# Vlasov-Poisson simulations of collisionless self-gravitating systems


Kohji Yoshikawa / 吉川 耕司

Center for Computational Sciences, University of Tsukuba

collaborators: Naoki Yoshida (Dept. of Phys., Univ. of Tokyo)

Masayuki Umemura (CCS, Univ. of Tsukuba)


"Direct Integration of the Collisionless Boltzmann Equation in Six-dimensional Phase Space: Self-gravitating Systems" (arXiv:1206.6152)
KY, Naoki Yoshida, Masayuki Umemura, submitted to ApJ

東京大学 大学院 理学系研究科・理学部

東京大学 THE UNIVERSITY OF TOKYO

筑波大学 計算科学研究センター Center for Computational Sciences

筑波大学 University of Tsukuba

# Numerical Simulation of Collisionless Self-Gravitating Systems

- N-body simulations
  - a “de facto standard” method to simulate the nonlinear evolution of self-gravitating systems for more than 30 years.
  - the mass distribution is sampled by particles in the 6D phase-space volume in a Monte-Carlo manner
  - very large number of particles can be treated with the aid of sophisticated Poisson solvers such as Tree and TreePM methods.
- Self-Consistent Field (SCF) method
  - particles are followed under the gravitational potential field obtained from the expansion series of the particles' density field.
  - applied only to some specific cases, such as secular evolution of nearly equilibrium systems or a collapse of spherical systems.

# Potential Drawbacks of N-body Simulations

- intrinsic contamination of shot noise in physical quantities

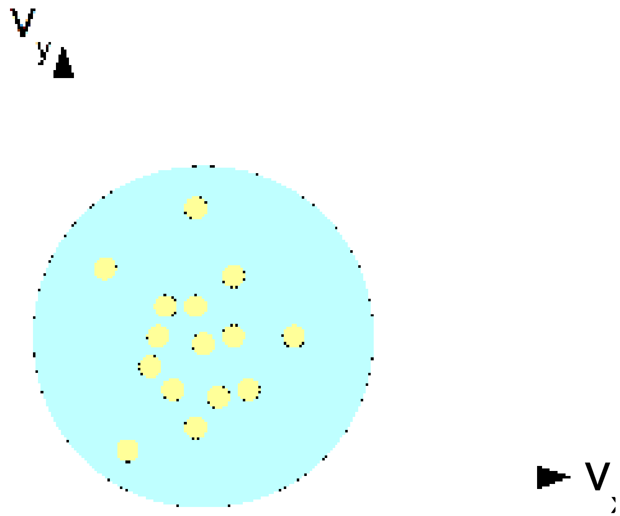
  shot noise term is only proportional to $N^{-1/2}$

- artificial two-body relaxation due to the super-particle approximation

  ⇒ introduces undesired collisional effect in a long-term evolution

- velocity space is rather sparsely sampled.

  ⇒ physical processes sensitive to the velocity structure such as the collisionless damping and the two-stream instability are not properly solved.



# Vlasov-Poisson Simulations

Vlasov-Poisson equations

$$\begin{cases} \dfrac{\partial f}{\partial t} + \vec{v} \cdot \dfrac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \dfrac{\partial f}{\partial \vec{v}} = 0 \\ \nabla^2 \phi = 4\pi G \rho = 4\pi G \displaystyle\int f d^3 \vec{v} \end{cases}$$

- an alternative way to solve the dynamics of collisionless self-gravitating systems
- treats the matter as continuum fluid in the phase space instead of sampling it by particles

  ⇒ free from shot noise contamination seen in the N-body approach

- so far limited to 1D or 2D simulations due to the huge amount of required memory space and huge computational costs.

We present the first 3D Vlasov-Poisson simulation in the 6D phase space volume.

# Numerical Methods

- Both of physical and velocity spaces are discretized with 3D regular mesh grids.

- Collisionless Boltzmann equation is solved using directional splitting scheme, in which following six 1D advection equations are sequentially integrated.

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} = 0 \qquad \frac{\partial f}{\partial t} - \frac{\partial \phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0 \quad (i = 1, 2, 3)$$

- Physical requirements for the scheme of 1D advection equations
  - positivity
  - mass conservation → Positive Flux Conservation (PFC) scheme
  - maximum principle

Filbet, Sonnendrucker, Bertrand, J. Comp. Phys. (2001) 172, 166-187

# Numerical Methods

- Poisson equation
  - Solved with the convolution method using the Fourier transform
  - Both for the periodic and isolated boundary condition

- Time integration

$$f(\vec{x},\vec{v},t^{n+1}) = T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2)$$

$$T_x(\Delta t)T_y(\Delta t)T_z(\Delta t)$$

$$T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2) \quad f(\vec{x},\vec{v},t^{n})$$

# Test Suite

- Stability of a stable solution of Vlasov-Poisson equations
- Merging of two self-gravitating systems

  + comparison with N-body methods
- Gravitational instability and collisionless damping of density fluctuations

More numerical tests are presented in our preprint.

# King Sphere

- **Initial condition**:

$$f_{\rm K}(\mathcal{E}) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}}\left[\exp(\mathcal{E}/\sigma^2) - 1\right] \qquad \mathcal{E} > 0$$

$$= 0 \qquad \mathcal{E} < 0$$

$$\mathcal{E} = \Psi - \frac{v^2}{2} \qquad \Psi(0)/\sigma^2 = 3$$

- **number of grids**:

