Resonances and responses of few-body systems

Y. Suzuki
(Niigata, RIKEN)

Outline

- Application of square-integrable basis to continuum problems
- Resonances and E1 & SD responses of ^{4}He

Resonance is a metastable state in continuum that has
enough energy to breakup into two or more subsystems (Moiseyev)

Application of square-integrable basis to continuum problems

Problems including continuum states

- **Decay of resonance**

  $A^* \longrightarrow B+b, \quad C+d+e$

- **Strength (response) function due to perturbation W**

  $A+W \longrightarrow A^*, \quad B+b, \quad C+d+e$

- **Radiative capture reactions**

  $A+a \longrightarrow C+\gamma$

  (Inverse process (photodisintegration): $C+\gamma \longrightarrow A+a$)

- **Two-body scattering and reactions**

  $A+a \longrightarrow B+b$

Spectrum of ^{4}He

^{4}He is doubly magic

The first excited state is 0^+

All the excited states are in continuum

A good system to study resonances

Mostly broad widths, thus challenging

(Though a special example, the idea should be general)

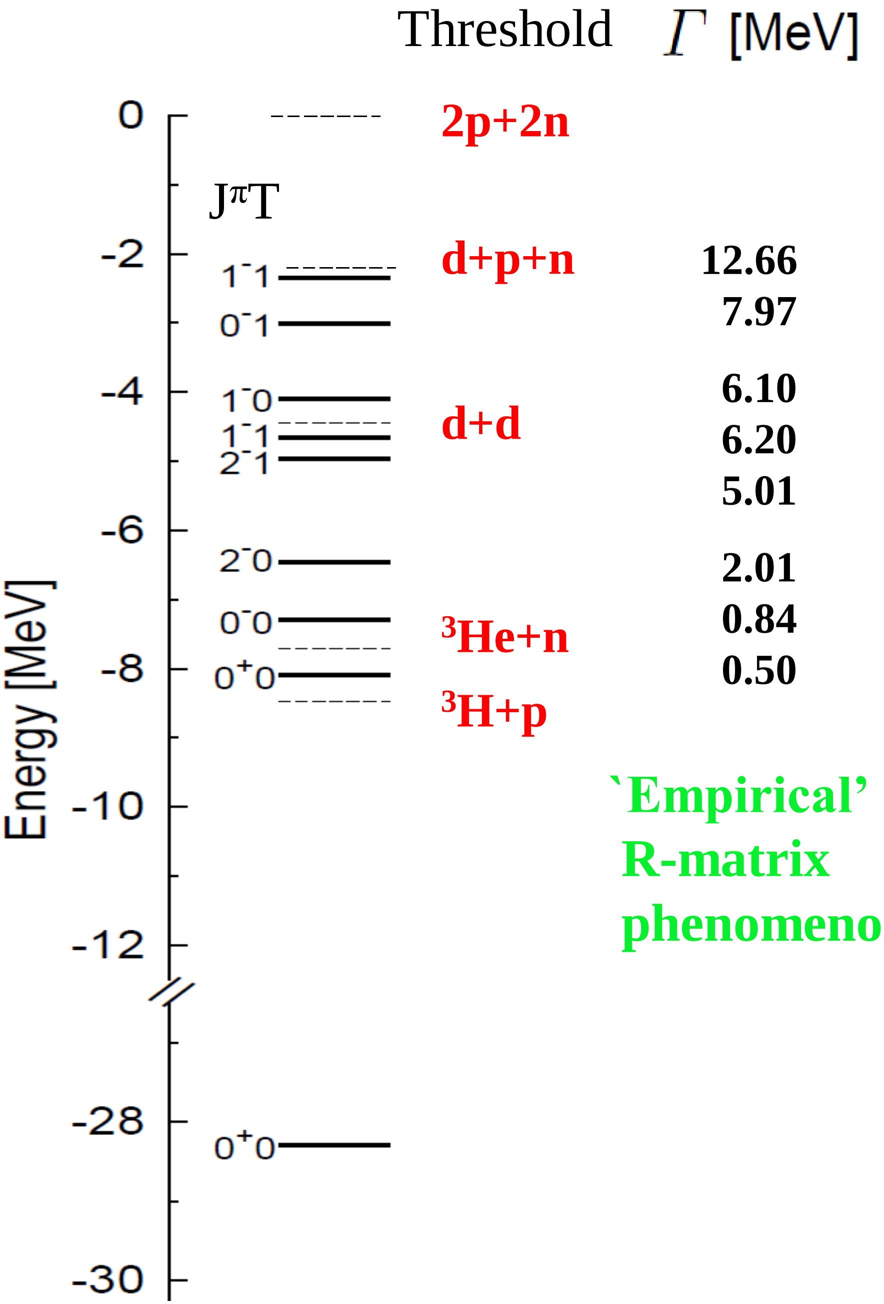
ACCC (Analytic continuation in coupling constant)

Real Hamiltonian H(λ)

CSM (Complex scaling method)

Complex Hamiltonian

Accurate solution of bound states is required



`Empirical' R-matrix phenomenology

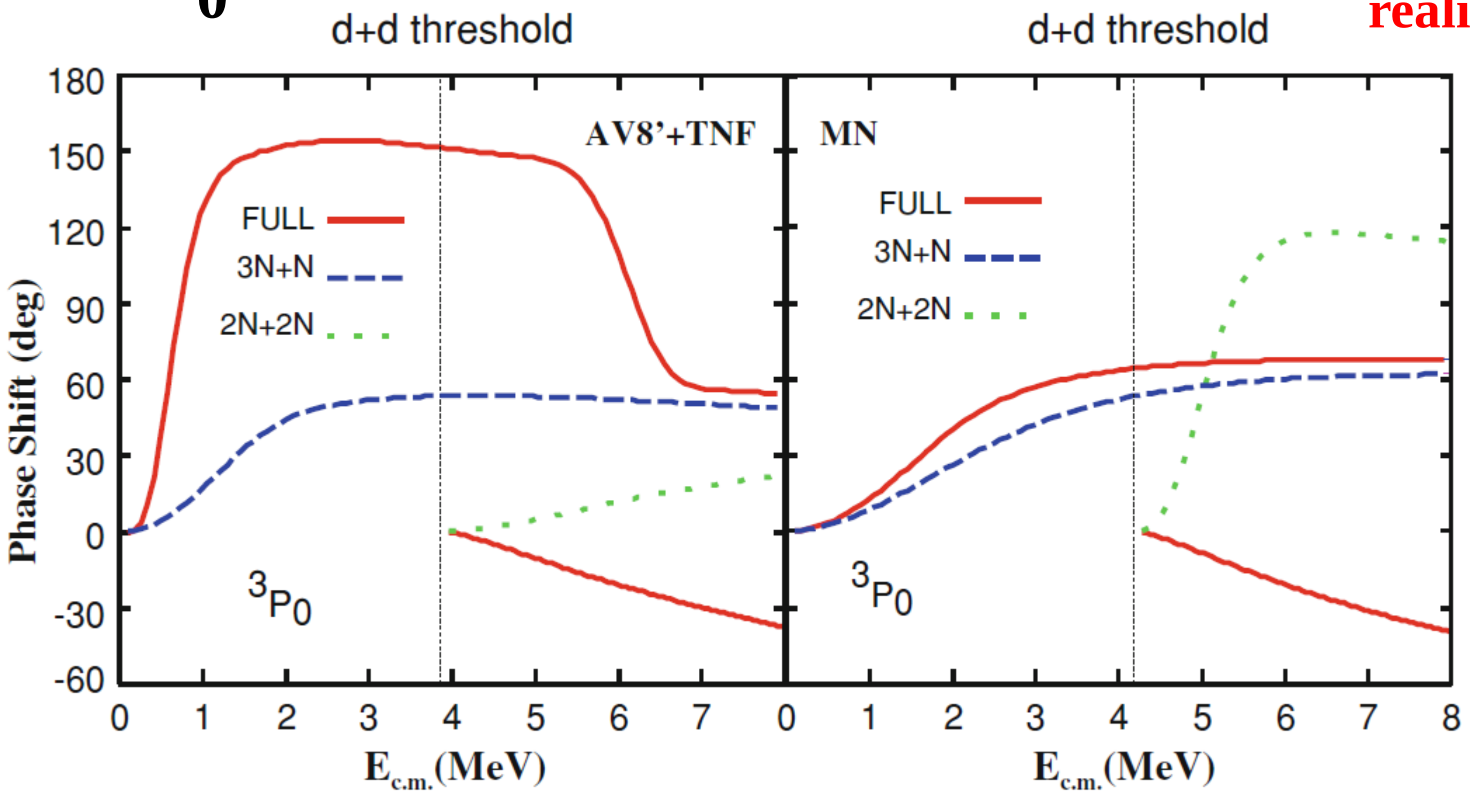
Scattering calculations produce negative-parity resonances of ^{4}He?

