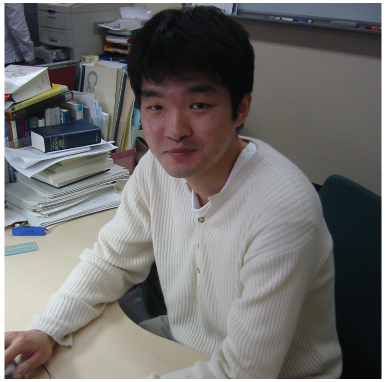
Many-body resonances in double quantum-dot systems

Akinori Nishino

Kanagawa University

Collaborators

Naomichi Hatano
(IIS, University of Tokyo)



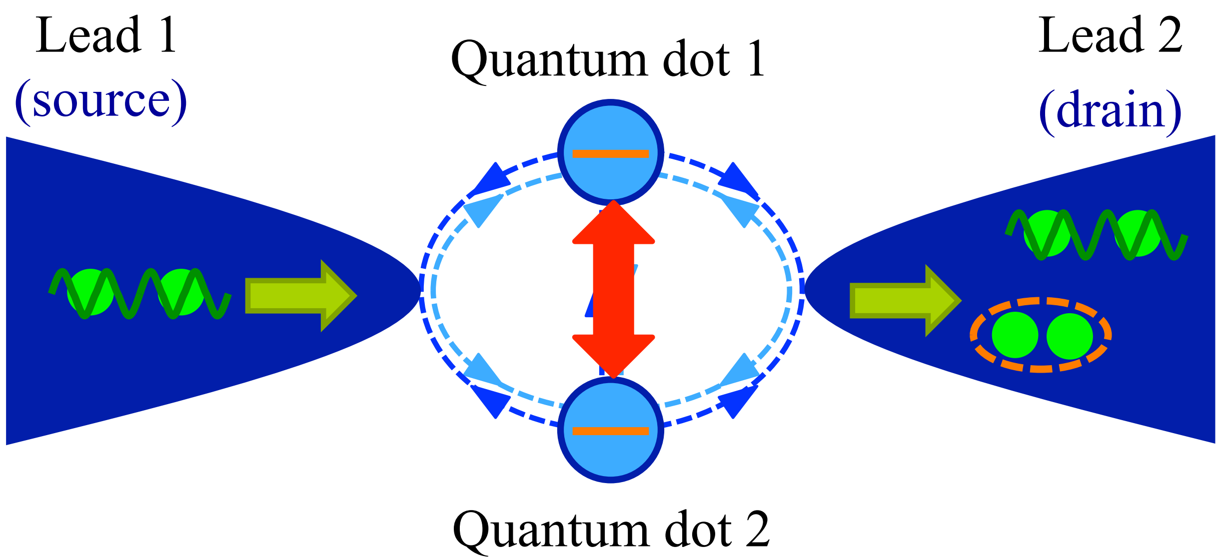
Takashi Imamura
(RCAST, University of Tokyo)

Gonzalo Ordonez
(Butler University)

Summary

We study double quantum-dot (DQD) systems with an interdot Coulomb interaction

- Exact many-electron scattering eigenstates
- Appearance of many-body bound states
  Binding strength ⟸ Resonant poles of scattering
- Many-body resonant states

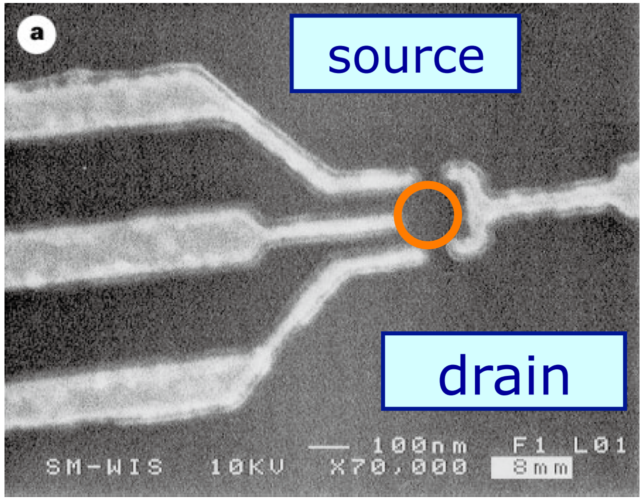


Part I:
Electron transport in open quantum-dot systems

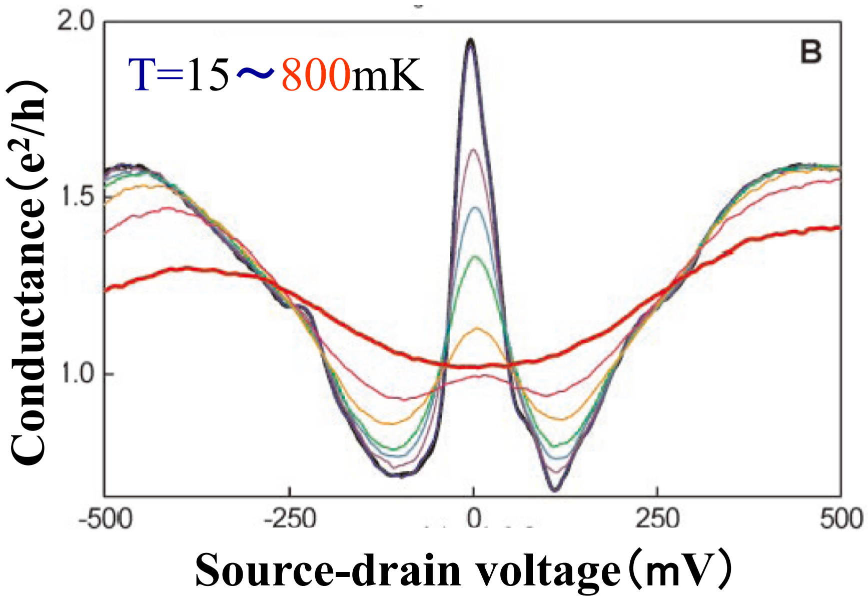
Experiments: Quantum dot (QD)

2D elec. systems are realized on heterostructures of semiconductors.

Quantum dot on GaAs/AlGaAs
[D. Goldhaber-Gordon et al.,
Nature391(1998)156]



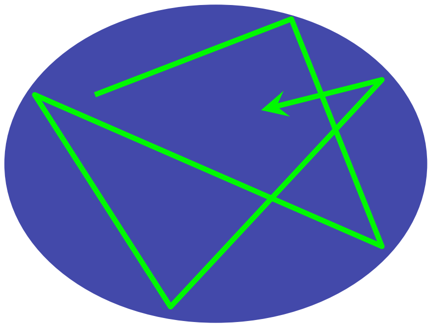
Kondo effect in QD
[W. G. van der Wiel et al.,
Science289(2000)2105]



