Nonlinear Effect of R-mode Instability in Uniformly Rotating Stars

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1. Introduction

Various Instabilities in Secular Timescale

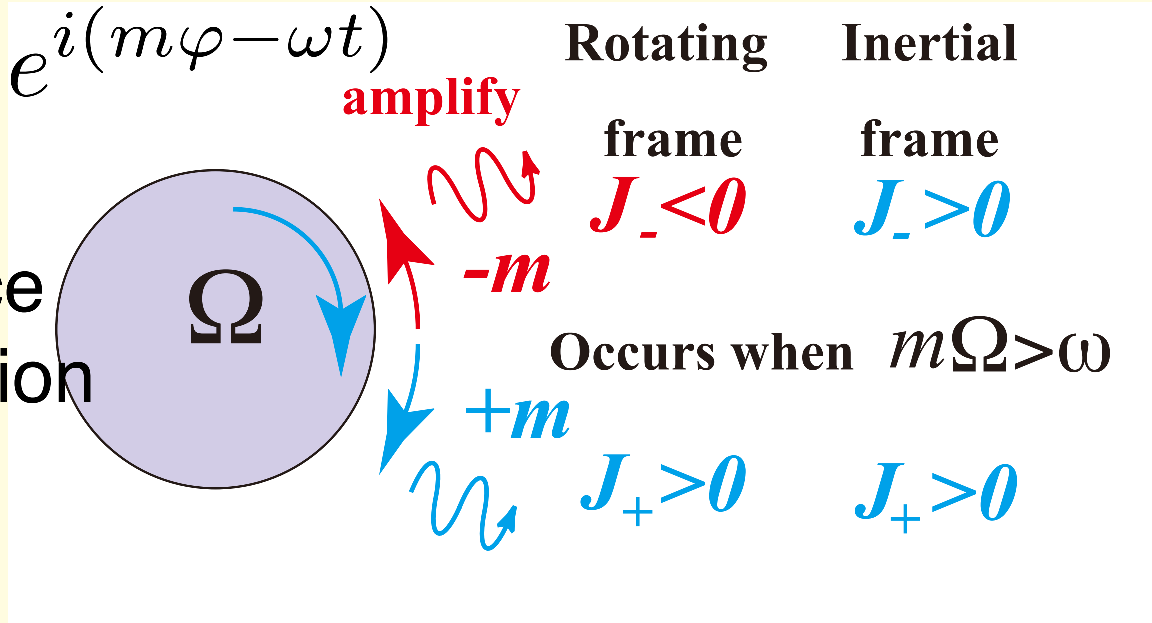
CFS instability

(Chandrasekhar 70, Friedman & Schutz 78)

- Fluid modes (f, p, g-modes) may becomes unstable due to gravitational radiation
- Instability occurs in dissipative timescale

r-mode instability

(Andersson 98, Friedman & Morsink 98)



- Fluid elements oscillate due to Coriolis force
- Instability occurs due to gravitational radiation

g-mode instability

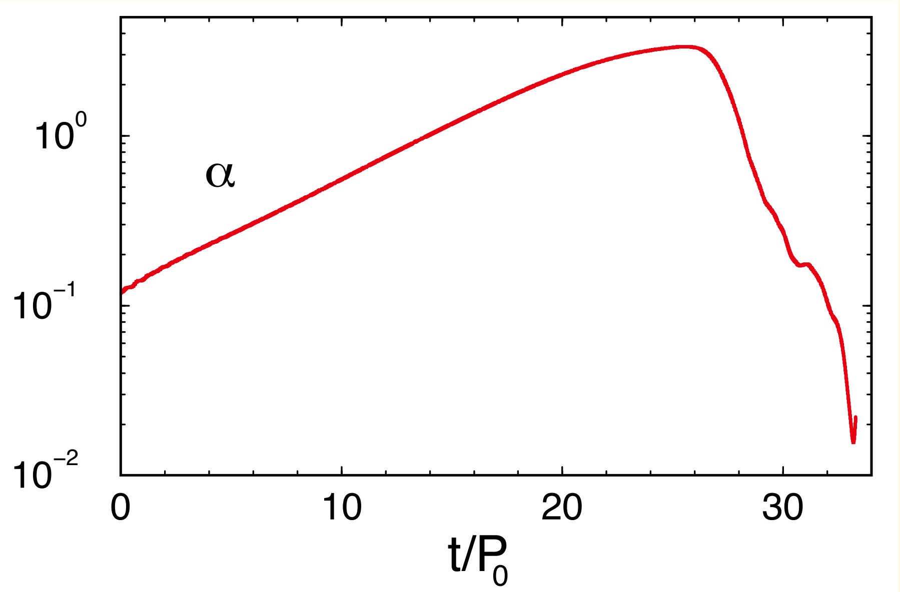
- Fluid elements oscillate due to restoring force of buoyancy
- Instability occurs in nonadiabatic evolution or in convective unstable cases

