Landau resonance and the core-cusp problem in cold dark matter halos

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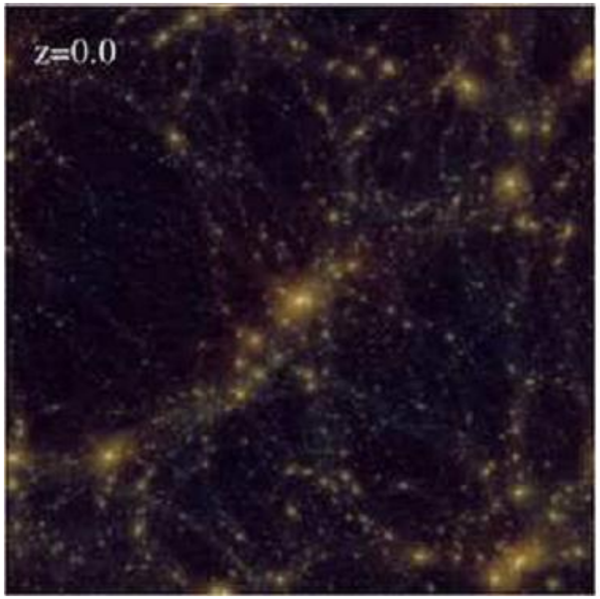
Structure Formation in the Universe

- Dark Matter (DM)
  - is the dominant element in mass
  - interacts only through gravity with others
  - assembles baryon (atoms) and DM
  - drives structure formation (DM halos, Galaxies etc.)

- Cold Dark Matter (CDM) cosmology
  - is the standard paradigm
  - matches observations in large scale
  - has serious problems in small scale

Core-Cusp problem

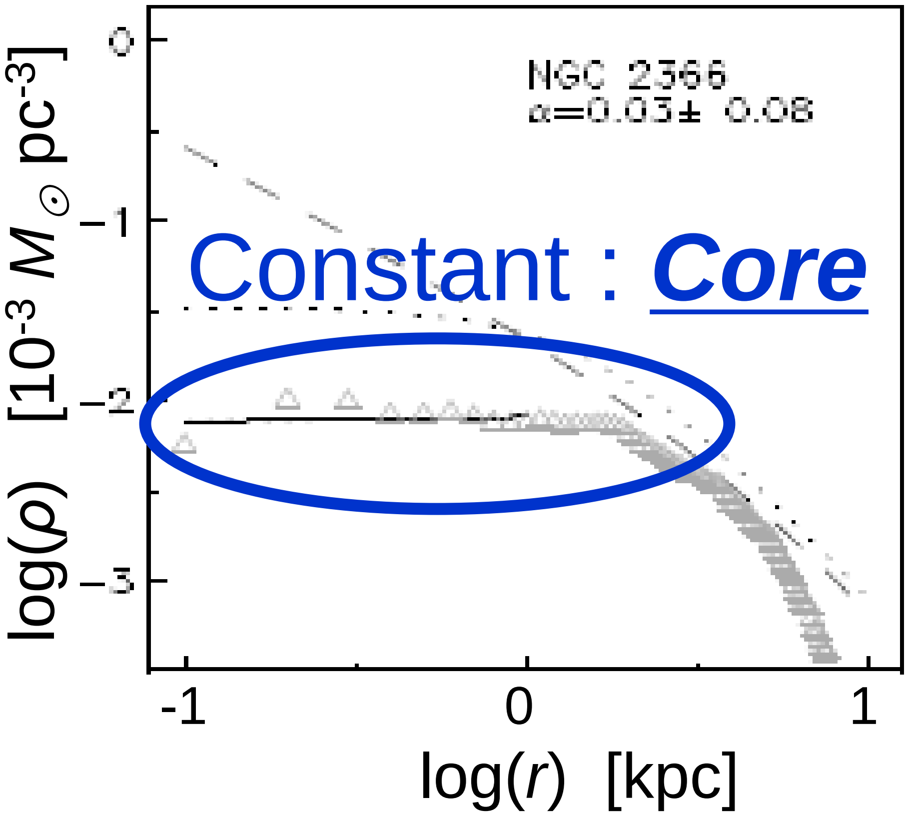


Ishiyama *et al*. (2011)

