Internal structure of the resonant $\Lambda(1405)$ state in chiral dynamics

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in collaboration with

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[1] T. S. , T. Hyodo and D. Jido, *Phys. Lett.* B669 (2008) 133-138.
[2] T. S. , T. Hyodo and D. Jido, *Phys. Rev.* C83 (2011) 055202.

Contents

1. Introduction

++ Abstract ++

$\hbar = c = 1$

- We want to measure the spatial size of the resonance systems.

--- Spatial size is one of the important properties of the system.

□ Stable systems: $\rho(r) = \psi^*(r)\psi(r)$ $\langle r^2 \rangle = \int d^3r\, \psi^*(r)\hat{r}^2\psi(r)$

- But, in general, wave functions of the unstable resonance states are not normalized due to the decay mode.

- Instead of evaluating the wave function, we determine the form factors, the Fourier transformation of the density, of unstable resonance states directly from the scattering amplitudes.

$$F(Q^2 \equiv -\vec{q}^{\,2}) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\rho(r)$$

--- On the resonance pole.

--- Form factor is well-defined in this approach, but complex value.

--> Mean squared radius is "defined" as: $\langle r^2 \rangle \equiv -6 \left. \frac{dF}{dQ^2} \right|_{Q^2=0}$

--- They become real values in small decay width limit.

1. Introduction

++ Strong interaction and hadrons ++

- Our research field: strong interaction and hadron physics.

--- Strong interaction is one of the 4 fundamental forces in the World.

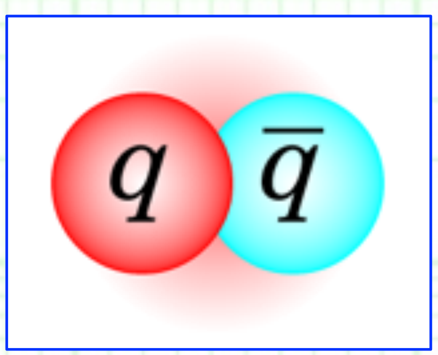
- □ Gravity
- □ Weak interaction
- □ Electromagnetic force
- □ Strong interaction

- Particles which interact with each other via the strong interaction = Hadrons. [ex.), proton (p), neutron (n), pion (π), ...]

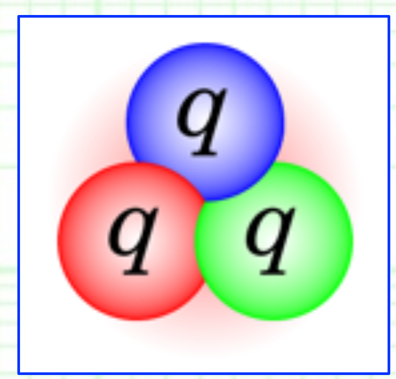
--- Many hadrons (about 300) have been discovered.

- It is commonly accepted that hadrons are composed of quarks, which carry color degrees of freedom. Greenberg ('64), Han-Nambu ('65).

--- Hadrons are "white" with respect to the color.



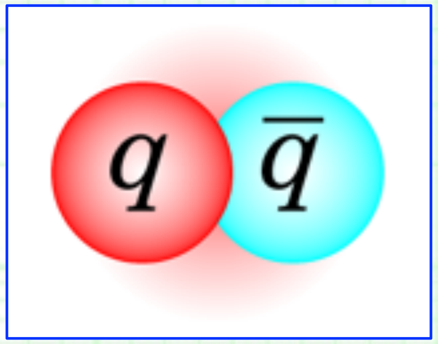
Mesons
(π, K, ρ, ...)



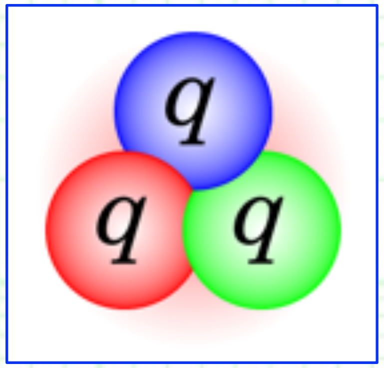
Baryons
(p, n, Λ, ...)

1. Introduction

++ Strong interaction and hadrons ++



Meson
(π, K, ρ, ...)

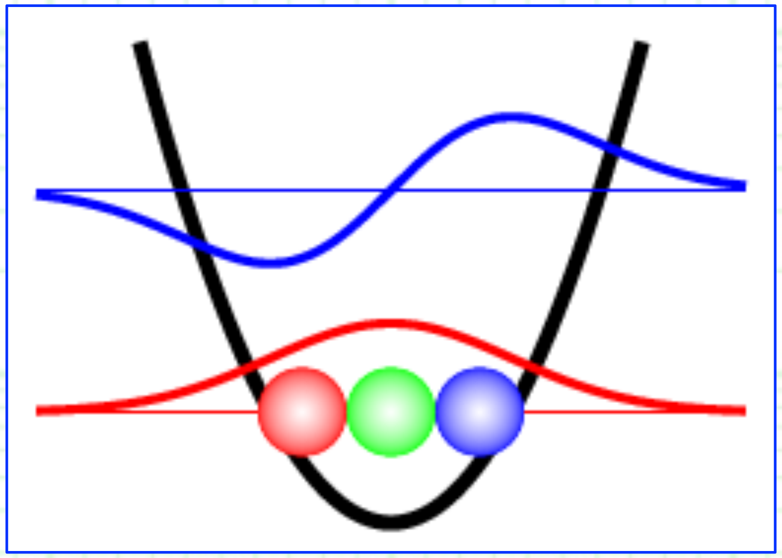


Baryon
(p, n, Λ, ...)

- From the modern viewpoint, **the strong interaction is consequence of the quantum quark(-gluon) dynamics**.

--- Quantum ChromoDynamics (QCD), $SU(3)$ gauge theory.

- **QCD** (or quantum quark dynamics) **confines quarks and generates hadrons.**

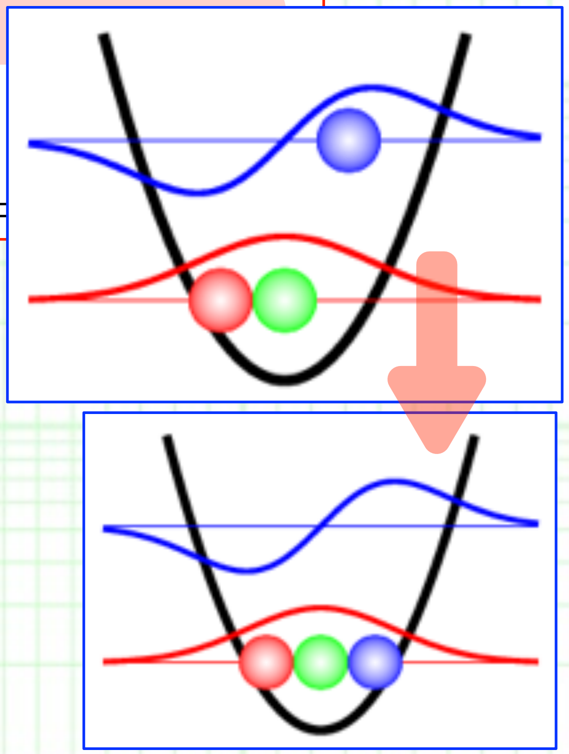


1. Introduction

++ Strong interaction and hadrons ++

- Let us see **the typical scales of the strong interaction**:

	Typical force range	Typical life time	Typical coupling strength (α)
Strong	$1\text{fm} = 10^{-15}\text{m}$	10^{-23}s	1
EM	∞	10^{-20}s	$1/137$
Weak	$1/M_W \sim 10^{-18}\text{m}$	$\gtrsim 10^{-12}\text{s}$	10^{-6}



--- Strong interaction is indeed "strong".

- **Almost all hadrons are unstable with respect to the strong interaction.**

  ex.) $\Delta^{++}(1232)$ --> $\pi^{+} + p$ via the strong interaction.

- Important and interesting point: Hadrons are generated by quantum quark dynamics, but it also decays the (almost) hadrons.

--- Due to the "strong" interaction, imaginary part of the system's eigenvalue is not negligible compared to the real part.

PDG

ex.) $\rho(770)$: eigenvalue $M - i\,\Gamma/2$ with M=775 MeV and Γ=149 MeV.

--> **One needs to treat (almost all) hadrons as resonance states.**

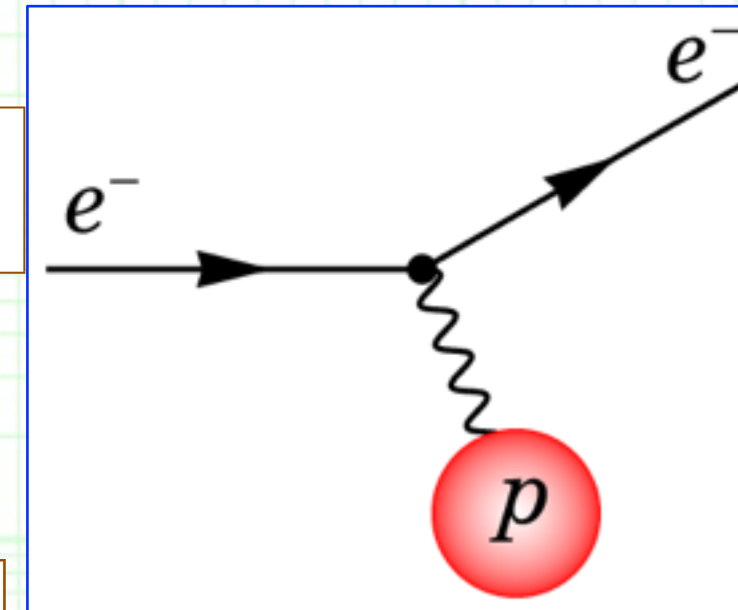
1. Introduction

++ Hadron structure ++

- **Ordinary hadrons are composed of qqq (baryon) or $q\bar{q}$ (meson).**

--- Consistent with the quark model.

