Astro-nuclear Reactions in the Microscopic R-matrix Method

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Introduction

Structure and reaction of light nuclei

- Few-nucleons correlation (cluster structure)
- Asymptotic behavior (halo nuclei)

⟹ ***Microscopic Cluster Model (RGM)***

Assume the cluster structure (e.g. ^{8}Be= a+a)

Cluster internal motions are fixed

but cluster relative motions are solved

very accurately by the variational method

Satisfy the Pauli principle exactly

for all the nucleons

Use only N-N (effective) interaction

(No cluster-cluster potential is used)

- **<u>Microscopic cluster model (RGM)</u>**

Cluster internal wave function

⟹ Assume the simple S-wave w.f. such as the $(0s)^n$ h.o function

Cluster relative wave function

⟹ Solved with the effective N-N interaction by the variational method

Effective N-N interaction

⟹ Central + LS +Coulomb (no tensor term),

Two or three range Gaussian pot.

e.g. Minnesota, Volkov pot.

Resonance excited states of ^{12}C

- **Extension of the microscopic cluster model**

Cluster internal wave function

⇨ Precise few-body w.f. solved by the realistic N-N interaction

W.f. includes higher partial wave through the tensor interaction

Cluster relative wave function

⇨ Solved with the same realistic interaction by the variational method

Large number of configuration, basis set

⇨ ***Ab initio calculation***

Astrophysical reaction of 2H(d,γ)4He, 2H(d,p)3H, 2H(d,n)3He

How to solve the cluster relative motion in the scattering state

$\Longrightarrow$ **Microscopic R-matrix method**

(Baye, Descouvemont)

a : channel raidus

$$\begin{cases} \rho_{\mathrm{rel}} < a \text{ --- Gaussian expansion} \\ \rho_{\mathrm{rel}} > a \text{ --- Exact Coulomb function} \end{cases}$$

Microscopic *R*-matrix method

(D.Baye, et al. NPA291 ('77)230)

- Schrodinger e.q. $\left(\hat{H}+\hat{L}-E\right)\Psi^{int}=\hat{L}\,\Psi^{ext}$
- Bloch operator $\hat{L}(E)=\left(\hbar^2/2\mu r\right)\delta(r-a)\left[\frac{d}{dr}r-b\right]$

$$\begin{cases} b=0 & \text{for open channel} \\ b=2ka\,W'(2ka)/W(2ka) & \text{for closed channel} \end{cases}$$

- W.F ($r<a$)

Gaussian expansion

$$\Psi^{int}=\sum_{\alpha k} f_{\alpha k} u_{\alpha k}\varphi_{\alpha}$$

$$\begin{cases} \alpha:(\ell, I) \\ u_{\alpha k} : \text{Gaussian basis function} \\ \varphi_{\alpha} : \text{Cluster internal function} \end{cases}$$

$$\Longrightarrow \quad \sum_{\alpha k} f_{\alpha k} \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{H} + \hat{L} - E \middle| u_{\alpha k} \varphi_k \right\rangle = \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{L} \middle| \Psi^{ext} \right\rangle$$

$$C_{\alpha' k', \alpha k} \equiv \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{H} + \hat{L} - E \middle| \Psi_{\alpha k} \right\rangle \qquad W_{\alpha' k'} \equiv \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{L} \middle| \Psi^{ext} \right\rangle$$

$$\Longrightarrow \quad \Psi^{int} = \sum_{\alpha k} f_{\alpha k} u_{\alpha k} \varphi_{\alpha} = \sum_{\alpha k \alpha' k'} C^{-1}_{\alpha k, \alpha' k'} W_{\alpha' k'} u_{\alpha k} \varphi_{\alpha}$$

$$\Psi^{ext} = \sum_{\alpha_1} r_{\alpha_1}^{-1} v_{\alpha_1}^{1/2} C_{\alpha_1} \left\{ I_{\alpha_1} \delta_{\alpha_1 \alpha_0} - U_{\alpha_1 \alpha_0} O_{\alpha_1} \right\} \varphi_{\alpha_1} + \sum_{\alpha_2} C_{\alpha_1} W_{-\eta, \ell+1/2}(2kr)/kr \; \varphi_{\alpha_2}$$

R-matrix

$$R_{\alpha\alpha'} = \hbar^2 a/2 \left(\mu_{\alpha} \mu_{\alpha'}\right)^{-1/2} \left(k_{\alpha}/k_{\alpha'}\right)^{1/2} \sum_{kk'} u_{\alpha k}(a) C^{-1}_{\alpha k, \alpha' k'} u_{\alpha' k'}(a)$$

S-matrix

$$U = \left(Z^*\right)^{-1} Z \qquad \because \quad \Psi^{ext}(a) = \Psi^{int}(a)$$

$$Z_{\alpha\alpha'} = I_{\alpha} \delta_{\alpha\alpha'} - R_{\alpha\alpha'} \left(k_{\alpha'} a\right) I'_{\alpha'} \left(k_{\alpha'} a\right)$$

How to derive Resonance parameters in the MRM

◦ **Iterative method**

(P.Descouvemont et al. PRA42('90)3835)

⟶ *Search the resonance*

on the Real Energy axis

(This can work only for a narrow resonance)

Iterative method (P.Descouvemont et al. PRA42('90)3835)

$$\sum_{m,n} C^{\lambda}_{m,n} \langle \varphi_m | H + L(E) | \varphi_n \rangle_P = e^{\lambda} \sum_{m,n} C^{\lambda}_{m,n} \langle \varphi_m | \varphi_n \rangle_P$$

$$\begin{cases} b = kaO'(ka)/O(ka) & \text{for open channel} \\ b = 2kaW'(2ka)/W(2ka) & \text{for closed channel} \end{cases}$$

