Implicit Lagrangian method on variable triangular grid for magnetorotational supernova simulations

Sergey Moiseenko

Space Research Institute,

Moscow, Russia

Basic equations: MHD +self-gravitation, infinite conductivity:

$$
\begin{cases}
\dfrac{dx}{dt} = \mathbf{u}, \dfrac{d\rho}{dt} + \rho \operatorname{div}\mathbf{u} = 0, \\
\rho \dfrac{du}{dt} = -\operatorname{grad}\left(p + \dfrac{\mathbf{H}\cdot\mathbf{H}}{8\pi} \right) + \dfrac{1}{4\pi}\operatorname{div}(\mathbf{H}\otimes\mathbf{H}) - \rho \operatorname{grad}\Phi \\
\rho \dfrac{d\varepsilon}{dt} + p\operatorname{div}\mathbf{u} + \rho F(\rho, T) = 0,\ p = P(\rho, T), \varepsilon = \mathrm{E}(\rho, T), \\
\Delta\Phi = 4\pi \mathrm{G}\rho, \\
\rho \dfrac{d}{dt}\left(\dfrac{\mathbf{H}}{\rho} \right) = \mathbf{H}\cdot\nabla\mathbf{u}.
\end{cases}
$$

Additional condition div**H**=0

Axis symmetry ($\dfrac{\partial}{\partial\phi} = 0$) and equatorial symmetry (z=0) are supposed.

Notations:

$\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + \mathbf{u}\cdot\nabla, \mathbf{x} = (\mathrm{r}, \varphi, \mathrm{z}), \mathbf{u}$ – velocity, ρ – density, p – pressure,

$\mathbf{H}$ – magnetic field, Φ – gravitational potential, ε – internal energy,

G – gravitational constant.

Operator-difference scheme

Ardeljan et al.

Lagrangian, implicit, triangular grid with rezoning, completely conservative

Method of **basic** operators (Samarskii) – grid analogs of basic differential operators:

GRAD(scalar) (differential) ~ GRAD(scalar) (grid analog)
DIV(vector) (differential) ~ DIV(vector) (grid analog)
CURL(vector) (differential) ~ CURL(vector) (grid analog)
GRAD(vector) (differential) ~ GRAD(vector) (grid analog)
DIV(tensor) (differential) ~ DIV(tensor) (grid analog)

Implicit scheme. Time step restrictions are weaker for implicit schemes (no CFL condition).

The scheme is Lagrangian=> conservation of angular momentum.

General properties of scalar, vector and tensor functions

$$\int_G p\nabla\cdot\vec{v}\,dV + \int_G \vec{v}\cdot\nabla p\,dV = \int_{\partial G} p\vec{v}\cdot d\vec{S},$$

$$\int_G \vec{E}\cdot(\nabla\times\vec{H})dV - \int_G \vec{H}\cdot(\nabla\times\vec{E})dV = \int_{\partial G}(\vec{H}\times\vec{E})\cdot d\vec{S},$$

$$\int_G \vec{v}\cdot\nabla A\,dV + \int_G A\cdot\nabla\vec{v}\,dV = \int_{\partial G}(A\cdot\vec{v})\cdot d\vec{S}.$$

Grid operators will be defined in analogy with formula:

$$(\nabla * A)(\vec{x}) = \frac{1}{V}\int_G(\nabla * A)dV = \frac{1}{V}\int_{\gamma(G)} d\vec{S} * A$$

The operator $\nabla_\triangle \cdot$ is a grid analog for the differential operator div, $\nabla_\times$ is a grid analog for grad. The following relation is valid for the introduced operators:

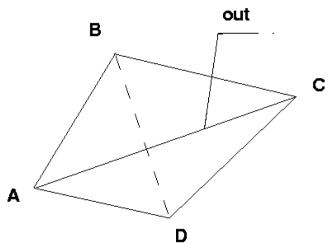
$$(\nabla_\times p, \mathbf{v})_\times + (\nabla_\triangle \cdot \mathbf{v}, p)_\triangle = (\Phi_\gamma p_\gamma, \mathbf{u}_\gamma)_\gamma$$

where $p \in B^0_{\triangle,0}$, $\mathbf{v} \in B_{\times,1}$, $p_\gamma \in B_{\gamma,0}$, $\mathbf{v}_\gamma \in B_{\gamma,1}$.

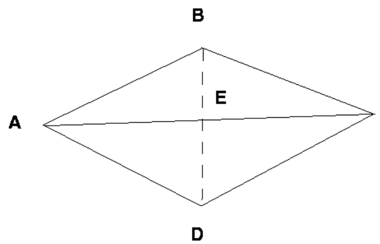
Operators $\nabla_\times$ and $-\nabla_\Delta \cdot$ are conjugated. $\Rightarrow$

operator $\nabla \cdot \nabla$ is self-conjugated

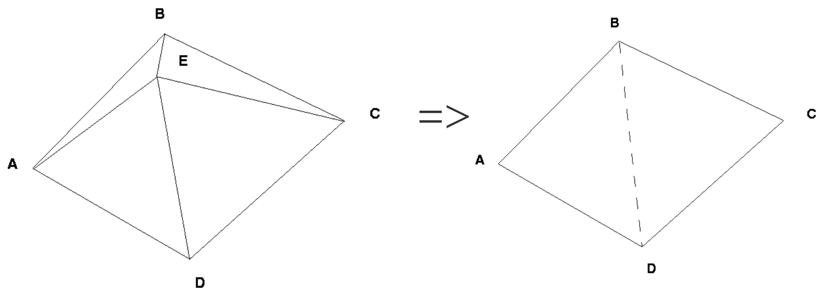
Grid reconstruction



Elementary reconstruction: BD connection is introduced instead of AC connection. The total number of the knots and the cells in the grid is not changed.



Addition a knot at the middle of the connection:
the knot E is added to the existing knots ABCD on the middle of the BD connection, 2 connections AE and EC appear and the total number of cells is increased by 2 cells.



Removal a knot: the knot E is removed from the grid and the total number of the cells is decreased by 2 cells

Interpolation of grid functions on a new grid structure (local):

Should be done in *conservative* way. Conditional minimization of special functionals.

