Numerical Simulation of Two Component Advective Flow (TCAF) : Is the Flow Stable ?

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Boundary Conditions for BHs:

- At the horizon

  Flow radial velocity $v = c \Rightarrow$ **Super-sonic**

  Flow angular momentum ($\lambda < \lambda_{kep}$)$\Rightarrow$sub- Keplerian

- **For r $\rightarrow \propto$**

  $v \sim 0 \Rightarrow$ sub-sonic

$$\Downarrow$$

- Accretion on to BH is necessarily transonic

Shocks in Advective Accretion Process

- $F_{GR} \sim -1/(r - r_g)^2$
- $F_{CEN} \sim \lambda^2/r^3$; λ=Constant
- $F_{GR} >> F_{CEN}$ at $r \sim r_g$ and $r \sim \infty$
- At intermediate distance $F_{GR} \sim F_{CEN}$

$$\Rightarrow v \downarrow \text{ and } \rho \uparrow$$

➢Slowed down inner part of the disk ≡ effective boundary layer of the BH.

➢ If slowing down is discontinuous

=> SHOCK!!!

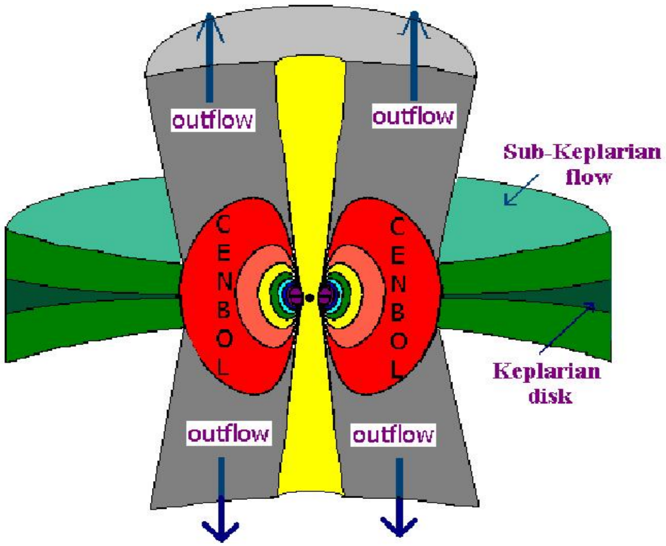
➢The hot puffed up post-shock region commonly known as CENBOL (CENtrifugal pressure supported Boundary Layer)

Black Hole Accretion

✓ Standard Accretion Disk: Geometrically thin and optically thick flow with a Keplerian angular momentum distribution

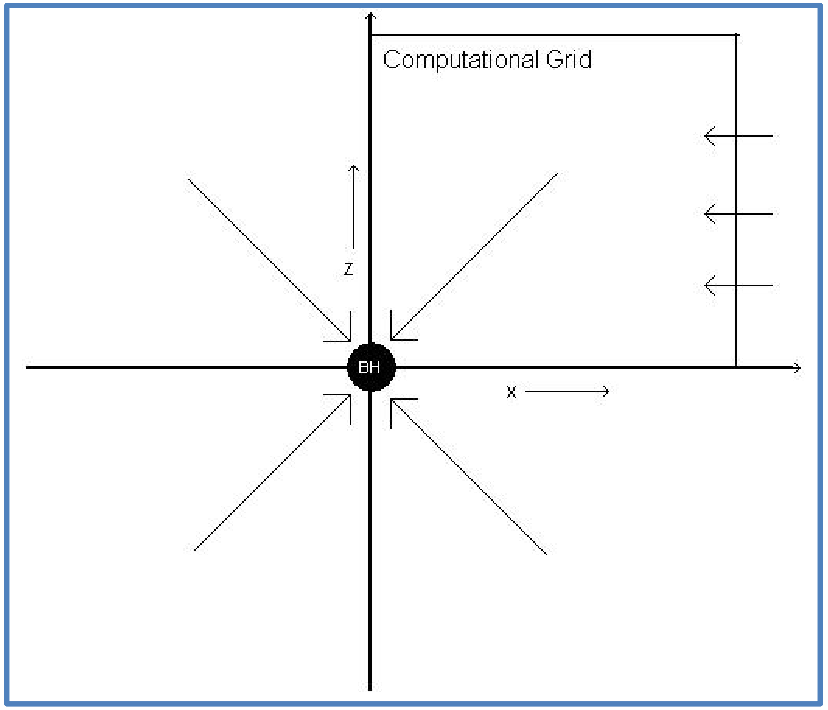
✓ Sub-Keplerian Disk: Optically thin and Geometrically thick flow and angular momentum is less than Keplerian angular momentum