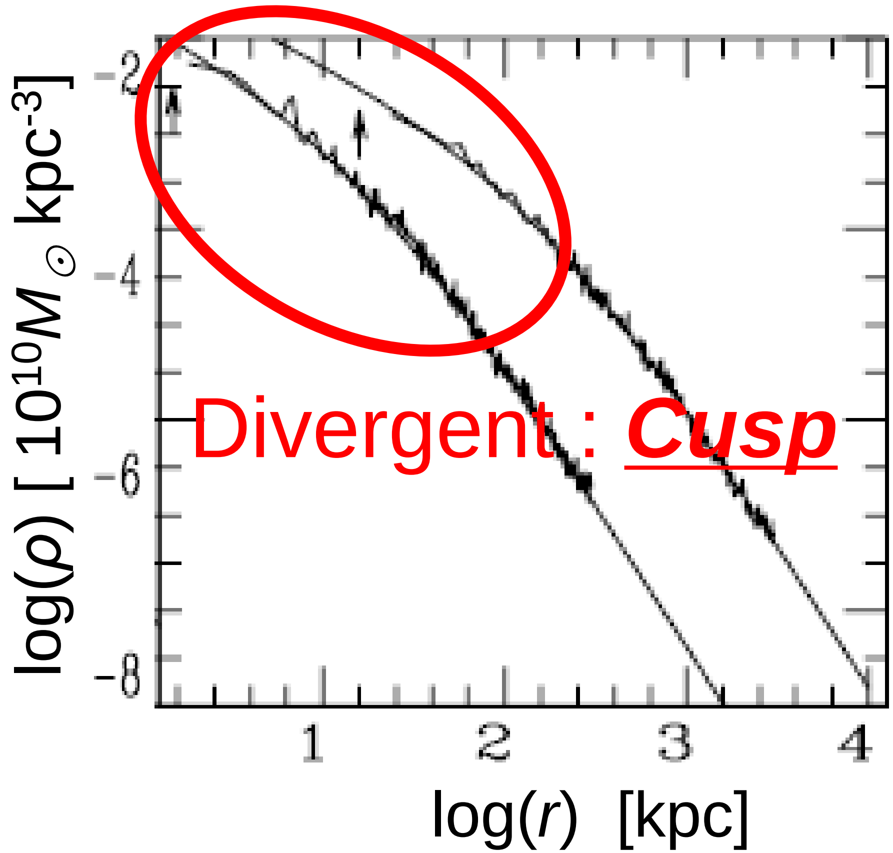
What is the Core-Cusp Problem?