$\rho_{old,k}$ – density on the old structure,

$\rho_{\mathrm{int},i}$ – old density, interpolated to the new structure,

$\rho_{new,i}$ – density on the new structure

Main functional:

$$\min_{i=\overline{1,N}} F(\rho_{new,i}) = (\rho_{new,i} - \rho_{\mathrm{int},i})^2$$

under condition:

$$\sum_k V_{old,k}\rho_{old,k} = \sum_i V_{new,i}\rho_{new,i} = m = const$$

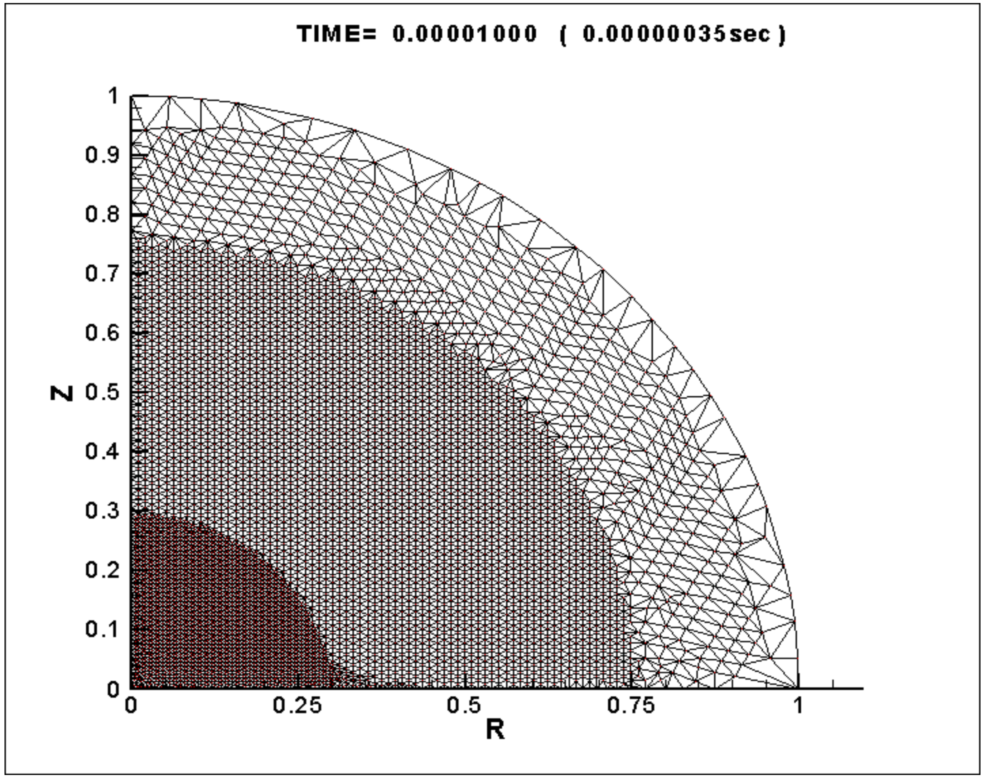
(mass conservation condition)

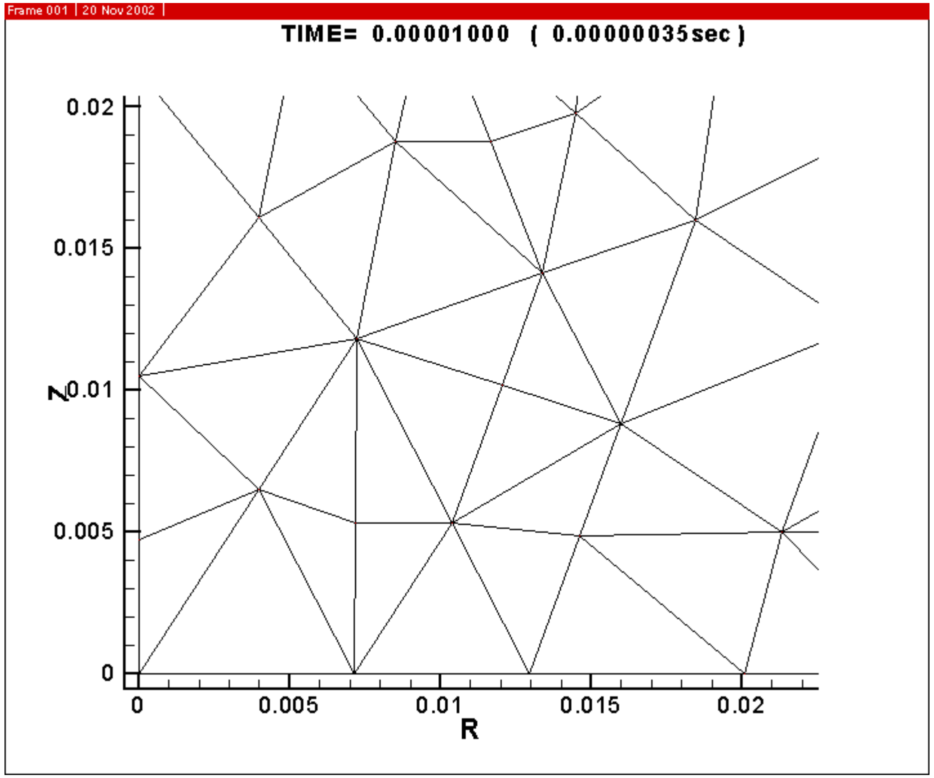
Old form of the functional:

$$\min_{i=\overline{1,N}} F(\rho_{new,i}) = \alpha(\rho_{new,i} - \rho_{\mathrm{int},i})^2 + \underline{(1-\alpha)(\mathrm{grad}(\rho_{new,i}) - \mathrm{grad}(\rho_{\mathrm{int},i}))^2}$$

Does not work on shocks☹

Example of the triangular grid





Magnetorotational mechanism for the supernova explosion Bisnovatyi-Kogan (1970)(original article was submitted: September 3, 1969)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion

First 2D calculations: LeBlanck&Wilson (1970))(original article was submitted: September 25, 1969) ->too large initial magnetic fields. $E_{mag0} \sim E_{grav} \Rightarrow$ axial jet

Bisnovatyi-Kogan et al 1976, Meier et al. 1976, Ardeljan et al.1979, Mueller & Hillebrandt 1979, Symbalisty 1984, Ardeljan et al. 2000, Wheeler et al. 2002, 2005, Yamada & Sawai 2004, Kotake et al. 2004, 2005, 2006, Burrows et al.2007, Sawai, Kotake, Yamada 2008,Barkov, Komissarov 2008,Kotake, Yamada 2010…

It is popular now!

