YITP workshop, Resonances and non-Hermitian systems in quantum mechanics, Des.11-13, 2012

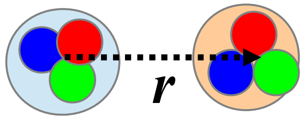
Scattering calculations of d+d, t+p and ^{3}He+n with realistic nuclear interactions

Niigata University S. Aoyama

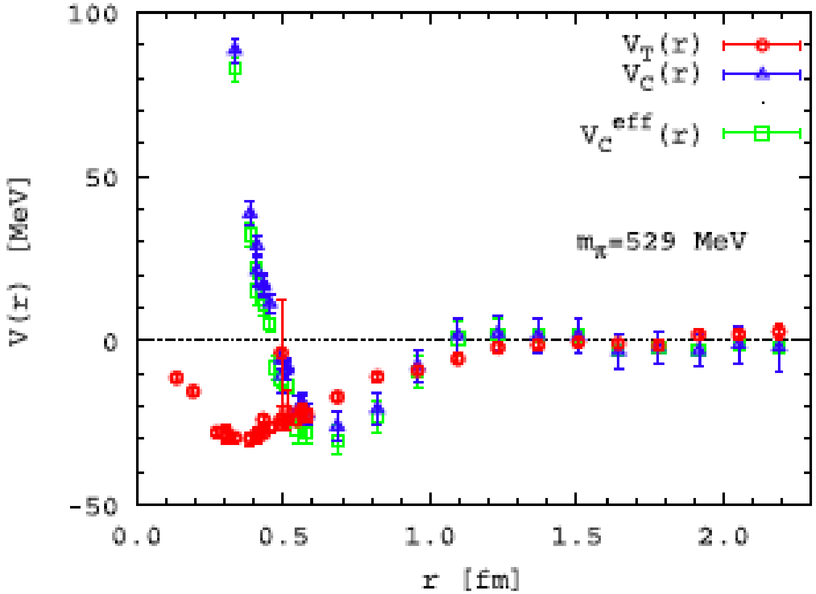
Main part of this talk

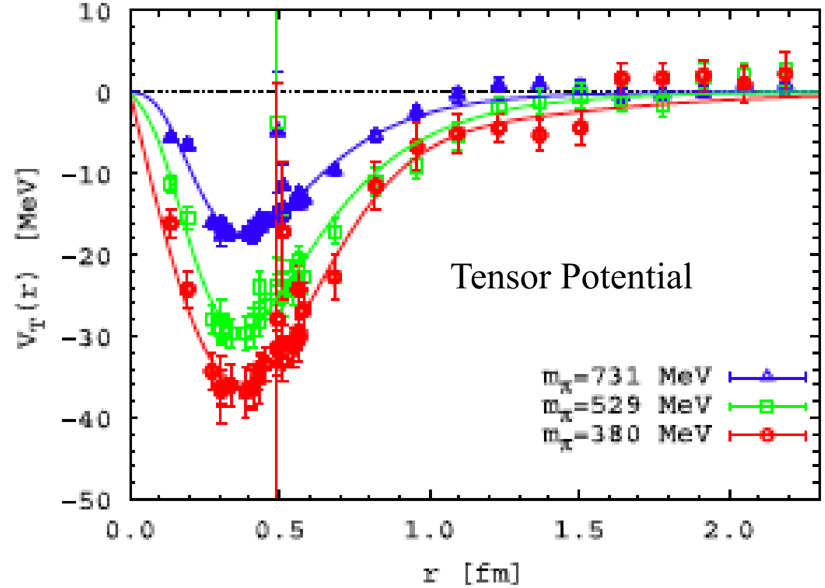
S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont and D. Baye, FBS52, (2012)97.

Central and Tensor Force in Lattice QCD



S. Aoki, T. Hatsuda, N. Ishi, PTP123(2010)89.





Tensor potential is a major ingredient of N-N interaction!

$$V_T(r)S_{ij}$$

$$S_{ij} = 3\frac{(\boldsymbol{\sigma}_\mathbf{i}\cdot\mathbf{r})(\boldsymbol{\sigma}_\mathbf{j}\cdot\mathbf{r})}{r^2} - \boldsymbol{\sigma}_\mathbf{i}\cdot\boldsymbol{\sigma}_\mathbf{j}$$

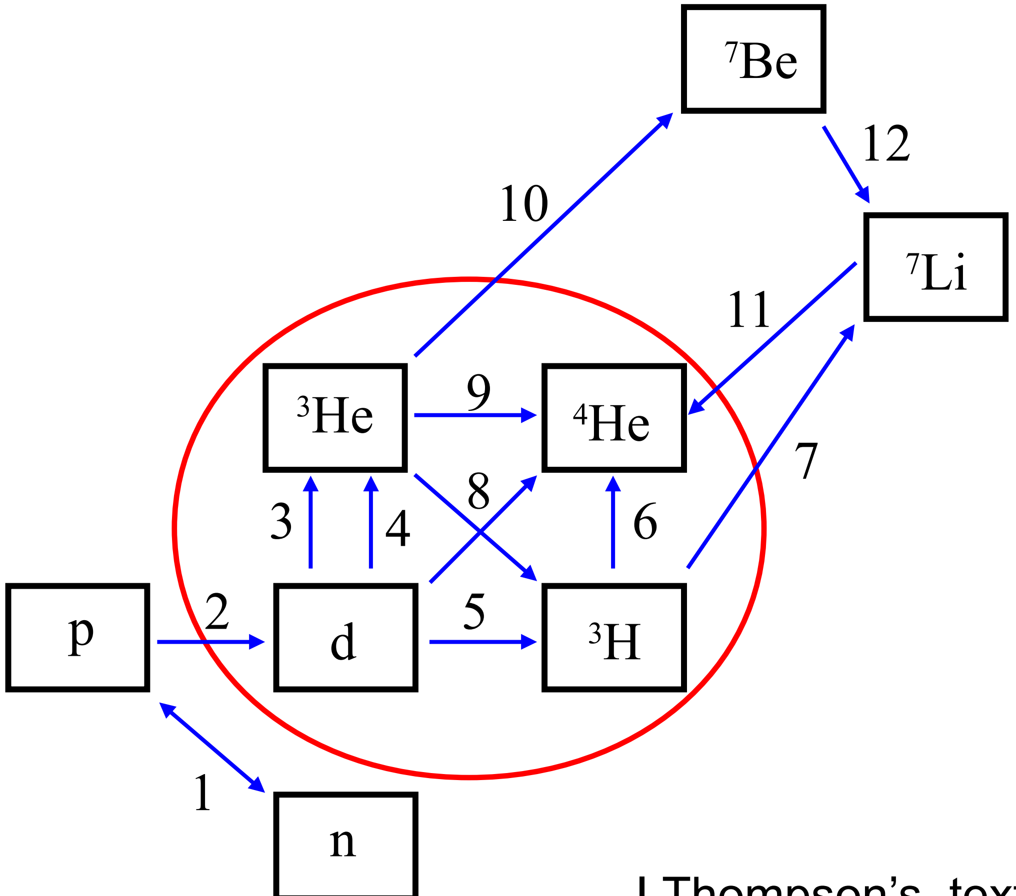
$$\propto \left[Y_2(\hat{r})[\boldsymbol{\sigma}_\mathbf{i}\times\boldsymbol{\sigma}_\mathbf{j}]_2\right]_0$$

ΔL＝2
(D-state component)

Spin dependence

Dominant reactions in primordial nucleosynthesis

Normally, the primordial nucleosybtesis is explained by the reaction chain calculation which is based on a simple nuclear model or a mere extrapolation from experiments.