$^3P_0$ elastic-scattering phase shifts

Microscopic R-matrix method

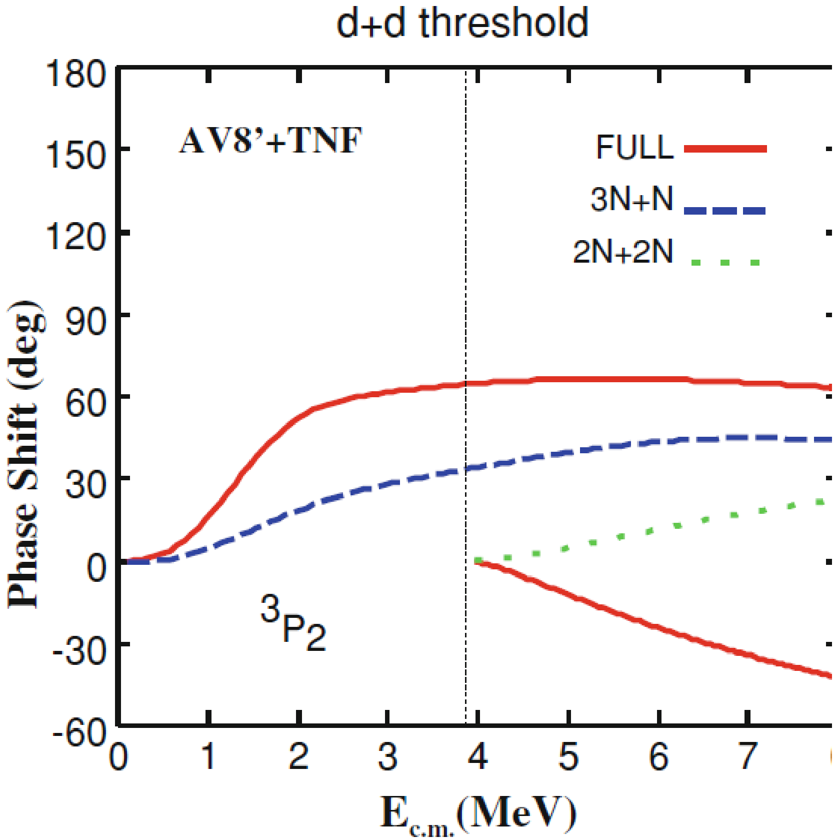
0^-0 resonance in realistic force

0^-

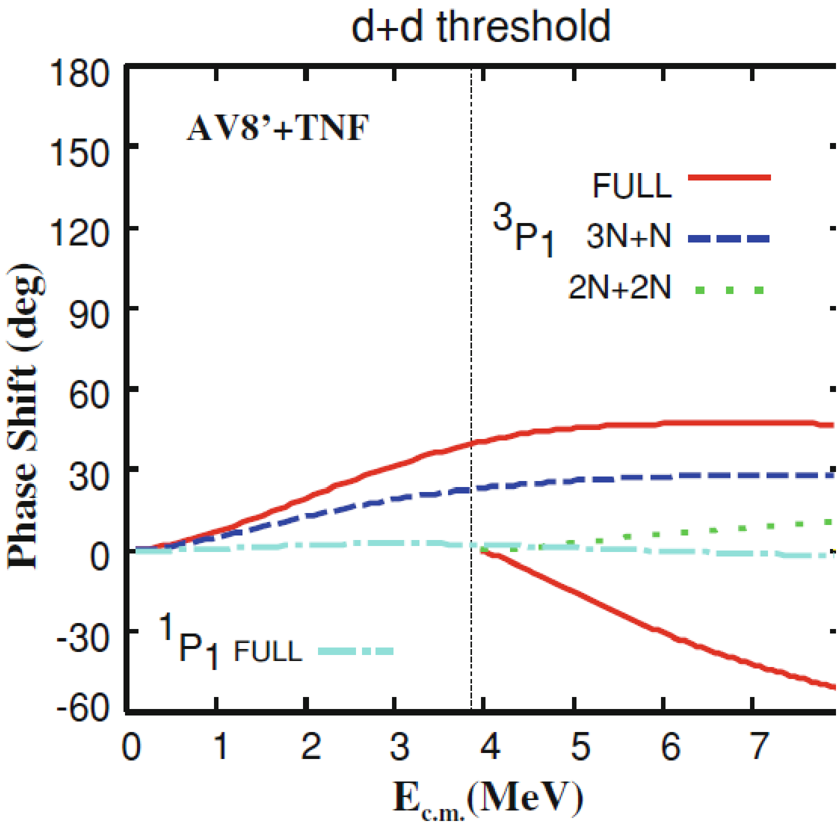


S.Aoyama, K.Arai, Y.S., P.Descouvemont, D.Baye, Few-Body Syst. 52 (2012)

$^3P_2$ **2^-**



$^3P_1$ and $^1P_1$ **1^-**



Nontrivial to determine resonance parameters
Some technique is needed? (cf. Shimamura)

Basis functions

LS coupling

$$\Psi^{\pi}_{JM_J,TM_T} = \sum_{LS} C_{LS,T}\,\Phi^{\pi}_{(LS)JM_J,TM_T}$$

$$\Phi^{\pi}_{(LS)JM_J,TM_T} = \mathcal{A}\left[\phi^{\pi}_{L}\,\chi_S\right]_{JM_J}\eta_{TM_T}$$

Spin part (Isospin part)

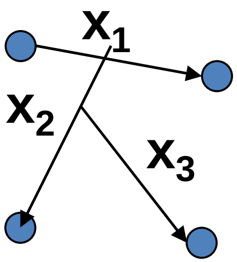
$$\chi_{(S_{12}S_{123}\ldots)SM_S} = [\ldots[[\chi_{\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)]_{S_{12}}\chi_{\frac{1}{2}}(3)]_{S_{123}}\ldots]_{SM_S}$$

Orbital part **ECG-GV** **(correlated basis)**

$$\phi^{\pi}_{(L_1L_2)LM_L}(A,u_1,u_2) = \exp(-\tilde{\boldsymbol{x}}A\boldsymbol{x})[\mathcal{Y}_{L_1}(\tilde{u}_1\boldsymbol{x})\mathcal{Y}_{L_2}(\tilde{u}_2\boldsymbol{x})]_{LM_L}$$

$\boldsymbol{x}=(\boldsymbol{x}_i)$ **A set of relative coordinates**

Extension to N-particle system



$$F_{\ell m}(\boldsymbol{r}) \approx \sum_a C_a \underline{\exp(-ar^2)}\underline{r^\ell Y_{\ell m}(\hat{\boldsymbol{r}})}$$