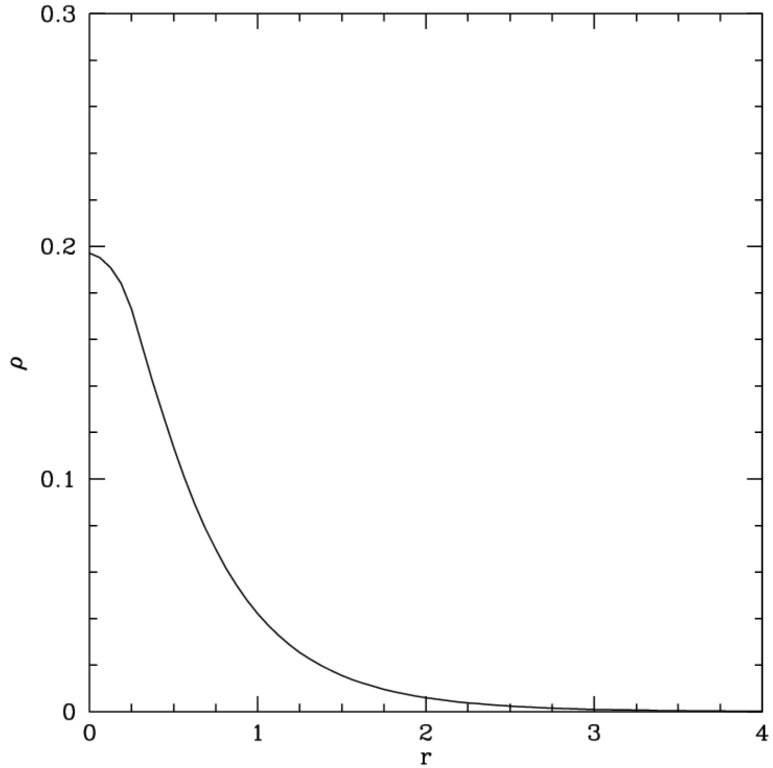
$64^3$ for physical space

$64^3$ or $32^3$ for momentum space

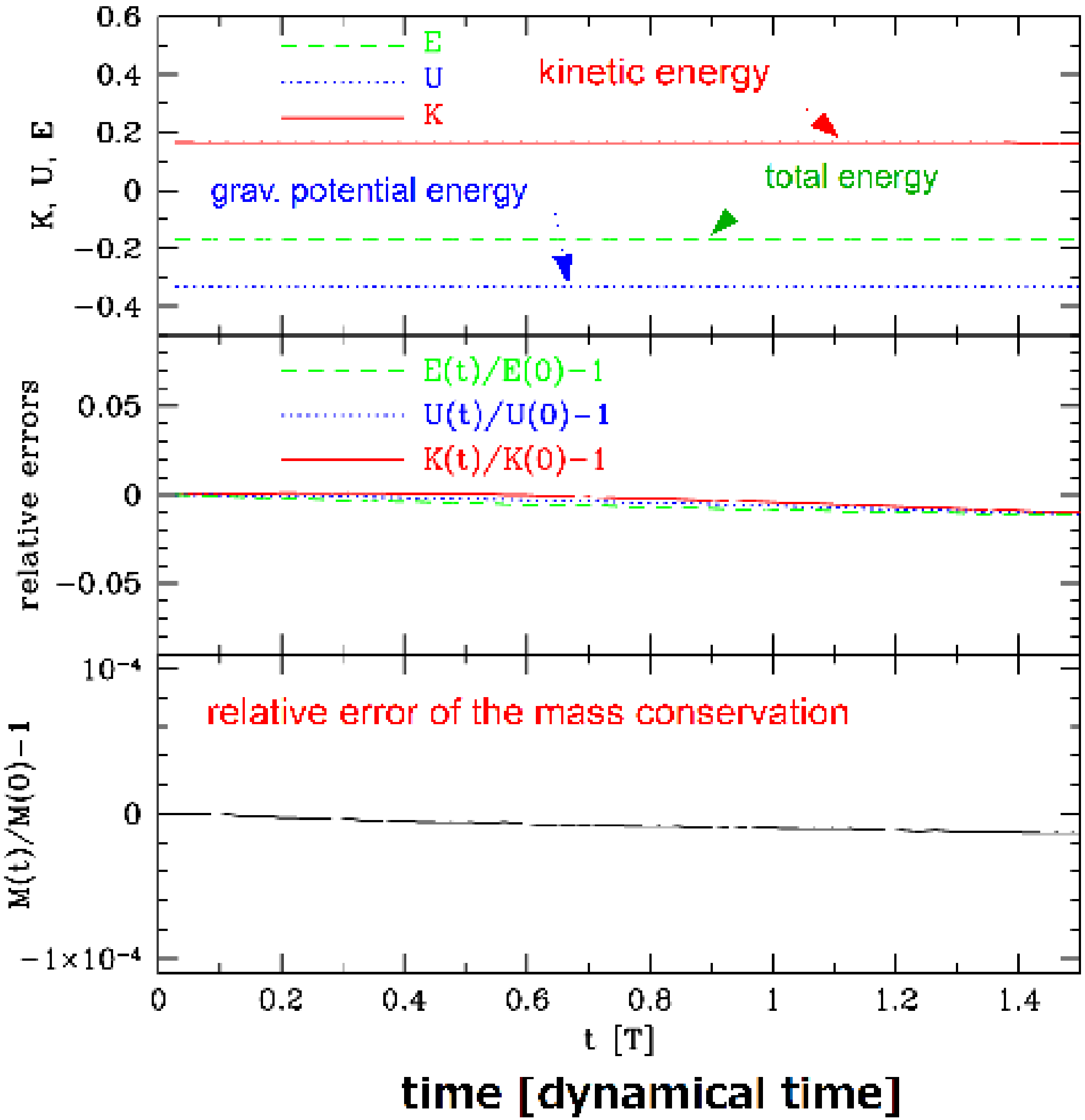
A stable solution of the Vlasov-Poisson eqs.

