Many-body resonances in He isotopes and those mirror nuclei

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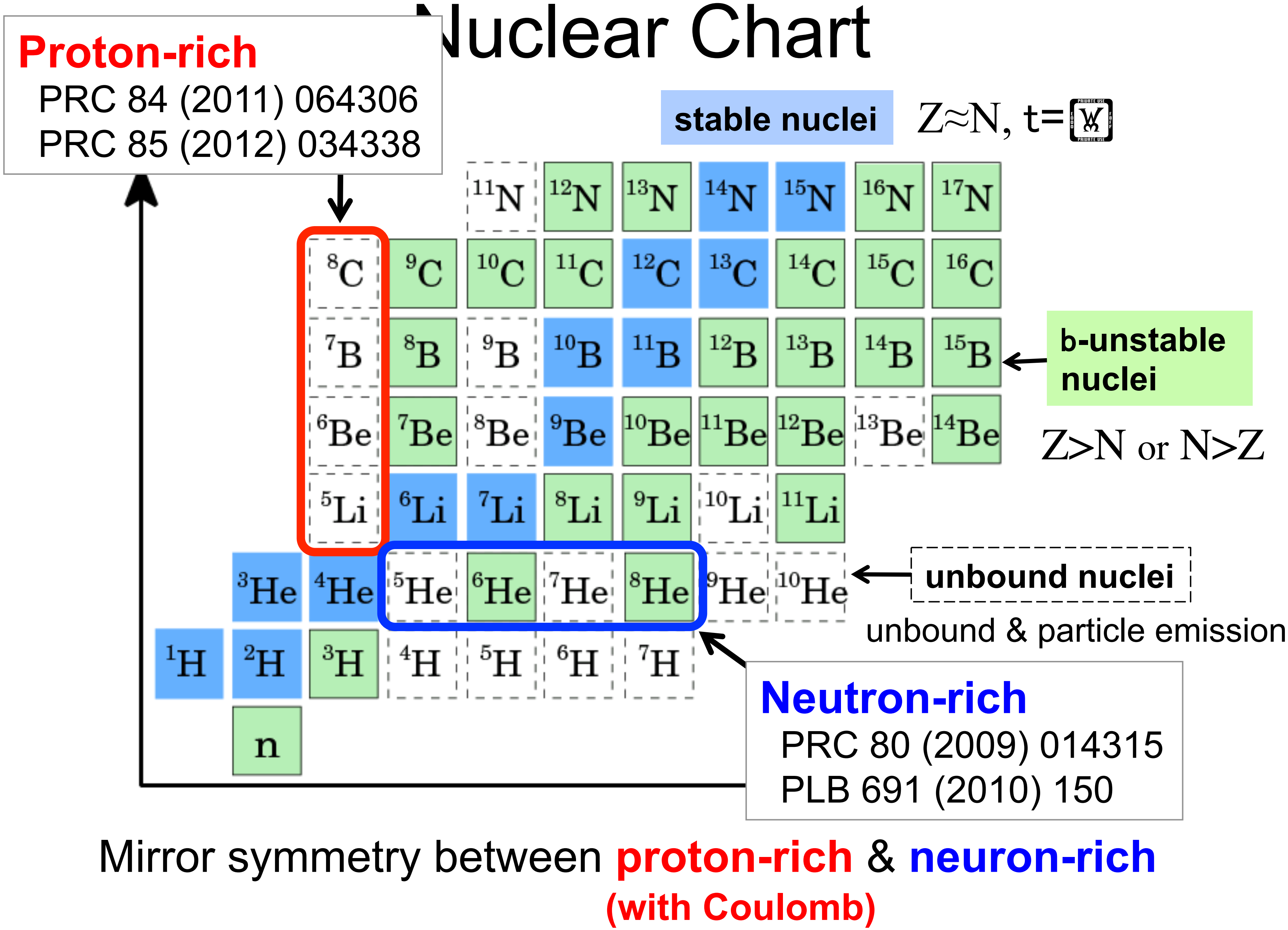
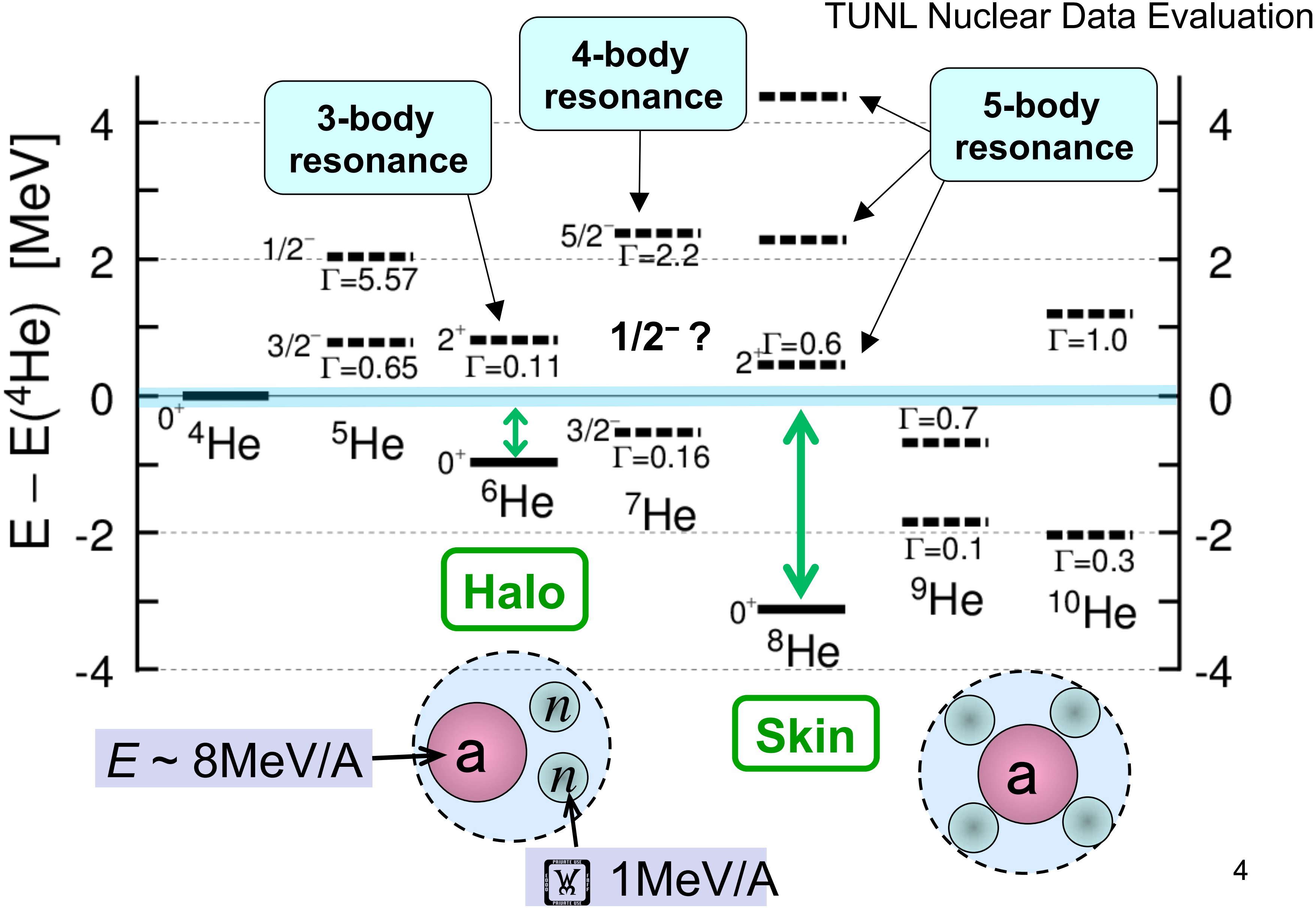
Outline

- Structure of Light Unstable Nuclei
  - He isotopes (neutron-rich)
  - Mirror nuclei (proton-rich)
- Cluster Orbital Shell Model (**COSM**)
  - Core nuclei + valence protons / neutrons
- Complex Scaling Method (**CSM**)
  - Many-body resonances & continuum states
  - Give continuum level density, Green's function
  - Strength functions


Nuclear Chart
Proton-rich
PRC 84 (2011) 064306
PRC 85 (2012) 034338
stable nuclei
Z≈N, t=
11N 12N 13N 14N 15N 16N 17N
8C 9C 10C 11C 12C 13C 14C 15C 16C
7B 8B 9B 10B 11B 12B 13B 14B 15B
6Be 7Be 8Be 9Be 10Be 11Be 12Be 13Be 14Be
5Li 6Li 7Li 8Li 9Li 10Li 11Li
3He 4He 5He 6He 7He 8He 9He 10He
1H 2H 3H 4H 5H 6H 7H
n
b-unstable nuclei
Z>N or N>Z
unbound nuclei
unbound & particle emission
Neutron-rich
PRC 80 (2009) 014315
PLB 691 (2010) 150
Mirror symmetry between proton-rich & neuron-rich
(with Coulomb)

Neutron-rich He isotopes : experiment

TUNL Nuclear Data Evaluation



Method

- Cluster Orbital Shell Model (COSM)
  - Open channel effect is treated.

