Bound state influence on long-time non-exponential decay in open quantum systems

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Outline:

Orientation: description of open quantum systems and prototype model

Deviations from exponential decay in quantum mechanics:

- Short time scales – quantum Zeno and anti-Zeno effects
- **Long time scales** – connection with continuum threshold

Survival Probability Formalism and Physical Motivation:

- Relevant studies in the literature

Prototype model – linear case:

- Bound state transition to anti-bound state (virtual state)
- Long time dynamics: long-time near zone $P(t) \sim t^{-1}$
- Long-time far zone $P(t) \sim t^{-3}$
- Timescale set by Δ_Q – gap between bound state and threshold

Side-coupled impurity model:

- Bound state trapping below threshold

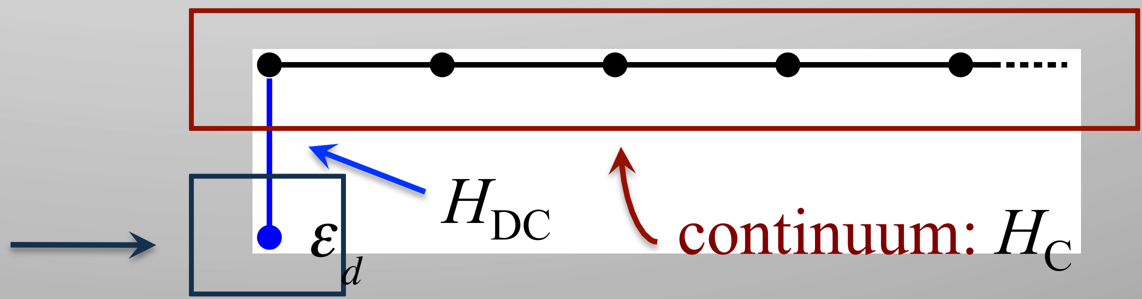
Open Quantum Systems

Open quantum system consists of:

- <u>Discrete</u> system H_{D}
- Embedded in a larger system (<u>continuum</u>) H_{C}
- Coupled via H_{DC}

<u>Prototype Model</u>: semi-infinite chain with end-point impurity

discrete component: H_{D}

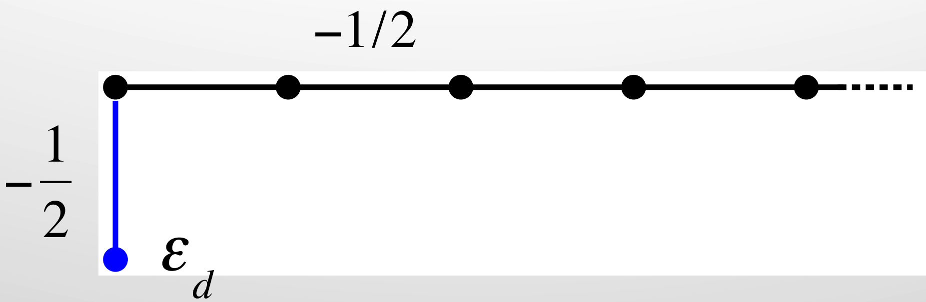


$$H_D = \varepsilon_d d^+ d$$

$$H_C = \frac{1}{2}\sum_{i=1}^{\infty}(c_i^+ c_{i+1} + c_{i+1}^+ c_i)$$

Open Quantum Systems: Prototype model

Prototype:



$$H = H_D + H_C + H_{DC}$$

$$H = \varepsilon_d d^+ d - \frac{1}{2}\sum_{i=1}^{\infty}(c_i^+ c_{i+1} + c_{i+1}^+ c_i) - \frac{1}{2}(c_1^+ d + d^+ c_1)$$

Short-time deviations from exponential decay

For decades it has been known that deviations from exponential decay exist in quantum systems *at least* on very short and very long time scales.

C. B. Chiu, B. Misra, and E. C. G. Sudarshan, Phys. Rev. D **16**, 520 (1977).

J. Martorell, J. G. Muga, and D. W. L. Spring, Lect. Notes. Phys. **789**, 239 (2009).

Short time scales typically give rise to parabolic decay: $P(t) \sim t^2$

- Quantum Zeno effect ➔ repeated measurements result in decelerated decay
- quantum anti-Zeno effect ➔ accelerated decay
- Experimental confirmation – ultra-cold sodium atoms initially trapped in accelerating optical potential:

S. R. Wilkinson, C. F. Bharucha, M. C. Fischer, K. W. Madison, P. R. Morrow, Q. Niu, B. Sunduram, and M. G. Raizen, Nature (London) **387**, 575 (1997).

M. C. Fischer, B. Gutiérrez-Medina, and M. G. Raizen, Phys. Rev. Lett. **87**, 040402 (2001).

Long-time deviations from exponential decay

Long time deviations intimately connected with the continuum threshold.