↓

Basic test for our Vlasov-Poisson solver

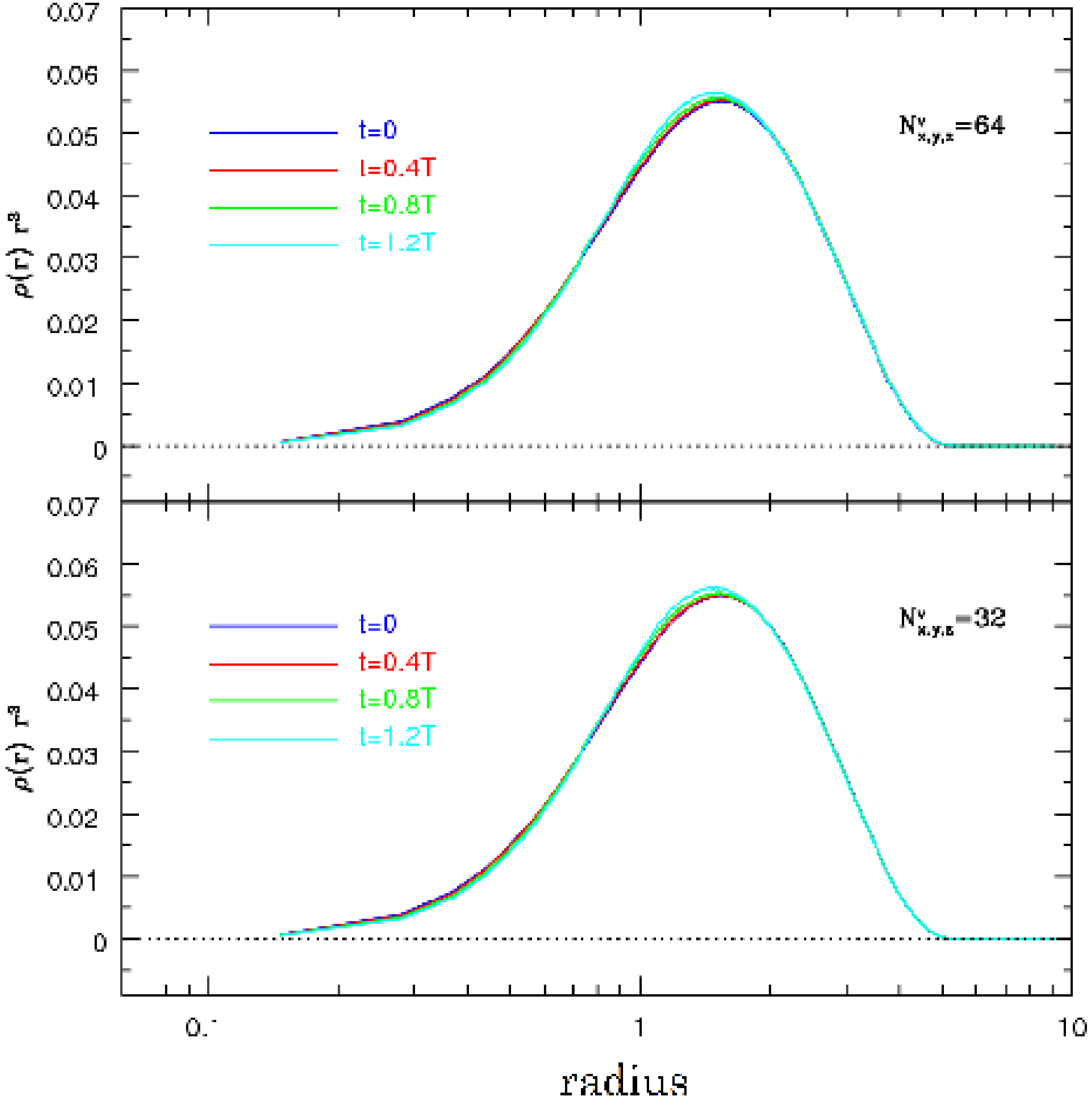


# King Sphere




- kinetic and grav. potential energies are almost constant over one dynamical timescale.
- time variation of the total energy is not larger than 1%.
- total mass is also well conserved with sufficiently good accuracy.

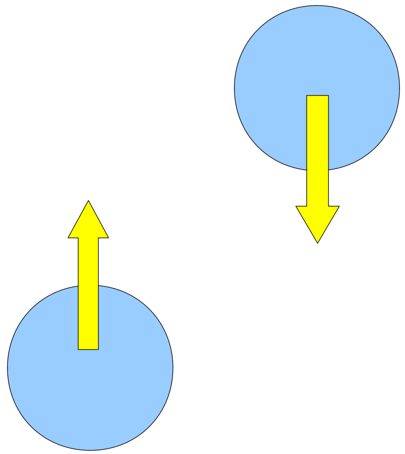
# King Sphere




- time evolution of the mass distribution
- the profiles are almost keep still.
- a slight mass transfer from center to outskirts, probably due to poor spatial resolution in the central region.
- no significant difference between different velocity resolutions.

# Merging of Two King Spheres

- initial condition

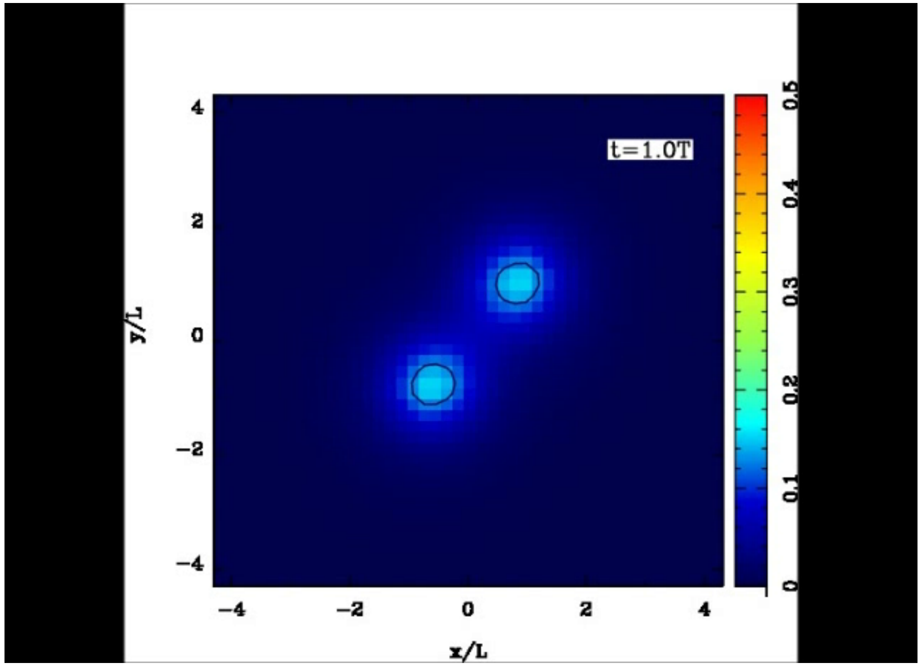


- offset merging of two King spheres
- $64^3$ mesh points for both the physical and velocity spaces.

