Compositeness of hadrons in field theoretical approach

Tetsuo Hyodo

Tokyo Institute of Technology

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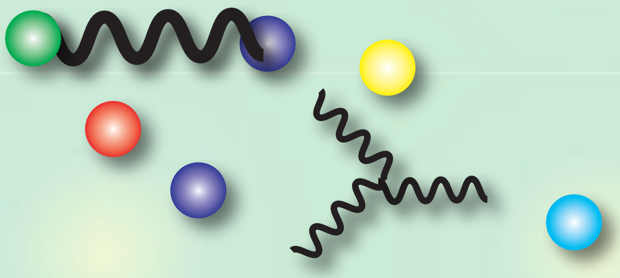
Excited hadrons

Fundamental fields in QCD: quarks and gluons



Asymptotic fields: hadrons (color singlet composites)

- **mesons ~** $q\bar{q}$, **baryons ~** qqq

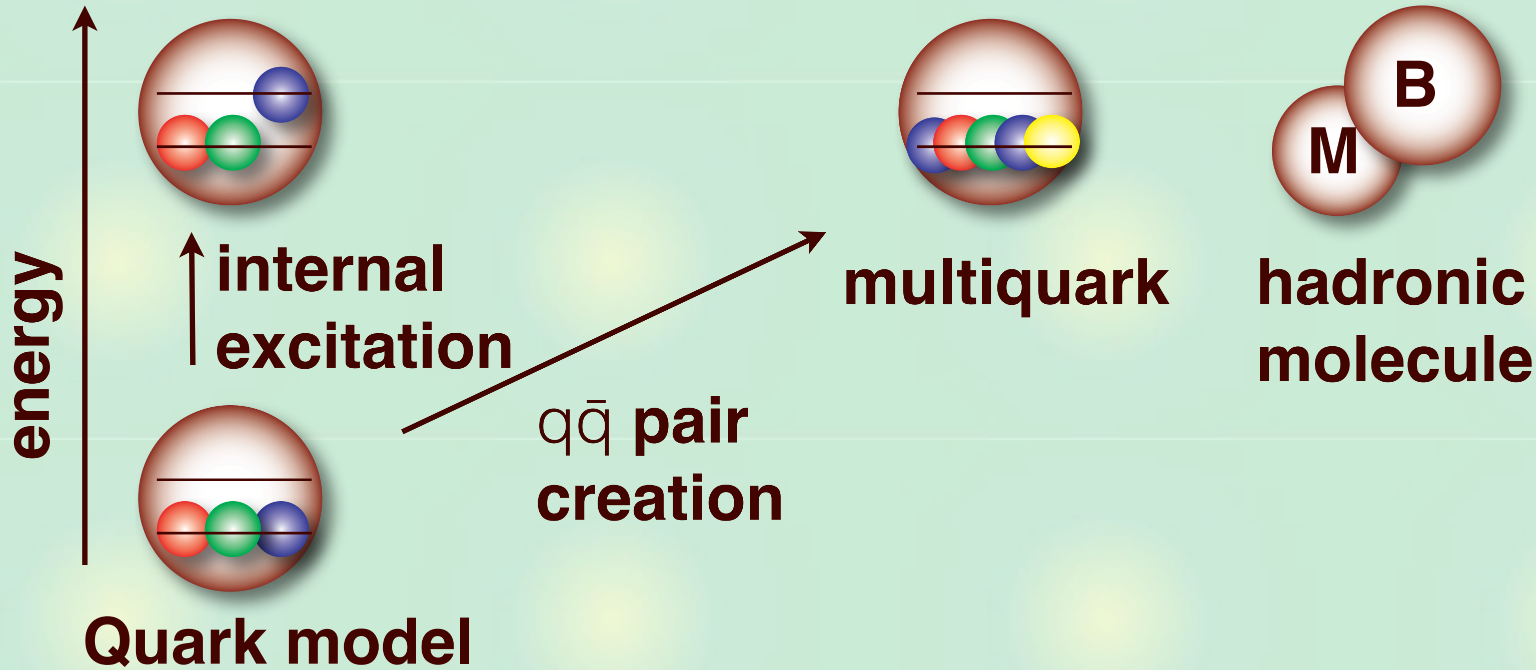
Excitation of hadrons (above two-hadron threshold):

- **Internal quark dynamics**
- **Inter-hadron dynamics (resonances)**

Structure of excited hadrons?