The realistic values of the magnetic field are: $E_{mag} << E_{grav}$ ($E_{mag}/E_{grav} = 10^{-8}$-$10^{-12}$)

Small initial magnetic field -is the main difficulty for the numerical simulations.

The problem has 2 different time scales.

The hydrodynamic time scale is much smaller than the magnetic field amplification time scale (*if magnetorotational instability is neglected*).

Explicit difference schemes require very big number of timesteps. (CFL restriction on the time-step).

Implicit schemes should be used.

The main difference between bounce shock, neutrino driven mechanisms and MR supernovae: the magnetic field works like a piston. This MHD piston supports the supernova MHD shock wave for some time.

Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas

.

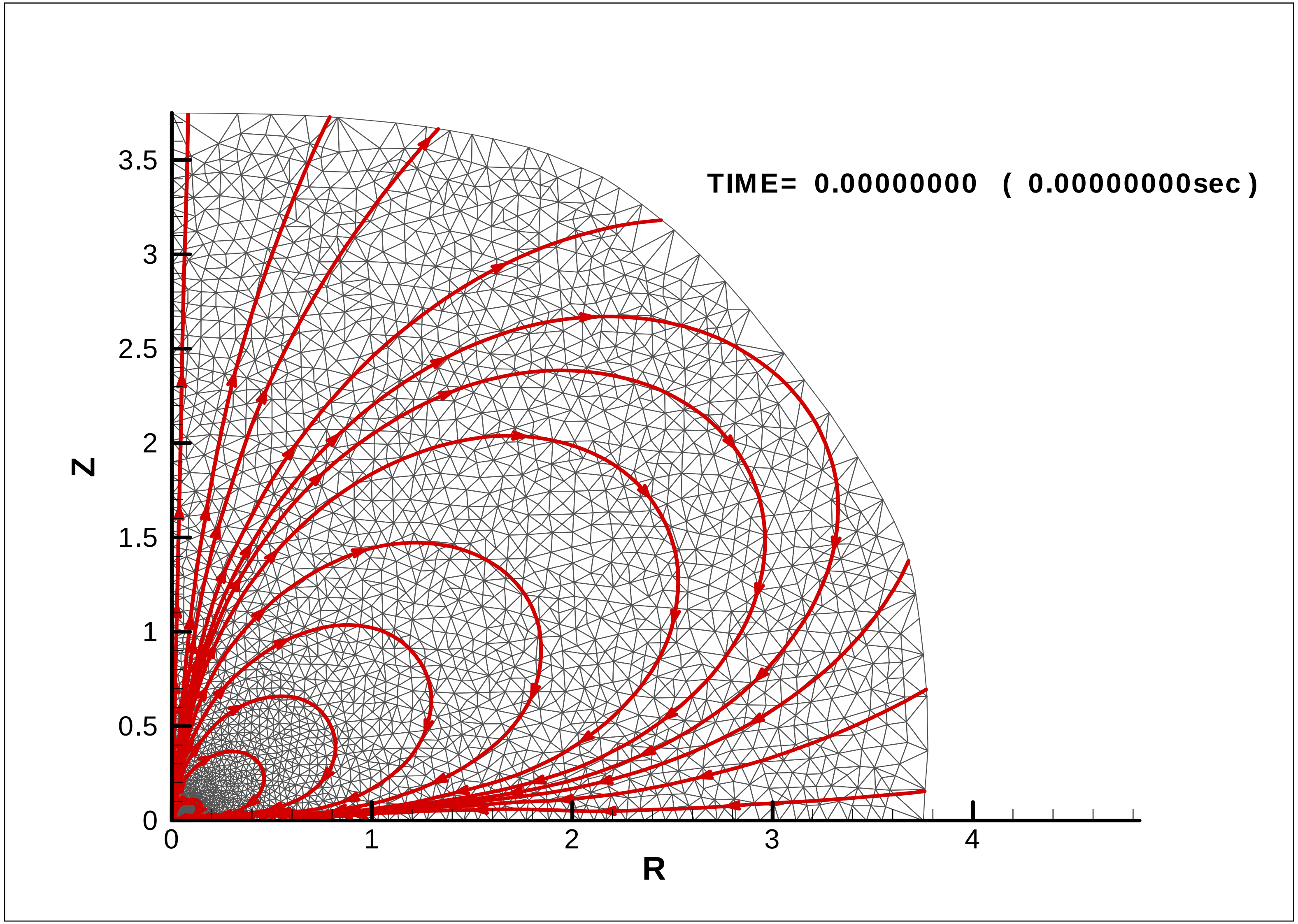
Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

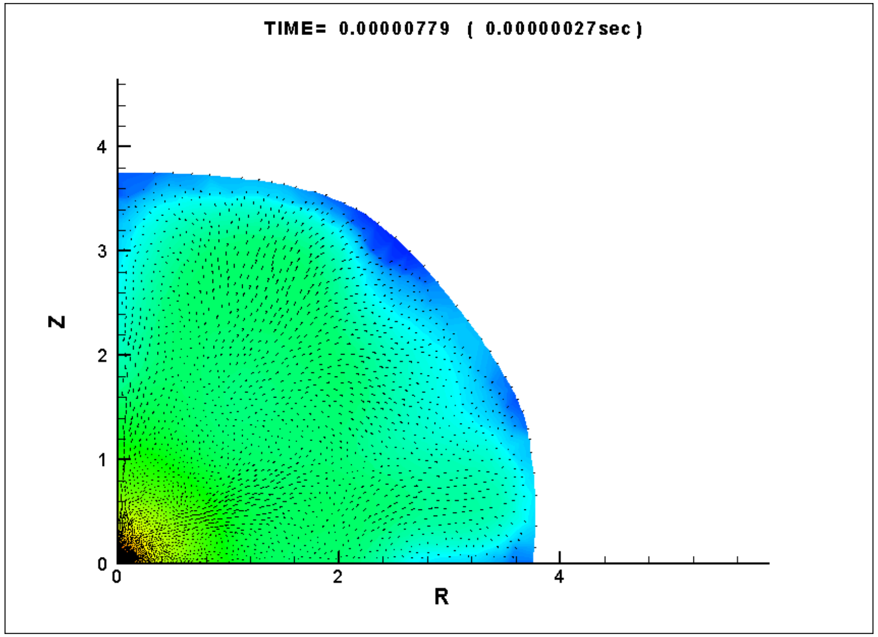
To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.