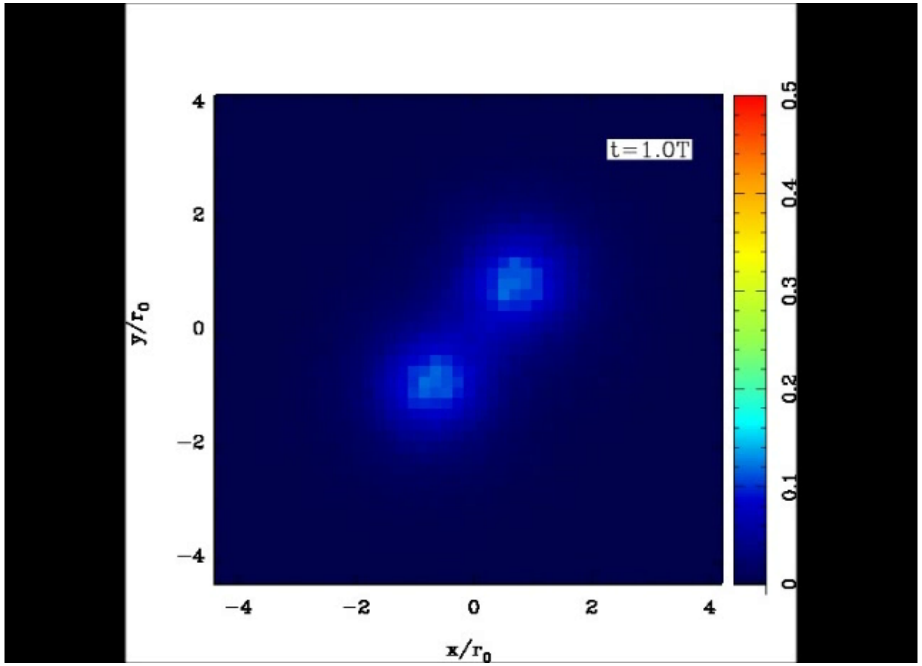
- N-body simulation for the comparison
  - each King sphere is represented with a million particles
  - simulated using the Particle – Mesh method with the same spatial resolution as the Vlasov – Poisson simulation.