Kelvin-Helmholtz instability

- Instability occurs when the deviation of the velocity between the different fluid layers exceeds some critical value

Dynamics of r-mode instabilities

Saturation amplitude of r-mode instability



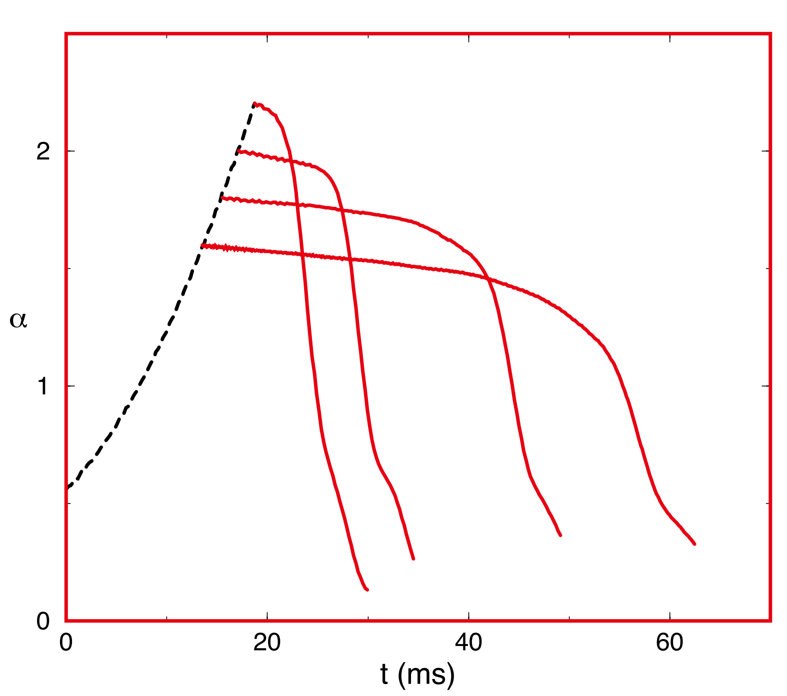
3D simulation

- Saturation amplitude of o(1)
- Imposing large amplitude of radiation reaction potential in the system to control secular timescale with dynamics (Lindblom et al. 00)

1D evolution with partially included 3 wave interaction

- Saturation amplitude of ~ o(0.001), which depends on interaction term (Schenk et al. 2001)

Final fate of r-mode instability



3D simulation

- Evolution starting from the amplitude o(1)
- Imposing large amplitude of radiation reaction potential
- Energy dissipation of r-mode catastrophically decays to differentially rotating configuration in dynamical timescale

1D evolution including mode couplings network

- After reaching the saturation amplitude ~o(0.001), Kolmogorov-type cascade occurs
- Destruction timescale is secular

(Gressman et al. 02, Lin & Suen 06)