Initial magnetic field –quadrupole-like symmetry

Ardeljan, Bisnovatyi-Kogan, SM, MNRAS 2005, 359, 333

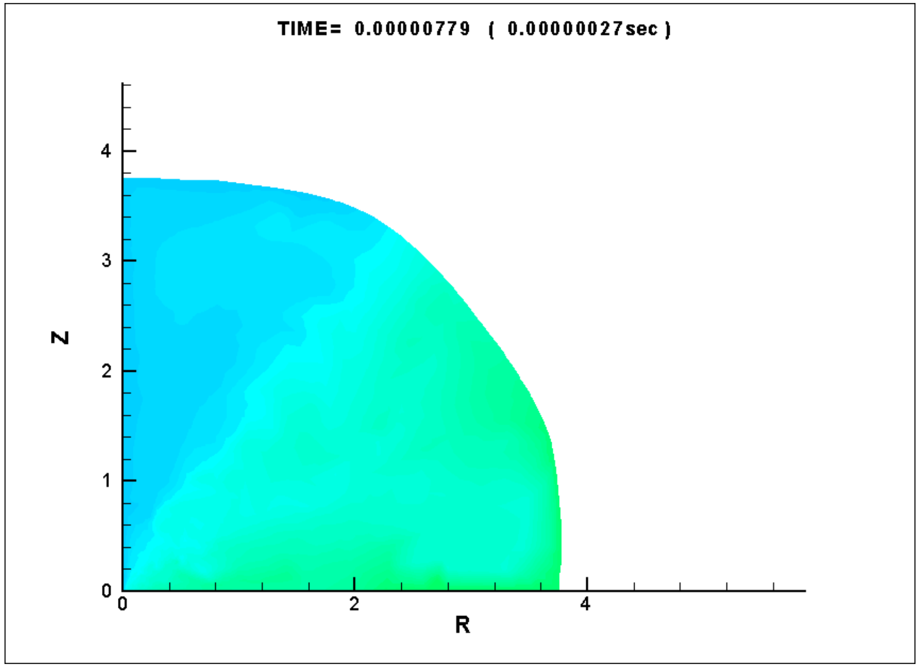


Temperature and velocity field



No jet.