Mass-density profile of DM halo

Observation vs **Theory (CDM)**

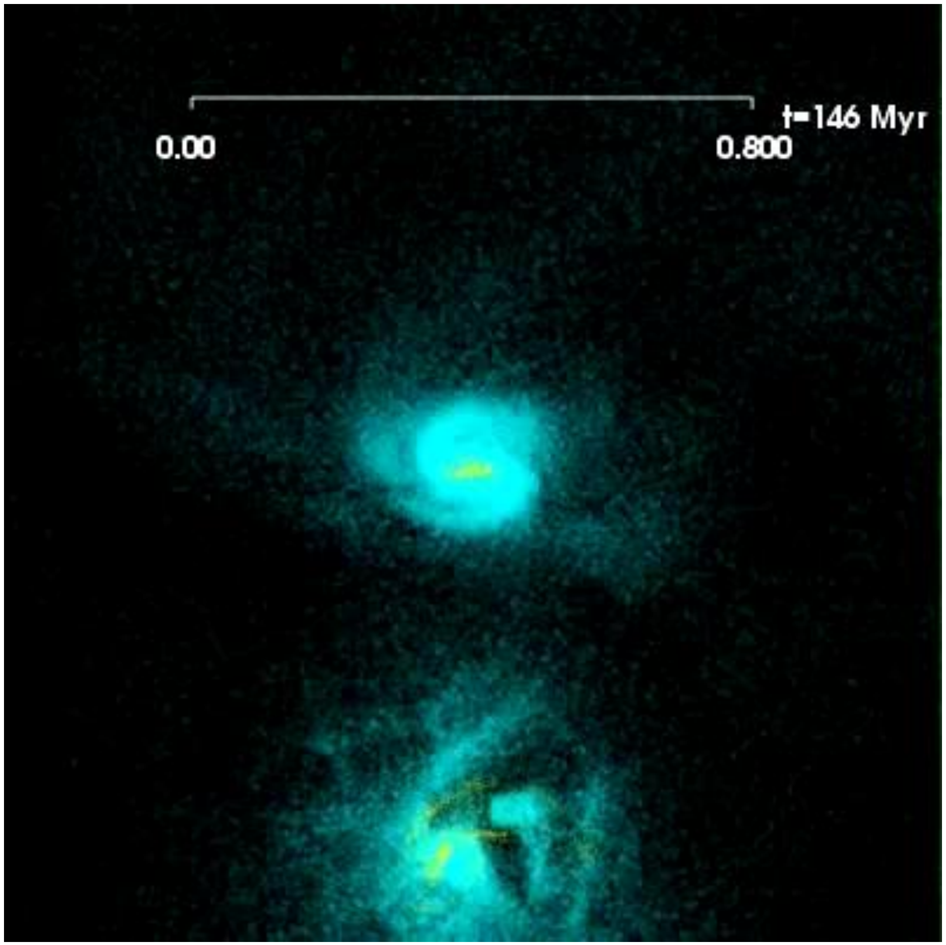


van Eymeren *et al.* (2009)



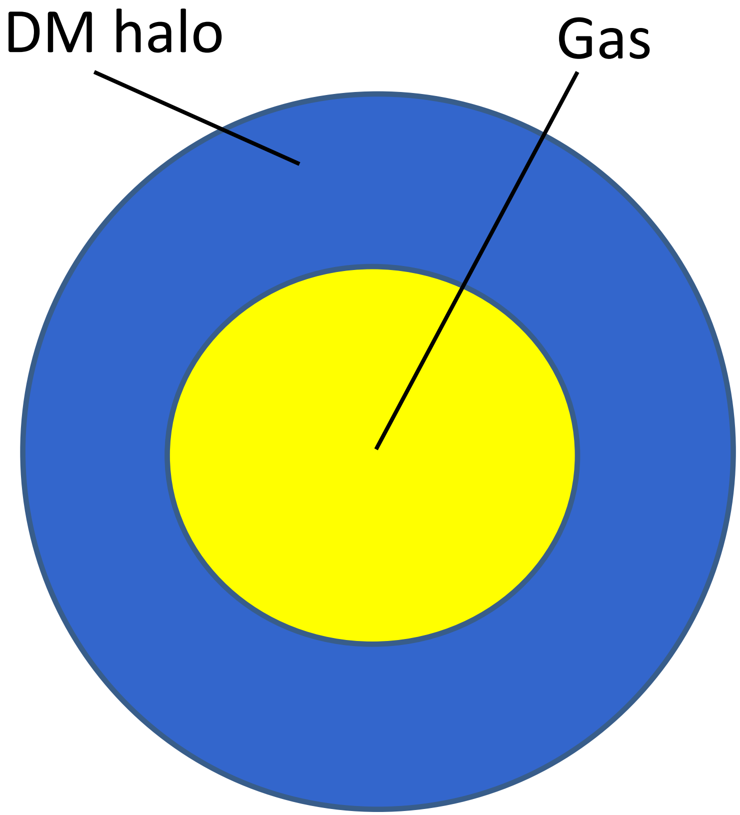
Navarro *et al.*(1997)

Realistic Simulation



- Mashchenko *et al*. (2008)
  - Cosmoligical *N*-body+SPH simulation
    - Supernova feedback etc.
    - Blue: gas, Yellow: star
  - Gas
    - Blown out (expansion)
    - Fall back towards center (contraction)
    - Repeat many times
    - Gas Oscillation

Idealized Model



1) Gas heating by supernovae
2) Gas expansion
3) Energy loss by radiative cooling
4) Contraction towards the center
5) Ignition of star formation again

Repetition of these processes

Gas Oscillation

Change of potential ⇒ DM halo is affected gravitationally

⇒ **Cusp to Core transition?**

Resonance Model

- Linear analysis of the resonance between the density wave and the particles: assumption and approximation
  - Perturb equilibrium system (0) by external force (ex) ⇒ induced values (ind)
  - Focus on the particle group with $\rho_0 = const.\ , v_0 = const.$
  - External force is oscillatory $-\dfrac{\partial \Phi_{\rm ex}}{\partial r} = \sum_n A_n \cos\left(kr - n\Omega t\right)$

  (A: strength, k: wavenumber, Ω: frequency)

- Linearized Euler eq.