Explicitly correlated Gaussian (ECG)

$$\exp(-ar^2) \rightarrow \ \exp\left[-\sum_{i<j} a_{ij}(\boldsymbol{r}_i - \boldsymbol{r}_j)^2\right] = \exp\left(-\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)$$

$$\boldsymbol{r}_i - \boldsymbol{r}_j = c_{ij}^{(1)}\boldsymbol{x}_1 + \ldots c_{ij}^{(N-1)}\boldsymbol{x}_{N-1}$$

$$\tilde{\boldsymbol{x}}A\boldsymbol{x} = \sum_{i,j} A_{ij}\boldsymbol{x}_i \cdot \boldsymbol{x}_j$$

$$A_{ji} = A_{ij}$$

Angular functions with global vectors (GV)

$$\boldsymbol{r} \rightarrow u_1\boldsymbol{x}_1 + u_2\boldsymbol{x}_2 + \ldots + u_{N-1}\boldsymbol{x}_{N-1} = \tilde{u}\boldsymbol{x}$$

$$r^\ell Y_{\ell m}(\hat{\boldsymbol{r}}) \rightarrow |\tilde{u}\boldsymbol{x}|^L Y_{LM}(\widehat{\tilde{u}\boldsymbol{x}}) = \mathcal{Y}_{LM}(\tilde{u}\boldsymbol{x})$$

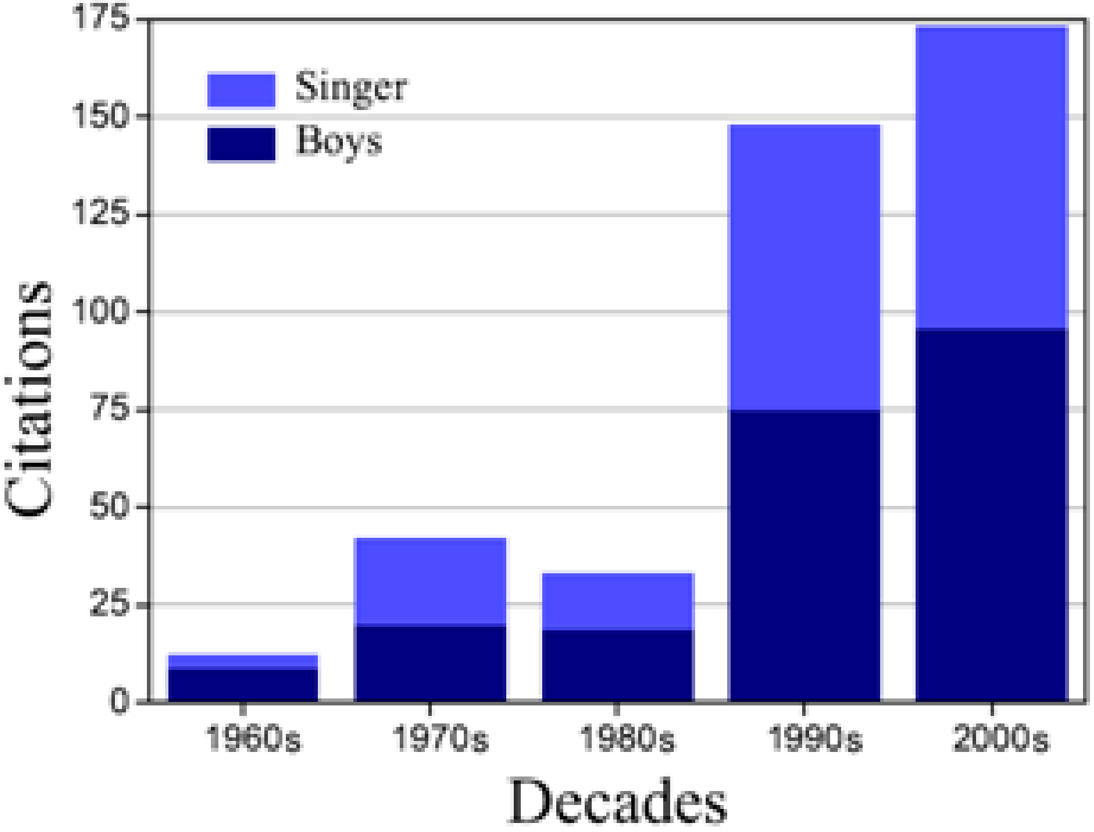
parameters a_{ij} u_i

K.Varga, Y.S., Phys. Rev. C52 (1995)
Y.S.,K.Varga, Lecture Notes in Physics 54 (Springer, 1998)

Correlated Gaussian: S.F. Boys K. Singer

Proc. R. Soc. London, Ser. A258 (1960)

Number of citations by decade to the original works (Boys, 1960) and (Singer, 1960)



Theory and application of explicitly correlated Gaussians

submitted to RMP atomic, molecular, condensed matter, nuclear

Resonances of Ps$^-$ ($e^-e^-e^+$): CSM

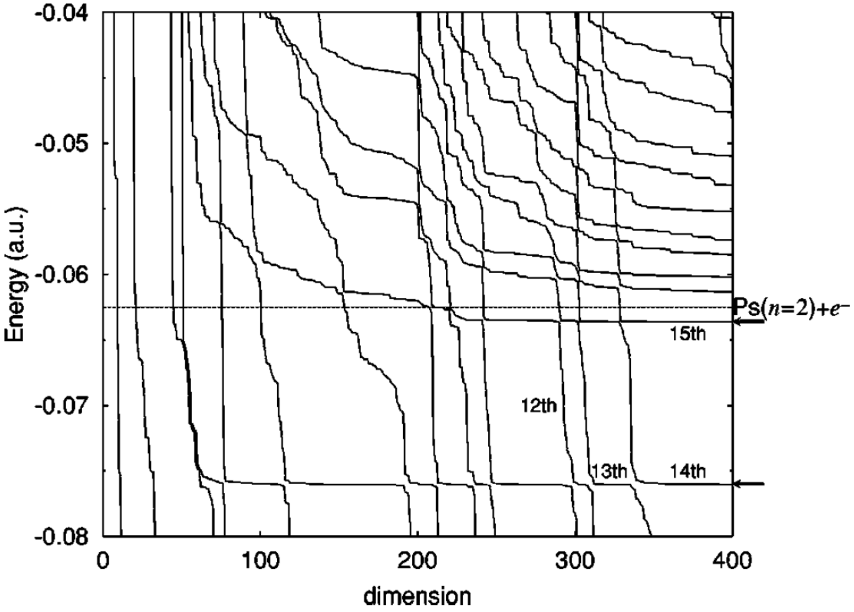
$$H(\theta) = Te^{-2i\theta} + Ve^{-i\theta}$$

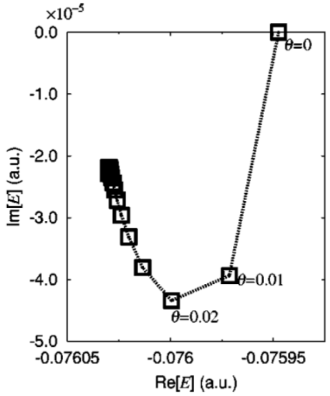
Basis functions to diagonalize H(θ)