- Mathematically proven for quantum systems:

L. A. Khalfin, Soc. Phys. JETP **6**, 1053 (1958).

M. N. Hack, Phys. Lett. A **90**, 220 (1982).

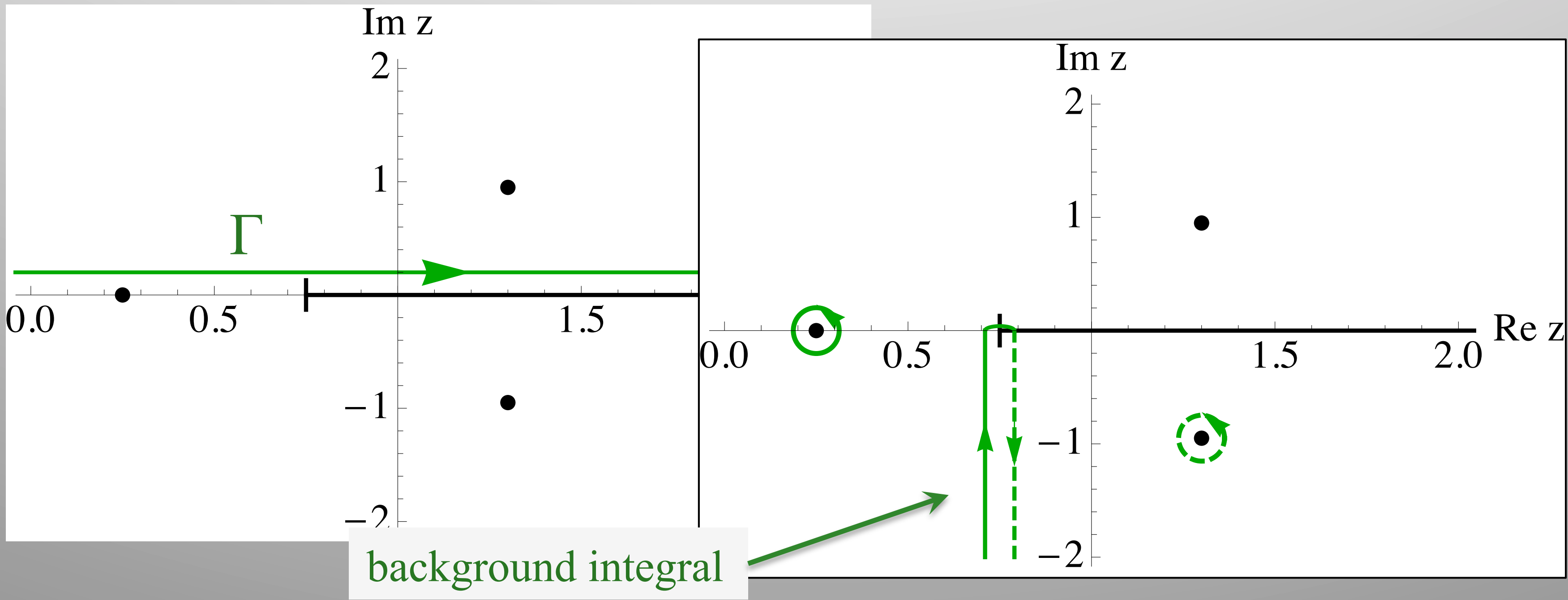
- Typically gives rise to inverse power law decay
- Typical asymptotic decay law: $P(t) \sim t^{-3}$
- Recent experimental verification: luminescence decay properties of dissolved organic materials following laser excitation:

C. Rothe, S. I. Hintschich and A. P. Monkman, Phys. Rev. Lett. **96**, 163601 (2006).

Formalism: survival probability for an initially prepared state

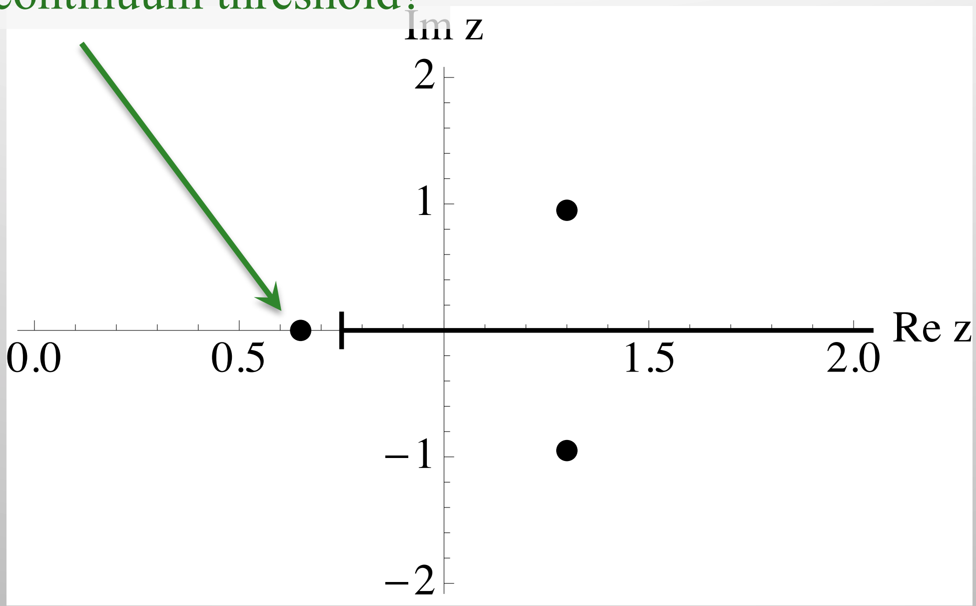
Survival probability: $P(t) = |A(t)|^2$

$$A(t) = \langle \psi_0 | e^{-iHt} | \psi_0 \rangle = \frac{1}{2\pi i} \int_\Gamma dz\, e^{-izt} \left\langle \psi_0 \left| \frac{1}{z-H} \right| \psi_0 \right\rangle$$



Physical motivations: bound state at threshold

question: what happens as bound state approaches continuum threshold?



Answer: the long-time non-exponential decay effects will be amplified as bound state approaches the threshold.

Note that bound state transitions to anti-bound state (2nd sheet) after reaching threshold

Relevant studies in the literature

T. Jittoh, S. Matsumoto, J. Sato, Y. Sako, and K. Takeda, Phys. Rev. A **71**, 012109 (2005).

Radial potential: for *s*-wave component, as energy of initially prepared state approaches threshold, **exponential decay suppressed completely**.

(However, they do not consider bound states).

Victor Dinu, Arne Jensen, and Gheorge Nenciu, J. Math. Phys. **50**, 013516 (2009).

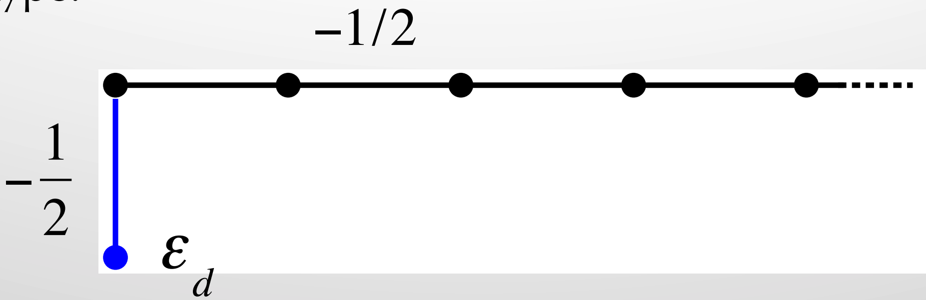
From mathematical physics perspective, authors study a bound state near threshold. Unfortunately, they make several unphysical assumptions.

(e.g. Discrete unperturbed energy appearing at threshold cannot form a bound state below threshold??)

Other relevant studies ➔ TBA

Prototype model: continuum and discrete spectra

Return to prototype:



$$H = H_D + H_C + H_{DC}$$

$$H = \varepsilon_d d^+ d - \frac{1}{2}\sum_{i=1}^{\infty}(c_i^+ c_{i+1} + c_{i+1}^+ c_i) - \frac{1}{2}(c_1^+ d + d^+ c_1)$$

Prototype model: continuum spectrum

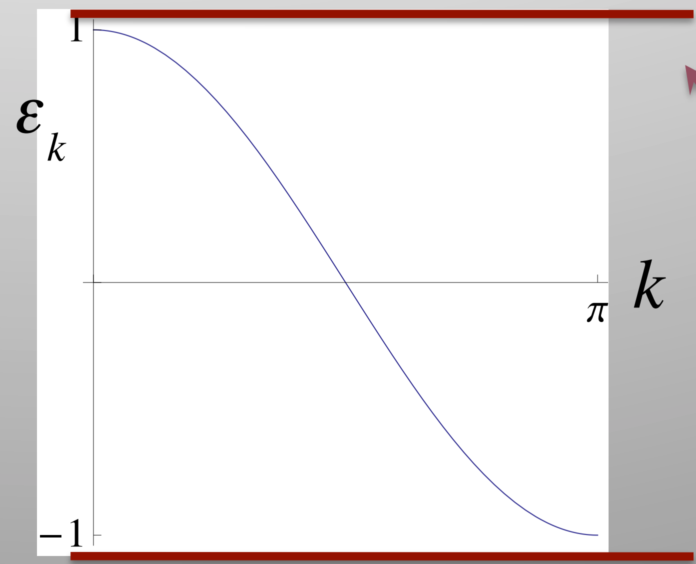
take continuum limit and introduce half-chain Fourier series:

$$H = \varepsilon_d d^\dagger d + \int_0^\pi \varepsilon_k c_k^\dagger c_k + \int_0^\pi V_k (c_k^\dagger d + d^\dagger c_k)$$

