Following Dark Matter into Phase Space

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Outline:

- *The dark matter sheet*
- *Vlasov Poisson system*
- *Analysis of N-body simulations*
- *Understanding artificial clumping*
- *Better density -> better potential*
- *New simulation techniques*
- *Summary*

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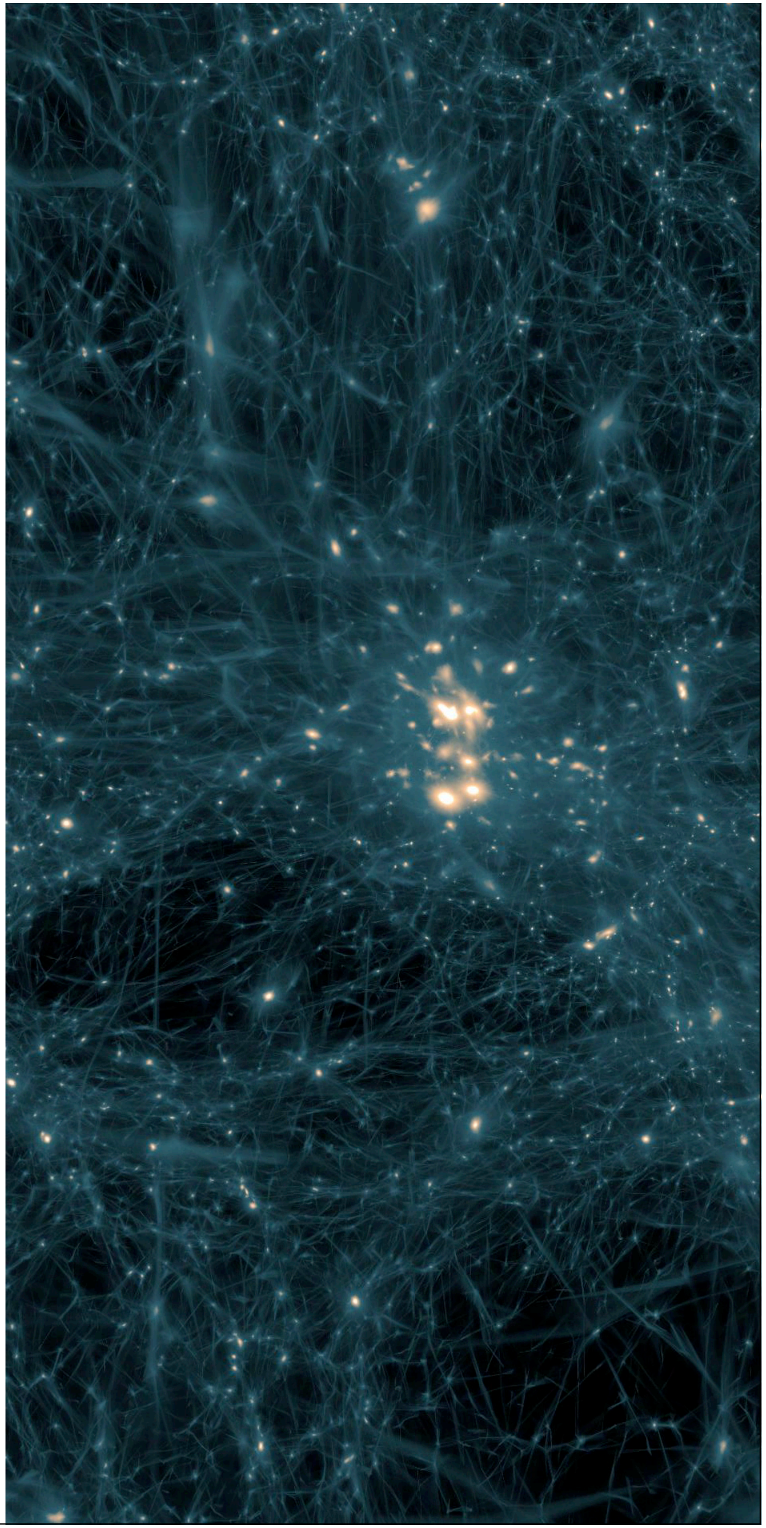
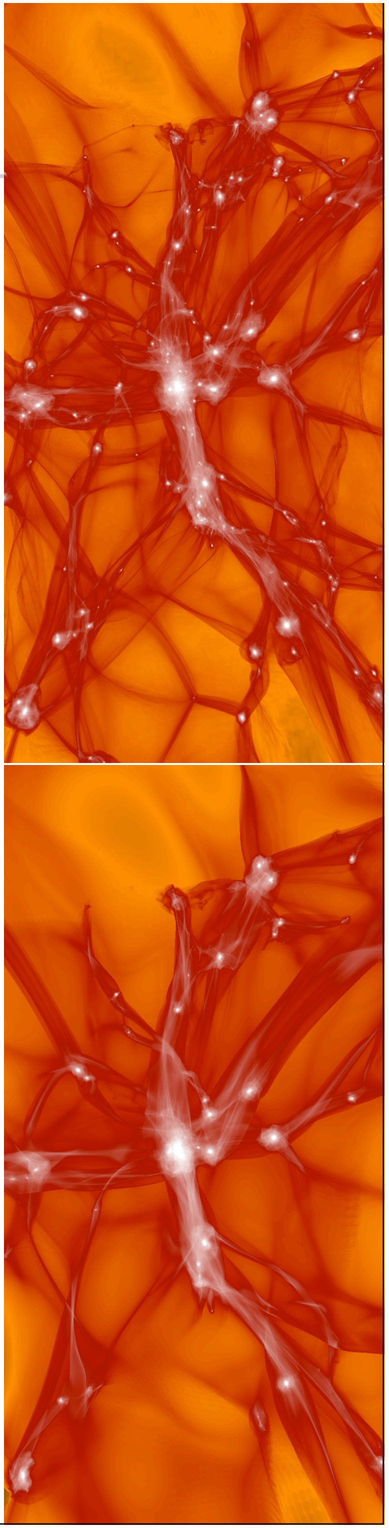
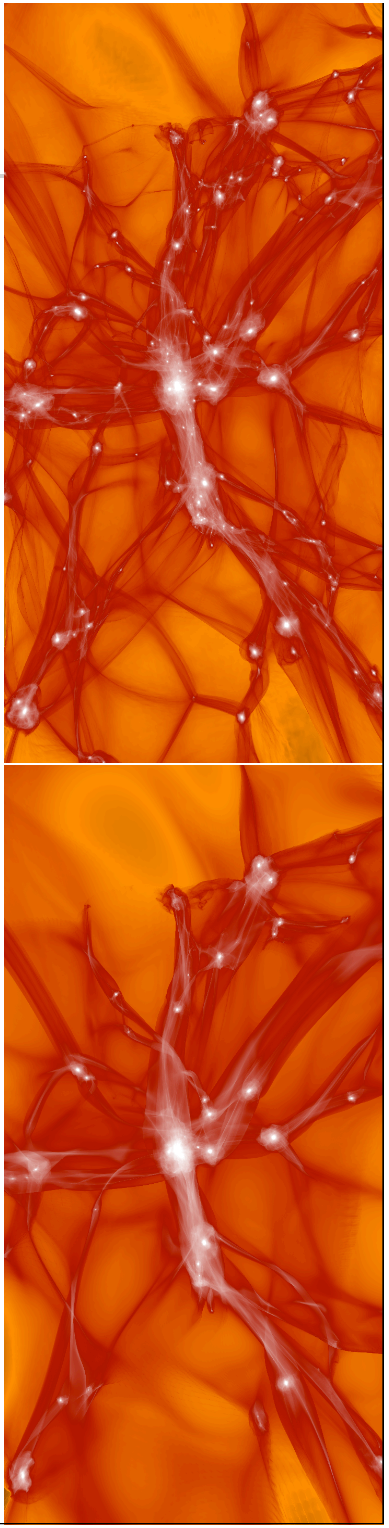


Cosmological N-body simulations

- Used to make predictions about the distribution of dark matter in the Universe
- Key results
    - Galaxies are arranged in cosmic web of voids/sheets/filaments/halos
    - Universal spherical Dark Matter density profile (NFW) [not understood from analytical arguments]
- Primary tool to study possible observational consequences of
    - initial conditions: warm vs cold DM, Gaussian vs non-Gaussian
    - sensitivity on global cosmological parameters such as the total matter content and amount of dark energy, etc.



Cosmological N-body simulations



$$\dot{\mathbf{x}} = \mathbf{v}(t) \qquad \dot{\mathbf{v}}_{\mathbf{i}} = -\sum_{i \neq j}^{N} G m_i m_j \frac{(\mathbf{x_j} - \mathbf{x_i})}{|\mathbf{x_j} - \mathbf{x_i}|^3}$$

- All modern cosmological simulation codes just differ in how they evaluate the sum over all particles to obtain the net force
- End result are simply the positions and velocities of all particles
- Softening of forces (add epsilon^2 in denominator) avoids singularities.
- Limit N goes to infinity must give correct answer

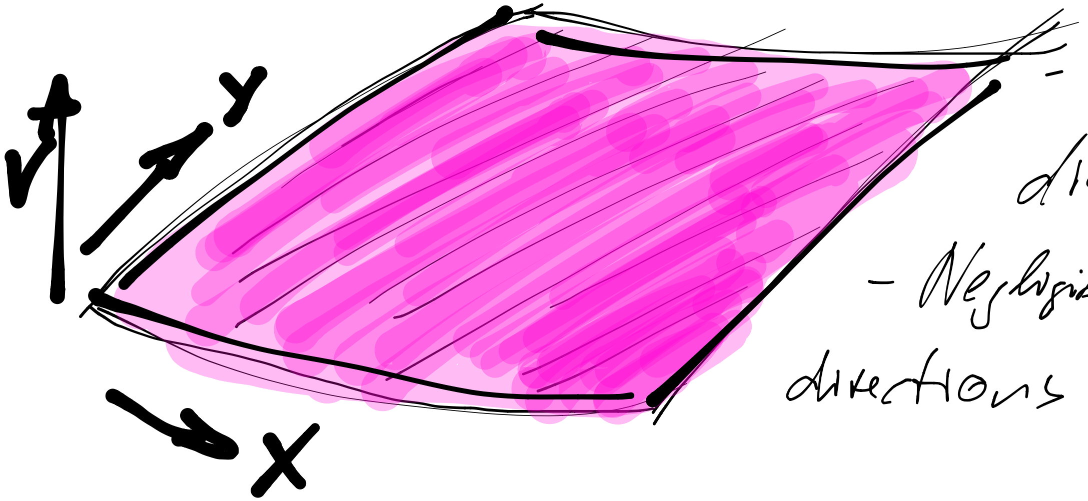
The Dark Matter Sheet?

COOL WIMPS

Dark Matter is commonly hypothesized to originate within seconds after the BIG BANG. If it were moving relativistically today, galaxies and other structures would not exist. We speak of **COLD DARK MATTER**.

Working HYPOTHESIS:

- Weakly interacting massive particle (say ≈ 100 GeV).
- Very cold. Even keV particles would only have ~ $\frac{m}{s}$ speeds today.



- Almost perfectly uniformly distributed initially.
- Negligible extent along velocity directions in phase space.

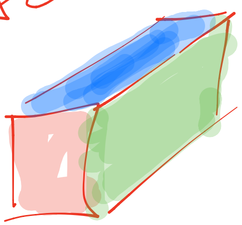
The Dark Matter Sheet ?

Phase space volume is conserved

$$\Delta V \cdot (\Delta v)^3 = \text{const}$$

spatial volume volume in velocity space

3D MANIFOLD MOVING IN SIX DIMENSIONAL PHASE SPACE



redshift when $E \approx 100\,\text{GeV}$: $1\,\text{eV}$ @ $z \sim 1000$

$100\,\text{GeV}$ @ $z \sim 10^{14}$

Matter density dropped by a factor $\sim 10^{42}$ since then.