S-matrix $U_{i,j}$ $\qquad U_{i,j} \propto \left[\delta_{i,j} + i \sum_{\lambda} \frac{\left(\Gamma^{\lambda}_i \Gamma^{\lambda}_j\right)}{e_{\lambda} - E} \right], \quad \Gamma^{\lambda}_i = 2P\gamma^2_i$

γ^2_i : Reduced width amplitude $\qquad \gamma^{\lambda}_i = \sqrt{\hbar^2 / 2\mu a} \sum_k C^{\lambda}_k a \chi_k(a)$

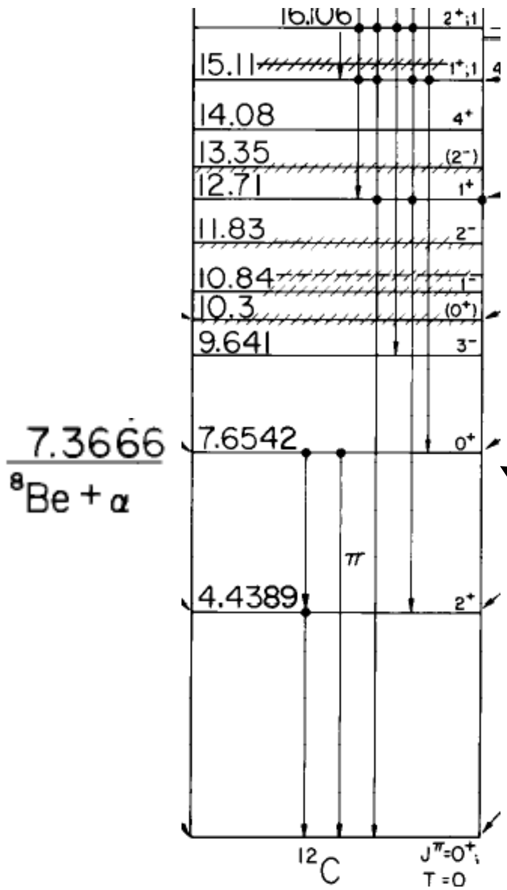
- Start the diagonalization with b=0 in $L(E)$
- Iterate the diagonalization with $E=\mathrm{Re}[e^l]$ in $L(E)$

$$\Longrightarrow \quad E_R = \mathrm{Re}[e^{\lambda}], \quad \Gamma_{\mathrm{R}} = -2\,\mathrm{Im}[e^{\lambda}]$$

- S-matrix is calculated on the real energy

Resonance excited states of ^{12}C

Purpose Search the three alpha resonance above the three-alpha threshold

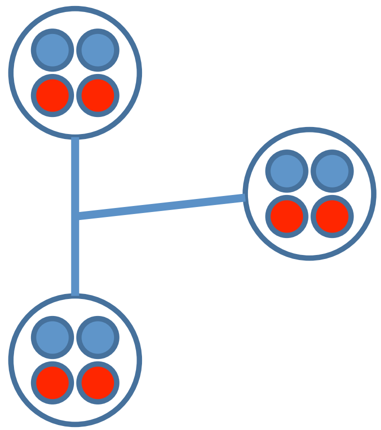


TUNL compilation

a+a+a reasonance

7.2748

a+a+a

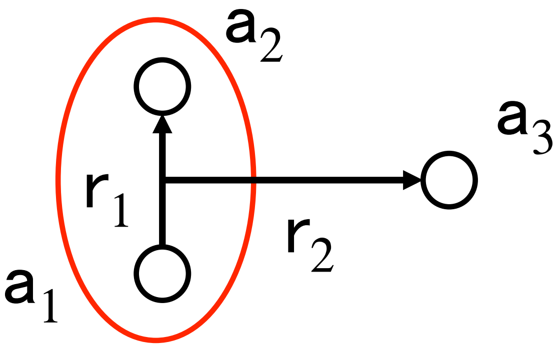


Hoyle state

triple a reaction

a condensation

12C = a+a+a three cluster model



^{8}Be(a+a)

Resonance states of ^{12}C is obtained by solving the ^{8}Be(0^+,2^+,4^+)+a two-body scattering problem

Total wave function $\Psi = A\{ \phi_{8Be}(\rho_1)\, \phi_{\alpha 3}\ \chi(\rho_2$

