act form of the Complex Spectral Representat of the Liouvillian for Weakly Coupled One-dimensional Quantum Lorentz Gas

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ersibility as an exact dynamics in terms of resonance states
reversibility is not a result of approximations or statistical operatio
ch as the coarse-graining

ulty of the problem: infinitely many degrees of freedom
far, there is no exact solution of the Liouville-von Neumann eq.
at shows irreversibility to approach to thermal equilibrium
space and momentum distribution

alk
ents an exact solution of irreversible process that exhibits
sport phenomena and approaches to equilibrium

ne-dimensional Weakly Coupled Quantum Lorentz G

ecially we discuss transport process for a non-hydrodynamic situat

Classical Model

Modern Model (Not Quantum)

hinko (Japanese Slot machine)

Lorentz Gas

$$H = H_0 + gH_V = \frac{p^2}{2m} + \sum_{j=1}^{N} \frac{p_j^2}{2M} + g \sum_{j=1}^{N} V(|x - x_j|)$$

$m/M \ll 1$

$g \ll 1$

Heavy particles are in equi

$$V(x) = \frac{2\pi}{L} \sum_k V_k e^{ikx}$$

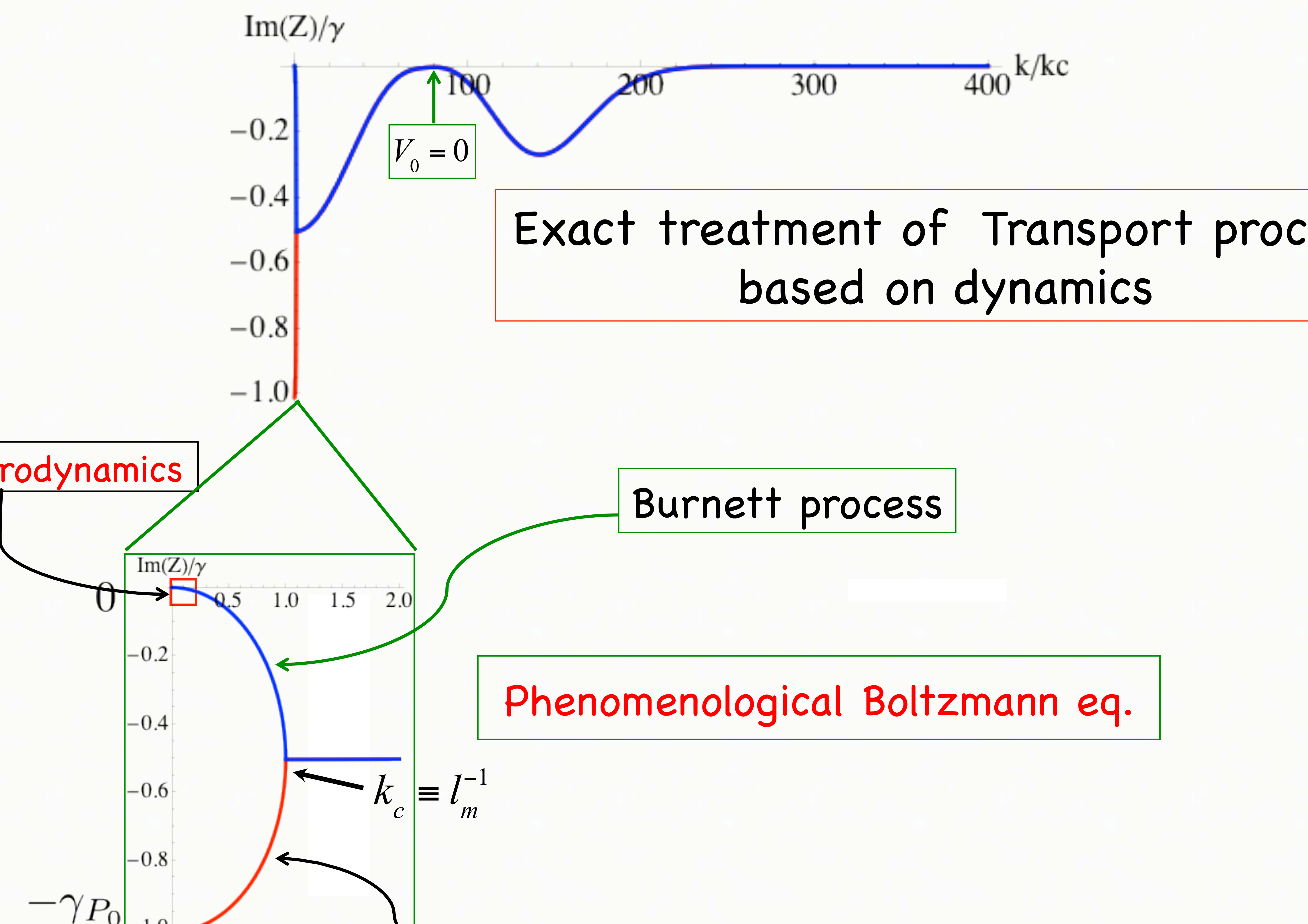
$V_0 = 0$ to avoid the 1st order secular ef