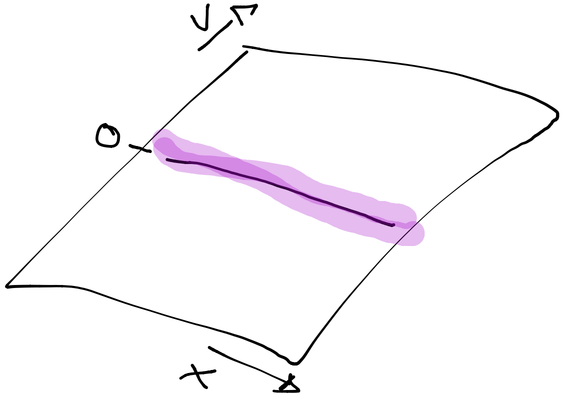
→ YES. VERY COLD.

Tiny initial peculiar velocities

⇒ distribution function $f(\vec{x},\vec{v}) = f_0\,\delta(\vec{v})$

is single valued at every $\vec{x}$

DIRAC DELTA



The Dark Matter Sheet ?

Fluid

\# OF DARK MATTER PARTICLES IN THE MILKY WAY :

$$N_{DM} = 10^{67}\left(\frac{100\,GeV}{m_{DM}}\right) \gg \text{\# OF STARS IN THE UNIVERSE}$$

$$\gg \text{\# OF PARTICLES THAT FIT ON A COMPUTER}$$

USING ALL THE COMPUTERS IN THE WORLD : $\lesssim 10^{17}$ particles

SOLVE VLASOV-POISSON SYSTEM INSTEAD.

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla_x f + \vec{a}\cdot\vec{\nabla}_v f = 0$$

$$\vec{a} = \vec{\nabla}\phi$$

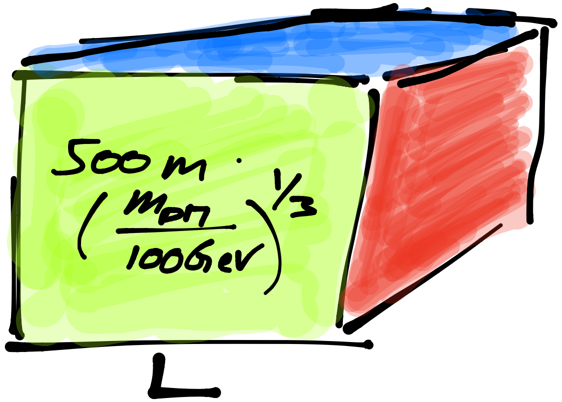
$$\nabla^2\phi = 4\pi G\rho$$

f: distribution function IN PHASE SPACE

ϕ: potential

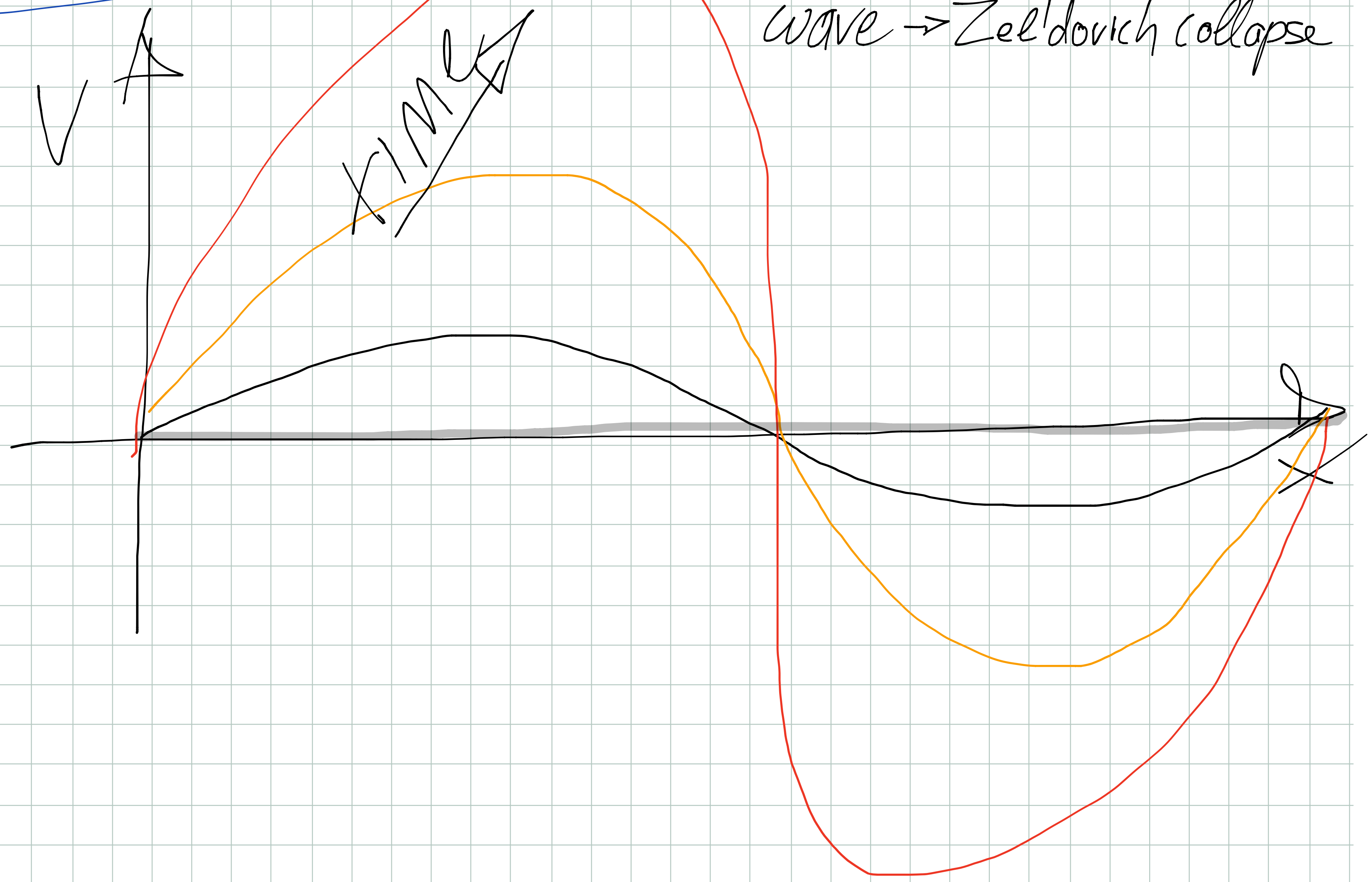
FOR PHASE SPACE ELEMENT TO CONTAIN 10^6 PARTICLES @ MEAN DENSITY IT HAS TO BE LARGER THAN

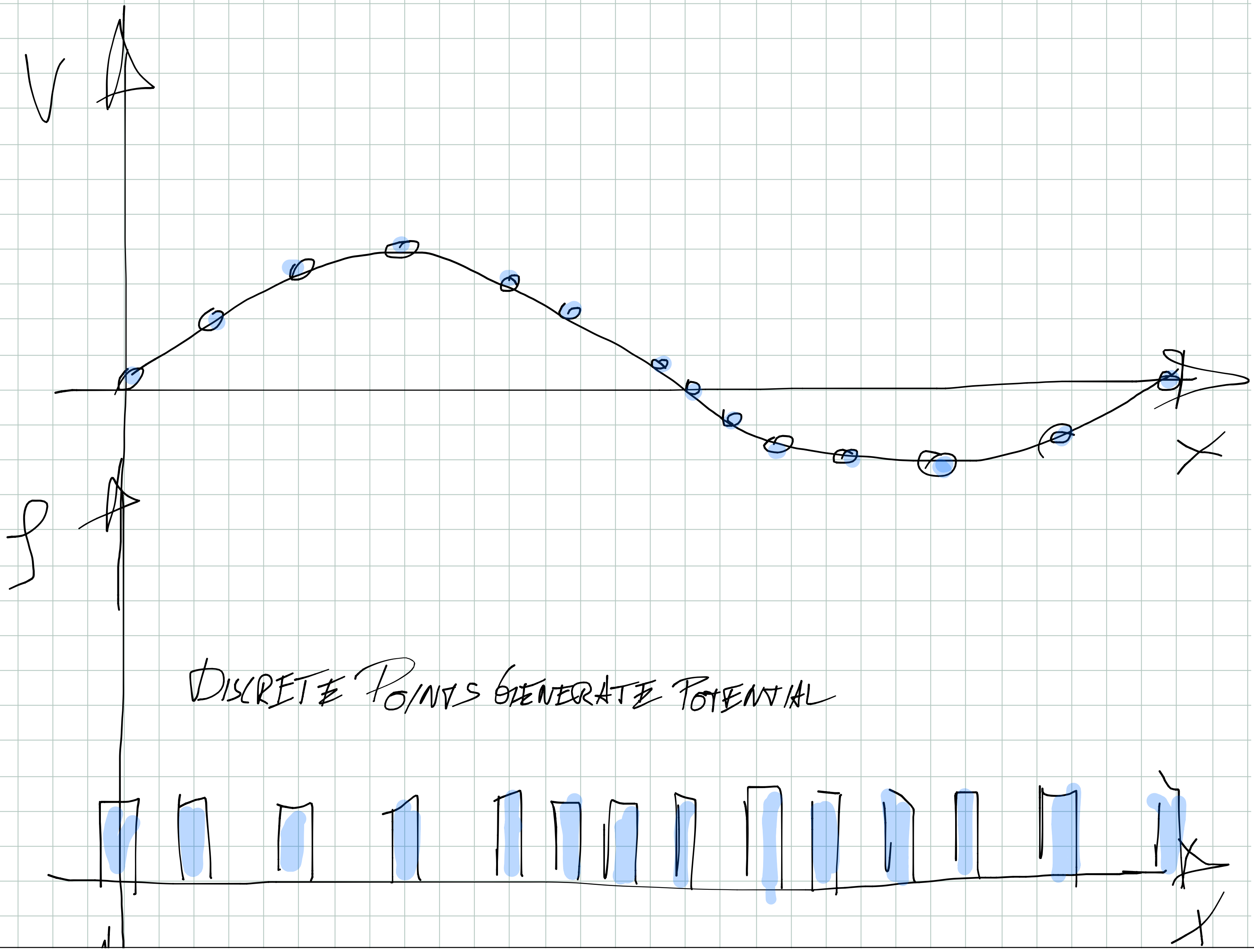
$$L \sim 500\,m\left(\frac{m_{DM}}{100\,GeV}\right)^{1/3}$$