Structure of hadron resonances

Example) baryon excited state

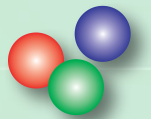
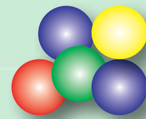


What are 3q state, 5q state, MB state, ...?

Clear (model-independent) definition of the structure?

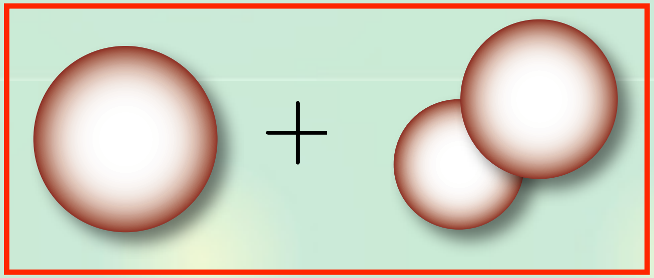
Definition of hadron structure

Number of quarks and antiquarks ($\neq$ quark number) ?

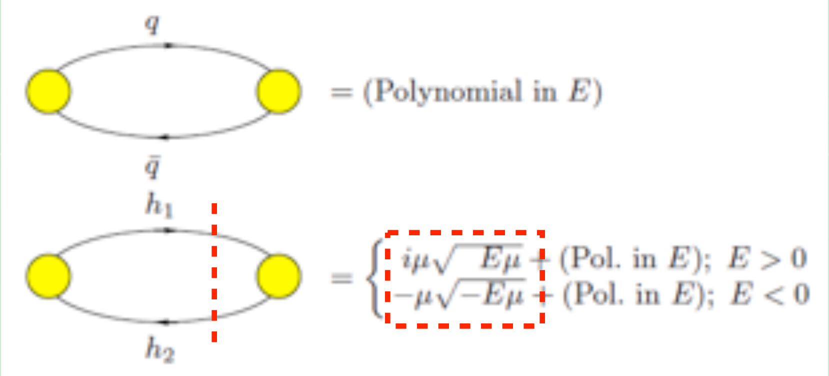
$|\,\Lambda(1405)\,\rangle =$  $+$  $+ \ldots$

may not be a good classification scheme.

Number of hadrons

$|\,\Lambda(1405)\,\rangle =$  $+ \ldots$

Hadrons are asymptotic states
--> different kinematical structure



C. Hanhart, Eur. Phys. J. A 35, 271 (2008)

Compositeness of hadrons?

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- Introduction
- Definition of compositeness

  S. Weinberg, Phys. Rev. 137, B672 (1965)

  D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

- Application to hadron models
  - Compositeness of bound states

    T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)

  - Generalization to resonances

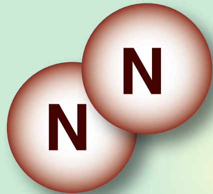
    T. Uchino, T. Hyodo, D. Jido, M. Oka, work in progress

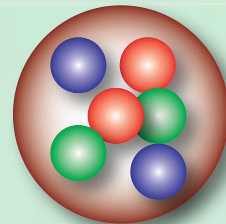
- Summary

Weinberg's compositeness and deuteron

Z: **probability of finding deuteron in a bare elementary state**

S. Weinberg, Phys. Rev. 137, B672 (1965)

$|\text{deuteron}\rangle =$ 

or 

$\notin$ NN **model space**

~ elementary particle

Z = 0 ⟷ Z = 1

model independent relation for weakly bound state

$$a_s = \left[\frac{2(1-Z)}{2-Z}\right] R + \mathcal{O}(m_\pi^{-1}), \quad r_e = \left[\frac{-Z}{1-Z}\right] R + \mathcal{O}(m_\pi^{-1})$$

a_s: **scattering length**

r_e: **effective range** **<-- Experiments**

R: **deuteron radius (binding energy)**

$$a_s = +5.41 \ [\text{fm}], \quad r_e = +1.75 \ [\text{fm}], \quad R \equiv (2\mu B)^{-1/2} = 4.31 \ [\text{fm}]$$