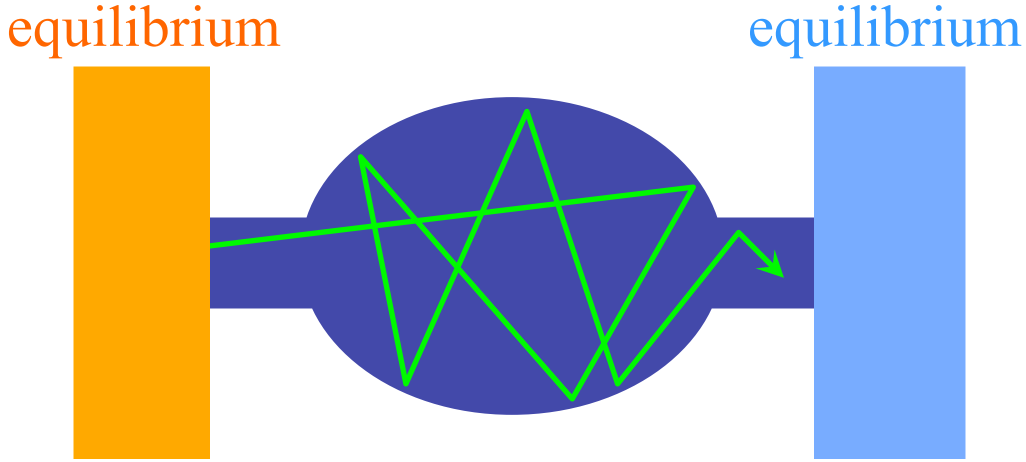
- ✓ Mesoscopic systems
- ✓ Finite bias voltage (The system is far from equilibrium.)
- ✓ Coulomb interactions for the electrons localized on QD

Open quantum systems

Closed systems | **Open systems**



Bound states



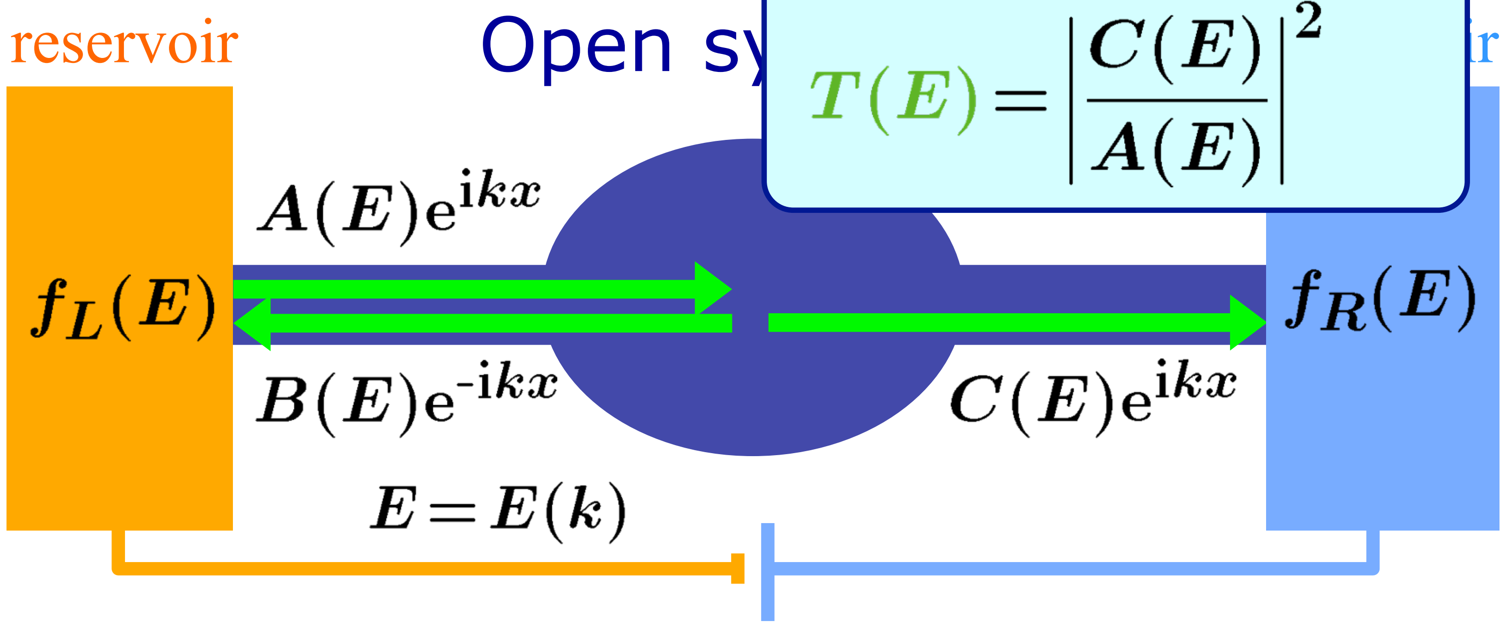
Scattering states

Equilibrium states in the TD limit ?

Nonequilibrium steady states between equilibrium states ?

Landauer formula

Coherent electron transport t



Transmission prob.

$$T(E) = \left|\frac{C(E)}{A(E)}\right|^2$$

Avera

$I = \frac{2}{}$ ... $(E))$

Electrical conductance

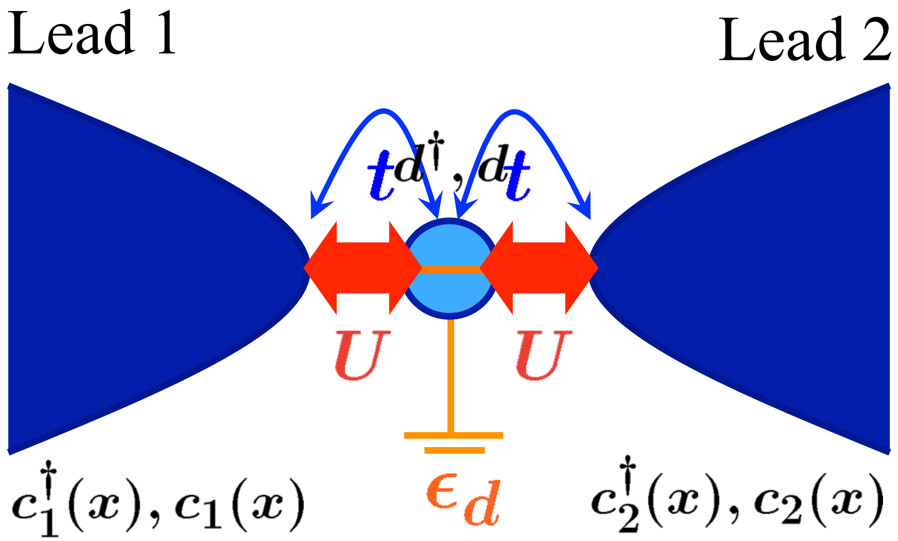
$$G(E) = \frac{2e^2}{h} T(E)$$

Interacting cases ➡ Many-electron scattering eigenstates

Open quantum-dot (QD) system

Interacting resonant-level model (IRLM)

("a minimal model" for QD systems with interactions)



- Spinless electrons
- Lead-dot coupling t
- Single energy level ϵ_d
- Coulomb interaction U

$$H = \sum_{\ell=1,2}\Big(\sum_k \epsilon(k)\tilde{c}_\ell^\dagger(k)\tilde{c}_\ell(k) + \frac{t}{\sqrt{2}}(c_\ell^\dagger(0)d + d^\dagger c_\ell(0))\Big)$$

$$+\epsilon_\mathrm{d} d^\dagger d + U\sum_{\ell=1,2} c_\ell^\dagger(0)c_\ell(0)d^\dagger d \quad \Big(\tilde{c}_\ell(k) = \int dx\, \mathrm{e}^{-\mathrm{i}kx} c_\ell(x)\Big)$$