Dynamics of r-mode instabilities

Saturation amplitude of r-mode instability

3D simulation

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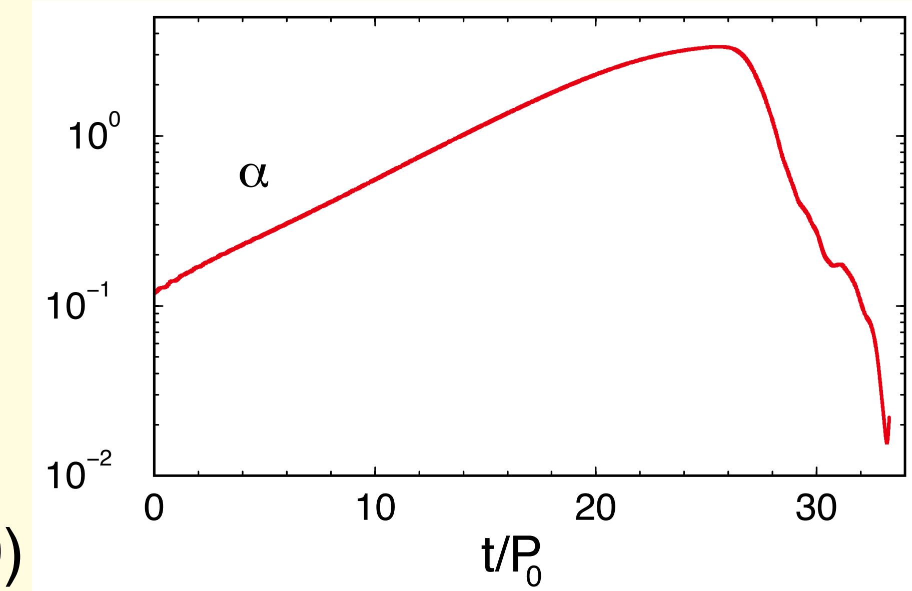


1D evol

- Satur n

Final fa 2001)

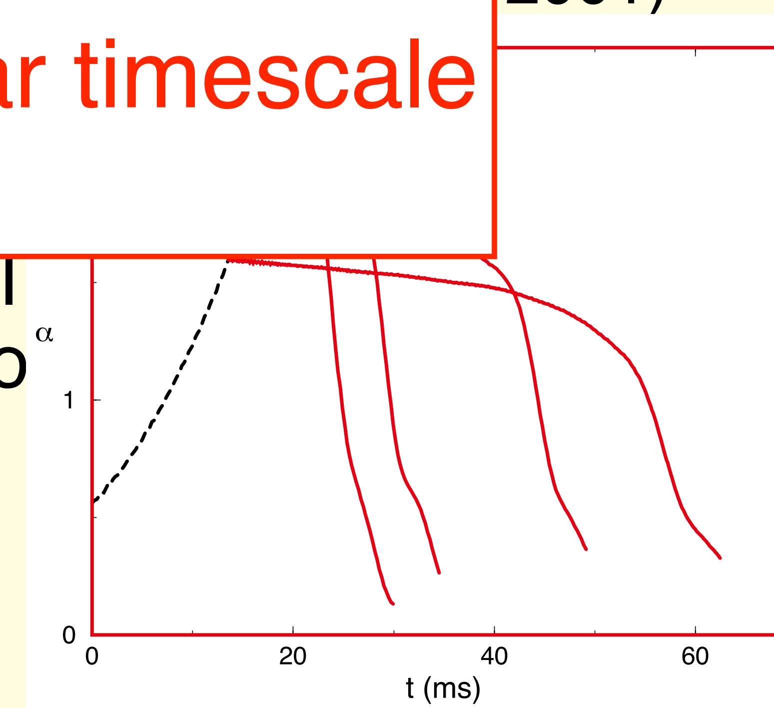
Alternative approaches

- From linear regime to nonlinear regime
- From dynamical timescale to secular timescale

are necessary!

3D si

- Evolut
- Imposing large amplitude of radiation reaction potential
- Energy dissipation of r-mode catastrophically decays to differentially rotating configuration in dynamical timescale



1D evolution including mode couplings network

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(Gressman et al. 02, Lin & Suen 06)

Amplitude of r-mode instability

(LIGO 10)

- Isolated neutron star in the supernova remnant Cassiopeia A
- Compelling evident that the central compact object is neutron star
- Restriction to the amplitude of the r-mode instability by not detecting gravitational waves

$$\alpha \approx 0.14 - 0.005$$

Possibility of gravitational wave source

(Bondarescu et al. 09)

- Possibility of parametric resonance by nonlinear mode-mode interaction
- Amplification to $\alpha \sim 1$

Necessary to obtain a common knowledge for the basic properties of r-mode instability !