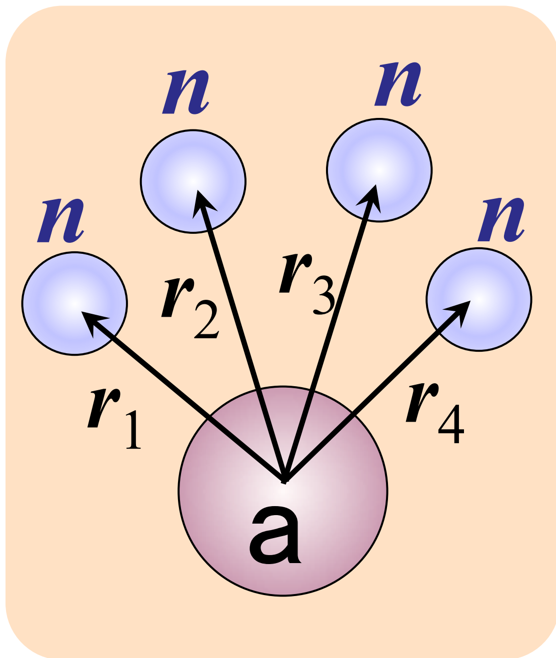
    ^{8}He : ^{7}He+n, ^{6}He+n+n, ^{5}He+n+n+n, ...
- Complex Scaling Method

  $$\mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Resonances with correct boundary condition as Gamow states

    $E=E_r-i$G$/2$
  - Give continuum level density (resonance+continuum)
  - Beyond drip-lines, a-cluster states

COSM



A. T. Kruppa, R. G. Lovas, B. Gyarmati, PRC37(1988) 383 (^{8}**Be as 2a**)

S. Aoyama, TM, K. Kato, K. Ikeda, PTP116(2006) 1 (**CSM review**)

C. Kurokawa , K. Kato, PRC71 (2005) 021301 (^{12}**C as 3a**)

Cluster Orbital Shell Model (*n*-rich)

- System is obtained based on RGM equation

$$H(^{A}\mathrm{He}) = H(^{4}\mathrm{He}) + H_{\mathrm{rel}}(N_V n) \qquad \Phi(^{A}\mathrm{He}) = A\left\{\psi(^{4}\mathrm{He}) \cdot \sum_{i=1}^{N} C_i \cdot \chi_i(N_V n)\right\}$$

valence neutron number

i : configuration

$\psi(^{4}\mathrm{He})$: $(0\mathrm{s})^4$ $\rightarrow$ No explicit tensor correlation

$\chi_i(N_V n) = A\{\varphi_{i1}\varphi_{i2}\varphi_{i3}\cdots\}$ $\qquad \varphi_i \ (L \le 2)$ Relative motion with Gaussian expansion

- Orthogonarity Condition Model (OCM) is applied.

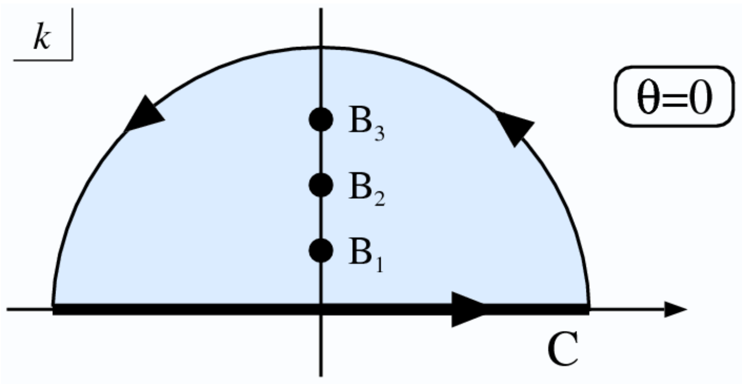
$$\sum_{i=1}^{N}\left\langle \chi_j \left| \sum_{k}^{N_V}\left(T_k + V_k^{cn}\right) + \sum_{k<l}^{N_V}\left(V_{kl}^{nn} + \frac{\hat{p}_i \cdot \hat{p}_j}{A_c m}\right)\right| \chi_i \right\rangle C_i = (E - E_{4\mathrm{He}})\, C_j$$

$\langle \varphi_i | \phi_{\mathrm{PF}} \rangle = 0$ Remove Pauli Forbidden states (PF)

Y. Suzuki, K. Ikeda, PRC38(1988)410, H. Masui, K. Kato, K. Ikeda, PRC73(2006)034318

Complex Scaling for 2-body case

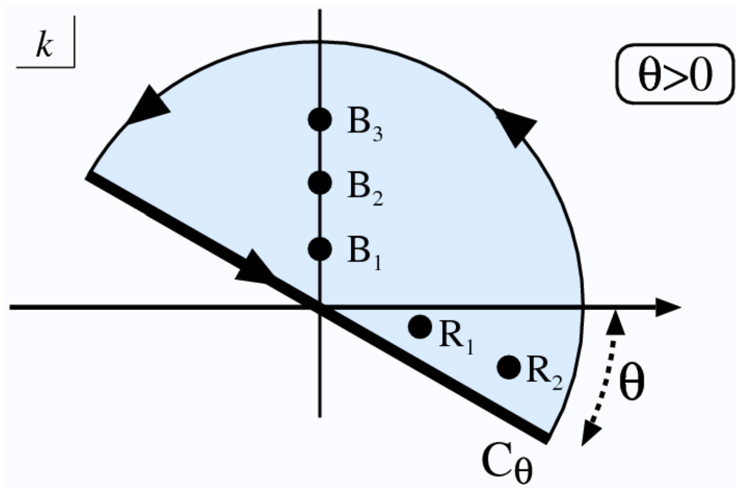
$$U(\theta)\ :\ \mathbf{r} \to \mathbf{r}\cdot\exp(i\theta)\ ,\quad \mathbf{k} \to \mathbf{k}\cdot\exp(-i\theta)\ ,\quad \theta \in \text{。}$$



Completeness relation

$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \int_C dk\, |\varphi_k\rangle\langle\tilde{\varphi}_k|$$

T. Berggren, NPA109('68)265.



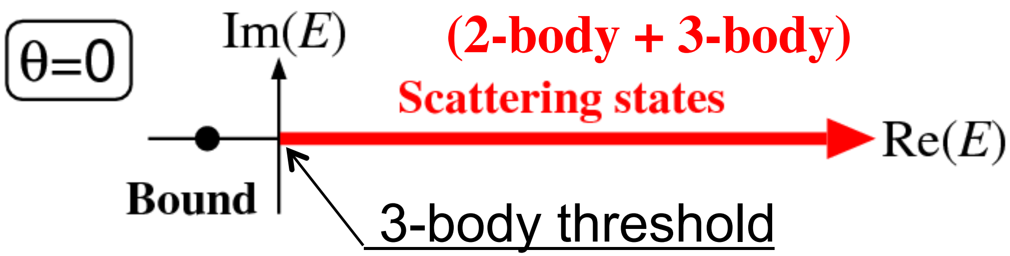
$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \sum_R |\varphi_R\rangle\langle\tilde{\varphi}_R| + \int_{C_\theta} dk_\theta \left|\varphi_{k_\theta}\right\rangle\left\langle\tilde{\varphi}_{k_\theta}\right|$$

J.Aguilar and J.M.Combes, Commun. Math. Phys.,22('71)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22('71)280.