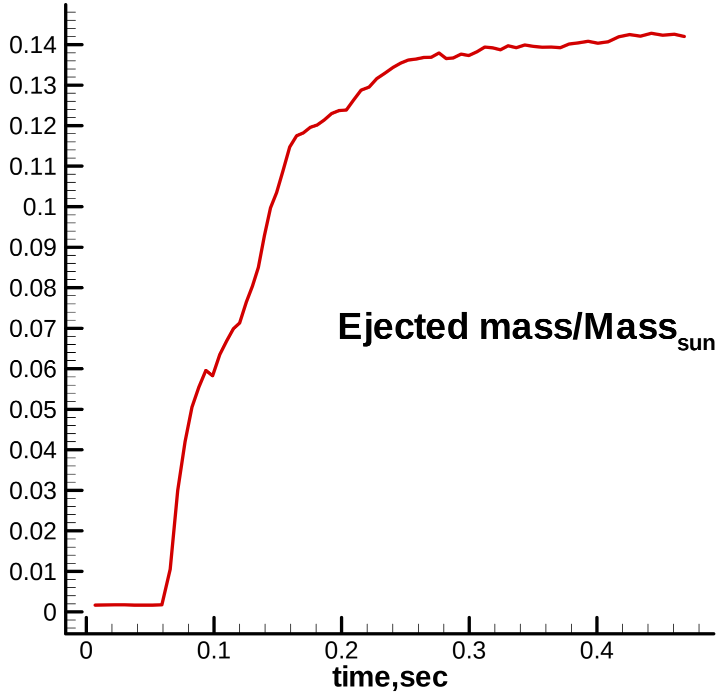
Specific angular momentum

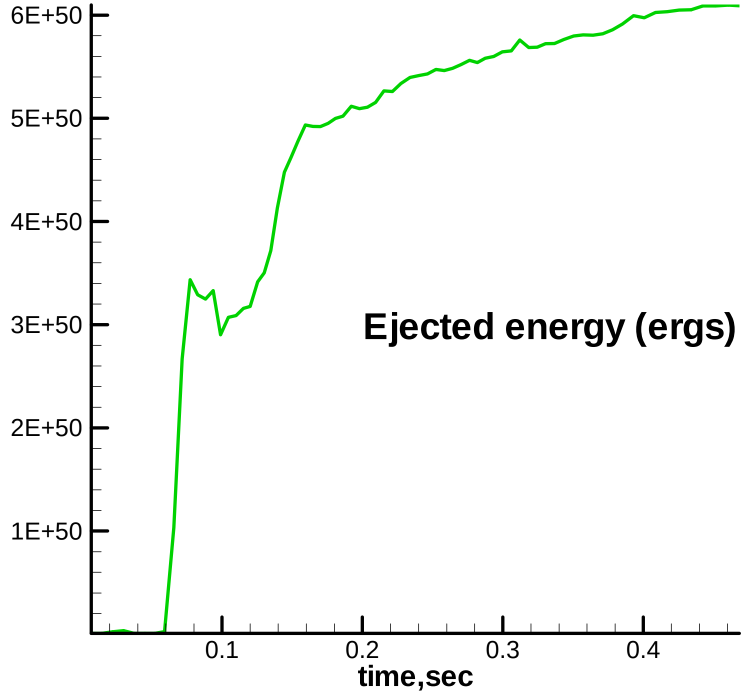


Ejected energy and mass

Ejected energy $0.6 \cdot 10^{51} erg$ Ejected mass $0.14 M_\odot$

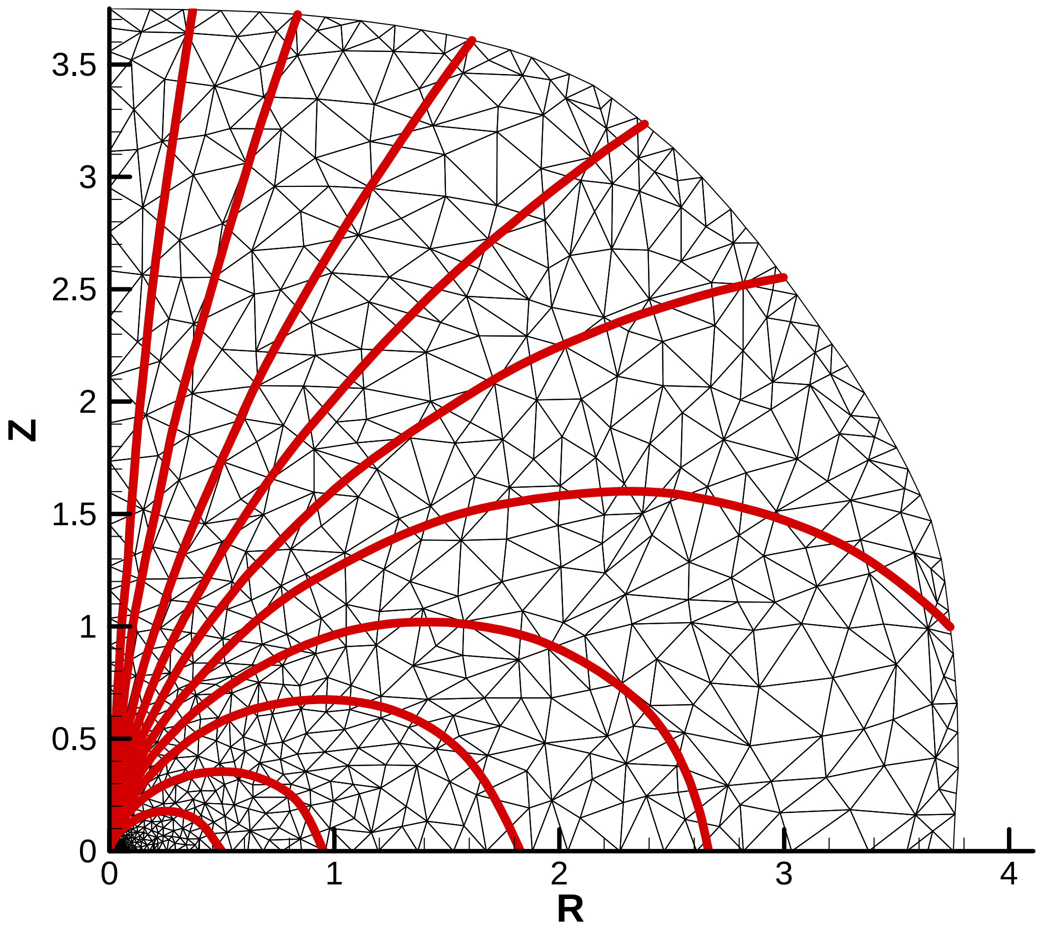
Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy



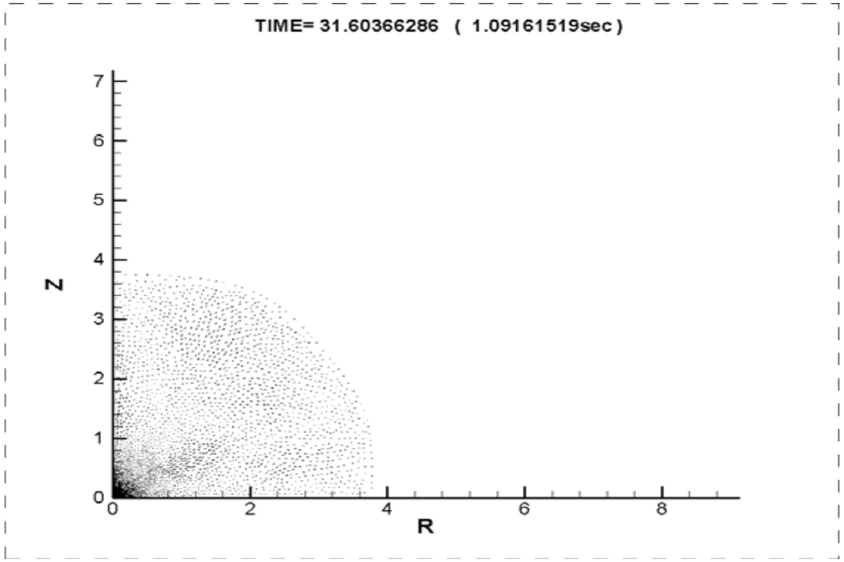


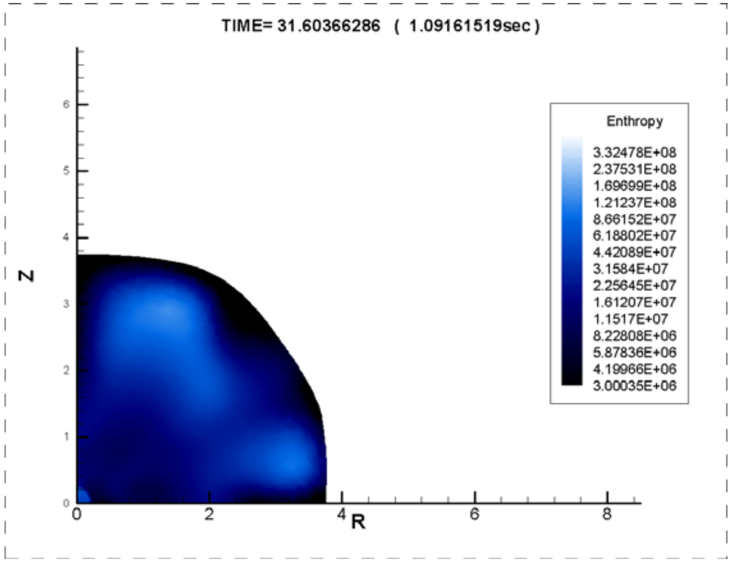
Initial magnetic field – dipole-like symmetry