PHASE SPACE

Steepening of initial sine wave → Zel'dovich collapse



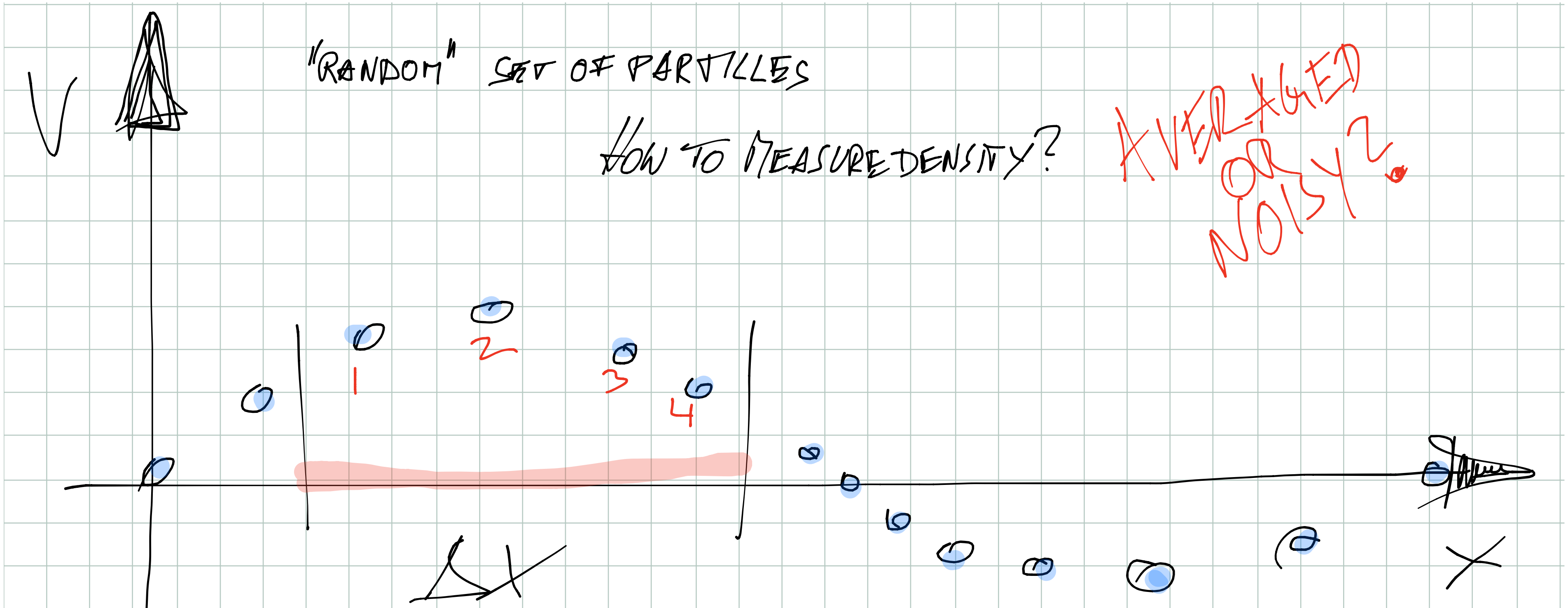


DISCRETE POINTS GENERATE POTENTIAL

"RANDOM" SET OF PARTICLES

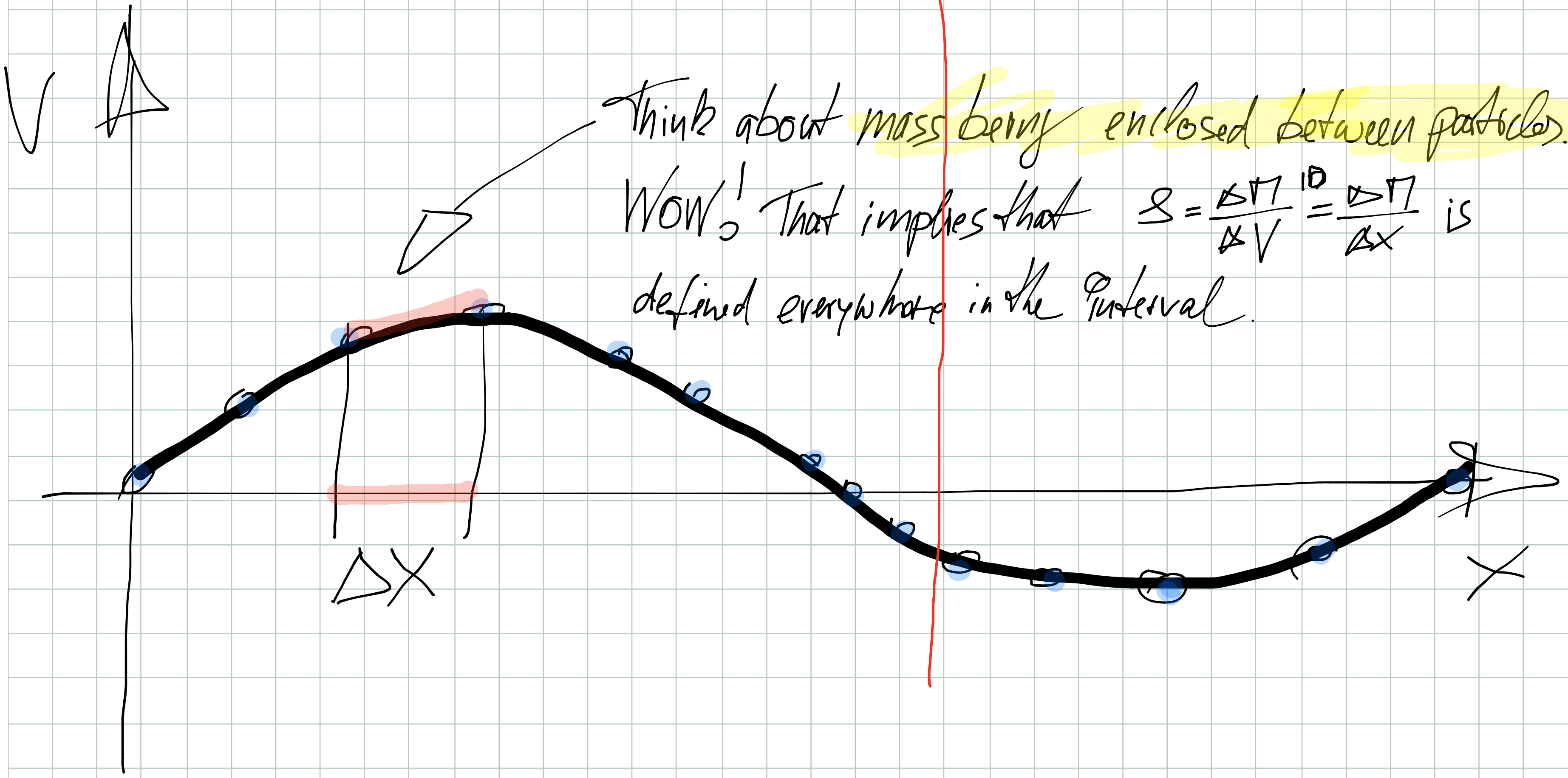
HOW TO MEASURE DENSITY?

AVERAGED OR NOISY?



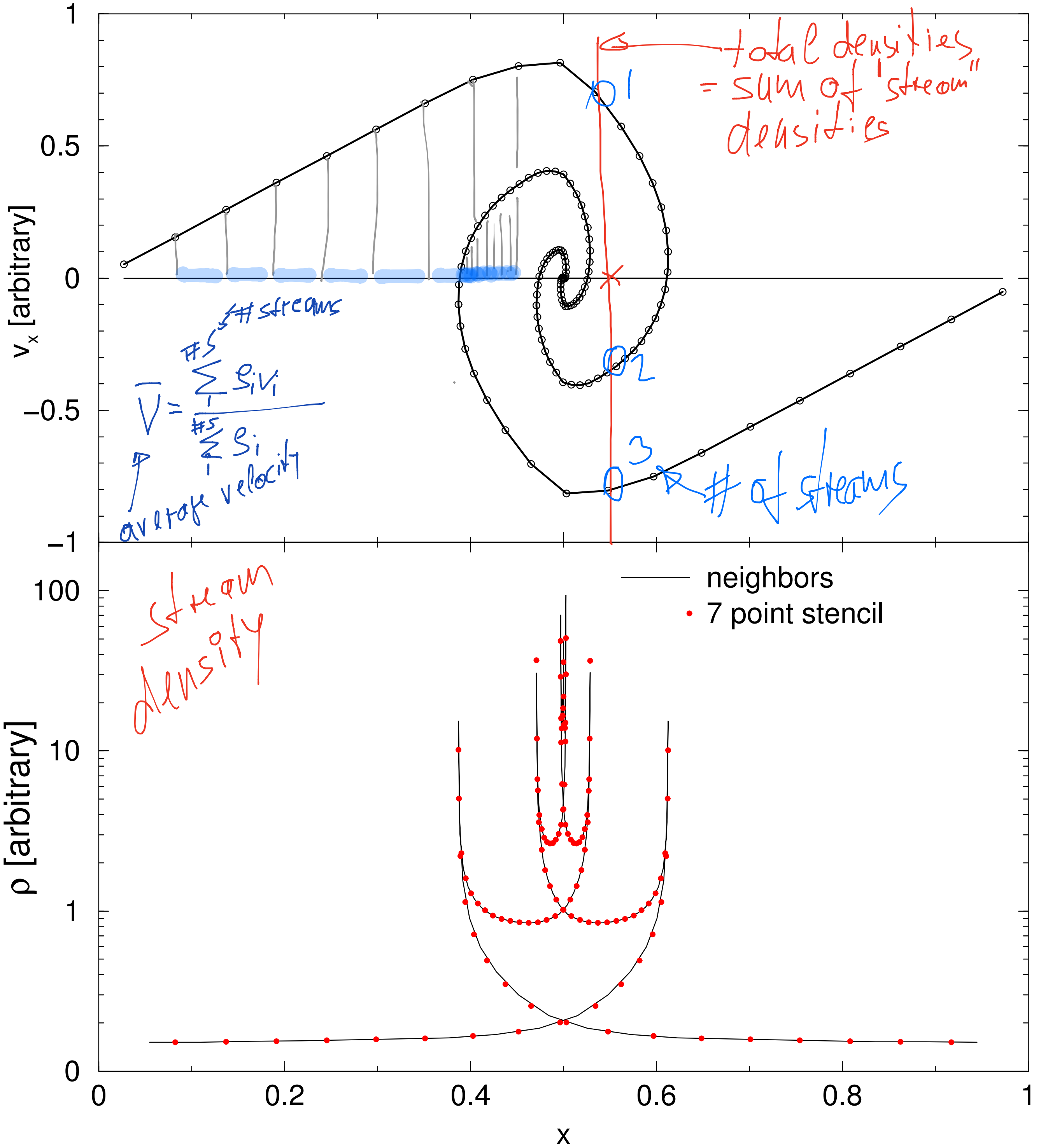
- PICK CONTROL VOLUME Δx & count # of particles : $\frac{N_{COUNT} \cdot m_i}{\Delta x} = \rho$

  ☹ that is an average density.
- Adaptive Kernel smoothing? Still an average...
- VORONOI? Assign every particle minimal volume around it. Not an average
  but potentially noisy...



Think about mass being enclosed between particles.

Wow! That implies that $\rho = \frac{\Delta M}{\Delta V} \overset{1D}{=} \frac{\Delta M}{\Delta X}$ is defined everywhere in the interval.



total densities = sum of "stream" densities

# of streams

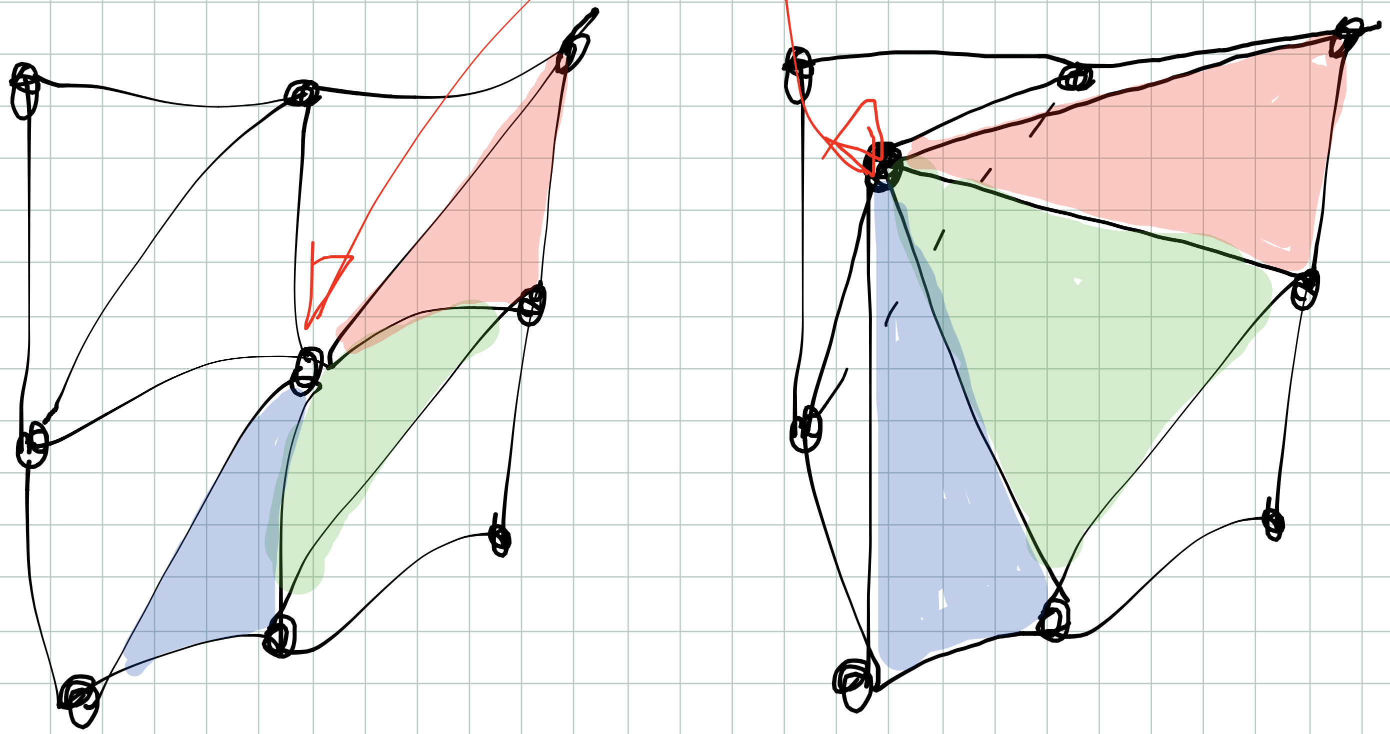
$$\bar{V} = \frac{\sum_{i}^{\#s} \rho_i v_i}{\sum_{i}^{\#s} \rho_i}$$

#streams

average velocity

stream density

A 2-D example



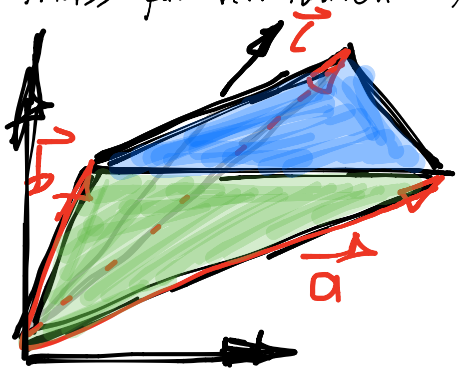
moved this point.