***Chakrabarti and Titarchuk* (1995) pointed out that the Spectral and Timing properties of black holes can be explained very well by a Two Component Advective Flow (TCAF) where the high viscosity Keplerian disk is surrounded by a low-viscosity sub-Keplerian flow**



We now show, using a numerical simulation, how this TCAF is naturally formed when viscosity decreases monotonically in the vertical direction

Schematic diagram of TCAF model with all components



Units

Distance (r_g) = $2GM_{BH}/c^2$
Velocity = c
Specific Angular Momentum = $2GM_{BH}/c$

- We consider the hydrodynamics of axisymmetric flows of gas under the Pseudo-Newtonian gravitational field of a point mass M_{BH} (representing a black hole) located at the centre in cylindrical coordinate [r,θ,z] system

- The gravitational field of the black hole can be described by Paczy´nski & Wiita (1980), $\phi(r, z) = -GM_{BH}/(R - r_g)$, where, G = Gravitational Constant, $R = \sqrt{(r^2 + z^2)}$ and the Schwarzschild radius is given by $r_g = 2GM_{BH}/c^2$, c = light velocity

- We also assume a polytropic equation of state for the accreting (or outflowing) matter, $P = K\rho^{\gamma}$.

The Equations for Non-Viscous Flow

$$\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{x}\frac{\partial (x\boldsymbol{F}_1)}{\partial x} + \frac{\partial \boldsymbol{F}_2}{\partial x} + \frac{\partial \boldsymbol{G}}{\partial z} = \boldsymbol{S},$$

where the state vector is

$$\boldsymbol{q} = \begin{pmatrix} \rho \\ \rho v_x \\ \rho v_\theta \\ \rho v_z \\ E \end{pmatrix},$$

the flux functions are

$$\boldsymbol{F}_1 = \begin{pmatrix} \rho v_x \\ \rho v_x^2 \\ \rho v_\theta v_x \\ \rho v_z v_x \\ (E+p)v_x \end{pmatrix}, \quad \boldsymbol{F}_2 = \begin{pmatrix} 0 \\ p \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{G} = \begin{pmatrix} \rho v_z \\ \rho v_x v_z \\ \rho v_\theta v_z \\ \rho v_z^2 + p \\ (E+p)v_z \end{pmatrix}$$

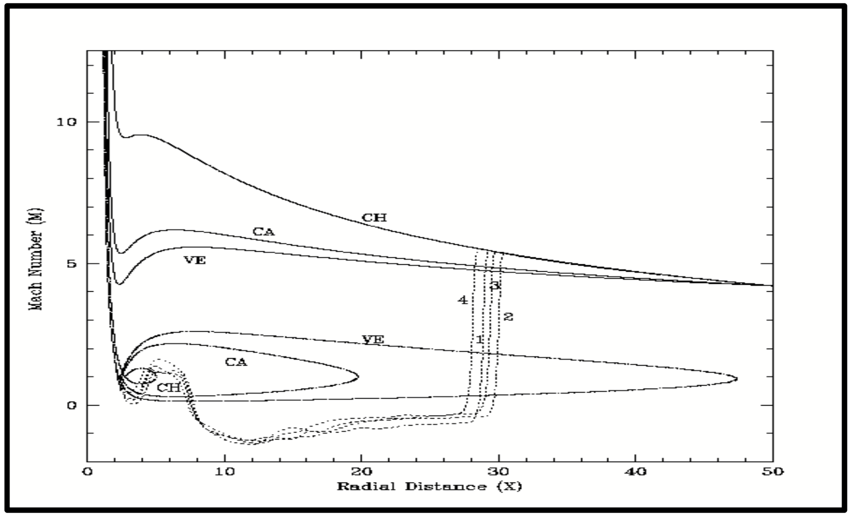
Here energy density E (without potential energy) is defined as $E = p/(\gamma - 1) + \rho(v_x^2 + v_\theta^2 + v_z^2)/2$.