I.Thompson's textbook

$$
\begin{aligned}
1:&\quad n \leftrightarrow p\\
2:&\quad p(n,\gamma)d\\
3:&\quad d(p,\gamma)^3\mathrm{He}\\
4:&\quad d(d,n)^3\mathrm{He}\\
5:&\quad d(d,p)^3\mathrm{H}\\
6:&\quad {}^3\mathrm{H}(d,n)^4\mathrm{He}\\
7:&\quad {}^3\mathrm{H}(^4\mathrm{He},\gamma)^3\mathrm{H}\\
8:&\quad {}^3\mathrm{He}(n,p)^3\mathrm{H}\\
9:&\quad {}^3\mathrm{He}(d,p)^4\mathrm{He}\\
10:&\quad {}^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}\\
11:&\quad {}^7\mathrm{Li}(p,^4\mathrm{He})^4\mathrm{He}\\
12:&\quad {}^7\mathrm{Be}(n,p)^7\mathrm{Li}
\end{aligned}
$$

$$d(d,\gamma)^4\mathrm{He}$$

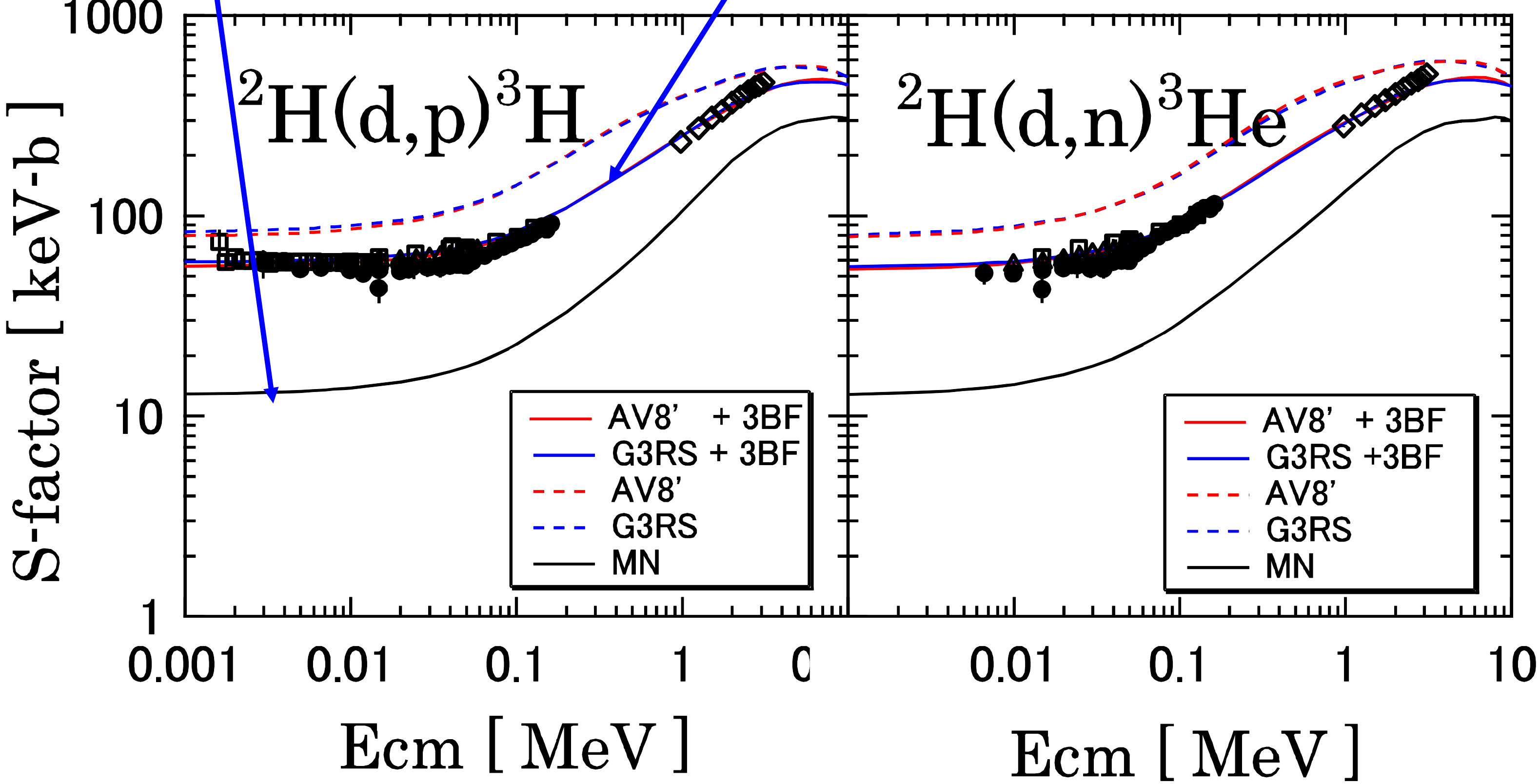
Can we understand these reactions in *ab-initio* way?

Is there some effects of tensor interaction in Big-Bang?

Transfer reaction

Effective interaction (without tensor)

Realistic interaction

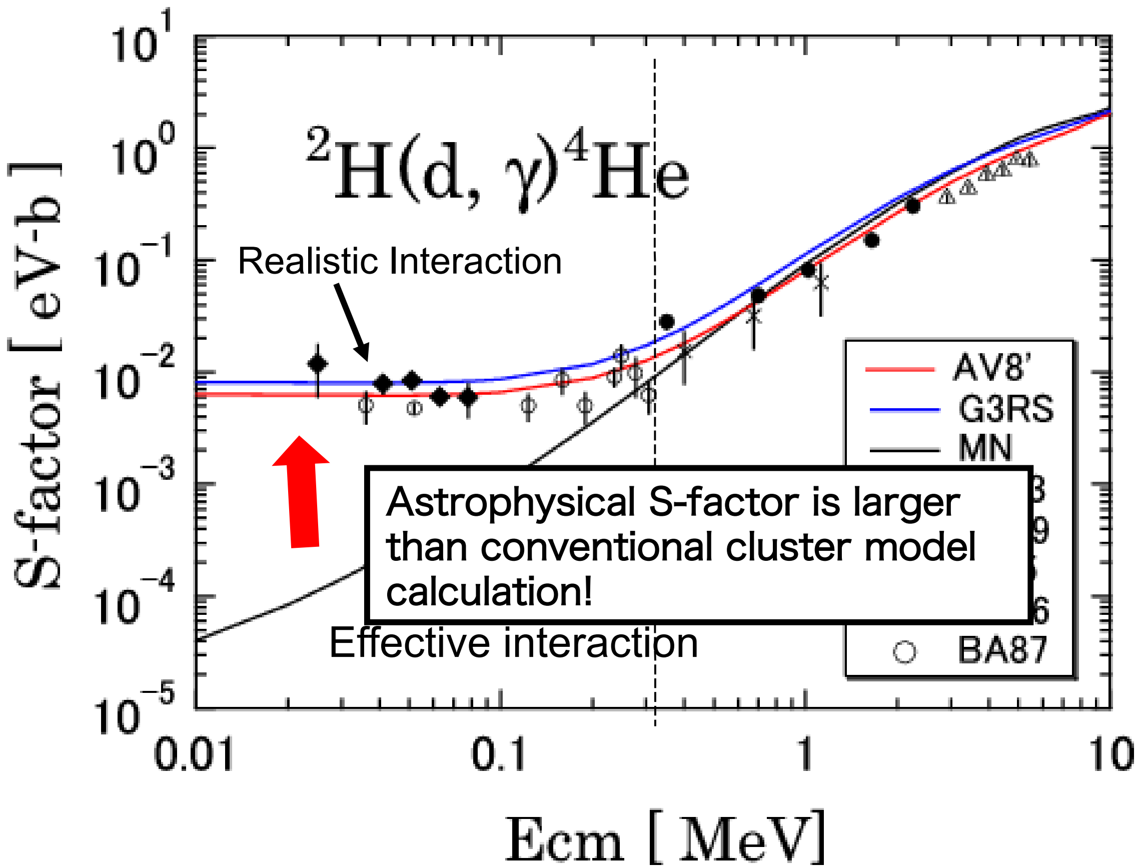


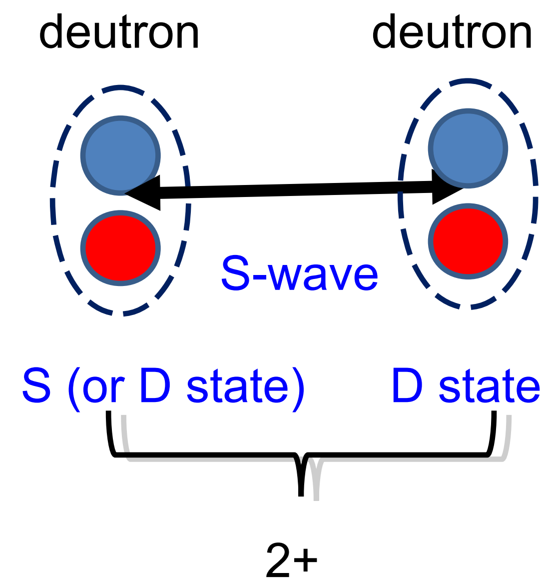
Tensor force accelerate the nuclear generation in Big-Bang.

K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont and D. Baye, PRL107 (2011) 132502.

Radiative capture （MRM）

K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont and D. Baye, PRL107 (2011) 132502.





E2 transtion is not reduced so much because of d-wave component.

d(1+;T=0) + d(1+; T=0) => ^{4}He (T=0) + γ

We can add a new evidence of D-wave components (tensor) of deutron and ^{4}He to the text book in nuclear physics.