Calculation of current in IRLM

- Bethe ansatz approach

  [P. Mehta & N. Andrei, PRL**96**(2006)216802]
- Numerical renormalization group (NRG)

  [L. Borda, K. Vladar & A. Zawadowski, PRB**75**(2007)125107]
- Nonequilibrium Greens function (NEGF) approach

  [B. Doyon, PRL**99**(2007)076806]

  [A. Golub, PRB**76**(2007)193307]
- Time-dependent DMRG (TDDMRG)

  [E. Boulat, H. Saleur & P. Schmitteckert, PRL**101**(2008)140601]
- Extension of Landauer formula with exact scattering eigenstates

  [A.N., T. Imamura & N. Hatano, PRL**102**(2009)146803]
- Functional RG (FRG) & Real time RG in frequency space

  [C. Karrasch et al., EPL**90**(2010)30003]

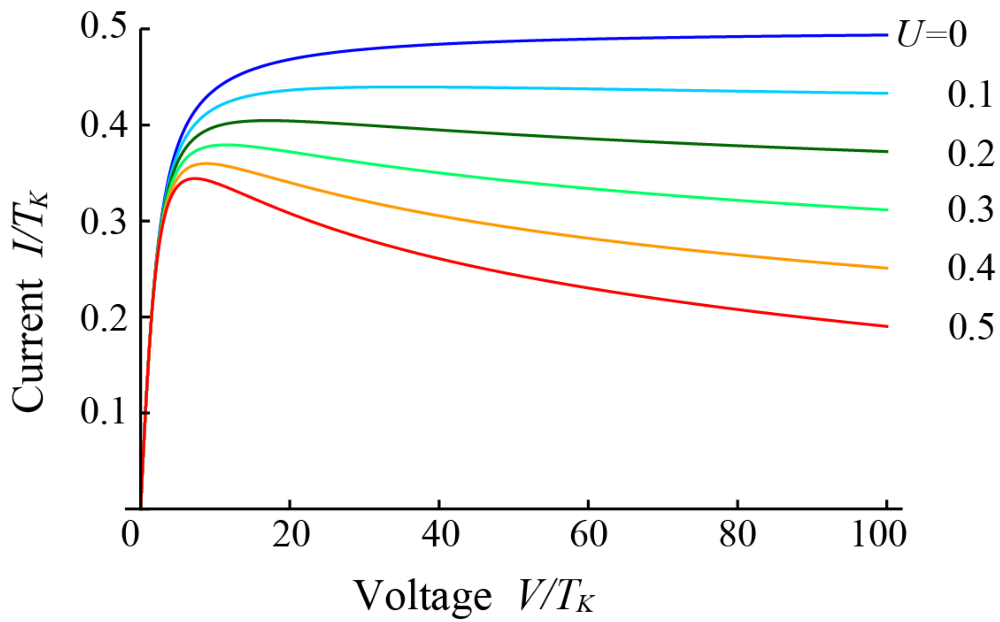
Universal electric current

[A.N., T. Imamura & N. Hatano,PRB**86**(2011)035306]

For the IRLM with linearized dispersion relations $\epsilon(k)=k$,

$$\frac{\langle I\rangle}{T_K}=\frac{2}{\pi}\arctan\left(\frac{V}{2T_K}\right) \qquad T_K=T_K(t,\epsilon_d,U)$$

$$-\frac{U}{\pi^2}\left(\arctan\left(\frac{V}{2T_K}\right)-\frac{\frac{V}{2T_K}}{\left(\frac{V}{2T_K}\right)^2+1}\right)\log\left(\left(\frac{V}{2T_K}\right)^2+1\right)+O(U^2)$$



Negative differential conductance appears for $U > 0$!

Formation of many-body bound states

Calculation of current in IRLM

- Bethe ansatz approach

  [P. Mehta & N. Andrei, PRL**96**(2006)216802]
- Numerical renormalization group (NRG)

  [L. Borda, K. Vladar & A. Zawadowski, PRB**75**(2007)125107]
- Nonequilibrium Greens function (NEGF) approach

  [B. Doyon, PRL**99**(2007)076806]

  [A. Golub, PRB**76**(2007)193307]

  Negative differential conductance for $U > 0$
- Time-dependent DMRG (TDDMRG)

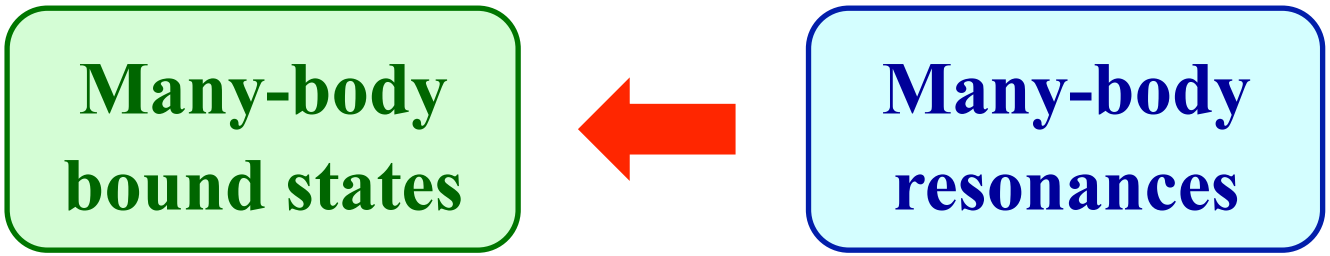
  [E. Boulat, H. Saleur & P. Schmitteckert, PRL**101**(2008)140601]
- Extension of Landauer formula with exact scattering eigenstates

  [A.N., T. Imamura & N. Hatano, PRL**102**(2009)146803]
- Functional RG (FRG) & Real time RG in frequency space

  [C. Karrasch et al., EPL**90**(2010)30003]

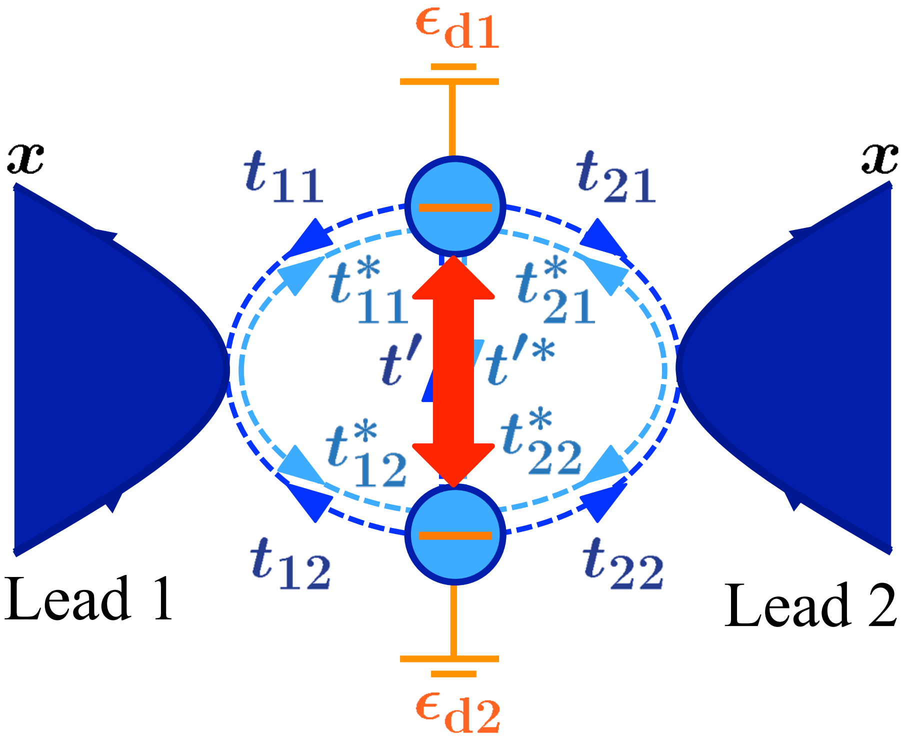
Summary of Part I and comments

- We need to consider the effect of the Coulomb interaction for the open QD systems.
- Many-electron scattering states play important roles to study the electron transport in the systems.
- The effect of the Coulomb interaction appear in physical quantities through many-body bound states.
- We apply the approach to other QD systems.



