Complex 2D Matrix Model
and
Internal Structure of Resonances

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In collaboration with

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and Atsushi Hosaka

[arXiv:1109.0426[hep-ph]]

CONTENTS

* **Nature transition** for
  - real energy states
  - complex energy states (resonace)

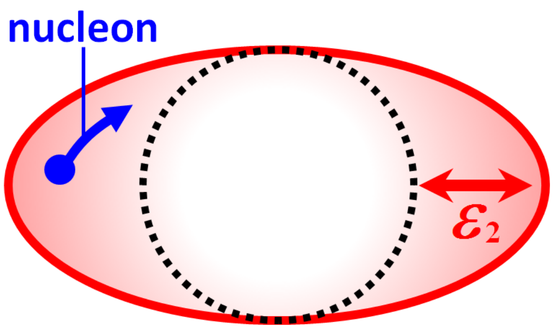
* Complex 2D Matrix Model

* Application to Hadron Physics

Introduction

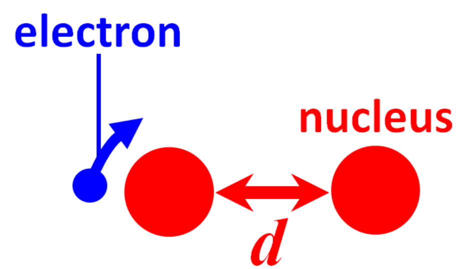
How do characters of quantum states ***change***
with ***variation of a parameter***
which specifies the property of the system or
of the environment where the system is placed ?

"deformed nuclei"



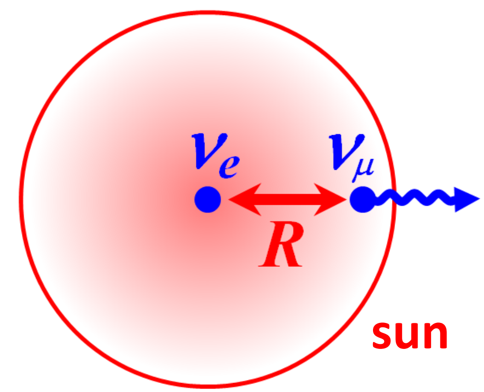
ε_2 (deformation parameter of nuclear MF potential)

"diatomic molecule"



d (inter-nuclear distance)

"solar neutrino"



R (distance from the center of the sun)

$\hat{\mathcal{H}}(\lambda)$: **Hamiltonian with** $\lambda \in \mathbf{R}$

Nature Transition

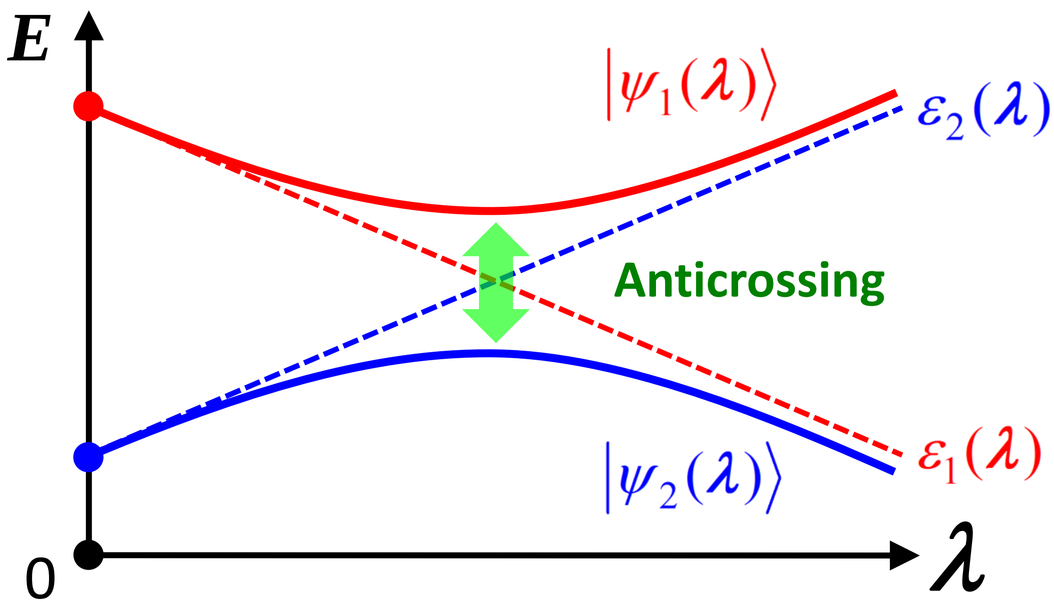
Important concept to **classify** the quantum states with variation of a parameter.

$\hat{\mathcal{H}}(\lambda)$: Hamiltonian with $\lambda \in \mathbf{R}$.

$|\phi_1\rangle, |\phi_2\rangle$: eigenstates at $\lambda = 0$.

Hamilton Matrix

$$\mathbf{H}(\lambda) = \begin{pmatrix} \langle\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_1\rangle & \langle\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_2\rangle \\ \langle\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_1\rangle & \langle\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_2\rangle \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

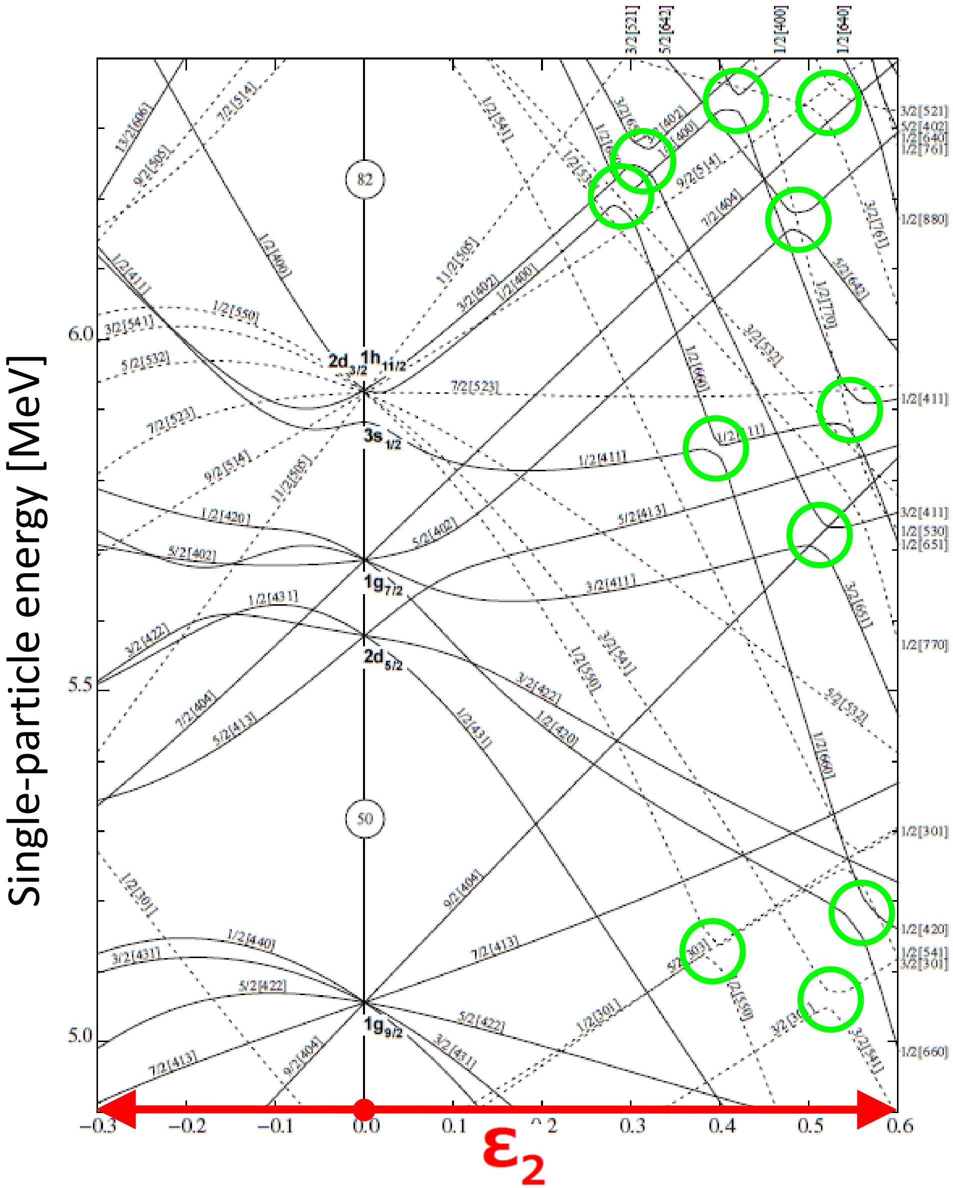


Character exchange with variation of λ.

"Nature Transition"

classification of quantum states with variation of $\lambda \in \mathbf{R}$.