$\phi_{8Be}(\rho_1) = A\{ \phi_{\alpha 1}\, \phi_{\alpha 2}\, \chi($

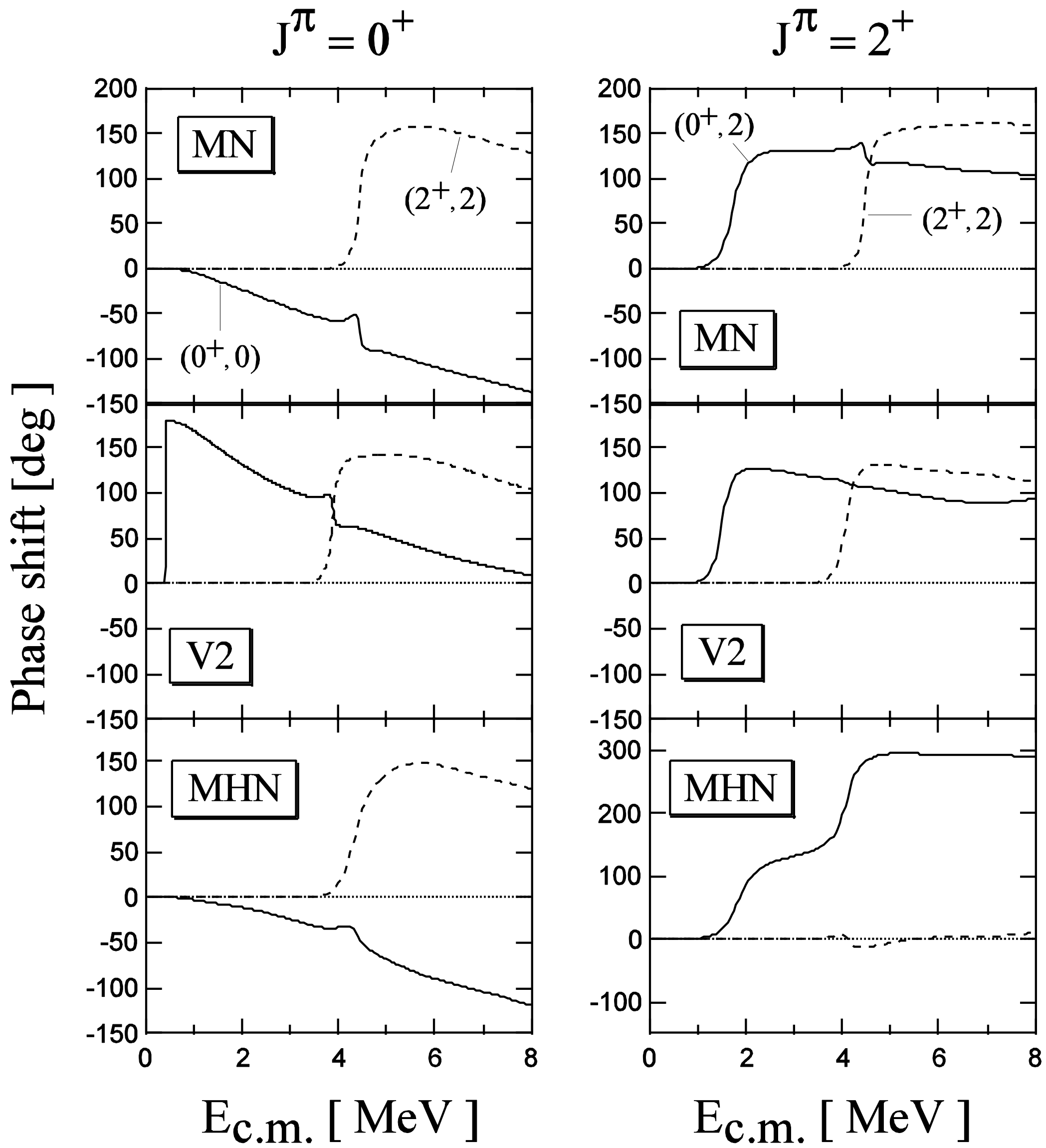
A means antisymmetrization the inter-cluster

ϕ_{α} cluster internal w.f. : $(0s)^4$ h.o. function

^{8}Be(0^+,2^+,4^+) w.f. : bound state approximation

Effective N-N pot. : MN, V2, MHN(Central+Coulomb)

Elastic scattering phase shifts in ^{12}C



Resonance parameters in ^{12}C

	Pot.	MRM E_R	MRM Γ_R	CSM(3-body) E_R	CSM(3-body) Γ_R
0_3^+	MN	~4.8	~0.3	4.7	1.0
	V2	~4.4	~0.3	4.3	1.1
	MHN	~5.2	~0.5	5.1	1.4
2_2^+	MN	2.0-2.1	~0.4	2.1	0.8
	V2	2.0-2.1	~0.3	1.9	0.9
	MHN	~2.7	~0.5	2.9	1.2
2_3^+	MN	~4.8	~0.3	4.9	0.9
	V2	~4.6	~0.3	5	2
	MHN	~5.0	0.4-0.5	5.0	0.8

Summary for ^{12}C calculation

MRM can give good agreement in the resonance energy with the 3-body CSM calculation
but gives in general smaller resonance width than the CSM calculation
because the MRM neglects the direct 3-body decay and the decay of ^{8}Be.