Real stabilization, SVM search

Search for complex energy poles

$^1S^e$





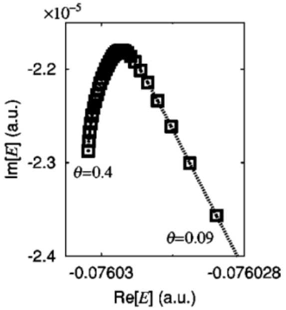


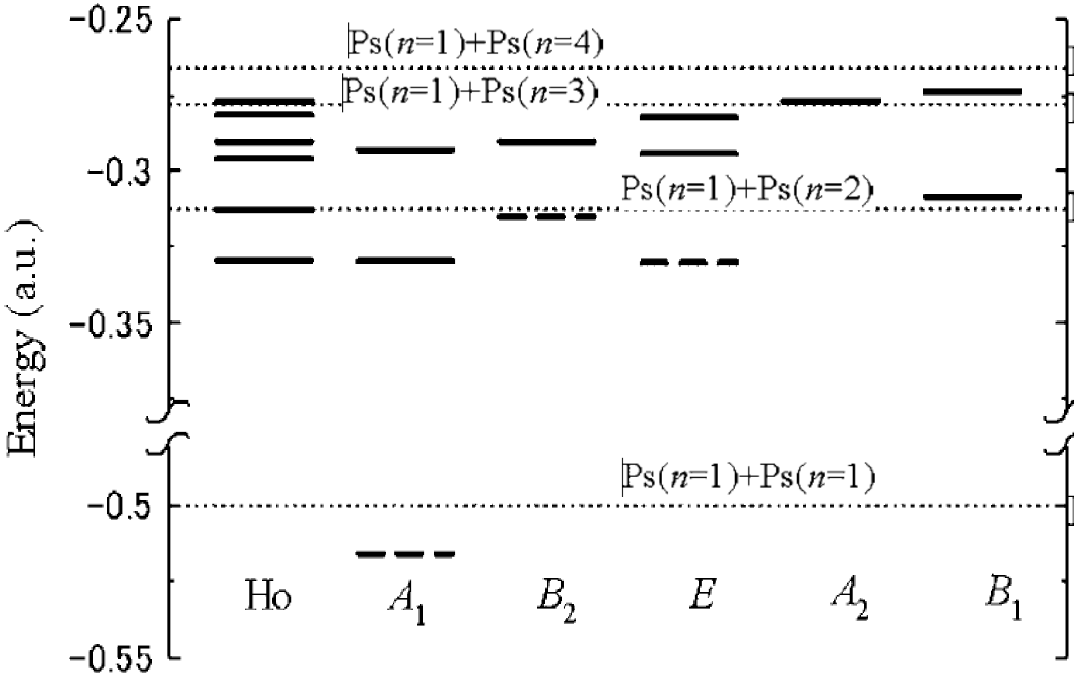
TABLE I. Resonances of Ps$^-$. E_R and Γ denote the resonance energy and width. The first four resonances are in $^1S^e$, while the last three in $^3S^e$.

Present		Ref. [9]	
$-E_R$ (a.u.)	Γ (a.u.)	$-E_R$ (a.u.)	Γ (a.u.)
0.0760297	4.236×10^{-5}	0.0760305(20)	$4.275(100)\times10^{-5}$
0.063667	8.99×10^{-5}		
0.0353329	7.68×10^{-5}	0.0353425(50)	$7.25(50)\times10^{-5}$
0.029845	5.52×10^{-5}		
0.0635373564	8.1×10^{-9}		
0.062505	7.4×10^{-5}		
0.02935176	2.0×10^{-5}		

J.Usukura,Y.S. PRA66(2002)

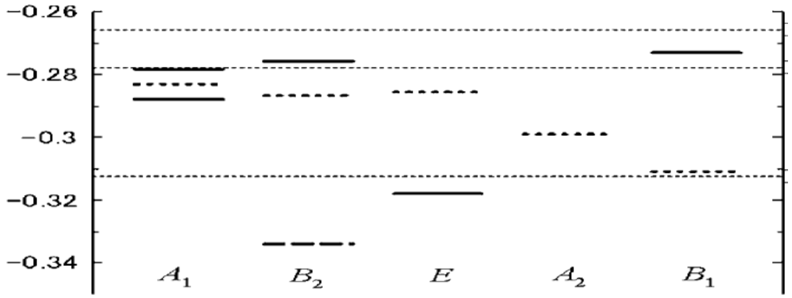
Predicted Energy levels of Ps$_2$ ($e^-e^-e^+e^+$): CSM

Coulomb four-body Hamiltonian: symmetry wrt exchanges of electrons, positrons, and charge reversal (D$_{2d}$ symmetry)



Solid line: 0$^+$ resonance

Dashed line: 0$^+$ bound state



Solid line: 1$^-$ resonance

Dotted line: 2$^+$ resonance

Dashed line: 1$^-$ bound state

K.Varga,J.Usukura,Y.S., PRL80(1998)

\`Bound’ againt autodissociation but unbound for annihilation

J.Usukura,Y.S. PRA66(2002), Nucl.Instr. and Meth. B221(2004)

LS channels for ^{4}He

J^π	(LS)
0^+	$(00), (22)\,;(11)$
1^+	$(01), (21), (22)\,;(10), (11), (12), (32)$
0^-	$(11)\,;(22)$ **(00) omitted**
1^-	$(10), (11), (12), (32)\,;(21), (22)$
2^-	$(11), (12), (31), (32)\,;(20), (21), (22), (42).$

AV8’ + Coulomb+3NF

$$V_q = \sum_{i<j} v^{(q)}(r_{ij})\mathcal{O}^{(q)}_{ij}$$

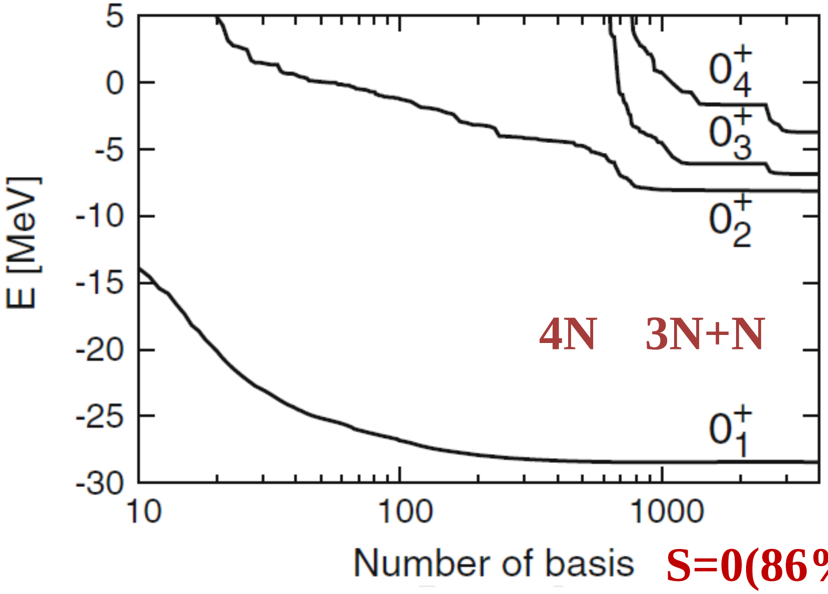
$$1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

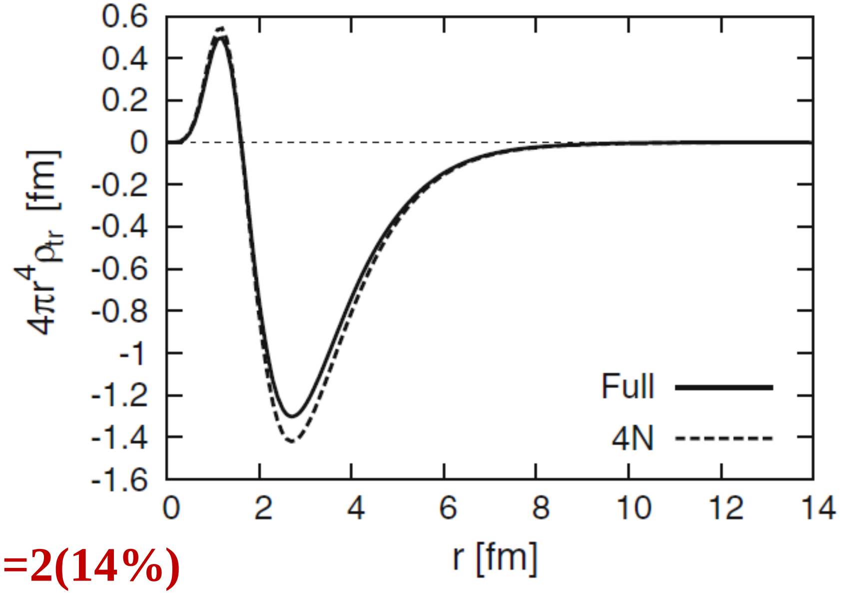
$$S_{ij}, S_{ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}, (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

Use of realistic forces is expensive numerically but has high predictive power

$$\rho_{\rm tr}(r) = \frac{1}{4\pi}\langle\Psi(0_2^+)|\sum_{i=1}^{4}\frac{\delta(|\boldsymbol{r}_i - \boldsymbol{x}_4| - r)}{r^2}|\Psi(0_1^+)\rangle$$