Part II:
Many-body resonances in double quantum-dot systems

Double quantum-dot (DQD) systems



- Spinless electrons
- Linear dispersion relation $\epsilon(k) = k$
- Lead-dot coupling $t_{\ell\alpha}$ $(\ell, \alpha = 1, 2)$
  Dot-dot coupling t'
- Single energy level $\epsilon_{\mathrm{d}\alpha}$
- Interdot Coulomb interaction U

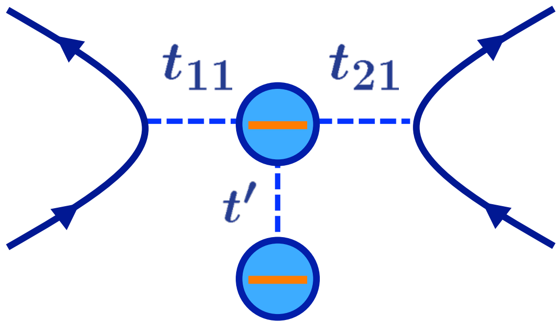
$$H = \sum_{\ell=1,2}\Big(\int dx\, c_\ell^\dagger(x)\frac{1}{\mathrm{i}}\frac{d}{dx}c_\ell(x) + \sum_{\alpha=1,2}\big(t_{\ell\alpha}c_\ell^\dagger(0)d_\alpha + t_{\ell\alpha}^* d_\alpha^\dagger c_\ell(0)\big)\Big)$$

$$+t' d_1^\dagger d_2 + t'^* d_2^\dagger d_1 + \sum_{\alpha=1,2}\epsilon_{\mathrm{d}\alpha} n_{\mathrm{d}\alpha} + U n_{\mathrm{d}1} n_{\mathrm{d}2} \qquad (n_{\mathrm{d}\alpha} = d_\alpha^\dagger d_\alpha)$$

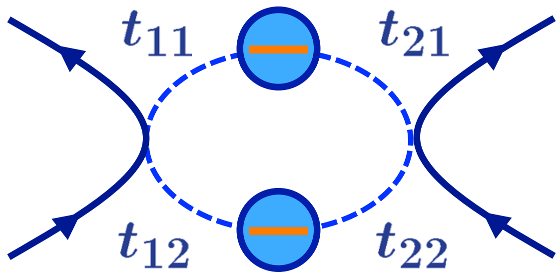
Relation to other DQD systems

Various DQD systems are reproduced at special values of parameters.
cf. [Tanaka & Kawakami, PRB 72 (2005) 085304]

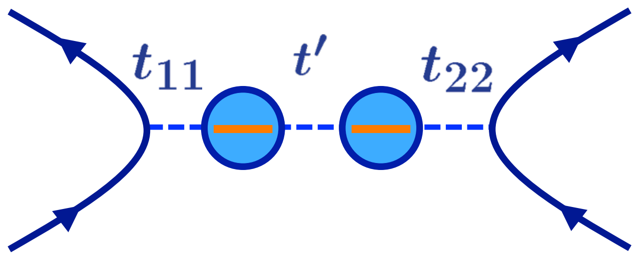
T-shaped: $t_{12}=t_{22}=0$

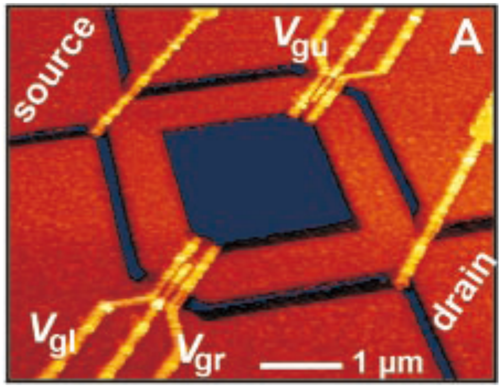


Parallel: $t'=0$

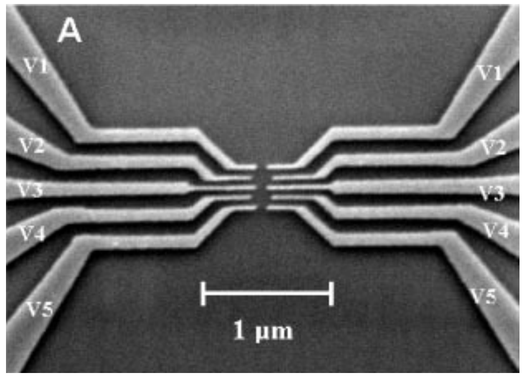


Serial: $t_{12}=t_{21}=0$





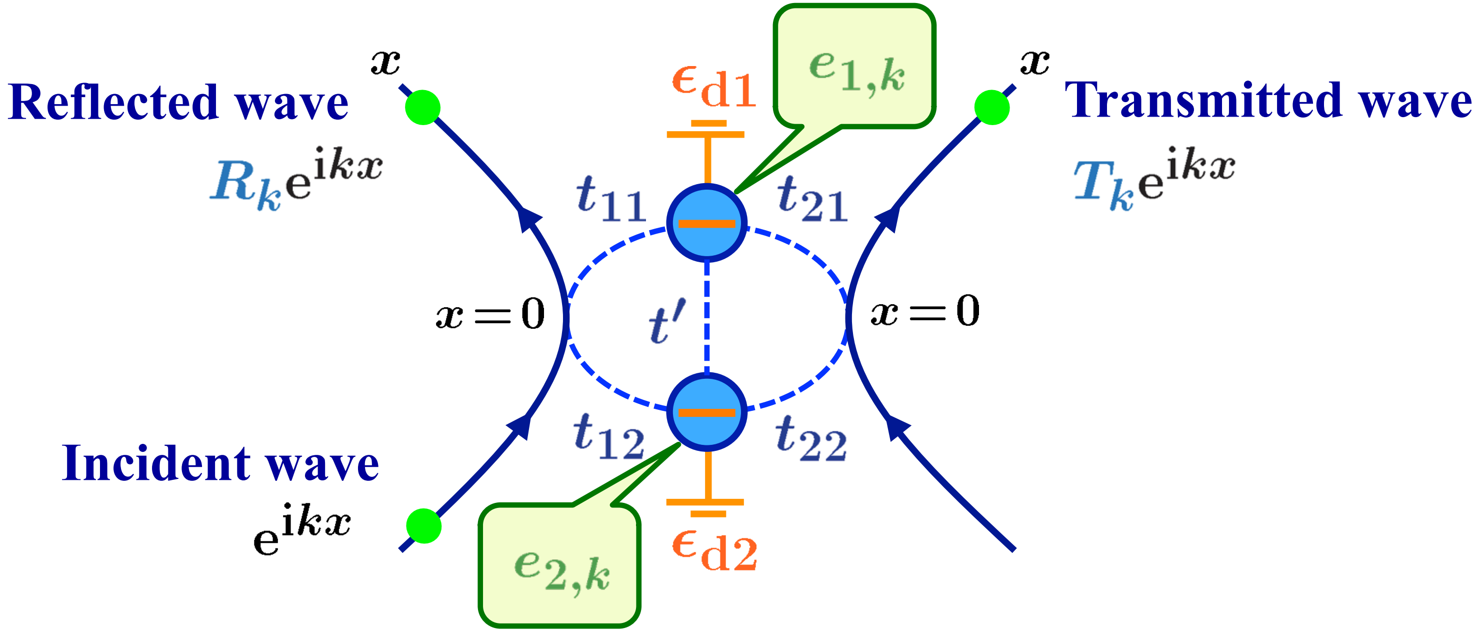
[W. G. van der Wiel et al., Science **289**, 2015 (2000)]