$(N \to \infty)$

Continuum:

$k \in [0,\pi]$ on $\varepsilon_k = -\cos k$



threshold

Prototype model: discrete spectrum

Obtain the discrete spectrum from:

$$\left\langle d \left| \frac{1}{z - H} \right| d \right\rangle = \frac{1}{z - \varepsilon_d - \Sigma(z)}$$

yields linear polynomial

Linear dispersion:

$$z - \varepsilon_d - \Sigma(z) = z - \varepsilon_d - \frac{1}{2}(z - \sqrt{z^2 - 1})$$

yields solution:

$$\boxed{\bar{z}_L(\varepsilon_d) = \varepsilon_d + \frac{1}{4\varepsilon_d}}$$

however, let's examine the dispersion more carefully…

Prototype model: Bound state absorption into continuum

Solution given from slightly re-written dispersion:

$$\frac{1}{2}z - \varepsilon_d - \sqrt{z^2 - 1} = 0$$

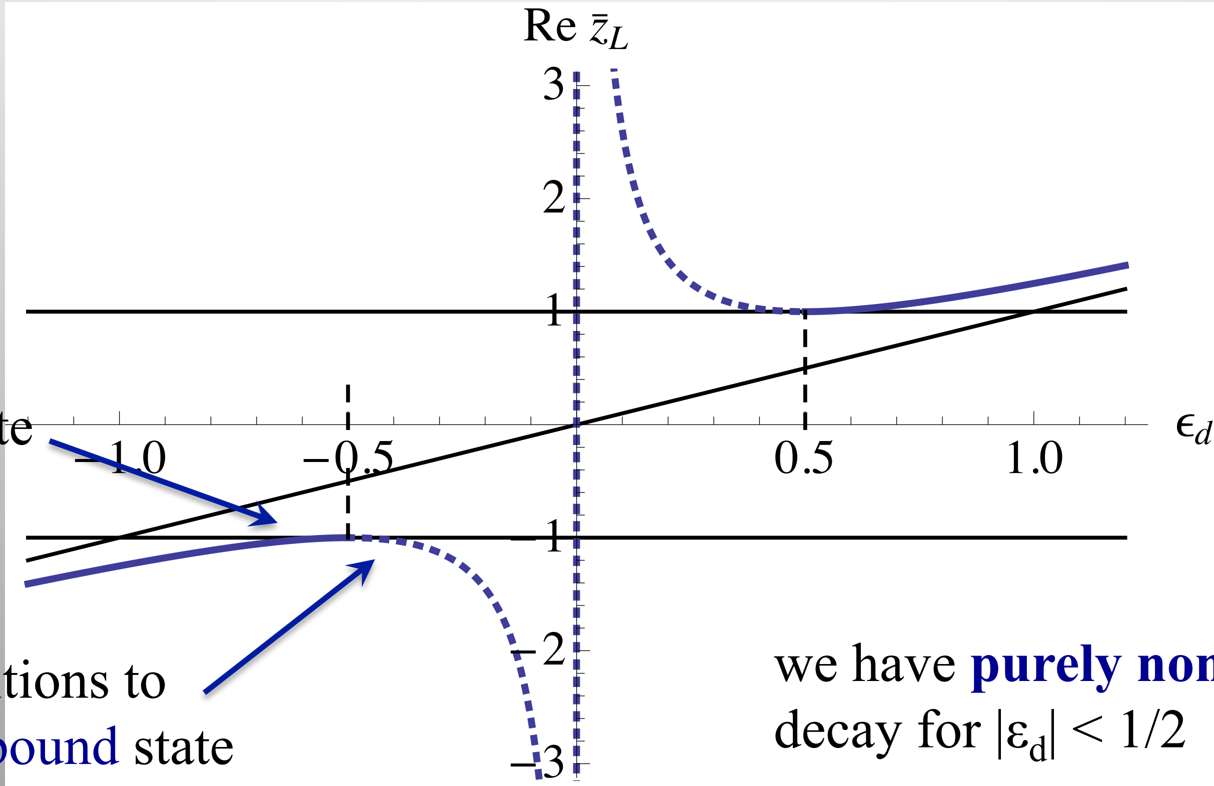
Note that the root vanishes for $z = \pm 1$, which occurs at:

$$\varepsilon_d = \pm\frac{1}{2}$$

These are the points where solution crosses one Riemann sheet into the other (localization/delocalization).

