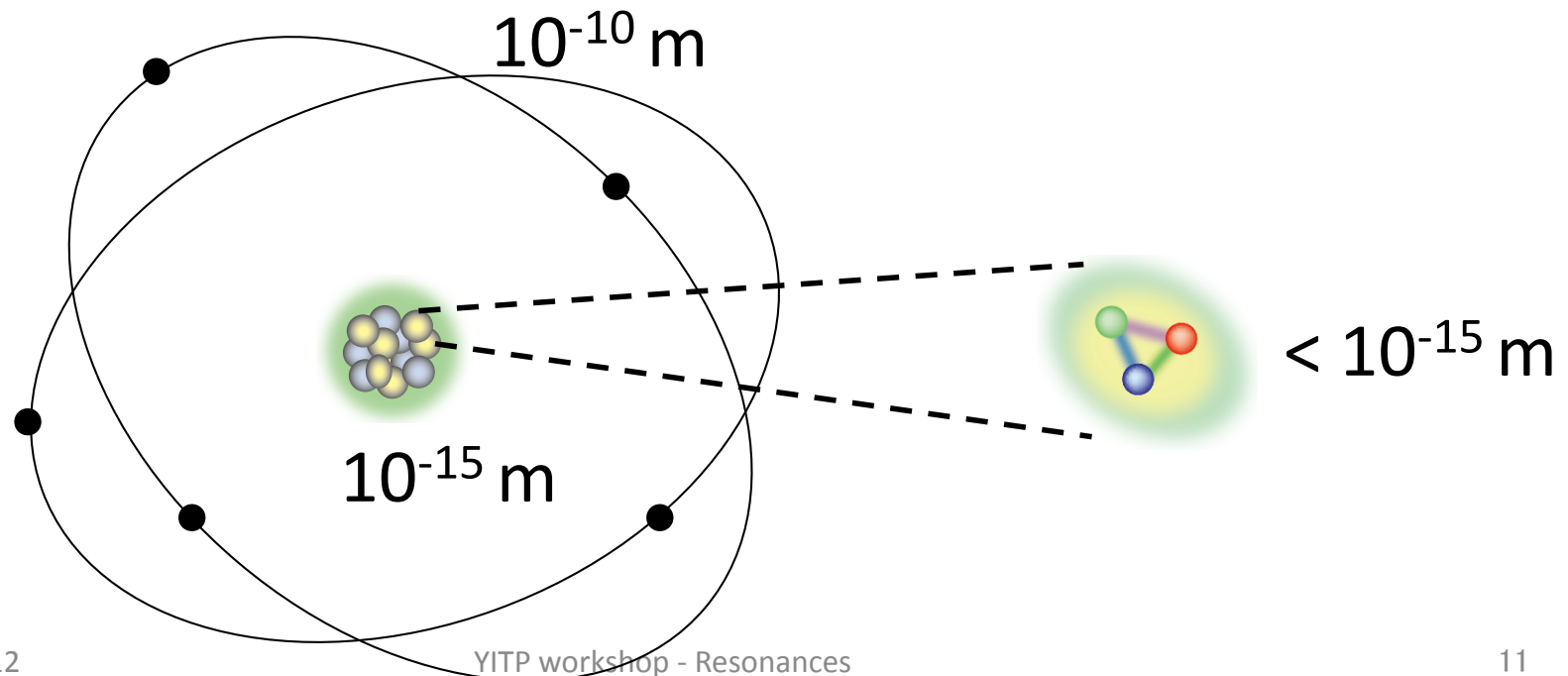
Scale

Atoms, molecules

$$\text{Energy} \sim m_e \alpha^2 \sim 20 \text{ eV}, \quad \text{size} \sim \alpha/m_e \sim \text{\AA}$$

Hadrons

$$\text{Energy} \sim \Lambda_{\text{QCD}} \alpha_s^2 \sim 300 \text{ MeV}, \quad \text{size} \sim \alpha_s/\Lambda_{\text{QCD}} \sim \text{fm}$$



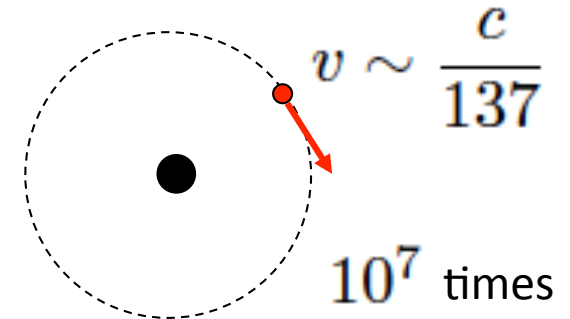
Resonances

Atoms, molecules

$$T \sim \frac{10^{-8}}{c/137} \sim 10^{-16} \text{ 秒}$$



$$\frac{\Delta\omega}{\omega} \sim 10^{-8}$$

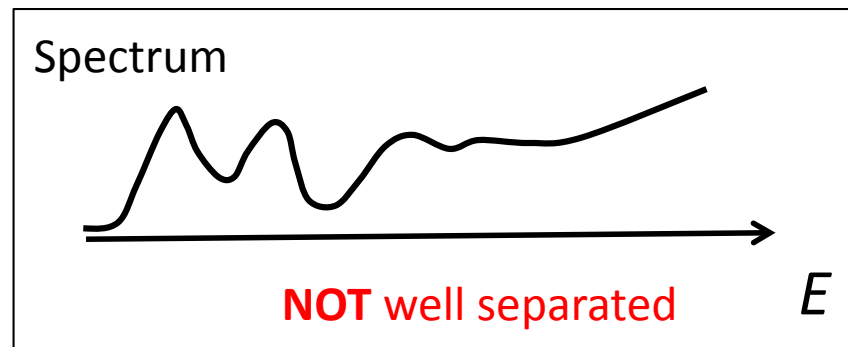
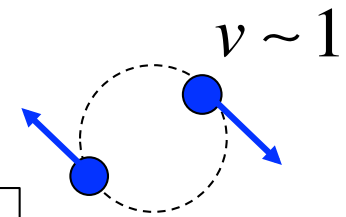


Hadrons

$$T \sim \frac{1 \text{ fm}}{c} \sim 1 \text{ fm}$$



$$\frac{\Delta\omega}{\omega} \sim 1$$



Only
Several times

From **quarks** to **hadrons**

$$L_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} \left(i\partial - eA^a \lambda^a - m \right) \psi$$



Color: 1965

ψ^a
 f



6 flavors: 1973

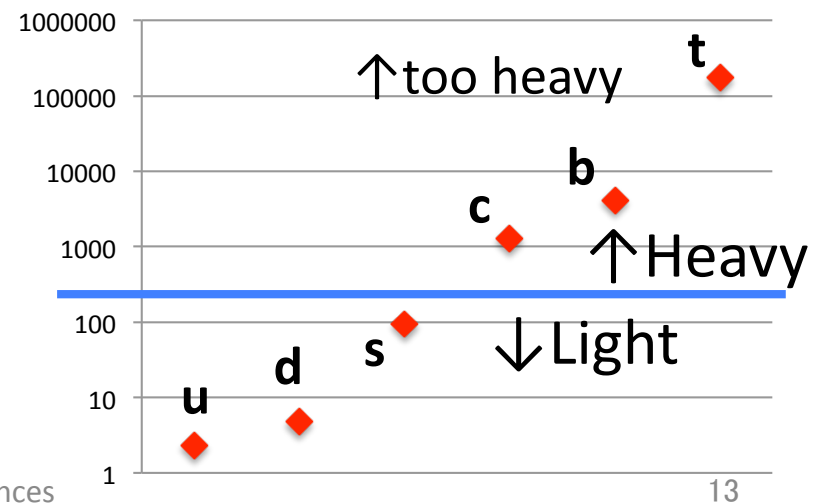
SU(3) with gluons

→

Color confinement, chiral SSB

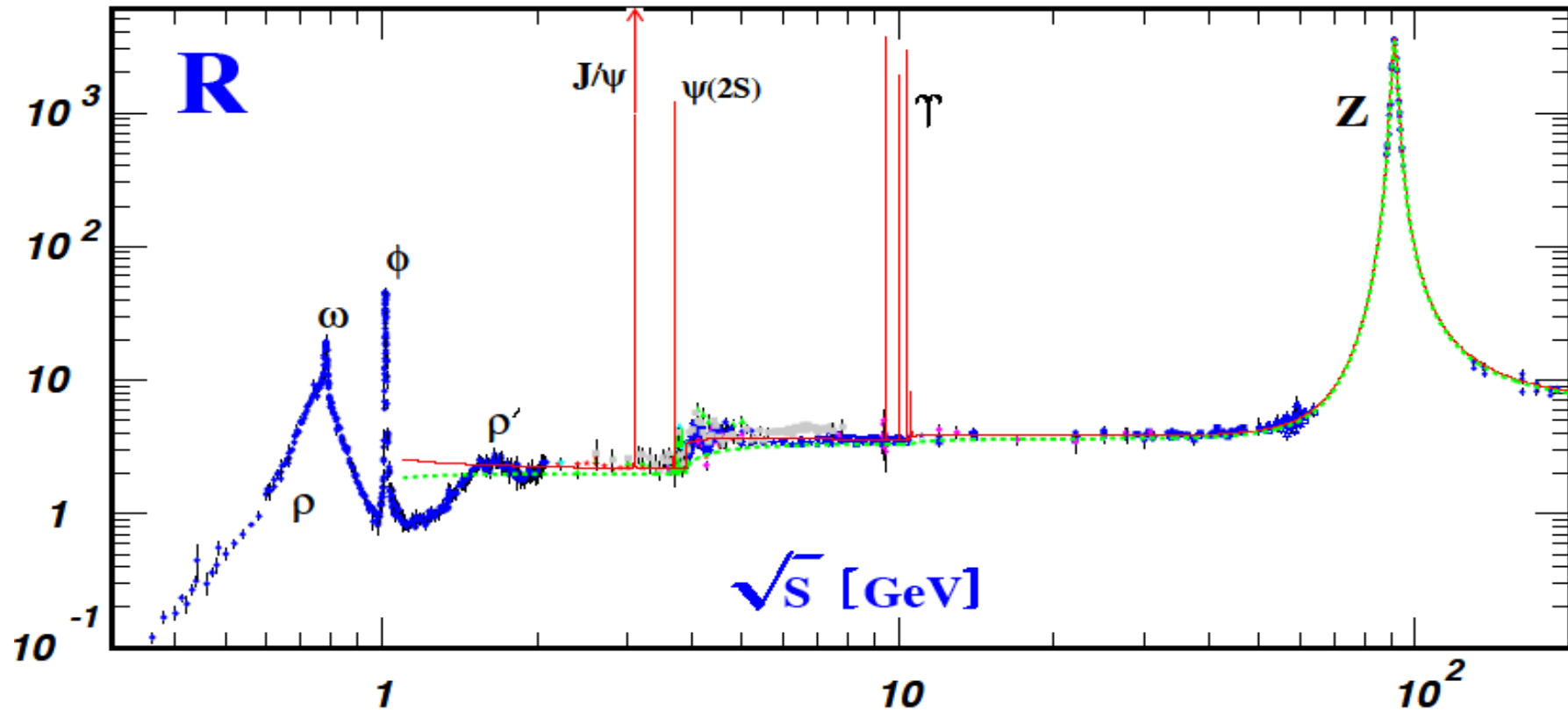
→

Mass generation of pn and massless NG pion (**force**)