[H. Jeong et al., Science **293, 2221**(2001)]

One-electron scattering eigenstates

Eigenstates with plane-wave incident states

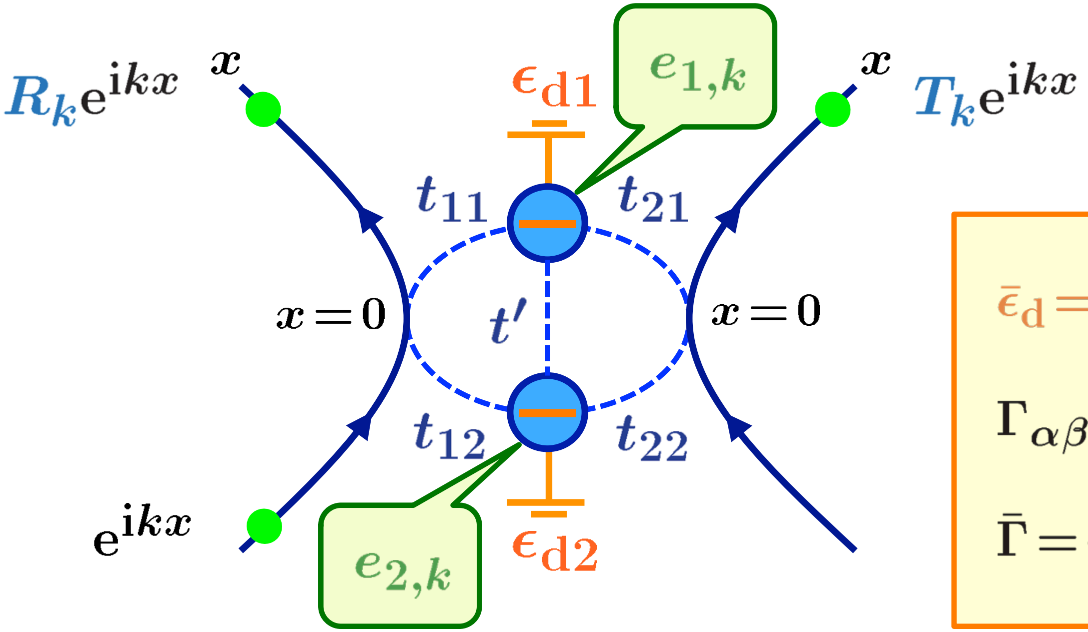


$$e_{\alpha,k} = \frac{1}{\sqrt{2\pi}} \frac{(k-\epsilon_{d\bar{\alpha}}+i\Gamma_{\bar{\alpha}\bar{\alpha}})t_{1\alpha} - (i\Gamma_{\alpha\bar{\alpha}} - t')t_{1\bar{\alpha}}}{(k-\epsilon_{d1}+i\Gamma_{11})(k-\epsilon_{d2}+i\Gamma_{22}) - (i\Gamma_{12} - t')(i\Gamma_{21} - t')}$$

$$R_k = 1 - i\sum_{\alpha} t_{1\alpha}\sqrt{2\pi}\, e_{\alpha,k}, \quad T_k = -i\sum_{\alpha} t_{2\alpha}\sqrt{2\pi}\, e_{\alpha,k}, \quad (\bar{\alpha} = 3 - \alpha)$$

$$\Gamma_{\alpha\beta} = \frac{1}{2}\sum_{\ell} t_{\ell\alpha} t_{\ell\beta}$$: level-width matrices

Resonances in one-electron scattering



$$\bar{\epsilon}_d = \frac{\epsilon_{d1}+\epsilon_{d2}}{2}, \qquad \Delta\epsilon = \epsilon_{d1}-\epsilon_{d2}$$

$$\Gamma_{\alpha\beta} = \frac{1}{2}(t^*_{1\alpha}t_{1\beta} + t^*_{2\alpha}t_{2\beta})$$

$$\bar{\Gamma} = \frac{\Gamma_{11}+\Gamma_{22}}{2}, \qquad \Delta\Gamma = \Gamma_{11}-\Gamma_{22}$$

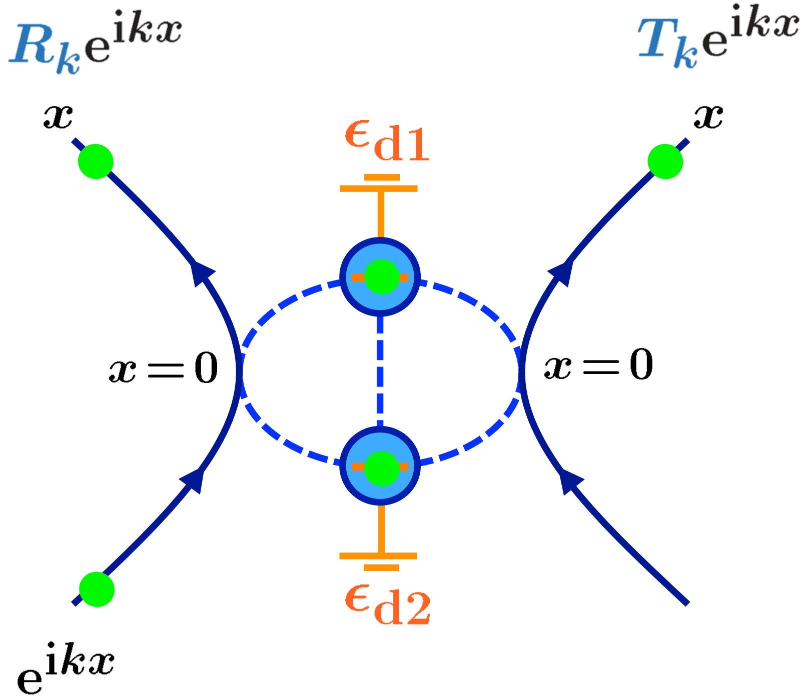
Resonant poles ⟺ Poles of T_k in the k-complex plane