Nature Transition in Nuclear Physics

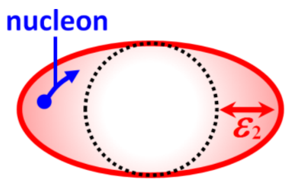


[Nilsson diagram for neutrons , 50 < N < 82]

Nilsson diagram of deformed shell model

Anisotoropic-Harmonic Oscillator

$$H = \frac{\mathbf{p}^2}{2m} + \frac{m}{2}\left\{\omega_T^2(\mathbf{x}^2 + \mathbf{y}^2) + \omega_z^2 \mathbf{z}^2\right\}$$



$$\omega_T^2 = \omega_0^2\left(1 + \tfrac{2}{3}\boldsymbol{\varepsilon_2}\right)$$

$$\omega_z^2 = \omega_0^2\left(1 - \tfrac{4}{3}\boldsymbol{\varepsilon_2}\right)$$

$$\lambda \sim \boldsymbol{\varepsilon}_2$$

deformation parameter of the mean field potential

Nature transition is important to know the structure of nuclei at each energy level **!!!**

Nature Transition for Resonance States

$$E \in \mathbf{C}$$ [G. Gamow, Z. Phys. 51, 204 (1928); 52, 510 (1928)]

$$\Rightarrow \mathsf{H}(\lambda) = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

Matrix elements become **complex**:

$$\varepsilon_i, V_{ij} \in \mathbf{C}$$

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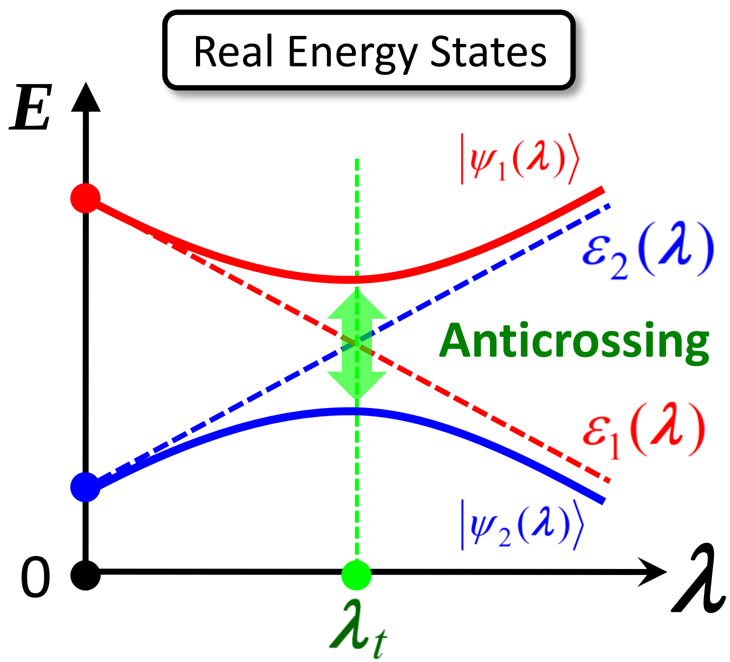

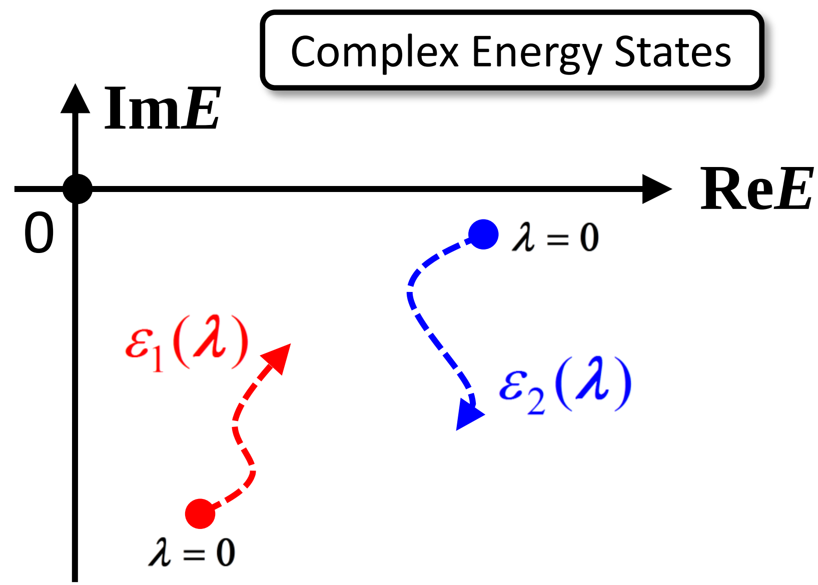

* What is the **simple criterion** of nature transition for resonances ?
* How do poles move two-dimensionally on complex-***E*** plane ?

History of Resonance States

1920〜1930 How to describe the **"α-decay of nuclei"** by quantum mechanics ?

1928 **Resonance as a complex energy state (Gamow state) :** $E \in \mathbf{C}$

[G. Gamow, Z. Phys. 51, 204 (1928); 52, 510 (1928)]

1965 **Biorthogonality** for resonance states

[N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)]

ket state : $|\phi) \equiv |\phi\rangle$, bra state : $(\phi| \equiv \langle\phi^*|$

1968 **Resonance shell model** (without completeness)

[W. Romo, Nucl. Phys. A116, 618 (1968)]

1968 **Extended completeness** with bound/resonance/continuum by modified contour integral [T. Berggren, Nucl. Phys. A109, 265 (1968)]

1971 **Complex Scaling Method (CSM)** [ABC theorem (1971)]

(Extended completeness with continuum effect controlled by a scaling paramete θ)

1970's〜 Application of CSM to **chemical physics with coulomb potential**

[N. Moiseyev, Phys. Rept. 302, 211 (1998)]

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Biorthogonal Representation of Resonance State

[N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)]
[W. Romo, Nucl. Phys. A116, 618 (1968)]
[T. Berggren, Nucl. Phys. A109, 265 (1968)]

● Biorthogonal basis :

ket state : $|\,\phi) \equiv \left|\phi\right\rangle$,   bra state : $(\phi\,| \equiv \left\langle\phi^{*}\right|$

↑ Dirac ket state   ↑ C.C. of Dirac bra state

● Biorthogonality :

$$(\phi_1\,|\,\phi_2) = \left\langle\phi_1^{*}\middle|\phi_2\right\rangle = \int d\boldsymbol{r}\;\phi_1\;\phi_2 = 0 \qquad \textbf{\textit{for}} \qquad E_1 \neq E_2 \in \mathbf{C}$$

Scalar metric product : $\left\langle\phi_1\middle|\phi_2\right\rangle = \int d\boldsymbol{r}\;\phi_1^{*}\phi_2 \neq 0$

● Complex norm:

$$(\phi_1\,|\,\phi_1) = \int d\boldsymbol{r}\;\phi_1\;\phi_1 \in \mathbf{C}$$

"**Complex probability**" implies that resonance is the object beyond the quantum mechanics.

Complex 2D Matrix Model

[K.N., et.al., arXiv:1109.0426[hep-ph]]

$\hat{\mathcal{H}}(\lambda)$: Hamiltonian with $\lambda \in \mathbf{R}$.

$|\phi_1), |\phi_2)$: eigenstates at $\lambda = 0$ in **"biorthogonal representation".**

$$|\phi) \equiv |\phi\rangle, \quad (\phi| \equiv \langle\phi^*|$$

[N. Hokkyo, Prog. Theor. Phys. 33, 1116 (1965)]

[T. Berggren, Nucl. Phys. A109, 265 (1968)]

Hamilton Matrix

$$\mathbf{H}(\lambda) = \begin{pmatrix} (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_2) \\ (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_2) \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

$$\varepsilon_i, V_{ij} \in \mathbf{C}, \quad \lambda \in \mathbf{R}$$

Complex 2D Matrix Model

Hamilton Matrix

$$\mathbf{H}(\lambda) = \begin{pmatrix} (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_1|\hat{\mathcal{H}}(\lambda)|\phi_2) \\ (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_1) & (\phi_2|\hat{\mathcal{H}}(\lambda)|\phi_2) \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

$$\varepsilon_i, V_{ij} \in \mathbf{C}$$

- Resonance eigenstates:

$$|\psi_1(\lambda)) = C_{11}(\lambda)|\phi_1) + C_{12}(\lambda)|\phi_2)$$

$$|\psi_2(\lambda)) = C_{21}(\lambda)|\phi_1) + C_{22}(\lambda)|\phi_2)$$

↑ Informations about internal structure of eigenstates

- Complex norm:

$$(\psi_i(\lambda)|\psi_i(\lambda)) = C_{i1}^2(\lambda) + C_{i2}^2(\lambda) \in \mathbf{C}$$

Assumption →

$\left|C_{ij}(\lambda)\right|^2$ can still be a guide of probability.

(suitable for narrow resonances)

Complex 2D Matrix Model

Hamilton Matrix

$$\mathbf{H}(\lambda) = \begin{pmatrix} (\phi_1 | \hat{\mathcal{H}}(\lambda) | \phi_1) & (\phi_1 | \hat{\mathcal{H}}(\lambda) | \phi_2) \\ (\phi_2 | \hat{\mathcal{H}}(\lambda) | \phi_1) & (\phi_2 | \hat{\mathcal{H}}(\lambda) | \phi_2) \end{pmatrix} = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