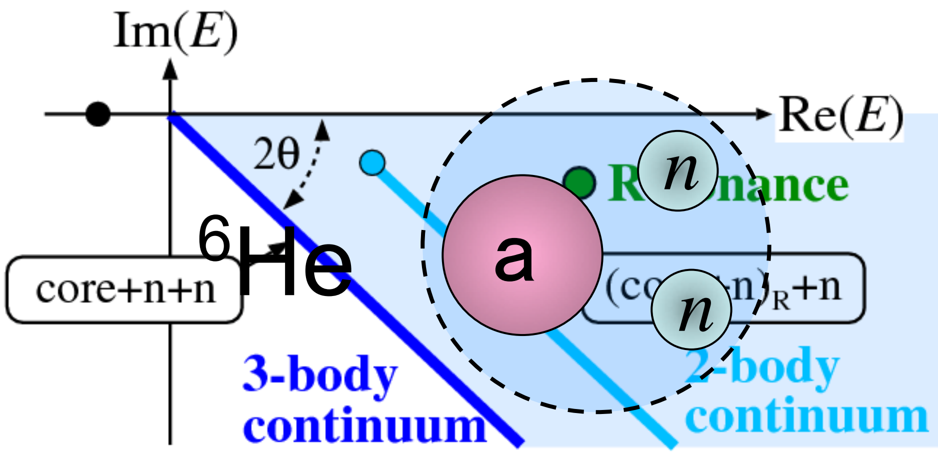
B.G.Giraud, K.Kato, A.Ohnishi
J. Phys. A **37** ('04)11575

Complex Scaling for 3-body case

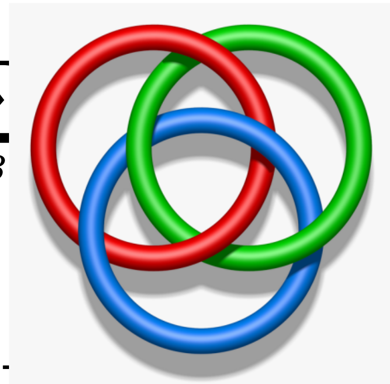
$$U(\theta)\ :\ \mathbf{r}\to\mathbf{r}\cdot\exp(i\theta)\ ,\quad \mathbf{k}\to\mathbf{k}\cdot\exp(-i\theta)\ ,\quad \theta\in\ \circ$$



$$1=\sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \int_C dE\, |\varphi_E\rangle\langle\tilde{\varphi}_E|$$



$$1=\sum_B \ldots + \ldots |\varphi_{E_\theta}\rangle\langle\tilde{\varphi}_{E_\theta}|$$



Borromean rings

Halo nuclei : "Core nuclei+n+n" with Borromean condition

^{6}He=^{4}He+n+n, ^{11}Li=^{9}Li+n+n, ^{14}Be=^{12}Be+n+n, …

Schrödinger Eq. and Wave Func. in CSM

$$U(\theta)HU^{-1}(\theta) = H_\theta = T_\theta + V_\theta \qquad T_\theta = e^{-2i\theta}\cdot T, \quad V = V(\mathbf{r}e^{i\theta})$$

$$H\Phi = E\Phi \;\rightarrow\; H_\theta\Phi_\theta = E\Phi_\theta, \qquad \Phi_\theta(\mathbf{r}) = e^{i3/2\cdot\theta}\cdot\Phi(\mathbf{r}e^{i\theta})$$

Asymptotic Condition in CSM ($r \rightarrow \infty$)

	No Scaling	Scaling
Bound	[illegible] [illegible] 0	[illegible] [illegible] [illegible] 0
Resonance	[illegible] [illegible] [illegible] [illegible]	[illegible] [illegible] [illegible] 0
Continuum	[illegible] [illegible] $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$	[illegible] [illegible] [illegible] $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$

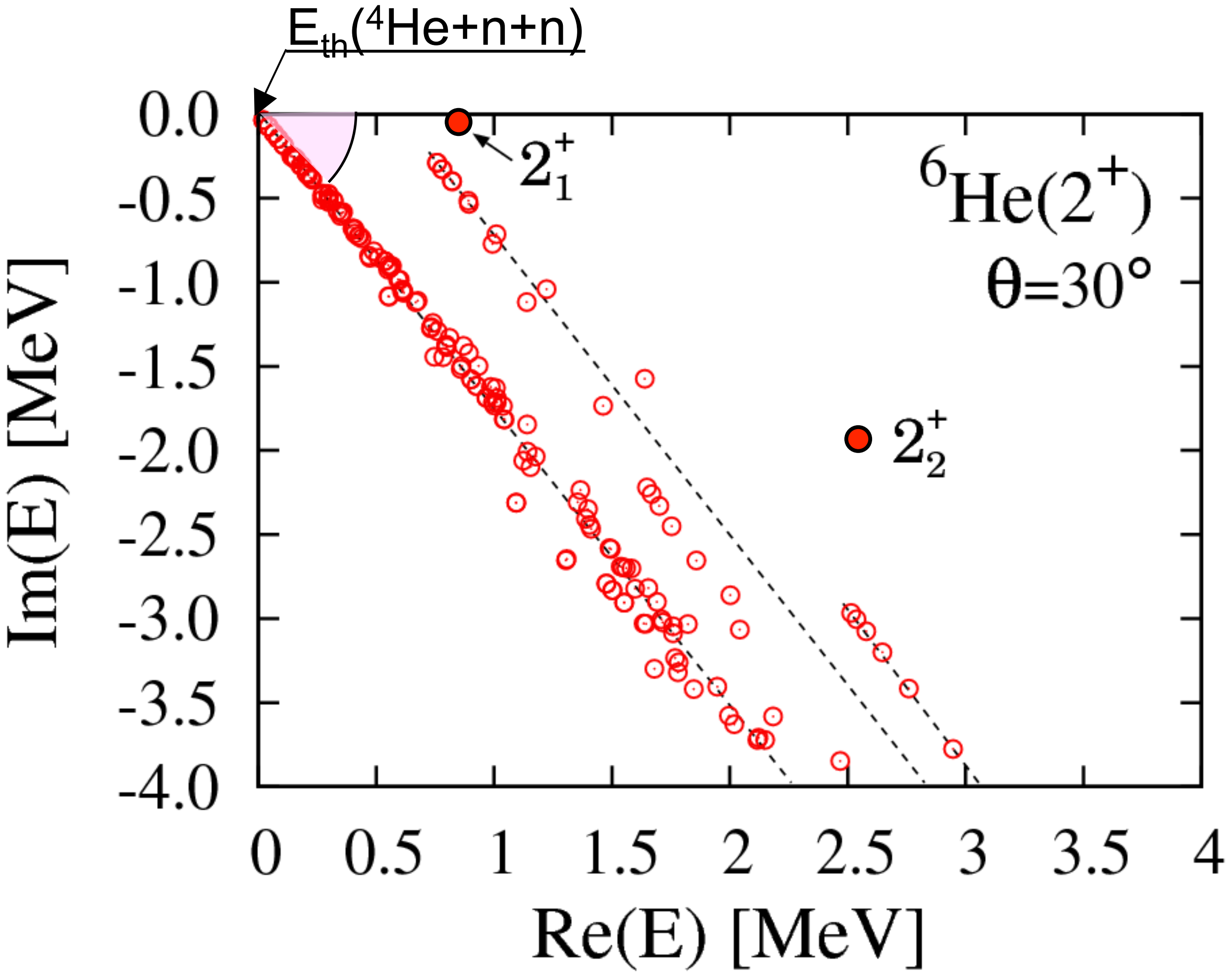
damping condition

$$\Phi^{res} \;:\; \exp(ik_r r) \;= \exp(i\mathrm{k}_r e^{-i\theta_r} r) \qquad k_r = \mathrm{k}_r \cdot e^{-i\theta_r}, \quad \theta_r > 0$$

$$\Phi^{res}_\theta \;:\; \exp(ik_r r_\theta) = \exp(i\mathrm{k}_r e^{i(\theta-\theta_r)} r)$$

$$= \exp\left[i\mathrm{k}_r r\cos(\theta-\theta_r)\right]\cdot\exp\left[-\mathrm{k}_r r\sin(\theta-\theta_r)\right]$$

Spectrum of ^{6}He with ^{4}He+n+n model



^{6}He$^{(*)}$
^{5}He+n
^{4}He+n+n

Continuum states are discretized using **Gaussian basis functions** (by M.Kamimura)

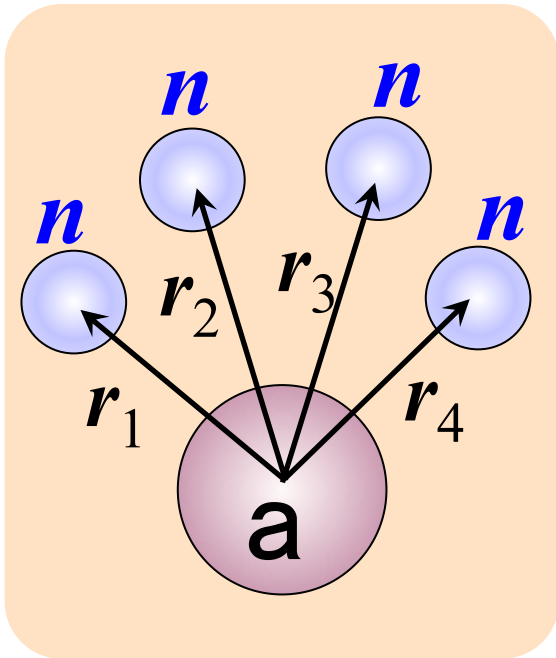
$$\phi_l(\mathbf{r}) = \sum_n C_n \cdot r^l e^{-(r/b_n)^2} Y_l(\hat{\mathbf{r}})$$