$$\left(V_k \sim o(k) \xrightarrow{k \to 0} 0 \right)$$

ille-von Neumann Eq.

$$i \frac{\partial}{\partial t} |\rho(t)\rangle\rangle = \mathcal{L} |\rho(t)\rangle\rangle \qquad \mathcal{L}\rho \equiv \frac{1}{\hbar}[H, \rho] = (\mathcal{L}_0 + g\mathcal{L}_V)\rho$$

igenvalue Problem the Liouvillian $\mathcal{L}|F_\alpha^{(k)}\rangle\rangle = Z_\alpha^{(k)} |F_\alpha^{(k)}\rangle\rangle$


Im(Z)/γ
k/kc
100
200
300
400
−0.2
−0.4
−0.6
−0.8
−1.0
$V_0 = 0$
Exact treatment of Transport proc
based on dynamics
rodynamics
Burnett process
Im(Z)/γ
0
0.5
1.0
1.5
2.0
−0.2
−0.4
−0.6
−0.8
Phenomenological Boltzmann eq.
$k_c \equiv l_m^{-1}$
$-\gamma P_0$

$$H = H_0 + gV \qquad H_0 = \sum_\alpha \hbar\omega_\alpha |\alpha\rangle\langle\alpha|$$

envalue problem: $H|\varphi_\alpha\rangle = z_\alpha|\varphi_\alpha\rangle$

with projection operators:

$$P_\alpha = |\alpha\rangle\langle\alpha|,\ Q_\alpha = 1 -$$

$$H\left(P_\alpha|\varphi_\alpha\rangle + Q_\alpha|\varphi_\alpha\rangle\right) = z_\alpha P_\alpha|\varphi_\alpha\rangle$$

$$H\left(P_\alpha|\varphi_\alpha\rangle + Q_\alpha|\varphi_\alpha\rangle\right) = z_\alpha Q_\alpha|\varphi_\alpha\rangle$$

$$\begin{cases} Q_\alpha|\varphi_\alpha\rangle = \dfrac{1}{z_\alpha - Q_\alpha H Q_\alpha} Q_\alpha gVP_\alpha|\varphi_\alpha\rangle \\ \left(P_\alpha HP_\alpha + P_\alpha gVQ_\alpha \dfrac{1}{z_\alpha - Q_\alpha HQ_\alpha} Q_\alpha gVP_\alpha\right)P_\alpha|\varphi_\alpha\rangle = z_\alpha P_\alpha|\varphi_\alpha\rangle \end{cases}$$

on of the eigenstate: $|\varphi_\alpha\rangle = N_\alpha\left(P_\alpha + \dfrac{1}{z_\alpha - Q_\alpha HQ_\alpha}Q_\alpha gVP_\alpha\right)P_\alpha|\varphi_\alpha\rangle$

ersion equation: $\hbar\omega_\alpha + \langle\alpha|gV|\alpha\rangle + \langle\alpha|gVQ_\alpha \dfrac{1}{z_\alpha - Q_\alpha HQ_\alpha} Q_\alpha gV|\alpha\rangle =$

self-energy part

ville-von Neumann equation: $\frac{\partial}{\partial t}\rho(t) = \mathcal{L}\rho(t) = \frac{1}{\hbar}[H, \rho(t)]$

villian: $\mathcal{L} = \mathcal{L}_0 + g\mathcal{L}_V$

$$\mathcal{L}_0|\alpha\rangle\langle\beta| = \frac{1}{\hbar}[H_0|\alpha\rangle\langle\beta| - |\alpha\rangle\langle\beta|H_0] = (\omega_\alpha - \omega_\beta)|\alpha\rangle\langle\beta|$$

$$\mathcal{L}_0|\alpha\rangle\langle\alpha| = \mathcal{L}_0|\beta\rangle\langle\beta| = \cdots = 0$$ intrinsic degeneracy

$\mathcal{L}_0 P^{(\mu)} = P$ ...

$P^{(\mu)\,2} = P$ ...