Convergence with SVM

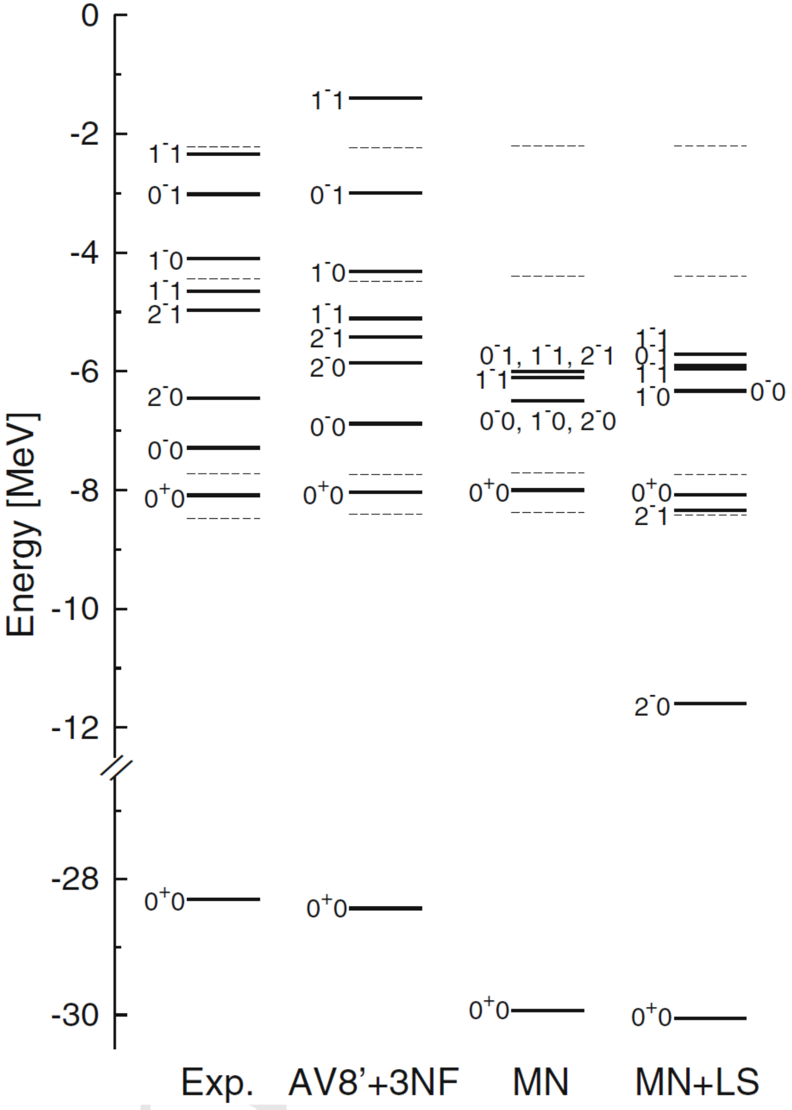




S=0(86%), S=2(14%)

Spectrum of ^{4}He

`Real stabilization’ (BSA)



- The first excited state is 0^+ but not a negative parity
- A variational calculation with realistic forces reproduces spectrum
- The tensor force is crucial to account for the level splitting
- Most levels are broad resonances that can be excited by spin-dipole operators (weak interaction)
  Study of EW response is interesting

W.Horiuchi, Y.S., PRC78 (2008)

Response and resonance

Response function for a suitable operator signals a resonance

$$S(p, \lambda, E) = \mathcal{S}_{f\mu} | \langle \Psi_f | \mathcal{O}^p_{\lambda\mu} | \Psi_0 \rangle |^2 \delta(E_f - E_0 - E)$$

Electric-dipole and spin-dipole excitations

E1 $\sum_{i=1}^{N} (\boldsymbol{r}_i - \boldsymbol{x}_N)_\mu \frac{1}{2}(1 - \tau_{0_i})$ **Photoabsorption**

SD $\sum_{i=1}^{N} [(\boldsymbol{r}_i - \boldsymbol{x}_N) \times \boldsymbol{\sigma}_i]_{\lambda\mu} \begin{pmatrix} 1 \\ \tau_{0_i} \\ t_{\pm_i} \end{pmatrix}$

Isoscalar

Isovector

Charge-exchange

ν-nucleus reaction

(no direct measurement)

J^π= 0^-,1^-,2^- states of ^{4}He with T=0 and 1 can be excited by SD
By charge-exchange SD operator, resonances of ^{4}H and ^{4}Li are probed

W.Horiuchi,Y.S.,K.Arai, PRC85(2012) E1 response of ^{4}He
W. Horiuchi,Y.S., SD response with CSM is in progress

From response function to resonance

Complex scaling method

$$U(\theta) \qquad \boldsymbol{x} \longrightarrow \mathrm{e}^{i\theta}\boldsymbol{x} \qquad \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} \longrightarrow \mathrm{e}^{(-\sin\theta+i\cos\theta)\boldsymbol{k}\cdot\boldsymbol{x}}$$

Continuum is made to damp asymptotically

$$S(p,\lambda,E) = -\frac{1}{\pi}\mathrm{Im}\sum_{\mu}\langle\Psi_0|\,\mathcal{O}^{p\dagger}_{\lambda\mu}U^{-1}(\theta)R(\theta)U(\theta)\mathcal{O}^{p}_{\lambda\mu}\,|\Psi_0\rangle$$

$$R(\theta) = U(\theta)RU^{-1}(\theta) = \frac{1}{E+E_0-H(\theta)+i\epsilon}$$

$$H(\theta) = U(\theta)HU^{-1}(\theta)$$

$$H(\theta)\Psi_\nu(\theta) = E_\nu(\theta)\Psi_\nu(\theta)$$

H(θ) may be diagonalyzed on square-integrable basis
No smoothing procedure is necessary
Stability of S(E) wrt θ is to be examined

Complex eigenvalues and response function

$$S(p,\lambda,E) = -\frac{1}{\pi}\sum_{\nu,\mu} \mathrm{Im}\frac{\tilde{\mathcal{D}}^{p,\nu}_{\lambda\mu}(\theta)\mathcal{D}^{p,\nu}_{\lambda\mu}(\theta)}{E+E_0-E_\nu(\theta)+i\epsilon}$$

$$\mathcal{D}^{p,\nu}_{\lambda\mu} = \langle(\Psi_\nu(\theta))^*|\,\mathcal{O}^p_{\lambda\mu}(\theta)\,|U(\theta)\Psi_0\rangle$$

$$\tilde{\mathcal{D}}^{p,\nu}_{\lambda\mu} = \langle(U(\theta)\Psi_0)^*|\,\tilde{\mathcal{O}}^p_{\lambda\mu}(\theta)\,|\Psi_\nu(\theta)\rangle$$

Contribution of eigenvalue ν to S(E)

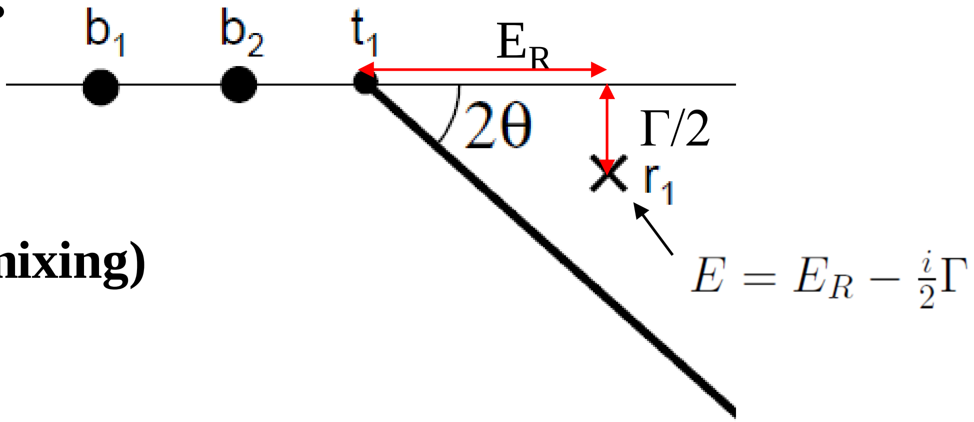
$$E_\nu(\theta) = \varepsilon_\nu(\theta) + E_0 - \frac{i}{2}\gamma_\nu(\theta)$$

$$\sum_\mu \tilde{\mathcal{D}}^{p,\nu}_{\lambda\mu}(\theta)\mathcal{D}^{p,\nu}_{\lambda\mu}(\theta) = \alpha^{p\lambda}_\nu(\theta) + i\beta^{p\lambda}_\nu(\theta)$$

$$S(p,\lambda,E) = \frac{1}{\pi}\sum_\nu \frac{\frac{1}{2}\alpha^{p\lambda}_\nu(\theta)\gamma_\nu(\theta) - (E-\varepsilon_\nu(\theta))\beta^{p\lambda}_\nu(\theta)}{(E-\varepsilon_\nu(\theta))^2 + \frac{1}{4}(\gamma_\nu(\theta))^2}$$