A. Csoto, PRC49 ('94) 3035,
S. Aoyama et al. PTP94('95)343, T. Myo et al. PRC63('01)054313

Hamiltonian

- $V_{a\text{-}n}$: microscopic KKNN potential
  - s,p,d,f-waves of a-n scattering
- V_{nn} : Minnesota potential with slightly strengthened (+ Coulomb for *p*-rich nuclei)

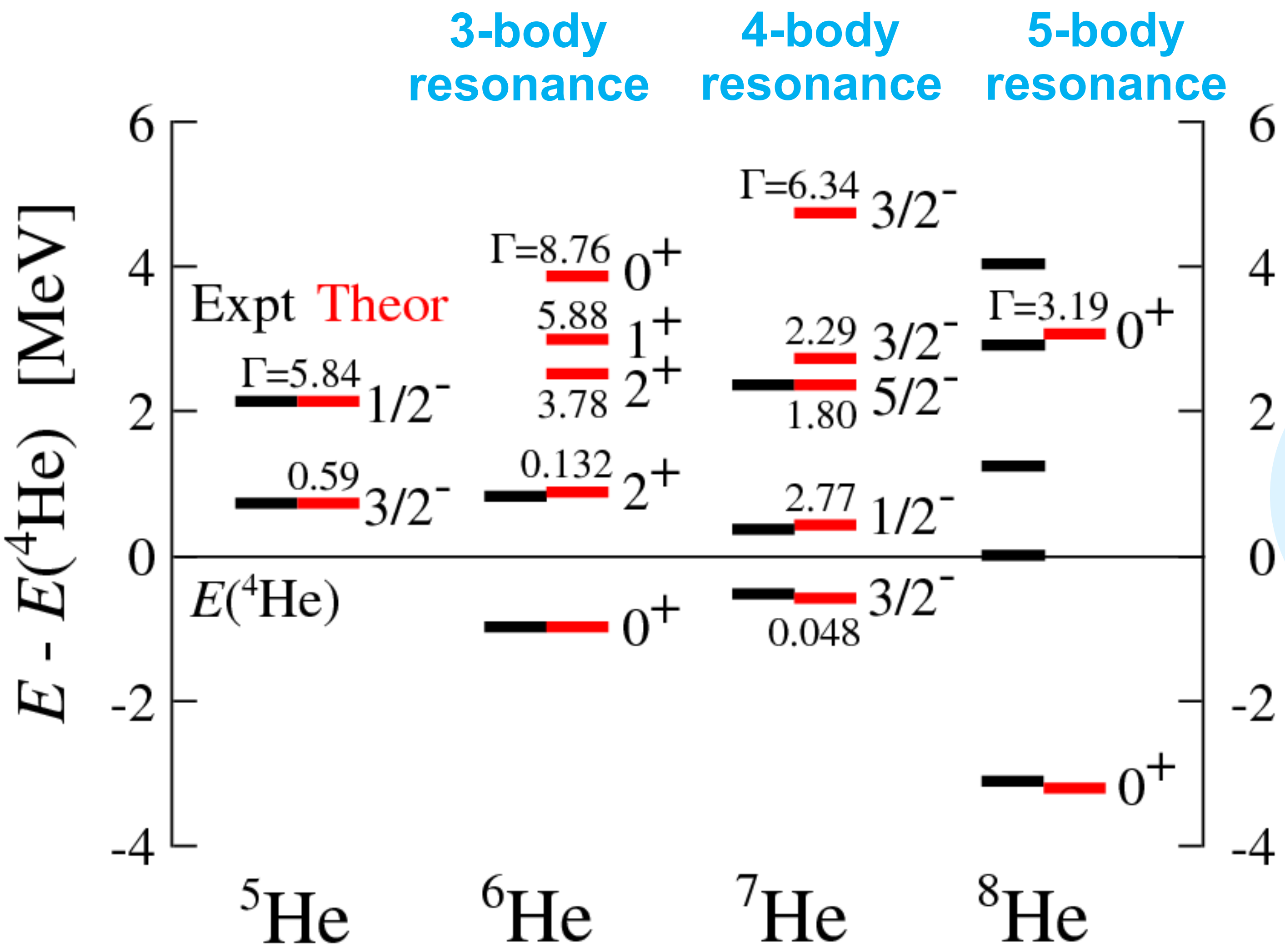
COSM

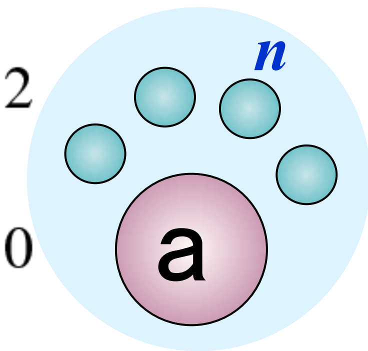


Fit energy of ^{6}He(0^+)

A. Csoto, PRC48(1993)165.
K. Arai, Y. Suzuki and R.G. Lovas, PRC59(1999)1432.
TM, S. Aoyama, K. Kato, K. Ikeda, PRC63(2001)054313.
TM et al. PTP113(2005)763.