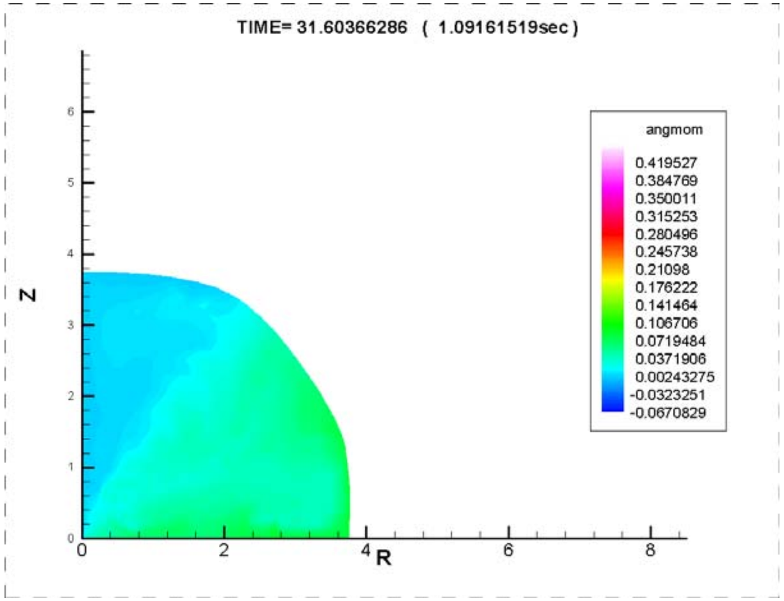
SM., Ardeljan & Bisnovatyi-Kogan MNRAS 2006, 370, 501



Magnetorotational explosion for the dipole-like magnetic field





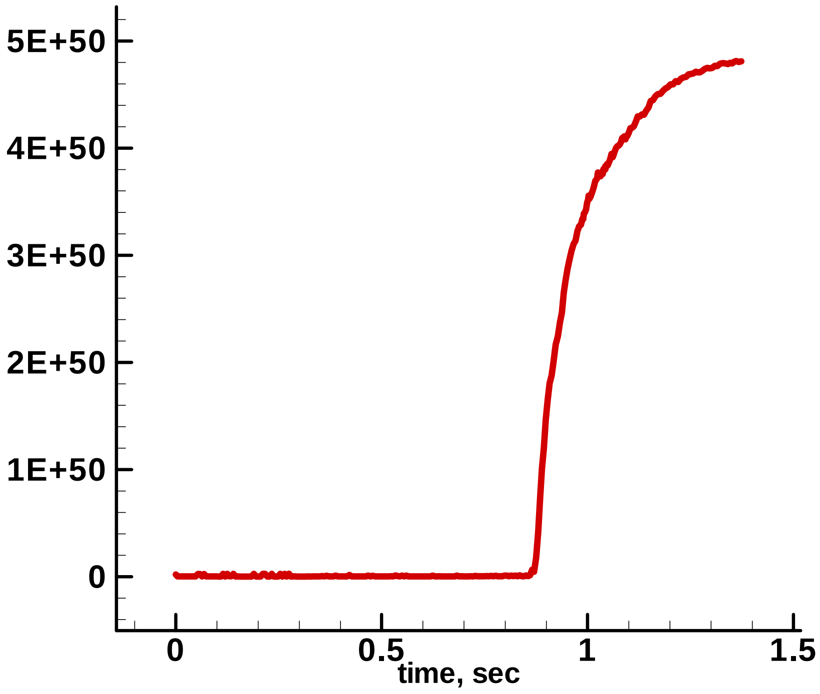


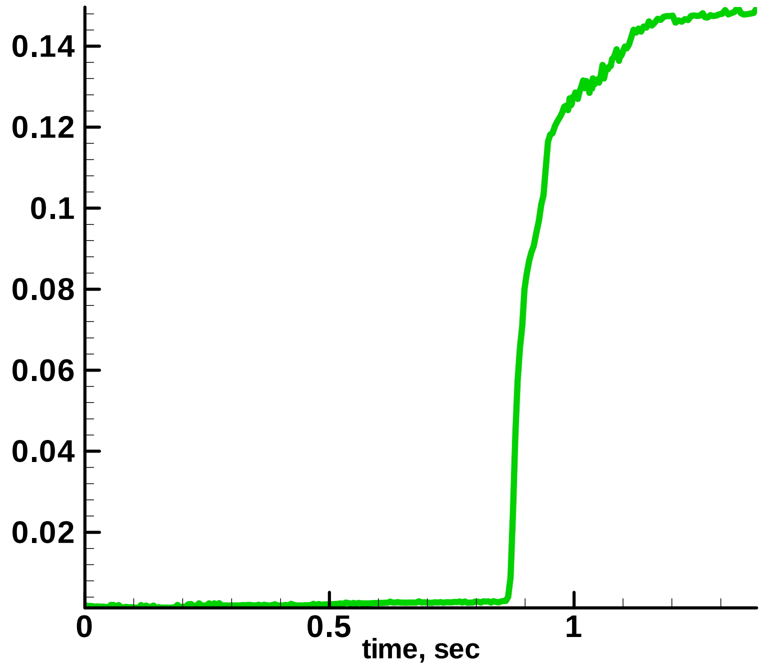
Mildly collimated jet.

Ejected energy and mass (dipole)

Ejected energy $\approx 0.5 \cdot 10^{51} erg$ Ejected mass $\approx 0.14 M_{\odot}$

Particle is considered "ejected" –
if its kinetic energy is greater than its potential energy





The magnetorotational supernova explosion is always asymmetrical.

but

Jet, kick and axis of rotation are **aligned** in MR supernovae.