$$\langle r^2 \rangle \equiv -6 \frac{dF}{dQ^2}\bigg|_{Q^2=0}$$



- **EM radii of ground state hadrons** are measured from **the form factors**:

$$\sqrt{\langle r_p^2 \rangle_{\rm E}} = 0.877 \pm 0.005 \text{ fm}$$

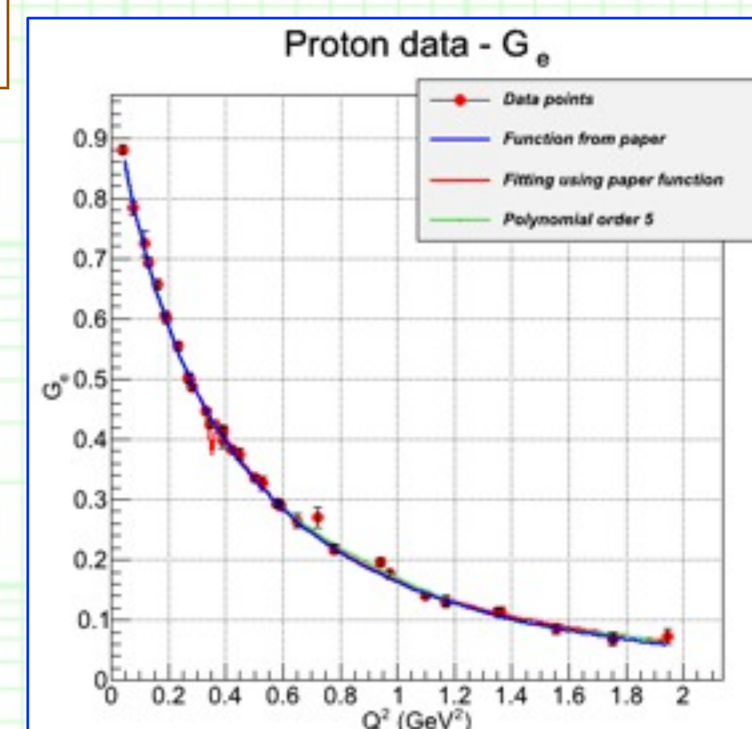
$$\sqrt{\langle r_n^2 \rangle_{\rm M}} = 0.862^{+0.009}_{-0.008} \text{ fm}$$

$$\sqrt{\langle r_{K^\pm}^2 \rangle_{\rm E}} = 0.560 \pm 0.031 \text{ fm}$$

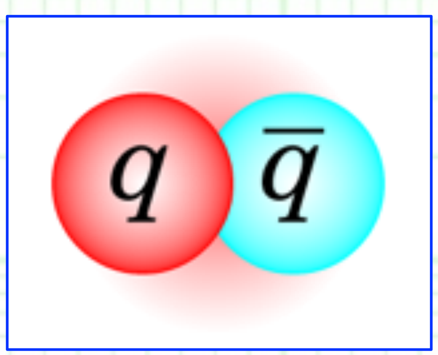
$$\sqrt{\langle r_{\Sigma^-}^2 \rangle_{\rm E}} = 0.78 \pm 0.10 \text{ fm}$$

PDG

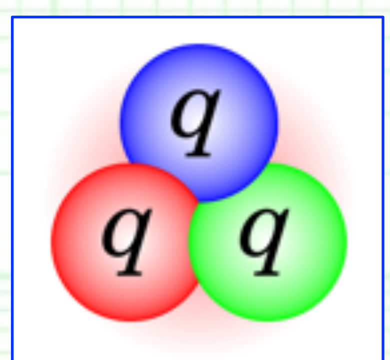
--- Typical hadron size is 0.8 fm.



The Catholic University of America Web page.



Mesons
(π, *K*, ρ, ...)



Baryons
(*p*, *n*, Λ, ...)

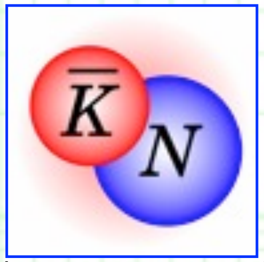
1. Introduction

++ Hadron structure ++

- However, some hadrons may have exotic structure rather than qqq or $q\bar{q}$ (but white!). --- Exotic hadrons.

--- Compact multi-quark systems, hadronic bound states, ...

  - Candidates: $\Lambda(1405)$, the lightest scalar mesons, $X\ Y\ Z$, ...



$\Lambda(1405)$?

- Problem: "number of valence quarks" is not a good classification.

ex.) $$|\Lambda(1405)\rangle = C_{uds}|uds\rangle + C_{\bar{K}N}|\bar{K}N\rangle + C_{uud\bar{u}s}|uud\bar{u}s\rangle + \cdots$$

--- One can observe mass, width, branching ratios, ... of hadrons, not the number quark of valence quarks.

- If a hadron is dominated by the bound state component of constituent hadrons, its coefficient [C_{KN} for $\Lambda(1405)$] will be large.

--- But what is the "good" observables to discriminate it?

--> The spatial size will be sensitive to the coefficient (C_{KN}).

--- Large spatial size <--> large coefficient C_{KN}.

- So, we want to determine the spatial structure of resonance states.

1. Introduction

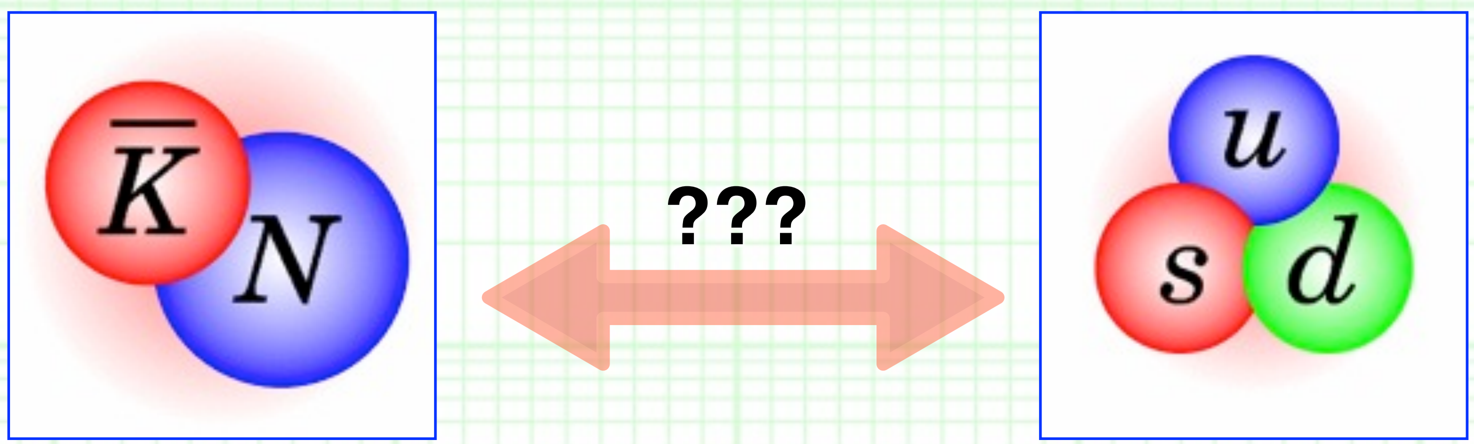
++ “Strange” baryon resonance Λ(1405) ++

- Λ(1405) --- Mass = 1406 ± 4 MeV, width = 1/(life time) = 50 ± 2 MeV, decay to $\pi\Sigma$ (100 %), $I\ (J^P) = 1\ (1/2^-)$. PDG

- Why is Λ(1405) the lightest excited baryon with $J^P = 1/2^-$?

--- Λ(1405) has a strange quark, which should be ~ 100 MeV heavier than up and down quarks.

--- Λ(1405) is a $\bar{K}N$ quasi-bound state ??? Dalitz and Tuan (’60), ...



- Our final goal: confirmation of the meson-baryon molecule picture in experiments (as well as pole position etc.).

<-- “Λ(1405) size” will be an important “observable”.

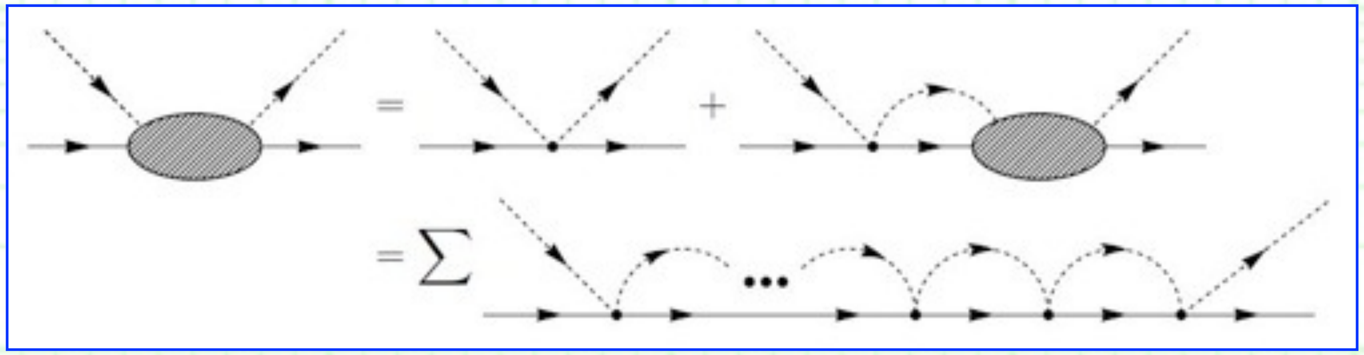
--- “Λ(1405) size” is also important for kaon-nucleus bound states.

1. Introduction

++ Dynamically generated Λ(1405) ++

- Chiral unitary model (ChUM) dynamically generates Λ(1405) in mesons and baryons degrees of freedom.

Kaiser-Siegel-Weise ('95), Oset-Ramos ('98), Oller-Meissner ('01), Jido *et al.* ('03), ...

T-matrix = 
--- Bethe-Salpeter Eq.

--- Based on spontaneous breaking of chiral symmetry in QCD + Scattering unitarity.

- In ChUM Λ(1405) is dynamically generated without explicit resonance poles. Hyodo *et al. Phys. Rev.* C78 025203.

--- Λ(1405) in the meson-baryon interaction picture.

- Then, how about internal structure, or spatial size of Λ(1405) ?

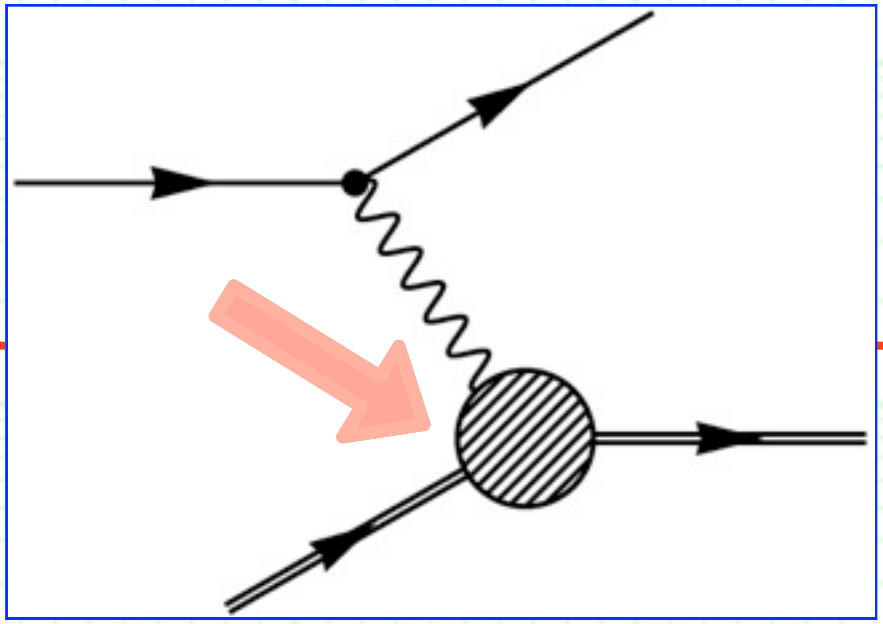
--> We probe internal structure of Λ(1405) in ChUM.