He isotopes : Expt vs. Complex Scaling

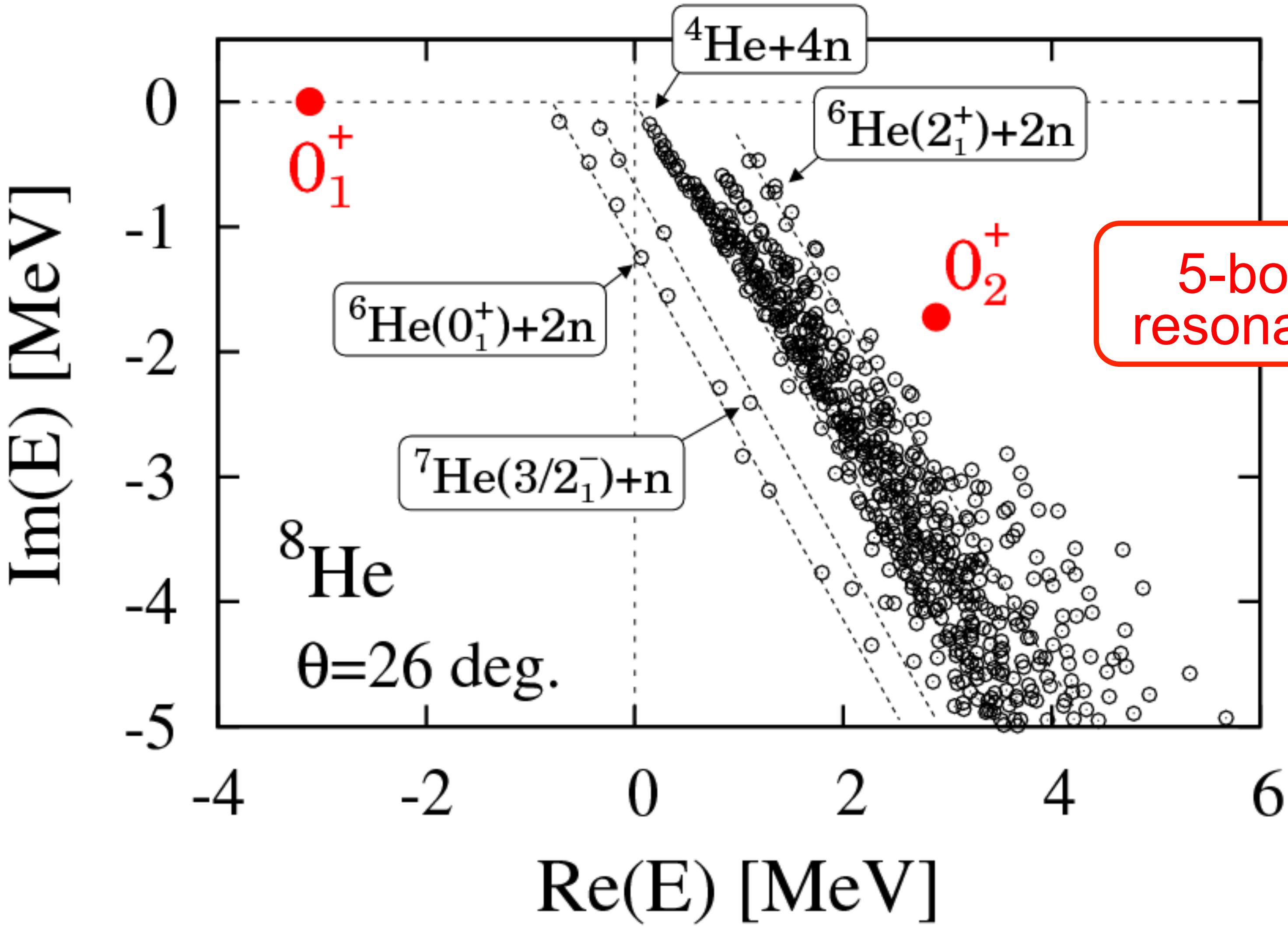




TM, K.Kato, K.Ikeda PRC76('07)054309
TM, R.Ando, K.Kato PRC80('09)014315

TM, R.Ando, K.Kato, PLB691('10)150
TUNL Nuclear Data Evaluation

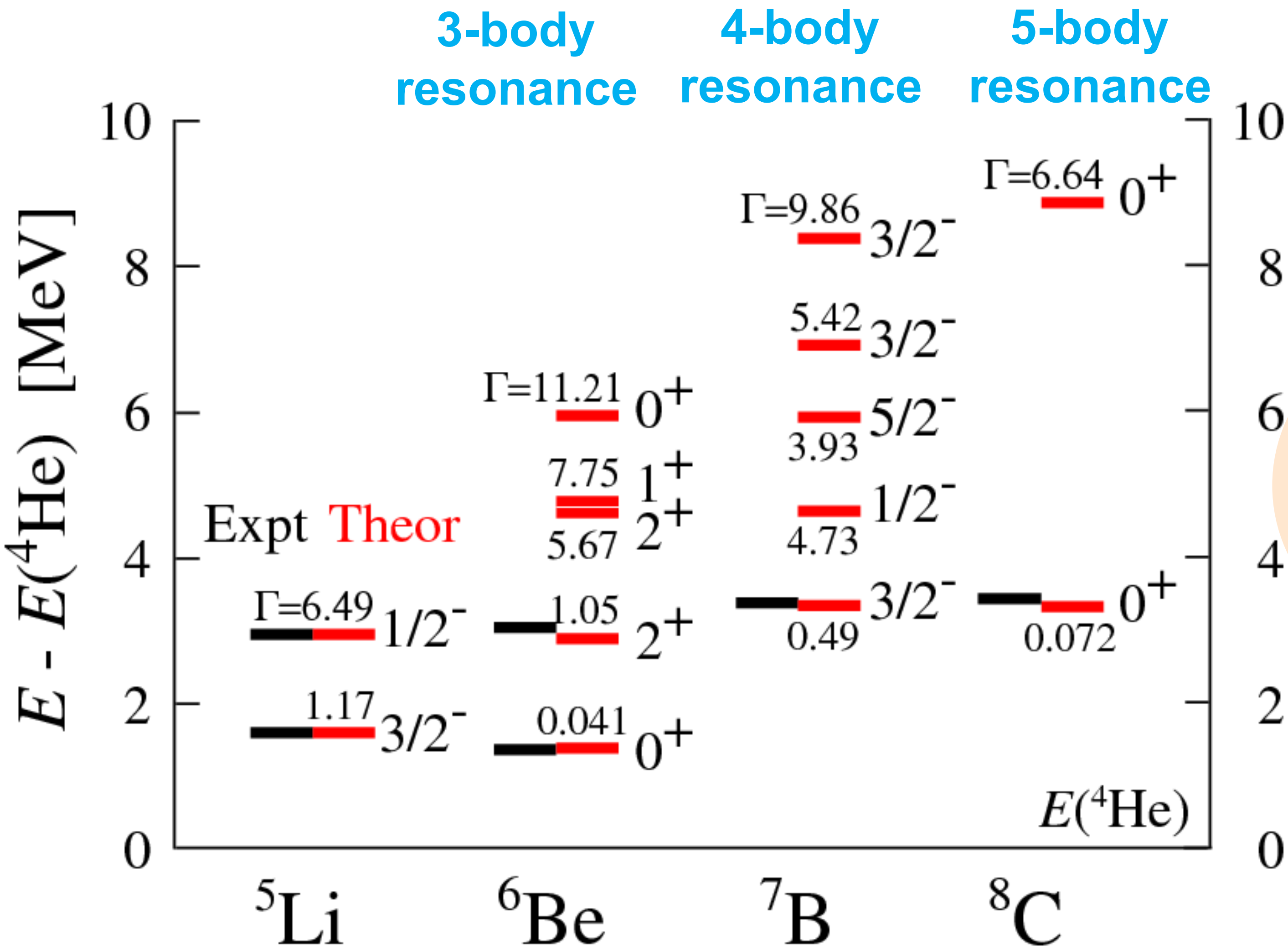
Energy of ^{8}He with complex scaling

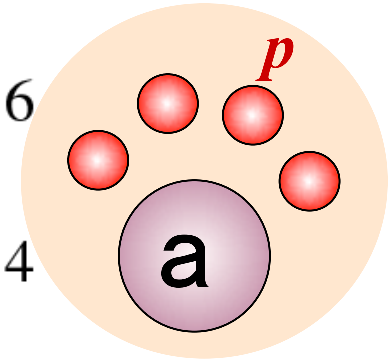


5-body resonance

Eigenvalue problem with 32,000 dim.
Full diagonalization of complex matrix @ SX8R of NEC

Proton-rich side : ^{4}He+p+p+p+p





TM, Kikuchi, Kato PRC84 (2011) 064306
PRC85 (2012) 034338

Expt: Charity et al. PRC84(2011)014320

S-factor of ^{6}He-n component in ^{7}He

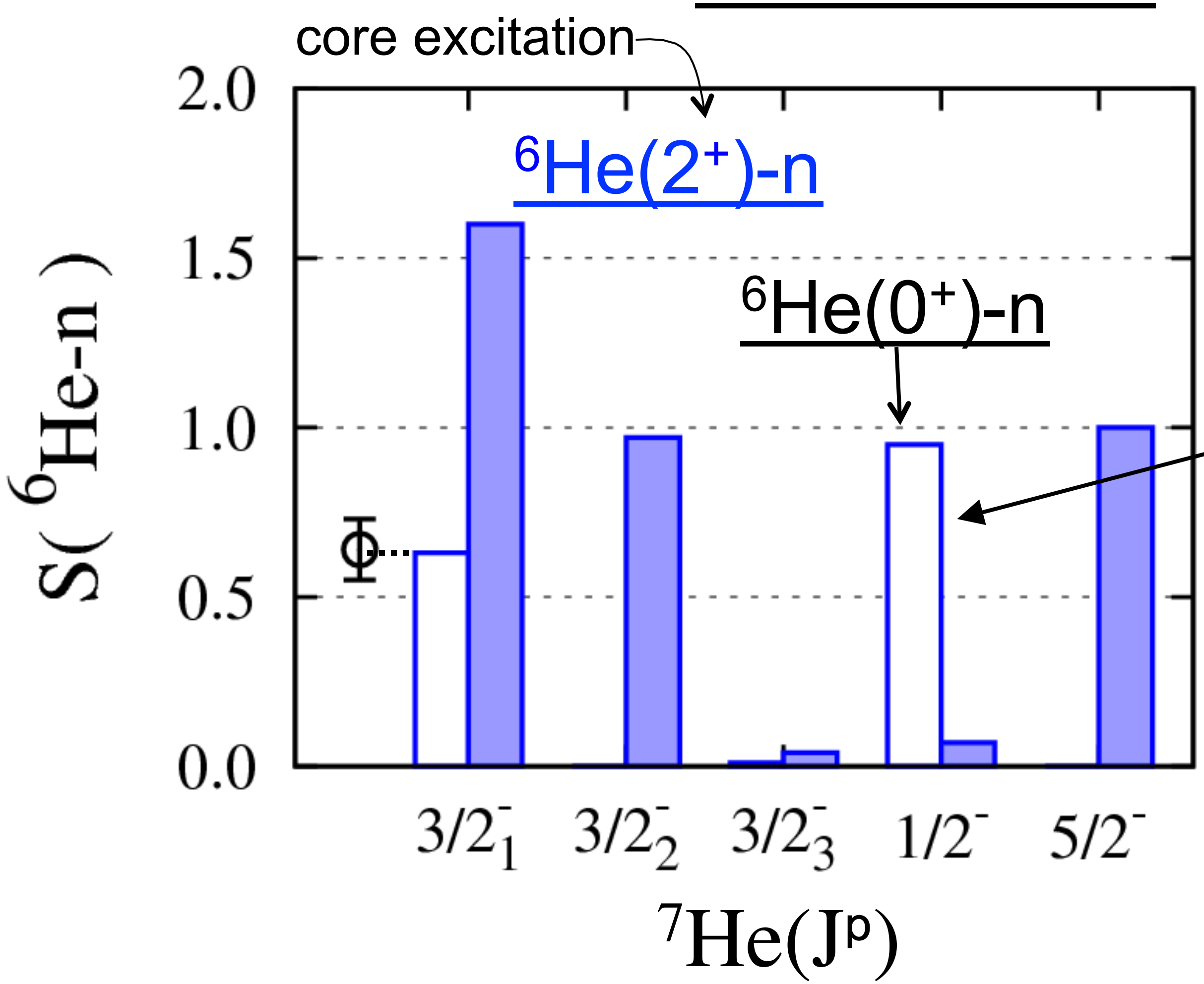
$$S_{J',J} = \sum_{nlj} \left| \left\langle {}^6\mathrm{He}(J') \middle| a_{nlj} \middle| {}^7\mathrm{He}(J) \right\rangle \right|^2$$