$Q^{(\mu)} = 1 - P$ ...

nvalue problem $\mathcal{L}|F_j^{(\mu)}\rangle\rangle = Z_j^{(\mu)}|F_j^{(\mu)}\rangle\rangle$, $\langle\langle\tilde{F}_j^{(\mu)}|\mathcal{L} = Z_j^{(\mu)}\langle\langle\tilde{F}_j^{(\mu)}|$

$$\psi^{(\mu)}\left(Z_j^{(\mu)}\right)P^{(\mu)}\left|F_j^{(\mu)}\right\rangle\rangle = Z_j^{(\mu)}P^{(\mu)}\left|F_j^{(\mu)}\right\rangle\rangle$$

↑

collision operator (self-frequency part of the Liouvillian) in kinetic equation

$$\psi^{(\mu)}(z) = P^{(\mu)}\mathcal{L}P^{(\mu)} + P^{(\mu)}g\mathcal{L}_V Q^{(\mu)}\frac{1}{z - Q^{(\mu)}\mathcal{L}Q^{(\mu)}}Q^{(\mu)}g\mathcal{L}_V P^{(\mu)}$$

complex eigenvalues ⇨ Transport coefficients

$$\left(P^{(k)}\mathcal{L}P^{(k)} + g^2 P^{(k)}\mathcal{L}_V Q^{(\mu)} \frac{1}{Z_\alpha^{(k)} - Q^{(\mu)}\mathcal{L}_0 Q^{(\mu)}} Q^{(\mu)}\mathcal{L}_V P^{(\mu)}\right)\left|u_\alpha^{(k)}\right\rangle\!\Big\rangle = Z_\alpha^{(k)}\left|u_\alpha^{(k)}\right\rangle\!\Big\rangle$$

Flow term

Non-Markov

$$\equiv g^2 \delta\psi_2^{(k)}\left(Z_\alpha^{(k)}\right)$$ Collision term

...al condition $\rho(0) = f(0) \otimes \rho_{heavy}^{eq}\left(p^N\right)$,

$$\rho_{heavy}^{eq}\left(p^N\right) = \left(\frac{1}{2\pi M k_B T}\right)^{N/2} \exp\left[\frac{-1}{2\pi M k_B T\ldots}\right.$$

...uced kinetic equation for $f(t) = \mathrm{Tr}_{heavy}\left[\rho(t)\right]$

Wigner distribution function

$$f^W(X,P,t) \equiv \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk\ e^{ikX} f_k(P,t)$$

$$f_k(P,t) \equiv \left\langle P + \frac{\hbar}{2}k \middle| f(t) \middle| P - \frac{\hbar}{2}k \right\rangle$$

$$\frac{\partial}{\partial t} f_k(P,t) = \left[-ikv + g^2 \delta\Psi_2^{(k)}\right] f_k(P,t)$$

$$v \equiv \frac{P}{m}$$

$$c \equiv N/\ldots$$

$$\Psi_2^{(k)}(z) \xrightarrow{m/M\to 0} g^2 \frac{2\pi c}{\hbar^2}\frac{2\pi}{L}\sum_l \left|V_l\right|^2 \left(e^{\frac{\hbar l}{2}\frac{\partial}{\partial P}} - e^{-\frac{\hbar l}{2}\frac{\partial}{\partial P}}\right)\frac{1}{z-(k-l)v}\left(e^{-\frac{\hbar l}{2}\frac{\partial}{\partial P}} - e\ldots\right.$$

$$\delta\Psi_2^{(k)}(Z_\alpha^{(k)}) \Rightarrow g^2\delta\Psi_2^{(0)}(+i0) = g^2 \frac{2\pi c}{\hbar^2}\frac{2\pi}{L}\sum_l |V_l|^2 (e^{\frac{\hbar l}{2}\frac{\partial}{\partial P}} - e^{-\frac{\hbar l}{2}\frac{\partial}{\partial P}})\frac{1}{lv+i0}(e^{-\frac{\hbar l}{2}\frac{\partial}{\partial P}} - e$$