1. Introduction

++ How to probe the structure ? ++

- **Usual approach:**
  Interaction --> (NR or Rel.) potential
  --> Schrödinger Eq. etc.
  --> wave function
  --> density distributions

e.g. Akaishi-Yamazaki ('02), Dote-Hyodo-Weise ('09).



- **Our approach:**
  Interaction --> scattering amplitude (T-matrix)
  --> form factors with respect to probe current (EM etc.)
  --> density distributions

--- Direct probe of the form factors from T-matrix.

- We keep analyticity for the scattering amplitudes.

--> Λ(1405) form factors on the resonance pole (1426 - 17 i MeV).

Λ(1405) form factors in cut-off scheme: Yamagata-Sekihara *et al.*, *Phys. Rev.* D83 (2011) 014003.

2. Photon-coupled amplitudes

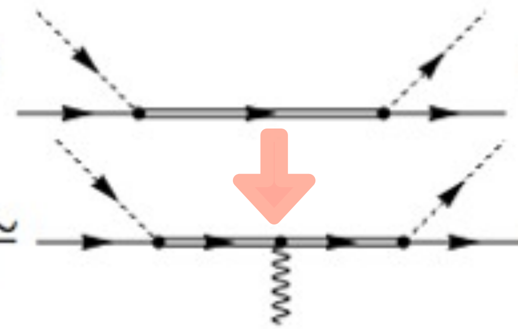
++ Matrix elements in scattering amplitude ++

- Define form factors as matrix elements of the resonance:

$$\langle Z_{\rm R}|J^\mu|Z_{\rm R}\rangle_{\rm Breit} = \left(F_{\rm E}(Q^2),\, F_{\rm M}(Q^2)\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\rm p}}\right) \qquad \left(\rho(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} F(Q^2) \right)$$

--- Straightforward extension from the stable systems.

- These matrix elements appear in the photon-coupled scattering amplitudes T_γ close to the pole position as:



$$T_{ij} \simeq \cdots \equiv \frac{g_i g_j}{\sqrt{s}-Z_{\rm R}}$$

$$T^\mu_{\gamma ij} \simeq \cdots \equiv -\frac{g_i}{\sqrt{s'}-Z_{\rm R}}\langle Z_{\rm R}|J^\mu|Z_{\rm R}\rangle\frac{g_j}{\sqrt{s}-Z_{\rm R}}$$

--- Here resonance vector is "formally" defined.

- So the matrix elements can be extracted from residue of pole:

$$\mathrm{Res}\left[-\frac{T^\mu_{\gamma ij}}{T_{ij}}\right]_{\rm Breit} = \left(F_{\rm E}(Q^2),\, F_{\rm M}(Q^2)\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\rm p}}\right)$$

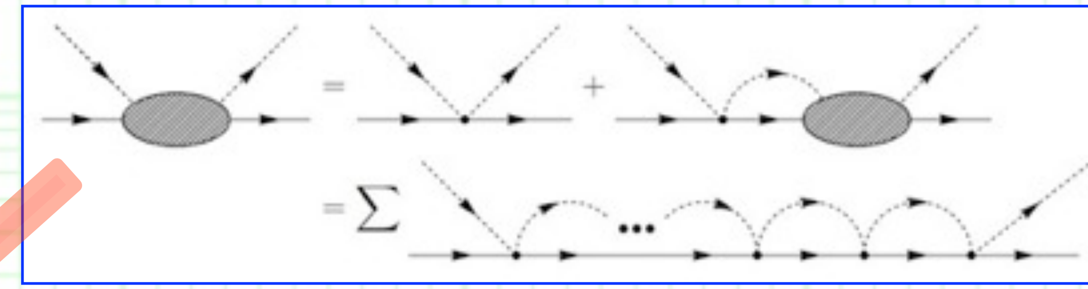
- Then, how T_γ (double pole !) is determined in ChUM ?

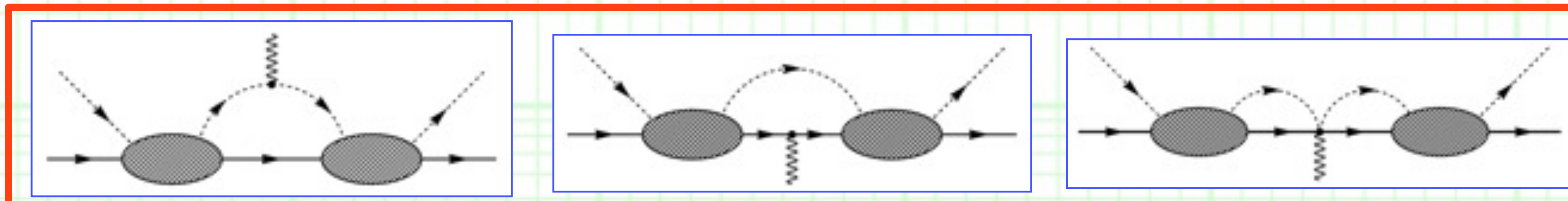
2. Photon-coupled amplitudes

++ Photon-coupled amplitudes in ChUM ++

- For $\Lambda(1405)$ in meson-baryon interaction picture, photon couples to **the intermediate mesons, baryons, and WT vertices**.

--> **Double-pole diagrams**, which contribute to T_γ on the pole, are:





Borasoy *et al.* *Phys. Rev.* C72 065201;
T. S. *et al. Phys. Lett.* B669 133.

$$\mathrm{Res}\left[-\frac{T^{\mu}_{\gamma ij}}{T_{ij}}\right]_{\mathrm{Breit}} = \left(F_{\mathrm{E}}(Q^2),\ F_{\mathrm{M}}(Q^2)\frac{i\boldsymbol{\sigma}\times\boldsymbol{q}}{2M_{\mathrm{p}}}\right)$$

- With this approach, we have Ward identity:

$$\mathrm{Res}\left[-\frac{q_\mu T^{\mu}_{\gamma ij}}{T_{ij}}\right] = 0$$

--> We have correct normalization:

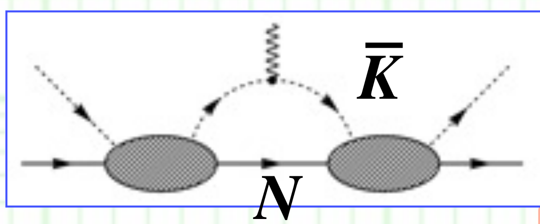
$$F_{\mathrm{E}}(Q^2=0) = Q_{\mathrm{EM}}, \quad F_{\mathrm{B}}(Q^2=0) = B, \quad F_{\mathrm{S}}(Q^2=0) = S$$

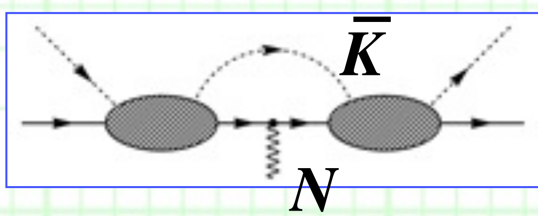
3. Results

++ Each channel contribution to charges ++

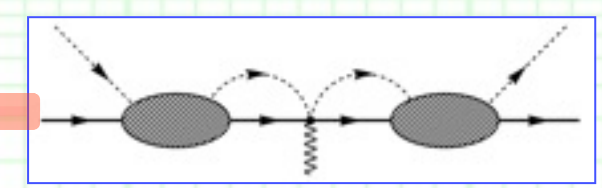
- Let us see the resonant $\Lambda(1405)$ structure, dynamically generated on 1426 - 17 i MeV [higher $\Lambda(1405)$ pole] in full coupled channel.

--- In order to see which channel is important for the $\Lambda(1405)$ structure, we make channel decomposition to the baryon / strangeness number for $\Lambda(1405)$.





Component	$F_{\mathrm{B}}(0) = -F_{\mathrm{S}}(0)$
Total	1
$\bar{K}N$	$0.994 + 0.048i$
$\pi\Sigma$	$-0.047 - 0.151i$
$\eta\Lambda$	$0.052 + 0.012i$
$K\Xi$	$-0.002 + 0.002i$
Contact	$0.002 + 0.089i$



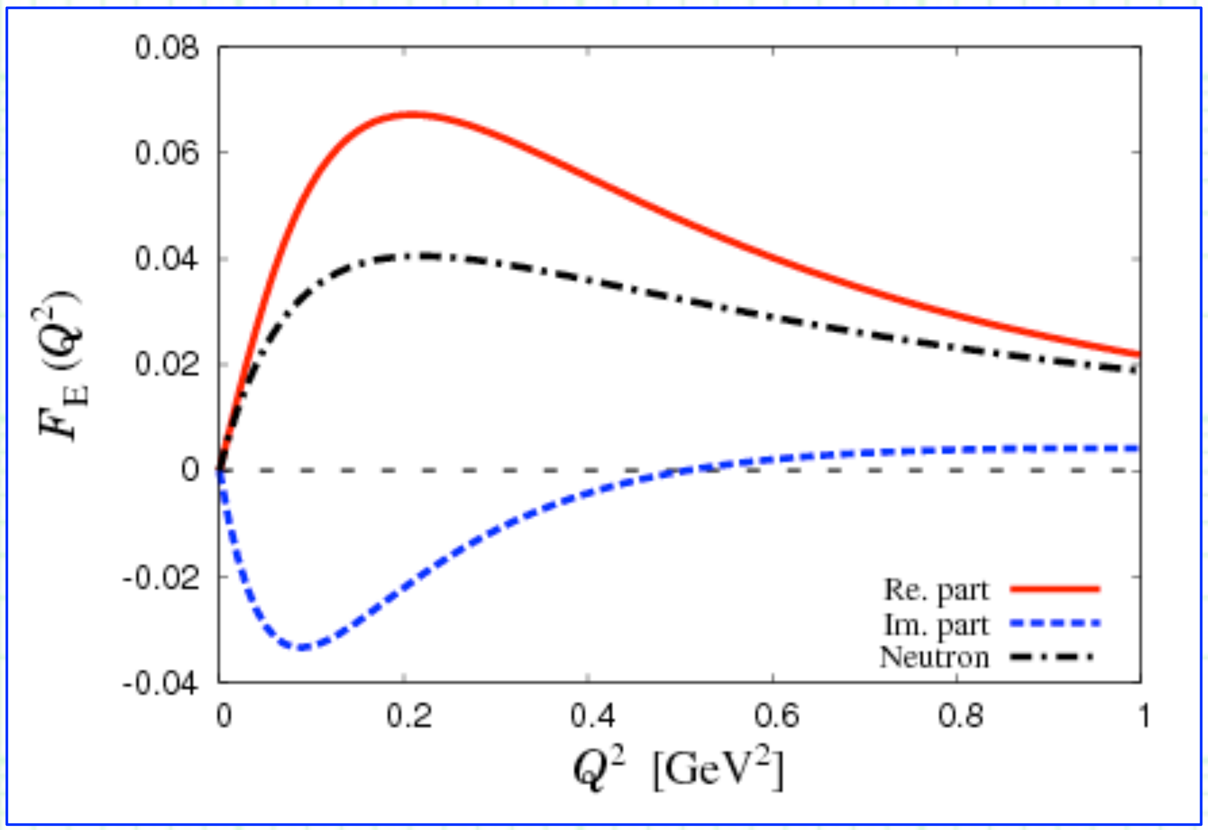
- The baryon / strangeness number is dominated by $\bar{K}N$.