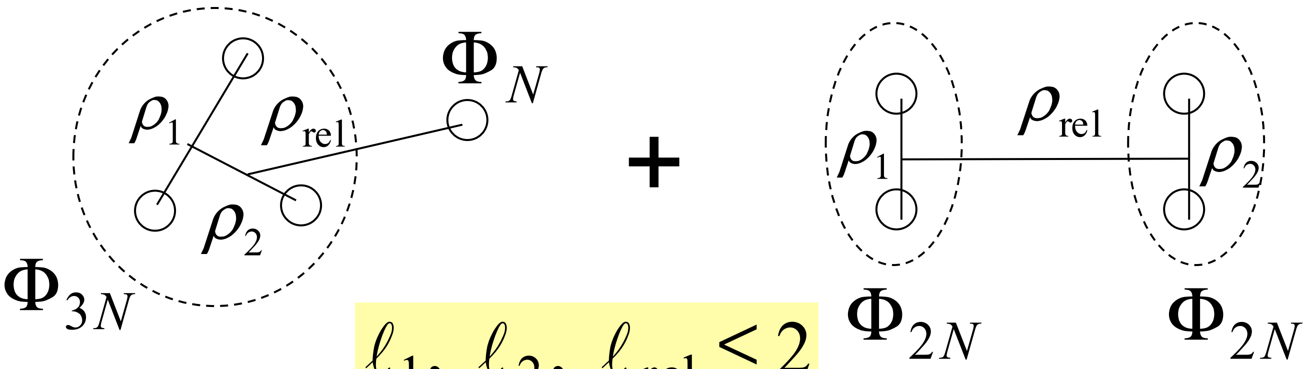
K. Arai, PRC74(2006)064311

Astrophysical reaction of ***d***
***(d,γ)*4He *d(d,p)*3H, *d(d,n)*3He**

[^{3}H(1/2$^+$)+p] + [^{3}He+n] + [d+d] + [pn(0$^+$) + pn(0$^+$)]
+ [2p(0$^+$) + 2n(0$^+$)] two-cluster model

Total wave function

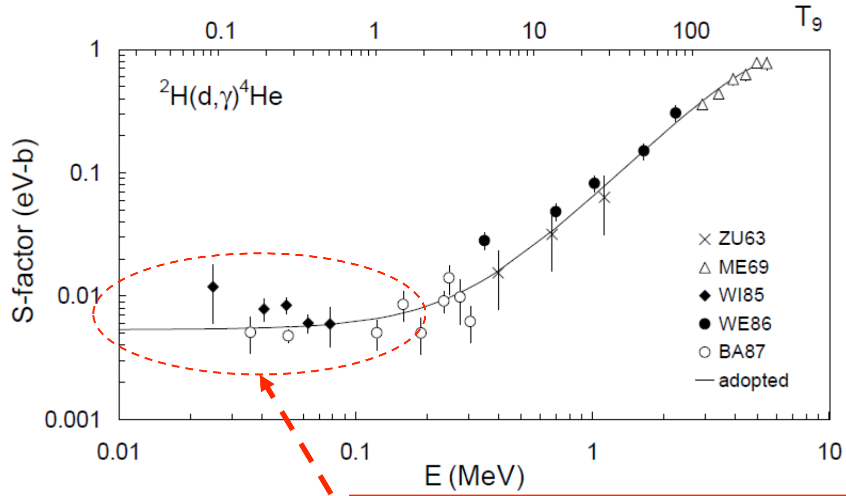
$$\Psi = A\{\Phi_{3N}(\rho_1,\rho_2)\,\Phi_N\,\chi(\rho_{\rm rel})\} + A\{\Phi_{2N}(\rho_1)\,\Phi_{2N}(\rho_2)\,\chi(\rho_{\rm rel})\}$$



$$\ell_1,\ \ell_2,\ \ell_{\rm rel} \le 2$$

Extended microscopic cluster model calculation

• Role of the tensor force

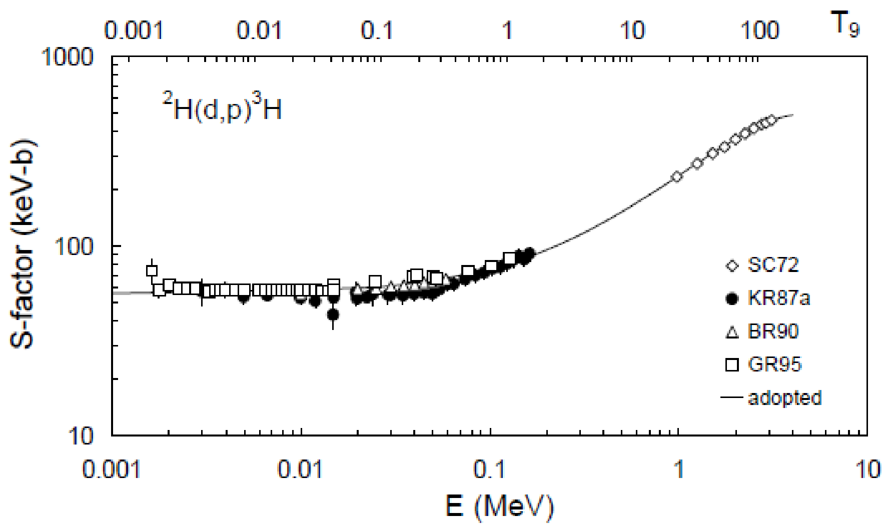


$^2H(d,\gamma)^4He$ Astrophysical S-factor

Nacre compilation
(C.Angulo et al. NPA656('99)p.3)

d+d S-wave → D-state in the 0^+ g.s. of ^4He

H. J. Assenbaum and K. Langanke, PRC36('87)p.17



$^2H(d,p)^3H$, $^2H(d,n)$3He

Role of the tensor force?

$\Phi_{3N}(\rho_1, \rho_2), \Phi_{2N}(\rho_1)$: Cluster intrinsic wave function

Precise three- and two-body wave function.

$$\Phi_{3N}(\rho_1, \rho_2) = A\{[\phi_{ST}[\chi_{\ell 1}(\rho_1)\chi_{\ell 2}(\rho_2)]_L]_J\}$$

W.f. is expanded by the Gaussian basis function.

Including the higher partial wave up to D-wave.

Basis set is selected by
the Stochastic Variational Method (SVM).