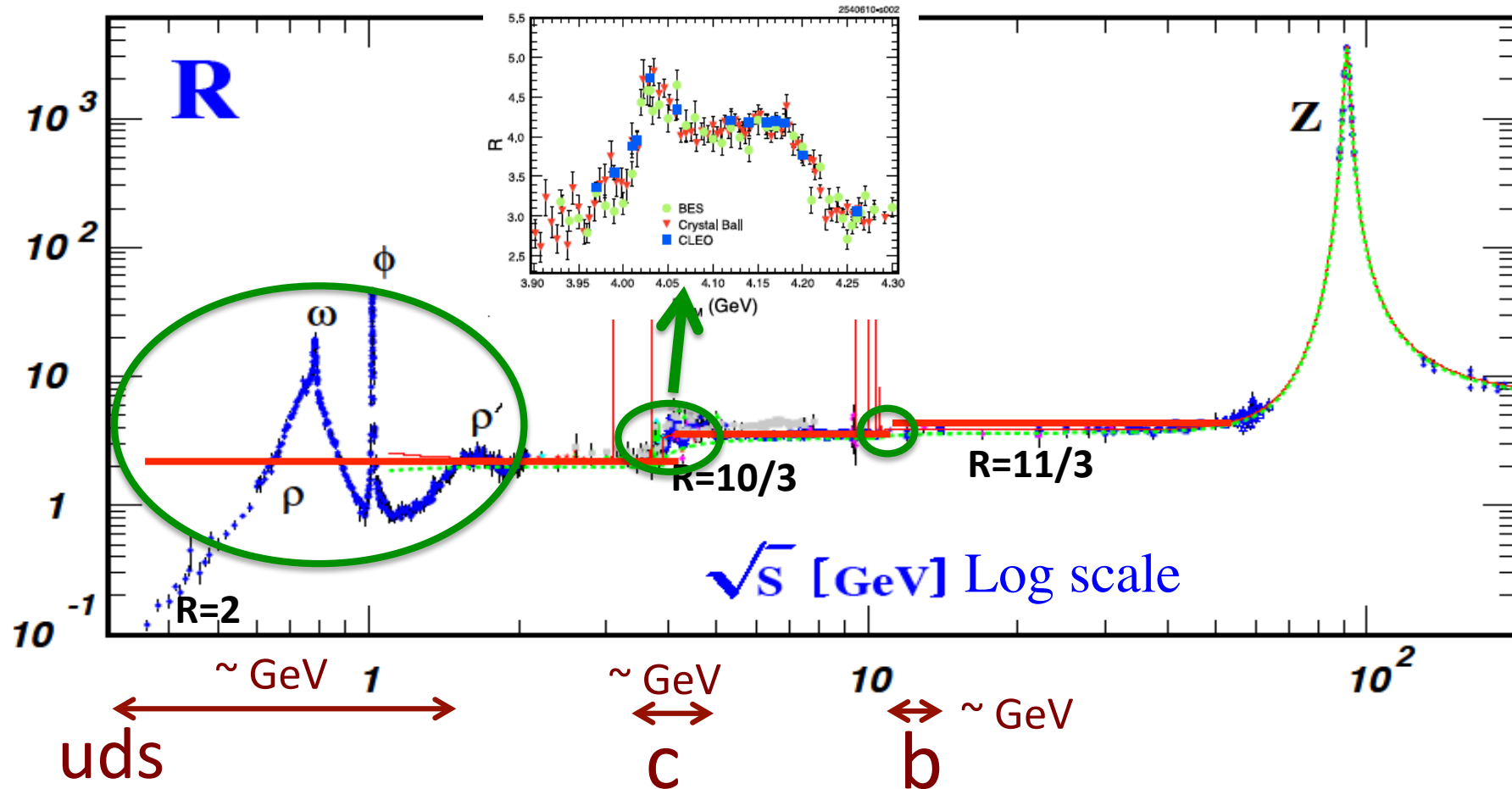
Creating quarks/hadrons (resonances)

$$R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}, s) / \sigma(e^+e^- \rightarrow \mu^+\mu^-, s).$$



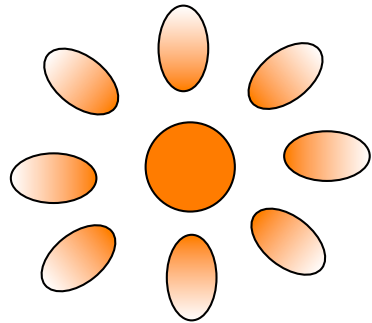
Creating quarks/hadrons (resonances)

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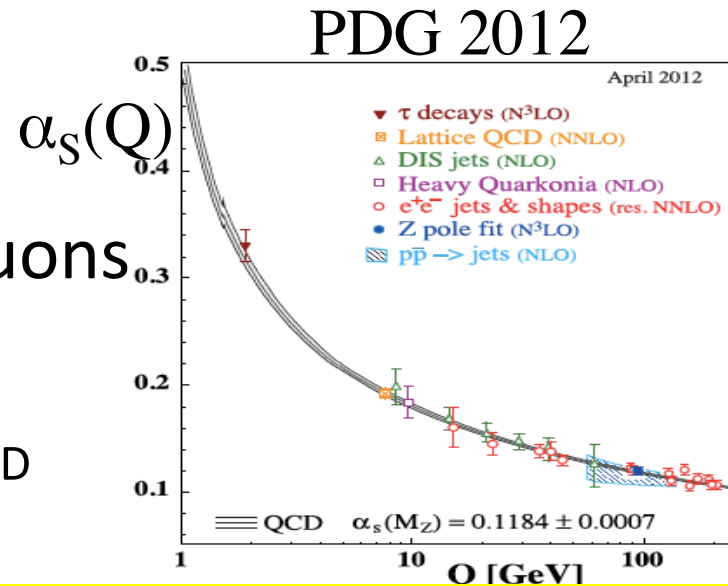
Resonances are **around the threshold** of quark creation

Light quark dynamics



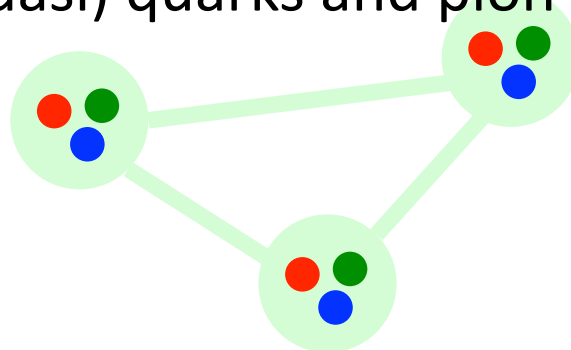
Anti-screening
(color) charged gluons

α_s diverges at Λ_{QCD}



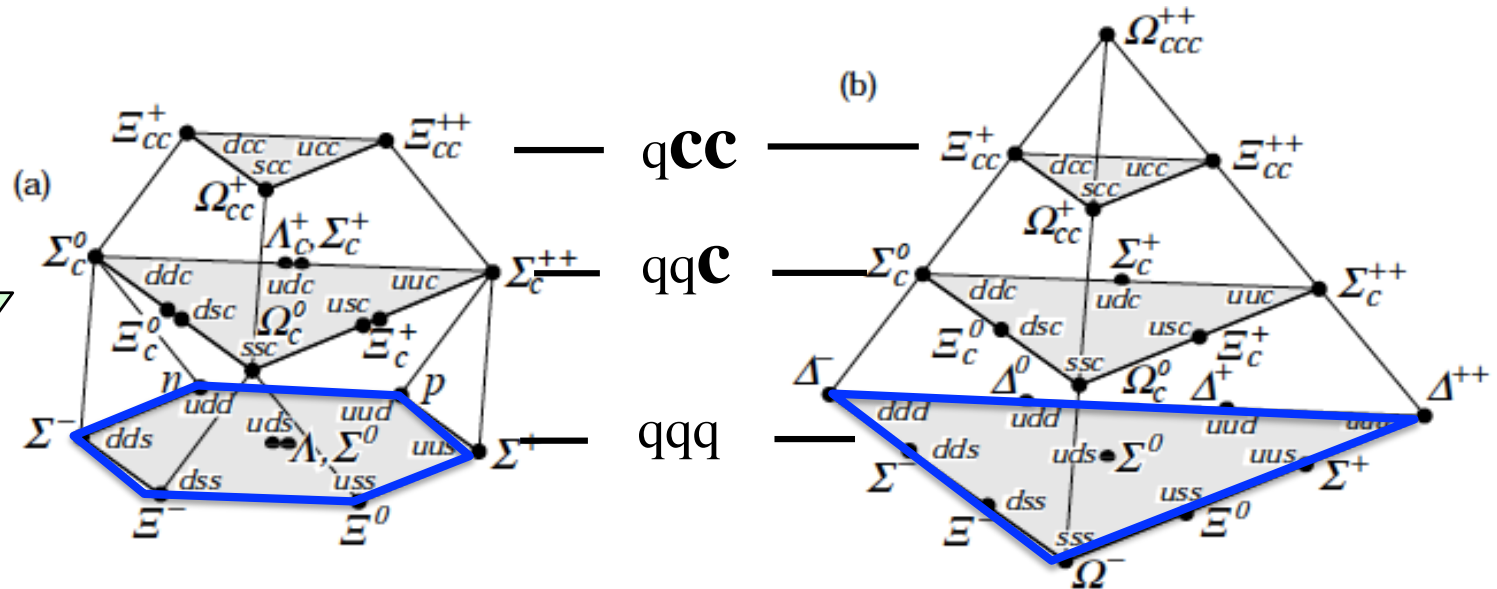
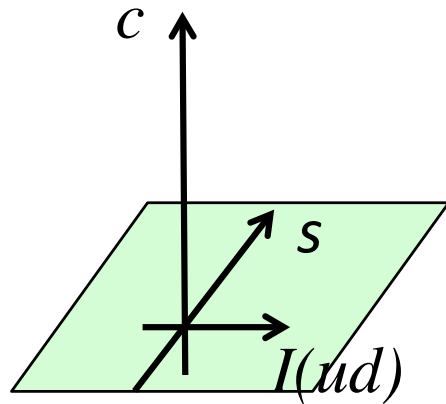
This generates the mass of ~ 300 MeV for light uds quarks

- Instability of the vacuum with finite $\langle q_L^\dagger q_R \rangle \rightarrow$ Dirac mass
 - This strong force acts also in 0^- channel for the massless pion
- \rightarrow Hadrons with (quasi) quarks and pion driven long range force