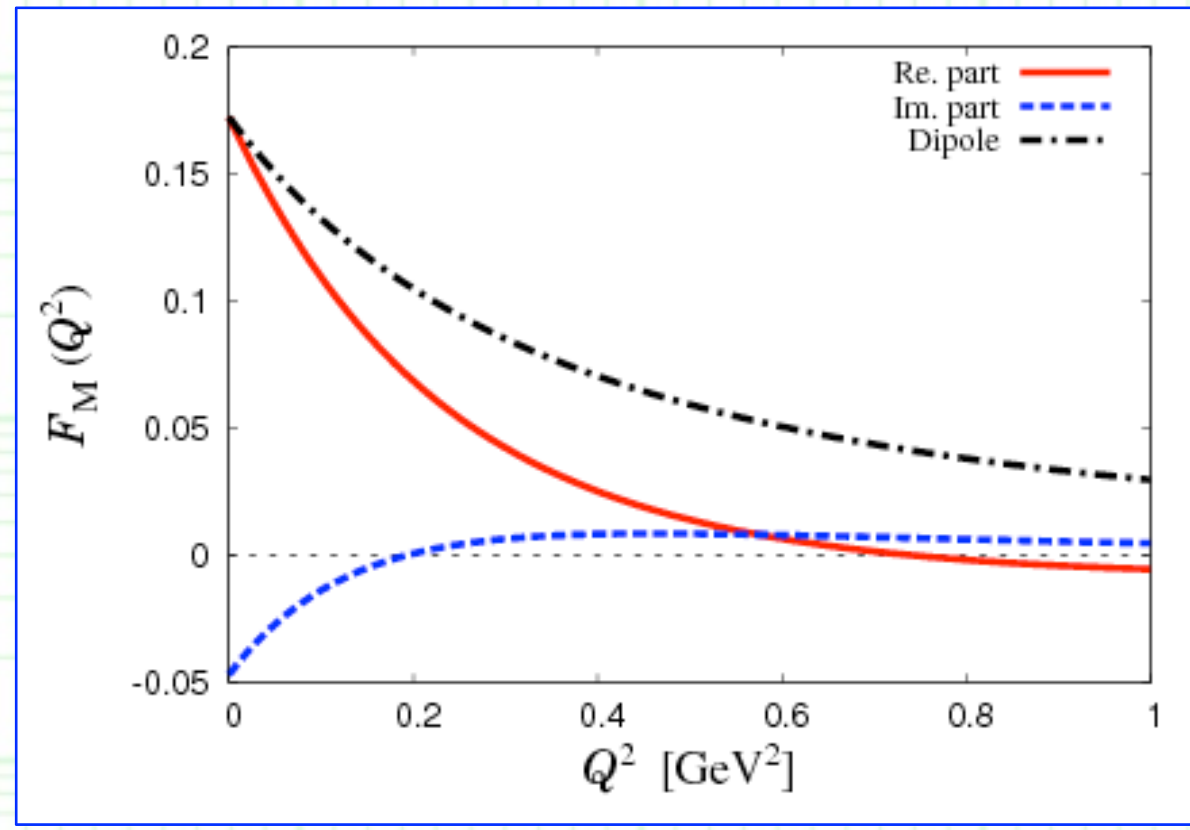
--- Consistent with the description of $\Lambda(1405)$ as a $\bar{K}N$ bound state.

3. Results

++ Structure of resonant Λ(1405) state ++

- Let us see **the resonant Λ(1405) structure**, dynamically generated on 1426 - 17 i MeV [higher Λ(1405) pole] in full coupled channel.





Neutron electric form factor fit: Platchkov et al. *Nucl. Phys.* A510, 740.

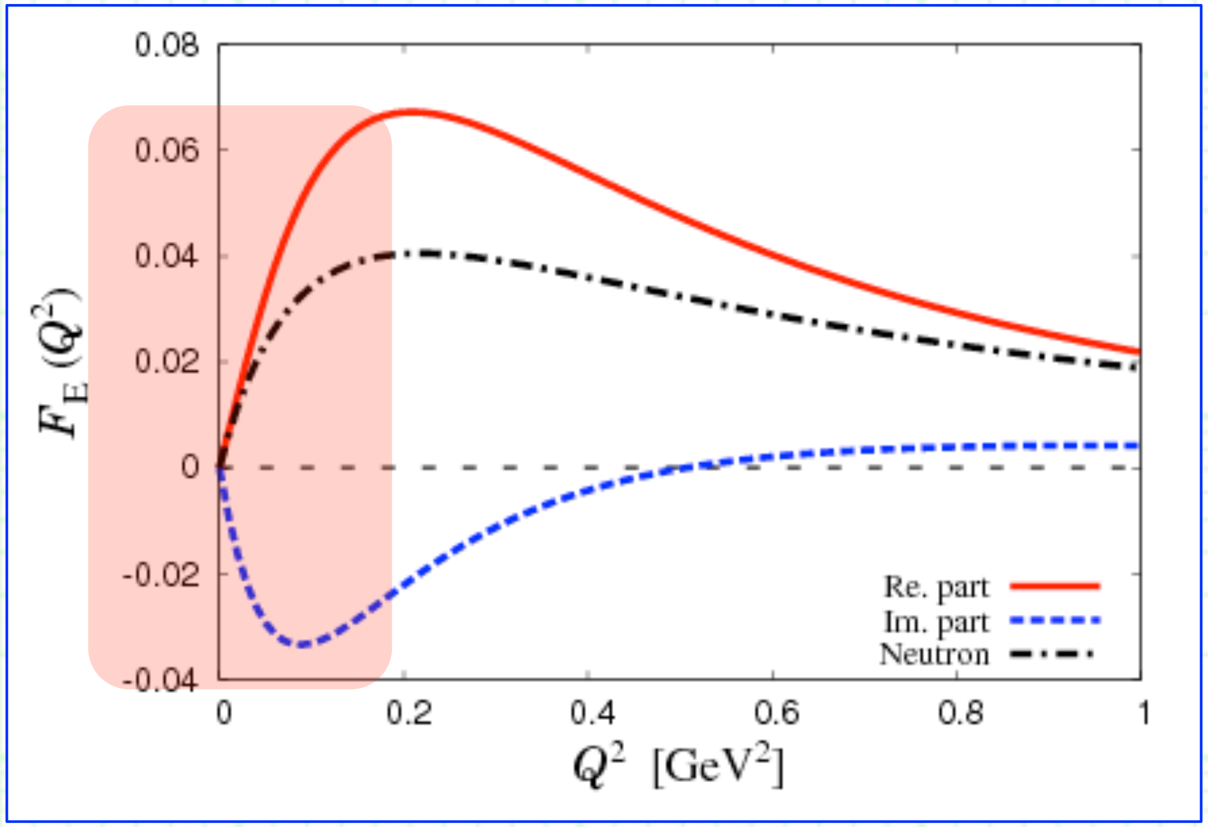
- **Complex form factors** for the resonant Λ(1405).

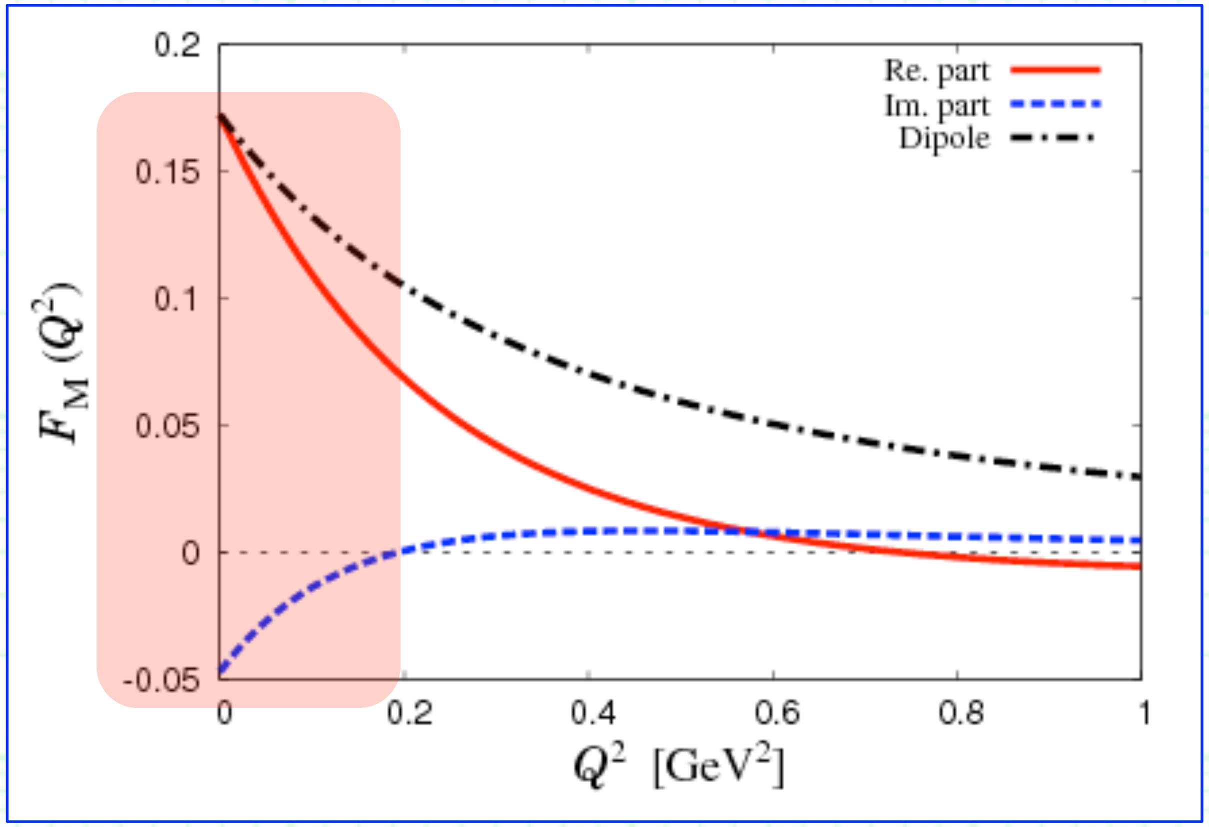
--- The imaginary parts are in smaller magnitude than the real parts reflecting relatively small imaginary part of the pole position.