1D → LINE ; 2D → triangle ; 3D → tetrahedron

- just "painting" triangles with a weight $\frac{1}{V} = \frac{2}{\vec{a} \times \vec{b}}$

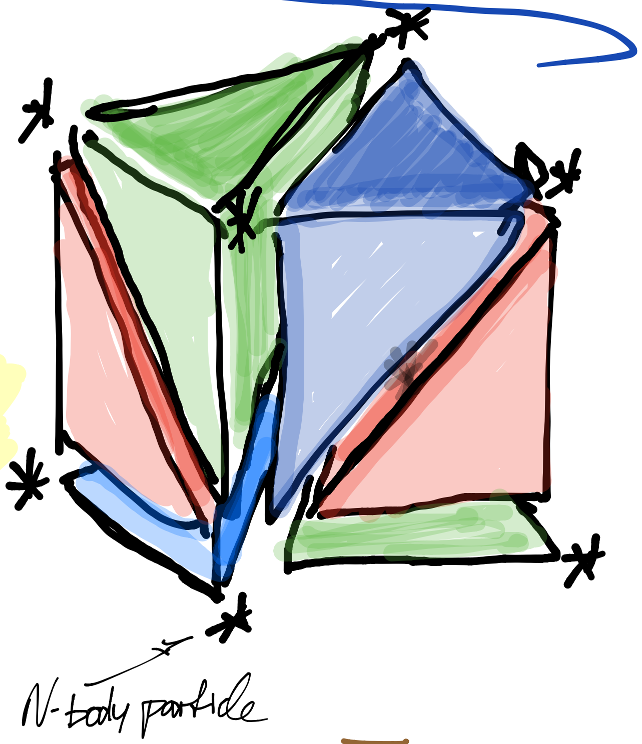
3 dimensional manifold in 6D Phase Space

- Natural tesselation takes unit cube & splits it into six equal size tetrahedra.
- mass per tetrahedron = 1/6 of DM particle mass.

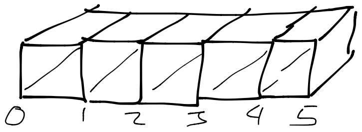


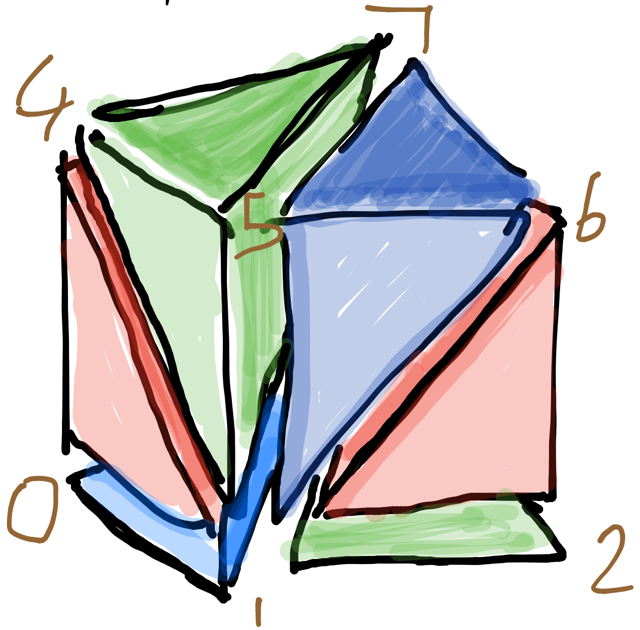
$$V = \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{6}$$

$$\Rightarrow \rho = \frac{m_p}{6V} = \frac{m_p}{|\vec{a}\cdot(\vec{b}\times\vec{c})|}$$

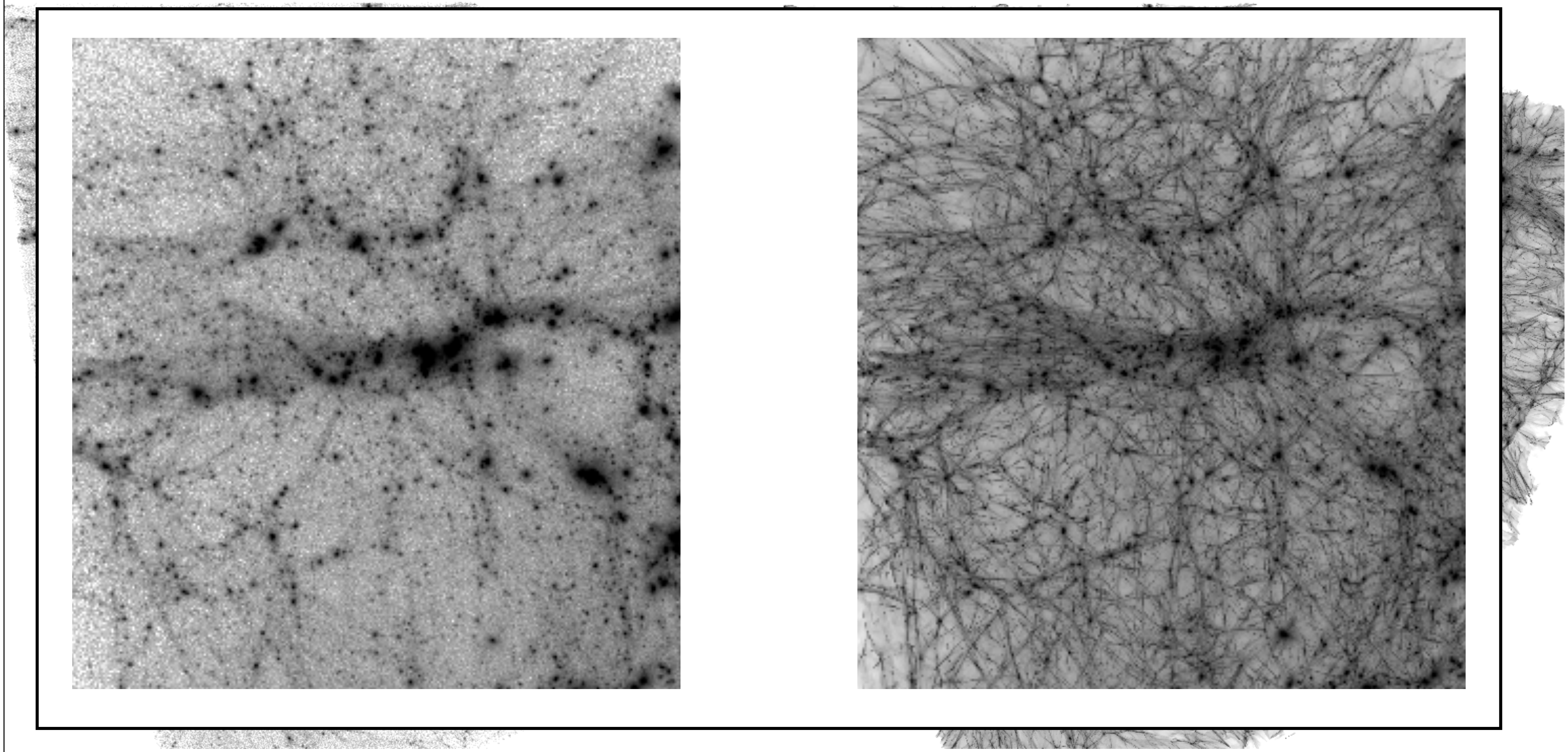


- Number the edges of the cube
- think of lattice
- Looping over the initial cartesian (LAGRANGIAN) lattice generates the 6N tetrahedra.





Much more intricate web structure...