⟺ Poles of $e_{\alpha,k}$ in the k-complex plane

$$k = \bar{\epsilon}_d - i\,\bar{\Gamma} \mp \frac{1}{2}\sqrt{(\Delta\epsilon_d - i\Delta\Gamma)^2 + 4(t'-\Gamma_{12})(t'-\Gamma_{21})}$$
$$= \zeta_\pm - i\,\eta_\pm$$

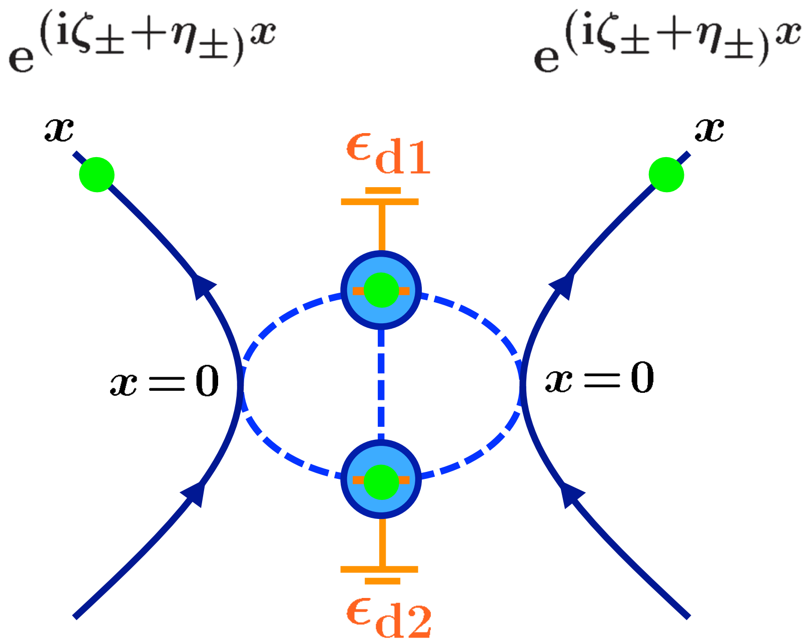
Scattering states & resonant states

Scattering eigenstates:
plane-wave incident states



Resonant states:
incident elec. is on DQD

Divergent in $x \to \infty$



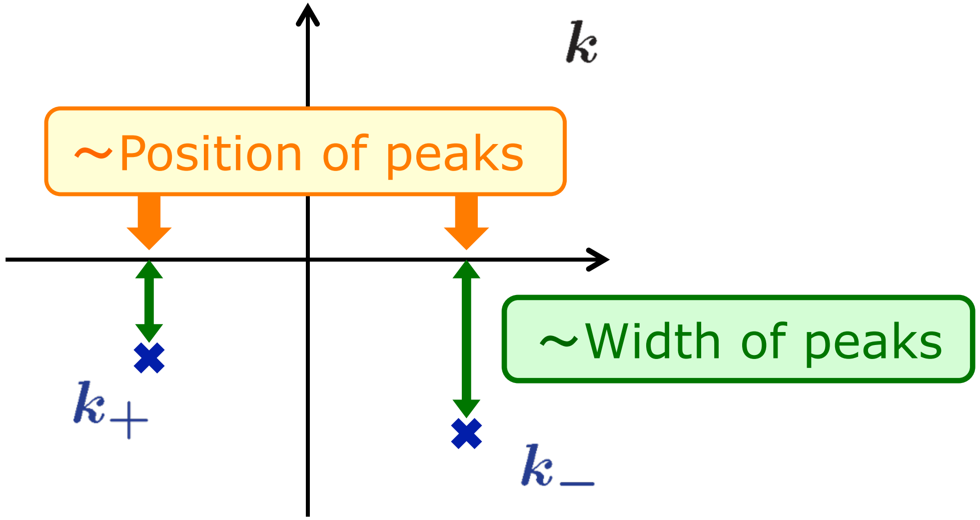
No incoming electron!

cf. Properties of resonant states in QD systems
[Hatano, Sasada, Nakamura & Petrosky, PTP**119**(2008)187]

Roles of resonant poles

Resonant poles

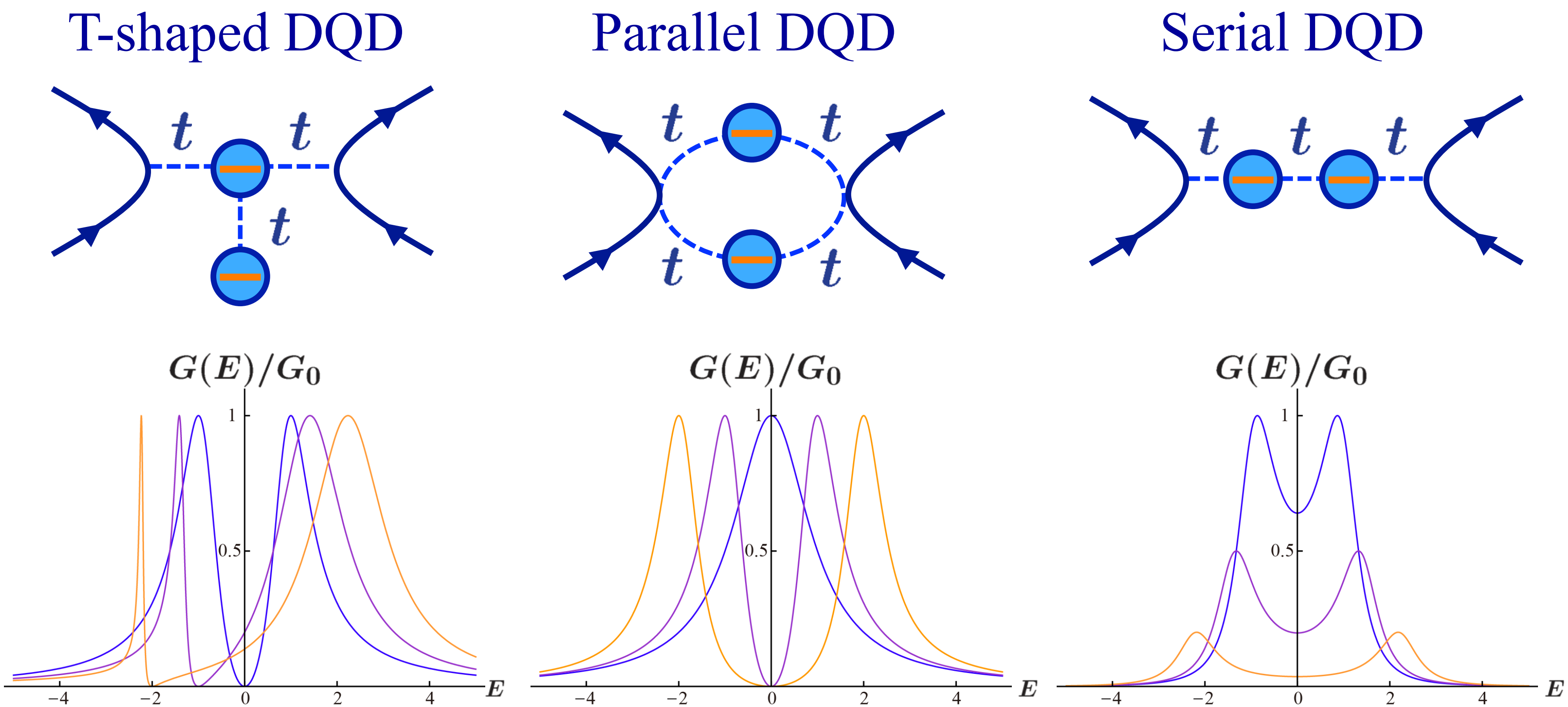
$$k_{\pm} = \zeta_{\pm} - i\,\eta_{\pm}$$