$\Rightarrow Z \lesssim 0.2$ **--> deuteron is almost composite!**

Definition of the compositeness 1-Z

Hamiltonian of two-body system: free + interaction V

$$\mathcal{H} = \mathcal{H}_0 + V$$

Complete set for free Hamiltonian: bare |B0 > + continuum

$$1 = |\, B_0 \,\rangle\langle\, B_0 \,| + \int d\boldsymbol{k} |\, \boldsymbol{k} \,\rangle\langle\, \boldsymbol{k} \,|$$

$$\mathcal{H}_0 |\, B_0 \,\rangle = E_0 |\, B_0 \,\rangle, \quad \mathcal{H}_0 |\, \boldsymbol{k} \,\rangle = E(\boldsymbol{k}) |\, \boldsymbol{k} \,\rangle$$

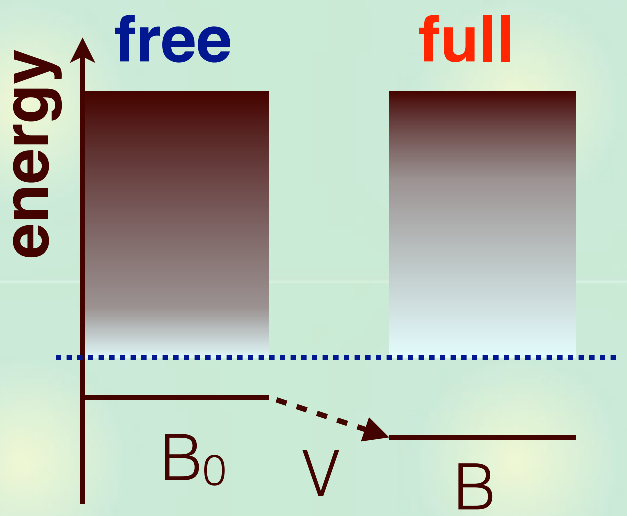
Physical bound state |B> : eigenstate of full Hamiltonian

$$(\mathcal{H}_0 + V) |\, B \,\rangle = -B |\, B \,\rangle$$

B: binding energy

Define Z as the overlap of B and B0 : probability of finding the physical bound state in the bare state |B>

$$Z \equiv |\langle\, B_0 \,|\, B \,\rangle|^2$$



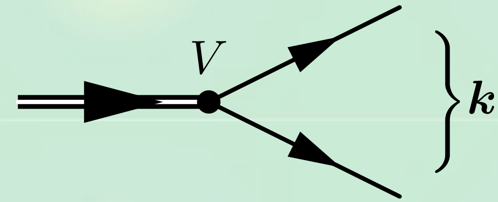
1 - Z : Compositeness of the bound state

Model-independent but approximated method

With the Schrödinger equation, we obtain

$$1 - Z = \int d\boldsymbol{k} \frac{|\langle\, \boldsymbol{k}\, |V|\, B\,\rangle|^2}{[E(\boldsymbol{k}) + B]^2}$$

$\langle\, \boldsymbol{k}\, |V|\, B\,\rangle :\quad B$



$$= 4\pi\sqrt{2\mu^3}\int_0^\infty dE \frac{\sqrt{E}|G_W(E)|^2}{(E+B)^2}$$

$\langle\, \boldsymbol{k}\, |V|\, B\,\rangle \equiv G_W[E(\boldsymbol{k})]$ **for s-wave**

Approximation: For small binding energy B<<1**, the vertex** $G_W(E)$ **can be regarded as a constant:** $G_W(E) \sim g_W$

Then the integration can be done analytically, leading to

$$1 - Z = 2\pi^2\sqrt{2\mu^3}\frac{g_W^2}{\sqrt{B}}$$

Compositeness <-- coupling g_W **and binding energy** B

S. Weinberg, Phys. Rev. 137 B672-B678 (1965)

- **Model-independent: no information of** V
- **Approximated: valid only for small** B

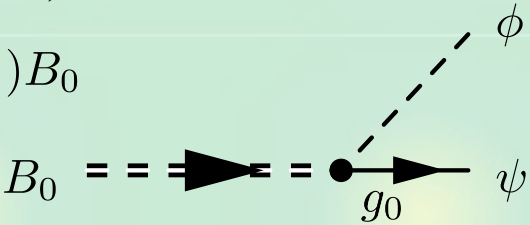
Z in Yukawa model

Field theory with Yukawa coupling (ψ, φ, B_0)

c.f. D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2\phi^2) + \bar{B}_0(i\partial\!\!\!/ - M_{B_0})B_0$$