This is valid only for $k \lesssim l_m^{-1}$ $\qquad$ l_m : mean free path

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ct eigenvalue of $\Psi_2^{(k)}\left(Z_\alpha^{(k)}\right) = -ikv + g^2\delta\Psi_2^{(k)}\left(Z_\alpha^{(k)}\right)$ for all value of

$$Z_{\pm;P_0}^{(k)} = -ig^2\frac{2\pi^2 mc}{\hbar^2}\left(\frac{V^2_{|k-\frac{2P_0}{\hbar}|}}{|P_0 - \frac{\hbar}{2}k|} + \frac{V^2_{|k+\frac{2P_0}{\hbar}|}}{|P_0 + \frac{\hbar}{2}k|}\right) \pm \frac{P_0}{m}\sqrt{k^2 - k_c^2}$$

$$\pm g^2\frac{2\pi mc}{\hbar^2}\mathcal{P}\int dl V_{|l|}^2\left\{\frac{1}{l(P_0 - \frac{\hbar}{2}k + \frac{\hbar}{2}l)} + \frac{1}{l(P_0 + \frac{\hbar}{2}k - \frac{\hbar}{2}l)}\right\}$$

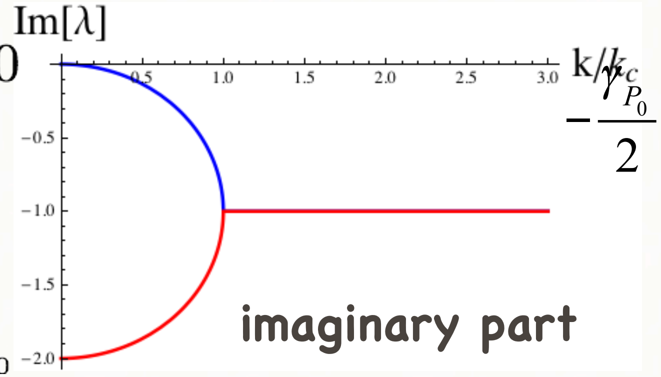
c

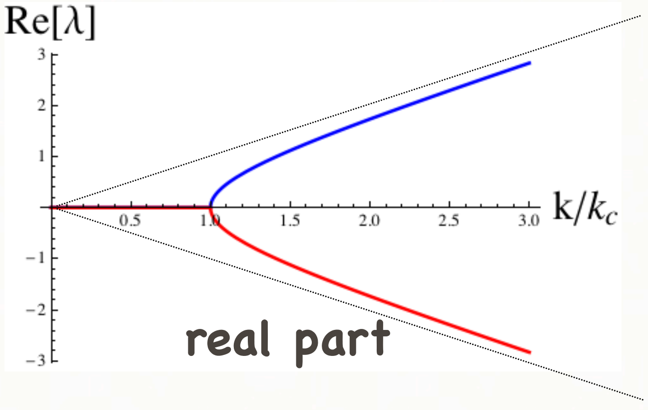
Eigenvalues of the phenomenological Boltzmann equation !

$$\gamma_P \equiv \frac{8\pi^2 mc\left|V_{|2P/\hbar|}\right|^2}{\hbar P}$$

$$k_c \equiv \frac{g^2\gamma_P}{2P/m}$$

γ_P and $k_c < \infty$ for $V_k \sim o(k) \xrightarrow{k\to}$





$$l_m = k_c^{-1}$$

Burnett expansion of the transport coefficients for k/k_c

$$\to \begin{cases} -ig^2\gamma_{P_0} + i\frac{g^2\gamma_{P_0}}{4}\left(\frac{k}{k_c}\right)^2 + i\frac{g^2\gamma_{P_0}}{16}\left(\frac{k}{k_c}\right)^4 + \cdots & \text{relaxation mode} \\ 0 - i\frac{g^2\gamma_{P_0}}{4}\left(\frac{k}{k_c}\right)^2 - i\frac{g^2\gamma_{P_0}}{16}\left(\frac{k}{k_c}\right)^4 + \cdots & \text{hydrodynamic mode} \end{cases}$$