$$\varepsilon_i, V_{ij} \in \mathbf{C}$$

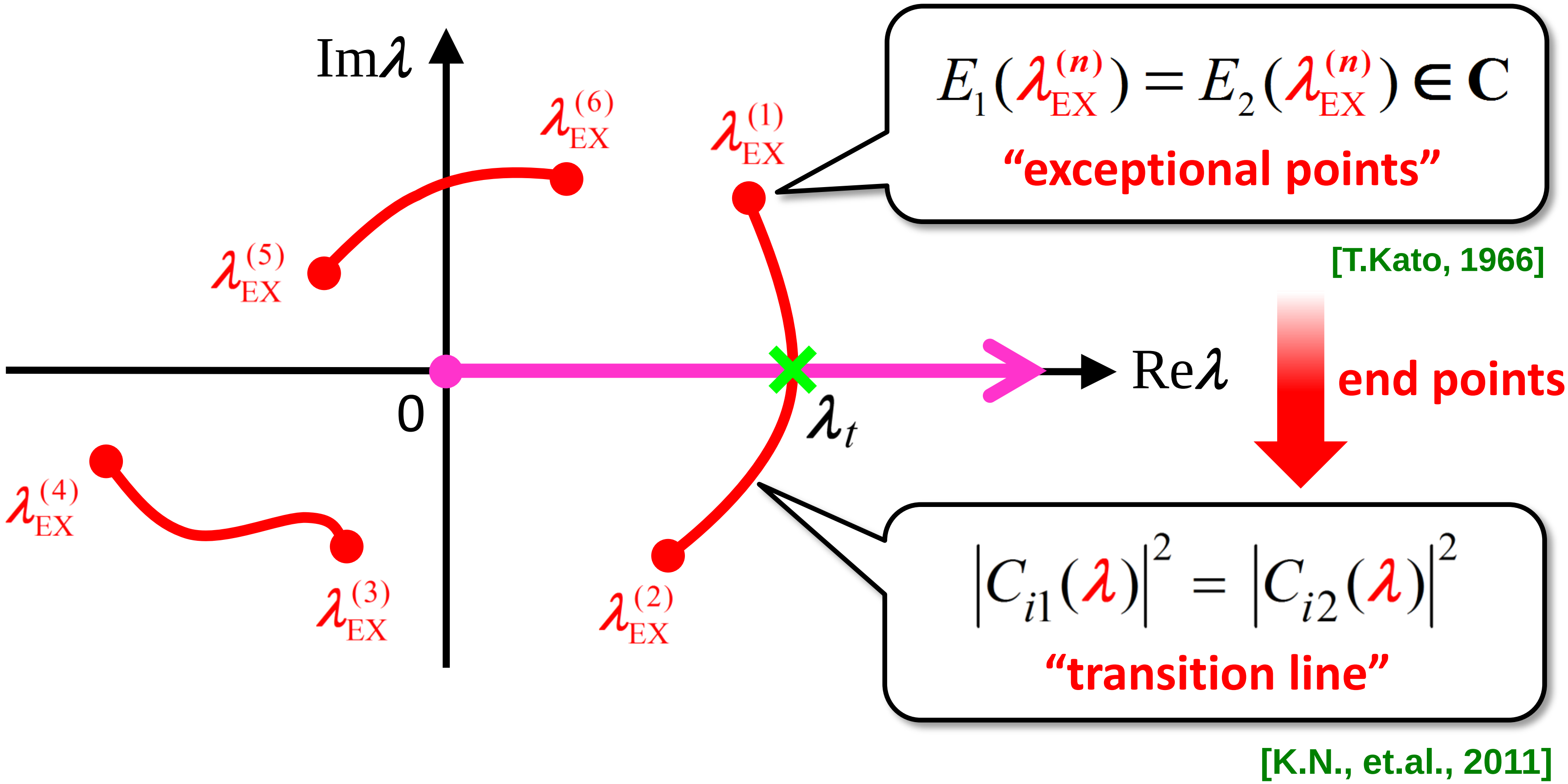
- Condition of nature transition :

$$\begin{matrix} |\psi_1(\mathbf{0})) = |\phi_1) \\ |\psi_2(\mathbf{0})) = |\phi_2) \end{matrix} \Leftrightarrow \begin{matrix} C_{11}(\mathbf{0}) = 1, \ C_{12}(\mathbf{0}) = 0 \\ C_{21}(\mathbf{0}) = 0, \ C_{22}(\mathbf{0}) = 1 \end{matrix}$$

Nature transition with character exchange:

$$\left|C_{i1}(\lambda)\right|^2 = \left|C_{i2}(\lambda)\right|^2$$

Geometry on Complex-λ Plane



"Geometry" with **"exceptional points"** and **"transition lines"**
on complex-λ plane is important
to judge the existence of nature transition
in the real parameter subspace : $\lambda \in \mathbf{R}$.

Exceptional Points

[T. Kato, *Perturbation Theory of Linear Operator*, 1966]

$$\mathbf{H}(\lambda) = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix} \qquad \varepsilon_i, V_{ij} \in \mathbf{R}, \quad V_{12} = V_{21}$$

Energy eigenvalues :

$$E_{i=1,2}(\lambda) = \frac{\varepsilon_1(\lambda)+\varepsilon_2(\lambda)}{2} \pm \sqrt{\left(\frac{\varepsilon_1(\lambda)-\varepsilon_2(\lambda)}{2}\right)^2 + V_{12}^2(\lambda)}$$

energy gap

$\lambda \in \mathbf{R}$ (1dim.) $\Longrightarrow$ $E_1(\lambda) \neq E_2(\lambda)$

(Neumann-Wigner non-crossing rule (1929))

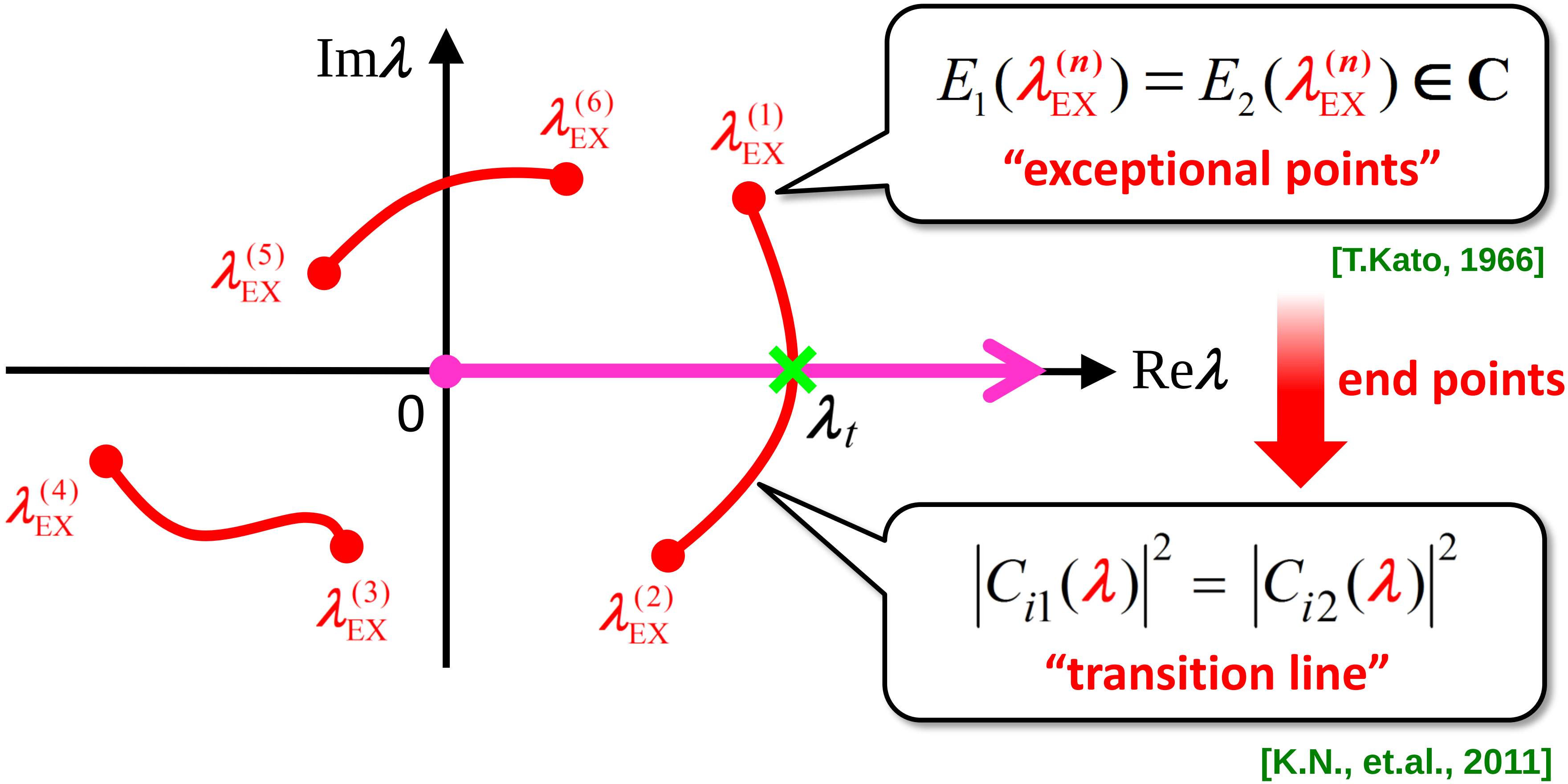
$\lambda \in \mathbf{C}$ (2dim.) $\Longrightarrow$ $E_1(\lambda_{\mathrm{EX}}^{(n)}) = E_2(\lambda_{\mathrm{EX}}^{(n)}) \in \mathbf{C}$

(isolated degenerating point = "**exceptional point**")

Dense exceptional points on complex-λ plane can be the criterion for the development of "**quantum chaos**" in the energy level statistics **!!!**

[W.D.Heiss and A.L.Sannino, PRA43,4159(1991); W.D.Heiss, PRE61, 929 (2000)]

Geometry on Complex-λ Plane



"Geometry" with **"exceptional points"** and **"transition lines"**
on complex-λ plane is important
to judge the existence of nature transition
in the real parameter subspace : $\lambda \in \mathbf{R}$.

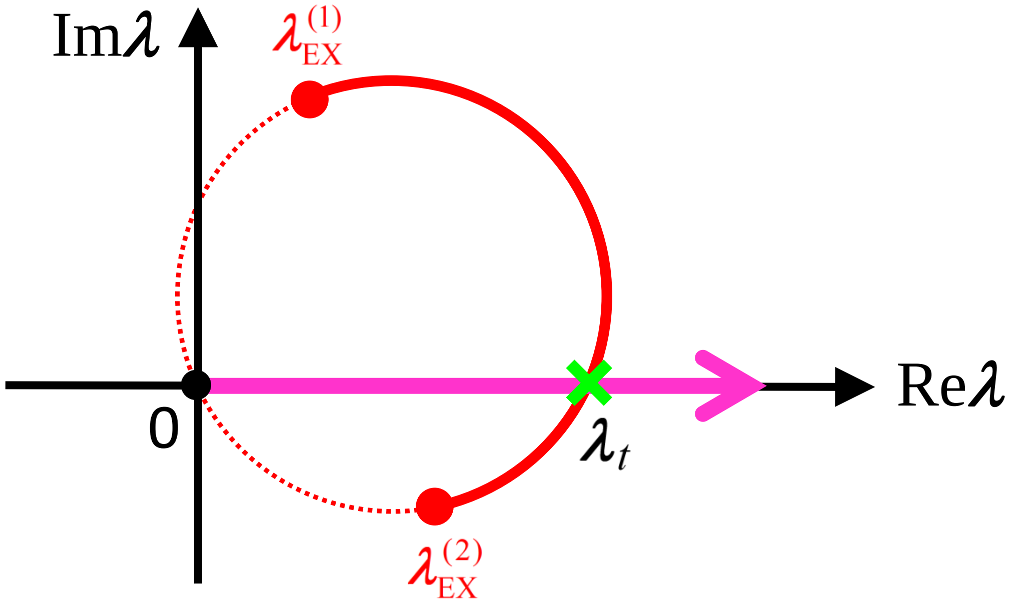
Linear-λ Model

[K.N. et.al., 2011]

$$\mathrm{H}(\lambda) = \begin{pmatrix} \varepsilon_1^{(0)} & 0 \\ 0 & \varepsilon_2^{(0)} \end{pmatrix} + \begin{pmatrix} \lambda \boldsymbol{v}_{11} & \lambda \boldsymbol{v}_{12} \\ \lambda \boldsymbol{v}_{21} & \lambda \boldsymbol{v}_{22} \end{pmatrix}$$