V.I. Kukulin and V. M. Krasnopol'sky, JPG3(1977) 795
K. Varga, Y. Suzuki, R. G. Lovas,NPA571(1994)447

Basis dimension ^3H, ^3He → N=30
^2H → N=8

N-N interaction

- Realistic N-N pot. (*Central*+*LS*+*Tensor*)

  AV8'

  G3RS (Tamagaki, PTP39('69)91)

  + Phenomenological 3BF (Hiyama et al., PRC70('04))

$$\sum_{i=1}^{2} V_i \, e^{-\alpha_i (r_{12}^2 + r_{23}^2 + r_{31}^2)}$$

- Effective N-N pot. (*Central* + *Coulomb*)

  Minnesota pot. (D. R. Thompson, NPA286('77)p.53)

  → 3-range Gaussian potential which reproduces np triplet and pp single s-wave scattering length and effective range

Cross section of the capture reaction

$$\sigma_{\gamma}^{E\lambda}(E)=\frac{2J_f+1}{(2I_1+1)(2I_2+1)}\frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)\frac{(\lambda+1)}{\lambda(2\lambda+1)!!^2}$$

$$\times\sum_{J_iI_i\ell_i}\frac{1}{(2\ell_i+1)}\left|\left\langle\Psi^{J_f\pi_f}\left\|M_\lambda^E\right\|\Psi_{\ell_iI_i}^{J_i\pi_i}\right\rangle\right|^2$$

Present cal.： E2 transition ($2^+ \rightarrow 0^+$ g.s.)