2. Dynamics beyond acoustic timescale

- Timescale which cannot be reached by GR hydrodynamics
  - ▶ Need to separate the hydrodynamics and the radiation term
- Instability driven by gravitational radiation
  - ▶ Need to impose gravitational waves

"Newton gravity + gravitaional radiation reaction" are at least necessary

Gravitational radiation reaction

(Blanchet, Damour, Schafer 90)

Quadrupole radiation metric

(includes 2.5PN term)

$$h_{ij} = -\frac{4G}{5c^5}\epsilon\frac{d^3 I_{ij}^{TT}}{dt^3}$$

amplification factor to control the radiation reaction timescale

Gravitational radiation reaction potential

$$\Phi^{(\mathrm{RR})} = \frac{1}{2}(-\psi + h_{ij}x^j\nabla_i\Phi)$$

$$\triangle\psi = 4\pi h_{ij}x^j\nabla^i\rho$$

Dynamics beyond the acoustic timescale

- Shortest timescale in the system restricts the maximum timestep for evolution
  ▶ Acoustic timescale in Newtonian gravity (control acoustic timescale)
- Relax the restriction from the rotation of the background star
  ▶ Introduce rotating reference frame

Anelastic approximation

Kill the degree of freedom of the sound wave propagation

$$v^j \nabla_j h + (\Gamma - 1) h \nabla_j v^j = 0$$

▼ No shocks

$$\boxed{\nabla_j (\rho v^j) = 0}$$

Linear regime

(Villain & Bonazzolla 02)

$$\nabla_j (\rho_{\mathrm{eq}} v^j) = 0$$

Propagation of the sound wave

$$\frac{\partial \rho}{\partial t} = \frac{1}{c_s^2} \frac{\partial P}{\partial t} = -\nabla_j (\rho v^j)$$

$$\frac{\partial}{\partial t} \nabla_j (\rho v^j) + \triangle P = S$$

▶

$$\left(\triangle - \frac{1}{c_s^2} \frac{\partial}{\partial t} \right) P = S$$

Imposing the anelastic approximation changes the structure of the pressure equation

Basic equations in rotating reference frame (Lie derivative)

Time evolution

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial}{\partial t}(\rho u_i) + \nabla_j(\rho u_i v^j) = -\nabla_i p - \rho \nabla_i (\Phi + \Phi^{(\mathrm{RR})})$$
$$-\rho(v^j_{(\mathrm{eq})} + v^j)\nabla_j u_{i\ (\mathrm{eq})} + \rho u_j \nabla_i v^j_{(\mathrm{eq})}$$

Spatial component of the momentum velocity

$$u_{i(\mathrm{eq})} + u_i = \tilde{\gamma}_{ij}(v^j_{(\mathrm{eq})} + v^j)$$

spatial metric $\tilde{\gamma}_{ij} = \delta_{ij} + h_{ij}$

up to 1st order of ϵ

Pressure poisson equation

$$\triangle p = S_p$$

Boundary condition:
P=0 at the stellar surface

Anelastic approximation (constraint)

$$\nabla_j(\rho v^j) = 0$$

Need a special technique to satisfy constraints throughout the evolution

Procedure

Similar procedure to SMAC method, which is used to solve Navie-Stokes incompressible fluid (McKee et al. 08)

1. Time update the linear momentum

$$(\rho\Delta u_i)^{(*)} = (\rho\Delta u_i)^{(n)} - \Delta t[\nabla_i p + \cdots]$$

Note that the velocity does not automatically satisfy anelastic condition

2. Introduce an auxiliary function ϕ and solve the following Poisson's equation

$$(\triangle\phi)^{(*)} = \partial_j(\rho\Delta v^j)^{(*)}$$

Boundary Condition: $\phi = 0$ at the stellar surface

3. Adjust the 3-velocity in order to satisfy the anelastic condition

$$(\rho\Delta v^i)^{(n+1)} = (\rho\Delta v^i)^{(*)} - (\partial^i\phi^{(*)})$$

4. Introduce another auxiliary function ψ and solve the following Poisson's equation

$$\triangle\psi = \delta^{ij}[\partial_j(\rho\Delta u_i)^{(*)} - \partial_j(\rho\Delta u_i)^{(n+1)}]$$