$$\mathcal{L}_{\rm int} = g_0 \bar{\psi}\phi B_0 + ({\rm h.c.})$$



Physical bound state B at total energy $W=M_B$

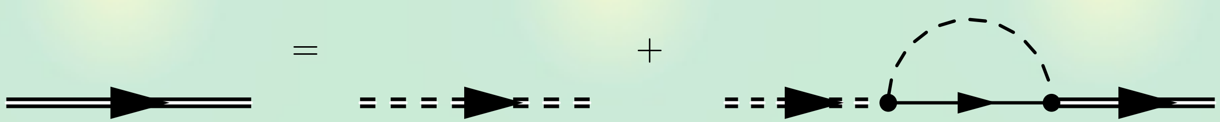
Free (full) propagator of B_0 (B) field (positive energy part)

$$\Delta_0(W) = \frac{1}{W - M_{B_0}}, \quad \Delta(W) = \frac{Z}{W - M_B}$$

Z: field renormalization constant

Dyson equation: relation between full and free propagators

$$\Delta(W) = \Delta_0(W) + \Delta_0(W) g_0 G(W) g_0 \Delta(W)$$



Master formula of compositeness

Solution of Dyson equation and renormalization

$$\Delta(W) = \frac{1}{W - M_{B_0} - g_0^2 G(W)} \to \frac{1}{W - g_0^2 G(W; a)}$$

Renormalization condition, pole at W=M_B : $M_B = g_0^2 G(M_B; a)$

The field renormalization constant: residue of the propagator

$$Z = \lim_{W \to M_B} \frac{W - M_B}{W - g_0^2 G(W; a)} = \frac{1}{1 - g_0^2 G'(M_B)}$$

Physical coupling constant: residue of T-matrix

$$g^2 = g_0^2 Z$$

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B)$$

Compositeness: summary

Compositeness of bound states

Method 1: nonrelativistic quantum mechanics

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2 (M + m - M_B)} \quad \text{for } M_B \to M + m$$

model independent, but valid only for weak binding

Method 2: field theory with Yukawa coupling

$$1 - Z = -g^2 G'(M_B)$$

exact (any M_B**), but Lagrangian dependent**

Compositeness <-- mass "M_B" **and coupling constant** "g"

↑

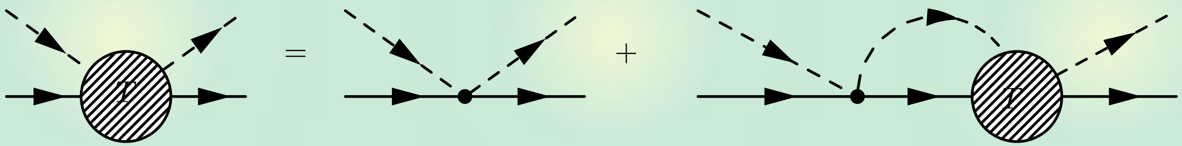
Experiments, Lattice QCD, Model calculation, ...

Dynamical chiral model

Chiral coupled-channel approach: MB **scattering,** B*

- Interaction <-- chiral symmetry

- Amplitude <-- unitarity in coupled channels



A review: [T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)]

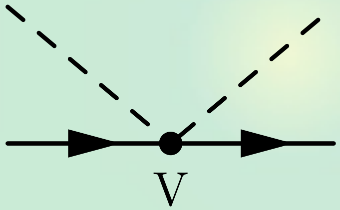
Test: single-channel scattering of meson m **and baryon** M.

$$T(W) = \frac{1}{1 - V(W)G(W;a)} V(W)$$

cutoff parameter

V: **4-point interaction, attractive**

$$V(W) = \begin{cases} V^{(\mathrm{const})} = Cm & \text{constant interaction} \\ V^{(\mathrm{WT})}(W) = C(W-M) & \text{WT interaction} \end{cases}$$



Natural renormalization condition

Mass and coupling of the bound state in dynamical model

Mass: bound state condition (pole at W=M_B)

$$1 - V(M_B)G(M_B;a) = 0$$

Coupling constant: residue of the pole

$$g^2 = \lim_{W \to M_B} (W - M_B)T(W) = \begin{cases} -[G'(M_B)]^{-1} & \text{constant interaction} \\ -\left[G'(M_B) + \dfrac{G(M_B;a)}{M_B - M}\right]^{-1} & \text{WT interaction} \end{cases}$$