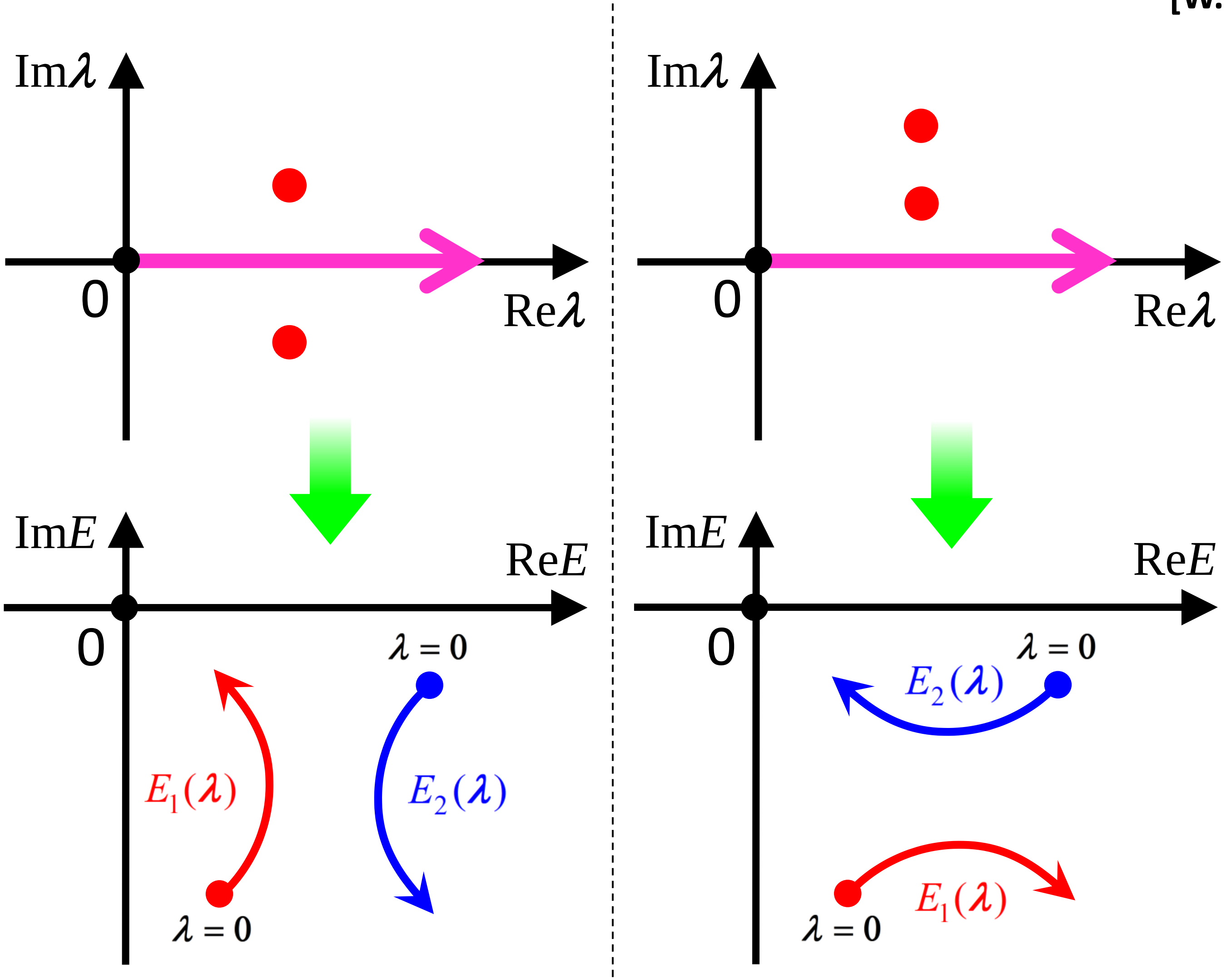
2 exceptional points : $\lambda_{\mathrm{EX}}^{(1)}$, $\lambda_{\mathrm{EX}}^{(2)}$

1 transition line (arc shape)



Exceptional Points with Pole Behavior

[W.D.Heiss, PRE61, 929 (2000)]



Level repulsion/
Width crossing

Level crossing/
Width repulsion

Exceptional Points with Pole Behavior

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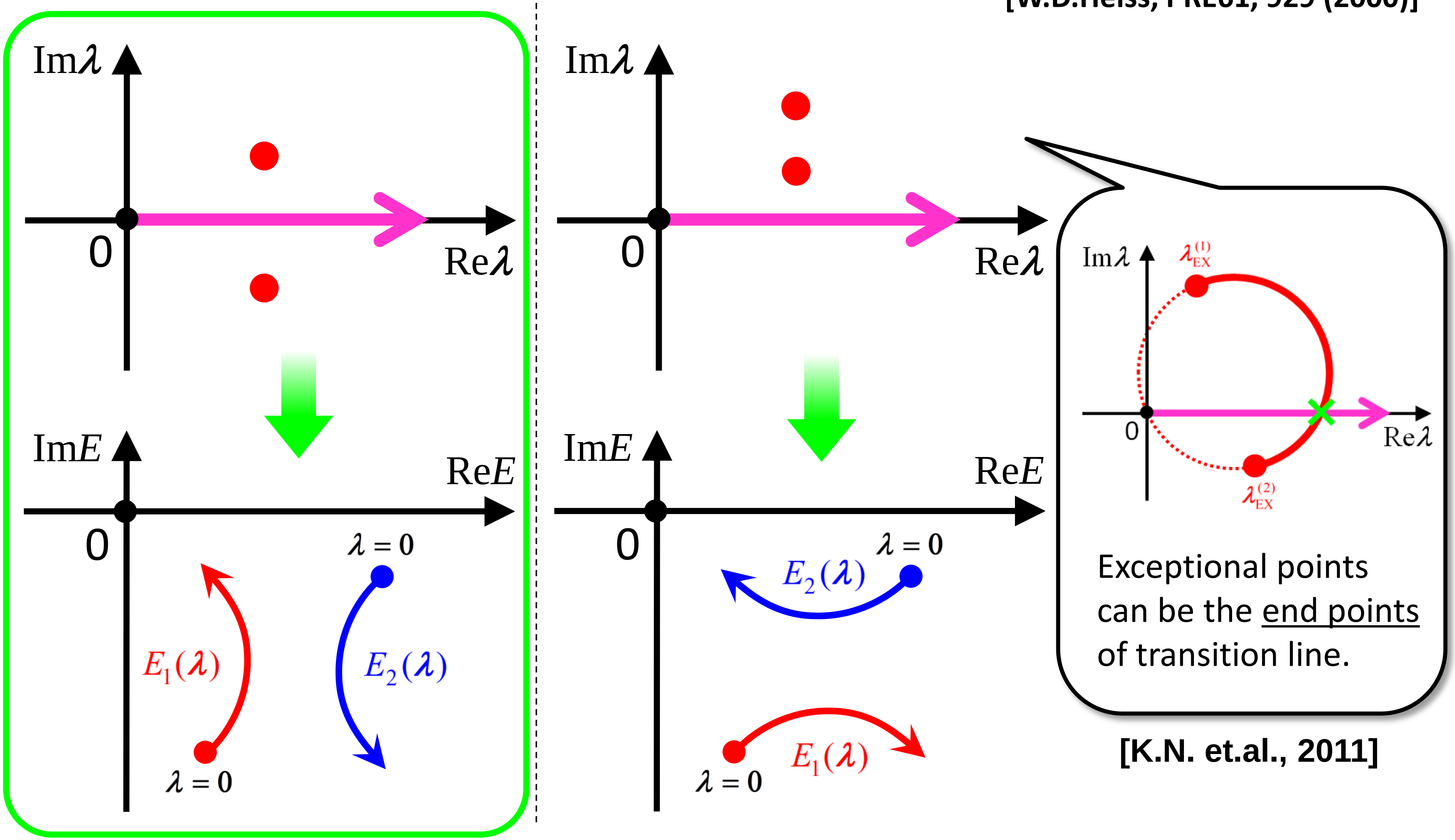
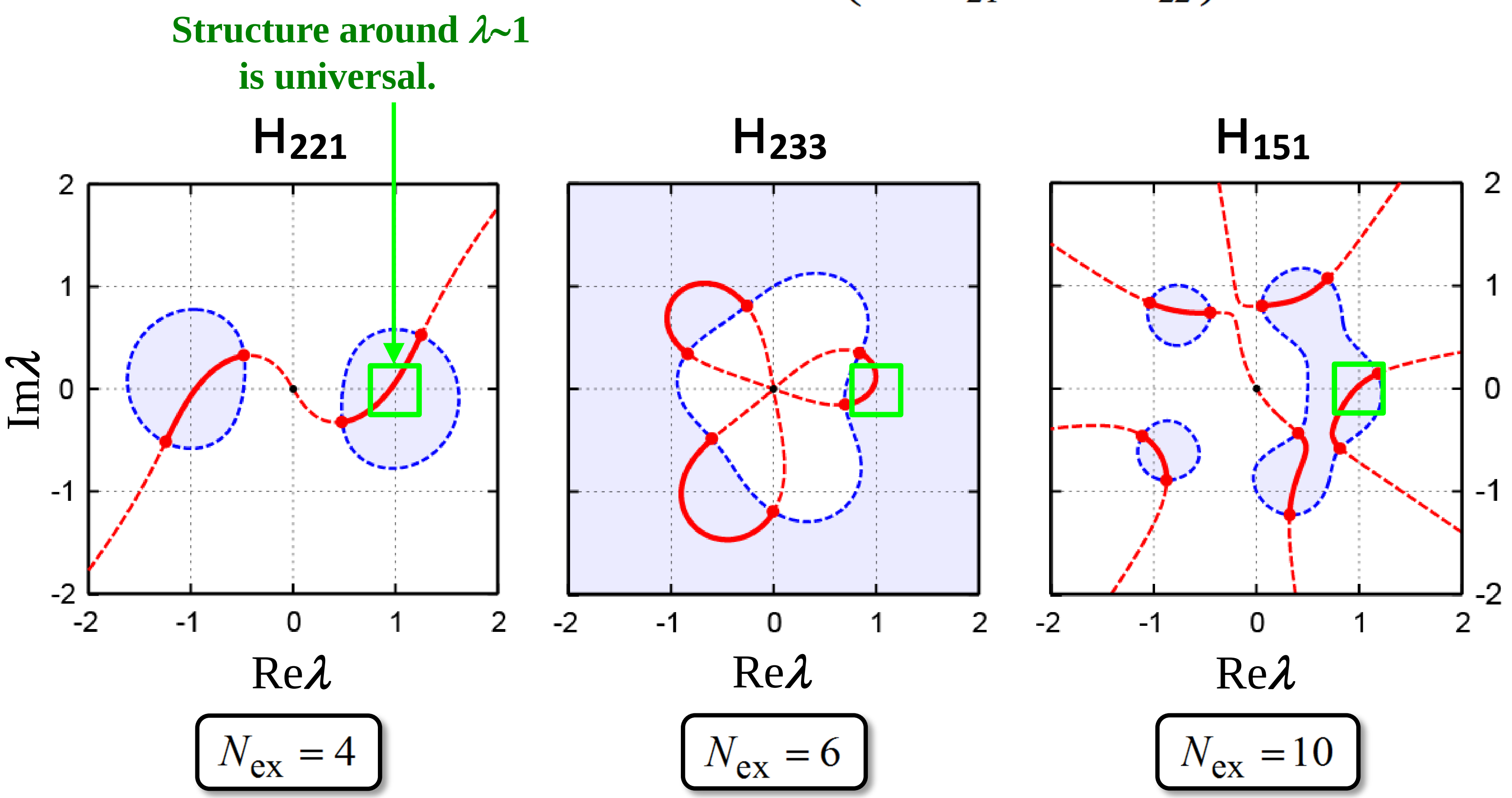