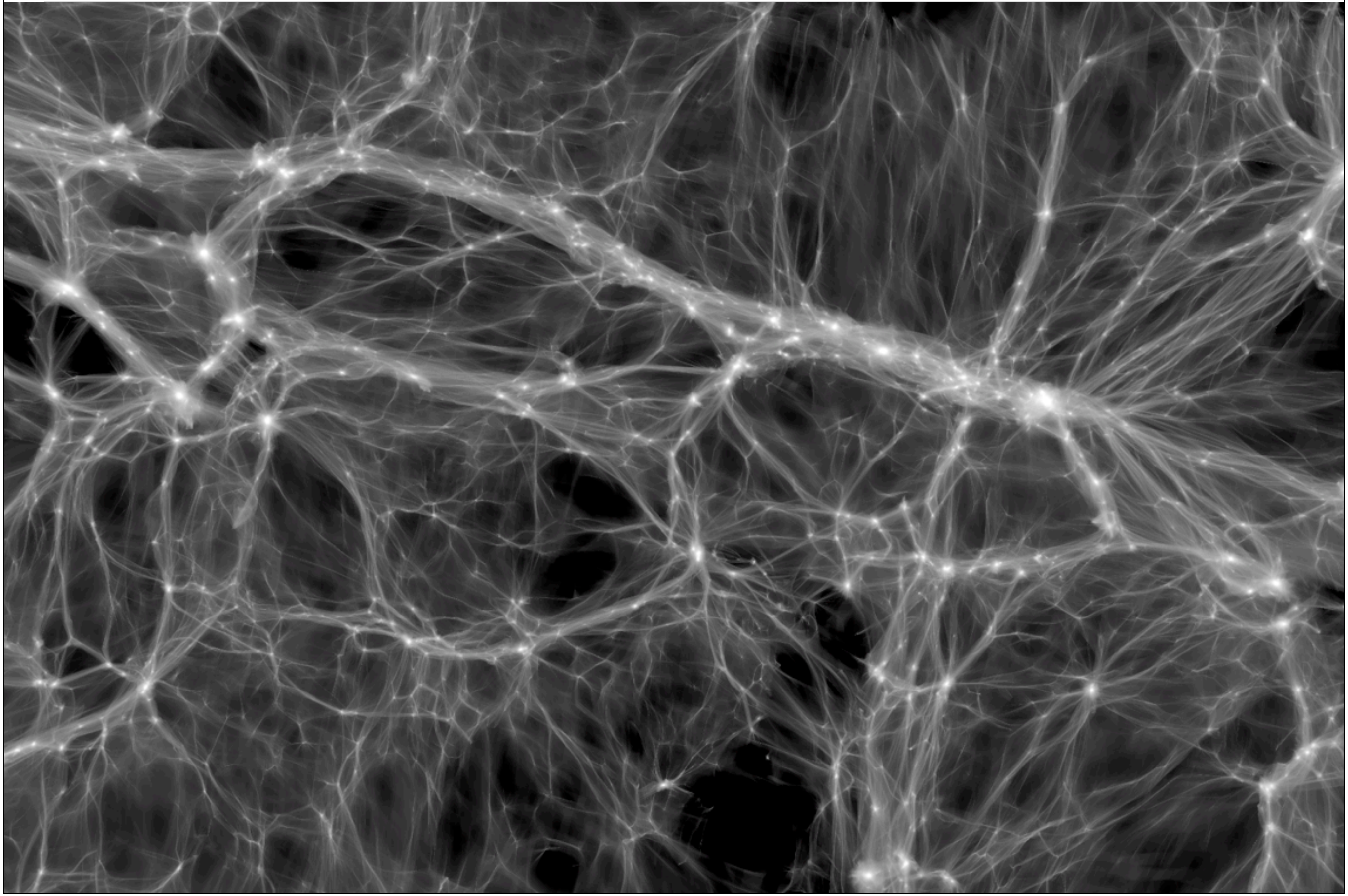


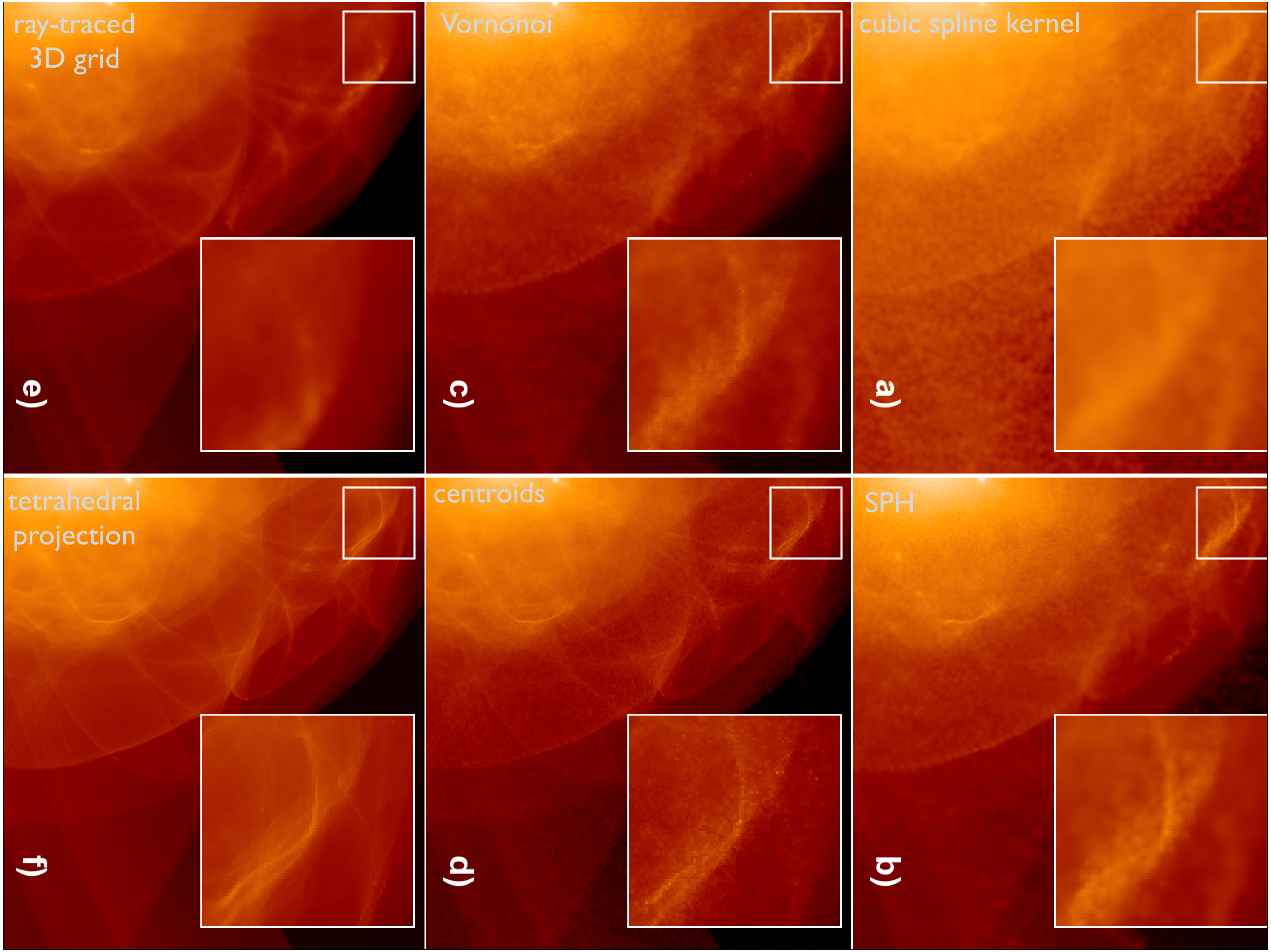
rendering points for particles.

rendering tetrahedral phase space cells.

Same simulation data!

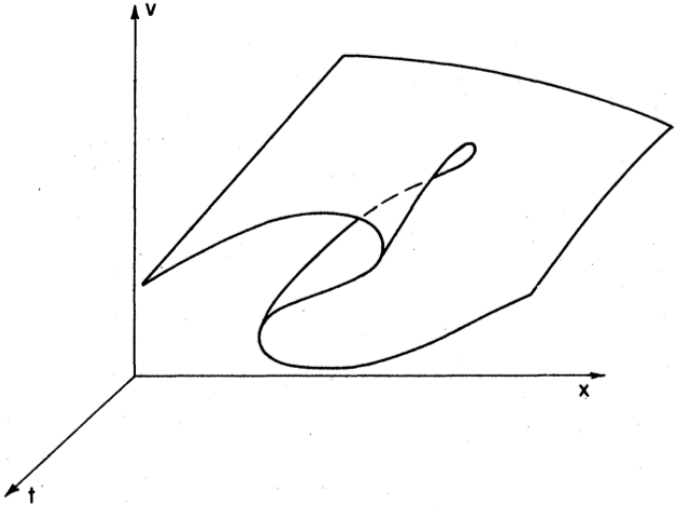
Density information everywhere in space



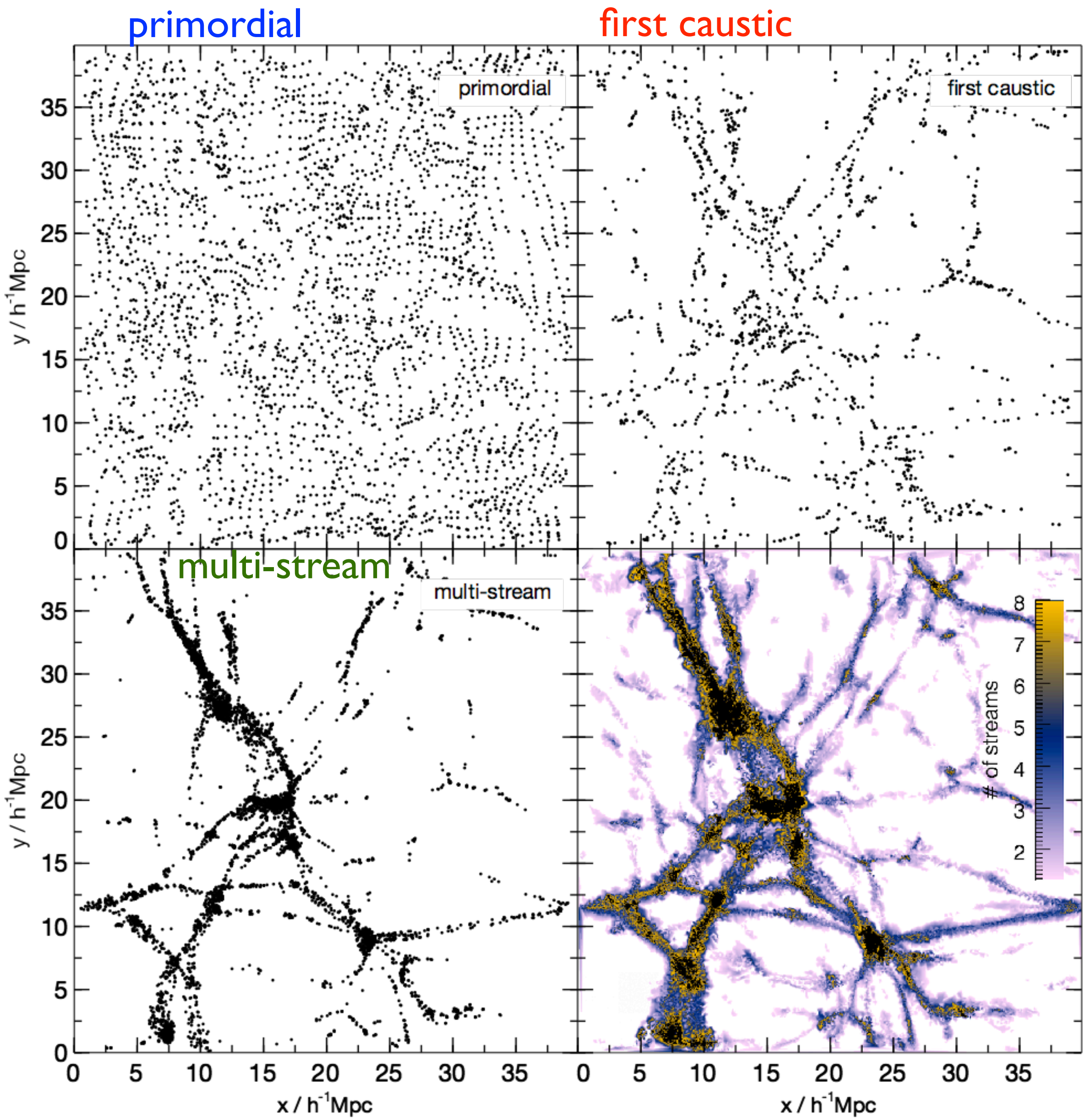

ray-traced
3D grid
Vornonoi
cubic spline kernel
e)
c)
a)
tetrahedral
projection
centroids
SPH
f)
d)
b)

From caustics to multistream...

Use the local number of foldings

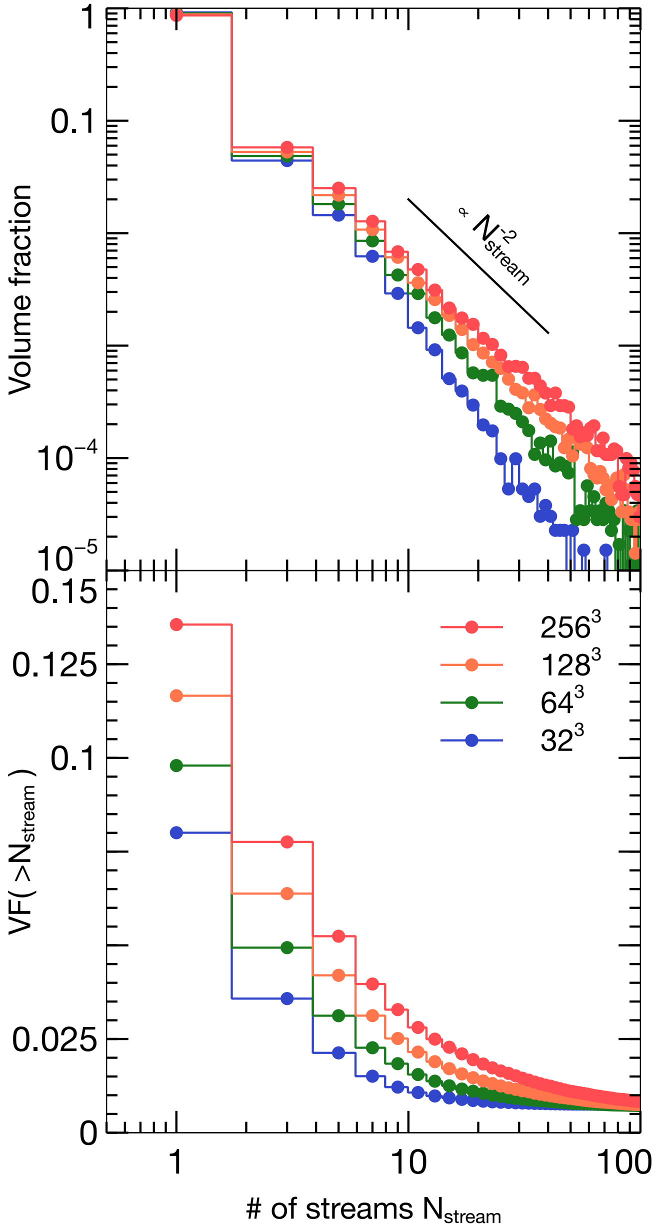


works remarkably well to understand dynamics of LSS



So, what volume fraction is multi-stream?