$$\frac{\partial v_{\rm ind}(t,r)}{\partial t} + v_0 \frac{\partial v_{\rm ind}(t,r)}{\partial r} = -\frac{\partial \Phi_{\rm ex}(t,r)}{\partial r} \Rightarrow v_{\rm ind}(t,r) = -\sum_n \frac{A_n}{n\Omega - kv_0}\left\{\sin\left(kr - \Omega t\right) - \sin\left(kr - kv_0 t\right)\right\}$$

- Linearized eq. of continuity

$$\frac{\partial \rho_{\rm ind}(t,r)}{\partial t} + v_0 \frac{\partial \rho_{\rm ind}(t,r)}{\partial r} = -\rho_0 \frac{\partial v_{\rm ind}(t,r)}{\partial r} \Rightarrow \rho_{\rm ind}(t,r) = -\sum_n \frac{A_n \rho_0 k}{(n\Omega - kv_0)^2}$$

$$\times\left\{\sin\left(kr - n\Omega t\right) - \sin\left(kr - kv_0 t\right) + (n\Omega - kv_0)t\cos\left(kr - kv_0 t\right)\right\}$$

- When $n\Omega \sim kv_0$, coefficients diverge **Resonance**

Resonance Condition

- Resonance between particles and density waves

$$n\Omega \sim k v_0$$

- Some arithmetic calculations (n = 1)

$$t_{\mathrm{d}}(r) \sim T$$

$$t_{\mathrm{d}}(r) = \sqrt{\frac{3\pi}{32G\bar{\rho}(r)}}$$

Resonance occurs when the condition is satisfied

→ efficient energy transfer

→ system expands

→ density change dramatically

⇒ ***Cusp to Core transition***

Our model ⇔ ***Observations***

T : Period of gas oscillation ⇔ Star formation histories of galaxies

$t_d(\mathrm{r})$: Local dynamical time ⇔ Density of DM halos

Core Scale

- Density profile of CDM halo $$\rho(r) = \frac{\rho_0 R_{DM}^3}{r^\alpha (r + R_{DM})^{3-\alpha}}$$
- Mass profile $$M(\alpha; x) = \frac{4\pi\rho_0 R_{\mathrm{DM}}^3}{3-\alpha} x^{3-\alpha} {}_2F_1[3-\alpha, 3-\alpha, 4-\alpha; -x]$$

↑ Gauss's hyper-geometric function

$$x \equiv r/R_{\mathrm{DM}}$$

- Resonant condition

$$t_{\mathrm{d}}(r) \sim T$$

Dynamical time

$$t_{\mathrm{d}}(r) = \sqrt{\frac{3\pi}{32G\bar{\rho}(r)}}$$

$$r_{\mathrm{core}} = R_{\mathrm{DM}}(T/T_{\mathrm{c}})^{2/\alpha}$$

$$T_{\mathrm{c}}^2 \equiv \frac{\pi^2}{8G} \frac{R_{\mathrm{DM}}^3 c^{3-\alpha} {}_2F_1[\alpha; -c]}{M_{\mathrm{vir}}}$$

- Virial mass M_{vir}
- Concentration c
- **Oscillation period T**

↓

We can predict the core scale created by resonance!!

Numerical Model

DM halo (*N*-body system)**:**
NFW model (Navarro *et al.* 1997)

$$\rho(r) = \frac{\rho_0 R_{\rm DM}^3}{r(r+R_{\rm DM}^2)}$$

Baryon (external potential)**:**
Hernquist potential (Hernquist 1990)