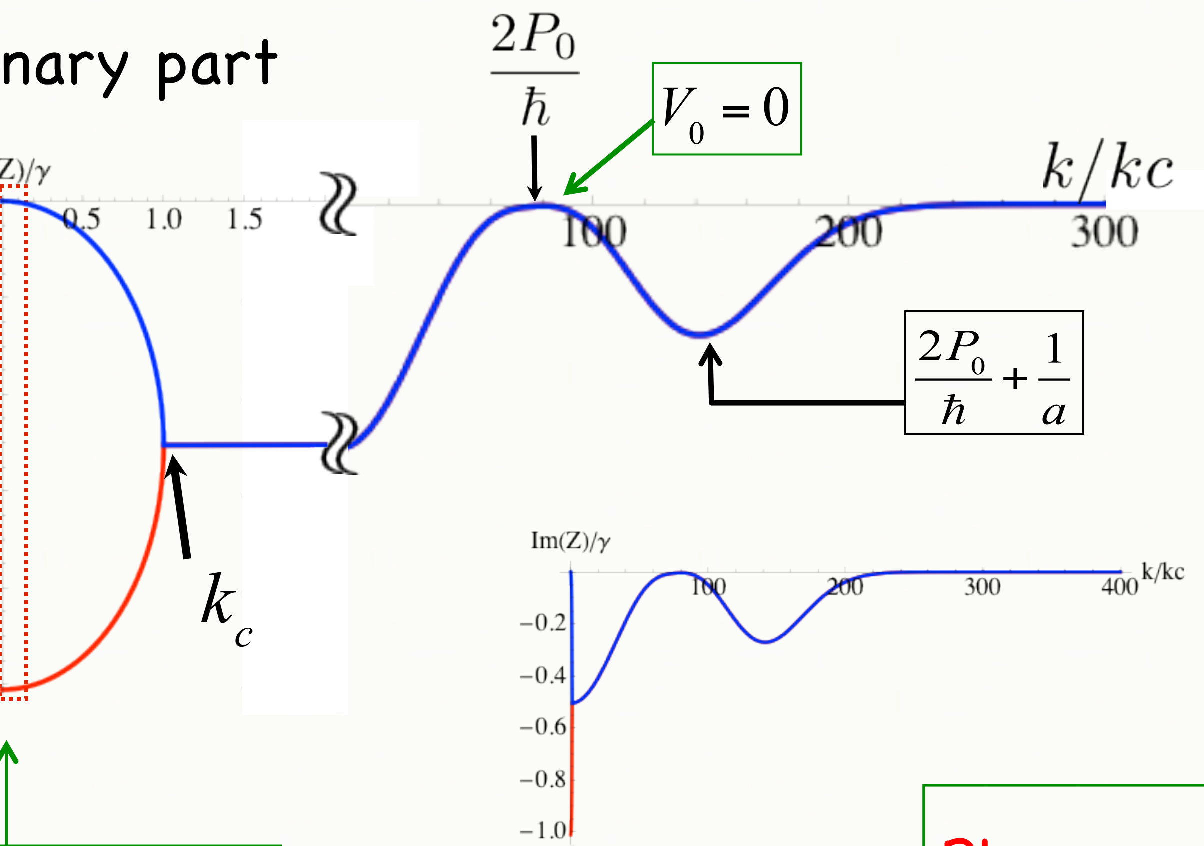
$$= -ig^2 \frac{2\pi^2 mc}{\hbar^2}\left(\frac{V^2_{|k-\frac{2P_0}{\hbar}|}}{|P_0-\frac{\hbar}{2}k|}+\frac{V^2_{|k+\frac{2P_0}{\hbar}|}}{|P_0+\frac{\hbar}{2}k|}\right) \pm \frac{P_0}{m}\sqrt{k^2-k_c^2}$$

$$\pm g^2\frac{2\pi mc}{\hbar^2}\mathcal{P}\int dl V^2_{|l|}\left\{\frac{1}{l(P_0-\frac{\hbar}{2}k+\frac{\hbar}{2}l)}+\frac{1}{l(P_0+\frac{\hbar}{2}k-\frac{\hbar}{2}l)}\right\}$$