or, how much volume is LSS?



again...,
approaches power-law

AGAIN,
Continues to change with resolution

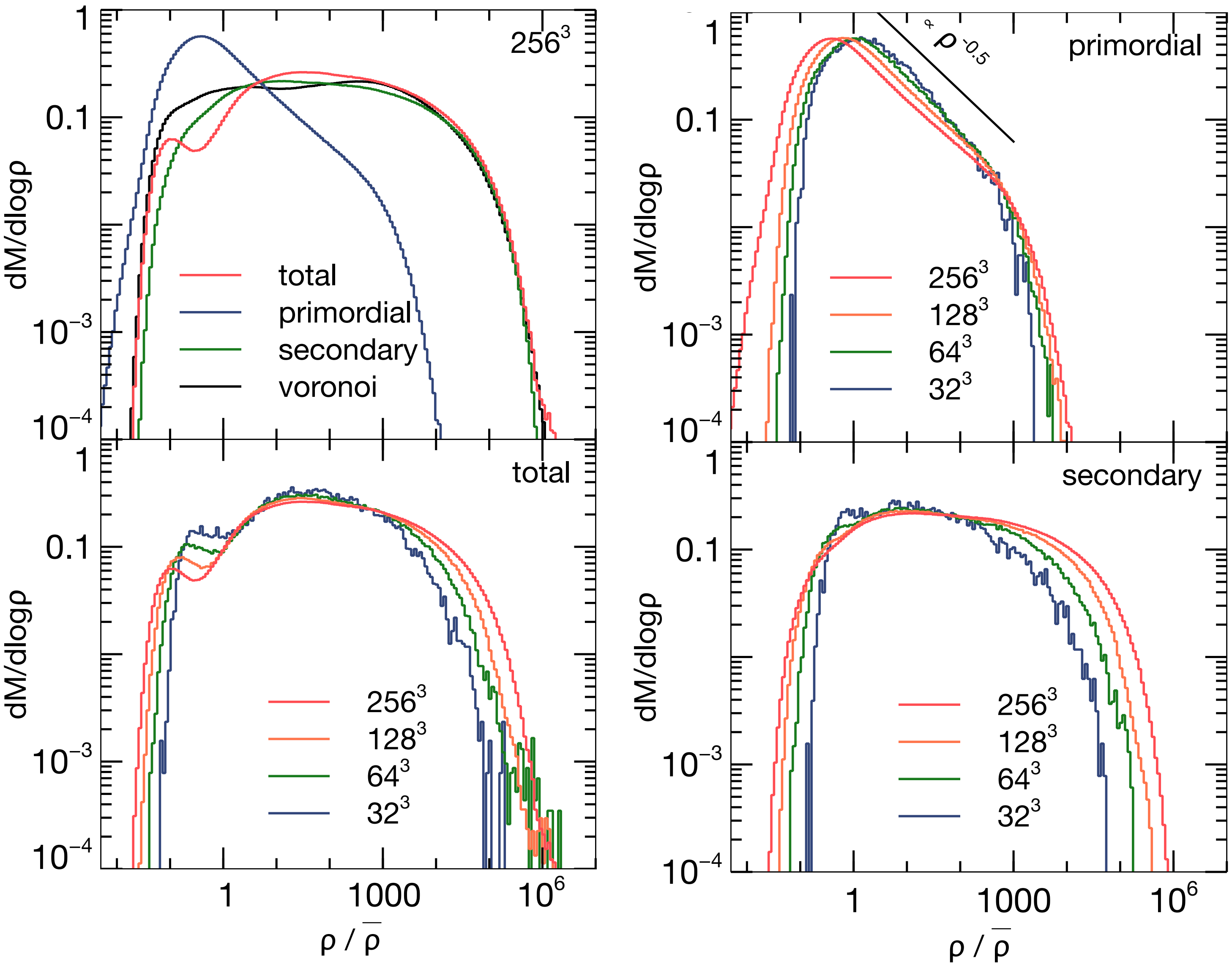
In particular:
The volume fraction of voids cannot even be determined.

This is CDM : clumps on all scales, maybe down to earth masses.

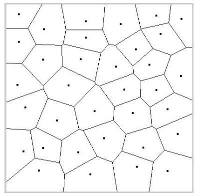
Voids, Sheets, Filaments can be sensibly defined only for a given spatial scale.

The density distributions

(mass weighted)

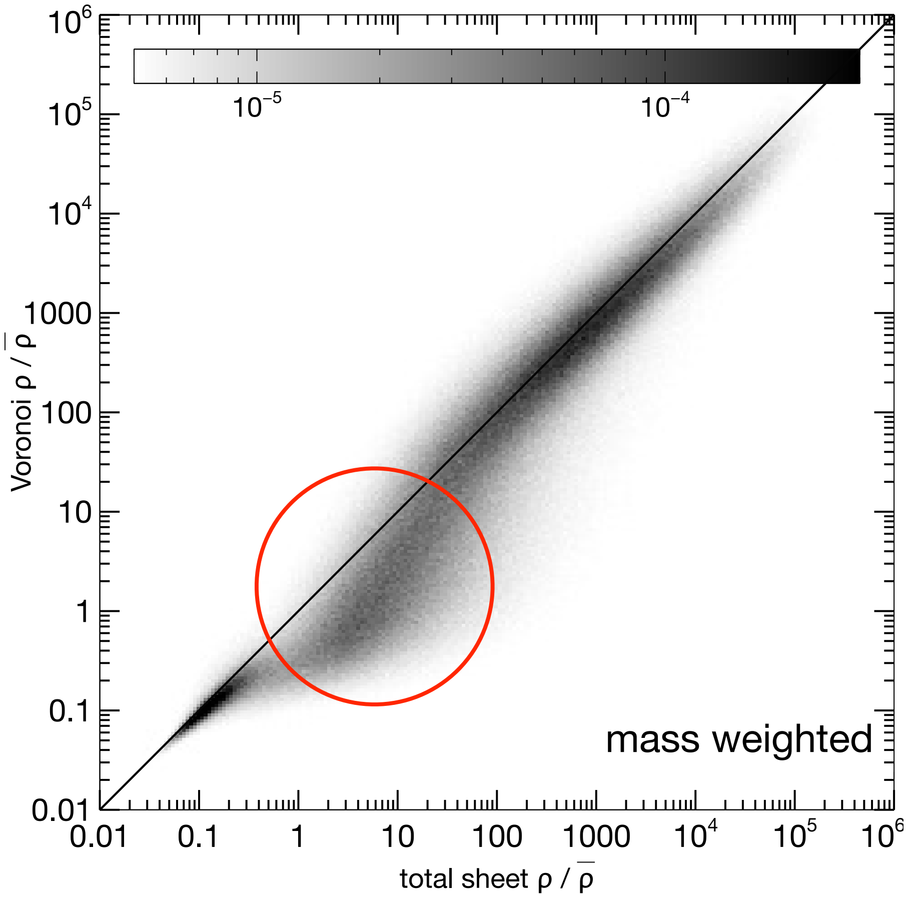


Voronoi:

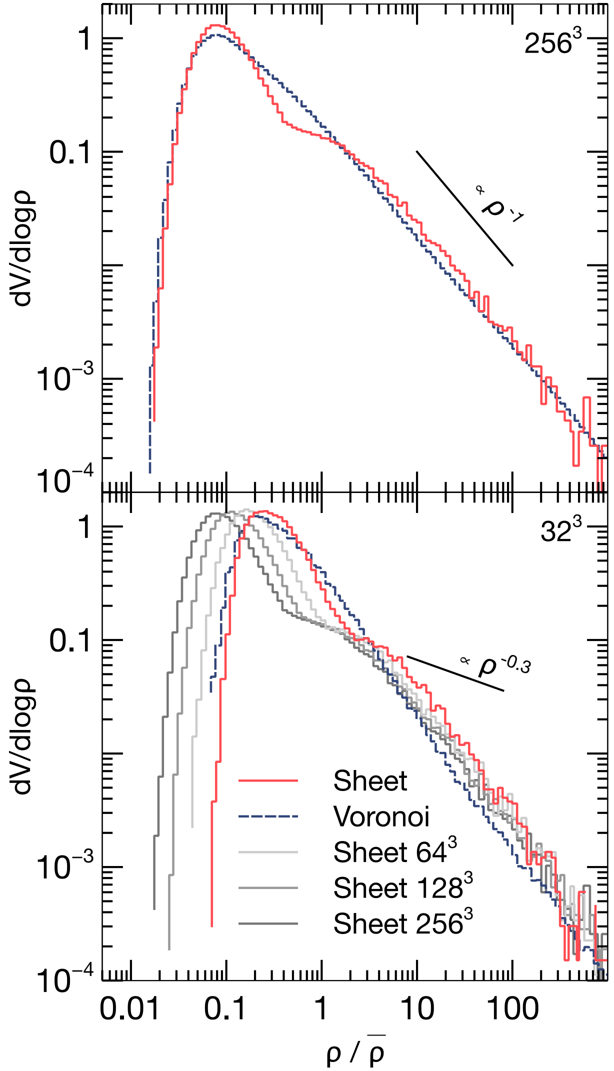


Comparison with Voronoi densities

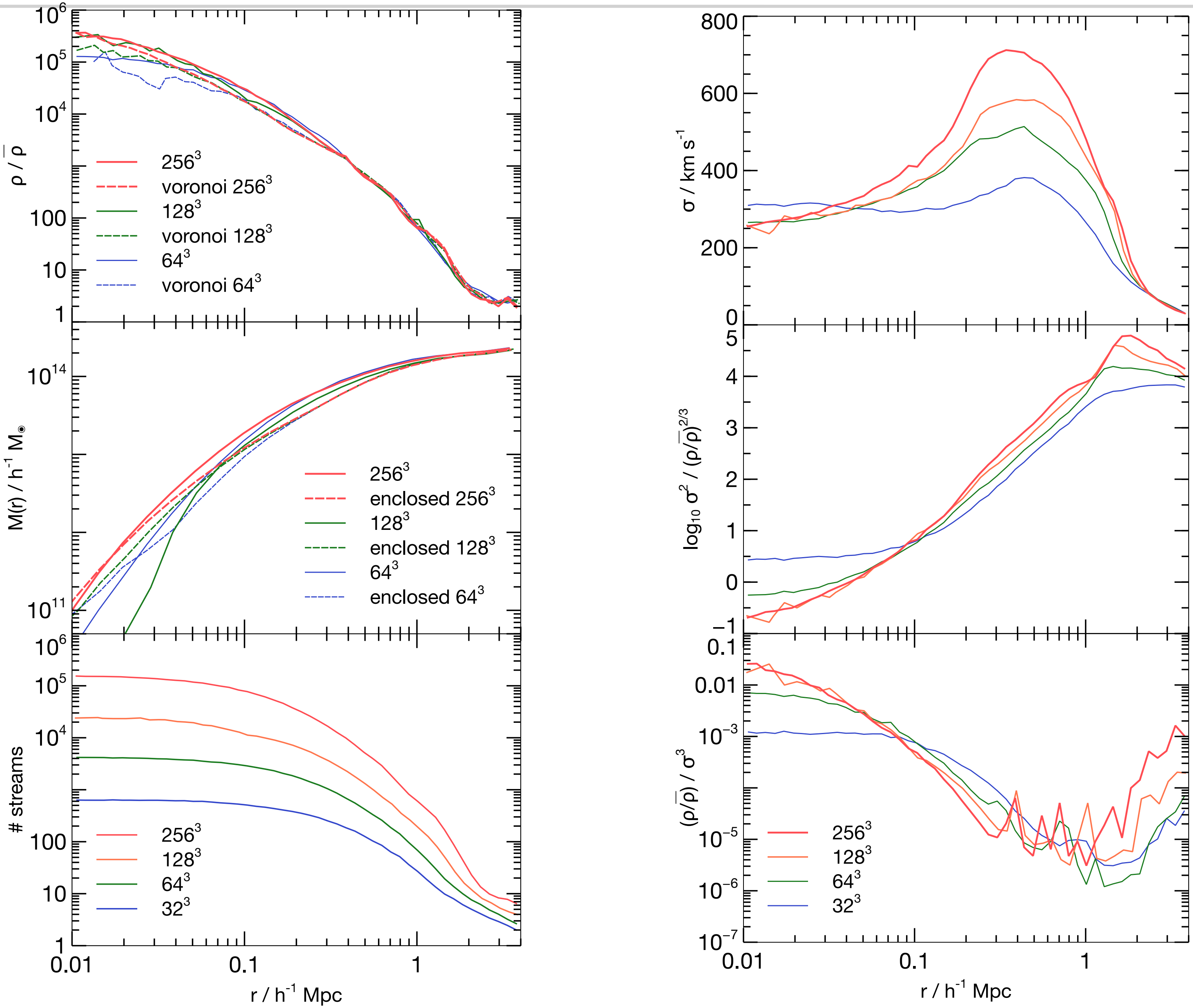
Much of the difference is at modest overdensities!