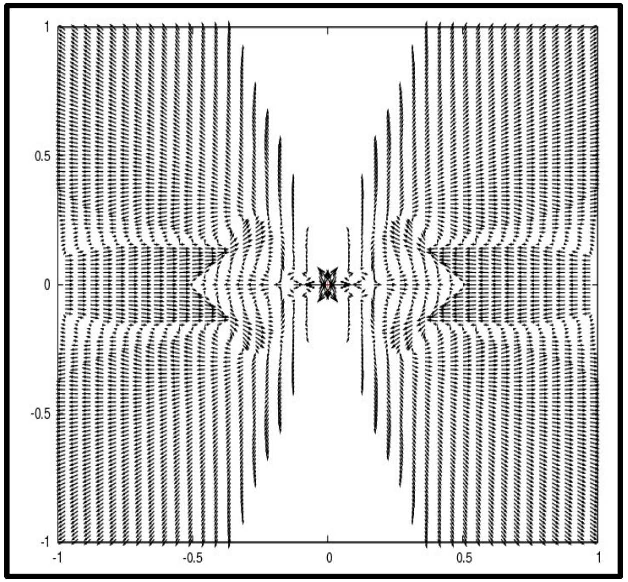
$$\boldsymbol{S} = \begin{pmatrix} 0 \\ \dfrac{\rho v_\theta^2}{x} - \dfrac{\rho x}{2(\sqrt{x^2+z^2}-1)^2\sqrt{x^2+z^2}} \\ -\dfrac{\rho v_x v_\theta}{x} \\ -\dfrac{\rho z}{2(\sqrt{x^2+z^2}-1)^2\sqrt{x^2+z^2}} \\ -\dfrac{\rho(x v_x + z v_z)}{2(\sqrt{x^2+z^2}-1)^2\sqrt{x^2+z^2}} \end{pmatrix}.$$

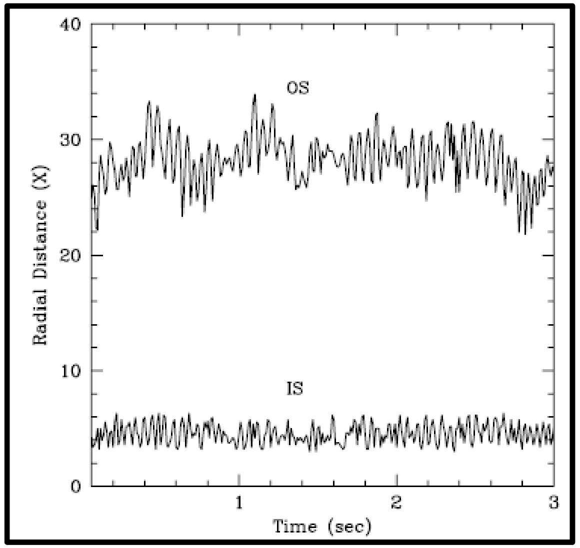
For non viscous solutions, We have used the mesh based finite difference TVD code which was originally developed Prof. Dongsu Ryu and his collaborators

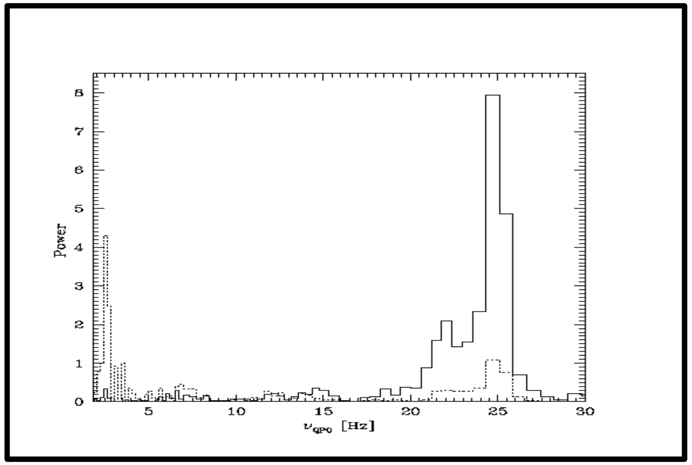
Mach Number distributions, shocks, velocity vectors, shock oscillations for

Non-Viscous Sub-Keplerian Flows









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Viscosity Prescription for a Thin Accretion Flow

The V_φ component of the momentum equation

$$\rho\left[\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\phi v_r}{r} + \frac{v_\phi}{r}\frac{\partial v_\phi}{\partial \phi} + v_z\frac{\partial v_\phi}{\partial z}\right] =$$

$$-\frac{1}{r}\frac{\partial P}{\partial \phi} + \mu\left[\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{\partial^2 v_\phi}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_\phi}{\partial \phi}\right] + F_\phi.$$