Hamiltonian for nuclear *ab-initio* calculation

$$H = \sum_{i=1} T_i - T_{\rm cm} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

N+N scattering data, deuteron

t, ^{3}He

Realistic Interaction: AV8' (+Coulomb+3NF)

V_{ij}: Central+LS+Tensor+Coulomb

Pudliner, Pandharipande, Carlson , Pieper, Wiringa: PRC56(1997)1720

V_{ijk}: Effective three nucleon force

Hiyama, Gibson, Kamimura, PRC 70(2003)031001

Effective Interaction: MN (+Coulomb)

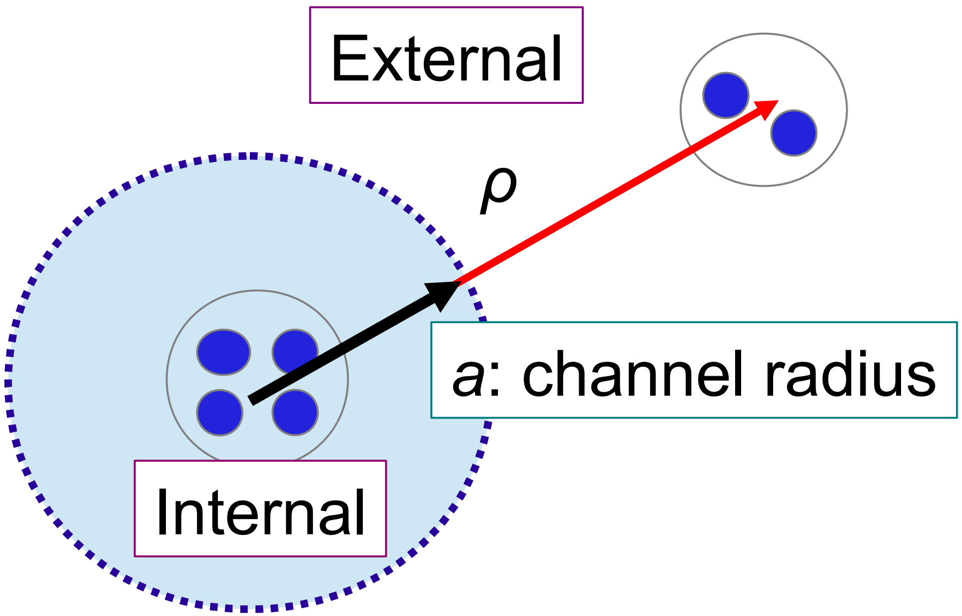
V_{ij}: Central+Coulomb

Thompson, LeMere, Tang, NPA(1977)286

V_{ijk}: = 0

Microscopic R-matrix Method

D. Baye, P. -H.Heenen, M. Libert-Heinemann, NPA291(1977).



$$\Psi_\alpha^{JM\pi} = \mathcal{A}\left[\left[\Phi^a_{I_a}\Phi^b_{I_b}\right]_I \chi_\alpha(\boldsymbol{\rho}_\alpha)\right]_{JM}$$

$$\Psi_{\rm int}^{JM\pi} = \sum_\alpha \Psi_\alpha^{JM\pi} = \sum_\alpha \sum_n f_{\alpha n}\mathcal{A}u_{\alpha n}(\rho_\alpha)\phi_\alpha^{JM\pi}$$

$$\chi_{\alpha m}(\boldsymbol{\rho}_\alpha) = \sum_n f_{\alpha n}u_{\alpha n}(\rho_\alpha)Y_{\ell m}(\widehat{\boldsymbol{\rho}}_\alpha)$$

$$\Psi_{\rm ext}^{JM\pi} = \sum_\alpha g_\alpha(\rho_\alpha)\phi_\alpha^{JM\pi}$$

$$\phi_\alpha^{JM\pi} = \frac{1}{\sqrt{(1+\delta_{I_aI_b}\delta_{ab})(1+\delta_{ab})}}\left\{\left[\left[\Phi^a_{I_a}\Phi^b_{I_b}\right]_I Y_\ell(\widehat{\boldsymbol{\rho}}_\alpha)\right]_{JM} + (-1)^{A_a+I_a+I_b-I+\ell}\left[\left[\Phi^b_{I_b}\Phi^a_{I_a}\right]_I Y_\ell(\widehat{\boldsymbol{\rho}}_\alpha)\right]_{JM}\delta_{ab}\right\}$$

$\rho < a$	Correlated Gaussian	<=Text book by Suzuki and Varga
$\rho \geq a$	$g_\alpha(\rho_\alpha) \sim I_\alpha(k_\alpha\rho_\alpha)\delta_{\alpha\,\alpha_0} - S^{J\pi}_{\alpha\,\alpha_0}O_\alpha(k_\alpha\rho_\alpha)$	Open channel
	$g_\alpha(\rho_\alpha) \sim W_{-\eta_\alpha,\ell+1/2}(2k_\alpha\rho_\alpha)$	Closed channel

α: channel

$$I_\alpha(k_\alpha\rho_\alpha) = O_\alpha(k_\alpha\rho_\alpha)^* = G_\ell(\eta_\alpha, k_\alpha\rho_\alpha) - iF_\ell(\eta_\alpha, k_\alpha\rho_\alpha)$$

S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont and D. Baye, FBS52(2012)97.

How to connect w.f. at channel radius ?

D. Baye, P. -H.Heenen, M. Libert-Heinemann, NPA291(1977).

S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont, D. Baye, FBS52(2012)97.

$$(H+\mathcal{L}-E)\Psi^{JM\pi}_{\rm int} = \mathcal{L}\Psi^{JM\pi}_{\rm ext}$$

$$\mathcal{L} = \sum_{\alpha} \frac{\hbar^2}{2\mu_\alpha R} |\phi^{JM\pi}_\alpha\rangle \delta(\rho_\alpha - R) \left(\frac{\partial}{\partial \rho_\alpha} - \frac{b_\alpha}{\rho_\alpha} \right) \rho_\alpha \langle \phi^{JM\pi}_\alpha |$$

Bloch Operator

open channel $b_\alpha = 0$

closed channel $b_\alpha = 2k_\alpha R W'_{-\eta_\alpha,\ell+1/2}(2k_\alpha R)/W_{-\eta_\alpha,\ell+1/2}(2k_\alpha R)$

$$\sum_{\alpha n} C_{\alpha' n',\alpha n} f_{\alpha n} = \langle \Phi^{JM\pi}_{\alpha' n'} | \mathcal{L} | \Psi^{JM\pi}_{\rm ext} \rangle$$

$$C_{\alpha' n',\alpha n} = \langle \Phi^{JM\pi}_{\alpha' n'} | H + \mathcal{L} - E | \mathcal{A} \Phi^{JM\pi}_{\alpha n} \rangle_{\rm int}$$

$$\Phi^{JM\pi}_{\alpha n} = u_{\alpha n}(\rho_\alpha) \phi^{JM\pi}_\alpha$$

$$\mathcal{R}_{\alpha' \alpha} \equiv \frac{\hbar^2 R}{2} \left(\frac{k_{\alpha'}}{\mu_{\alpha'} \mu_\alpha k_\alpha} \right)^{\frac{1}{2}} \sum_{n' n} u_{\alpha' n'}(R) (C^{-1})_{\alpha' n',\alpha n} u_{\alpha n}(R)$$

$$\mathcal{Z}_{\alpha' \alpha} \equiv I_\alpha(k_\alpha R) \delta_{\alpha' \alpha} - \mathcal{R}_{\alpha' \alpha} k_\alpha a I'_\alpha(k_\alpha R)$$

S-Matrix → $S^{J\pi} = (\mathcal{Z}^*)^{-1} \mathcal{Z}$

Elastic scattering case

$$S^{J\pi}_{\alpha\alpha} = \eta^{J\pi}_\alpha e^{2i\delta^{J\pi}_\alpha}$$

Correlated Gaussian function with triple global vectors for four nucleon system

Unnatural parity 0^-

L1=L2=L12=L3=1

$$F_{L_1L_2(L_{12})L_3LM}(u_1,u_2,u_3,A,x)$$
$$=\exp\left(-\frac{1}{2}\widetilde{x}Ax\right)[[\mathcal{Y}_{L_1}(\widetilde{u_1}x)\mathcal{Y}_{L_2}(\widetilde{u_2}x)]_{L_{12}}\mathcal{Y}_{L_3}(\widetilde{u_3}x)]_{LM}$$

Single gloval vector

Double global vector

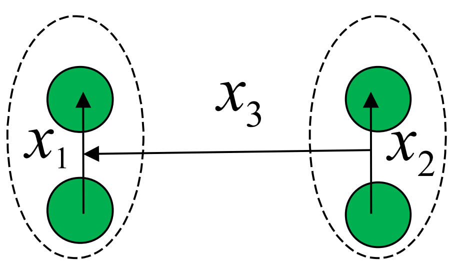
New extension(triple)

$$\mathcal{Y}_{L_iM_i}(\widetilde{u_i}x)=|\widetilde{u_i}x|^{L_i}Y_{L_iM_i}(\widehat{\widetilde{u_i}x}) \qquad \widetilde{u_i}x=\sum_{j=1}^{N-1}(u_i)_jx_j$$