Apply the master formula of compositeness

$$1 - Z = -g^2 G'(M_B)$$

Compositeness of bound states

Compositeness in Yukawa theory

$$1 - Z = -g^2 G'(M_B) = \begin{cases} 1 & \text{constant interaction} \\ \left[1 + \dfrac{G(M_B; a)}{(M_B - M) G'(M_B)}\right]^{-1} & \text{WT interaction} \end{cases}$$

- constant interaction --> purely composite bound state
- WT interaction --> mixture of composite and elementary
- Purely composite bound state for WT interaction:

$$G'(M_B) = -\infty \quad \text{or} \quad G(M_B; a) = 0$$

$$M_B = M + m \quad \text{or} \quad C \to -\infty$$

1) zero energy bound state
2) infinitely strong two-body attraction

Model space ≠ structure of generated resonances

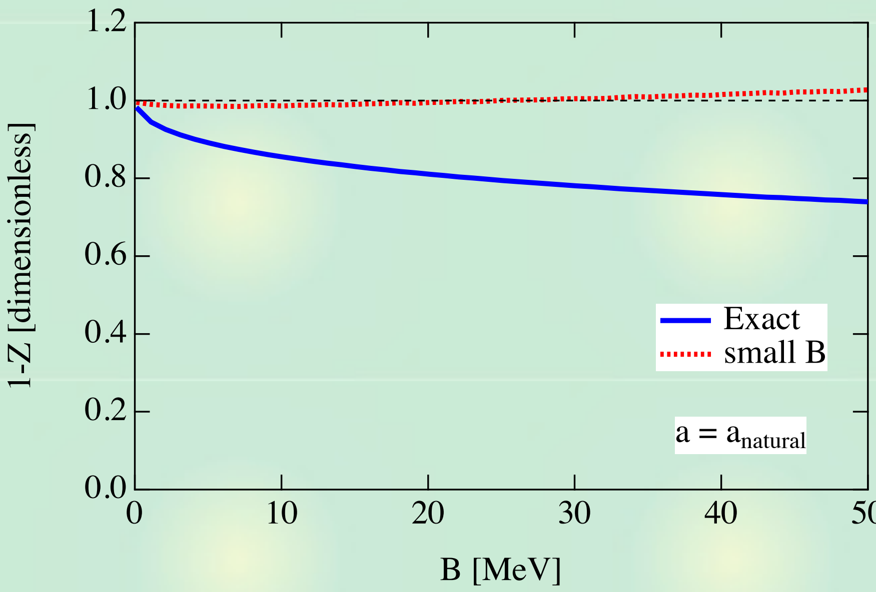
Check of natural renormalization scheme

WT Natural renormalization condition
<-- to exclude elementary contribution from the loop function

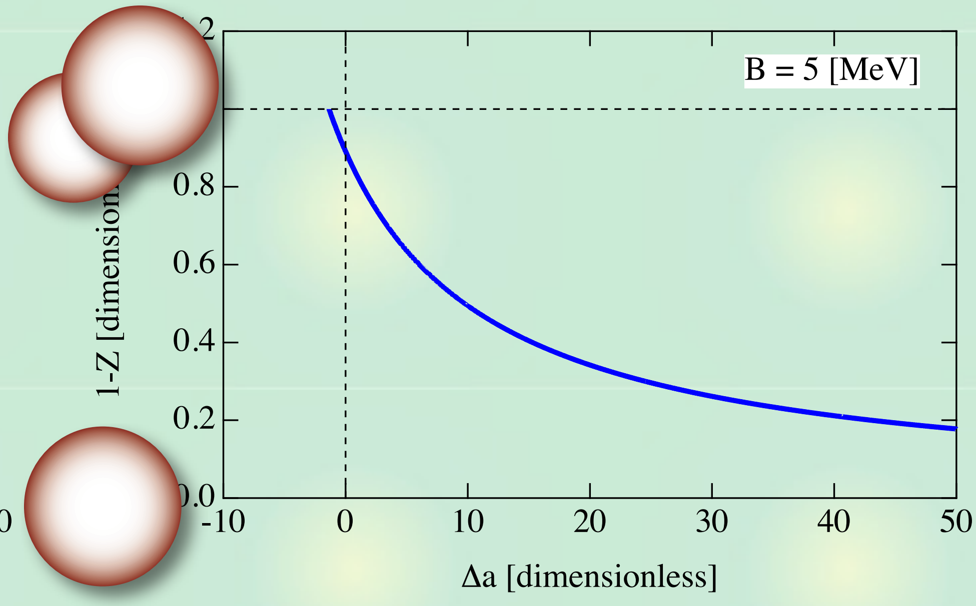
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C78, 025203 (2008)

$$G(W = M; a_{\text{natural}}) = 0$$

1) a = a_{natural}, **vary** B



2) B = 5 **MeV, vary** a



natural scheme --> Z ~ 0

large deviation --> Z ~ 1

Application to resonances

Naive generalization: input pole position and residue

$$1 - Z = -g^2 G'(M_B)$$

- **no completeness:** Z **is not normalized**
- g, M_B **are complex:** Z **is complex**