# Merging of Two King Spheres

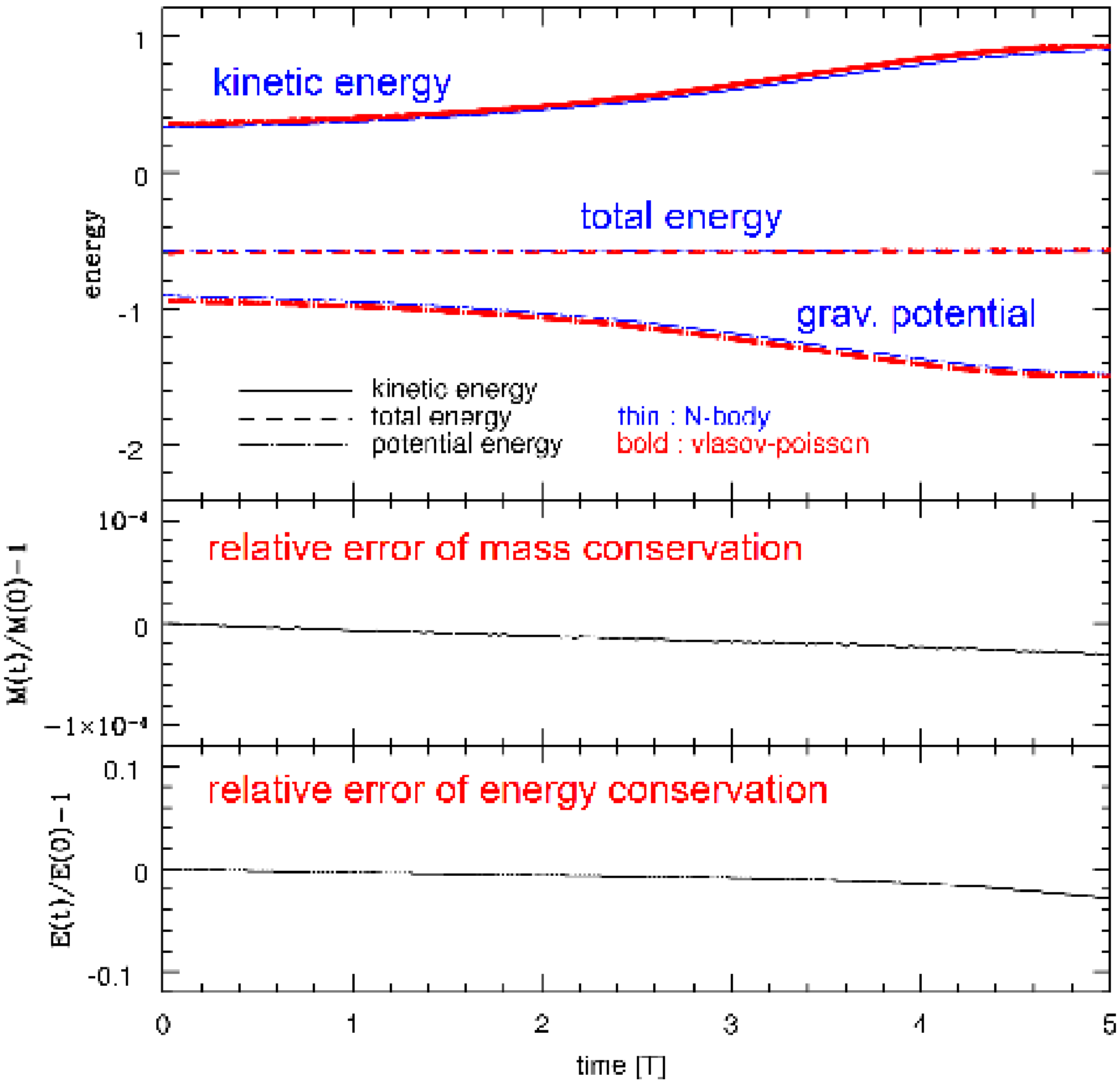
Vlasov – Poisson simulation




N-body simulation



# Merging of Two King Spheres



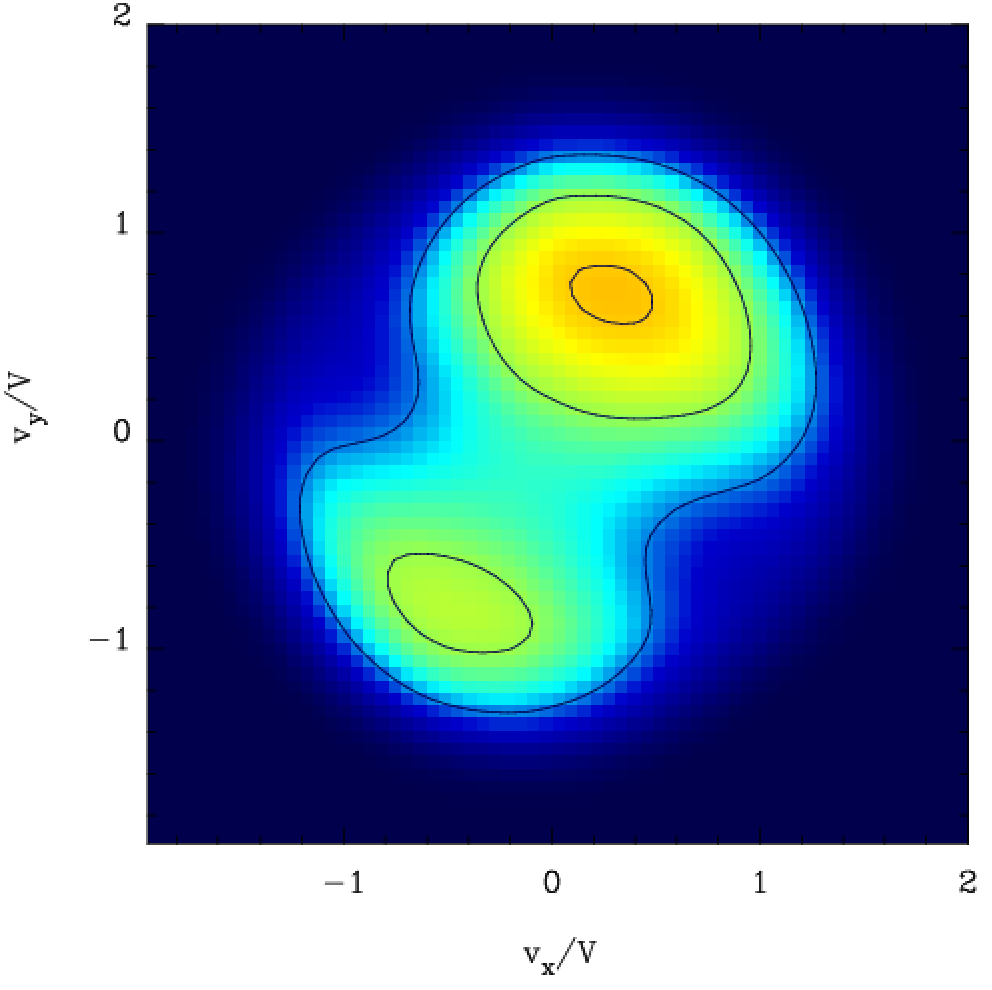

time [dynamical time]

- time evolution of the kinetic, grav. potential and total energy in Vlasov – Poisson and N-body simulations
- good agreement between Vlasov – Poisson and N-body simulations.
- energy conservation is assured within 1% error at t<**4**.5T.