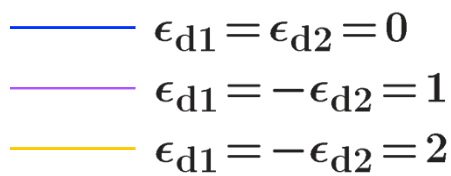


i) Resonant peaks of conductance at *U*=0 (known)

$$G(k) = \frac{1}{2\pi}|T_k|^2$$ (Landauer formula)

Conductance at U =0





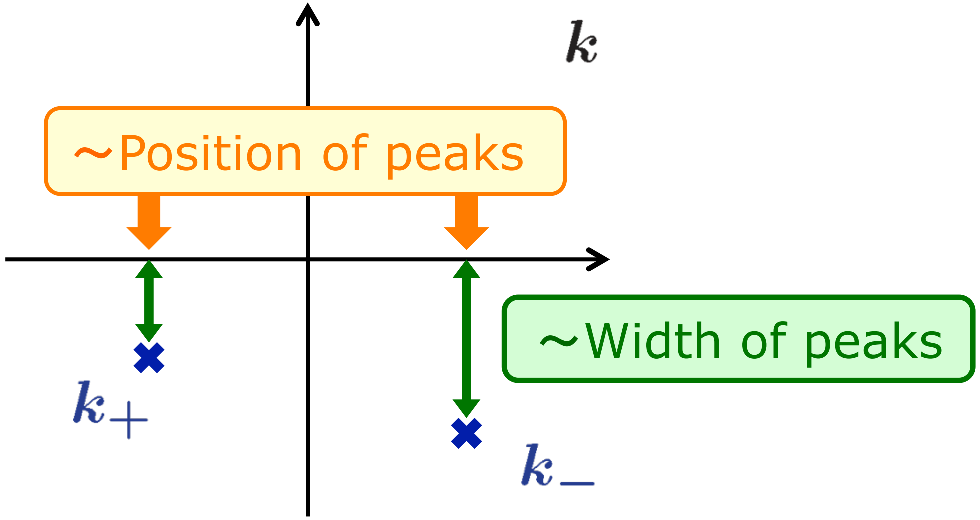
Asymmetric peak
(Fano resonance)

cf.[K. Kobayashi et al., PRB**70**(2004)035319]
[N. Hatano, K. Sasada & G. Ordonez, JPSJ**80**(2011)104707.]

Roles of resonant poles

Resonant poles

$$k_{\pm} = \zeta_{\pm} - i\,\eta_{\pm}$$



i) Resonant peaks of conductance at U=0 (known)

$$G(k) = \frac{1}{2\pi}|T_k|^2 \qquad \text{(Landauer formula)}$$

ii) Binding strength of two-body bound states in interacting cases ($U > 0$)

Exact two-electron scattering eigenstates

Eigenstates with two-electron plane-wave incident states

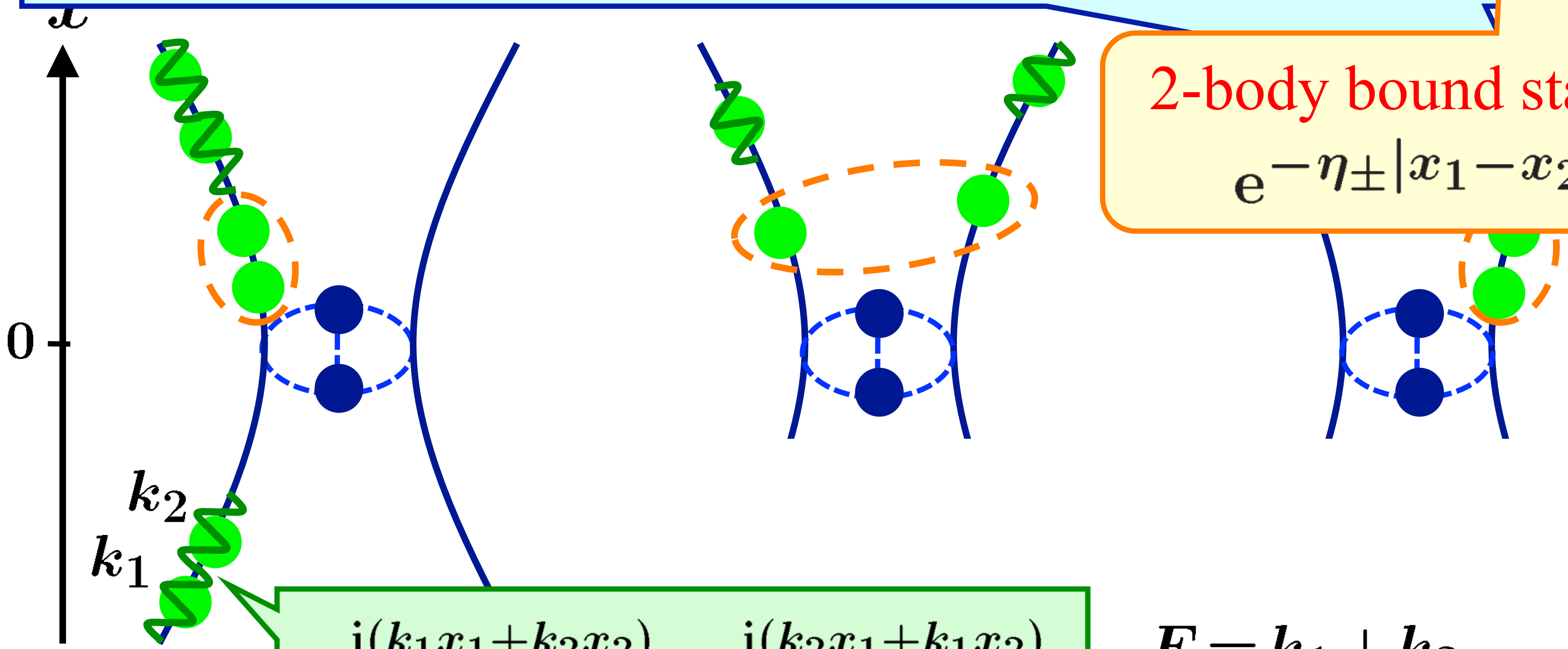
Transmitted wave

$$\left(\bar{x} = \frac{x_1 + x_2}{2},\ \bar{\Gamma} = \frac{\Gamma_{11} + \Gamma_{22}}{2}\right)$$

$$T_{k_1} T_{k_2} \left(e^{i(k_1 x_1 + k_2 x_2)} - e^{i(k_2 x_1 + k_1 x_2)} \right) + \frac{iU}{E - 2\bar{\epsilon}_{\mathrm{d}} - U + 2i\bar{\Gamma}} \sum_{s=\pm} Z^{s}_{k_1 k_2} \operatorname{sgn}(x_1 - x_2) e^{\left(iE\bar{x} + i\left(\frac{E}{2} - (\zeta_s - i\eta_s)\right)|x_1 - x_2|\right)}$$