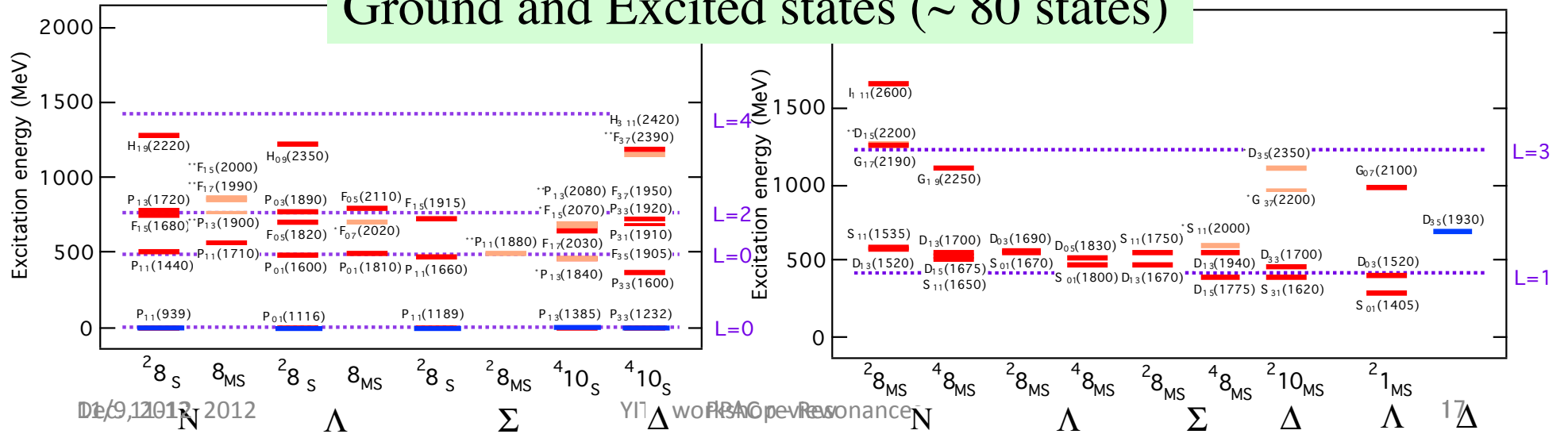


Hadrons (baryons)

Ground state



Ground and Excited states (~ 80 states)



Hadrons are composite

Many resonant states

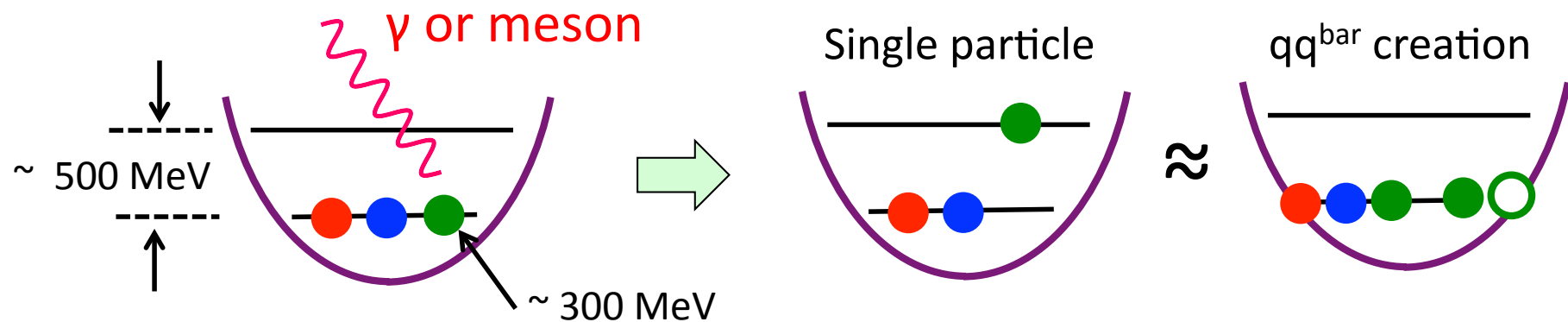
Particle data book (PDG)

BARYONS				MESONS				CHARMED, STRANGE		c \bar{c}													
Σ^+				LIGHT UNFLAVORED (S = C = B = 0)				STRANGE (S = ± 1 , C = B = 0)		CHARMED, STRANGE (C = S = ± 1)		c \bar{c}											
P_{11}				$F(J^{PC})$				$I(J^{PC})$		$I(J^{PC})$		$F(J^{PC})$											
p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****	Λ_c^+	****	π^\pm	1 $^-(0^-)$	$\pi_2(1670)$	1 $^-(2^-)$	K^\pm	1/2(0 $^-$)	D_s^\pm	0(0 $^-$)	$\eta_c(1S)$	0 $^+(0^-)$
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	Σ^0	P_{11}	****	Ξ^-	P_{11}	****	Λ_c^0	****	π^0	1 $^-(0^-)$	$\phi(1680)$	0 $^-(1^-)$	K^0	1/2(0 $^-$)	D_s^0	0(?)	$J/\psi(1S)$	0 $^-(0^-)$
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****	$\Lambda_c(2625)^+$	***	η	0 $^+(0^-)$	$\rho_3(1690)$	1 $^+(3^-)$	K_S^0	1/2(0 $^-$)	$D_{s1}^0(2317)^\pm$	0(0 $^+$)	$\chi_{c0}(1P)$	0 $^+(0^+)$
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$	*	*	$\Lambda_c(2765)^+$	*	η'	0 $^+(0^-)$	$\rho(1700)$	1 $^+(1^-)$	K_L^0	1/2(0 $^-$)	$D_{s1}(2460)^\pm$	0(1 $^+$)	$\chi_{c1}(1P)$	0 $^+(1^+)$
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Sigma(1480)$	*	*	$\Xi(1690)$	***	***	$\Lambda_c(2880)^+$	***	$\eta(600)$	0 $^+(0^+)$	$\rho(1700)$	1 $^+(1^-)$	$K_S^0(800)$	1/2(0 $^-$)	$D_{s1}(2536)^\pm$	0(1 $^+$)	$\chi_{c2}(1P)$	0 $^+(1^+)$
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Sigma(1560)$	**	**	$\Xi(1820)$	D_{13}	***	$\Lambda_c(2940)^+$	***	$\rho(770)$	1 $^+(1^-)$	$a_2(1700)$	1 $^-(2^+)$	$K^*(892)$	1/2(1 $^-$)	$D_{s2}(2573)^\pm$	0(?)	$\eta_c(2S)$	0 $^+(0^-)$
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Sigma(1580)$	**	**	$\Xi(1950)$	***	***	$\Lambda_c(2940)^+$	***	$\omega(782)$	0 $^-(1^-)$	$\omega(1710)$	0 $^+(0^+)$	$K_1^*(1270)$	1/2(1 $^+$)	$D_{s1}(2700)^\pm$	0(1 $^+$)	$\psi(2S)$	0 $^-(0^-)$
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$	***	***	$\Sigma_c(2520)$	***	$\eta'(958)$	0 $^+(0^-)$	$\eta(1760)$	0 $^+(0^+)$	$K_1^*(1400)$	1/2(1 $^+$)	$\psi(2S)$	0 $^-(0^-)$		
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$	*	*	$\Sigma_c(2800)$	***	$\phi(980)$	0 $^+(0^+)$	$\pi(1800)$	1 $^-(0^-)$	$K_1^*(1410)$	1/2(1 $^-$)	$\psi(3770)$	0 $^-(1^-)$		
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$	**	**	Ξ_c^+	***	$a_0(980)$	1 $^-(0^+)$	$f_2(1810)$	0 $^+(2^+)$	$K^*(1430)$	1/2(0 $^+$)	$X(3872)$	0 $^+(2^+)$		
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Sigma(1690)$	**	**	$\Xi(2370)$	**	**	Ξ_c^0	***	$\phi(1020)$	0 $^-(1^-)$	$X(1835)$??(?)	$K_1^*(1430)$	1/2(0 $^+$)	$\chi_{c2}(2P)$	0 $^+(2^+)$		
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Sigma(1750)$	S_{11}	***	$\Xi(2500)$	*	*	Ξ_c^+	***	$h_1(1170)$	0 $^-(1^+)$	$\pi(1850)$	0 $^-(3^-)$	$K_2^*(1430)$	1/2(2 $^+$)	$X(3940)$??(?)		
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Sigma(1770)$	P_{11}	*	Ω^-	****	****	Ξ_c^0	***	$h_2(1235)$	1 $^+(1^+)$	$\eta_2(1870)$	0 $^+(2^+)$	$K(1460)$	1/2(0 $^-$)	$X(3945)$??(?)		
$N(2000)$	F_{15}	**	$\Delta(2100)$	S_{31}	*	$\Sigma(1775)$	D_{15}	****	$\Omega(2250)^-$	***	***	Ξ_c^0	***	$a_1(1260)$	1 $^-(1^+)$	$\pi_2(1880)$	1 $^-(2^+)$	$K_2(1580)$	1/2(2 $^-$)	B^\pm	1/2(0 $^-$)		
$N(2080)$	D_{13}	**	$\Delta(2200)$	G_{37}	*	$\Sigma(1840)$	P_{13}	*	$\Omega(2380)^-$	**	**	Ξ_c^0	***	$f_2(1270)$	0 $^+(2^+)$	$\rho(1900)$	1 $^+(1^-)$	$K_2(1630)$	1/2(2 $^-$)	B^0	1/2(0 $^-$)		
$N(2090)$	S_{11}	*	$\Delta(2300)$	H_{39}	**	$\Sigma(1880)$	P_{11}	**	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$b_1(1235)$	1 $^+(1^+)$	$\eta_2(1870)$	0 $^+(2^+)$	$K_1(1650)$	1/2(1 $^+$)	B^0	1/2(0 $^-$)		
$N(2100)$	P_{11}	*	$\Delta(2350)$	D_{35}	**	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$a_2(1260)$	1 $^-(1^+)$	$f_2(1910)$	0 $^+(2^+)$	$K_1(1650)$	1/2(1 $^+$)	B^0	1/2(0 $^-$)		
$N(2190)$	G_{17}	****	$\Delta(2390)$	F_{37}	*	$\Sigma(1940)$	D_{13}	***	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\eta(1300)$	1 $^-(0^-)$	$\rho_3(1990)$	1 $^+(3^-)$	$K_1^*(1680)$	1/2(1 $^-$)	B^*	1/2(1 $^-$)		
$N(2200)$	D_{15}	**	$\Delta(2400)$	G_{39}	**	$\Sigma(2000)$	S_{11}	*	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$a_2(1320)$	1 $^-(2^+)$	$f_2(2010)$	0 $^+(2^+)$	$K_2^*(1770)$	1/2(2 $^-$)	B^*	1/2(1 $^-$)		
$N(2220)$	H_{19}	****	$\Delta(2420)$	$H_{3,11}$	****	$\Sigma(2030)$	F_{17}	****	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1370)$	0 $^+(0^+)$	$f_2(2020)$	0 $^+(0^+)$	$K_3^*(1780)$	1/2(3 $^-$)	$B_1^*(5732)$	1/2(1 $^+$)		
$N(2250)$	G_{19}	****	$\Delta(2750)$	$h_{3,11}$	****	$\Sigma(2070)$	F_{15}	*	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$h_1(1380)$?(1 $^+$)	$a_4(2040)$	1 $^-(4^+)$	$K_2(1820)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
$N(2600)$	$h_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**	$\Sigma(2080)$	P_{13}	*	$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\pi_3(1400)$	1 $^-(1^+)$	$K(2050)$	0 $^+(4^+)$	$K_1^*(1950)$	1/2(0 $^+$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
$N(2700)$	$K_{1,13}$	**				$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\eta(1405)$	0 $^+(0^+)$	$\pi_2(2100)$	1 $^-(2^+)$	$K_2^*(1980)$	1/2(2 $^+$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1420)$	0 $^+(1^+)$	$f_2(2100)$	0 $^+(0^+)$	$K_1^*(2045)$	1/2(2 $^+$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\omega(1420)$	0 $^-(1^-)$	$f_2(2150)$	0 $^+(2^+)$	$K_2(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1430)$	0 $^+(2^+)$	$\rho(2150)$	1 $^+(1^-)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$a_0(1450)$	1 $^-(0^+)$	$\phi(2170)$	0 $^-(1^-)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\rho(1450)$	1 $^+(1^-)$	$f_2(2200)$	0 $^+(0^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\eta(1475)$	0 $^+(0^+)$	$f_2(2220)$	0 $^+(2^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1500)$	0 $^+(0^+)$	$\eta(2225)$	0 $^+(0^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1510)$	0 $^+(1^+)$	$\rho_3(2250)$	1 $^+(3^-)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2^*(1525)$	0 $^+(2^+)$	$f_2(2300)$	0 $^+(2^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1565)$	0 $^+(2^+)$	$f_4(2300)$	0 $^+(4^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\rho(1570)$	1 $^+(1^-)$	$f_2(2330)$	0 $^+(0^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$h_1(1595)$	0 $^-(1^+)$	$f_2(2340)$	0 $^+(2^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\pi_3(1600)$	1 $^-(1^+)$	$\rho_3(2350)$	1 $^+(5^-)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$a_2(1640)$	1 $^-(1^+)$	$a_0(2450)$	1 $^-(6^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$f_2(1640)$	0 $^+(2^+)$	$f_2(2510)$	0 $^+(6^+)$	$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\eta_2(1645)$	0 $^+(2^-)$			$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\omega(1650)$	0 $^-(1^-)$			$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***	$\omega_3(1670)$	0 $^-(3^-)$			$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***					$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**	Ξ_c^0	***					$K_1(2250)$	1/2(2 $^-$)	$B_2^*(5747)^0$	1/2(2 $^+$)		
						$\Sigma(2100)$			$\Omega(2470)^-$	**	**												