Difficulty of nuclear CSM:

Complex energy plane



Nuclear Hamiltonian is complicated

Short-range strong repulsion

Long-range OPEP attraction (S-D mixing)

Accurate solution is hard

- To cover the resonance

$$\theta \sim \tfrac{1}{2}\arctan(\Gamma/2E_R)$$

- $e^{-\rho r} \longrightarrow e^{-\rho r(\cos\theta + i\sin\theta)}$

Potential range increases to $\rho\cos\theta$

Rotation by large angles leads to numerical instability

Only few excited states with same quantum number are required
cf. atomic case (several excited states but narrow widths)

Reviews on CSM **Y.K.Ho, Phys.Rep99(1983)**
N.Moiseyev, Phys. Rep.302(1998)
S.Aoyama,T.Myo,K.Kato,K.Ikeda, PTP116(2006)

Photoabsorption of ^{4}He

W.Horiuchi, Y.S., K.Arai, PRC85 (2012)

(1)Use the realistic interaction
(2)Include coupling with final decay channels
(3)Check CSM results with microscopic R-matrix method (MRM)

Photoabsorption and radiative capture

Detailed balance $$\frac{v_1\sigma_{1\to2}}{\rho_2}=\frac{v_2\sigma_{2\to1}}{\rho_1}$$

From radiative capture cross section
to photoabsorption cross section

$$\sigma_{\rm cap}(E)=\frac{2J_f+1}{(2I_1+1)(2I_2+1)}\frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}\frac{(\lambda+1)}{\lambda(2\lambda+1)!!^2}$$
$$\times\sum_{J_iI_i\ell_i}\frac{1}{(2\ell_i+1)}\left|\langle\Psi^{J_f\pi_f}||\mathcal{M}^E_\lambda||\Psi^{J_i\pi_i}_{\ell_iI_i}(E)\rangle\right|^2,$$

$$\rightarrow\quad\sigma_\gamma(E)$$

Photoabsorption

$\gamma+{}^4\text{He}\rightarrow{}^3\text{H+p}$
${}^3\text{He+n}$
${}^2\text{H+p+n}$

Radiative capture

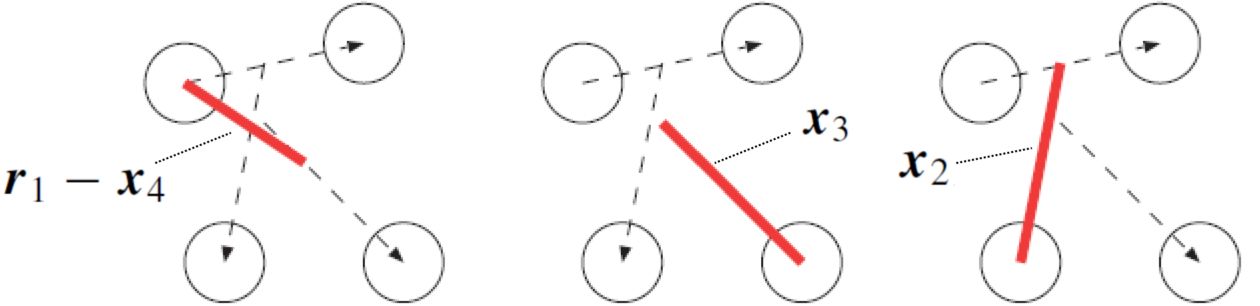
${}^3\text{H+p}\rightarrow{}^4\text{He}+\gamma$
${}^3\text{He+n}\rightarrow{}^4\text{He}+\gamma$
${}^2\text{H+p+n}\rightarrow{}^4\text{He}+\gamma$
(difficult to evaluate)

Construction for continuum discretized basis

$$\mathcal{M}_{1\mu}=\sum_{i=1}^{4}\frac{e}{2}(1-\tau_{3i})(\boldsymbol{r}_i-\boldsymbol{x}_4)_\mu$$

basis states for 1^-

- ‘Goldhaber –Teller’ type (ED)
  (E1 sum rule)



(i) Single-particle excitation

(ii) 3N+N two-body disintegration

(iii) d+p+n three-body disintegration

1200 bases **3000** **3200**

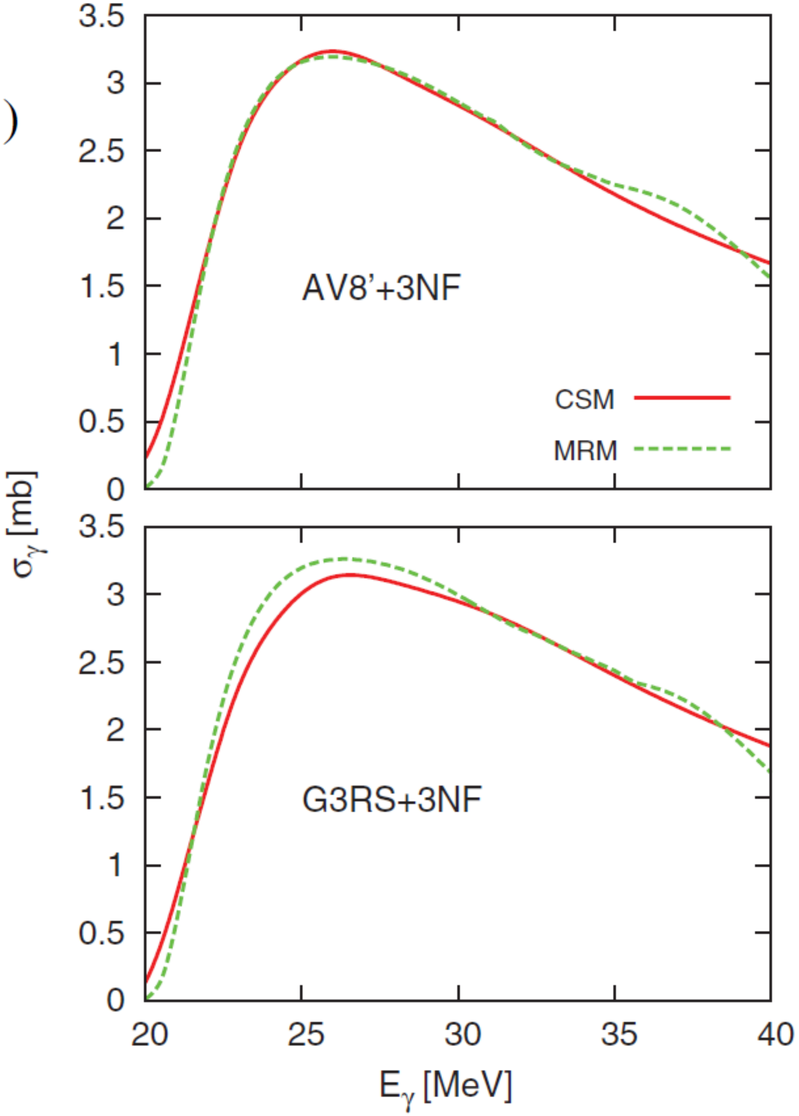
- 3N + N cluster type
- 3N* + N cluster type
  (Final state asymptotics)

$$\mathcal{A}\big[\Phi_0^{(4)}(i)\mathcal{Y}_1(\boldsymbol{r}_1-\boldsymbol{x}_4)\big]_{1M}\eta^{(4)}_{T_{12}T_{123}10}$$

$$\mathcal{A}\big[\Phi^{(3)}_{J_3}(i)\exp\big(-a_3x_3^2\big)[\mathcal{Y}_1(\boldsymbol{x}_3)\chi_{\frac{1}{2}}(4)]_j\big]_{1M}\big[\eta^{(3)}_{T_{12}\frac{1}{2}}\eta_{\frac{1}{2}}(4)\big]_{10}$$