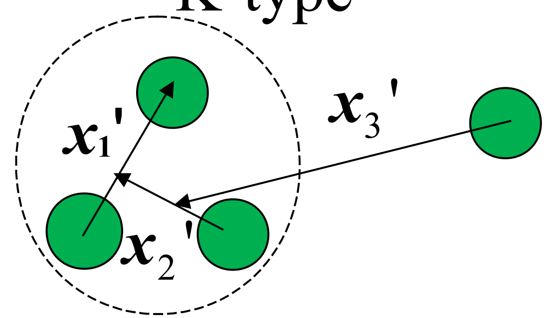
Double global vector representation (DGVR)

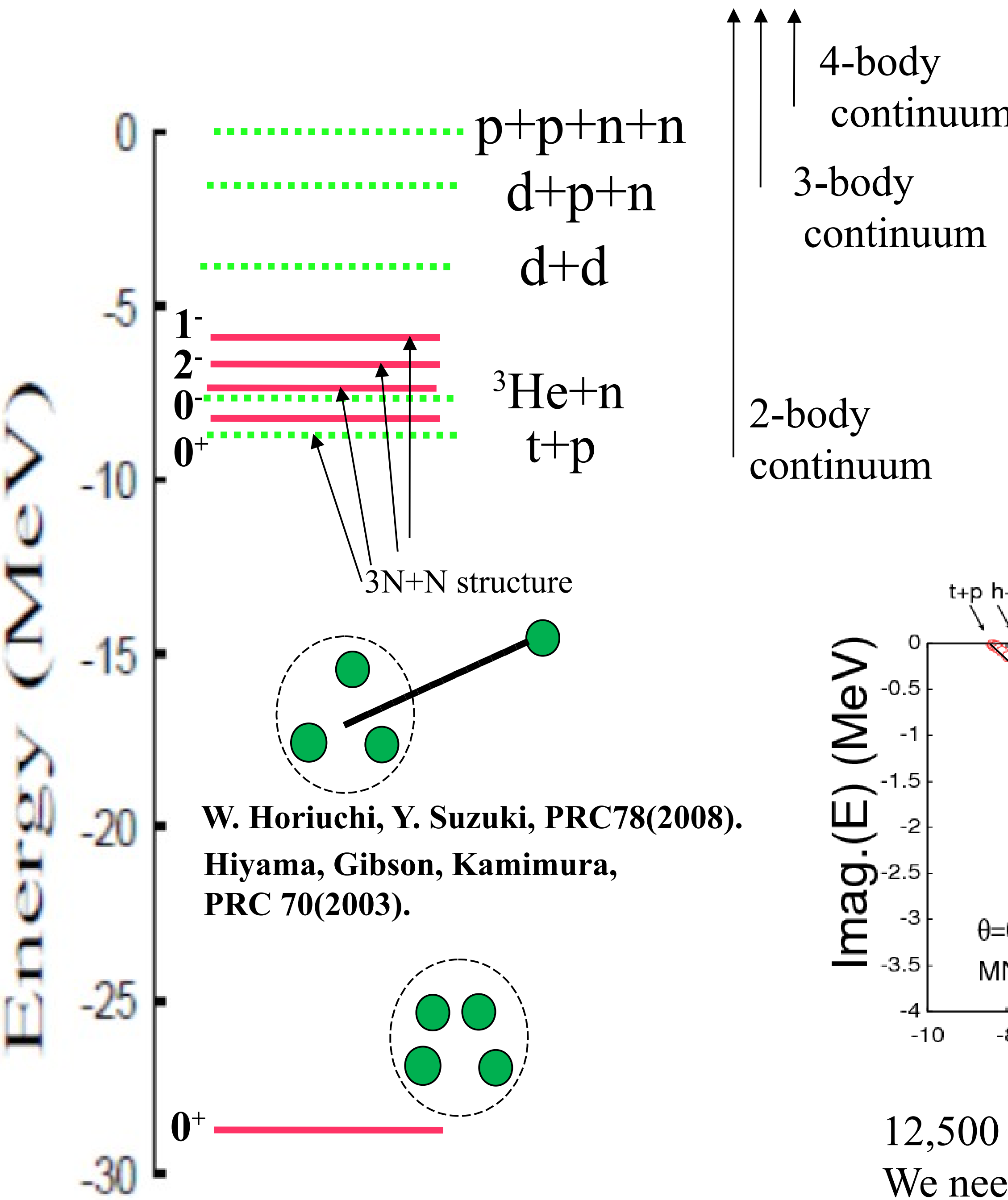
Y. Suzuki, W. Horiuchi and W. Orabi, K. Arai, FBS42(2008)33

H-type



K-type



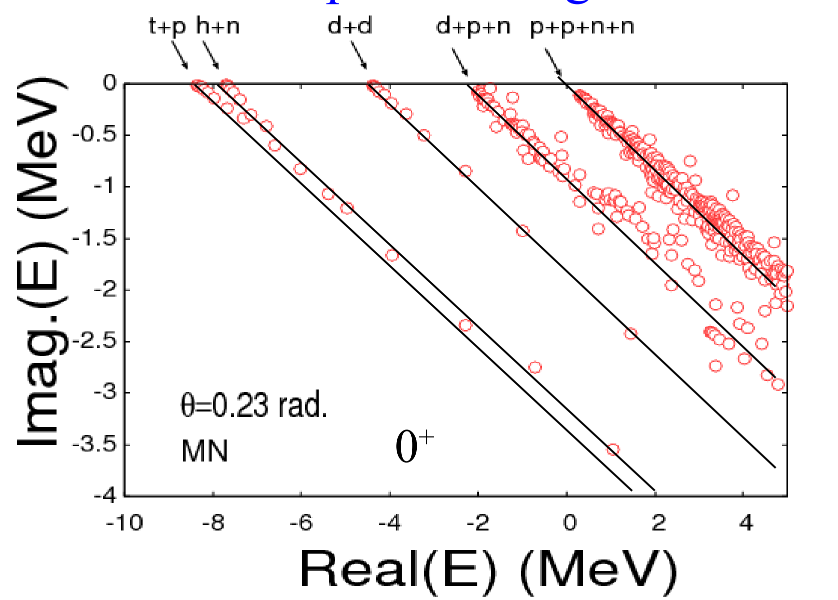


W. Horiuchi, Y. Suzuki, PRC78(2008).

Hiyama, Gibson, Kamimura, PRC 70(2003).

Review of CSM in nuclear physic
S. Aoyama, T. Myo, K. Kato, K. Ikeda, PTP116(2006)1 .

Complex Scaling Method



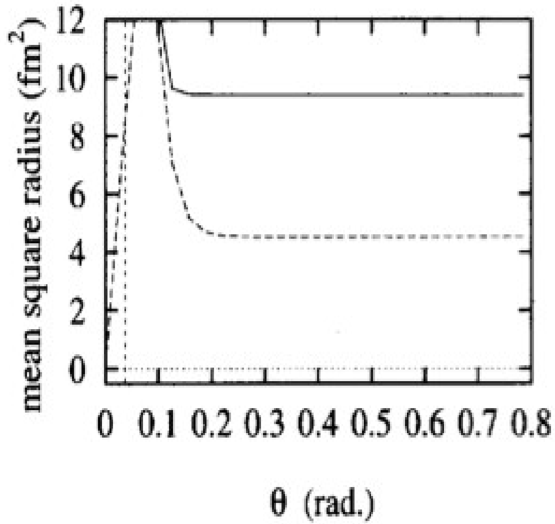
12,500 dimension (even for simple case)

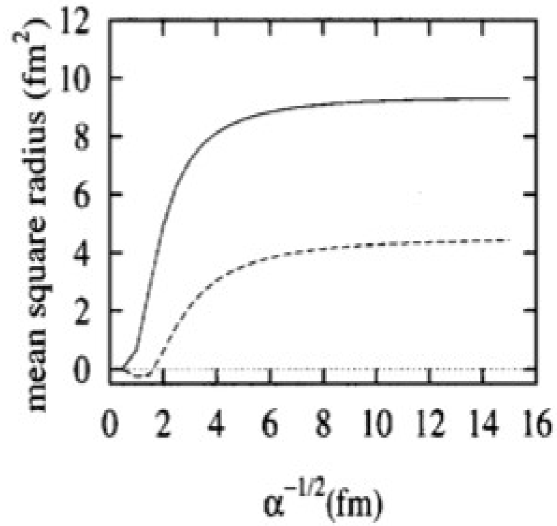
We need so much computational time!