[K.N. et.al., 2011]

Nature transition occurs only for **level repulsion/width crossing** case !!!

Geometrical Maps

$$\mathbf{H}_{PQR}(\lambda) = \begin{pmatrix} \varepsilon_1^{(0)} & 0 \\ 0 & \varepsilon_2^{(0)} \end{pmatrix} + \begin{pmatrix} \lambda^P v_{11} & \lambda^R v_{12} \\ \lambda^R v_{21} & \lambda^Q v_{22} \end{pmatrix}$$



Application of Complex 2D Matrix Model to Hadron Physics

* **QCD** is SU(N_c) gauge theory with $N_c = 3$.

[Y. Nambu, 1966]

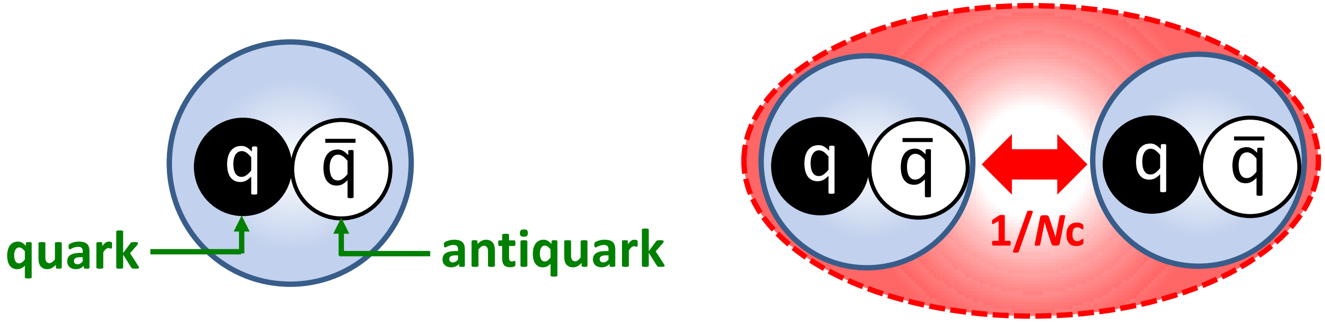
Fundamental theory of **the strong interaction** with **quarks** and **gluons**.

(QCD becomes strong coupling at low-energy scale.)

* **Large-N_c QCD** with $N_c = \infty$ gets lots of phenomenological successes.

[G.'tHooft, NPB72,461(1974);B75,461(1974); E. Witten, NPB160,57(1979)]

Mesons in Large-N_c QCD



$M_{\text{meson}} \propto O(N_c^0)$ $\quad M_{\text{exotics}} \to \infty$ ✕

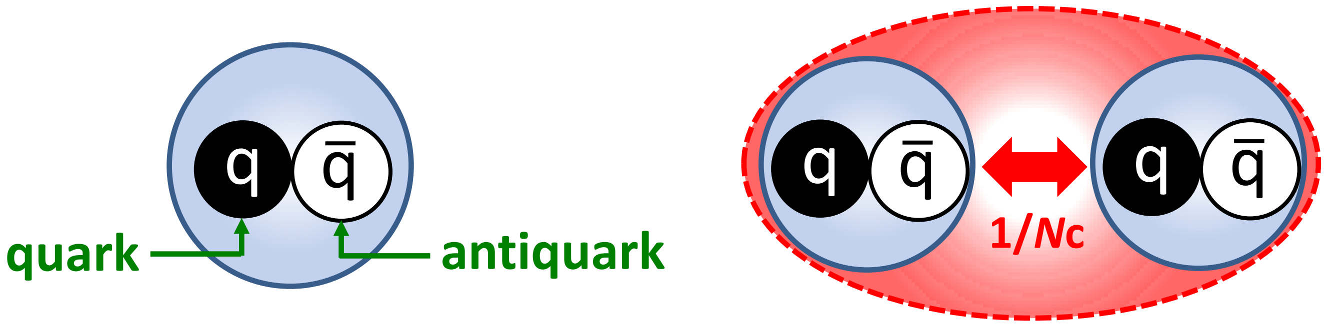
Exotics are probably not entirely absent in the real world, but they are certainly suppressed – they are certainly not conspicuous in phenomenology. The only known field theoretic reason for this suppression is the $1/N$ expansion.

[E. Witten, NPB160,57(1979)]

∗ Internal structure of hadrons should depend on *N*c.

(**It can drastically change due to the development of hadron dynamics scaled by 1/Nc.**)

Mesons in Large-N_c QCD



$M_{\text{meson}} \propto O(N_c^0)$ $\qquad$ $M_{\text{exotics}} \to \infty$ ✕

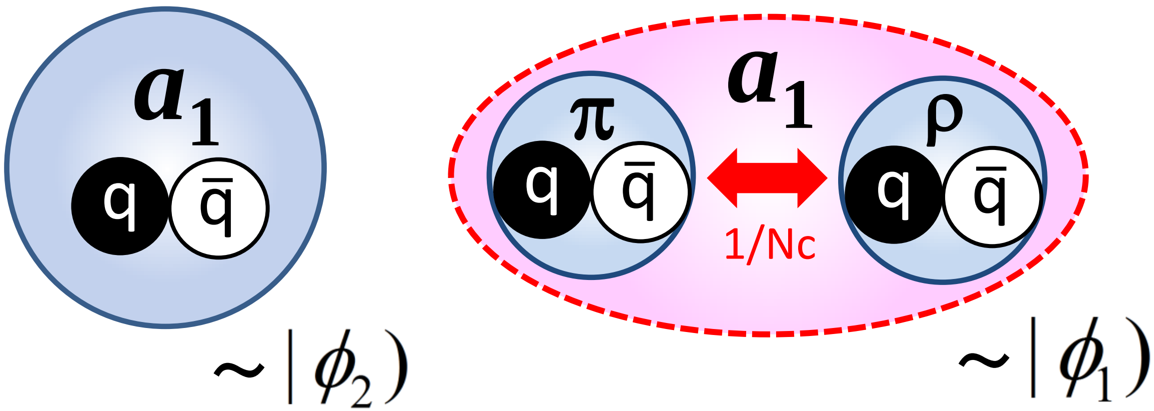
*** How does the internal structure of hadrons change from $N_c=\infty$ to $N_c=3$?**