Boundary Condition: $\psi = 0$ at the stellar surface

5. Time update the pressure

$$p^{(n+1)} = p^{(n)} + \frac{\psi^{(*)}}{\Delta t}$$

3. Nonlinear r-mode instability

r_p/r_e	T/W
0.55	0.102
0.65	0.088
0.70	0.076
0.75	0.062

Equilibrium configuration of the star

- Rapidly rotating neutron star
- Uniformly rotating, n=1 polytropic equation of state

Eigenfunction and eigenvector of r-mode in incompressible star

Eigenfunction of the velocity

$$\delta v = \alpha \Omega R \left(\frac{r}{R}\right)^l Y_{ll}^{(B)}$$

$$\alpha = 1 \times 10^{-4}$$

Impose eigenfunction type perturbation on the equilibrium velocity to trigger r-mode instability

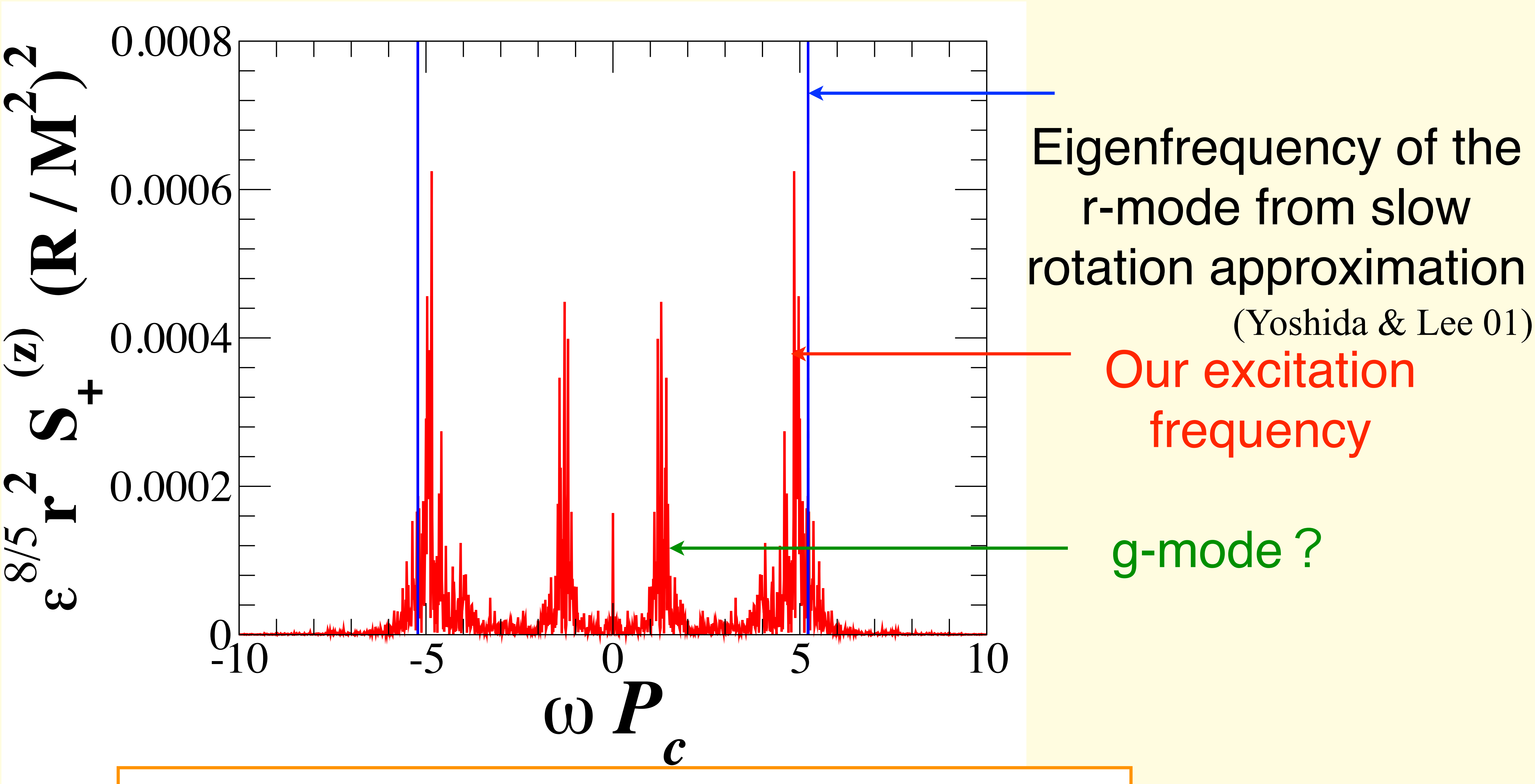
Eigenfrequency (rotating reference frame)