2-body bound states

$$e^{-\eta_{\pm}|x_1 - x_2|}$$



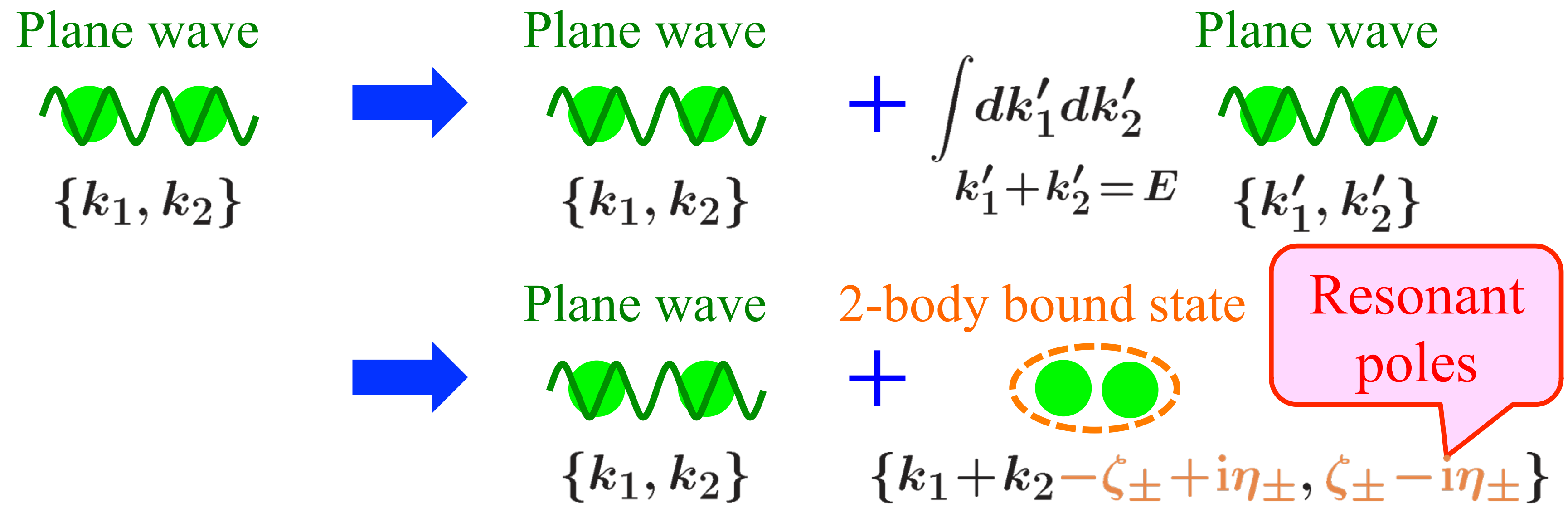
$$e^{i(k_1 x_1 + k_2 x_2)} - e^{i(k_2 x_1 + k_1 x_2)}$$

$$E = k_1 + k_2$$

Two-body bound states & resonant poles

Two-electron transmitted wave

$$\langle 0|c_2(x_2)c_2(x_1)|k_1,k_2\rangle$$
$$= T_{k_1}T_{k_2}\left(e^{i(k_1x_1+k_2x_2)} - e^{i(k_2x_1+k_1x_2)}\right)$$
$$+\frac{iU}{E-2\bar{\epsilon}_d-U+2i\bar{\Gamma}}\sum_{s=\pm} Z^s_{k_1k_2}\,\mathrm{sgn}(x_1-x_2)\,e^{(iE\bar{x}+i(\frac{E}{2}-(\zeta_s-i\eta_s))|x_1-x_2|)}$$



Resonant states appear as two-body bound states!

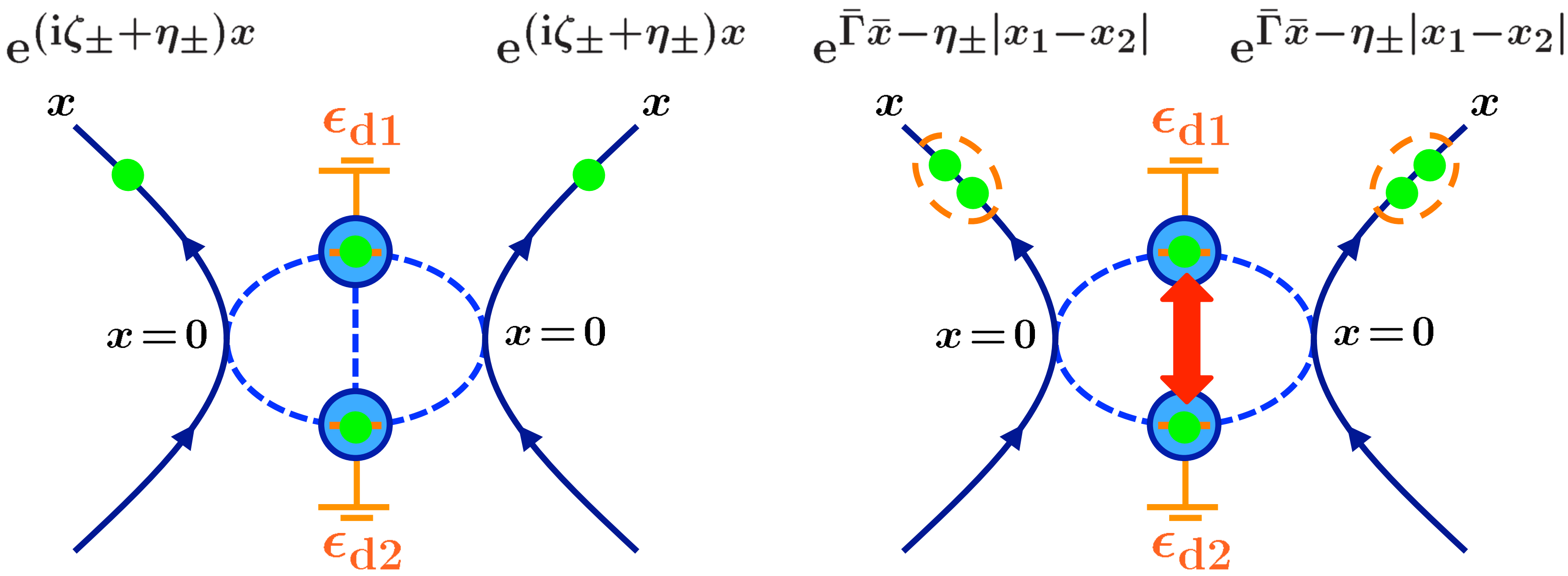
Two-body resonant states

One-body resonant states

an incident elec. on DQD

Two-body resonant states

two incident elec. on DQD



No incoming electron!

$$\bar{x}=\frac{x_1+x_2}{2},\qquad \bar{\Gamma}=\frac{\Gamma_{11}+\Gamma_{22}}{2},\qquad \Gamma_{\alpha\alpha}=\frac{1}{2}\sum_{\ell}|t_{\ell\alpha}|^2$$

The 2-body resonant state is the resonant state of 2-body bound states!

Summary

We studied the DQD systems with arbitrary lead-dot and dot-dot couplings and an interdot Coulomb interaction.

- Exact many-electron scattering eigenstates
- Relation between many-body bound states and many-body resonant poles
- One-body and two-body resonant states

Future problems

- Calculation of electric current for the DQD systems
- How can we observe many-body resonant states?