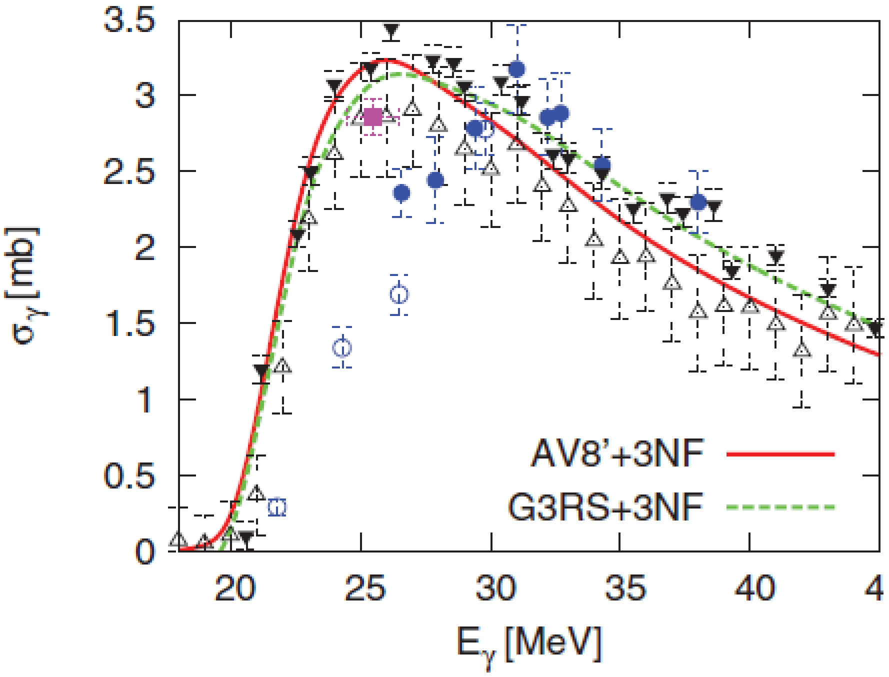
Comparison between CSM and MRM

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$



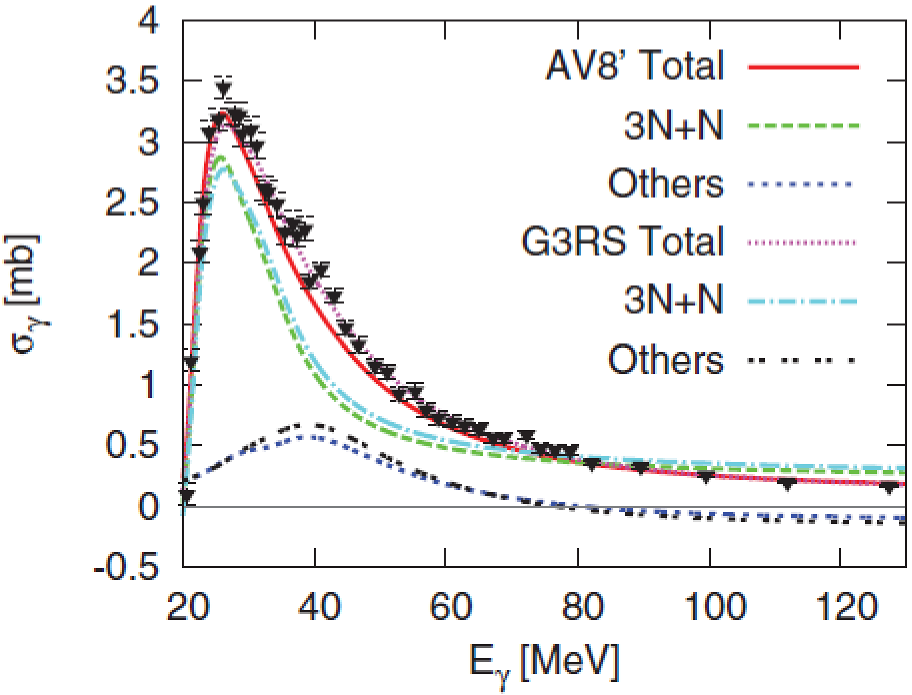
MRM results:
Sum of two-body channels of t+p and h+n

Comparison between CSM and experiment



Good agreement with most data except for low-energy data of Shima et al.

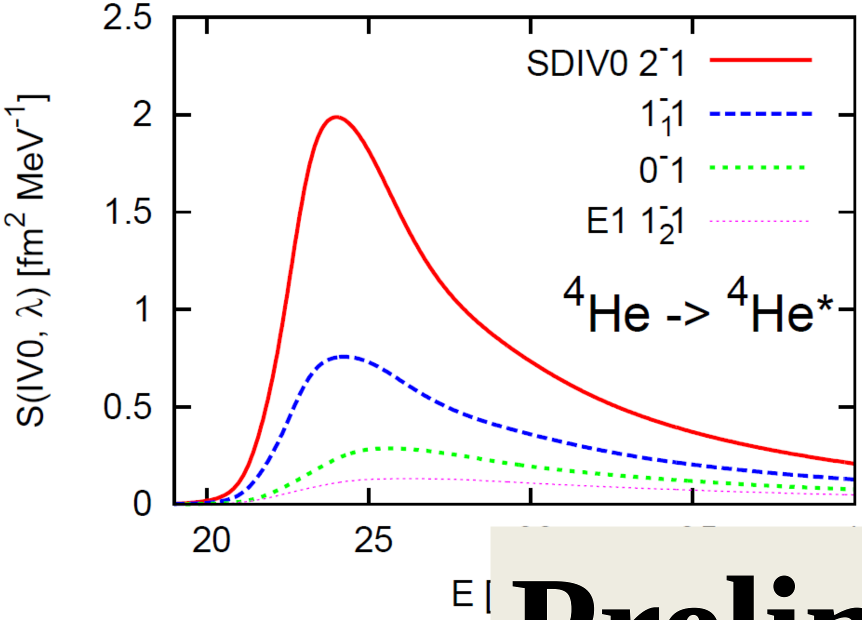
`Giant resonance' (collective motion of protons against neutrons)

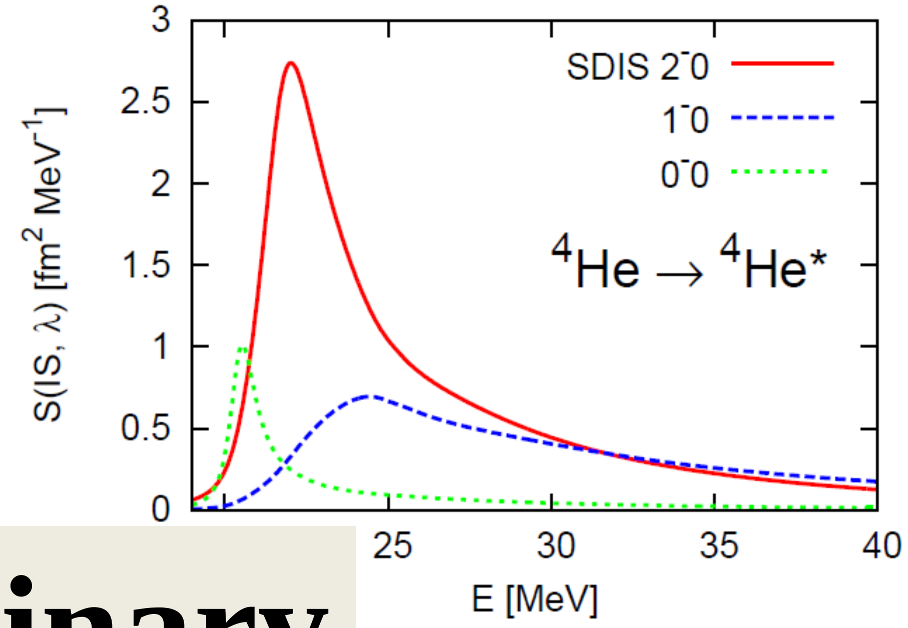


IV and IS SD strength functions

$$\sum_{i=1}^{N}[(\boldsymbol{r}_i - \boldsymbol{x}_N)\times\boldsymbol{\sigma}_i]_{\lambda\mu}\begin{pmatrix}1\\ \tau_{0_i}\\ t_{\pm_i}\end{pmatrix}$$