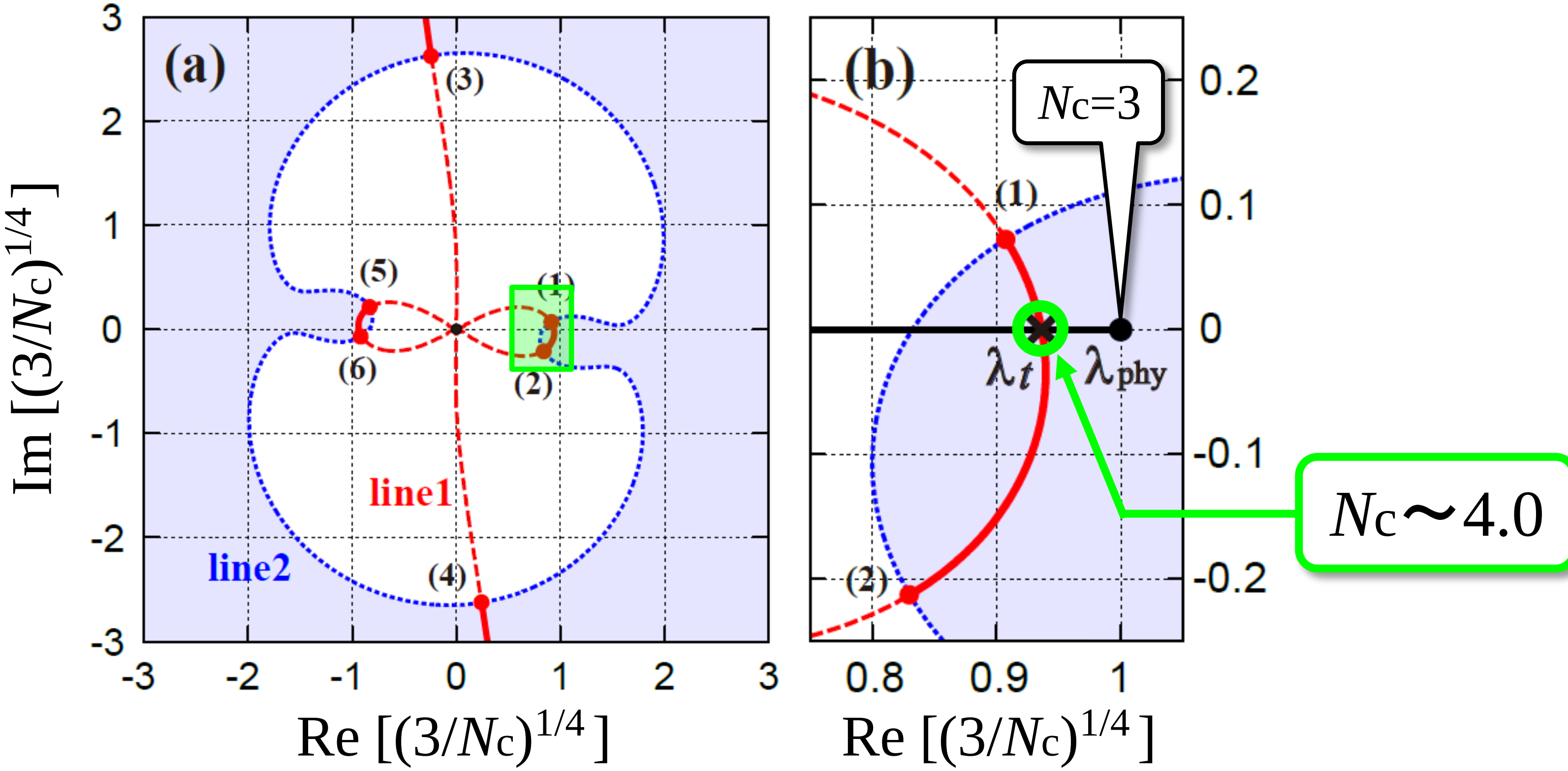
$$1/N_c \sim \lambda$$

Internal structure of hadrons can drastically changes at the "critical color number" of nature transitions.

[K.N., et.al., arXiv:1109.0426[hep-ph]]

Geometrical Map for a_1 Meson





Summary

We formulate **Complex 2D Matrix Model**
to discuss **the parameter dependence of**
the internal structure of resonances.

* **"Nature transition" with character exchange** occurs
at a critical value λ_{t}, which can be estimated
from the **geometrical map on complex-λ plane**.

* **Pole behavior on complex-E plane** and **their internal structure**
can be successfully related.

* Internal structure of hadrons can drastically change
at a **"critical color number" of nature traisition**.

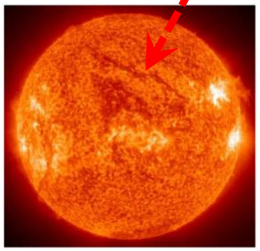
$\lambda \sim 1/N_{\mathrm{c}}$ [arXiv:1109.0426[hep-ph]]

Adiabatic Transition in Astrophysics

$$4\mathrm{P} + 2e^{-} \rightarrow {}^{4}\mathrm{He} + 6\gamma + 2\nu_e + 26.65\mathrm{MeV}$$

Less detection of solar neutrino ?

[R.Davis, 1968]



Sun

"Solar Neutrino Puzzle"

Earth

Adiabatic Transition in Astrophysics

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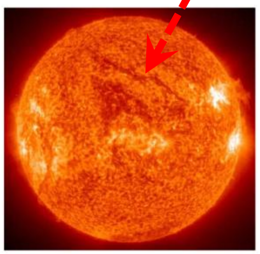
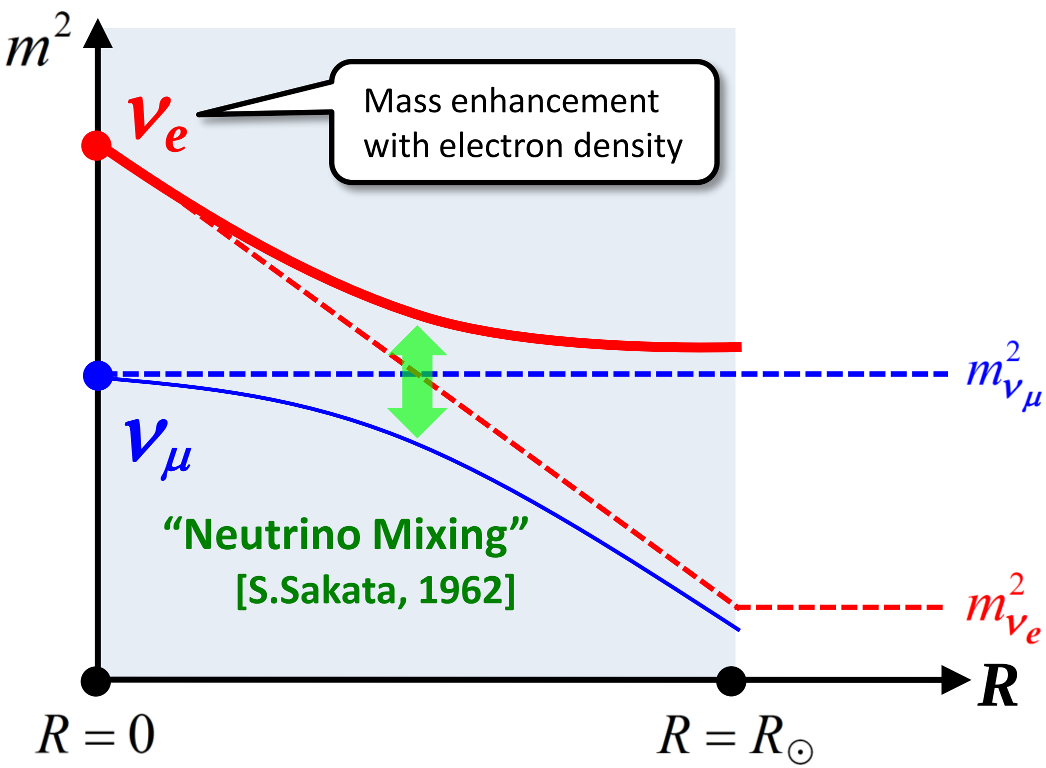


Sun

"Solar Neutrino Puzzle"

Earth



$$\lambda \sim R$$

distance from the center of the Sun

Adiabatic transition can solve the **"solar neutrino puzzle"** as the conversion of the neutrinos within the Sun !!!

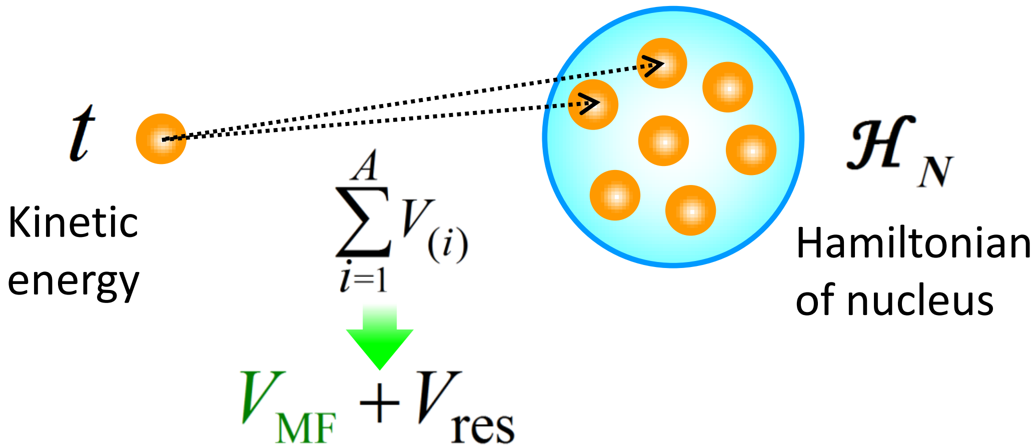
➡ **Finite mass of neutrino.**

[H.A.Bethe, PRL56,1305(1986)]

Resonance Shell Model

[W. Romo, Nucl. Phys. A116, 618 (1968)]

"a nucleon" interacting with "a nucleus" composed by A-nucleons



Solving the effective Hamiltonian in MF basis

$$\det\left(s \,|\, E - \mathcal{H}_0 - V_{\mathrm{eff}}(E) \,|\, s'\right) = 0$$

"Important" bound/resonance states of $\mathcal{H}_0 \equiv \mathcal{H}_N + t + V_{\mathrm{MF}}$

$$V_{\mathrm{eff}}(E) = V_{\mathrm{res}}\left[1 - \left(\frac{1}{E + i\varepsilon - \mathcal{H}_0} - \sum_s \frac{|s)(s|}{E - E_s}\right) V_{\mathrm{res}}\right]^{-1}$$

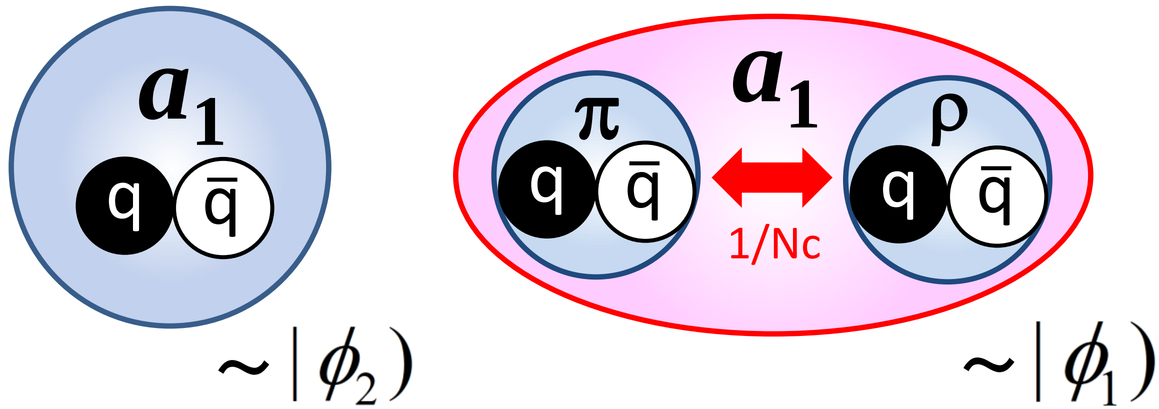
1st point

Diagonalization of effective Hamiltonian over **finite set** of shell-model states, including the effects of **neglected states** in the effective interaction.