$$\Phi_b(r,t) = -\frac{GM_b}{r+R_b(t)}$$

$$R_{\rm b}(t) \propto \cos(2\pi t/T)$$

Property of DM halo

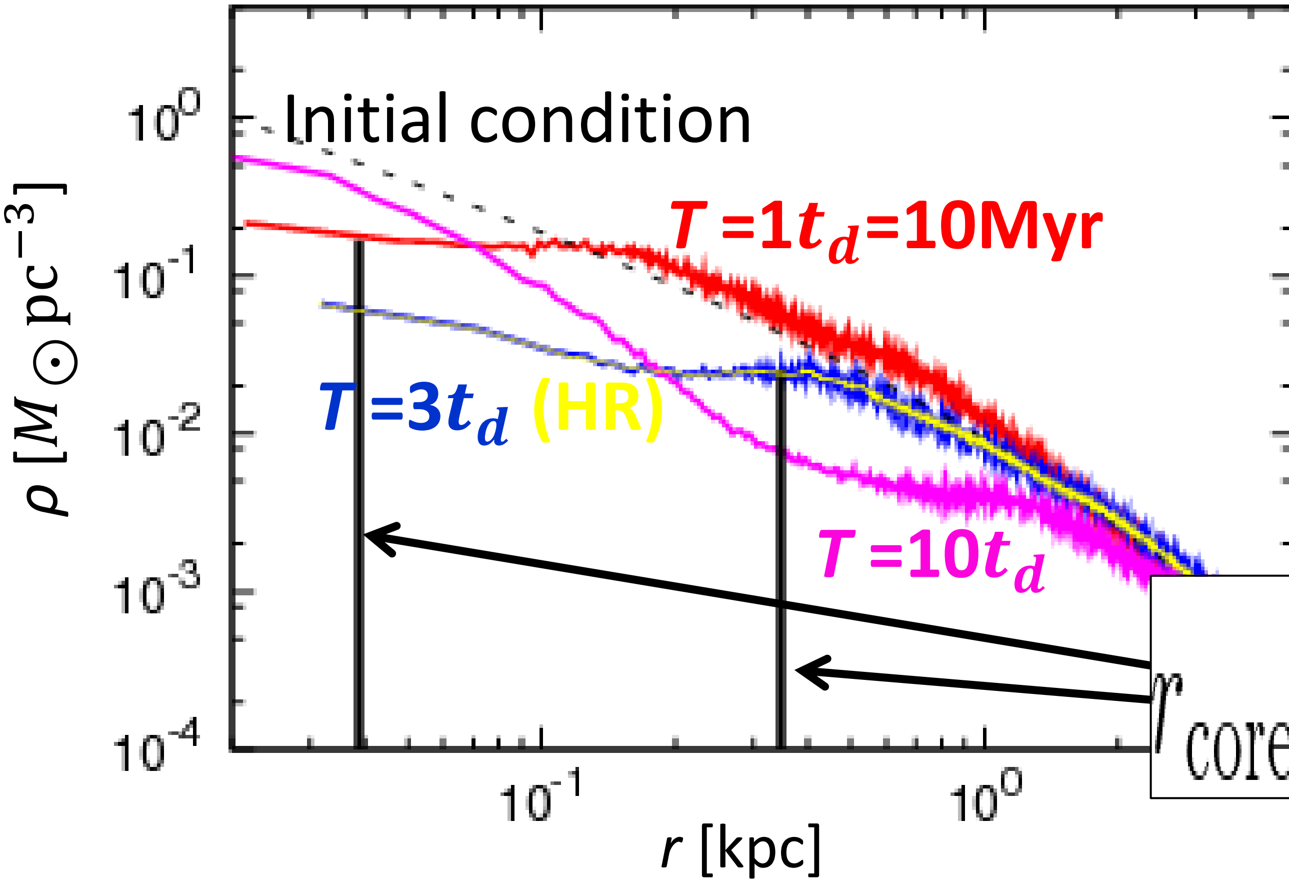
Number of particles N	16M, 128M
Softening parameter ε	0.004kpc
Virial mass of a DM halo $M_{\rm vir}$	$10^9 M_\odot$
Virial radius of a DM halo $R_{\rm vir}$	10kpc
Scale radius of a DM halo $R_{\rm DM}$	2kpc
Total baryon mass $M_{\rm b,tot}$	$1.7\times10^8 M_\odot$

Oscillation period of the external force, $\boldsymbol{T}$

$\boldsymbol{T}$ = 1, 3, 10 $t_{\rm d}$(0.2kpc)

Results1 -Density Profile-

The cusp-core transition and the resultant core scale depends on the oscillation period of the external potential, T.