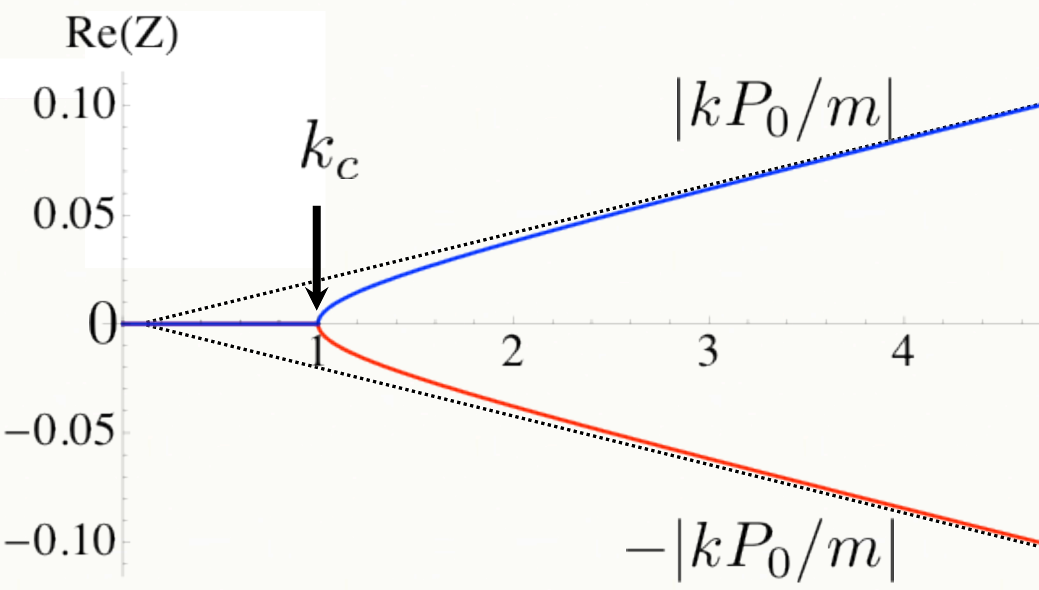
$$V_{|k|} = V_0\, k^2 \exp[-(a\, k)$$

a : interaction range

nary part



Real part

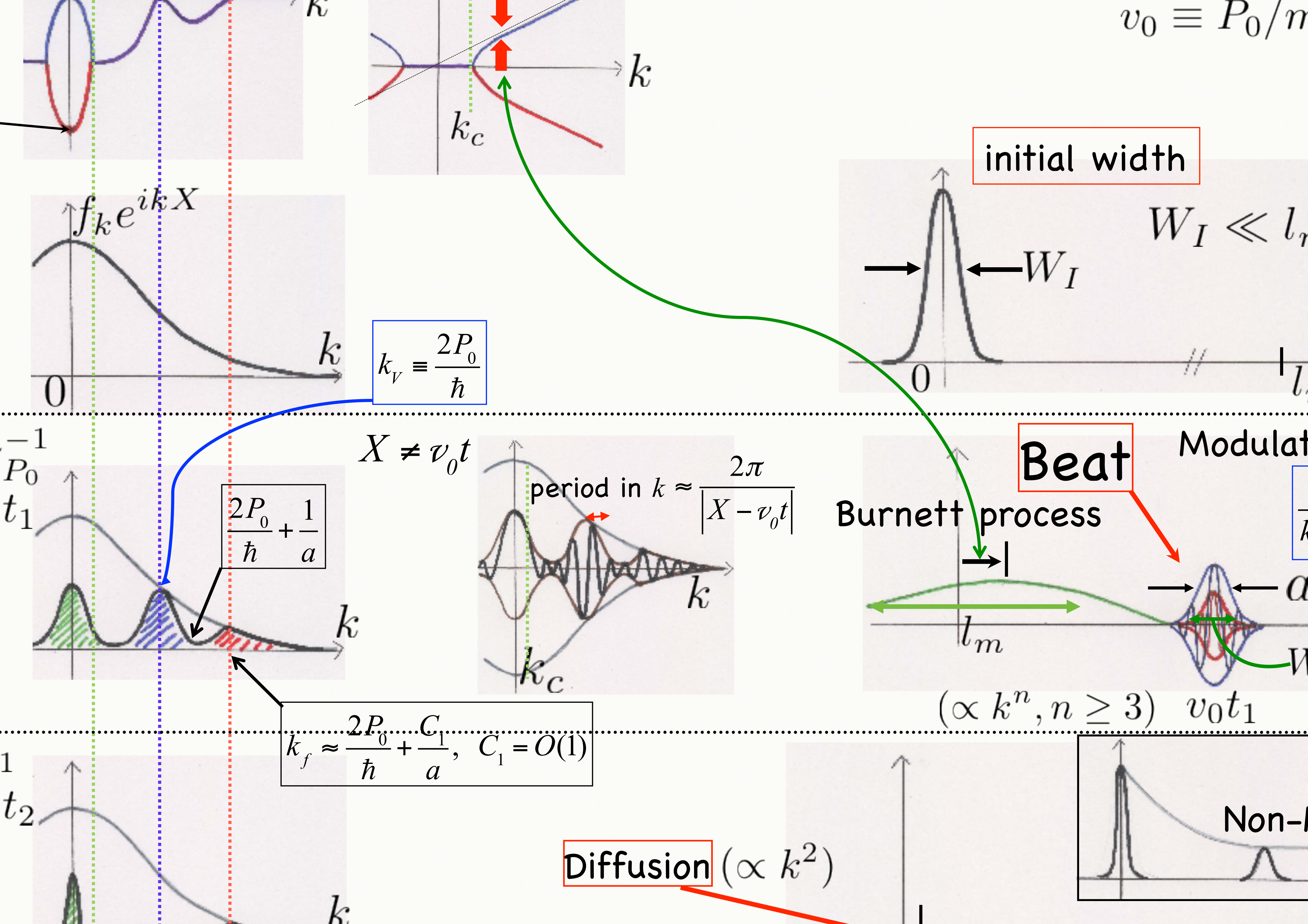


dynamics

Phenomenological Boltzmann eq.