Evidence for alignment of the rotation and velocity vectors in pulsars

S. Johnston et al. MNRAS, 2005, 364, 1397

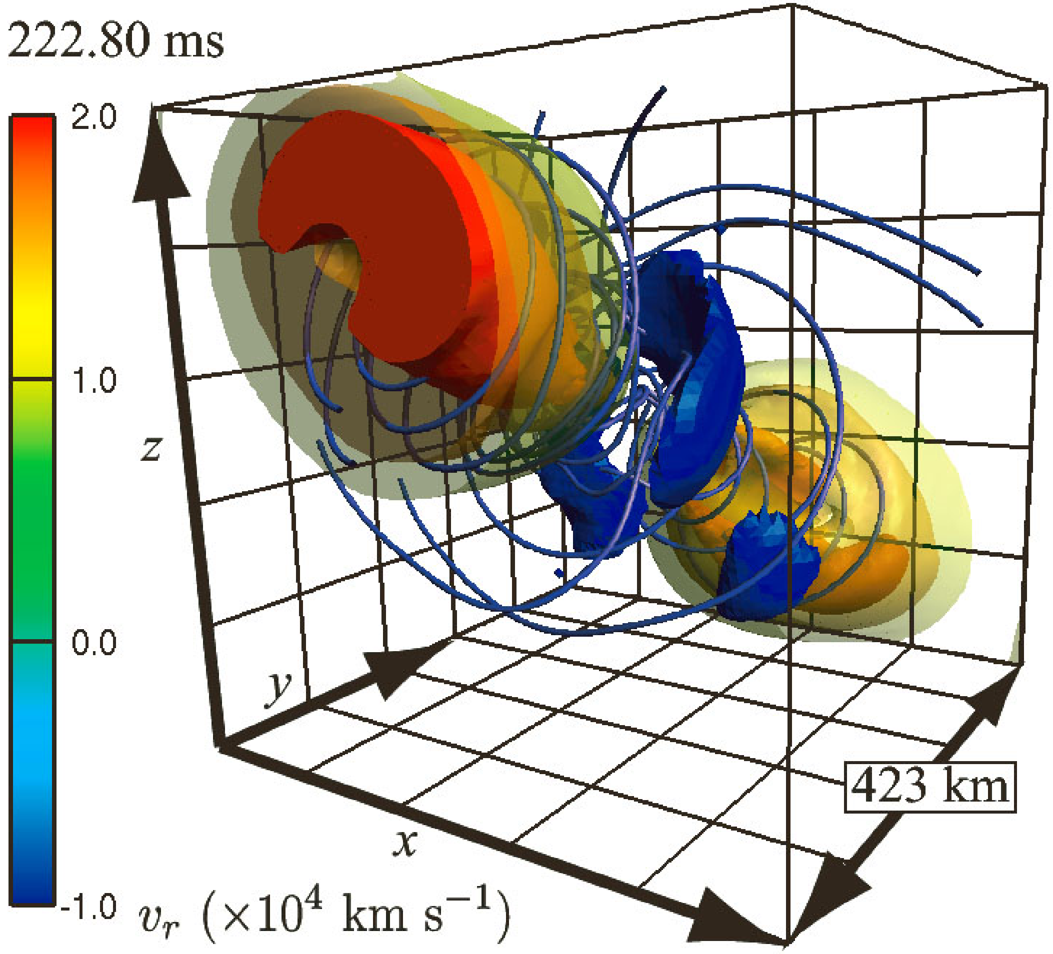
“We present strong observational evidence for a relationship between the direction of a pulsar's motion and its rotation axis. We show carefully calibrated polarization data for 25pulsars, 20 of which display linearly polarized emission from the pulse longitude at closest approach to the magnetic pole…

we conclude that the velocity vector and the rotation axis are aligned at birth“.

First 3D simulations of MR supernova (simplified)

Hanawa et al. ApJ 2008

(strong initial magnetic field, simple EoS, no neutrino transport



Rotational axis and jet axes are **aligned** !

Recent results

(in collaboration with K.Kotake,T.Takiwaki, K.Sato)

Implementation of modified (Shen et al., 1998) equation of state.

Approximate treatment of electron captures and neutrino transport. (Kotake et.al.2003) . Neutrino leakage scheme.

$$Y_l = Y_e + Y_\nu$$

$$\frac{dY_e}{dt} = -\gamma_e$$ Equation for electron fraction

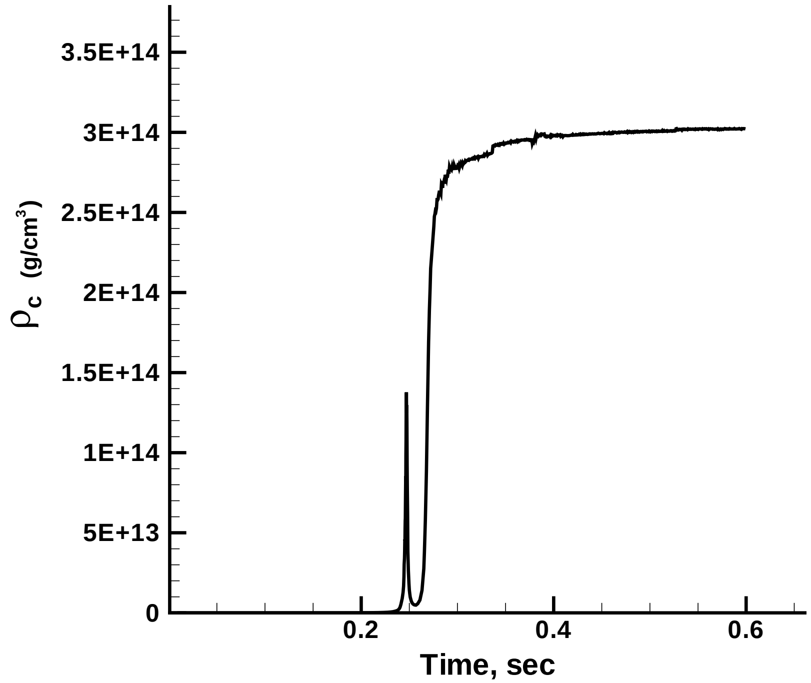
$$\frac{dY_l}{dt} = -\frac{Y_\nu}{\tau_{\rm esc}}$$ Equation for lepton fraction

$$L_\nu = \int dV \left(\epsilon_\nu \frac{Y_\nu}{\tau_{\rm esc}} \frac{\rho}{m_u} + \epsilon_\nu \gamma_e \frac{\rho}{m_u} \right)$$ - neutrino luminosity

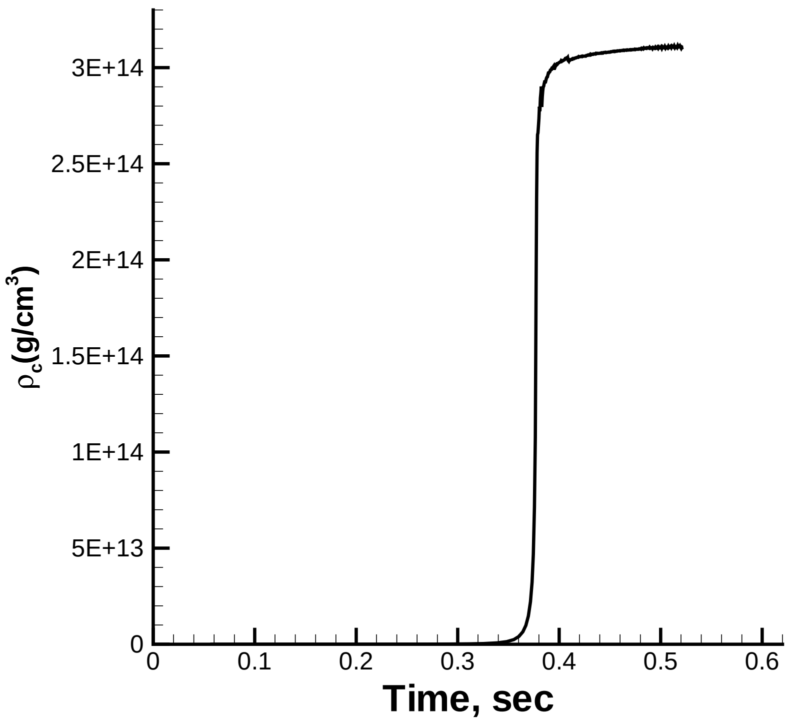
Neutrino pressure was taken into account.