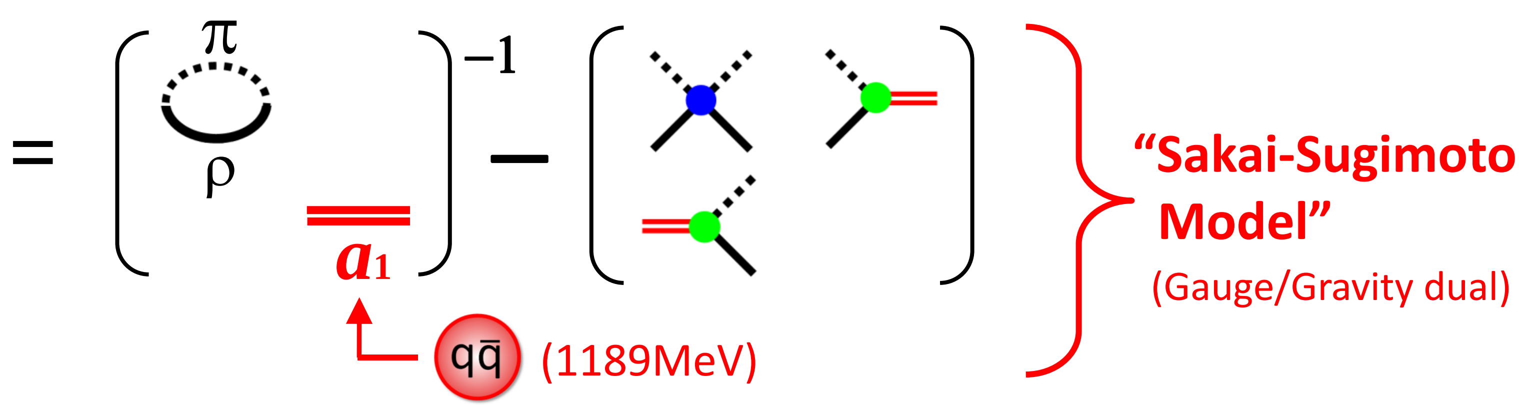
2nd point

- One may take the **traditional approximation** of shell-model as $V_{\mathrm{eff}}(E) \sim V_{\mathrm{eff}}^{\mathrm{const}}$
- Absorptive effect can be represented by $V_{\mathrm{eff}}^{\mathrm{const}} \in \mathbf{C}$

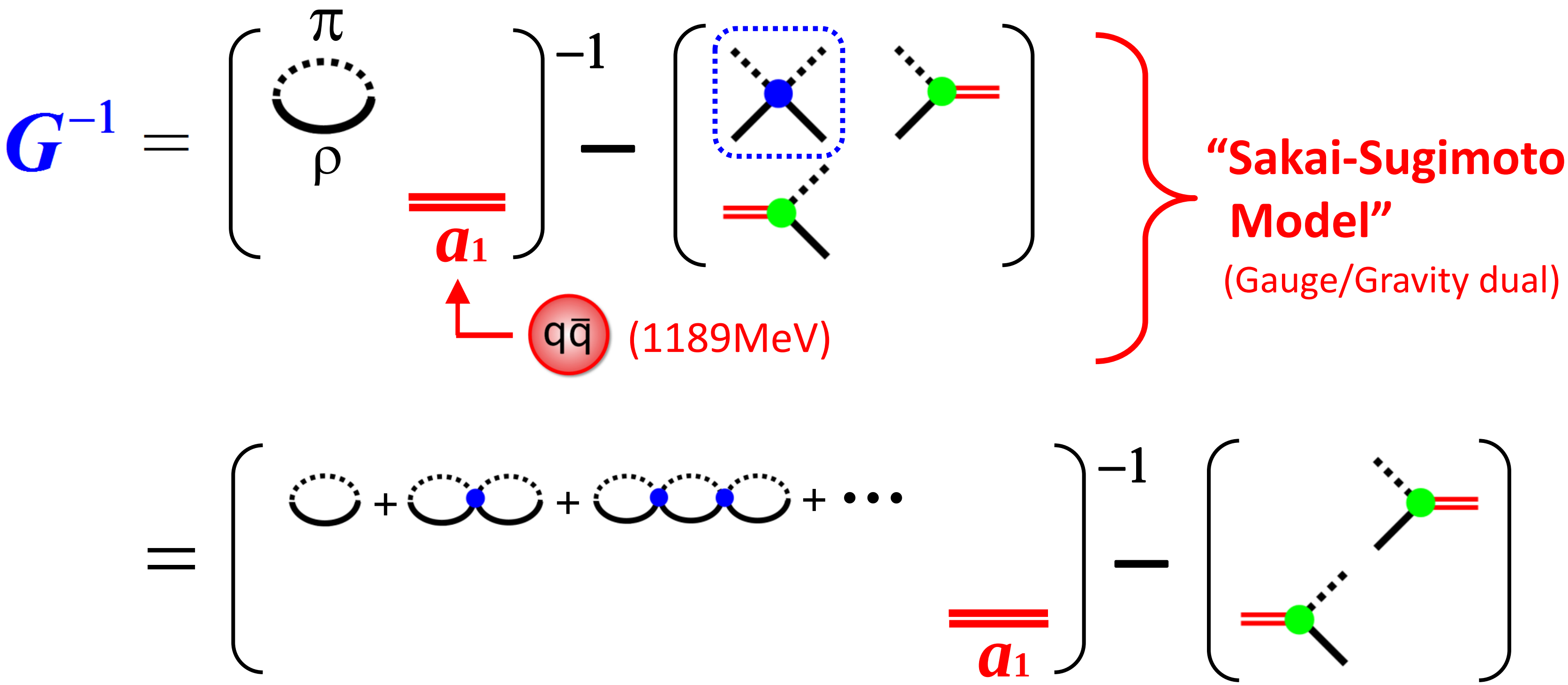
G-function Formalism for a_1 Meson



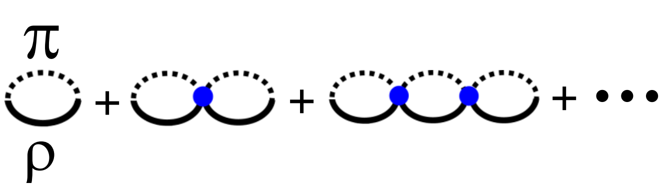
$$
\begin{aligned}
G^{-1} &= G_0^{-1} - V \\
&= \begin{pmatrix} G_{\pi\rho} & 0 \\ 0 & G_{a_1} \end{pmatrix}^{-1} - \begin{pmatrix} v_{\mathrm{WT}} & v_{a_1\pi\rho} \\ v_{a_1\pi\rho} & 0 \end{pmatrix}
\end{aligned}
$$



G-function Formalism for a_1 Meson



"Pole approximation" with non-perturbative $\pi\rho$ dynamics :



$$= \frac{Z(s)}{s - s_p}$$

: "πρ molecule state"

Geometrical Map on Complex-N_c Plane

π ρ qq̄ (large Nc, 1189MeV)

$$\bar{G} = \left[\begin{pmatrix} s - s_p & 0 \\ 0 & s - m_{a_1}^2 \end{pmatrix} - \begin{pmatrix} 0 & \sqrt{Z} v_{a_1\pi\rho} \\ \sqrt{Z} v_{a_1\pi\rho} & 0 \end{pmatrix} \right]^{-1}$$

$$\det \bar{G}^{-1} = 0$$ **[Relativistic eigenvalue Eq.]**

Non-rela. Approx. with energy fixing.