Progress of Theoretical Physics, Vol. 97, No. 4, April 1997

Matrix Elements of Physical Quantities Associated with Resonance States

Mitsuo HOMMA, Takayuki MYO and Kiyoshi KATŌ





$$< \widetilde{\Phi_1}(k_1)|\hat{O}|\Phi_2(k_2) >= \lim_{\alpha\to 0}\int \Phi_1^*(-k_1^*,\ \vec{r}\ )\hat{O}\Phi_2(k_2,\ \vec{r}\ )e^{-\alpha r^2}d\vec{r}, \qquad (2\cdot 1)$$

Norm =>Hokkyo, PTP33(1965)1116

Comlex expectation value =>Romo, NPA116(1968)618

Complex root mean square radius=>Gyarmati, Vertse, NPA 160(1971)523

Interpretation =>Berggren, Phys.Lett.B33(1970)547, B373(1996)1

Norm Density $N = \tilde{\chi}^* \chi$

$$\frac{\partial}{\partial t}\int_R \tilde{\chi}^* \chi dr = -(\frac{\hbar}{2mi})\left[\tilde{\chi}^*(\frac{\partial \chi}{\partial r}) - (\frac{\partial \tilde{\chi}}{\partial r})^* \chi\right]_{r=R} = 0$$

$$\Rightarrow \quad (\chi|\chi) \equiv \int \tilde{\chi}^*(r,t)\chi(r,t)dr = 1$$

Complex Scaling is a useful method to easily define the norm density!

Probability Density $P = \chi^* \chi$

$$\frac{\partial}{\partial t}\int_R \chi^* \chi dr \neq 0 \quad \Rightarrow$$

$$\langle\chi|\chi\rangle \equiv \int \chi^*(r,t)\chi(r,t)dr = \exp\left(-\frac{2Im(E)}{\hbar}t\right) = \exp\left(-\frac{\Gamma}{\hbar}t\right)$$

The basis function for the sub-system is determined by SVM

potential	cluster	present				literature			
		N_k	E (MeV)	$R^{\rm rms}$ (fm)	P_D (%)	E (MeV)	$R^{\rm rms}$ (fm)	P_D (%)	
	$d(1^+)$	8	−2.18	1.79	5.9	−2.24	1.96	5.8	
AV8′	$t(\frac{1}{2}^+)$	30	−8.22	1.69	8.4	−8.41	-	-	Hiyama
(with TNF)	$h(\frac{1}{2}^+)$	30	−7.55	1.71	8.3	−7.74	-	-	Hiyama
	^{4}He(0^+)	(2370)	−27.99	1.46	13.8	−28.44	-	14.1	Hiyama
	$d(1^+)$	4	−2.10	1.63	0	−2.20	1.95	0	Horiuchi
MN	$t(\frac{1}{2}^+)$	15	−8.38	1.70	0	−8.38	1.71	0	Horiuchi
	$h(\frac{1}{2}^+)$	15	−7.70	1.72	0	−7.71	1.74	0	Horiuchi
	^{4}He(0^+)	(1140)	−29.94	1.41	0	−29.94	1.41	0	Horiuchi

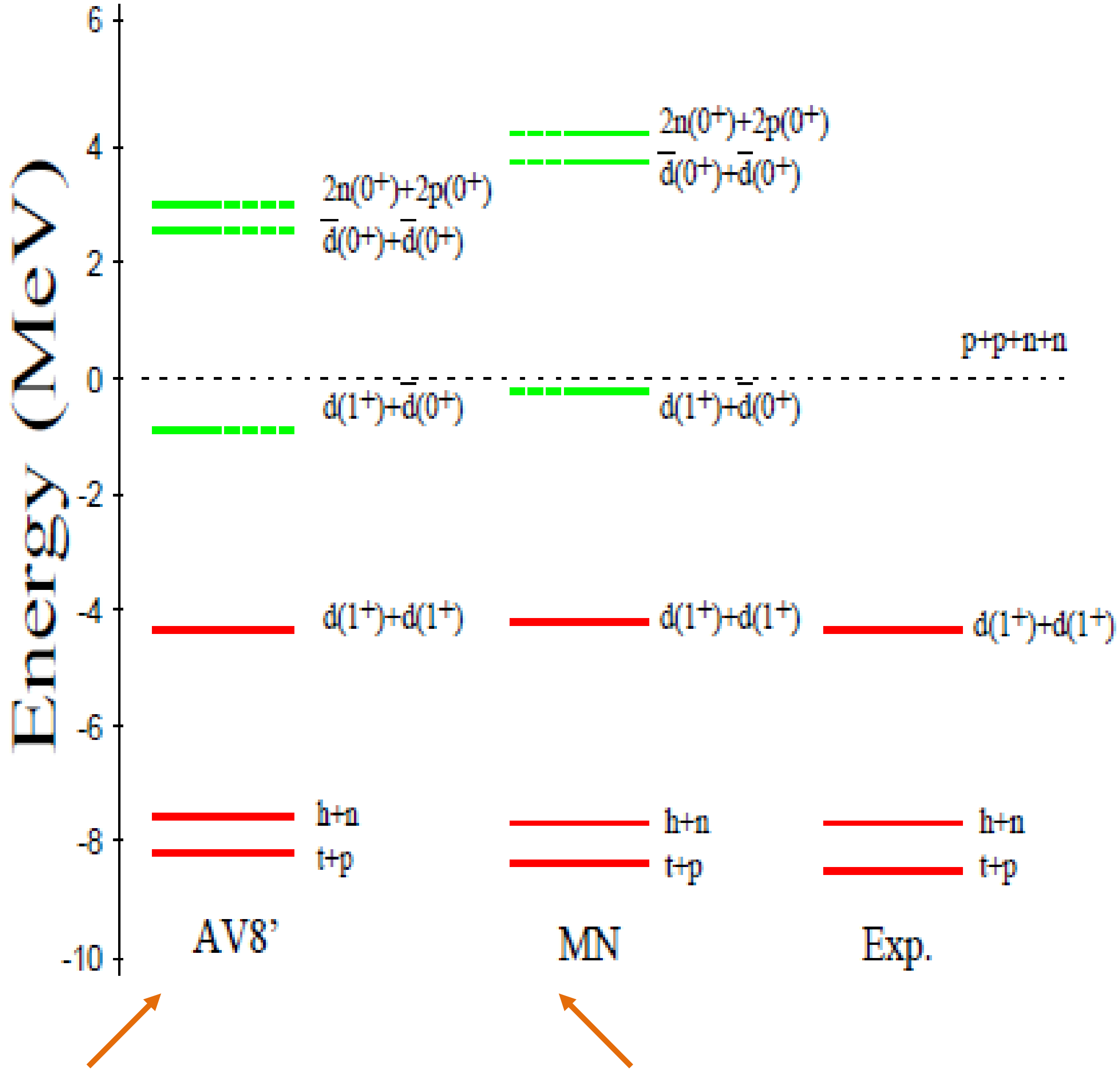
present

SVM (Stochastic Variational Method)

Y. Suzuki, K. Varga, Stochastic variational approach to quantum-mechanical few-body problems (Lecture notes in physics, Vol. 54). Springer, Berlin Heidelberg New York

K. Varga, Y. Suzuki, Phys Rev C 52(1995).

Threshold positions in the present calculation



with three-nucleon force

without three-nucleon force

S. Aoyama, K. Arai, Y. Suzuki , P. Descouvemont , D. Baye , FBS52(2012)97

Included channels in the present calculation

model			channel
FULL	$2N+2N$	I	$d(1^+)+d(1^+)$
			$d(1^+)+d^*(1^+)$
			$d^*(1^+)+d^*(1^+)$
		II	$\bar{d}(0^+)+\bar{d}(0^+)$
			$\bar{d}(0^+)+d^*(0^+)$
			$d^*(0^+)+d^*(0^+)$
		III	$d^*(2^+)+d^*(1^+)$
			$d^*(2^+)+d^*(2^+)$
		IV	$d^*(3^+)+d^*(1^+)$
			$d^*(3^+)+d^*(2^+)$
			$d^*(3^+)+d^*(3^+)$
		V	$2n(0^+)+2p(0^+)$
			$2n(0^+)+2p^*(0^+)$
			$2n^*(0^+)+2p(0^+)$
			$2n^*(0^+)+2p^*(0^+)$
	$3N+N$	1	$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
			$t^*(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
		2	$h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$
			$h^*(\frac{1}{2}^+)+n(\frac{1}{2}^+)$

Thanks to the reduction of basis function by SVM for the sub-system. We can reduce the dimension of matrix elements very much!