$$\omega = \frac{2m}{l(l+1)}\Omega$$

Incompressible star case

Check the excitation of the eigenfrequency

Spectrum

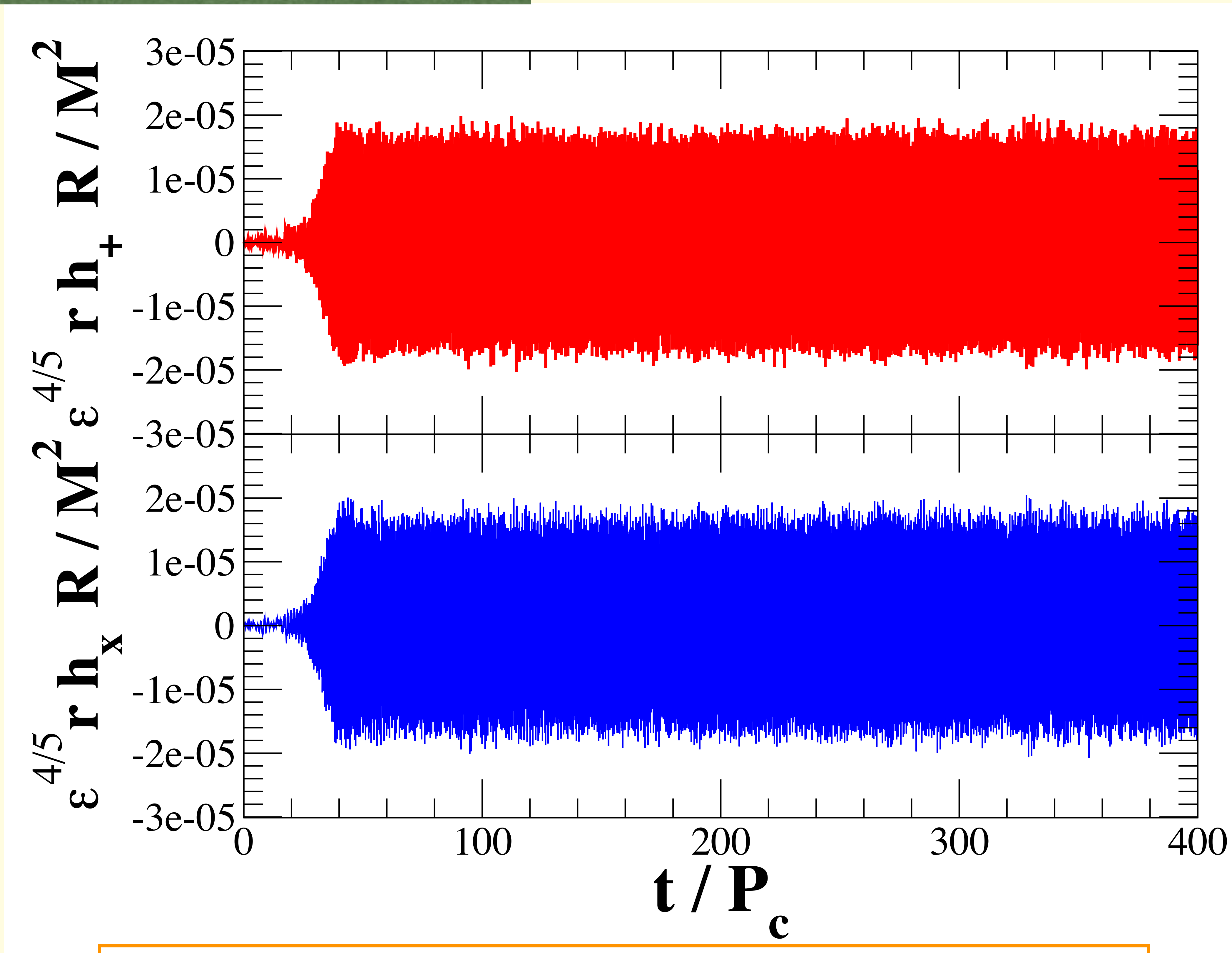