3. Results

++ Structure of resonant Λ(1405) state ++

- Let us see the resonant Λ(1405) structure, dynamically generated on 1426 - 17 i MeV [higher Λ(1405) pole] in full coupled channel.





Neutron electric form factor fit: Platchkov et al. *Nucl. Phys.* A510, 740.

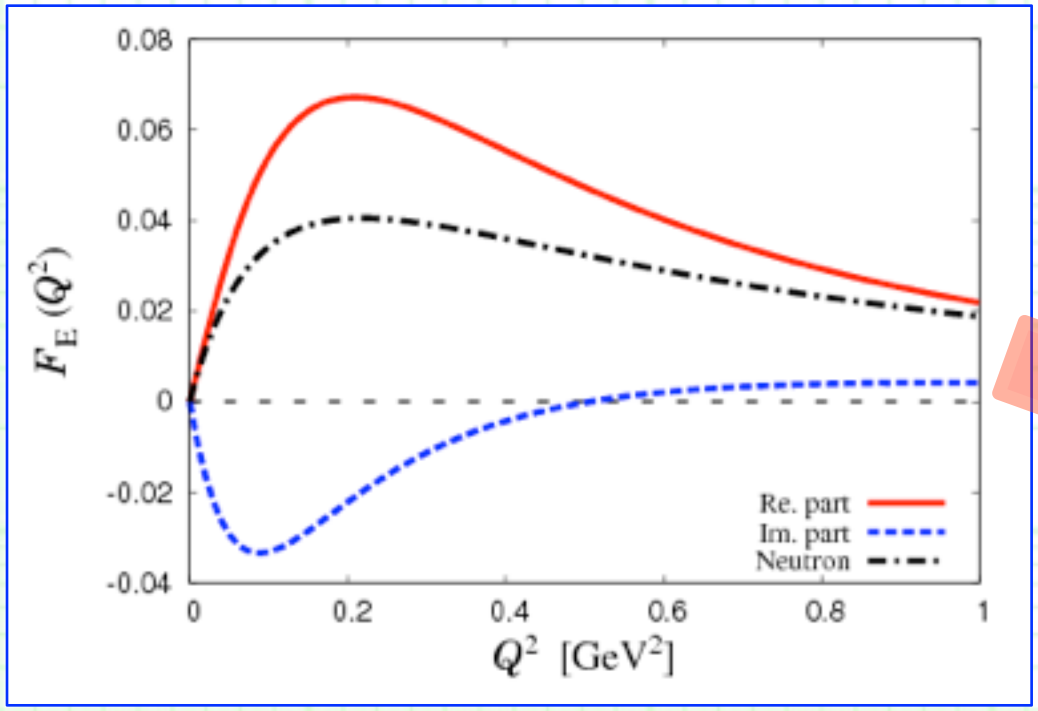
- Rapid increase / decrease of the EM form factors at small Q^2.

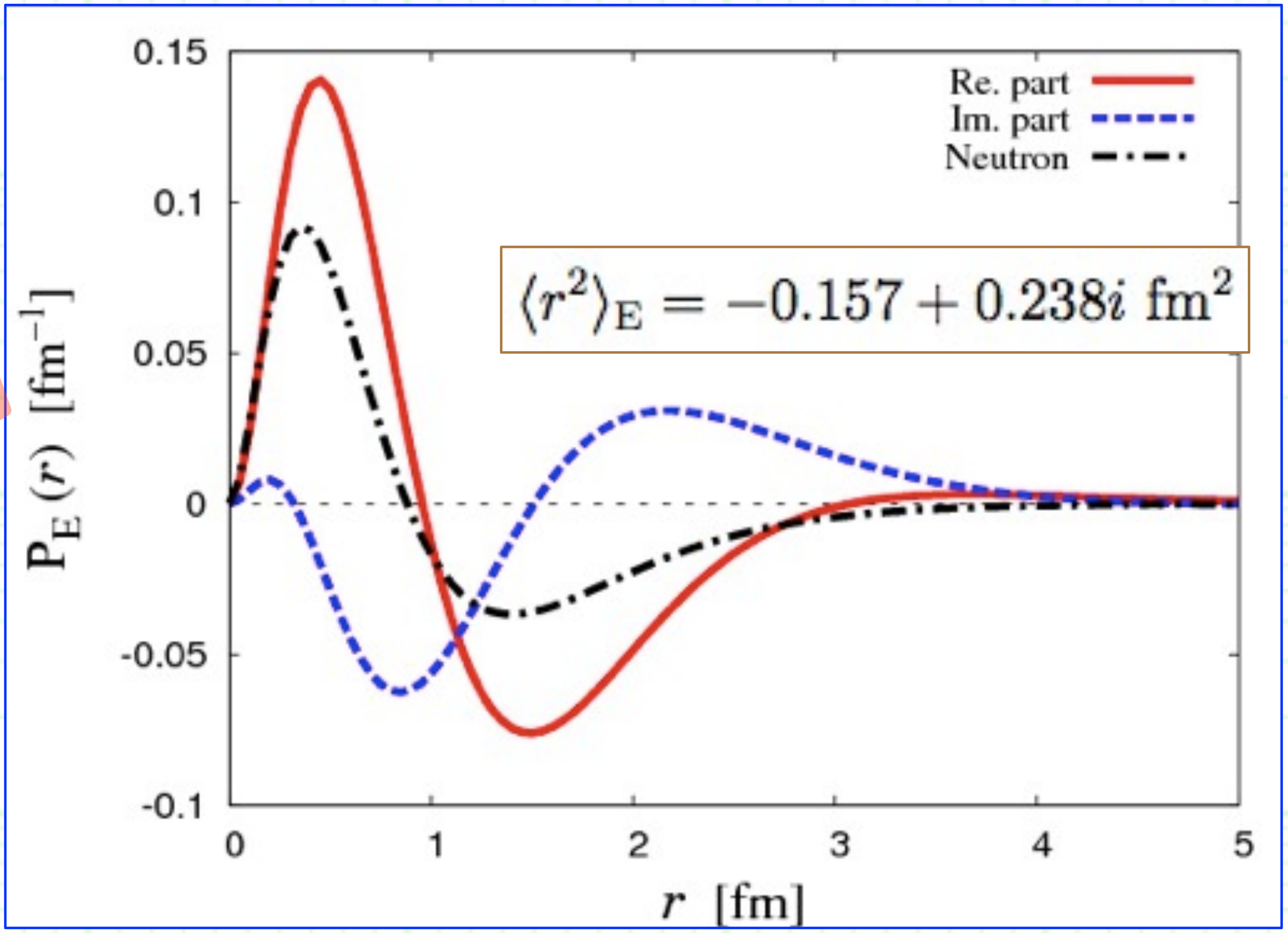
--> This implies characteristic structure of EM density for Λ(1405) !

3. Results

++ Structure of resonant Λ(1405) state ++

- Fourier transformation --> charge density ($P = 4\pi r^2 \rho$).



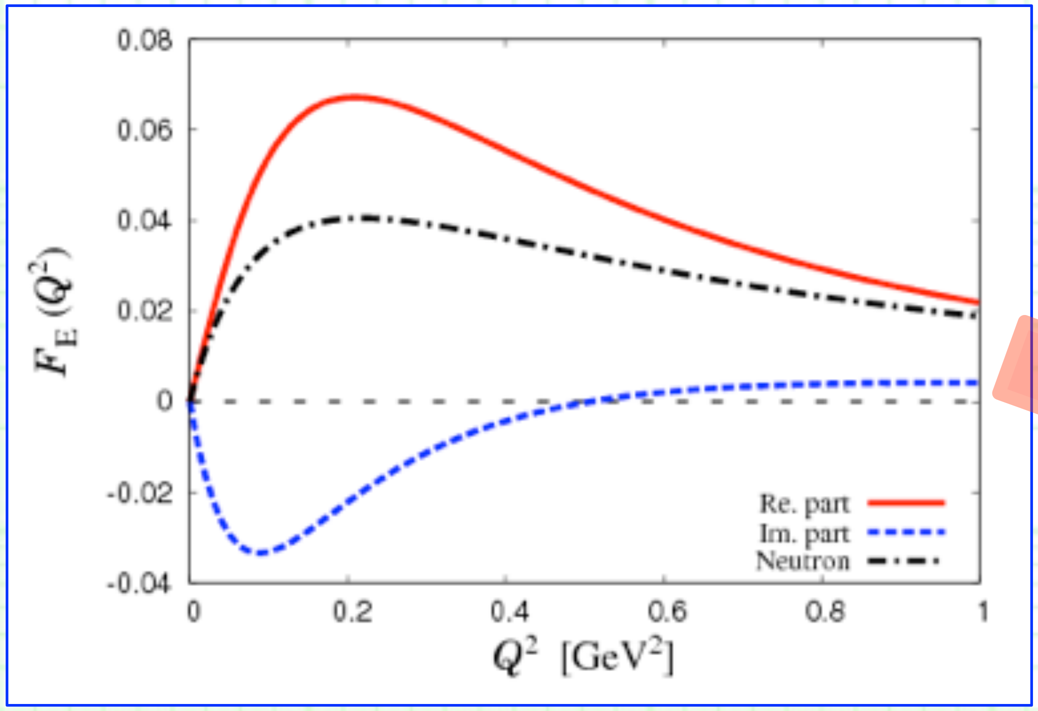


(radial coordinate in CM)