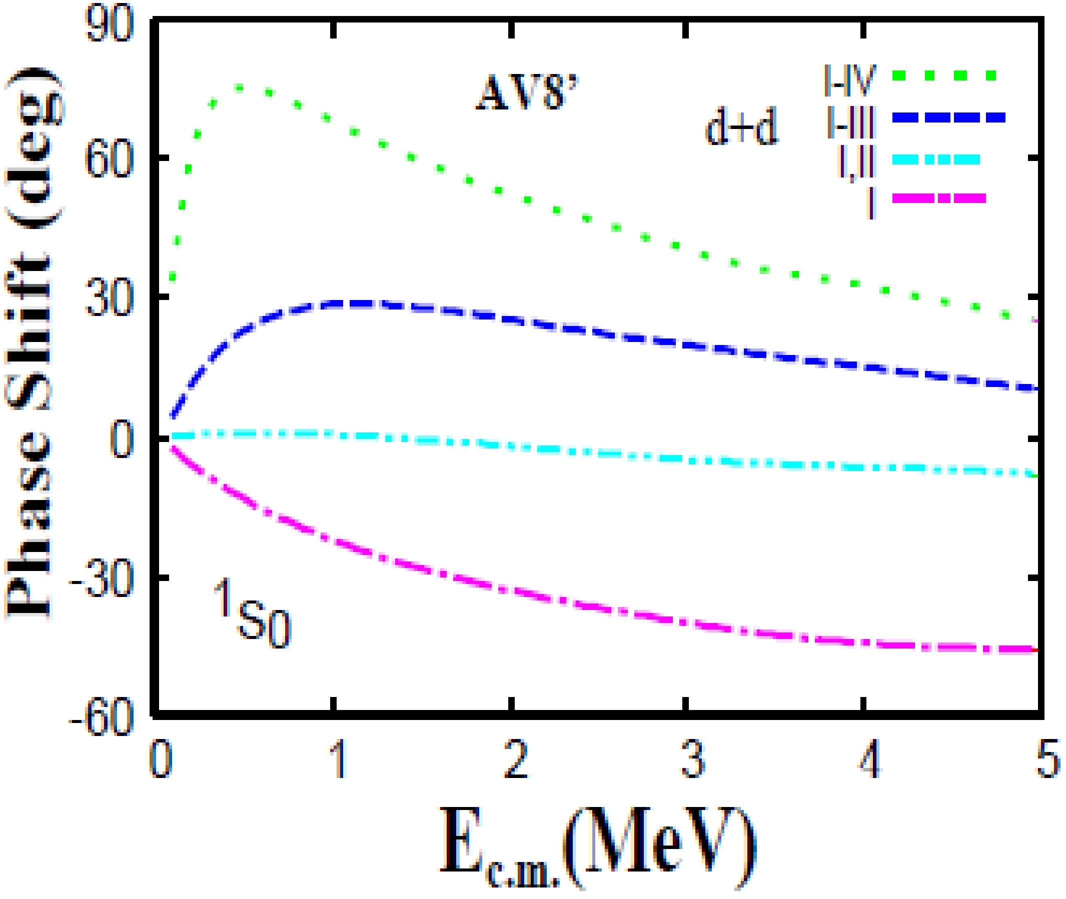
Dimensions of matrix elements for FULL in the LS-coupled case

0+	6660
1+	16680
2+	22230
0-	4200
1-	11670
2-	12480

For 2+, it takes about 200 days with 1CPU(1Core). And we need about 20Gbyte memory for the MRM calculation.

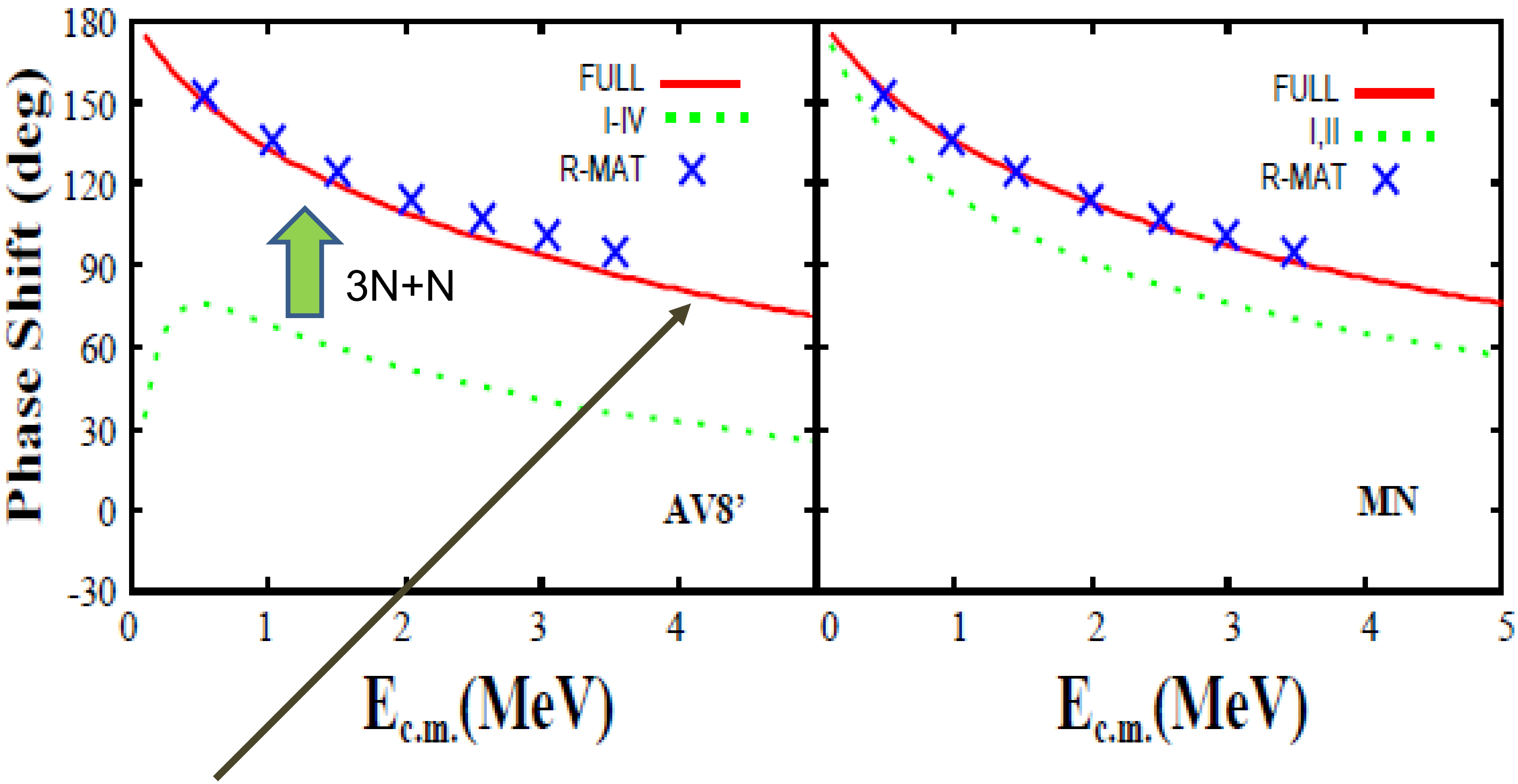
All pseudo states (discretized continuum state) are employed in the MRM calculation.

$^1S_0$ d+d elastic phase shift within d+d channel



	channel
I	$d(1^+)+d(1^+)$
	$d(1^+)+d^*(1^+)$
	$d^*(1^+)+d^*(1^+)$
II	$\bar{d}(0^+)+\bar{d}(0^+)$
	$\bar{d}(0^+)+d^*(0^+)$
	$d^*(0^+)+d^*(0^+)$
III	$d^*(2^+)+d^*(1^+)$
	$d^*(2^+)+d^*(2^+)$
IV	$d^*(3^+)+d^*(1^+)$
	$d^*(3^+)+d^*(2^+)$
	$d^*(3^+)+d^*(3^+)$

$^{1}S_{0}$ d+d elastic phase shift (0+)