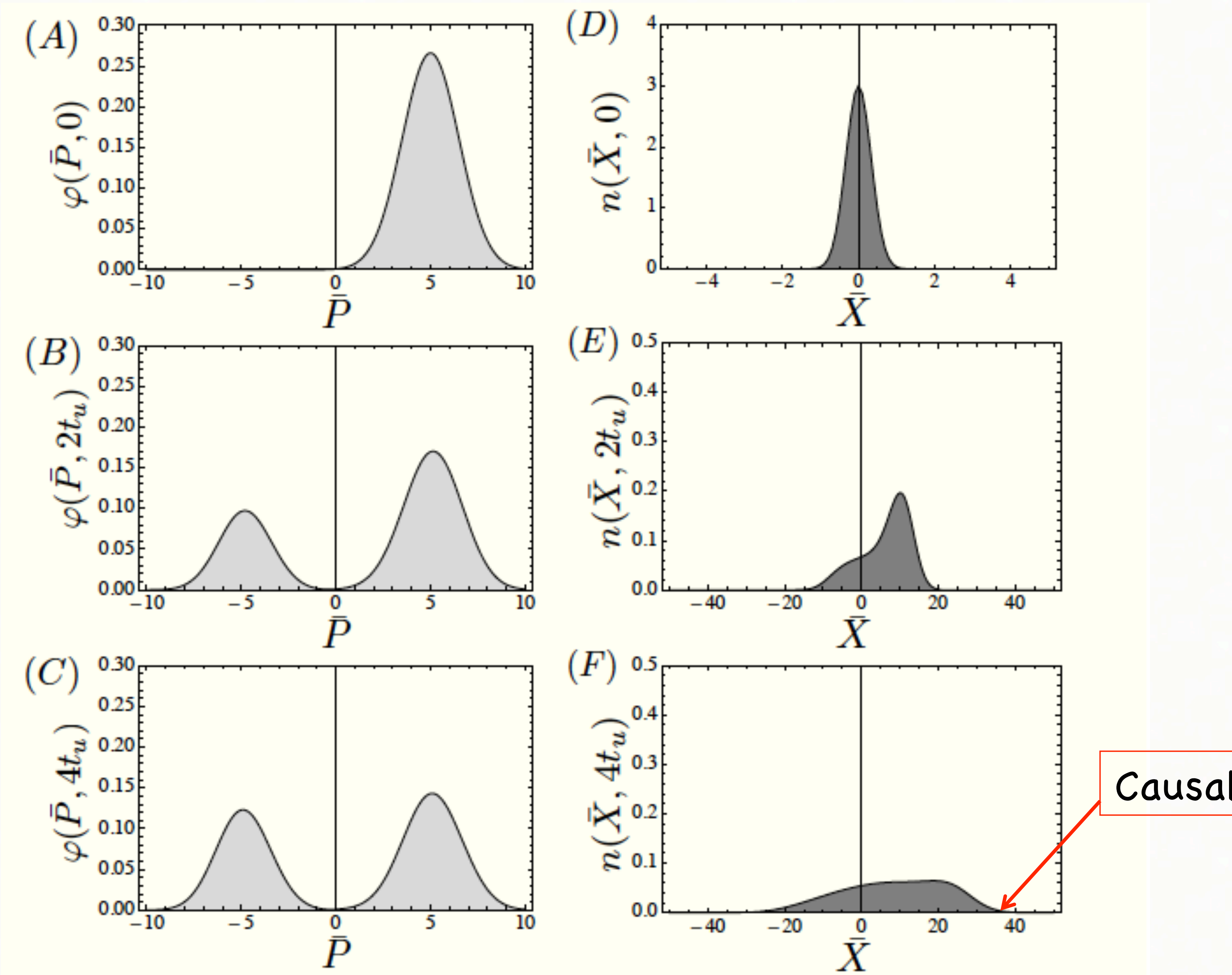
ree spatial scales:

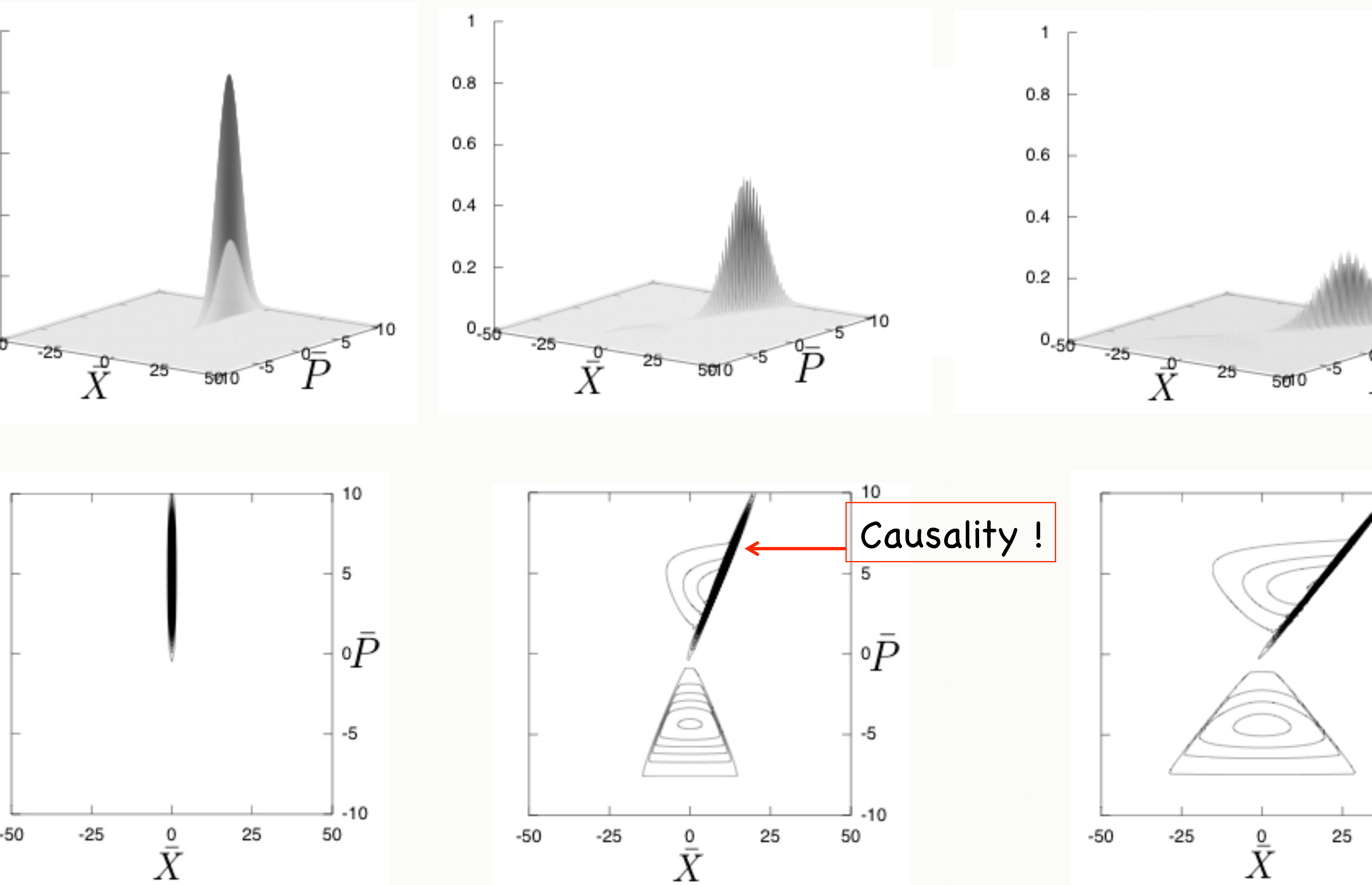
1



$v_0 \equiv P_0/m$
k
k_c
initial width
$f_k e^{ikX}$
$W_I \ll l$
W_I
k
0
0
$k_V \equiv \frac{2P_0}{\hbar}$
$X \neq v_0 t$
period in $k \approx \frac{2\pi}{|X - v_0 t|}$
Beat
Modulat
Burnett process
t_1
$\frac{2P_0}{\hbar} + \frac{1}{a}$
k
k
k_c
l_m
$(\propto k^n, n \geq 3)$
$v_0 t_1$
$k_f \approx \frac{2P_0}{\hbar} + \frac{C_1}{a}, \quad C_1 = O(1)$
t_2
k
Diffusion $(\propto k^2)$
Non-

$$\varphi(P,t) \equiv \langle P | f(t) | P \rangle \qquad n(X,t) \equiv \langle X | f(t) | X \rangle$$




Causality !
$\bar{X}$
$\bar{P}$

Summary

We could described irreversible process
as an purely dynamical process
as a solution of
the Liouville-von Neumann equation!

終わり

Thank you very much!