"Resonance Shell Model" **[Romo, 1968]**

$$\det[\mathcal{H} - E] = 0$$ **[Schrodinger Eq.]**

$$\mathcal{H} = \begin{pmatrix} \sqrt{s_p} & 0 \\ 0 & m_{a_1} \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2\tilde{m}} \sqrt{Z} v_{a_1\pi\rho} \\ \frac{1}{2\tilde{m}} \sqrt{Z} v_{a_1\pi\rho} & 0 \end{pmatrix}$$

Geometrical Map on Complex-Nc Plane

$$\mathcal{H} = \begin{pmatrix} \sqrt{s_p} & 0 \\ 0 & m_{a_1} \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} \\ \frac{1}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} & 0 \end{pmatrix}$$

'tHooft-Witten: $m_{a_1} \sim O(N_c^0),\ v_{a_1\pi\rho} \sim O(1/N_c^{1/2})$

Our estimates: $\sqrt{s_p} \sim O(N_c^{1/2}),\quad \tilde{m} \sim O(N_c^{1/4}),\ \sqrt{Z} \sim O(N_c^{1/2})$

$$\mathcal{H} = \begin{pmatrix} \frac{1}{\lambda^2}\sqrt{s_p} & 0 \\ 0 & m_{a_1} \end{pmatrix} + \begin{pmatrix} 0 & \lambda\frac{1}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} \\ \lambda\frac{1}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} & 0 \end{pmatrix}$$

$$\lambda \equiv \left(\frac{3}{N_c}\right)^{1/4}$$

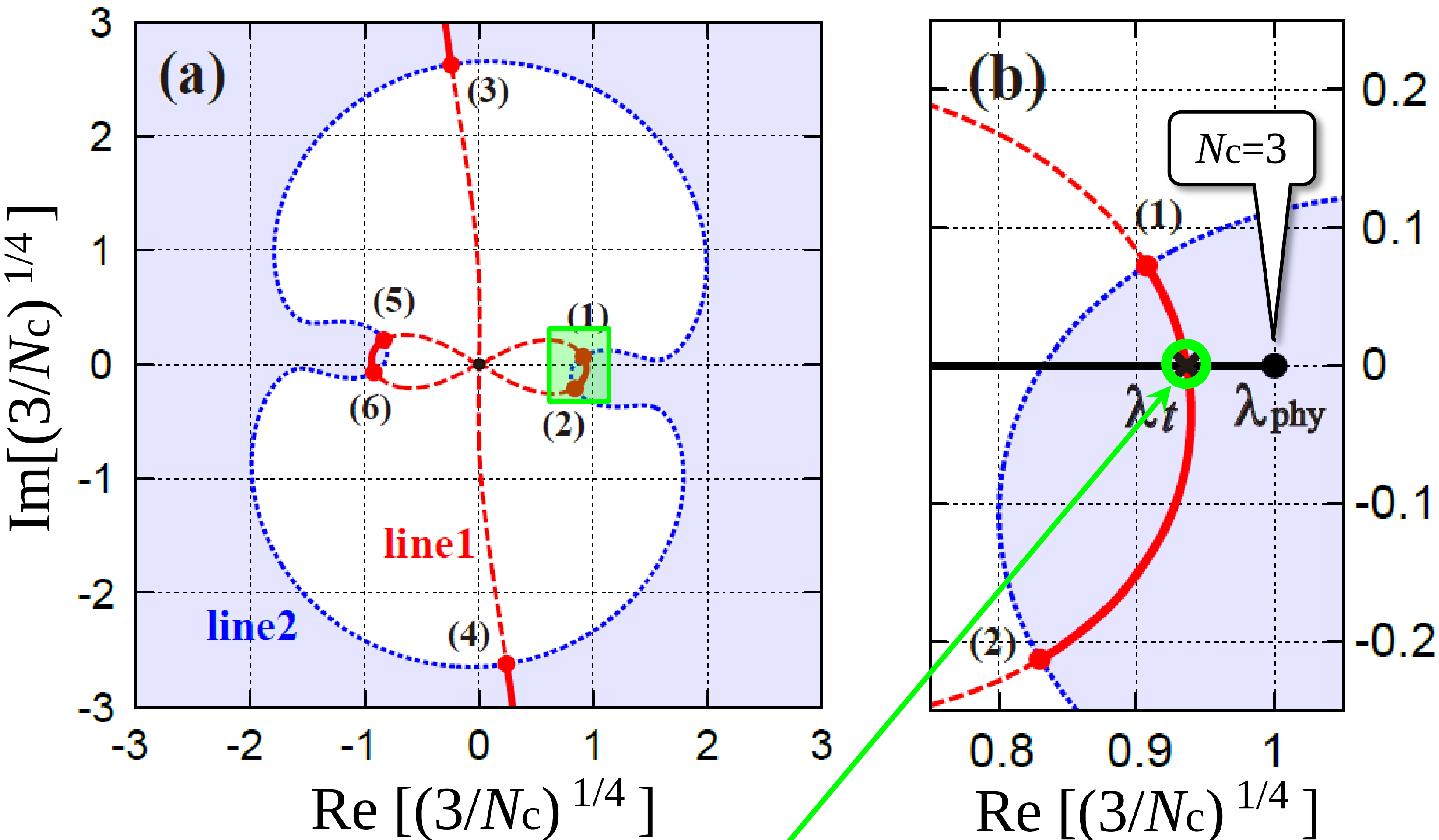
Complex 2D Matrix Model

$$\mathcal{H}(\lambda) = \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}$$

$\mathrm{Re}[A(\lambda)^* V_{12}(\lambda)] = 0$

$|A(\lambda)|^2 \leq |V_{12}(\lambda)|^2$

Geometrical Map on Complex-N_c Plane

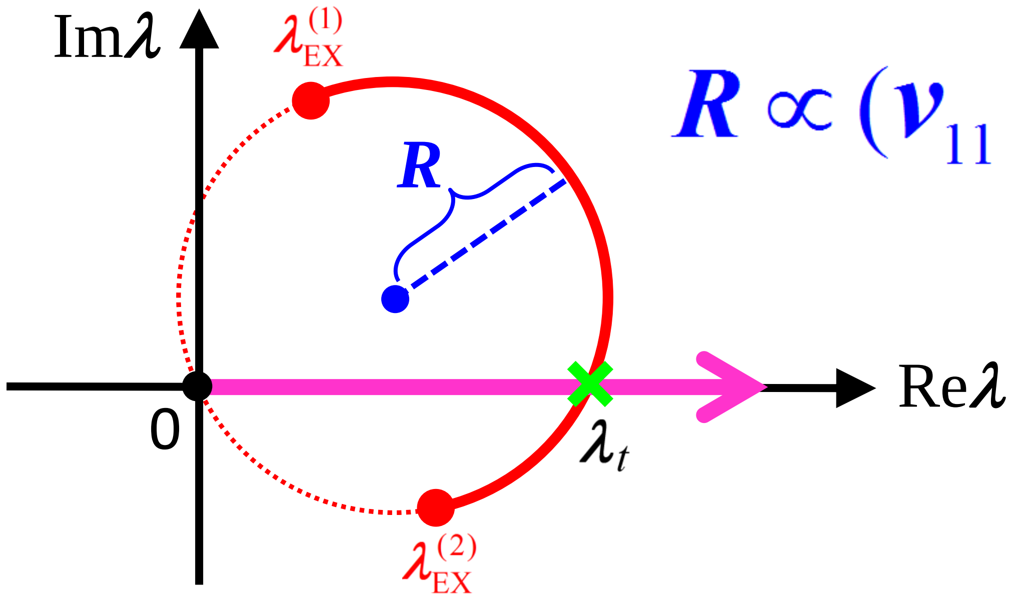


$(3/N_c)^{1/4} \sim 0.93 \rightarrow N_c \sim 4.0$

Linear-λ Model

[K.N. et.al., 2011]

$$\mathrm{H}(\lambda) = \begin{pmatrix} \varepsilon_1^{(0)} & 0 \\ 0 & \varepsilon_2^{(0)} \end{pmatrix} + \begin{pmatrix} \lambda v_{11} & \lambda v_{12} \\ \lambda v_{21} & \lambda v_{22} \end{pmatrix}$$



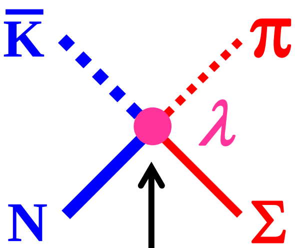
$$R \propto (v_{11} - v_{22})^{-1}$$

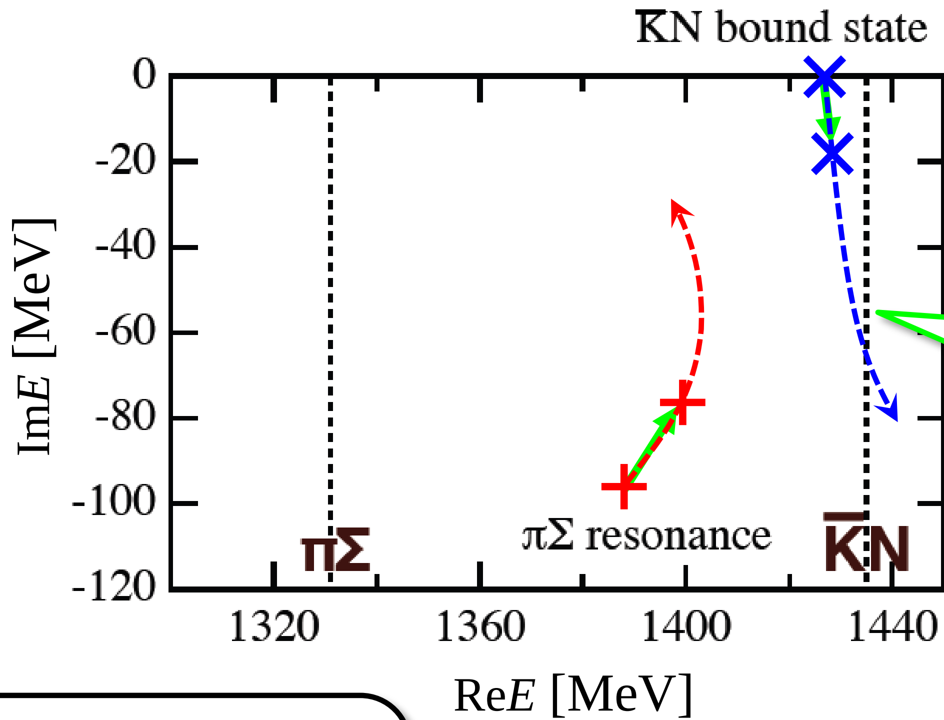
$$v_{11} = v_{22} = 0 \Rightarrow R \to \infty$$

There is no nature transition for finite λ .

(Mixing of basis component up to 50% at most.)

Application to Λ(1405)





Level repulsion/ Width crossing ?

[T. Hyodo and W. Weise, Phys. Rev. C 77, 035204 (2008)]

λ controls only the **mixing** between two levels.

$$H(\lambda) = \begin{pmatrix} \varepsilon_1^{(0)} & 0 \\ 0 & \varepsilon_2^{(0)} \end{pmatrix} + \begin{pmatrix} \lambda v_{11} & \lambda v_{12} \\ \lambda v_{21} & \lambda v_{22} \end{pmatrix}$$

$$v_{11} = v_{22} = 0$$

⇒

$$R \rightarrow \infty$$

(Transition Arc with infinite radius)

There is no nature transition for arbitrary finite value of λ !!!