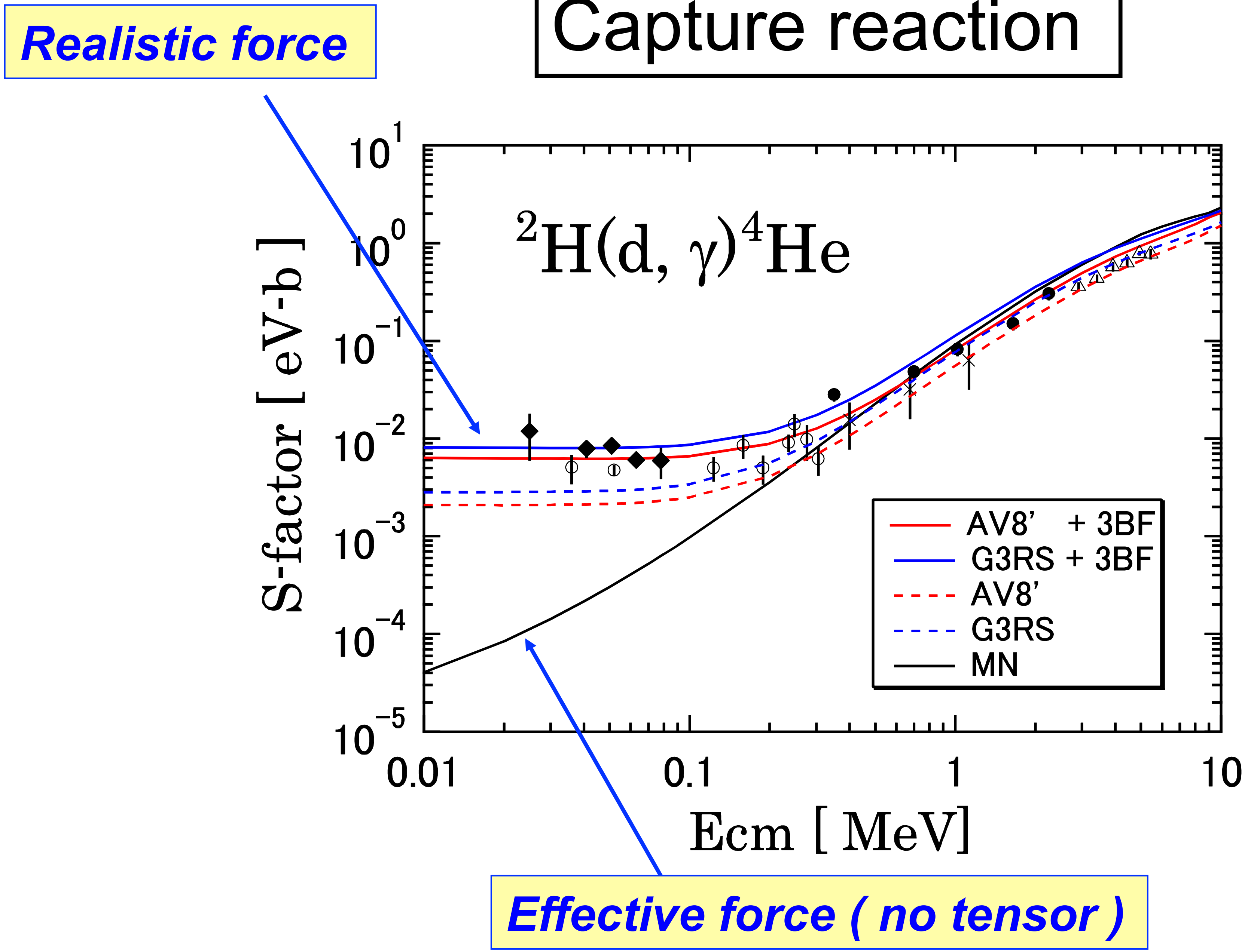
Cross section of the transfer reaction

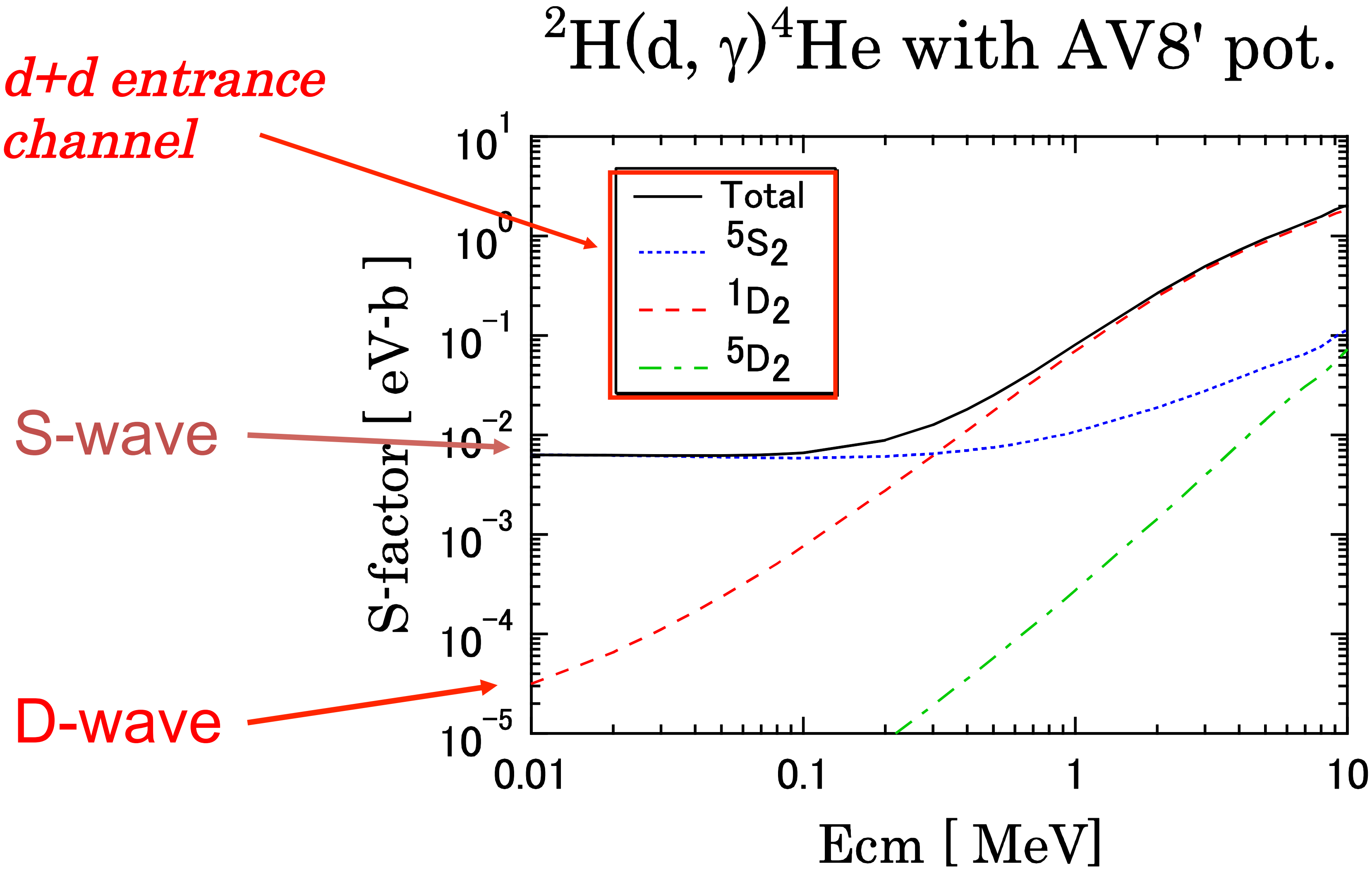
$$\sigma(E)=\frac{\pi}{k^2}\sum_{J\pi}\frac{2J+1}{(2I_1+1)(2I_2+1)}\sum_{\ell_i\ell_fI_iI_f}\left|U^{J\pi}_{i\,\ell_iI_i,f\,\ell_fI_f}(E)\right|^2$$