Prototype model: linear dispersion plot

$$\bar{z}_L(\varepsilon_d) = \varepsilon_d + \frac{1}{4\varepsilon_d}$$



bound state

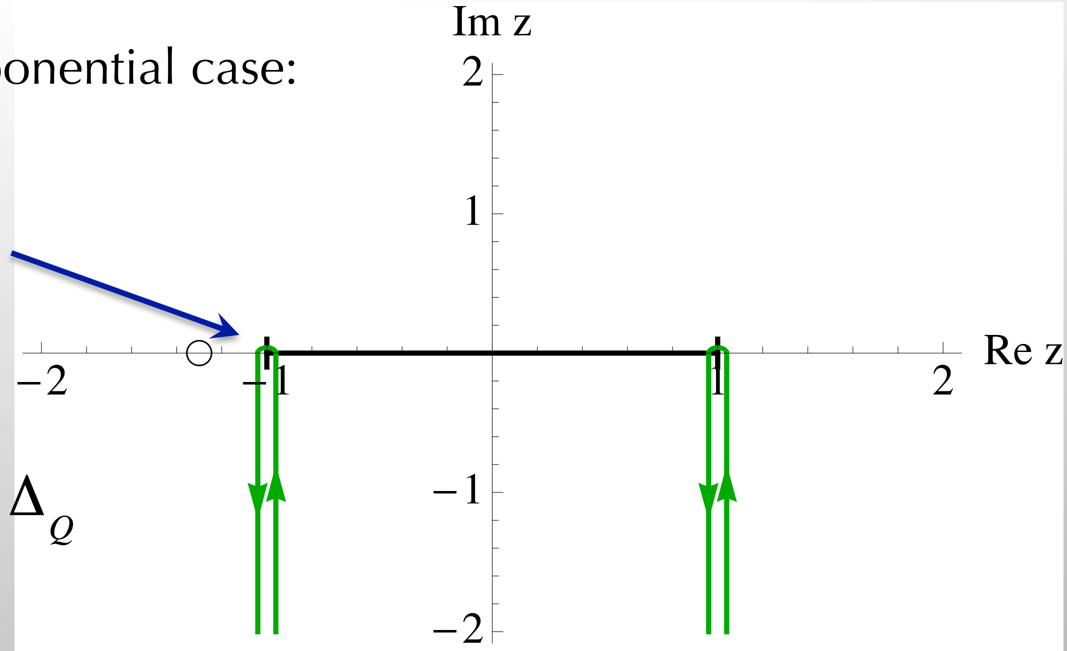
transitions to anti-bound state

we have **purely non-exponential** decay for $|\varepsilon_d| < 1/2$

Long-time dynamics for prototype model

Focus on the purely non-exponential case:

vicinity of lower threshold



define $\bar{z}_L \equiv -1 + \Delta_Q$

$$A_{th}^{-}(t) = \frac{1}{2\pi i}\left[\int_{-1}^{-1-i\infty} dz \, \frac{e^{-izt}}{\eta^{\mathrm{I}}(\mathrm{z})} - \int_{-1-i\infty}^{-1} dz \, \frac{e^{-izt}}{\eta^{\mathrm{II}}(\mathrm{z})}\right]$$

$$= \frac{1}{2\pi i \varepsilon_d} \int_{-1}^{-1-i\infty} dz \, e^{-izt} \, \frac{\sqrt{z^2 - 1}}{z - \bar{z}_L(\Delta_Q)}$$

Long-time dynamics: near zone and far zone

$$A^{-}_{th}(t) = \frac{e^{-it}}{2\pi i \varepsilon_d t^2} \int_0^\infty dz\, e^{-s} \frac{\sqrt{s^2 - 2its}}{\Delta_Q + i\frac{s}{t}}$$

Consider the timescale: $1 << t << (\Delta_Q)^{-1}$ (Long-time 'near zone')