Number of degrees of freedom

Bound states: g, M_B **<-->** g_0**,** a

2 2

Resonances: Re g**,** Im g**,** Re z_R**,** Im z_R **<-->** g_0**,** a

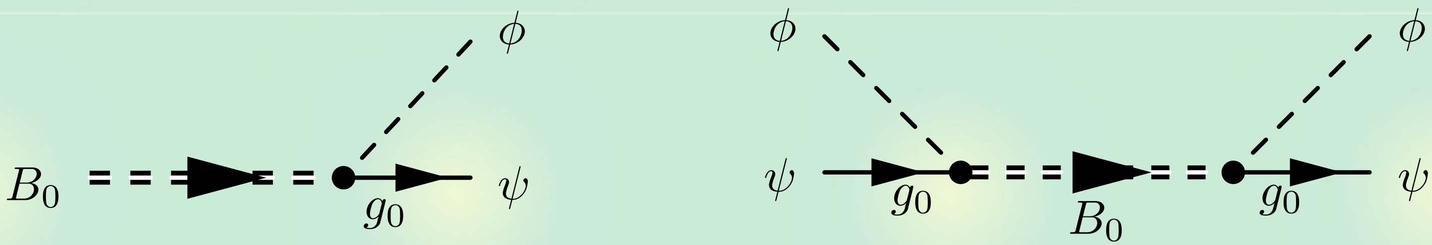
4 2

Definition with simple Yukawa model is insufficient?

Extension of the Yukawa model

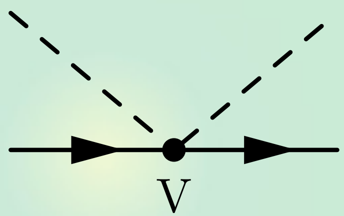
Yukawa model:

g_0 controls both $B_0\psi\varphi$ coupling and $\psi\varphi$ --> $\psi\varphi$



Add contact interaction to control $\psi\varphi$ --> $\psi\varphi$

$$V = \frac{g_0^2}{W - M_0} + \boxed{V_{\rm con}}$$



- wavefunction renormalization + vertex renormalization

$$g^2 = Z Z_3 g_0^2$$

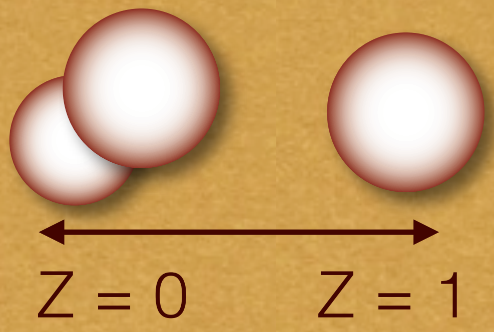
Origin of the phase of the residue?

T. Uchino, T. Hyodo, D. Jido, M. Oka, work in progress

Summary 1

Compositeness of the bound state

- **Field renormalization constant Z: compositeness**



- **Model independent formula**

$$1 - Z_{NR} = g^2 \frac{M|\lambda^{1/2}(M_B^2, M^2, m^2)|}{16\pi M_B^2 (M + m - M_B)} \quad \text{for } M_B \to M + m$$

S. Weinberg, Phys. Rev. 137 B672 (1965)

- **Exact formula**

$$1 - Z = -g^2 G'(M_B)$$

D. Lurie and A. J. Macfarlane, Phys. Rev. 136, B816 (1963)

- **Expressed in terms of physical quantities**

Summary 2

Application to hadron models

- Bound state by energy-indep. int.

  --> purely composite state

- Bound state by energy-dep. (chiral) int.

  --> mixture of composite and elementary

- Natural scheme corresponds to $Z \sim 0$

  --> composite particle is generated

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)