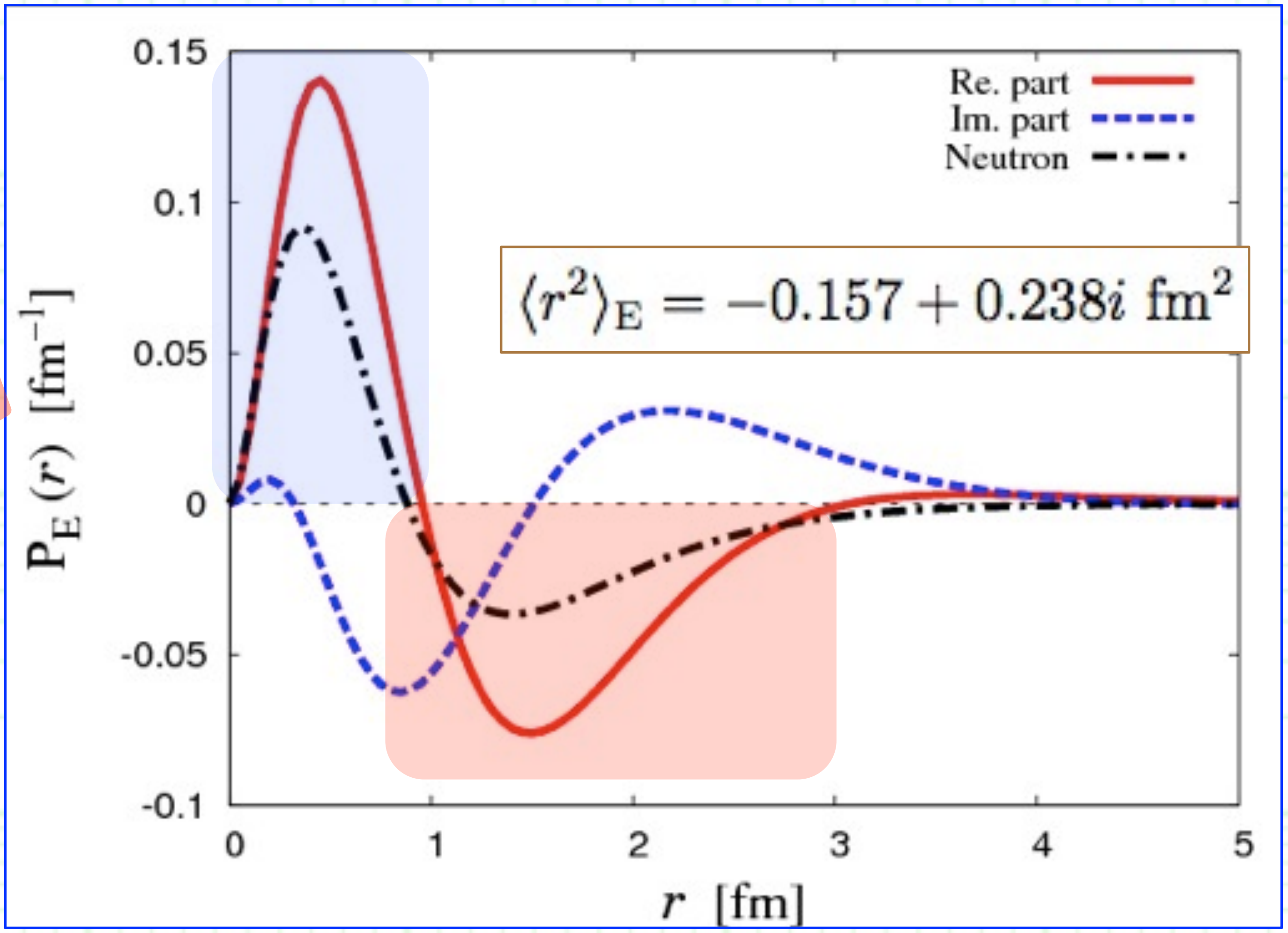
3. Results

++ Structure of resonant Λ(1405) state ++

- Fourier transformation --> charge density ($P = 4 \pi r^2 \varrho$).



(radial coordinate in CM)



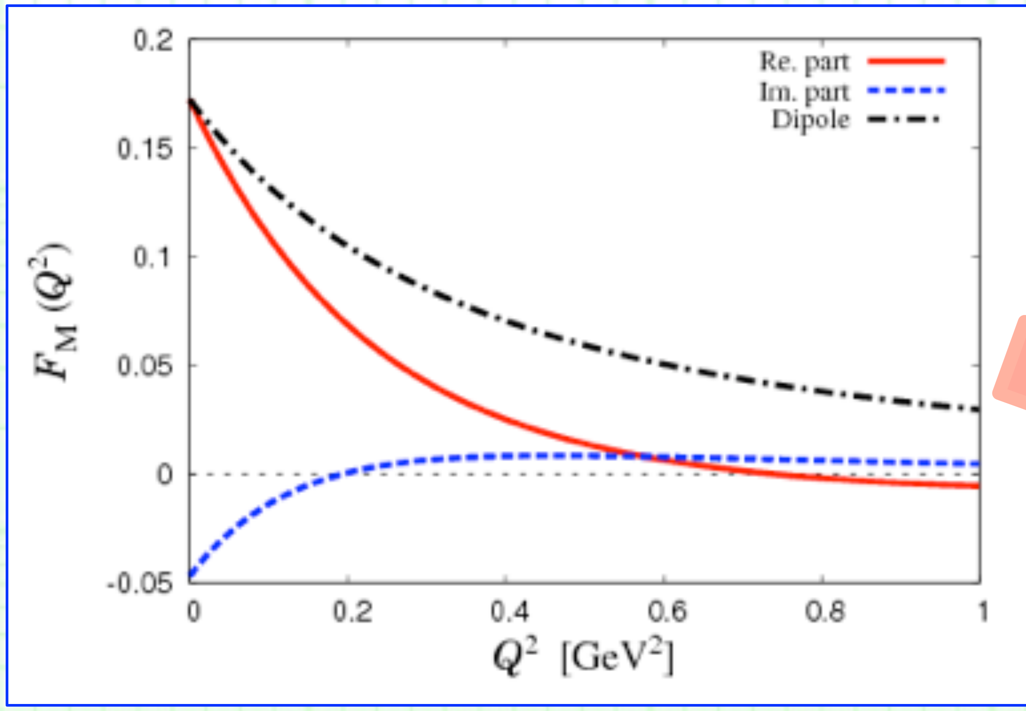
- Negative (positive) charge appears in outer (inner) region.

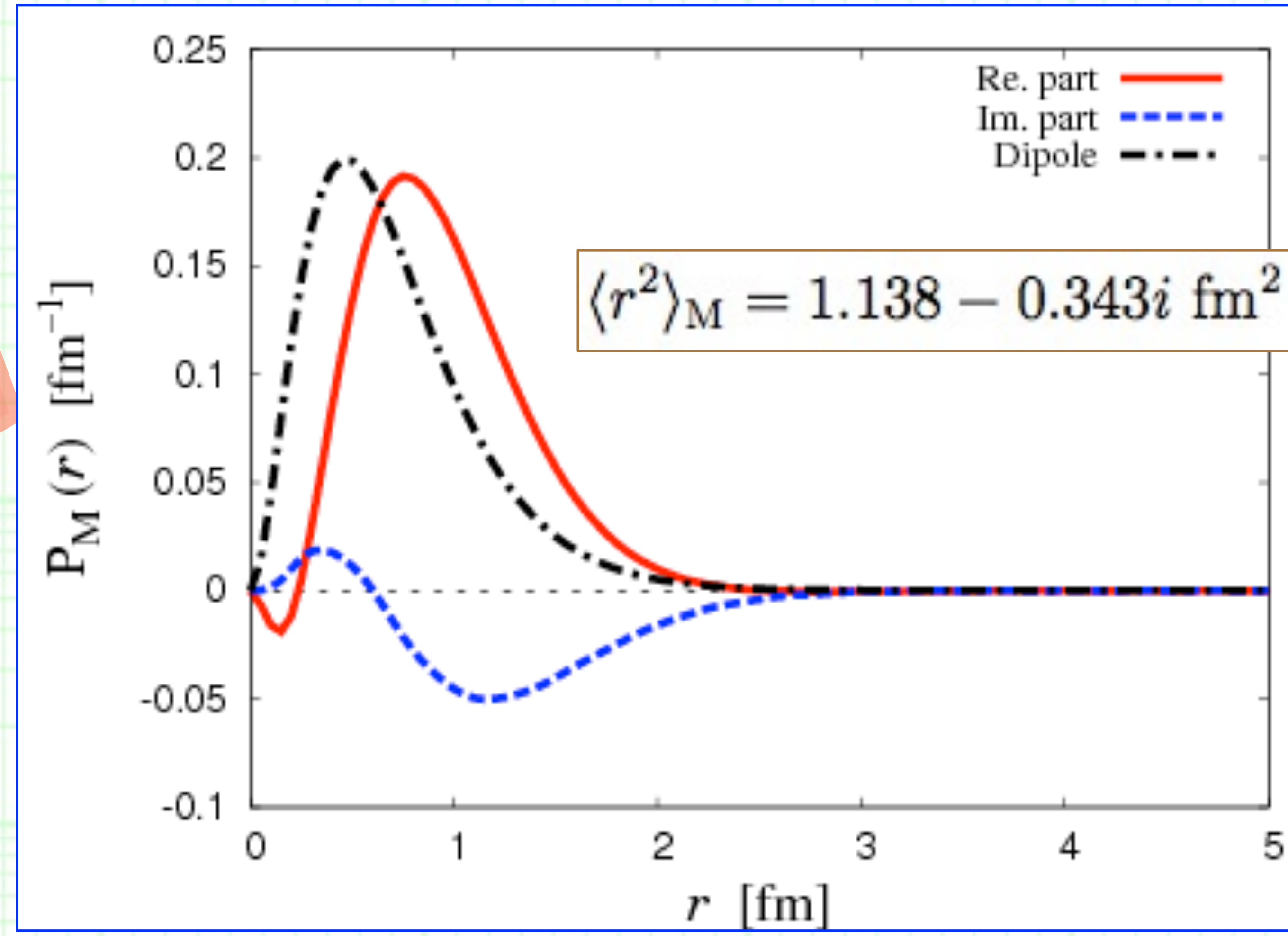
--- Interpreted as that the lighter K^- surrounds the heavier p, recalling the large $\bar{K}N$ component for the conserved charge.

3. Results

++ Structure of resonant Λ(1405) state ++

- Fourier transformation --> magnetic moment density ($P = 4\pi r^2 \rho$).





(radial coordinate in CM)

- Spatially larger structure of Λ(1405).

--- Magnetic moment distribution beyond ~ 1 fm.

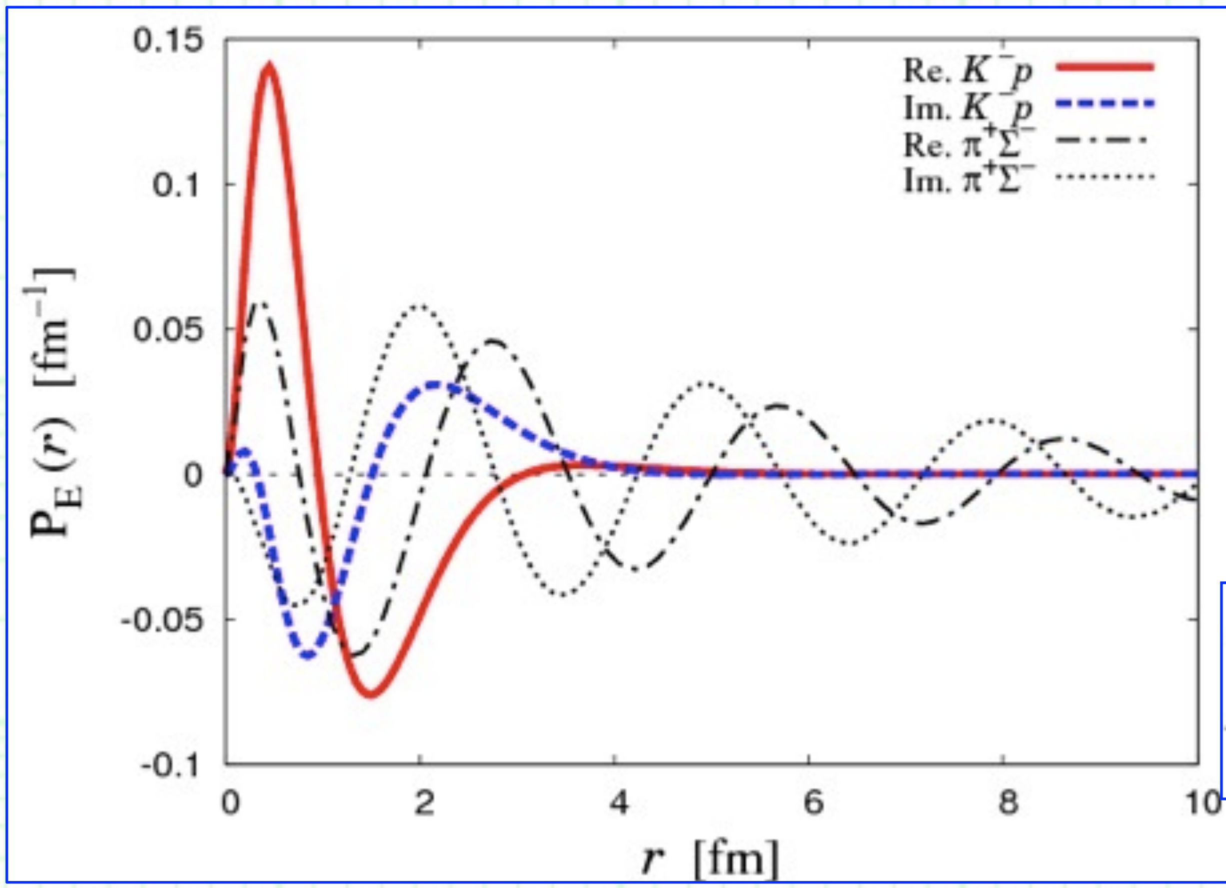
<-- Large distribution of nucleon inside Λ(1405).