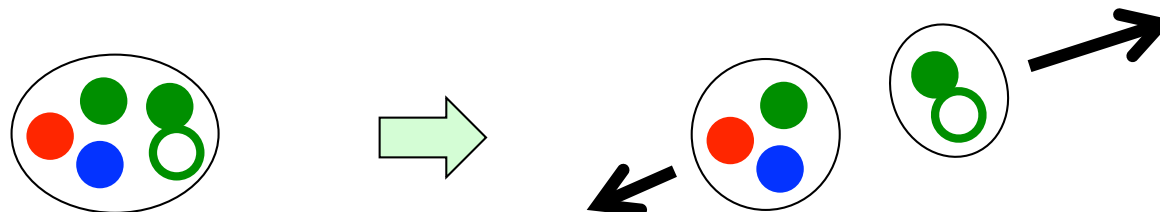
Excited states

Quark model

Bare (current) quarks \rightarrow Potential (confinement \sim HO) model

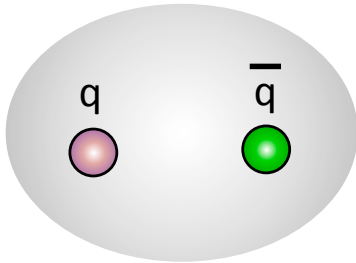


These couple to decay channel(s) = Feschbach resonance



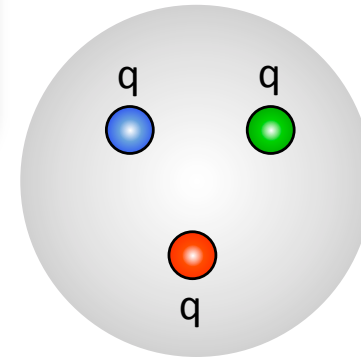
Possible hadrons (Kamano)

Meson

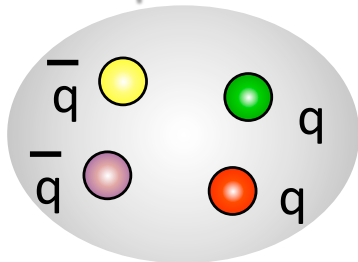


How we determine the difference

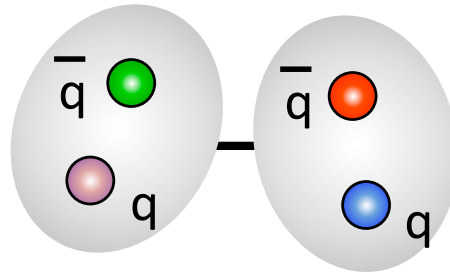
Baryons



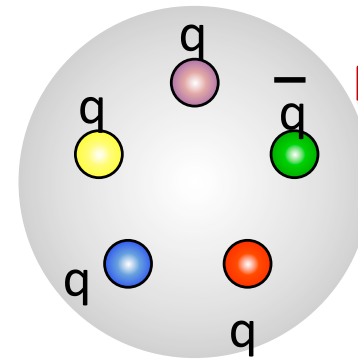
Multiquarks



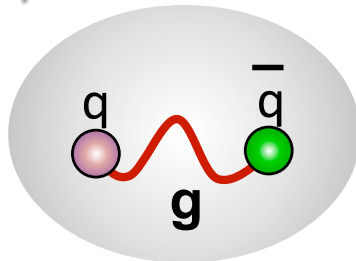
Hadronic molecule



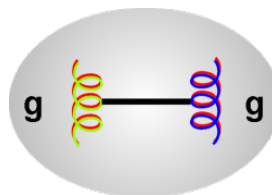
Multiquarks



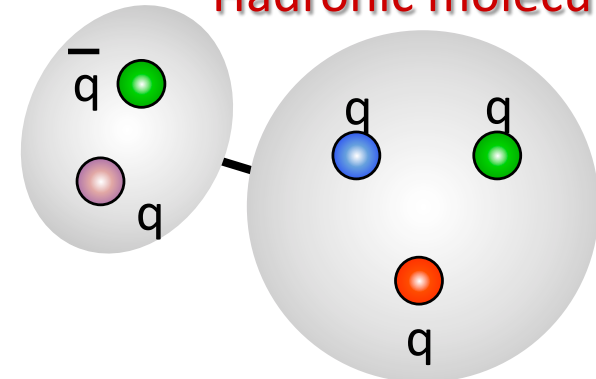
Hybrid



Glueball



Hadronic molecules



Scattering experiments

Meson and photon induced reactions

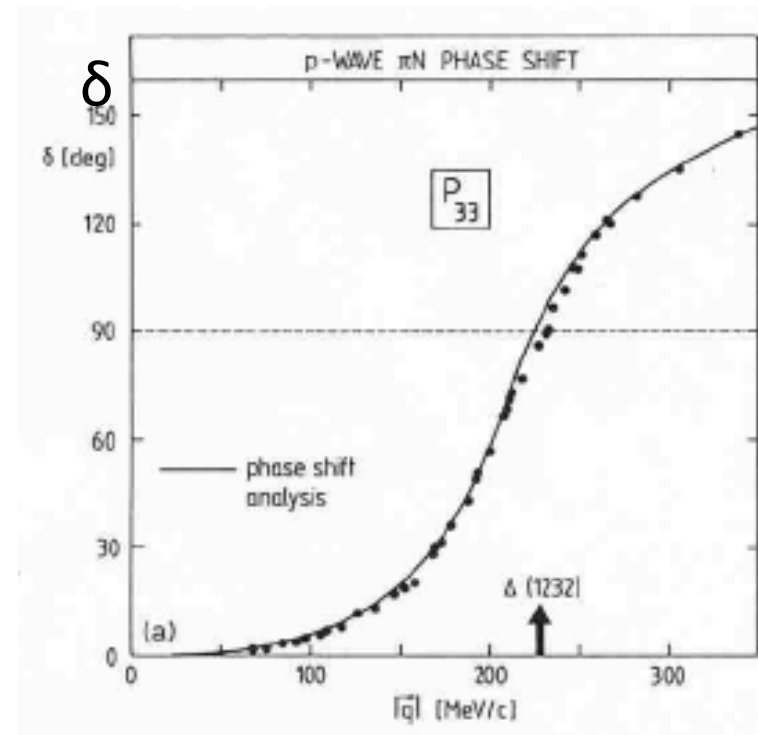
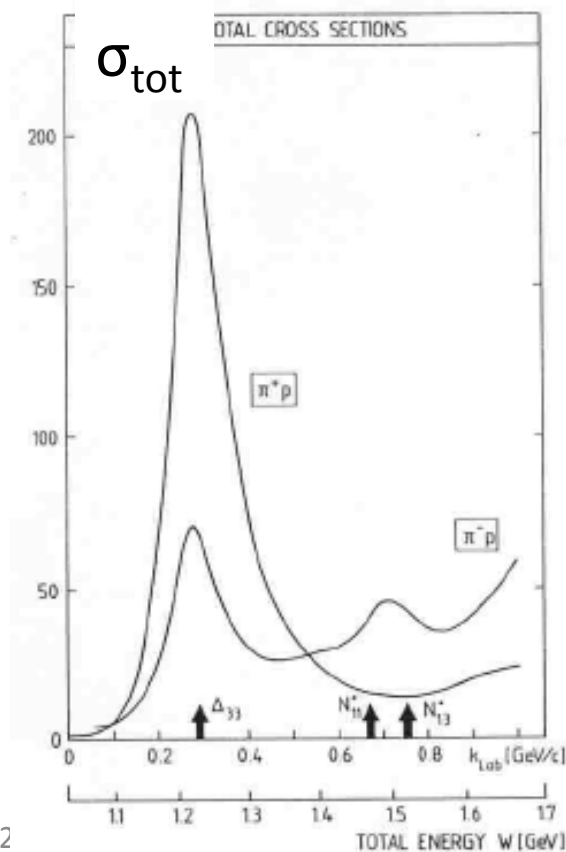
Create hadron resonances and measure their decays

Scattering experiments

Meson and photon induced reactions

Create hadron resonances and measure their decays

$\Delta(1232)$ in πN scattering, first example of hadron resonance



Super Photon ring-8 GeV SPring-8

- Third-generation synchrotron radiation facility
- Circumference: 1436 m
- 8 GeV
- 100 mA
- 62 beamlines