Due to

- slow rotation approximation
- anelastic approximation

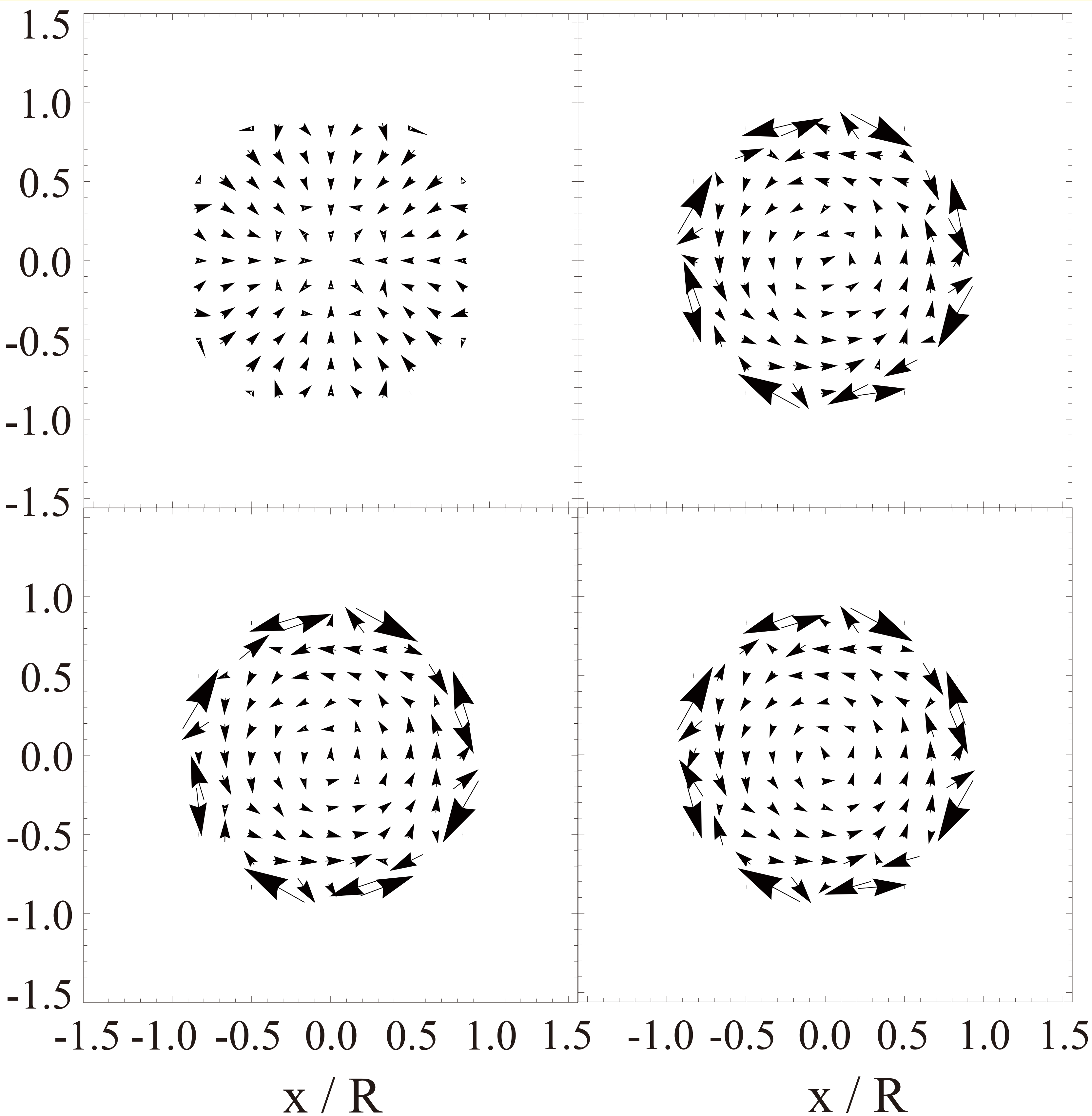
the eigenfrequency does not perfectly agree