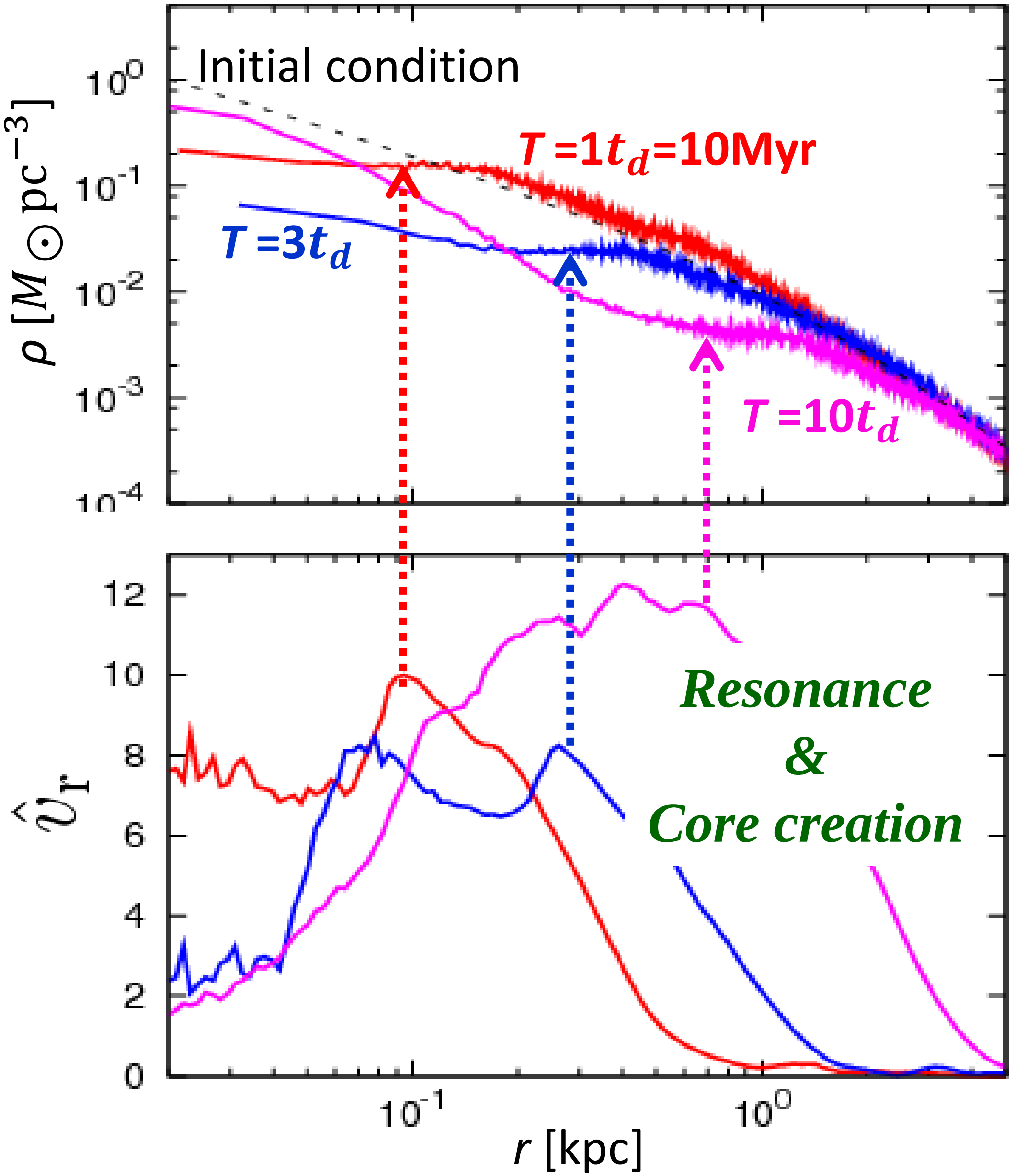


Prediction of Core Scale

- Virial mass M_{vir}
- Concentration c
- Oscillation period T

$$r_{\mathrm{core}} = R_{\mathrm{DM}}\left(T/T_{\mathrm{c}}\right)^{2/\alpha}$$

Result2 -Fourier Spectrum of Velocity-



Density profiles of DM halos after several oscillation periods

Fourier spectrum of radial velocity

$$v_\mathrm{r}(t, r) \rightarrow \hat{v}_\mathrm{r}(\omega, r)$$

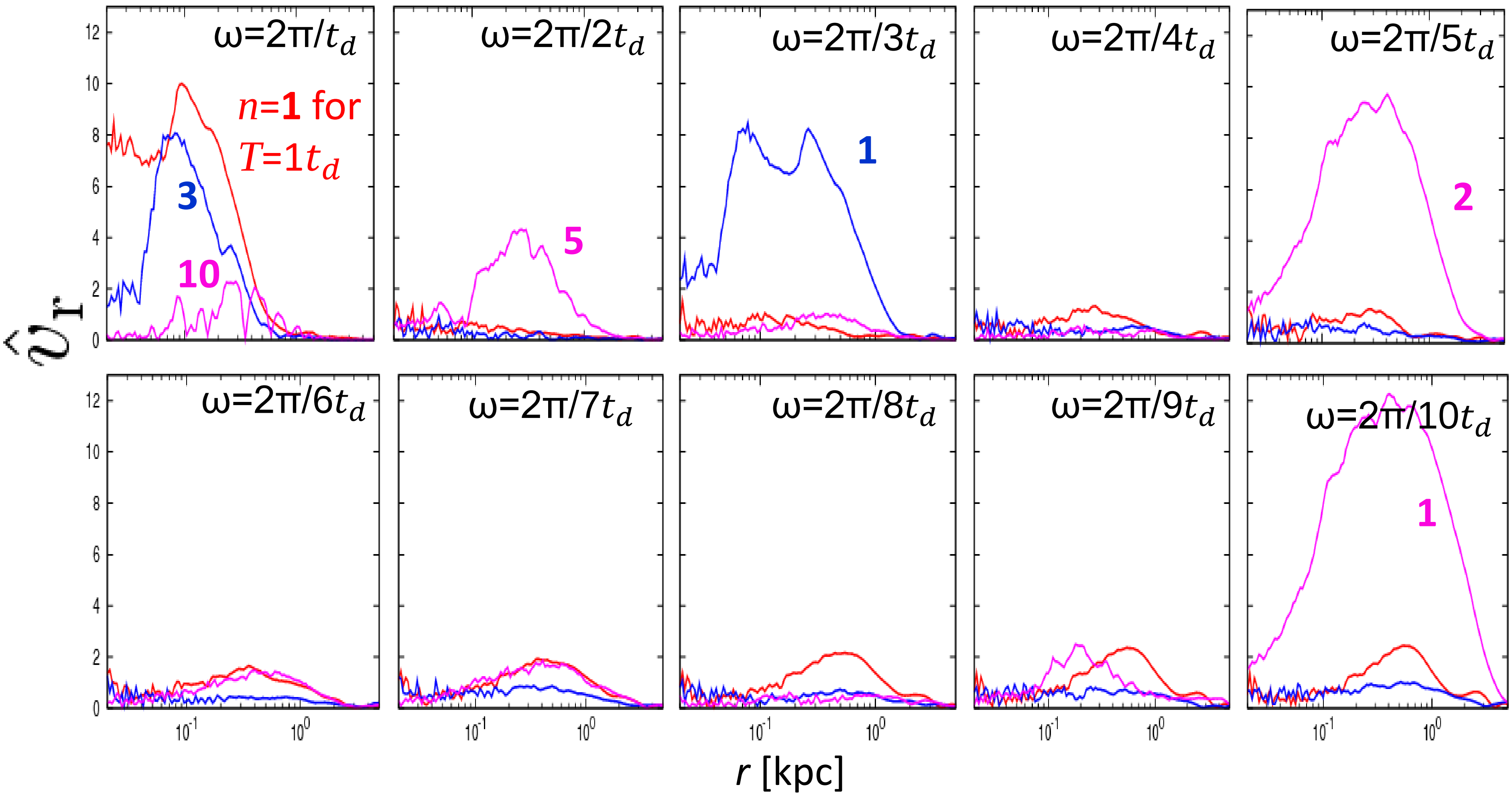
Peaks appear when $\omega = 2\pi/T$.

Each position of the peaks matches the core scale.

Result3 -Overtones-

$$v_{\mathrm{ind}}(t,r) = -\sum_{n} \frac{A_n}{n\Omega - kv_0}\left\{\sin\left(kr - \Omega t\right) - \sin\left(kr - kv_0 t\right)\right\}$$

$\omega = n\Omega$ → Spectrum with peak (Resonance)



Summary

- Study the dynamical response of a DM halo
  - The Core-Cusp problem
  - Oscillatory change of gravitational potential
- Analytical Model
  - Resonance between particles and density waves
  - Resonant condition $t_{\rm d}(r) \sim T$

    dynamical time oscillation period
- *N*-body simulations
  - Resonance plays a significant role to flatten central cusp
    - Resonance → Efficient energy transfer
    - → Cusp to Core transition
  - Core scale is well matched to our predictions