neutron removal

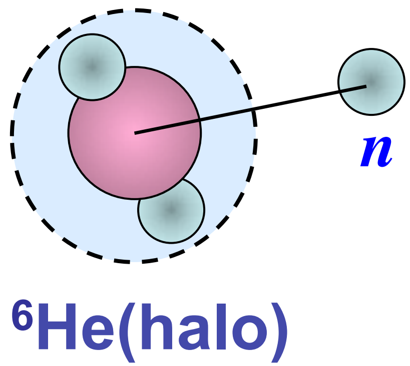
Bi-orthogonal relation

T. Berggren, NPA109(1968)265

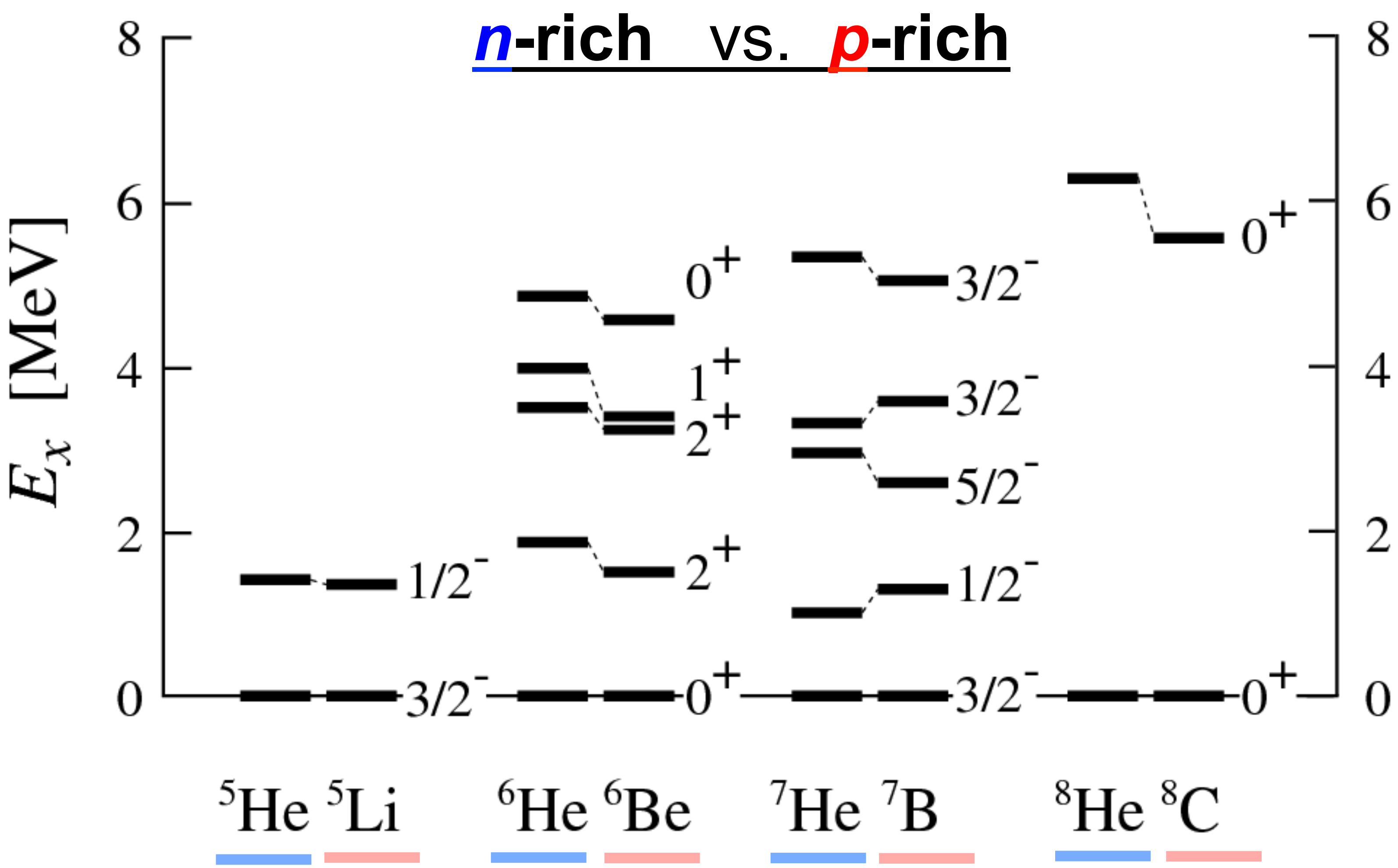
TM, K.Kato, K.Ikeda, PRC76(2007)054309



weak coupling of ^{6}He+*n*



Mirror Symmetry in resonances

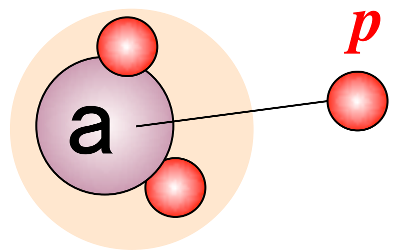


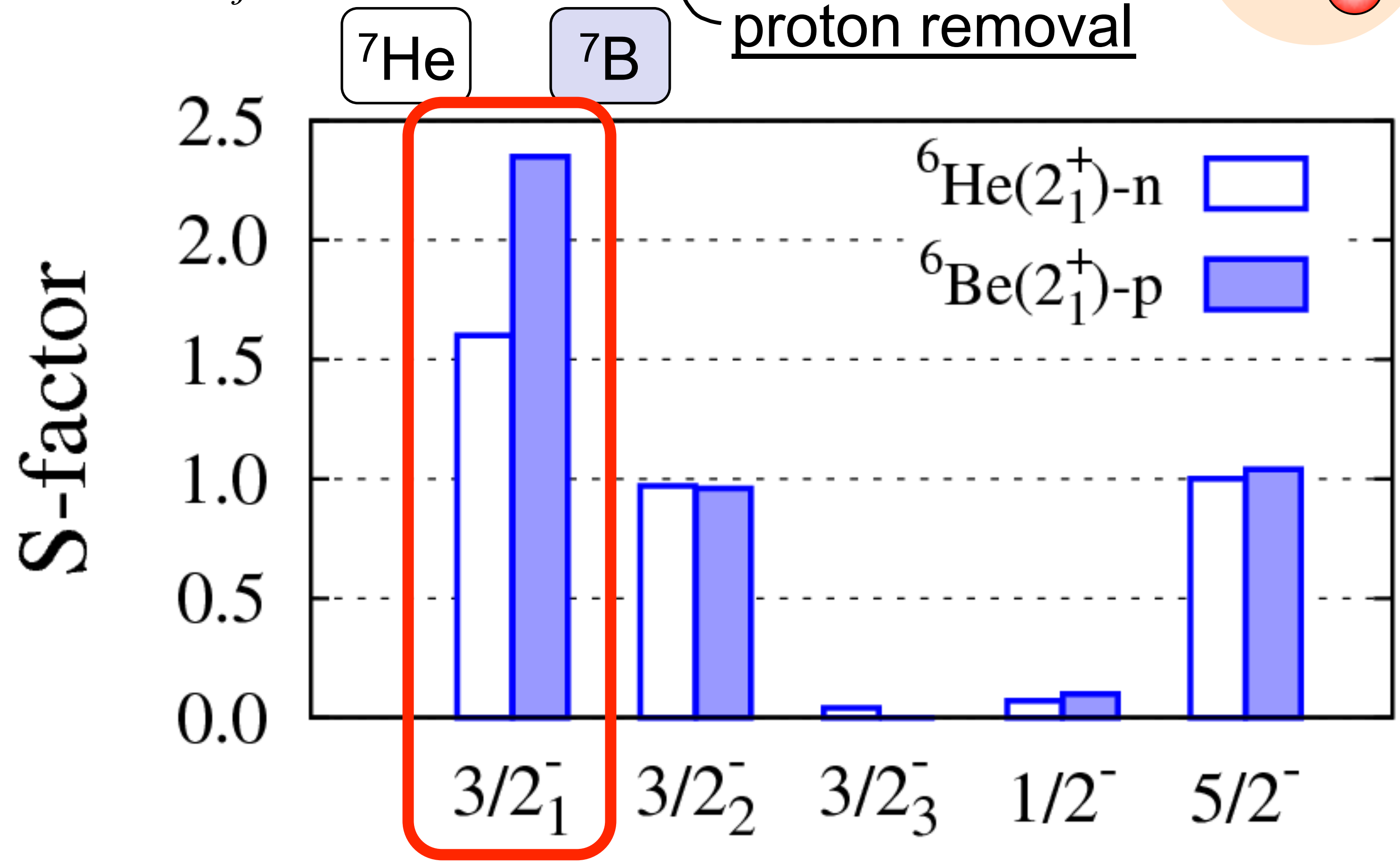
TM, Kikuchi, Kato PRC84 (2011) 064306 , 85 (2012) 034338