概観

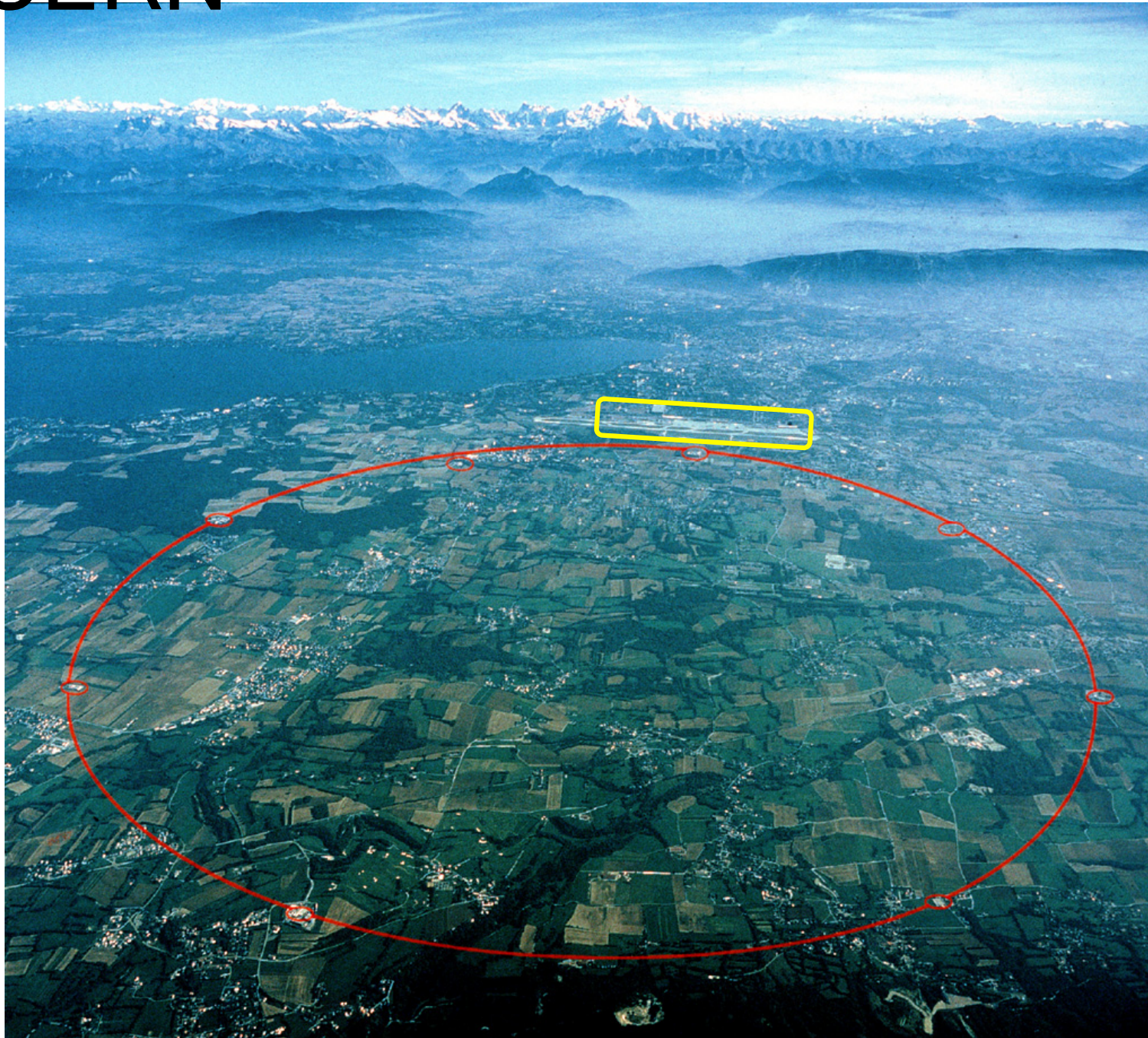
Heavy quark mesons from Belle KEK



e^+e^- collider at 11.5 GeV (CM)



CERN The Large Hadron Collider (LHC)



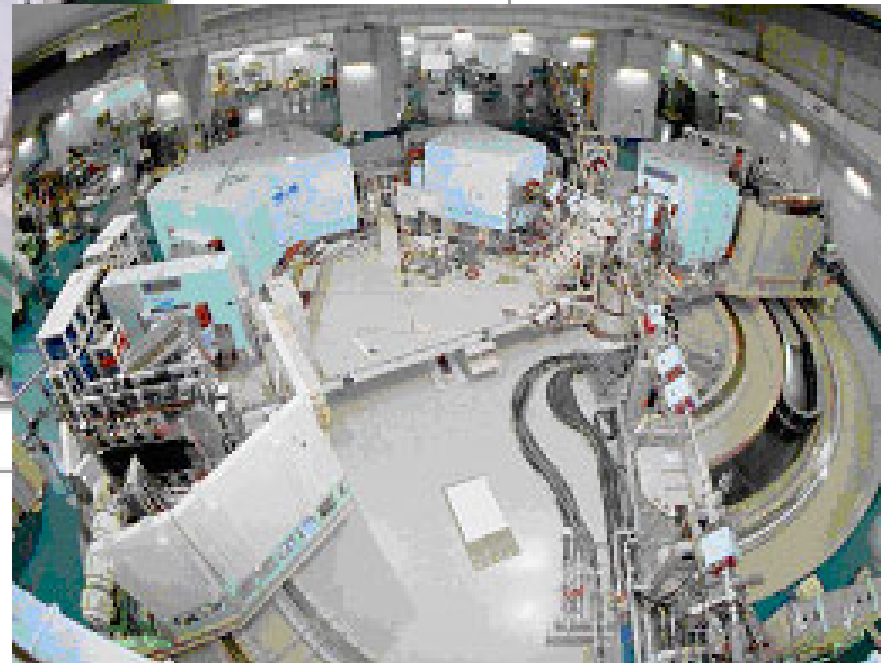
実験装置

サイクロトロン@RCNP



世界中の物理学者に開かれた
原子核物理学の共同利用研究所

陽子400 MeV



2. T-matrices and poles

Schrodinger eq.

$$(H_0 + V) \psi = E \psi \quad \Leftrightarrow$$

Lippmann-Schwinger eq.

$$T(E) = V + V G_0(E) T(E)$$

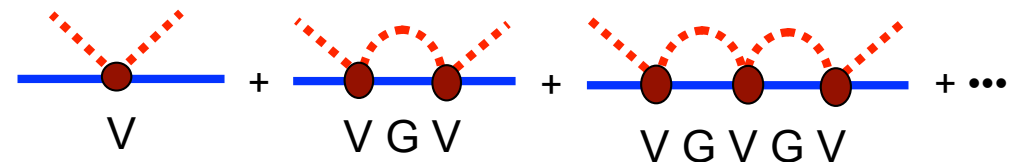
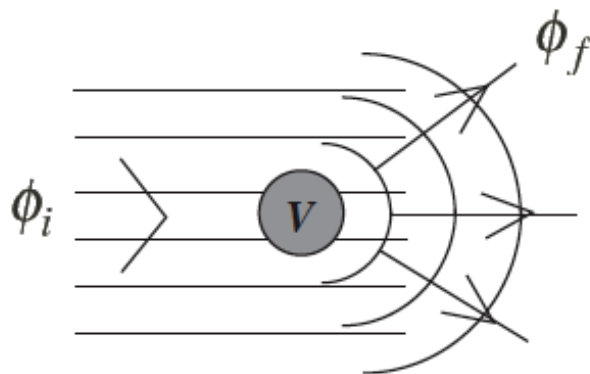
$$G_0(E) = \frac{1}{E - H_0 + i\epsilon}$$

Solution

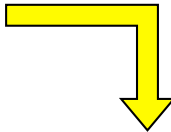
$$\psi_i = \phi_i + \frac{1}{E - H_0 + i\epsilon} V \psi_i$$

$$T = \frac{1}{1 - V G_0} V$$

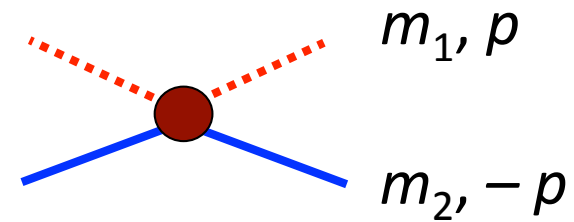
$$= V + V G_0 V + V G_0 V G_0 V + \dots$$



Eigenvalue equation and pole(s) of the T-matrix

$$(H_0 + V)\psi = E\psi$$


$$\det(E - H_0 - V) = \det(1 - VG_0(E)) \det(E - H_0) = 0$$



$$\det(1 - G_0(E)V) = 0$$

Determines pole(s) of the T-matrix

$$T = \frac{1}{1 - VG_0} V$$

Separable interaction and G-function

$$\langle \vec{q} | V | \vec{p} \rangle = v g(\vec{q}) g(\vec{p})$$

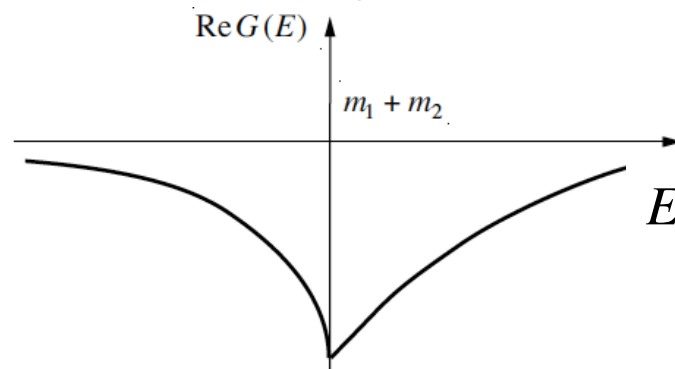
$$G_0(E) = \sum_{\vec{q}} \frac{1}{E - \sqrt{\vec{q}^2 + m_1^2} - \sqrt{\vec{q}^2 + m_2^2} + i\epsilon}$$

$$\sim i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_T}{(P - q)^2 - M_T^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon}$$

Bound states

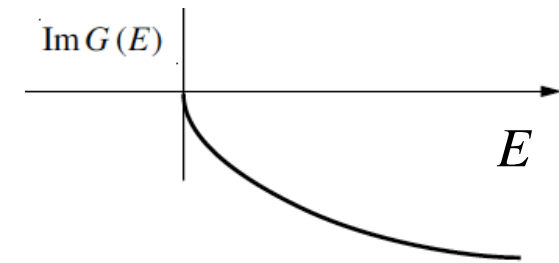
$$G(E) = \sum_{\vec{q}}^{\Lambda} \left(\frac{P}{E - \sqrt{\vec{q}^2 + m_1^2} - \sqrt{\vec{q}^2 + m_2^2}} - i\pi\delta \left(E - \sqrt{\vec{q}^2 + m_1^2} - \sqrt{\vec{q}^2 + m_2^2} \right) \right)$$