Presnt cal ： $J^\pi = 0^\pm,\ 1^\pm,\ 2^\pm$

- Astrophysical S factor $S(E)=\sigma(E)\,E\exp(2\pi\eta)$

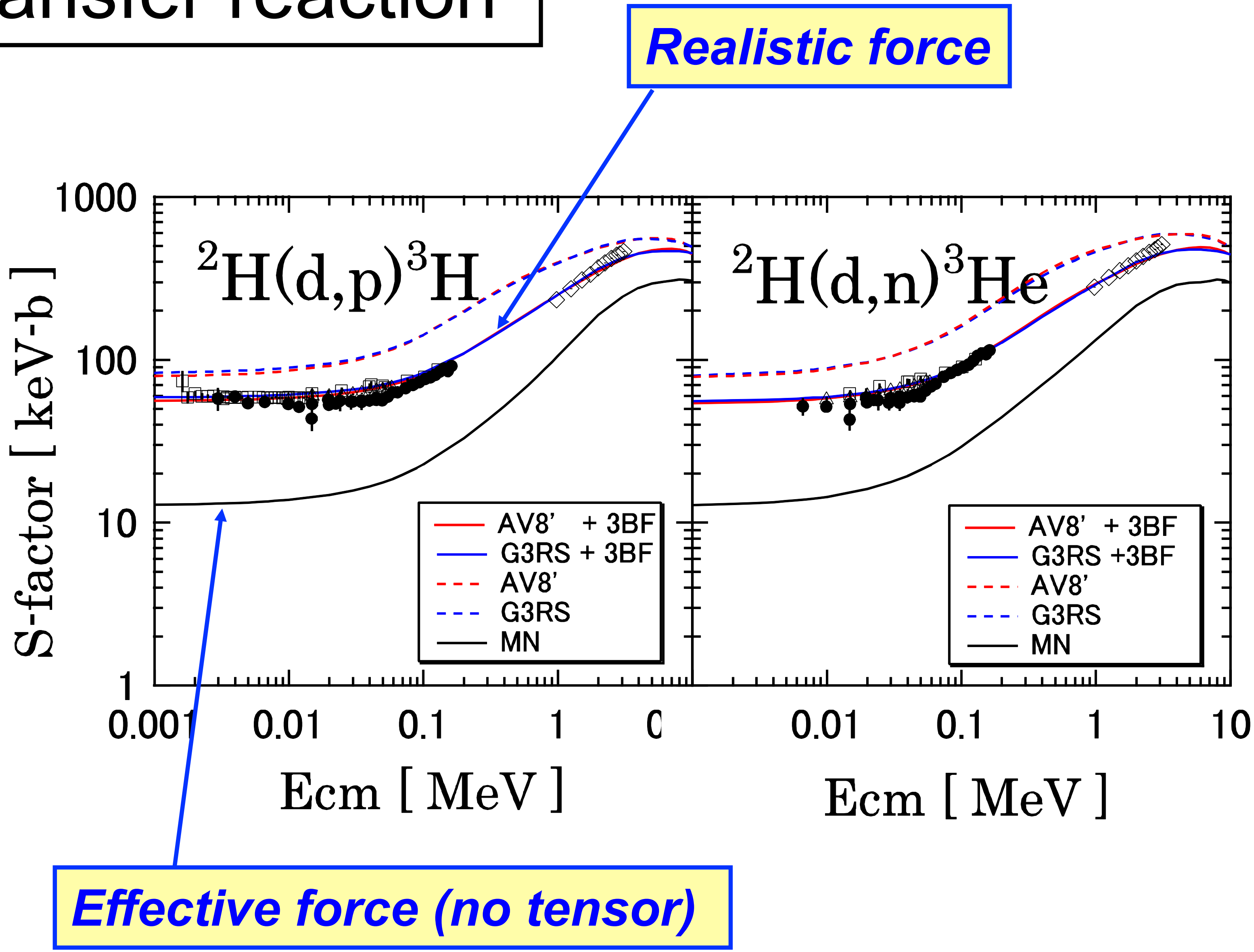
Capture reaction





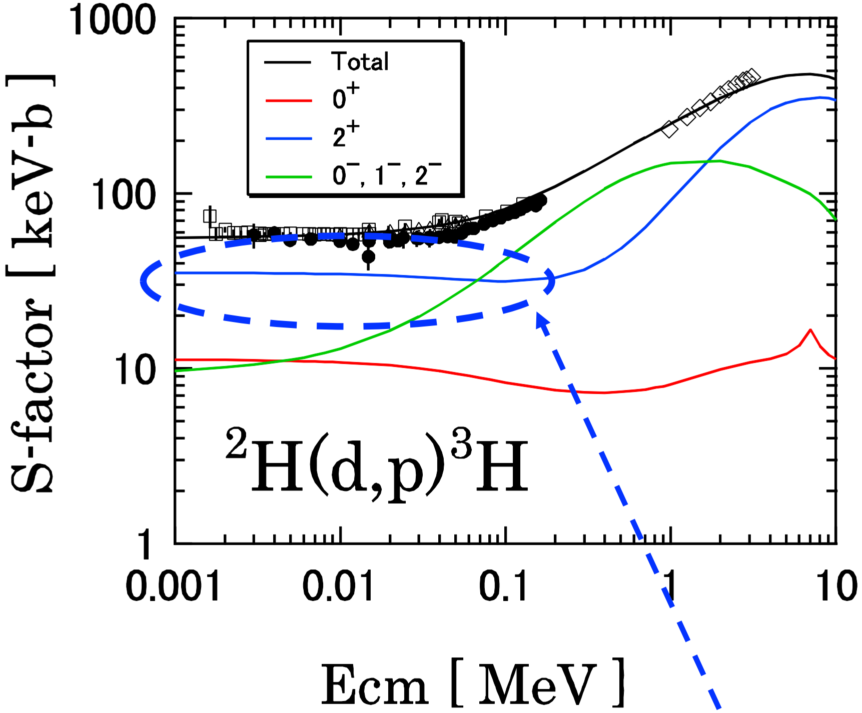
- d+d ***S-wave*** → ^{4}He 0^+ (***L=2, S=2***) ***D-wave*** component
- d+d D-wave → ^{4}He 0^+ (L=0, S=0) S-wave component