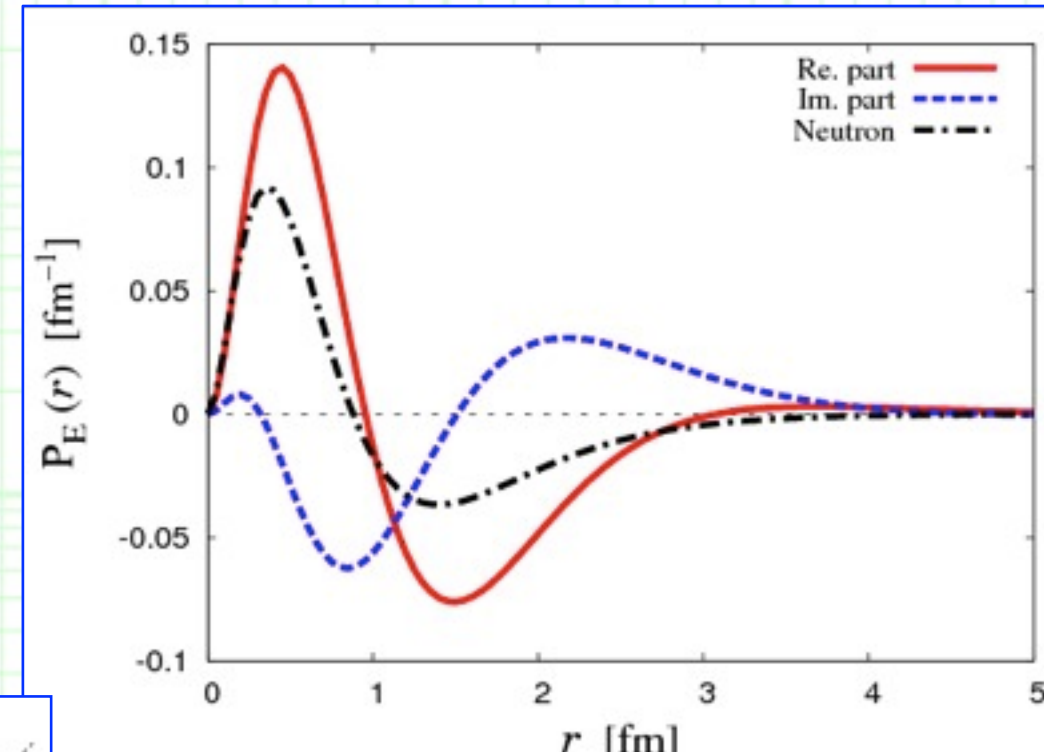
3. Results

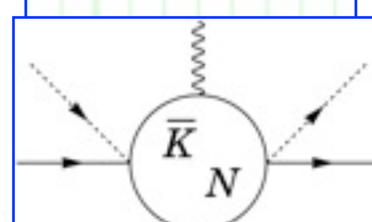
++ Structure of resonant Λ(1405) state ++

- Channel decomposition shows component of structure.

--- Electric $\bar{K}N$ component:







decomposition.

- Distribution is dominated by the $\bar{K}N$ component.

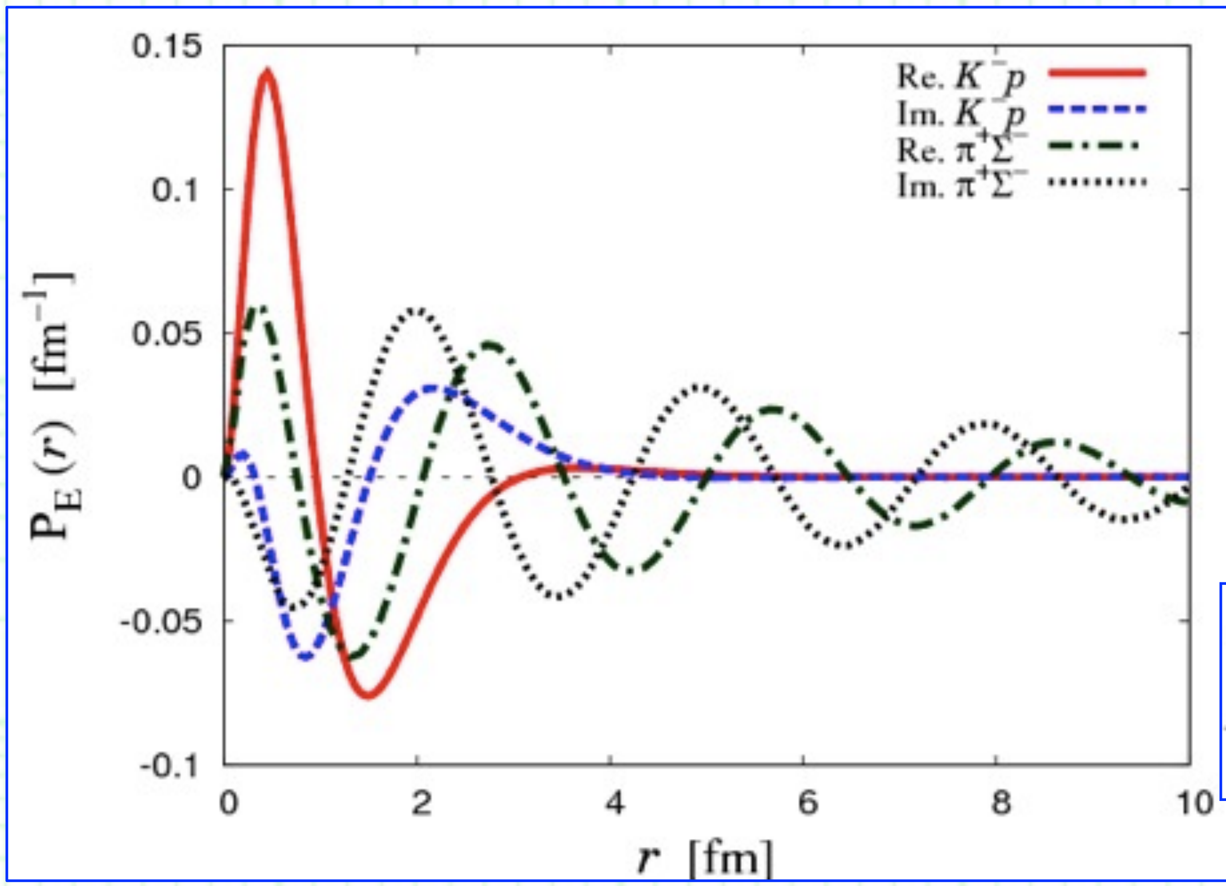
--> Indeed the lighter K^- surrounds the heavier p, as expected.

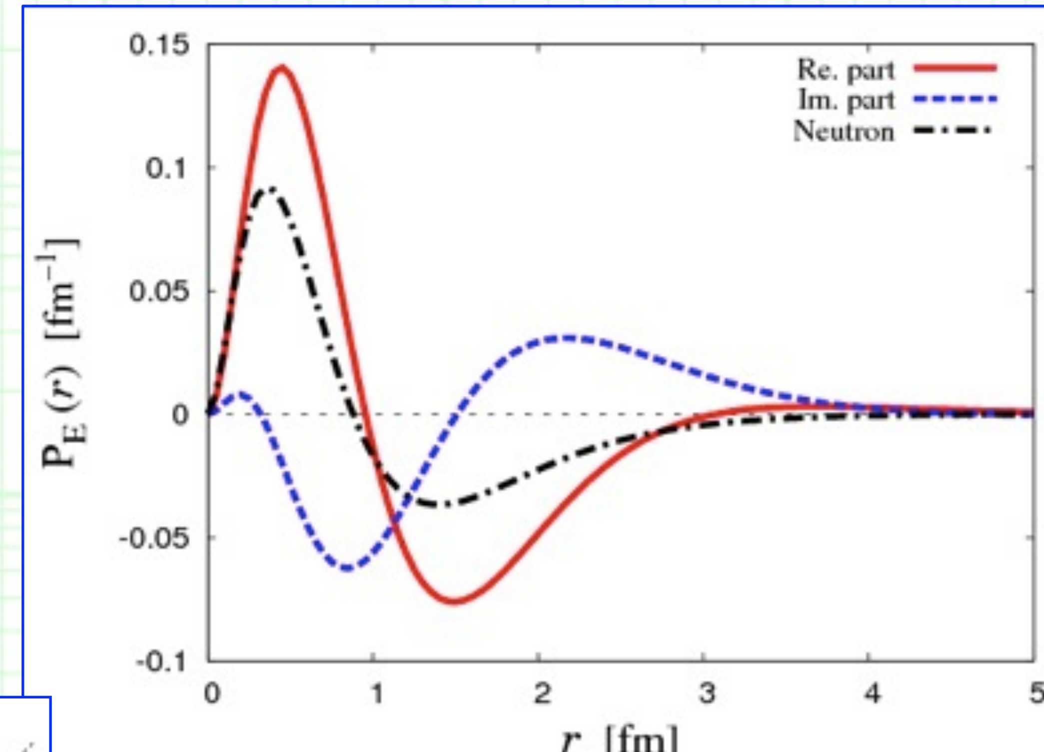
3. Results

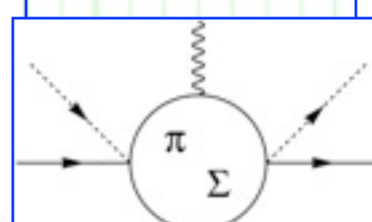
++ Structure of resonant Λ(1405) state ++

- **Channel decomposition** shows component of structure.