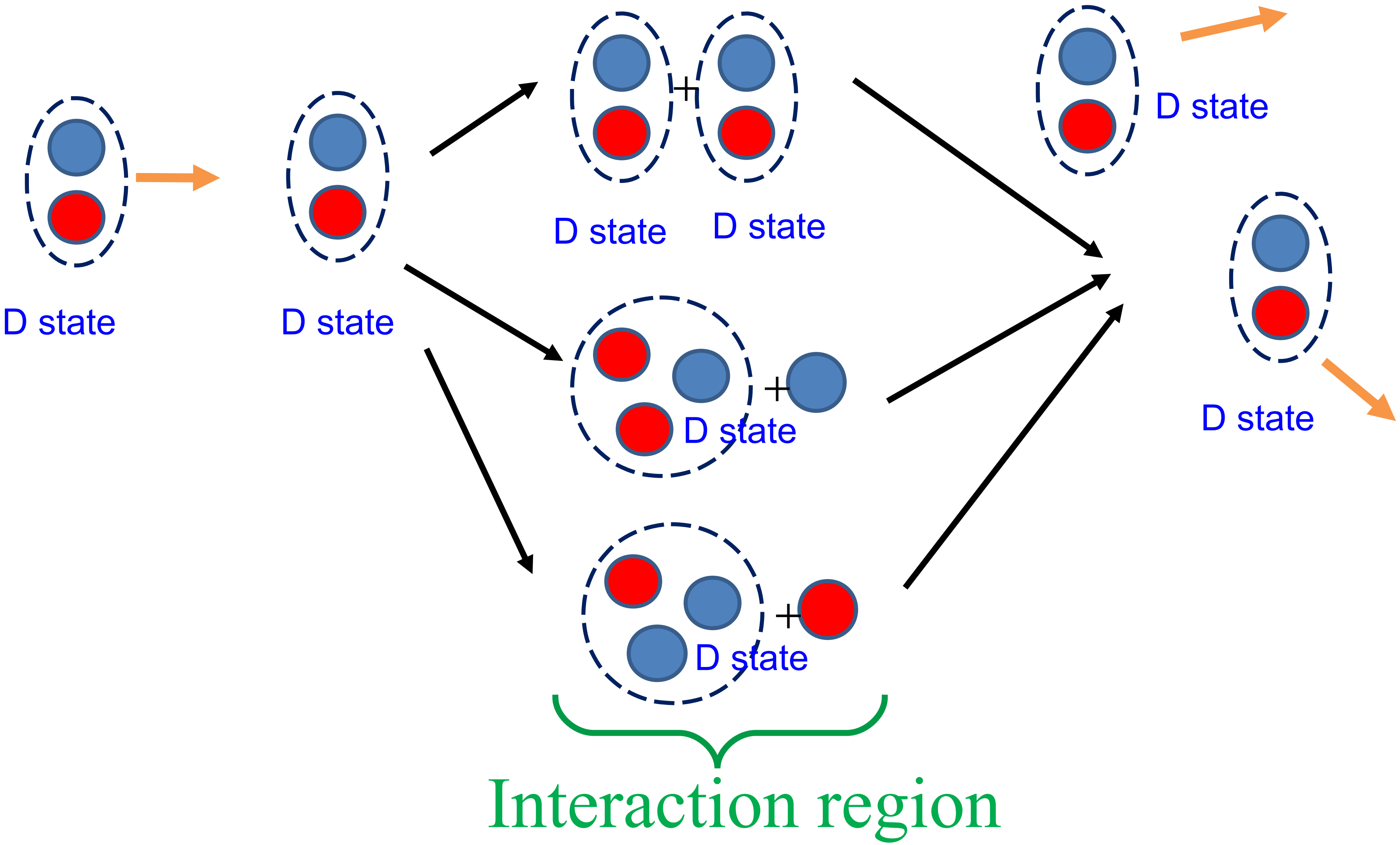


For effective interaction, d+d scattering picture is good!

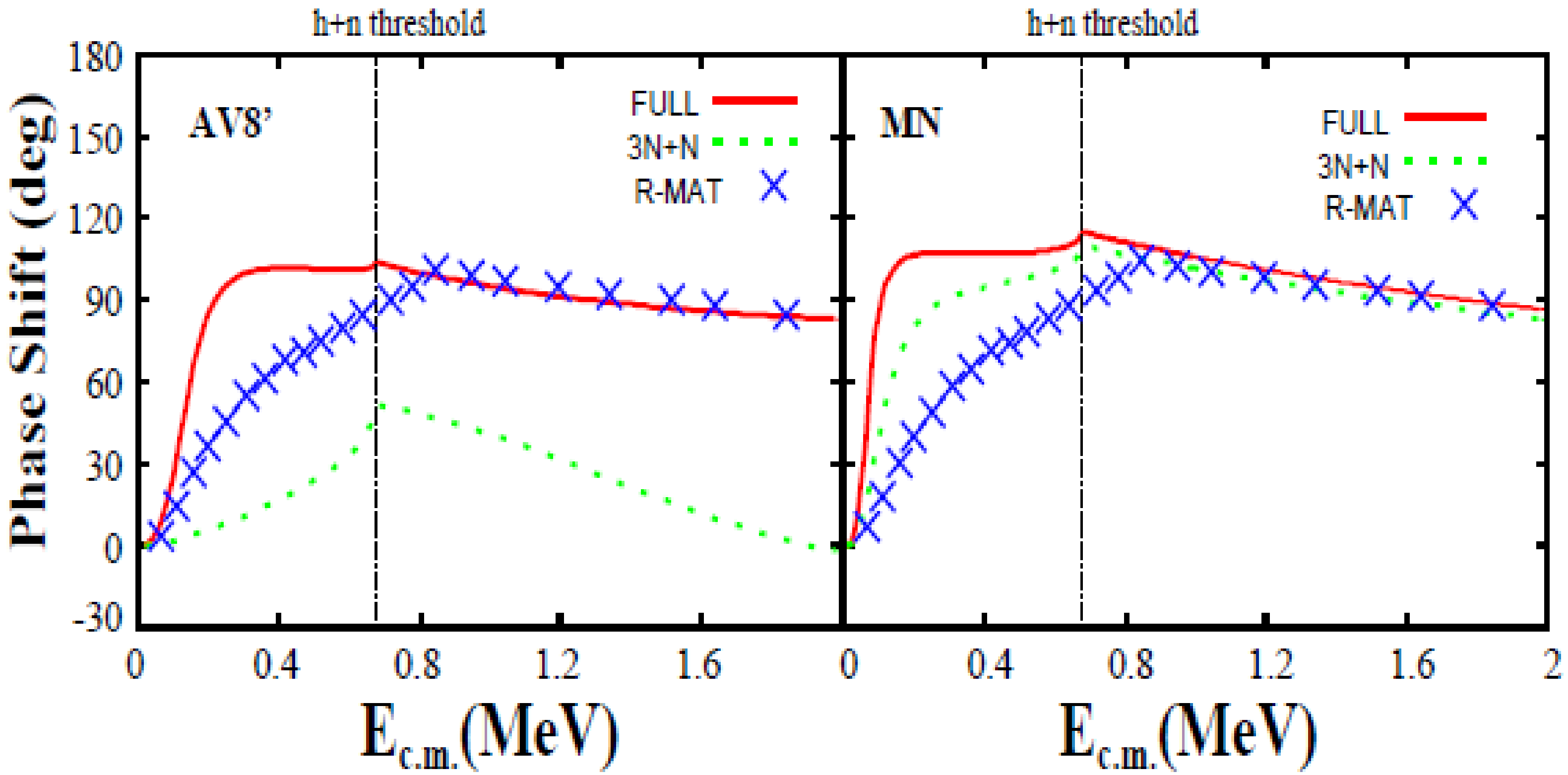
R.-Matrix analyses : Hofmann, Hale, PRC77(2008)044002.

Coupling between d+d channel and 3N+N channels

Tensor force makes the coupling of rearrangement strong

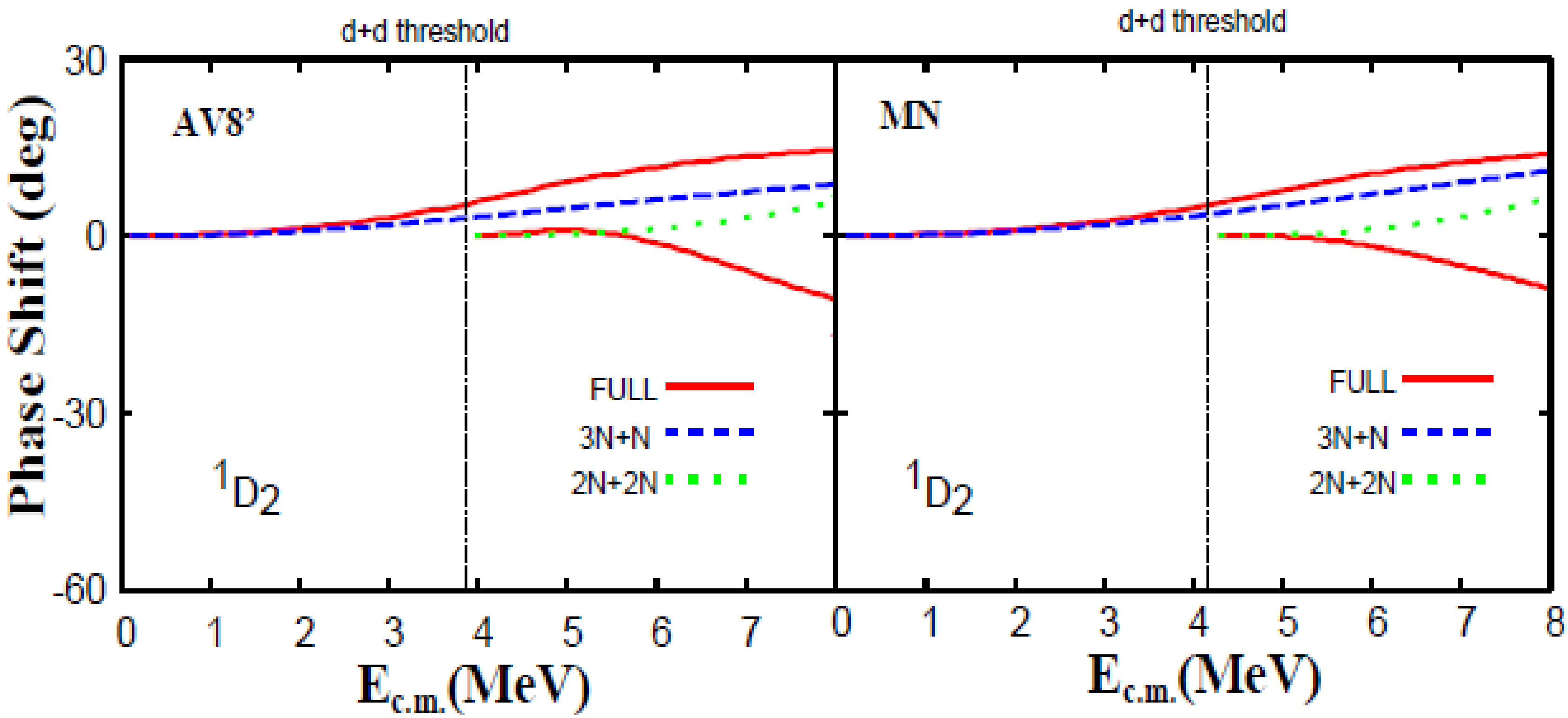


$^1S_0$ t+p elastic phase shift (0+)