Real part



Imaginary

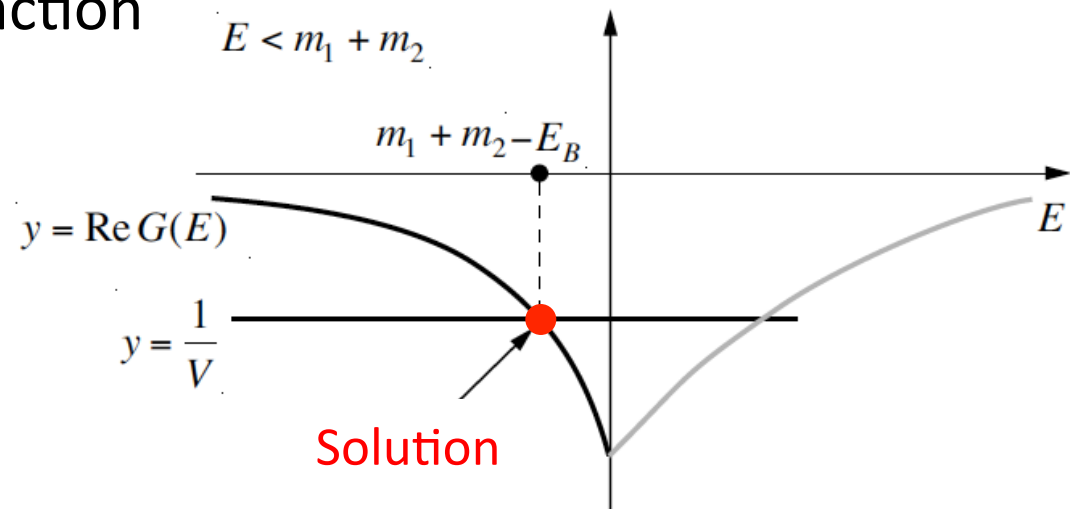
only when $E > m_1 + m_2$



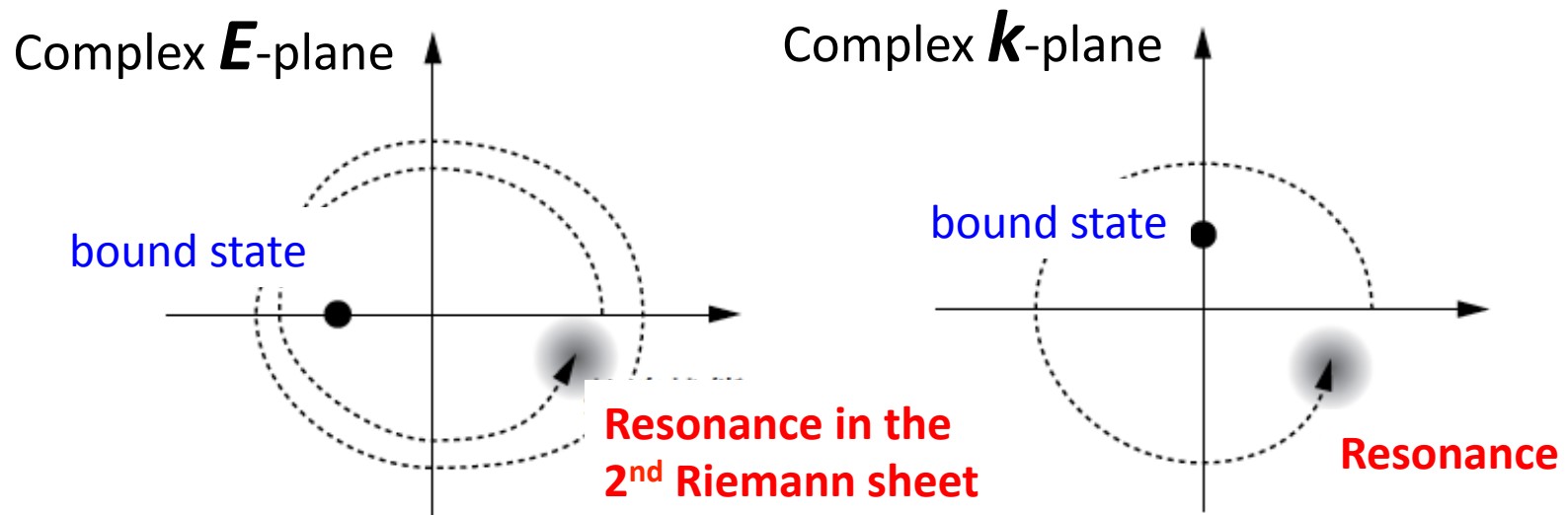
Separable, constant interaction

$$1 - VG(E) = 0$$

$$\Rightarrow G(E) = \frac{1}{V}$$



Resonance

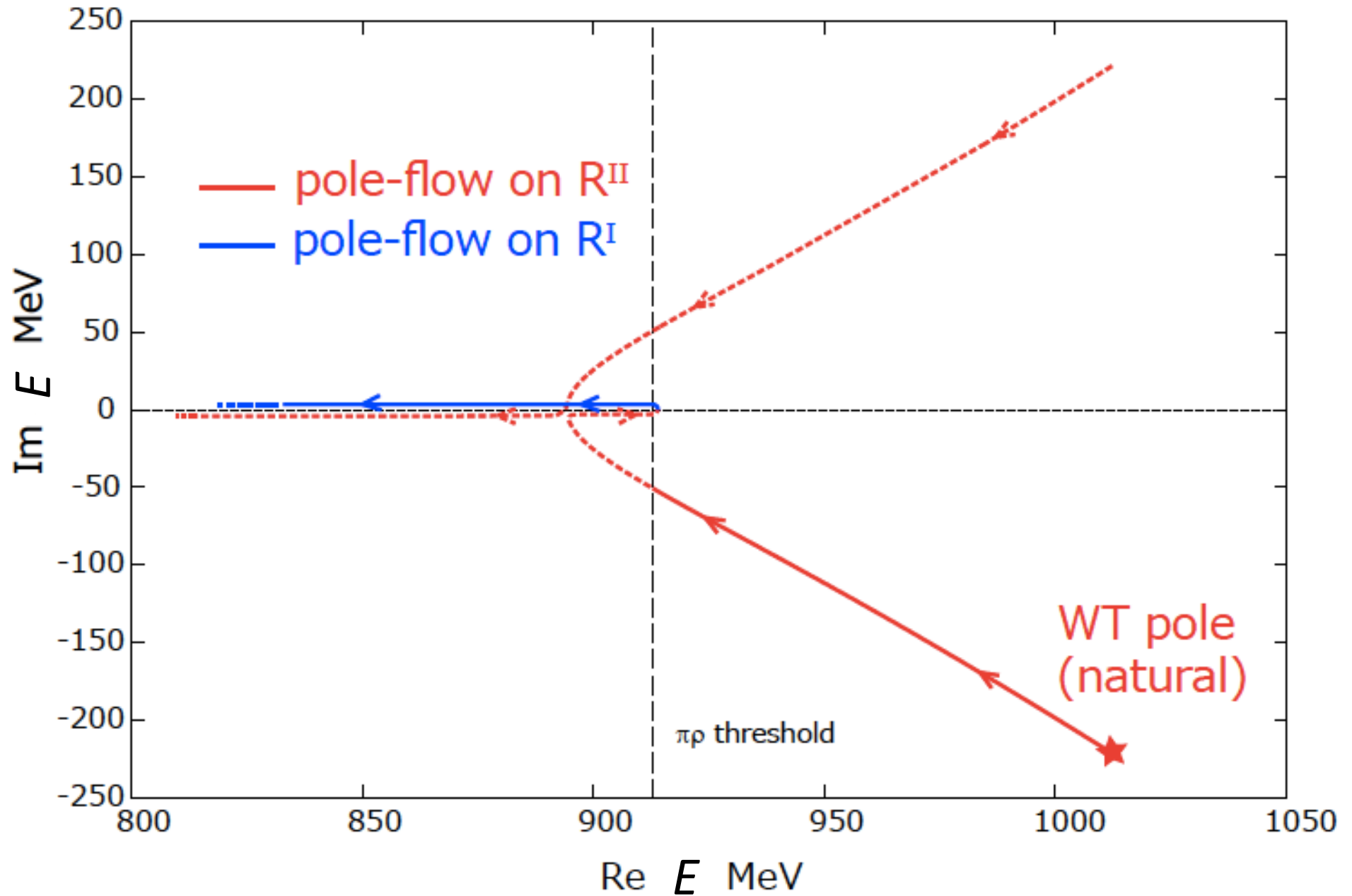


Let $G(k) = -\alpha - i\beta k$ Then $\frac{1}{V} + \alpha + i\beta k = 0$

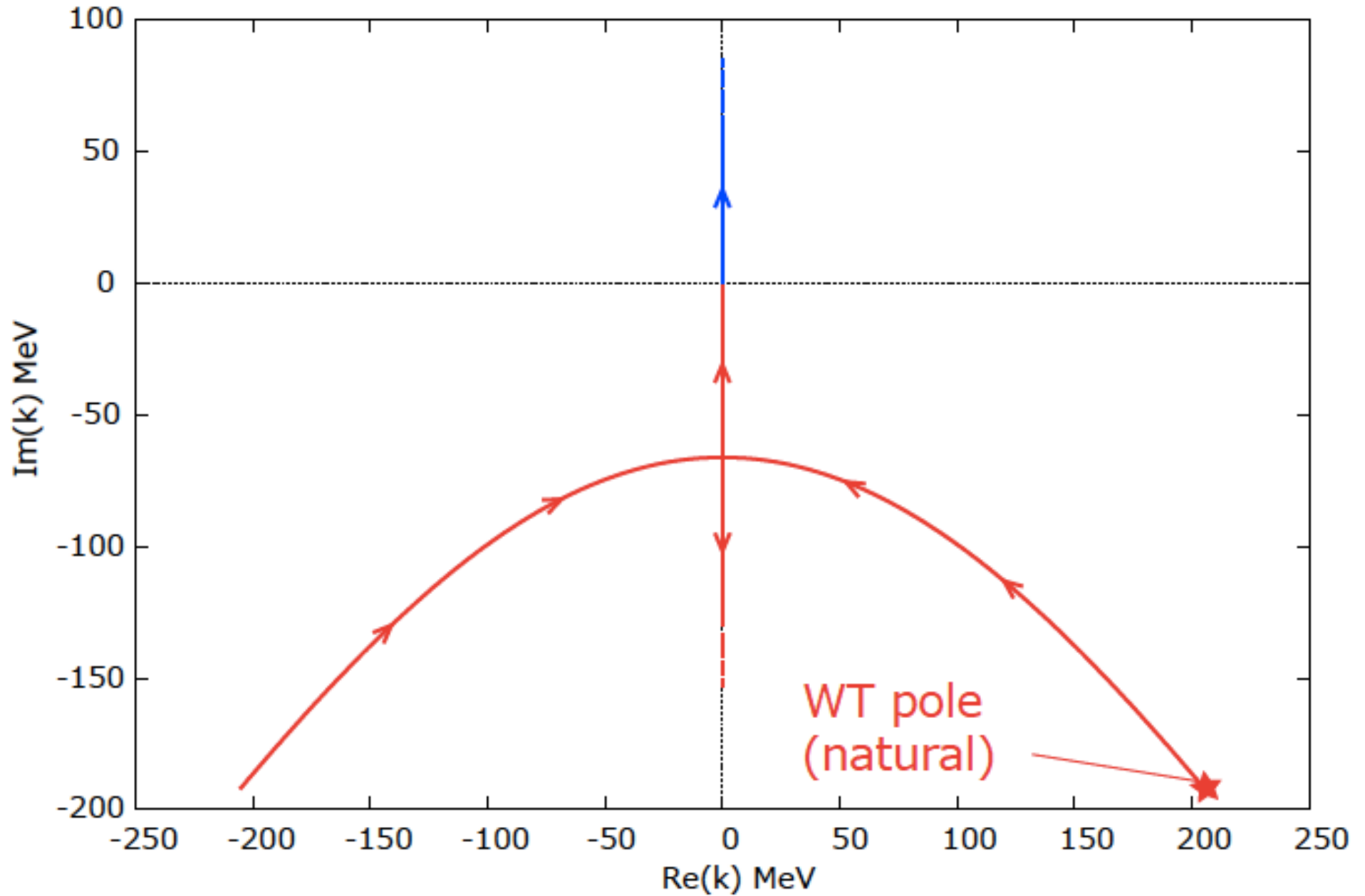
- For constant V , there is no resonance solution
- For energy dep $V = -cE$, **cubic** eq in k allows complex solutions