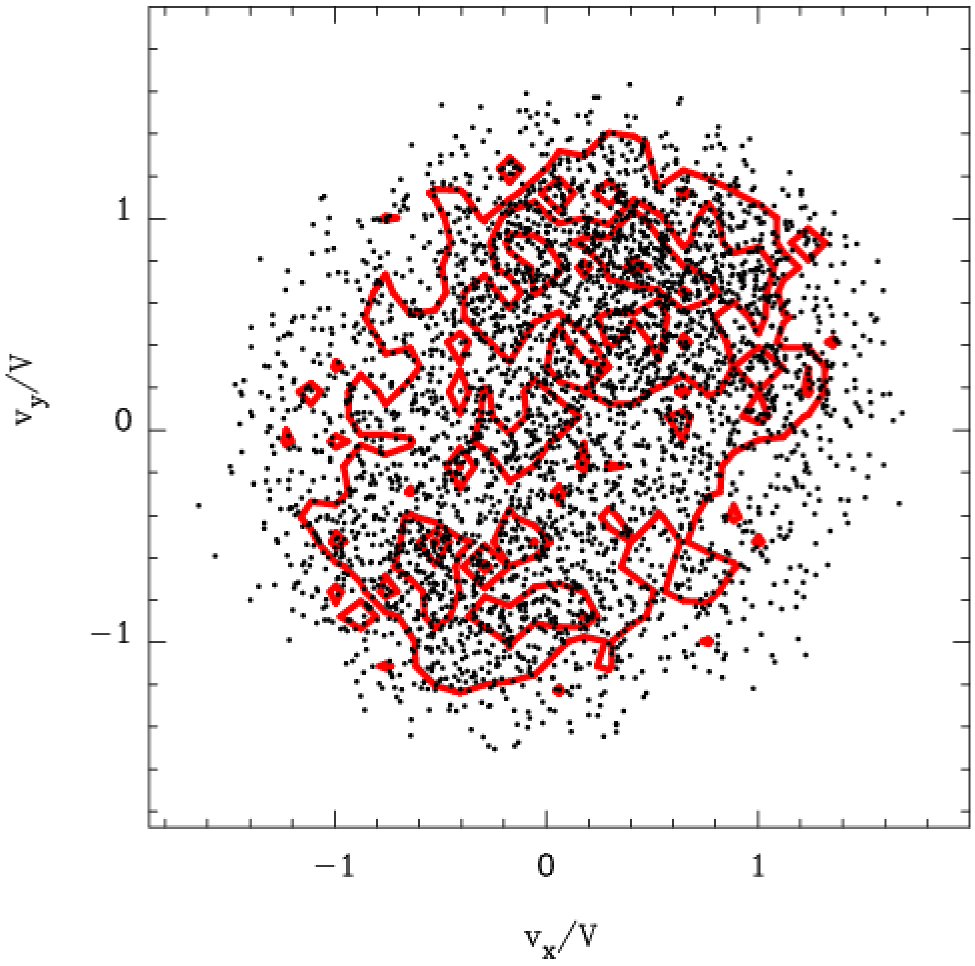
# Velocity Distribution

- phase space density in the central region at a time of the closest approach.