But they occupy a lot of volume.

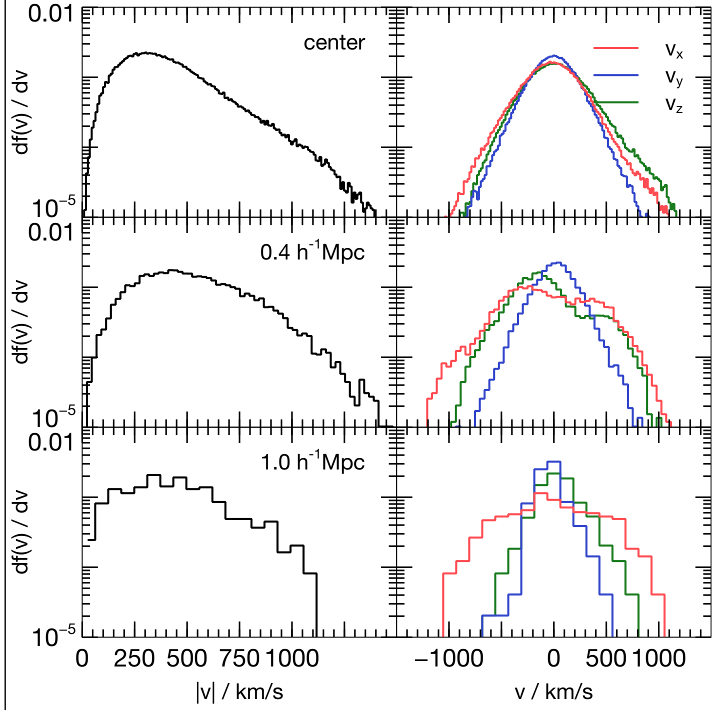


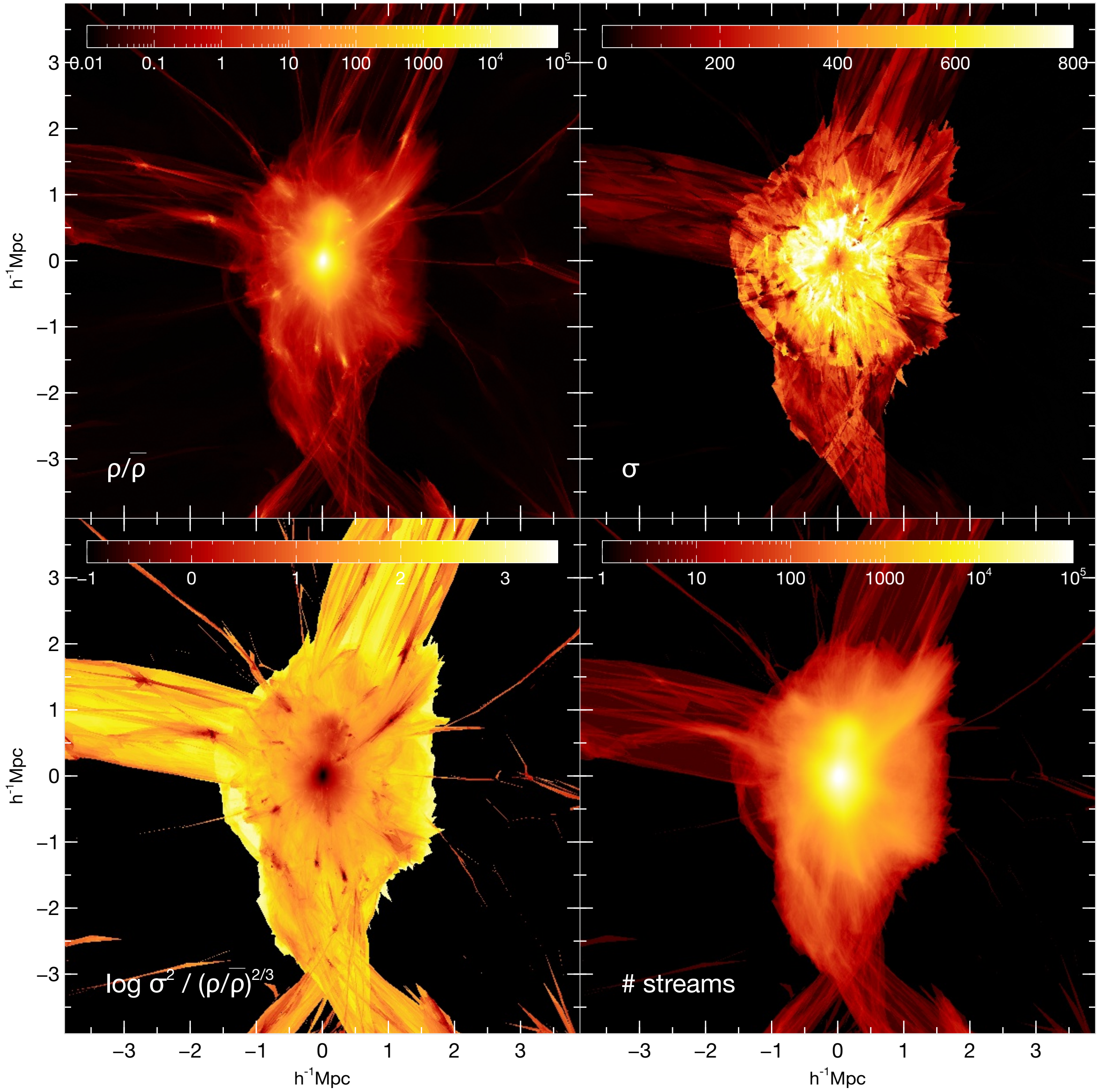
Radial profiles



A first glimpse: analyzing phase space

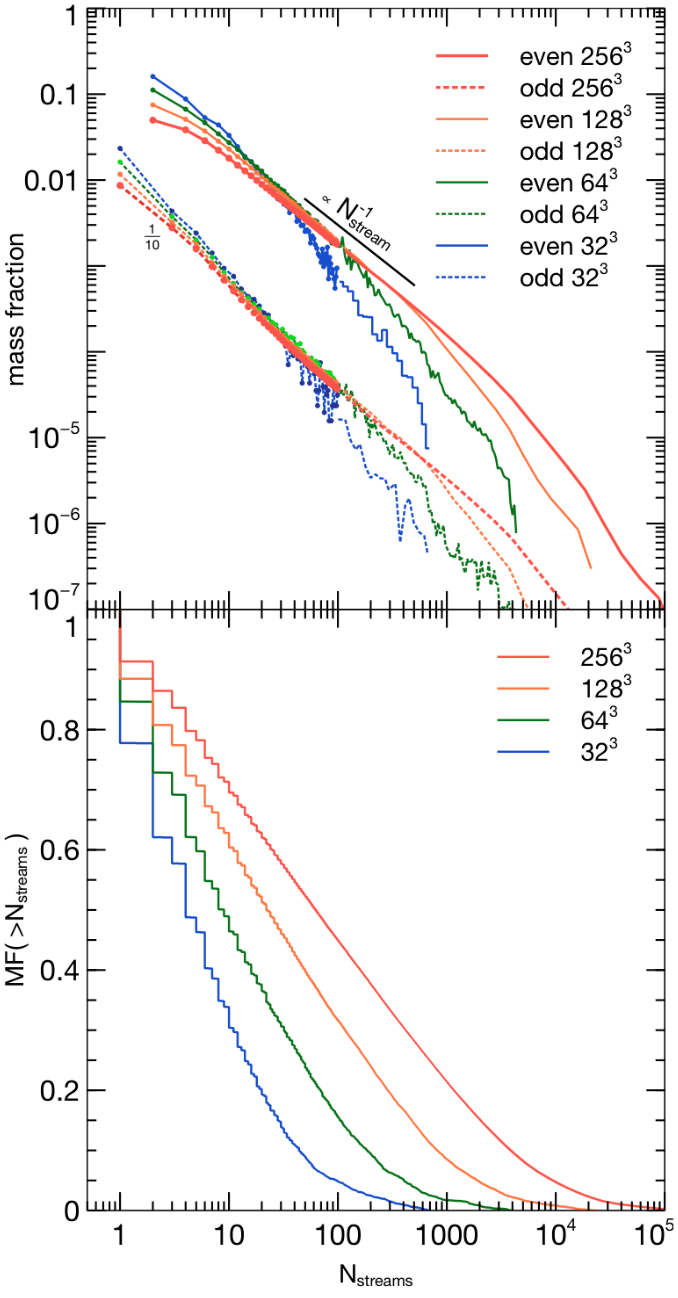
can probe fine-grained phase space structure.





What mass fraction is multi-stream?

or, how much mass is collapsed?



almost everything,
a few% in caustics

approaches power-law?
(cf. also Shandarin et al. 2011)

Numbers increase
with resolution
just as you expect for CDM

New numerical methods?

Problems of the N-body method

Vlasov-Poisson system

$$\frac{\partial f}{\partial t} = -\frac{\mathbf{p}}{m} \cdot \boldsymbol{\nabla}_x f - \boldsymbol{\nabla}_x \phi \cdot \boldsymbol{\nabla}_p f$$

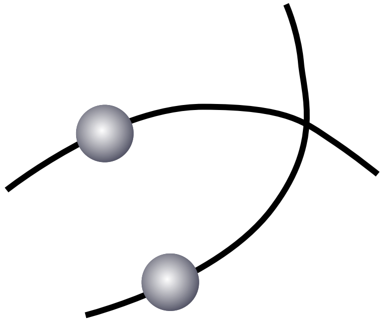
Distribution function

$$f(\mathbf{x}, \mathbf{p}, t) = \sum_{i=1}^{N} \delta_D(\mathbf{x} - \mathbf{x}_i(t))\, \delta_D(\mathbf{p} - \mathbf{p}_i(t))$$

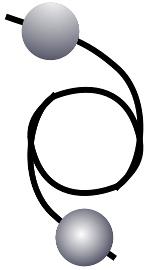
⇒ eq. of motion for N massive particles, not a continuum

Main Problem: two-body effects, can be reduced by force softening

Scattering

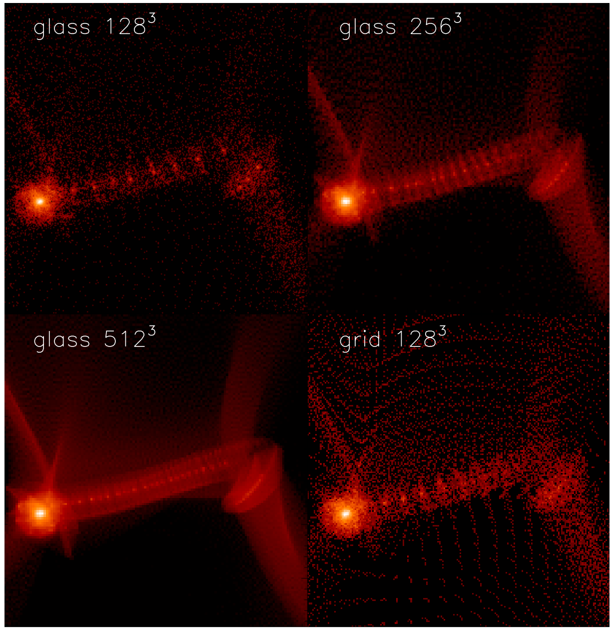


Clumping/
Fragmentation



Most problematic for non-CDM simulations!