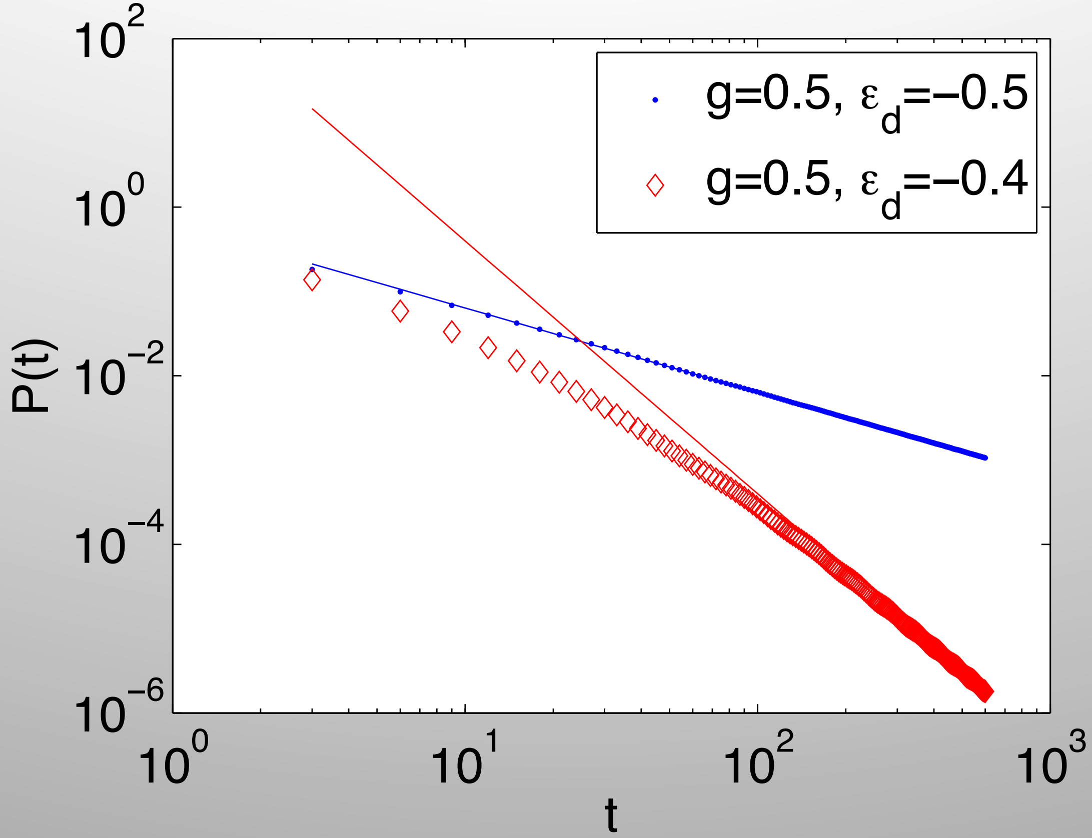
$$\left|A^{-}_{th}(t)\right|^2 \sim t^{-1}$$

Asymptotic limit: $(\Delta_Q)^{-1} << t$ (Long-time 'far zone')

$$\left|A^{-}_{th}(t)\right|^2 \sim t^{-3}$$

Note that for $\Delta_Q = 0$, the near zone becomes fully asymptotic

Long-time dynamics: numerical results for prototype model



S. Garmon, T. Petrosky, L. Simine, and D. Segal, Fortschritte der Physik, DOI: 10.1002/prop.201200077; **arXiv:1204.6141**.

Long-time dynamics for general open quantum systems

Similar effect observed in the following works:

S. Longhi, Phys. Rev. Lett. **97**, 110402 (2006).

S. Garmon, Ph.D. thesis, University of Texas at Austin (2007).

Axel D. Dente, Raúl A. Bustos-Marún, and Horacio M. Pastawski, Phys. Rev. A **78**, 062116 (2008).

Straight-forward to demonstrate: time scale separating near and far zones should *always* be inversely related to Δ_Q in OQS

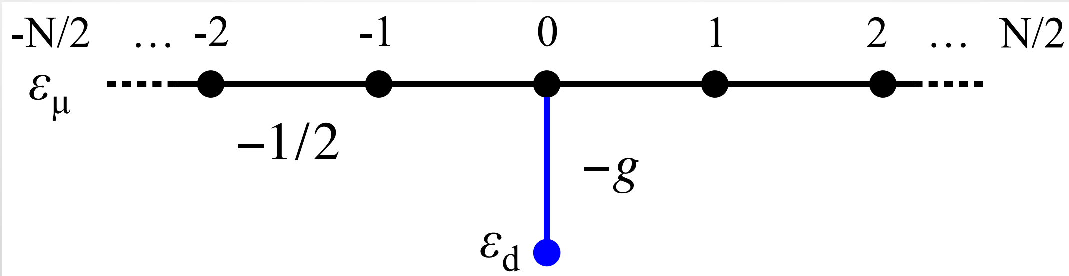
S. Garmon, T. Petrosky, L. Simine, and D. Segal, Fort. Physik, DOI: 10.1002/prop.201200077; **arXiv:1204.6141** (2012).

The transition from bound to anti-bound states also leads to a maximum in the local density of states:

Raúl A. Bustos-Marún, Eduardo A. Coronado, and Horacio M. Pastawski, Phys. Rev. B **82**, 035434 (2010).

Further demonstration: bound state trapped below threshold

Side-coupled impurity model (or T-model):



$$H = \varepsilon_d d^\dagger d + \int_{-\pi}^{\pi} \varepsilon_k c_k^\dagger c_k + \int_{-\pi}^{\pi} V_k (c_k^\dagger d + d^\dagger c_k)$$