S-factors of ^{7}He & ^{7}B

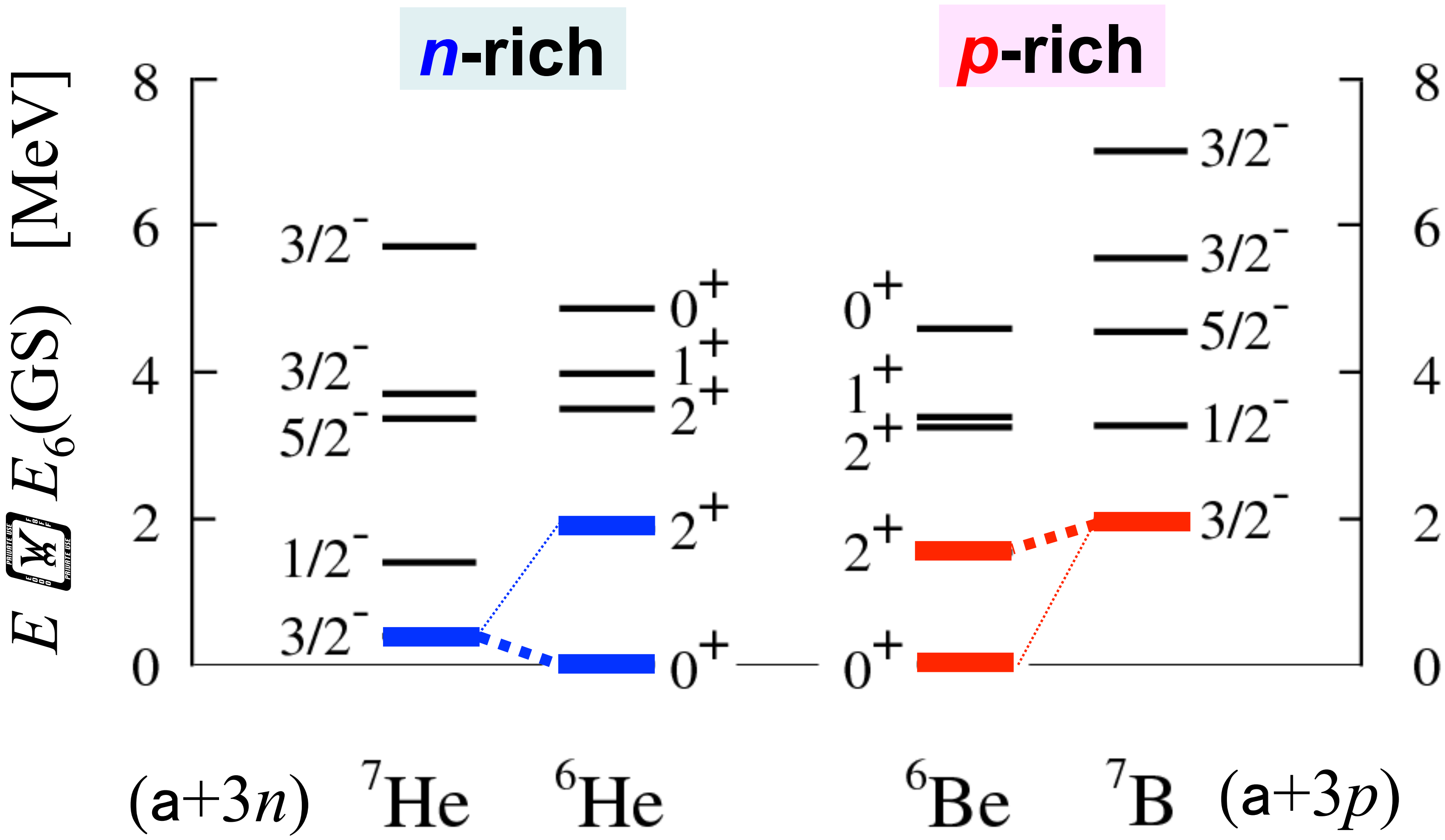
$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(2^+) \middle| a_{nlj} \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$

proton removal





Thresholds of $[A=6]+N$ system



Mirror symmetry breaking due to the channel coupling effect caused by Coulomb force

Continuum Level Density (CLD) in CSM

$$\Delta E = -\frac{1}{\pi}\,\mathrm{Im}\left[\mathrm{Tr}\left[G(E) - G_0(E)\right]\right], \qquad G_{(0)} = \frac{1}{E - H_{(0)}},$$

$$\Delta E = \frac{1}{2i\pi}\,\mathrm{Tr}\left[S(E)^{\dagger}\frac{d}{dE}S(E)\right] \quad \to \quad \frac{1}{\pi}\frac{d\delta}{dE} \quad \text{(single channel case)}$$

S. Shlomo, NPA539('92)17

K. Arai and A. Kruppa, PRC60('99)064315

R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

CLD in CSM

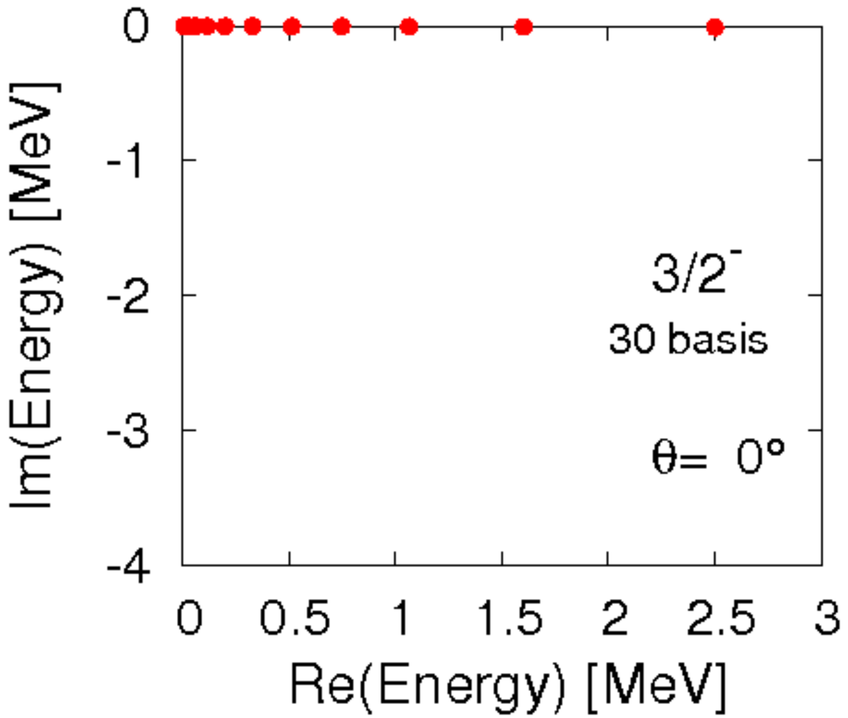
$$\Delta E = -\frac{1}{\pi}\,\mathrm{Im}\left[\mathrm{Tr}\left[G^{\theta}(E) - G_0^{\theta}(E)\right]\right]$$