Recent results
(in collaboration with K.Kotake,T.Takiwaki, K.Sato)

B_0=10(9)G



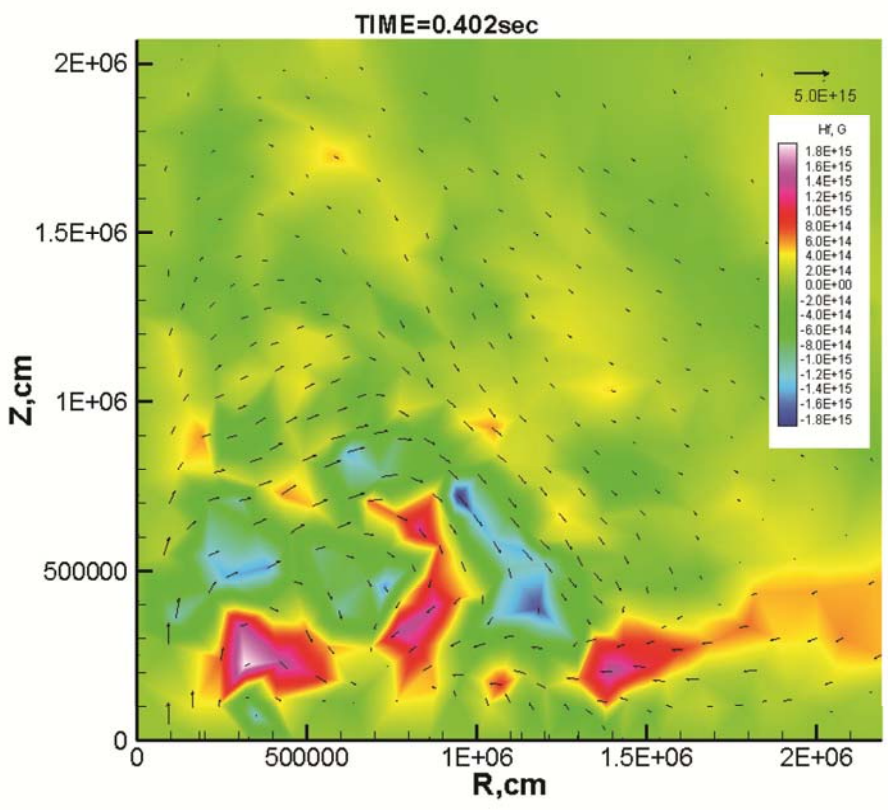
B_0=10(12)G



Recent results
(in collaboration with K.Kotake,T.Takiwaki, K.Sato)

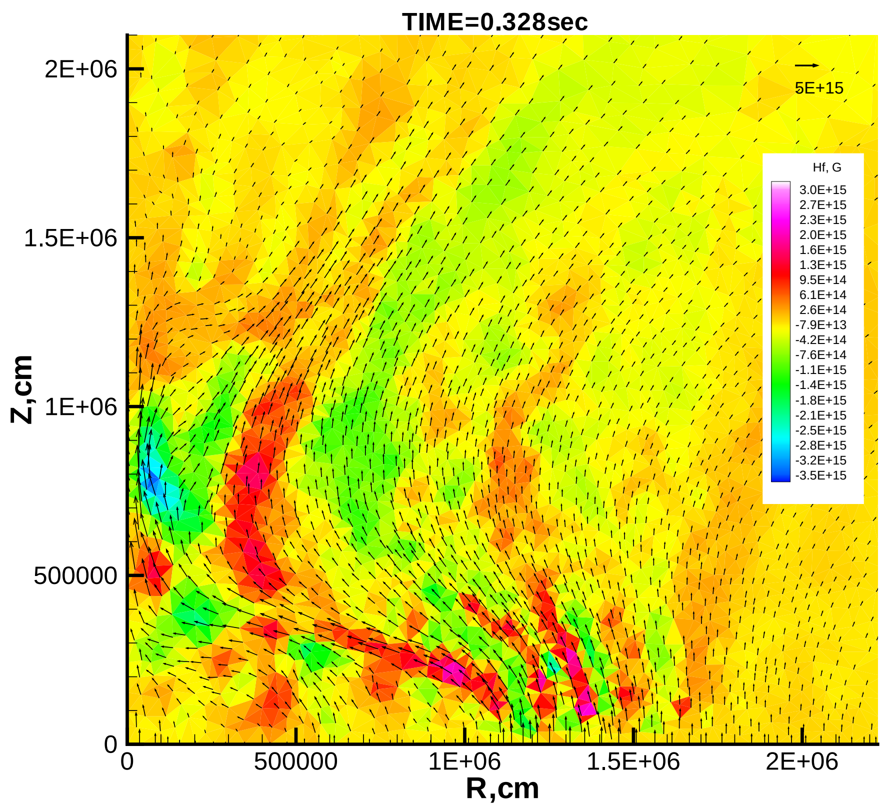
MRI

B_0=10(9)G



No MRI

B_0=10(12)G



Recent results

(in collaboration with K.Kotake,T.Takiwaki, K.Sato)

•Neutrino burst during core-collapse stage is very similar to previous result.

•Maximum neutrino luminosity is ~2.5 10(53) erg/sec at t~0.17sec.

•After core collapse angular velocity steeply decreases outwards.

•The MRI is developed as it was for ‘old’ equation of state.

•The MR supernova explosion energy is ~ 0.6 10(51) erg.

Conclusions

- Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.
- The MRM is weakly sensitive to the equation of state and details of neutrino cooling mechanism.
- MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.
- MRI developes in MR supernova explosion.
- One sided jets and rapidly moving pulsars can appear due to MR supernovae.
- 3D simulations of MR supernova with full physics are necessary.