Gravitational Waveform



Saturation amplitude is around $\alpha \approx 10^{-3}$

Velocity profile



- No velocity profile appears in the equatorial plane in linear and slow rotation regime of r-mode instability

Effect of rapid rotation and nonlinearity

- Shock wave seems to form at the surface as the times goes on

"Destruction" of r-mode instability

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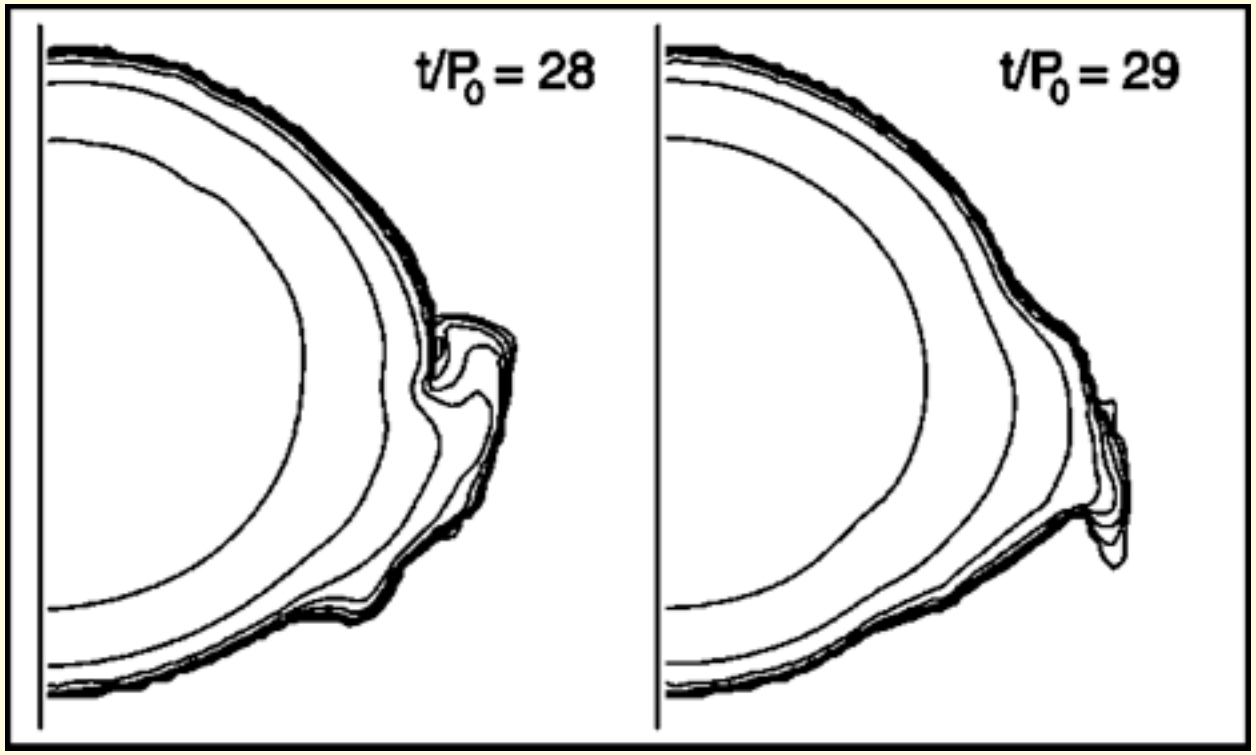
Comment to the Catastrophic Decay?

- Lindblom et al. shows in their paper that the catastrophic decay is due to the shocks and the breaking waves at the surface
- Anelastic approximation kills the dominant contribution of the density fluctuation
- Computation with small amplitude of velocity perturbation with Newtonian hydrodynamics may answer the question



(Lindblom et al. 02)

Might be very difficult

4. Summary

We investigate the r-mode instability of uniformly rotating stars by means of three dimensional hydrodynamical simulations in Newtonian gravity with radiation reaction

- We have succeeded in constructing a nonlinear anelastic approximation in the rotating reference frame, which kills the propagation of sound speed, in order to evolve the system beyond the dynamical timescale.
- When the current multipole contribution is dominant to the r-mode instability (density fluctuation effect is negligible), the instability seems to last for at least hundreds of rotation periods
- Studies of no anelastic approximation with small amplitude of velocity perturbation may help us for a better understanding