$$G = \frac{1}{E - H^{\theta}}$$

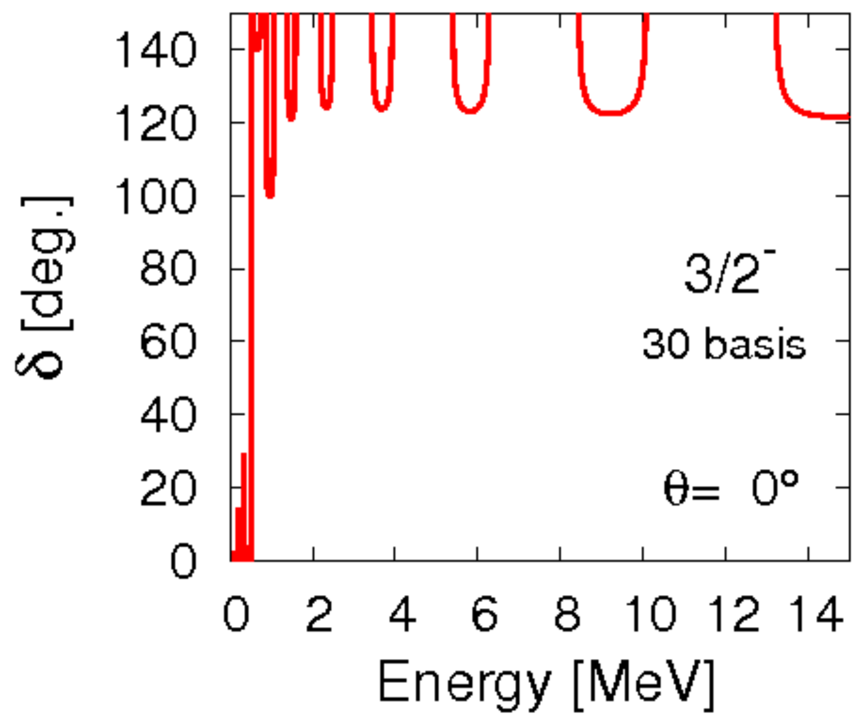
$$G_0 = \frac{1}{E - H_0^{\theta}} \quad \text{(asymptotic)}$$

a+n scattering with complex scaling using discretized continuum states

energy eigenvalues



$P_{3/2}$ scattering phase shift



30 Gaussian basis functions

Strength function $S(E)$ in CSM

- Strength function and response function

Bi-orthogonal relation

$$S(E)=\sum_i \langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle \cdot \delta(E-E_i)$$

initial state

$$= -\frac{1}{\pi}\,\mathrm{Im}\left[R(E)\right]$$

$$R(E)=\sum_i \frac{\langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle}{E-E_i}$$

Response function

- Complex-scaled Green's function

complete set in CSM

$$G^\theta(E)=\frac{1}{E-H_\theta}=\sum_i \frac{|\varphi_i^\theta\rangle\langle\tilde{\varphi}_i^\theta|}{E-E_i^\theta}$$

Bound+Resonance+Continuum

Reaction theory

- LS-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote($K^{bar}N$))

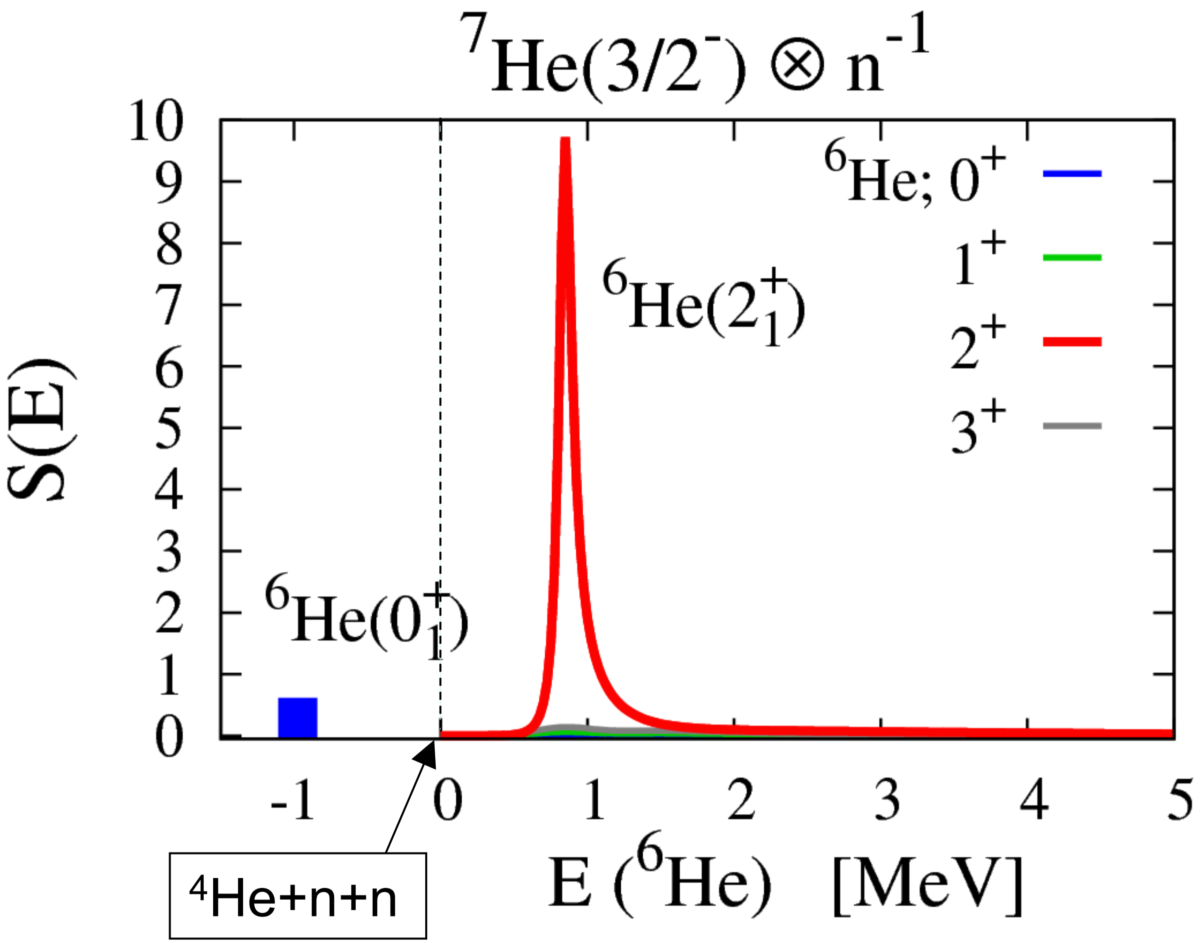
T. Berggren, NPA109('68)265, TM, A. Ohnishi and K. Kato, PTP99('98)801

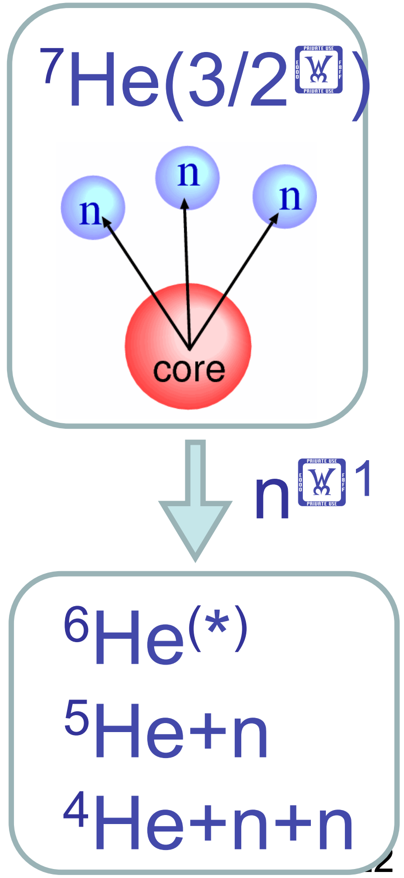
One-neutron removal strength of ^{7}He

$$S_{J',J}(E) = \sum_{nlj} \left\langle {}^6\mathrm{He}^{J'}(E) \middle| a_{nlj} \middle| {}^7\mathrm{He}^{J} \right\rangle^2$$

TM, Ando, Kato
PRC80(2009)014315

" ^{4}He+n+n" complete set using CSM





Summary

- **Light Unstable Nuclei**
  - He isotopes (***n*-rich**) & Mirror nuclei (***p*-rich**)
  - Mirror symmetry & Channel coupling (threshold)
- **Complex Scaling**
  - Many-body resonance spectroscopy
  - Continuum level density (resonance+continuum)
  - Strength functions using Green's function
    - Coulomb breakups, Nucleon removal, ...
    - Application to reaction theory (CDCC, LS eq.,…)