$$1 - c \frac{k^2}{2\mu} (\alpha + i\beta k) = 0$$

Pole flow as function of c , $\mathbf{V} = -c\mathbf{E}$, made by H. Nagahiro



Pole flow as function of c , $\mathbf{V} = -c\mathbf{E}$, made by H. Nagahiro



Hadron Talks

- **Hiroyuki Kamano**
How parameters of light-flavor baryon resonances are extracted from experimental data
- **Tetsuo Hyodo**
Compositeness of hadrons in field theoretical approach
- **Kanabu Nawa**
Complex 2D Matrix Model and Internal Structure of Resonances
- **Takayasu Sekihara**
Internal structure of the resonant $\Lambda(1405)$ state in chiral dynamics

Key words: Dynamically generated \sim hadronic molecules
and Quark (gluon) originated \sim static (excited but no decay)

Kamano: Hadrons as resonances

Intrinsic (Static) hadron

Actual hadrons = Resonance

Spectrum (Mass)

Real Energy

“pole” mass (Complex)

Physical resonance

$$\frac{1}{E - M_0 - \Sigma(E)}$$

=

$$\frac{1}{E - M_0} + \underbrace{\left[\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \right]}$$

Intrinsic (static ~ bare)

Coupling to reaction channels (Reaction dynamics)

$$E = M_{\text{pole}}$$

$$E - m_{N^*}^0 - \Sigma(E) = 0$$

$$M_0 \rightarrow \underline{M_{\text{pole}}}$$

Complex

Many resonances with many open channels ← Shumamura

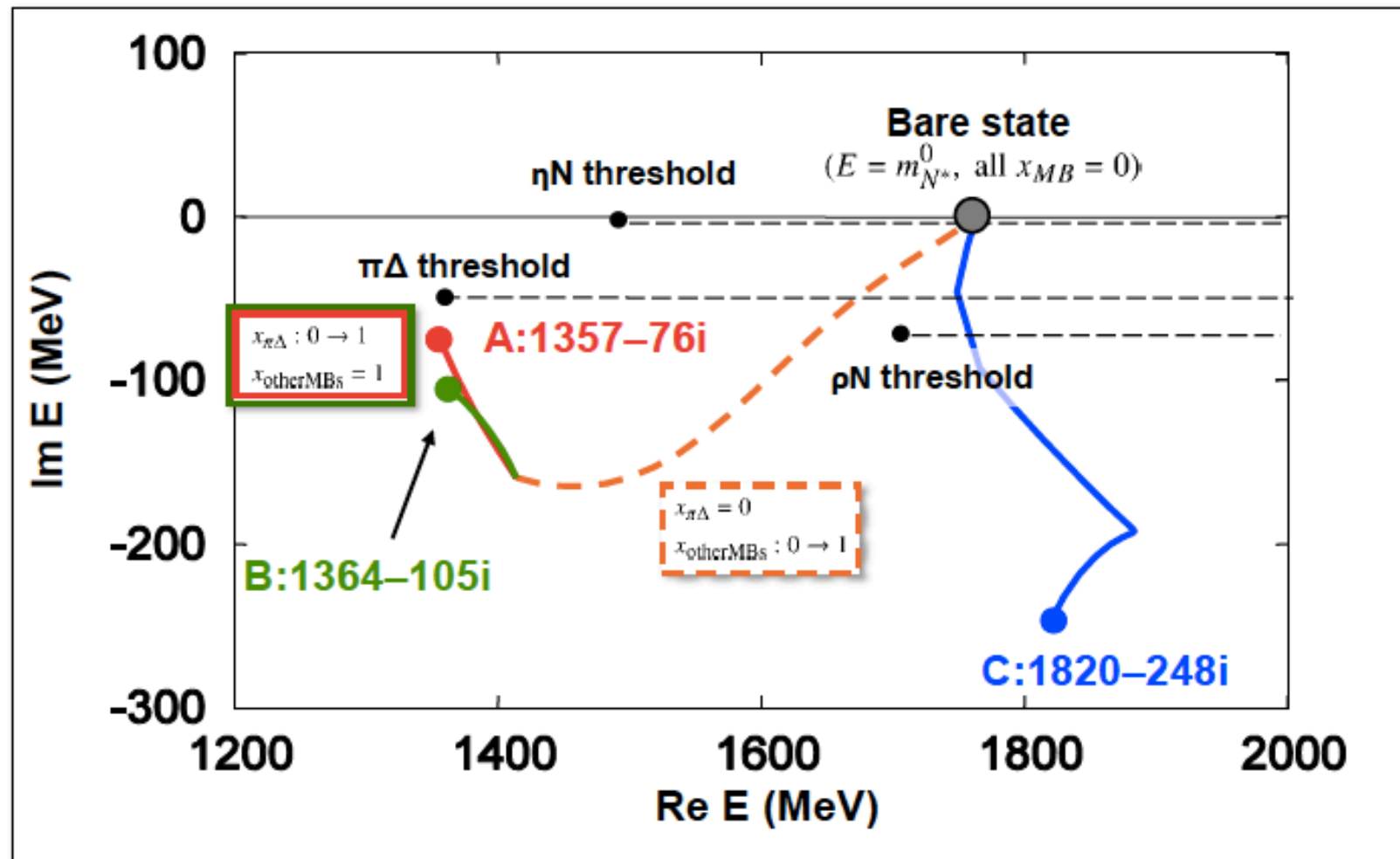
And compare with data to extract M_{pole}

Dynamical origin of P11 resonances

Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama, Sato, PRL104 042302 (2010)

Pole trajectory
of N^* propagator

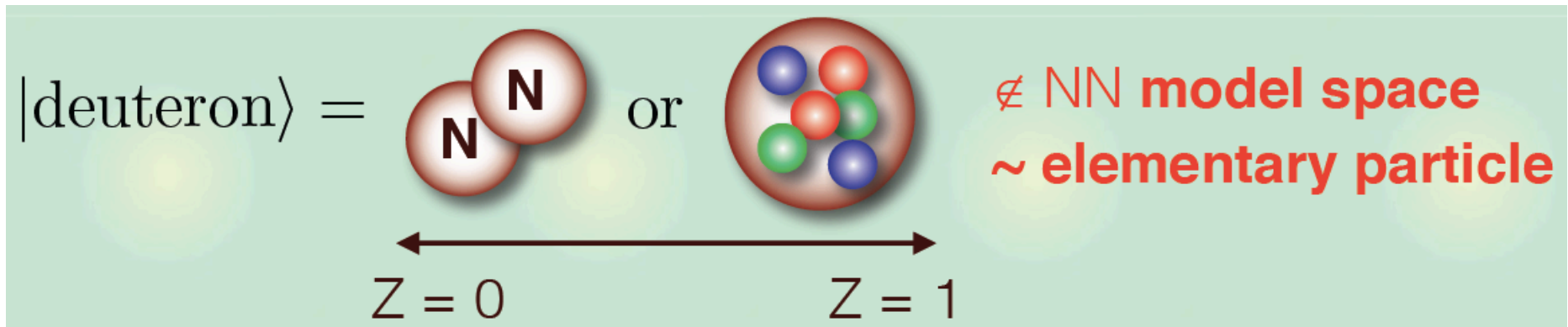
$$\frac{1}{E - m_{N^*}^0 - \sigma(E)} \rightarrow \frac{1}{E - m_{N^*}^0 - \sum_{MB} x_{MB} \sigma_{MB}(E)} \quad x_{MB} : 0 \rightarrow 1$$



Hyodo: Compositeness

Question: S. Weinberg, Phys. Rev. 137, B672 (1965)

How often does the deuteron merge into a six-quark bag?



$$a_s = +5.41 \text{ [fm]}, \quad r_e = +1.75 \text{ [fm]}, \quad R \equiv (2\mu B)^{-1/2} = 4.31 \text{ [fm]}$$

$\Rightarrow Z \lesssim 0.2 \quad \text{--> deuteron is almost composite!}$

Apply this idea to the chiral theory

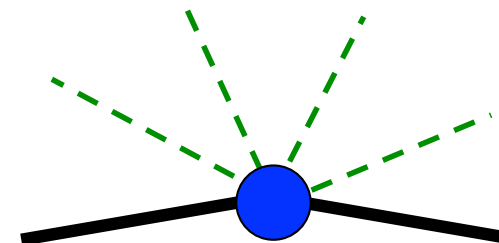
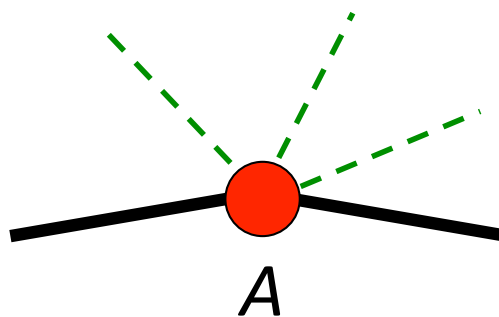
Chiral theory

Chiral symmetry is spontaneously broken

→ Broken symmetry is dynamic (not algebraic)

→ Low energy theorems

$$\frac{g_A}{f_\pi} \mathbf{A}_a^\mu \partial_\mu \phi^a, \quad \frac{1}{f_\pi^2} \mathbf{V}_c^\mu f_{abc} (\partial_\mu \phi^a) \phi^b$$