Isoscalar
Isovector
Charge-exchange to ^{4}H, ^{4}Li





Preliminary

Estimation of resonance parameters from the behavior near peak

Sum rules

Non energy-weighted sum rule

$$\underbrace{\int_0^\infty S(p,\lambda,E)dE}_{m_0(p,\lambda)} = \underbrace{\langle\Psi_0|\sum_\mu \mathcal{O}^{p\dagger}_{\lambda\mu}\mathcal{O}^{p}_{\lambda\mu}|\Psi_0\rangle}_{\mathrm{SR}} \qquad \mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{N}[(\boldsymbol{r}_i - \boldsymbol{R}_N)\times\boldsymbol{\sigma}_i]_{\lambda\mu} \left(\begin{array}{c} 1 \\ \tau_z(i) \\ t_+(i) \\ t_-(i) \end{array}\right) \begin{array}{l} \mathrm{IS} \\ \mathrm{IV0} \\ \mathrm{IV+} \\ \mathrm{IV-} \end{array}$$

- **Both sides are calculated independently**
- **Accurate, correlated ground-state wave function is used**
- **Check of the adequacy of basis for SD excited configurations**

Preliminary

	AV8′+3NF					
	IS		IV0		IV±	
λ	$m_0(p,\lambda)$	SR	$m_0(p,\lambda)$	SR	$m_0(p,\lambda)$	SR
0	2.71	2.72	4.59	4.59	2.48	2.48
1	12.16	12.17	9.35	9.36	4.66	4.68
2	17.98	18.02	18.36	18.38	9.18	9.19

(fm^2)

Resonance properties of ^{4}He

obtained from strength functions and complex energies

	E_R (MeV)			Γ (MeV)		
$J^\pi T$	$S(E)$	Exp.	$E(\theta)$	$S(E)$	Exp.	$E(\theta)$
0^-0	20.54	21.01	20.42	1.1	0.84	0.96
2^-0	22.04	21.84	21.66	3.1	2.01	2.12
2^-1	23.09	23.33	23.62	5.6	5.01	5.00
1_1^-1	23.34	[illegible]	[illegible]	[illegible]	[illegible]	5.31
1^-0	24.44	[illegible]	[illegible]	[illegible]	[illegible]	5.40
0^-1	24.69	[illegible]	[illegible]	[illegible]	[illegible]	7.56
1_2^-1	25.30	25.95	—	13.4	12.66	—

Preliminary

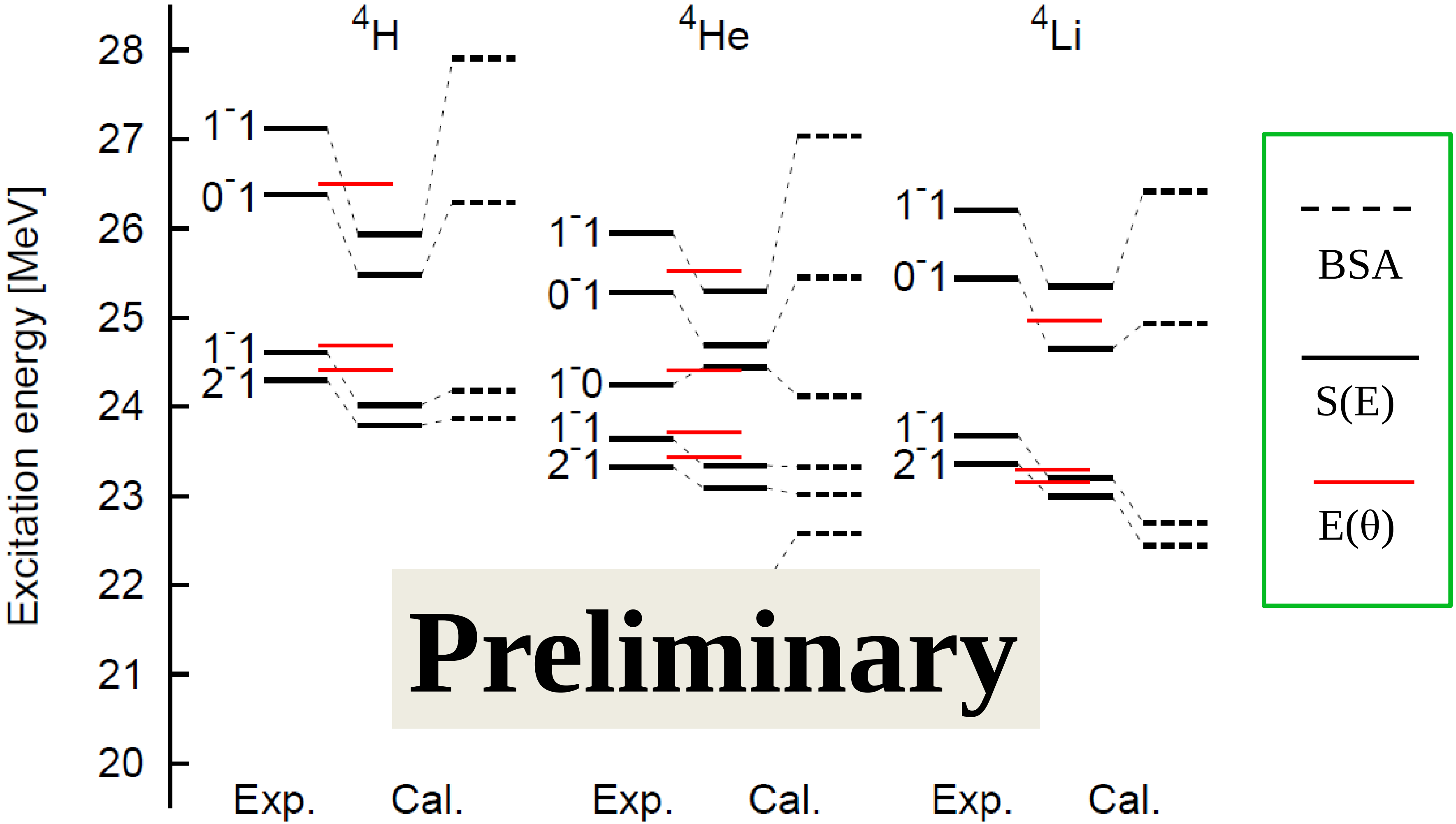
Good overall agreement

Average deviation of E_R: 0.4 MeV in S(E); 0.3 MeV in E(θ)

Γ tends to be slightly larger with S(E)

Difficulty for broad resonance in E(θ)

Isobar diagram for A=4 nuclei



Preliminary

Summary

- ECG-GV basis functions to study electric- and spin-dipole responses of ^{4}He with realistic nuclear forces
- Use of square-integrable basis is made possible through CSM
- All the negative-parity resonances of ^{4}He below 4N threshold are satisfactorily reproduced
- A combined use of S(E) and E(θ) is robust even for broad resonances

Outlook

- Three T=0 resonances with 1^-, 2^-, 0^- exist slightly above 2n+2p threshold, decaying dominantly to d+d channel (I=1, P-wave)
  - Are these excited by IS SD operators?
  - Can further inclusion of d+d (d*) configurations reproduce them?
- 2^+ and 1^+ T=0 resonances just below 2n+2p threshold: IS SQ ($Y_2\boldsymbol{\sigma}$) operator?

in collaboration with W. Horiuchi (Hokkaido)