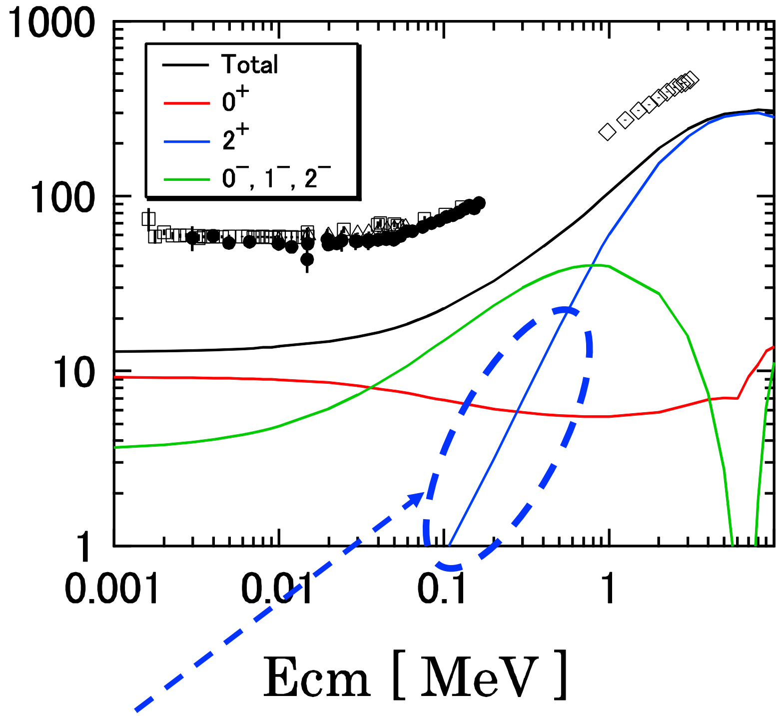
Transfer reaction



Contribution of each spin parity state in $^2H(d,p)^3H$

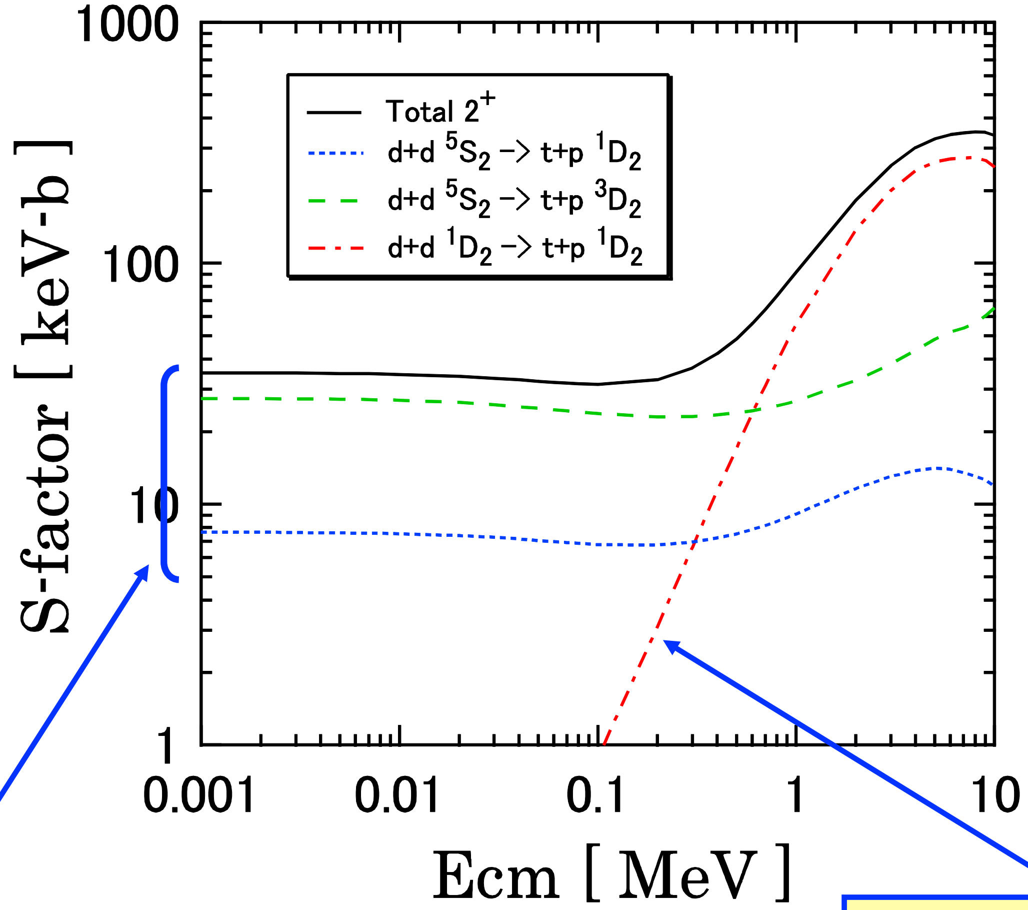
AV8' pot, **MN pot.**





S-factor by the 2^+ state

2^+ contribution in $^2H(d,p)^3H$ is decomposed according to the entrance & exit channel



AV8' pot.

d+d S-wave → t+p D-wave

d+d D-wave → t+p D-wave

Summary for d(d,γ), d(d,p), d(d,n)

- $^{2}H(d,\gamma)^{4}He$

D-wavev component

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d\ \ S\text{-wave} \rightarrow (\mathrm{L,S}) = (2,2) \\ E_{cm} > 0.3\text{MeV} & d+d\ \ D\text{-wave} \rightarrow (\mathrm{L,S}) = (0,0) \end{cases}$$

S-wave component

- $^{2}H(d,p)^{3}H$, $^{2}H(d,n)^{3}He$

$J^{p}=2^{+}$ contribution

Coupled by tensor force

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d\ \ S\text{-wave} \rightarrow t+p\ D\text{-wave} \\ E_{cm} > 0.3\text{MeV} & d+d\ \ D\text{-wave} \rightarrow t+p\ D\text{-wave} \end{cases}$$

- Tensor force plays an essential role to reproduce the astrophysical S-factor not only in the capture reaction, $^{2}H(d,g)^{4}He$, but also in the transfer reaction, $^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$.
  And this effect of the tensor force can be seen only at very low energy .

K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont D. Baye, PRL107(2011)132502

Summary

MRM is powerful tool not only to discuss the scattering and reaction problem in the light nuclei but also to search the resonance state.