These are determined by a few parameters

→ Energy dependent force of NB and matter → $V \sim cE$

Sekihara

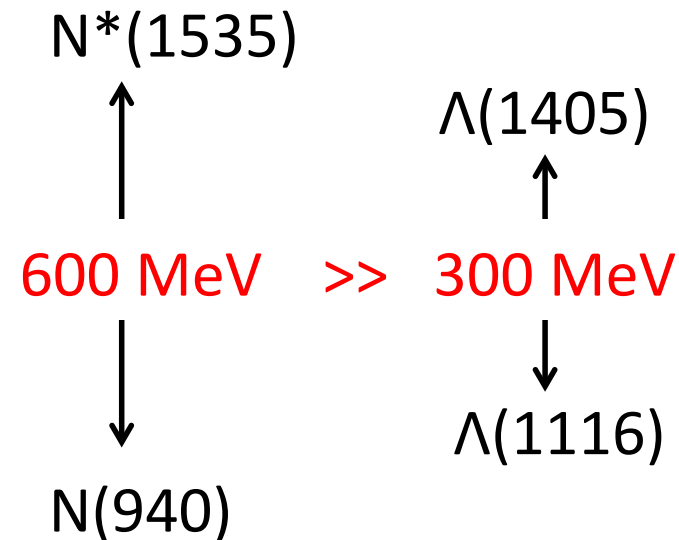
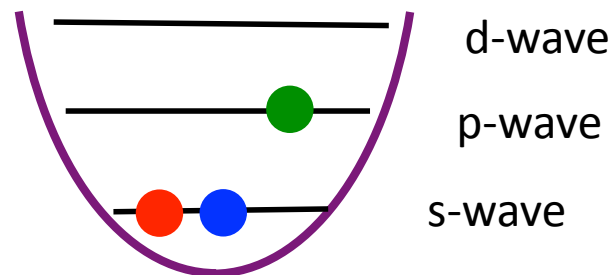
Physical observables of $\Lambda(1405)$

Strong candidate of baryon resonance with molecular structure

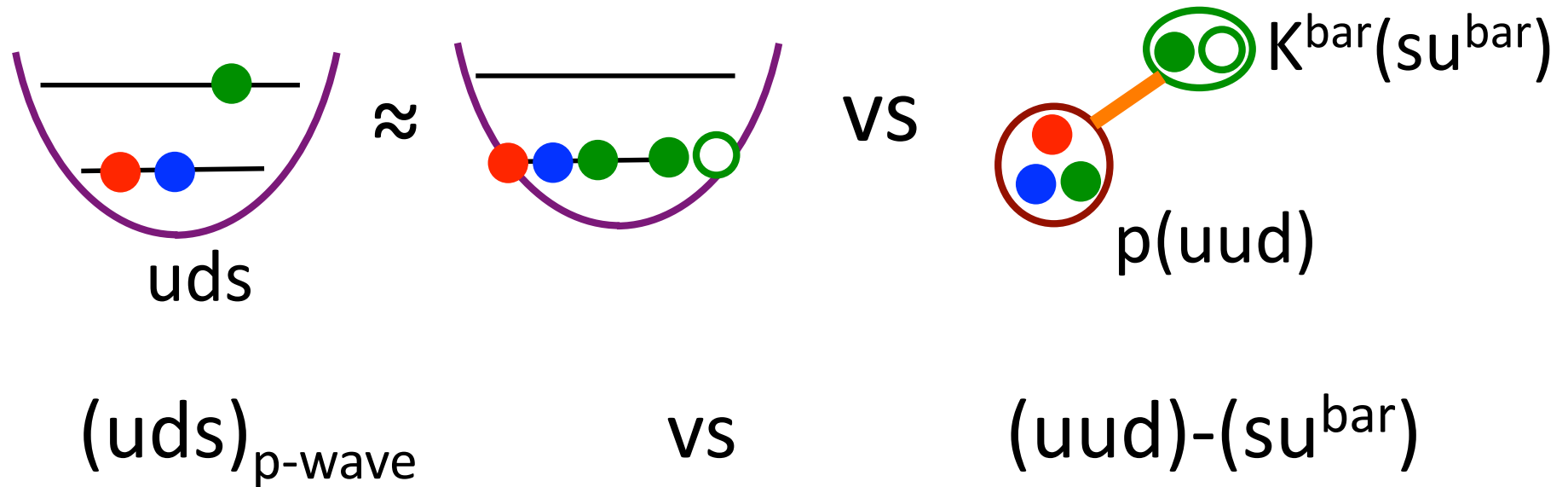
The lightest p-wave baryon, though it contains s-quark

$$m_s > m_u \sim m_d$$

p-wave excitation
in a quark model



KN molecule for $\Lambda(1405)$



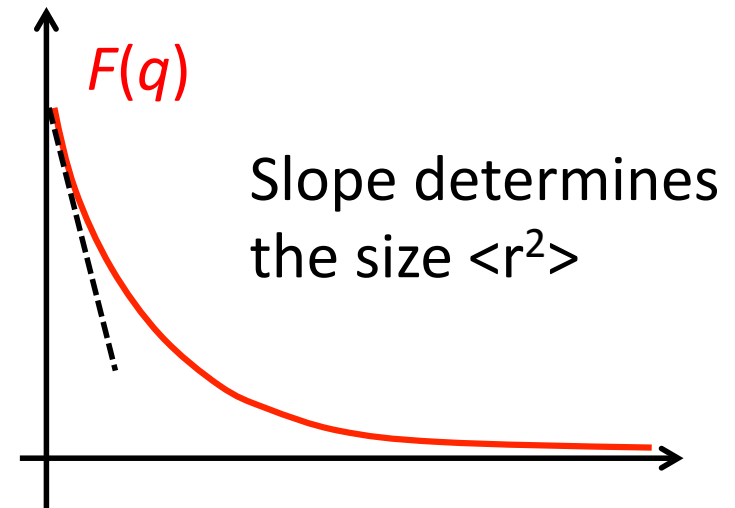
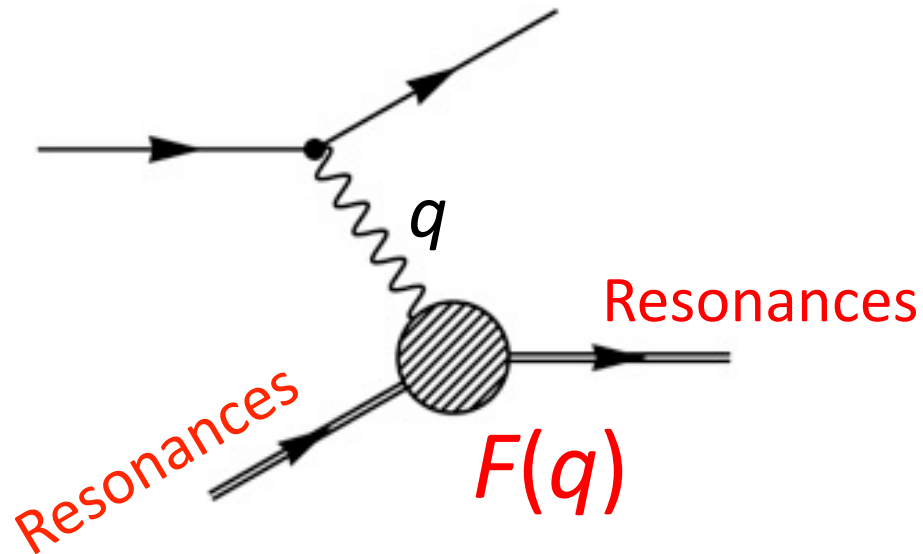
$K\bar{u}$ -N form a bound state,
 which couples to $\pi\Sigma$ as an open channel



Feshbach resonance

Properties of $\Lambda(1405)$ as a $K^{\text{bar}}\text{-N}$ molecule

For example: EM form factors

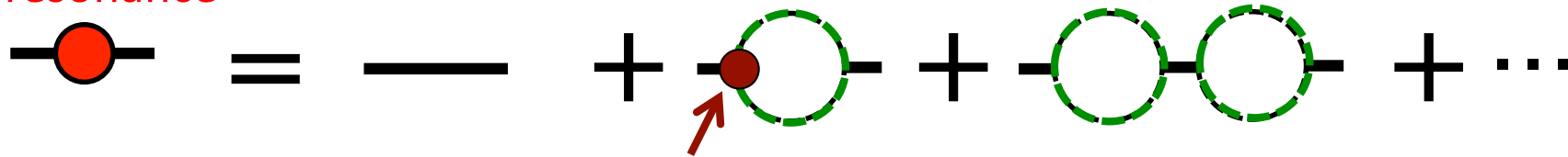


Real \rightarrow Complex quantity
Resonance wave functions

Nawa: Two level problem

Intrinsic and/or composite

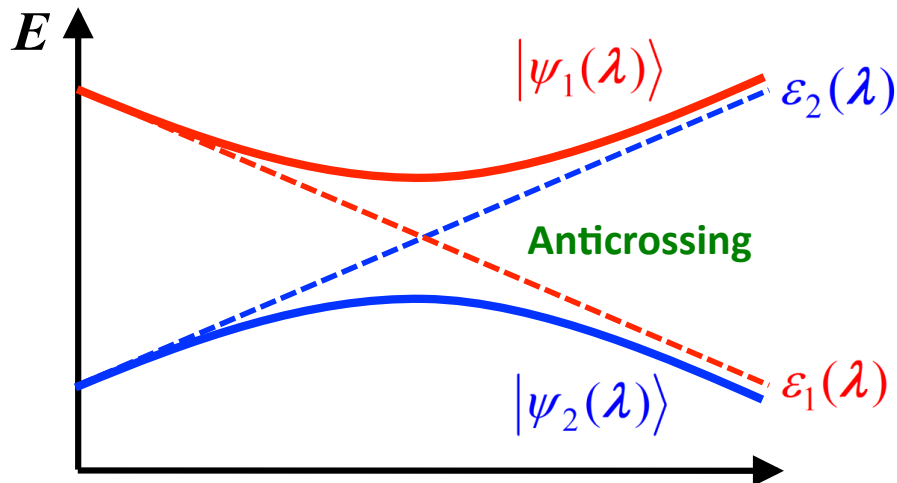
Physical resonance



This coupling determines the strength of the two components

Hamilton Matrix

$$\mathbf{H}(\lambda) = \begin{pmatrix} \langle \phi_1 | \hat{H}(\lambda) | \phi_1 \rangle & \langle \phi_1 | \hat{H}(\lambda) | \phi_2 \rangle \\ \langle \phi_2 | \hat{H}(\lambda) | \phi_1 \rangle & \langle \phi_2 | \hat{H}(\lambda) | \phi_2 \rangle \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$



Character exchange with variation of λ

“Nature Transition”

classification of quantum states with variation of $\lambda \in \mathbb{R}$.

Summary

Hadrons

- Hadrons are composite objects of quarks and gluons
 - Due to Non-perturbative QCD, structure is non-trivial
 - Quark *intrinsic* or Hadronic *composite (dynamical)*
 - Appear in scattering phenomena → *T-matrix poles*
-
- Extraction of T-matrices from exp. (Kamano)
 - Compositeness of hadronic molecules (Hyodo)
 - Geometric aspects of poles of two level model (Nawa)
 - Lambda(1405) and complex matrix elements (Sekihara)

Summary

- We have seen **Yukawa, Nambu, Skyrme**, where various aspects of the **pion** were emphasized.
- **Pion** plays not only light flavor sectors but also heavy quark sector, as long as our world **breaks chiral symmetry spontaneously**
- Other application of chiral symmetry
vector mesons => Talk by Kaneko