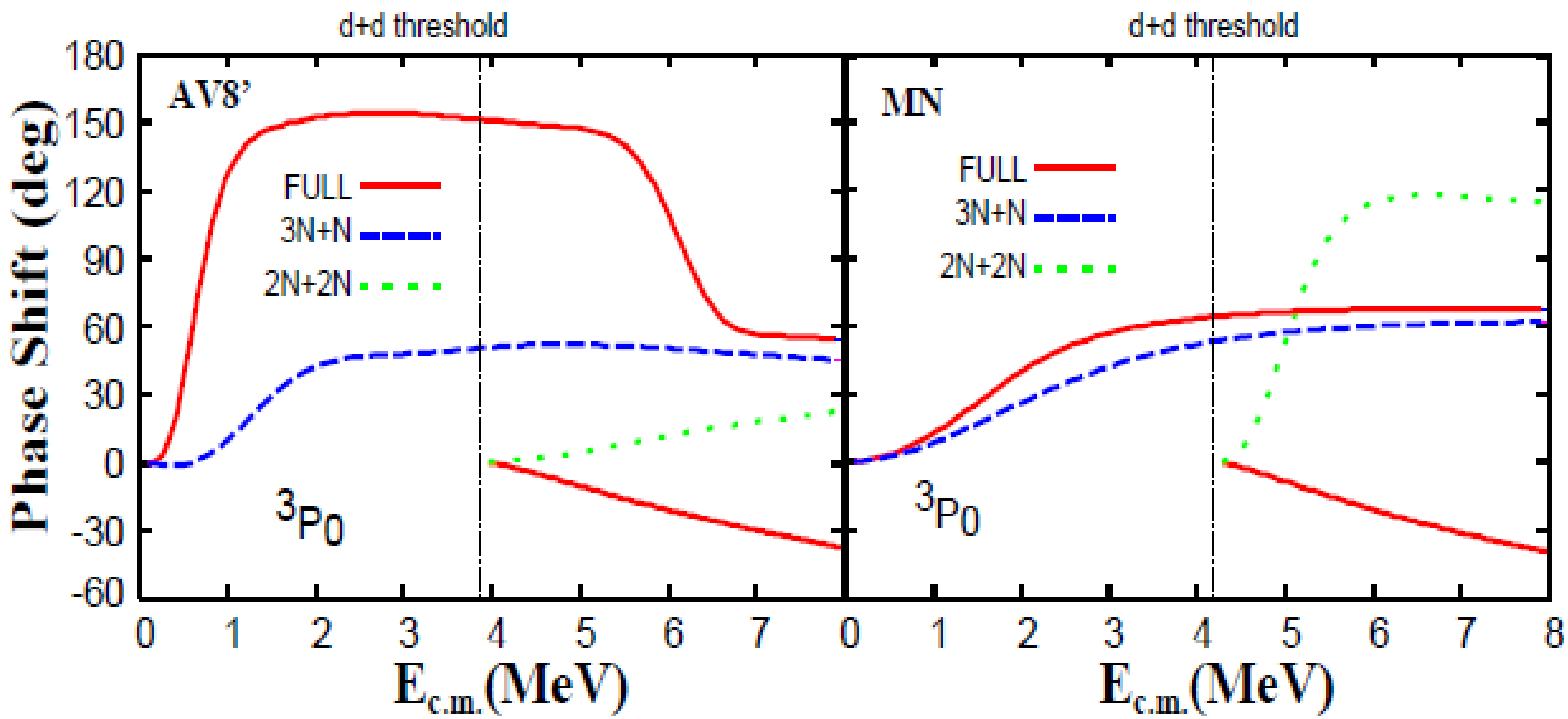
For effective interaction, t+p scattering picture is good!

$^1D_2$ elastic phase shift (2+)



Phase shift with Realistic interaction is not different so much from effective interaction for $^1D_2$

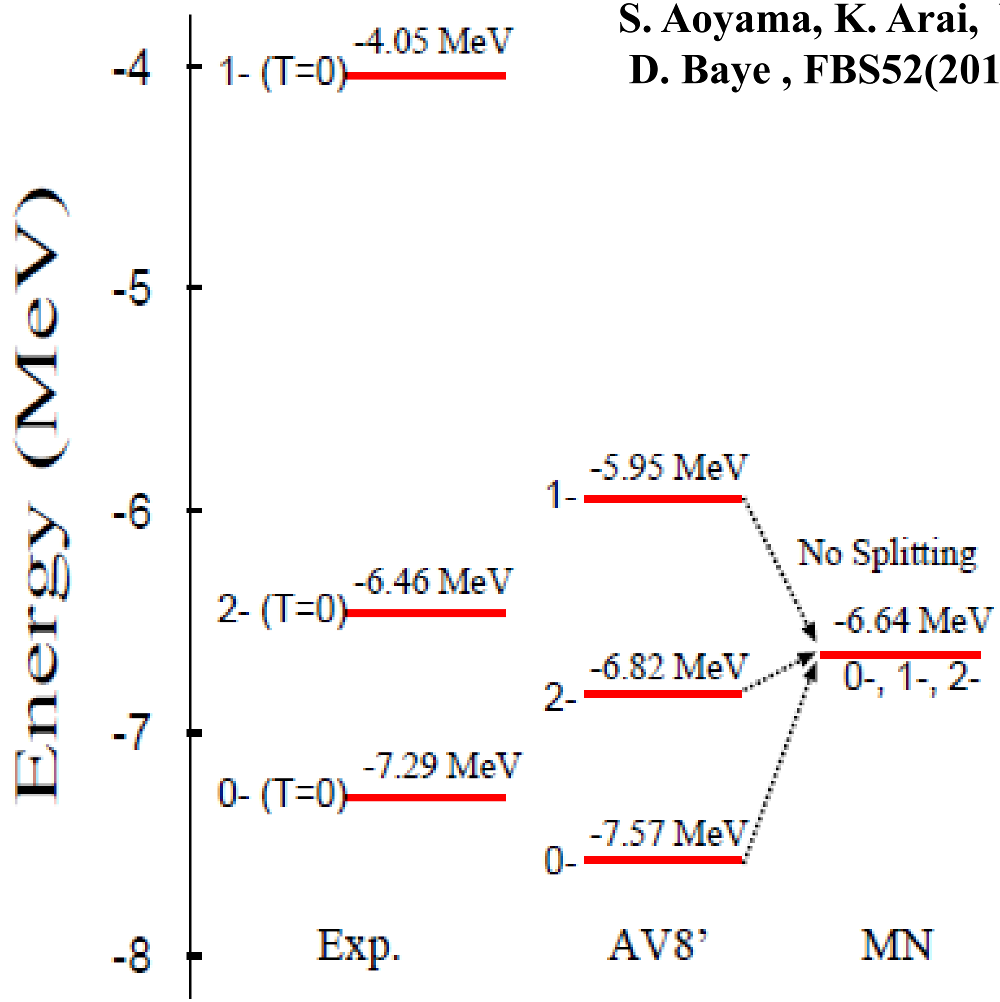
$^3P_0$ elastic phase shift (0-)



channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$ $^5D_0$	$^5D_1$	$^5S_2$ $^1D_2$ $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$ $^3D_1$	$^1D_2$ $^3D_2$	$^3P_0$	$^1P_1$ $^3P_1$	$^3P_2$

Energy levels for negative parity states

S. Aoyama, K. Arai, Y. Suzuki , P. Descouvemont , D. Baye , FBS52(2012)97



Effective interaction (MN) gives same phase shift for 0-.1-.2- !

W. Horiuchi, Y. Suzuki PRC78(2008)034305

Summary

By using the triple global vector representation method with MRM, we calculated the four nucleon scattering phase shifts with a realistic interaction (AV8'+3NF) and an effective interaction (MN).

The distortion of the deuteron cluster for $^1S_0$ due to the tensor interaction is very large.

For negative parity states, the energy splitting of ^3P_J is very large for the realistic interaction, but they are degenerating for the effective interaction.

In progress

Determining resonant pole position in the excited region of ^4He with complex scaling is in progress.
(We got much computational time of HPCI from Oct.)