--- Electric πΣ component:







decomposition.

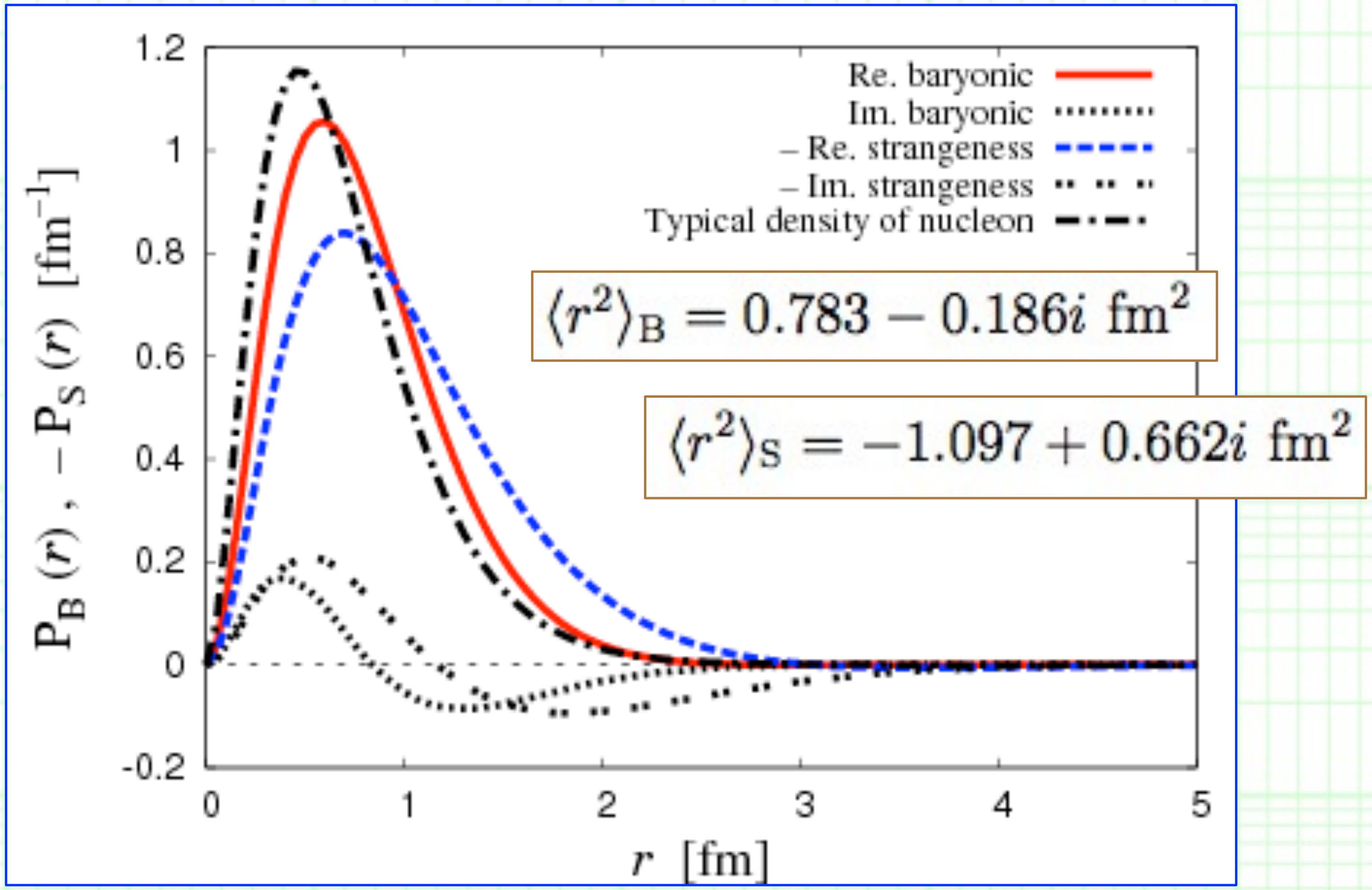
- **πΣ component shows dumping oscillation** as decay channel.

--> **Observe the decaying component in coordinate space**, originating from that Λ(1405) exists above the πΣ threshold.

3. Results

++ Structure of resonant Λ(1405) state ++

- **Baryonic and strangeness structure for Λ(1405):**



- **We observe widely spread $\bar{K}$ around N, and both distributions are larger than the typical nucleon size ~ 0.8 fm.**
- **Consistent with the EM structure.**

--- Small imaginary part and decaying πΣ part (very tiny).

4. Summary

++ Summary ++

- We have shown why we are interested in the spatial size of unstable resonance state.

--- Spatial size of hadrons may be a key to discriminate their component. --- Hadronic bound state or others.

$$|\Lambda(1405)\rangle = C_{uds}|uds\rangle + C_{\bar{K}N}|\bar{K}N\rangle + C_{uud\bar{u}s}|uud\bar{u}s\rangle + \cdots$$

- Hadrons are generated by quantum quark dynamics, but it also decays the (almost) hadrons.

--- Due to the “strong” interaction, imaginary part of the system’s eigenvalue is not negligible compared to the real part.

--> One needs to treat (almost all) hadrons as resonance states.

- If a hadron is dominated by the bound state component of constituent hadrons, its coefficient [C_{KN} for $\Lambda(1405)$] will be large.

--> The spatial size will be sensitive to the coefficient (C_{KN}).

--- Large spatial size <--> large coefficient C_{KN}.

4. Summary

++ Summary ++

- We calculate electromagnetic, baryonic, and strangeness form factors and internal density distributions of $\Lambda(1405)$ in chiral unitary approach, in which we have meson-baryon interaction picture based on chiral symmetry with Bethe-Salpeter equation.

--- Structure from our form factor is consistent with expectation from quantum mechanics.

- $\Lambda(1405)$ is composed of widely spread $\bar{K}$ around N (dominant) + escaping $\pi\Sigma$ oscillation component.

- Both $\bar{K}$ and N distributions are larger than typical nucleon size ~ 0.8 fm.

--- Our description of $\Lambda(1405)$ in chiral dynamics is consistent with meson-baryon interaction picture.

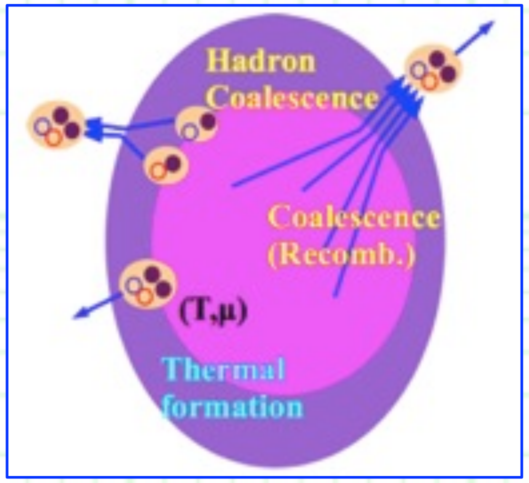
Thank you very much for your kind attention !

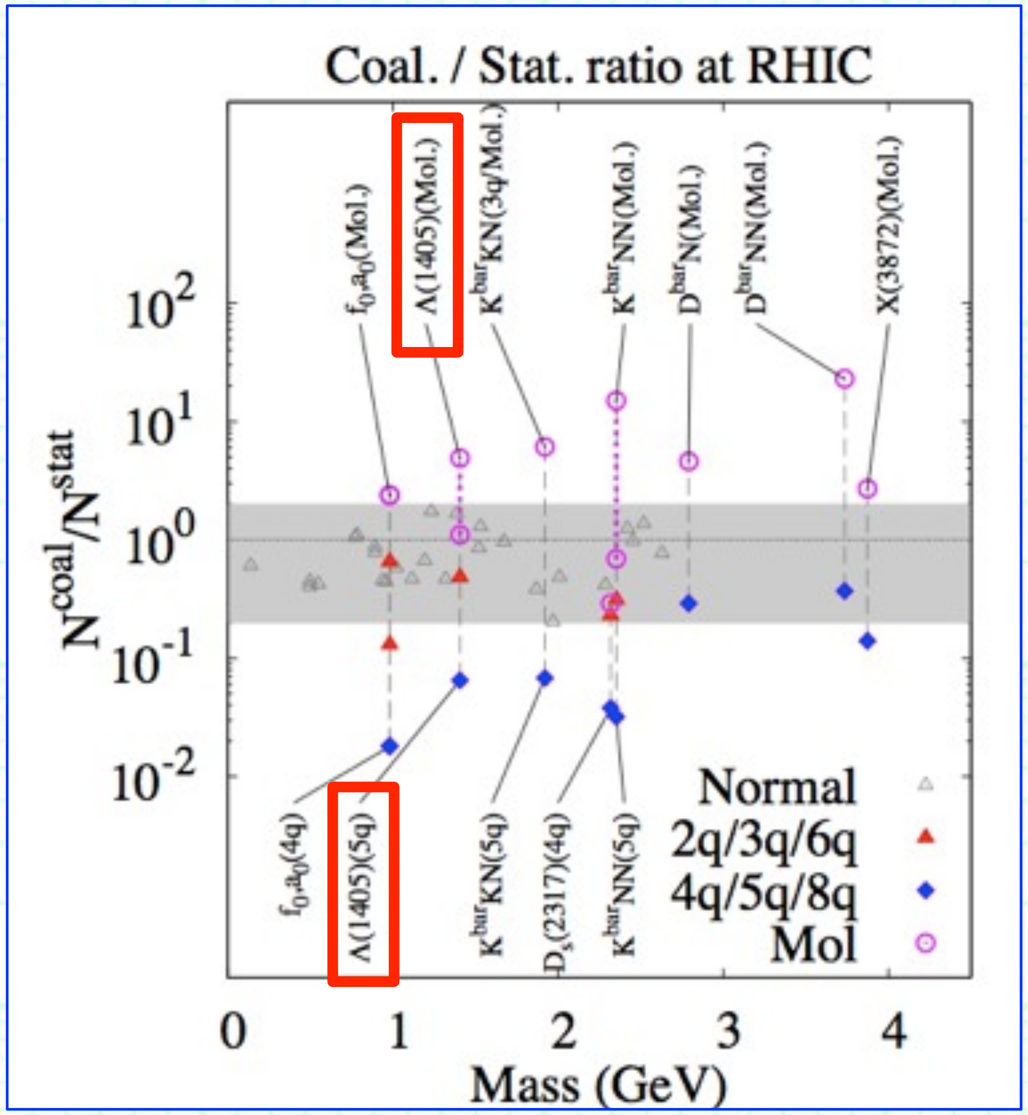
Appendix

Appendix

++ How to probe "Λ(1405) size" in Exp.? ++

- There is some possibility to obtain
  information of "Λ(1405) size" from heavy ion collisions.
  --- Λ(1405) yields estimated by the coalescence model.
  <-- Sensitive to the structure!





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