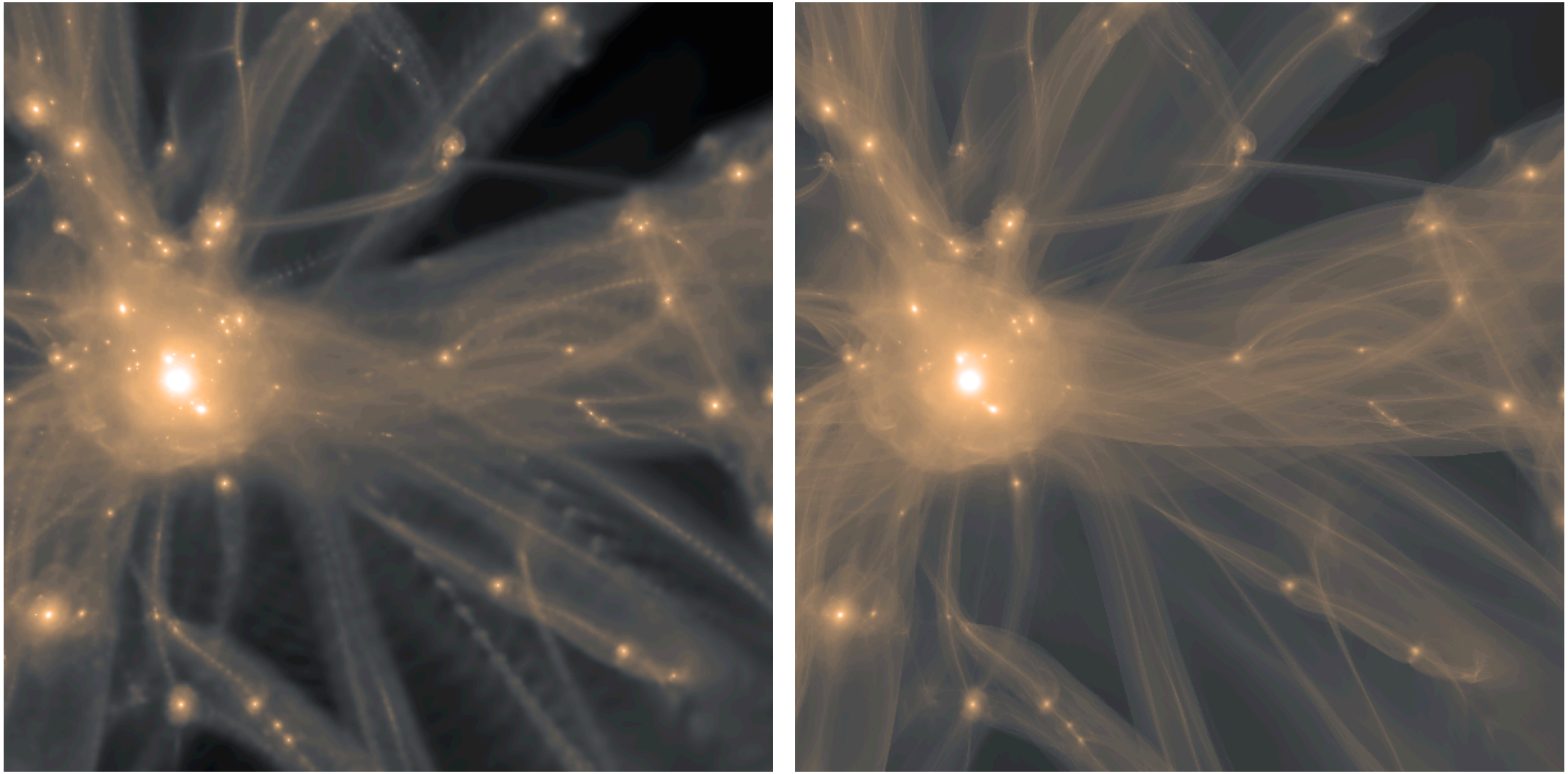
(cf. Wang&White 2007, Mellott 1983-



Wang&White 2007

Use DM sheet to get space density

Renderings of **same** warm DM simulation data



Mass is spread out ⇒ fragmentation reduced

Adaptive kernel filtered

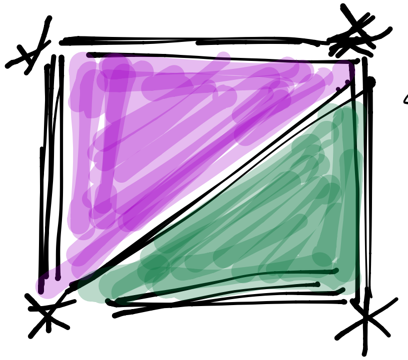
Kaehler, Hahn, Abel 2012 full tet rendering

TET-PM

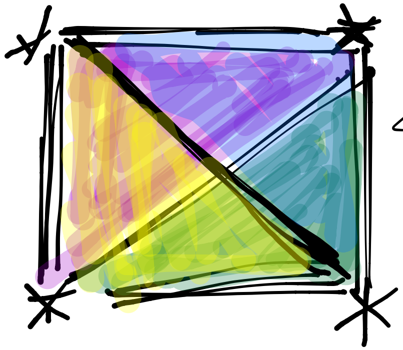
- NEW WAY TO DO N-BODY SIMULATIONS
- Massless TRACERS moving along characteristics.
- These span tetrahedra.
- FIRST IMPLEMENTATION: Monopole approximation
  - → MASS OF TET DEPOSITED AS POINT PARTICLE @ CENTROID LOCATION
  - OTHERWISE IDENTICAL TO A PARTICLE MESH CODE

NEW

2 TYPES OF PARTICLES



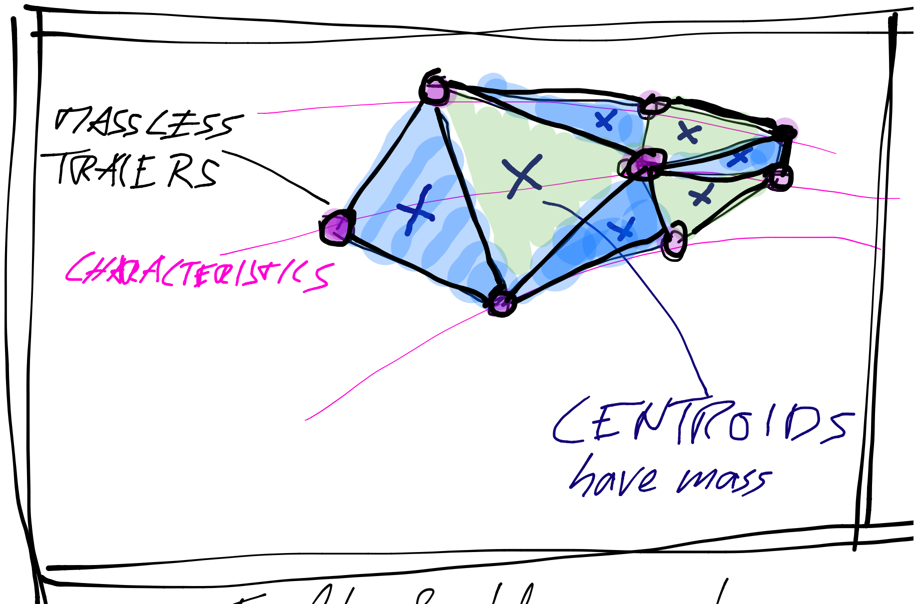
⇐ PROPER TESSELATION



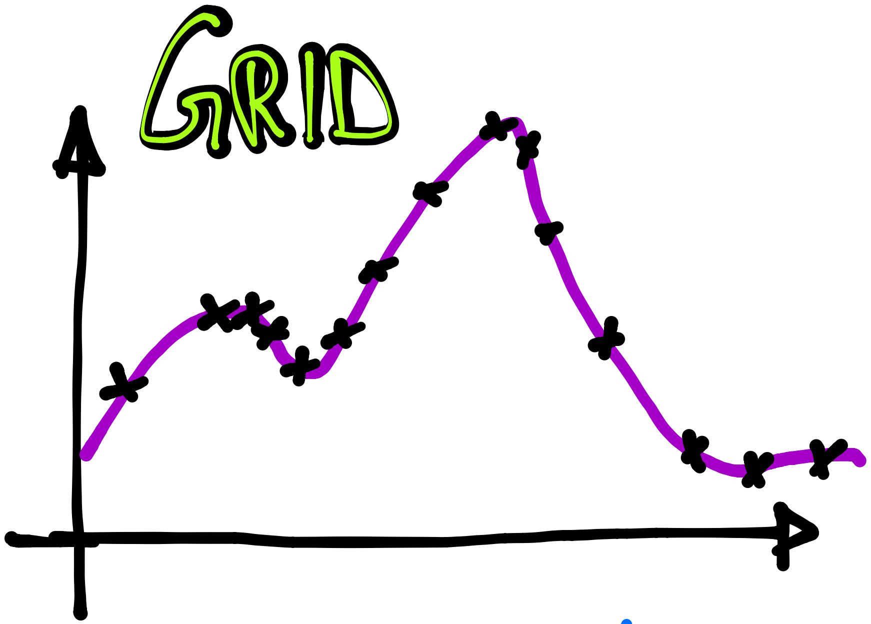
⇐ SYMMETRIC VERSION

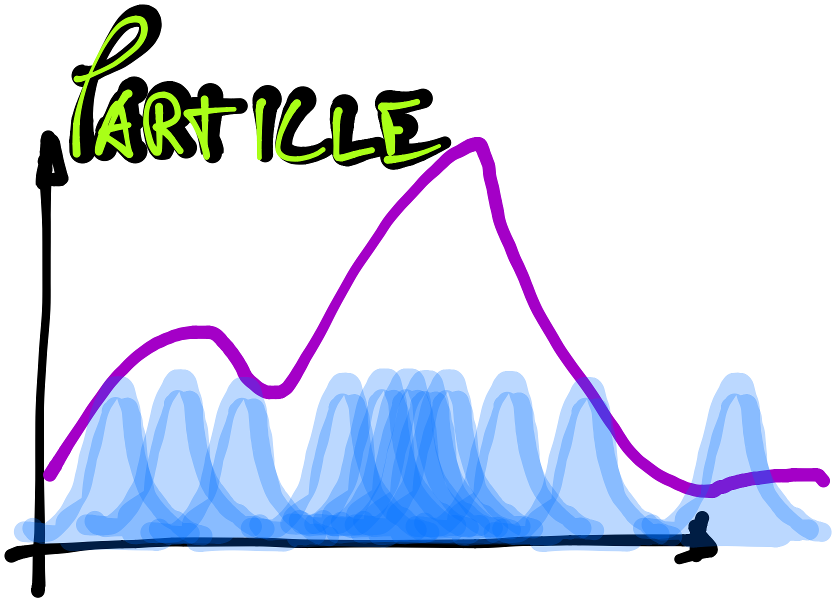
2D : 4 △'s per □

3D : 8 △ per □



Tesselation & particle arrangement in TET-PM





Representing the Solution, e.g. $\rho(\vec{x})$

Uniform, adaptive,
moving, (un)structured,

MESH

fixed, adaptive,
high order, asymmetric, ...

KERNEL