Vlasov – Poisson simulation



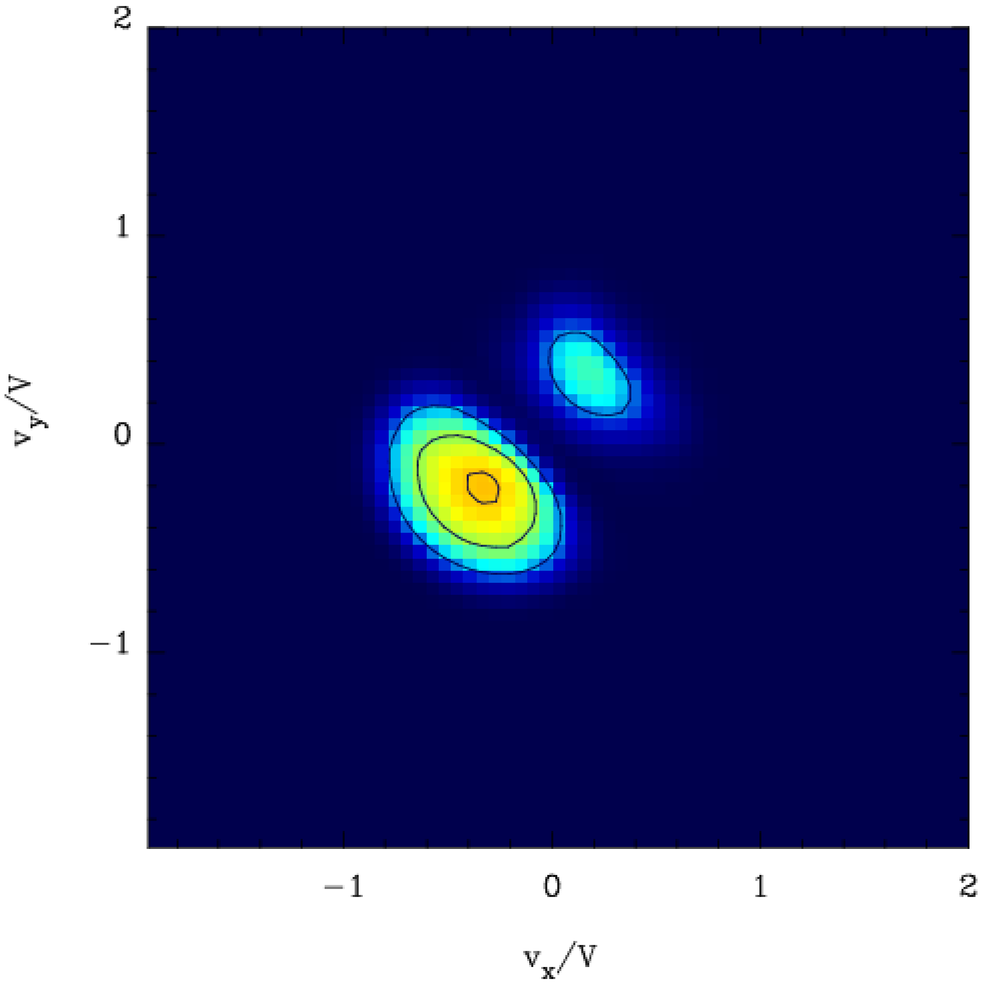

N-body simulation



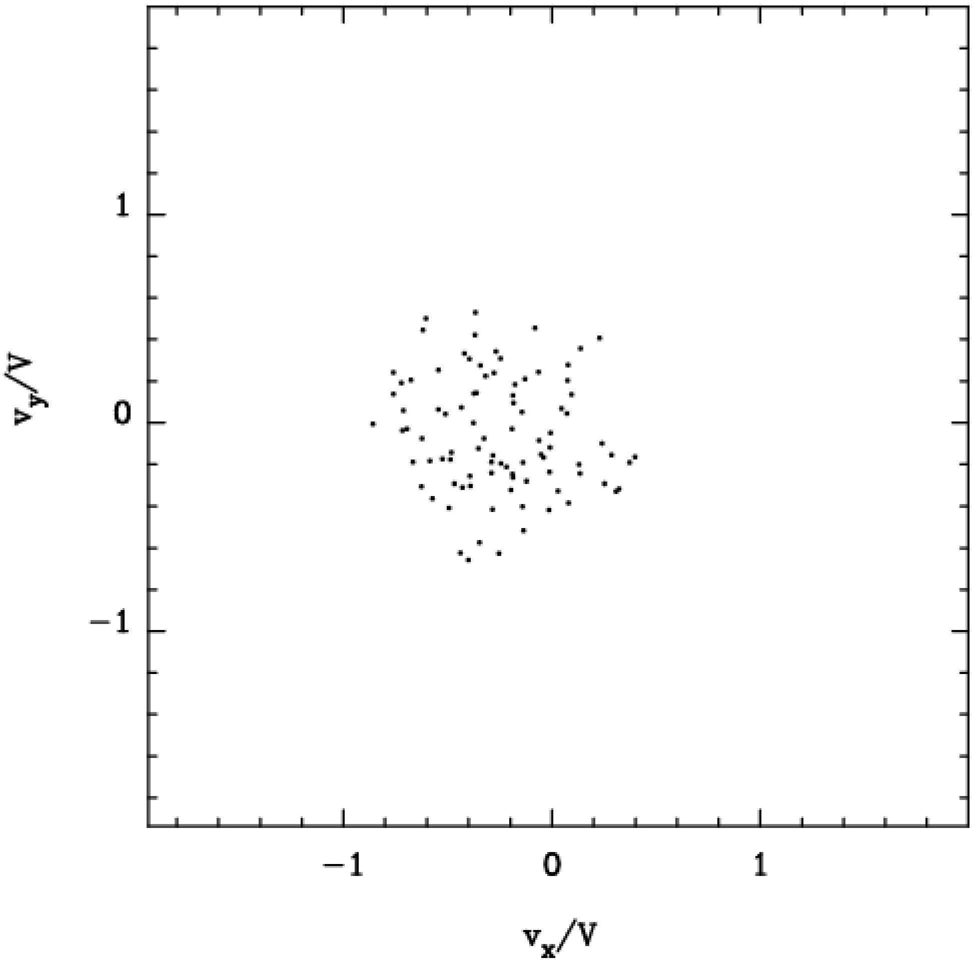
# Velocity Distribution

- phase space density in the outskirts at a time of the closest approach.

Vlasov – Poisson simulation




N-body simulation



# 3D Gravitational Instability and Collisionless Damping

**Initial condition**

$$
\begin{cases}
f(\vec{x}, \vec{v}, t=0) = \dfrac{\bar{\rho}(1+\delta(x))}{(2\pi\sigma^2)^{3/2}} \exp\left(-\dfrac{|\vec{v}|}{2\sigma^2}\right) \\
\rho(x, t=0) = \bar{\rho}(1+\delta(x))
\end{cases}
$$

- The density fluctuation δ(x) is given so that it has a white noise power spectrum.
- periodic boundary condition
- Jeans wave number

$$k_{\mathrm{J}} = \frac{\sqrt{4\pi G\bar{\rho}}}{\sigma}$$

$k > k_{\mathrm{J}}$ → collisionless damping

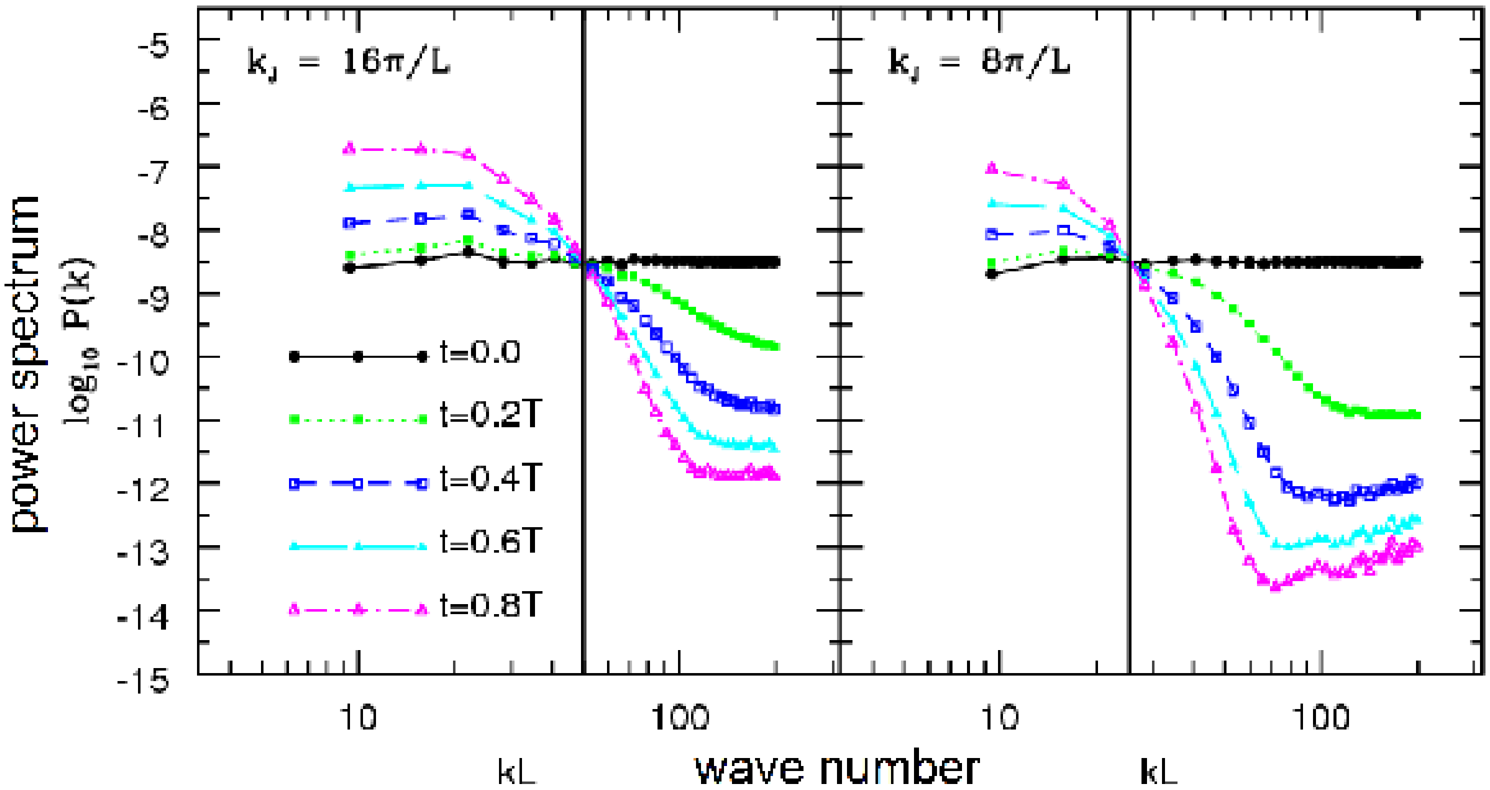
$k < k_{\mathrm{J}}$ → gravitational instability