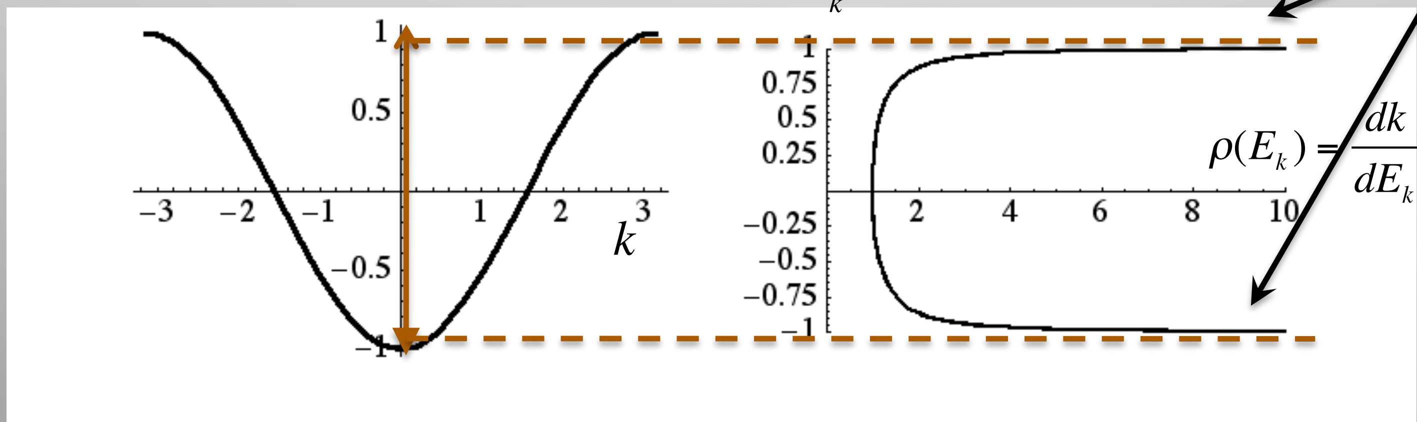
integration over the full chain

Side-coupled impurity model: van Hove singularities

$$H = \varepsilon_d d^\dagger d + \int_{-\pi}^{\pi} \varepsilon_k c_k^\dagger c_k + \int_{-\pi}^{\pi} V_k (c_k^\dagger d + d^\dagger c_k)$$

DOS singularities at $E_k = \pm B$

$E_k = -\cos k$

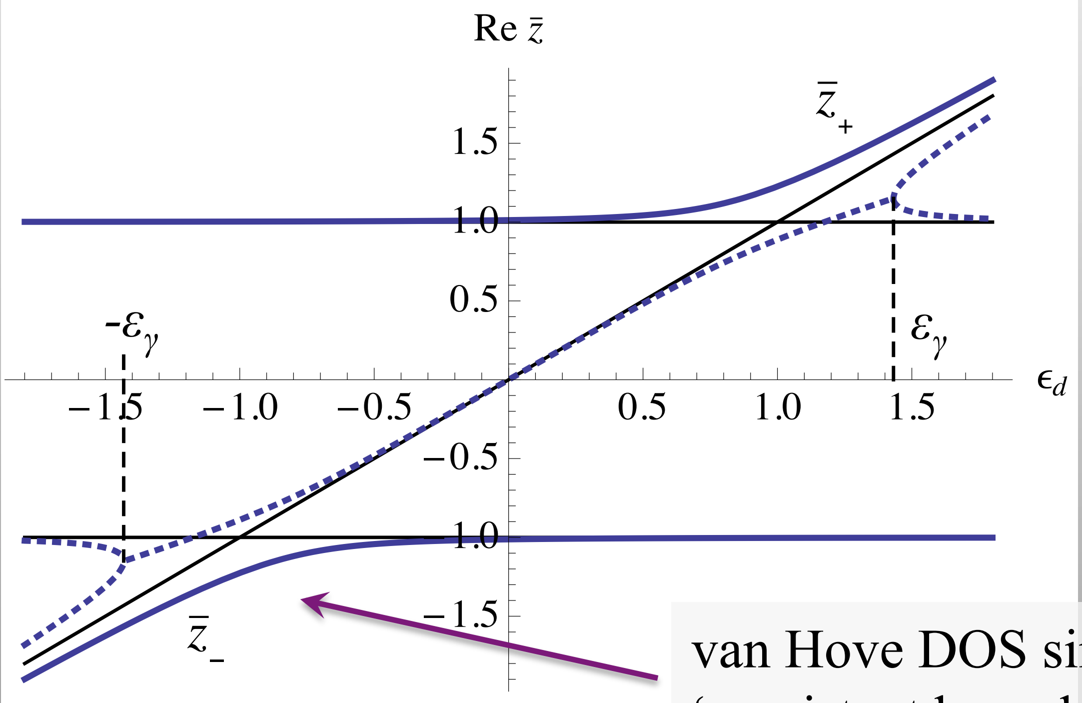


$$\rho(E_k) = \frac{dk}{dE_k} = \frac{1}{\sqrt{B^2 - E_k^2}}$$

Side-coupled impurity: bound state trapped below threshold

We again find (quartic) dispersion from resolvent method:

$$z - \varepsilon_d = \frac{g^2}{\sqrt{z^2 - 1}}$$



S. Garmon, H. Nakamura, N. Hatano, and T. Petrosky, Phys. Rev. B **80**, 115318 (2009).

van Hove DOS singularity prevents 'persistent bound state' reaching threshold

Long-time dynamics for bound state trapped below threshold

Bound state eigenvalue expansion:

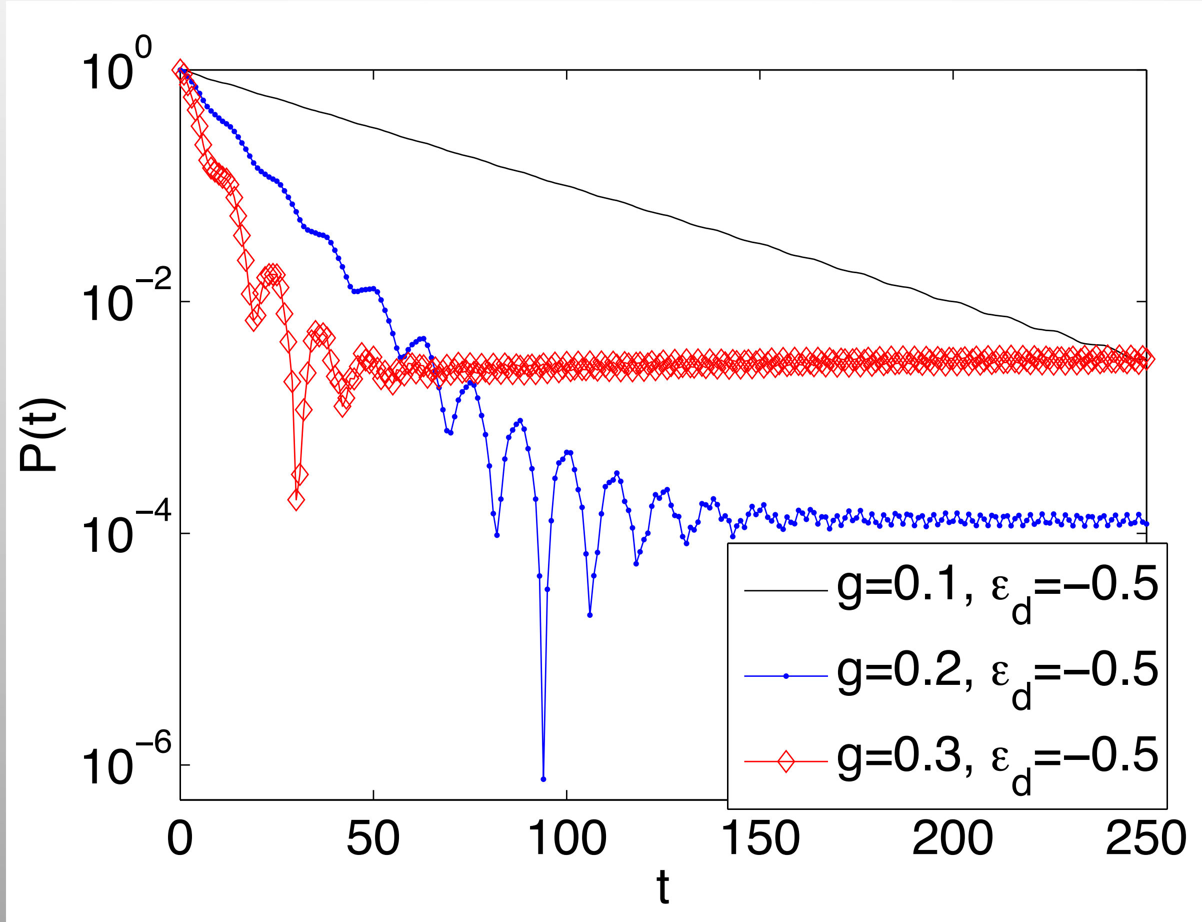
$$\bar{z}_{-} = -1 - \tilde{\Delta}_Q + O(g^8) \ \text{ with } \ \tilde{\Delta}_Q \equiv \frac{1}{2(1+\varepsilon_d)^2} g^4$$

In the near zone:

$$\left| A_{th}^{-}(t) \right|^2 = \frac{\tilde{\Delta}_Q^2}{2\pi g^2 t} = \frac{g^4}{2\pi (1+\varepsilon_d)^4 t}$$

Hence closing the gap requires ε_d goes to infinity, which kills the effect in any case.

Long-time dynamics: numerical results for side-coupled impurity model



Due to bound states, resonance, difficult to see much

Conclusions

Bound state influence on long time dynamics in OQS:

- Bound state transition to anti-bound state at continuum threshold
- Purely non-exponential dynamics when only anti-bound states are present
- Long time dynamics for prototype model:
  - Long-time near zone: $P(t) \sim t^{-1}$
  - Long-time far zone: $P(t) \sim t^{-3}$
- Amplification of non-Markovian decay as bound state transitions to anti-bound state; near zone becomes asymptotic dynamics