- number of mesh points

  $64^3$ for the physical space

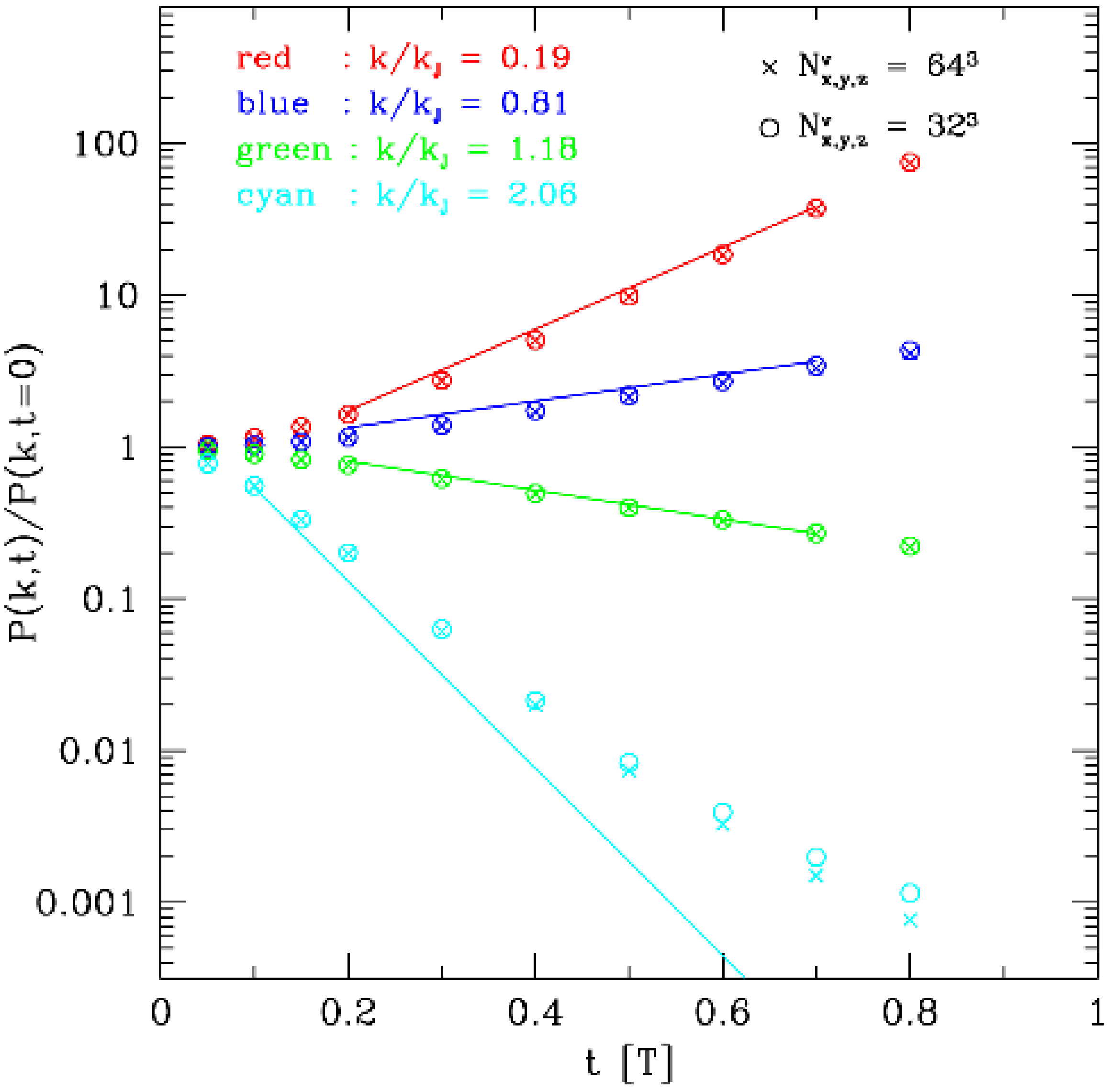
  $64^3$ or $32^3$ for the velocity space

# Power Spectra




- Growth and damping of the density fluctuations switch each other clearly at the Jeans wave number.

# 3-D Self-Gravitating System




- Growth / damping rates of fluctuations
- Symbols : results for different velocity resolutions
- Lines: theoretical predictions of growth and damping rates based on the linear perturbation theory
- overshoot of the power spectra for k/k_J=2 is due to the nonlinear effect.

# Advantage and Disadvantage

- The resolution in the velocity space is significantly better than that of N-body methods

  ⇒ Physical processes sensitive to the velocity distribution such as collisionless damping and two-stream instability can be simulated accurately.

- It is free from the shot noise contamination and artificial two–body effect.

  ⇒ suitable to follow the long-term evolution of the self-gravitating systems

- Current spatial resolution of the Vlasov – Poisson simulation is rather poor compared with the conventional N-body simulations, due to the required large amount of memory.

  ⇒ needs for hierarchical or adaptive mesh structure

- In simulating the dynamical problem, we need to determine the boundary of the simulation regions beforehand the simulations.