For a thin, axisymmetric flow the viscous term reduces to

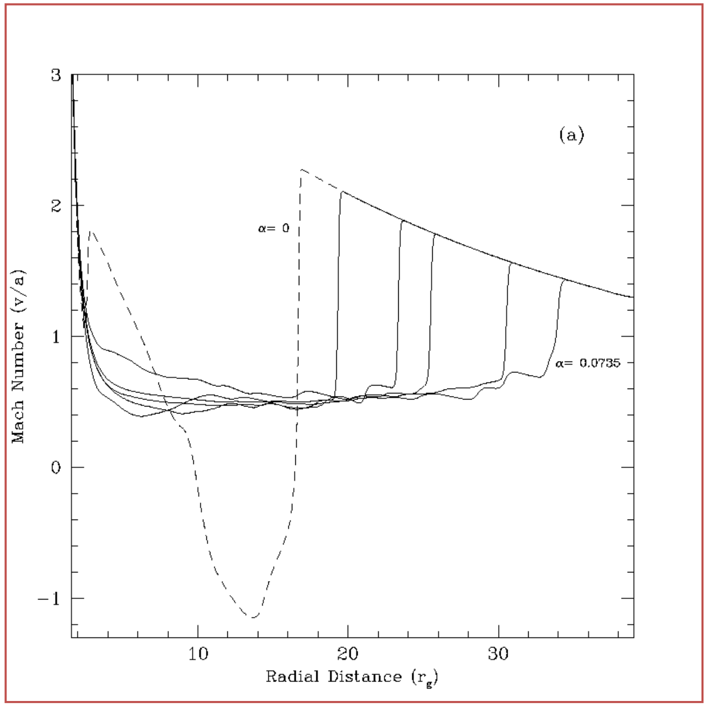
$$\mu\left[\frac{\partial^2 v_\phi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r^2}\right]$$

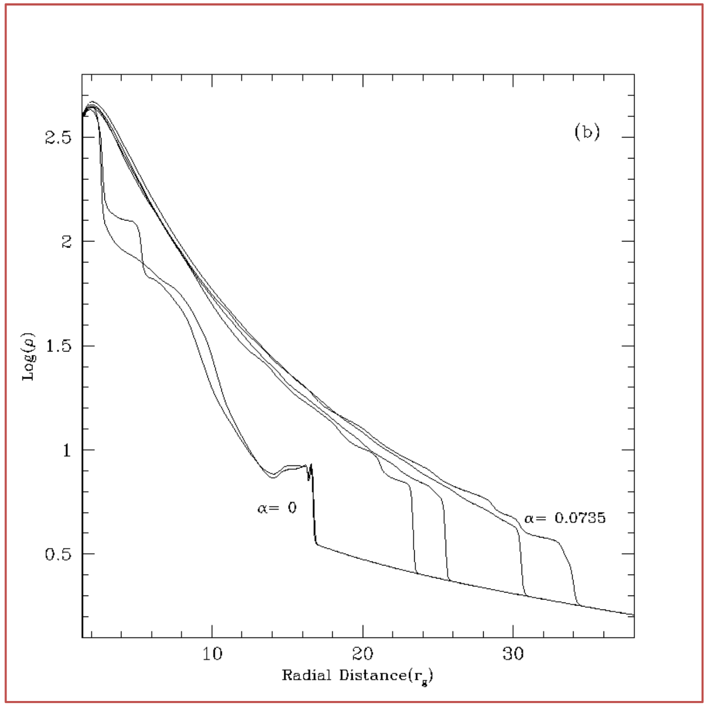
$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \tau_{r\phi}),$$

Shakura & Sunyaev alpha prescription for the shearing viscosity stress

For a thin flow, $T_{r\varphi} = -\alpha P$, where, P = thermal pressure and α is the viscous parameter with $0 \leq \alpha < 1$

Variation of (a) Mach number and (b) density with radial distance in a viscous transonic flow on the equatorial plane. As the viscosity parameter is increased, the angular momentum is transported outward shifting along with it the centrifugal pressure supported shock wave. The α parameters are [left to right in (a) and bottom to top in (b)] : 0.0, 0.0175, 0.035, 0.0525, 0.06125, 0.07, 0.0735 respectively.





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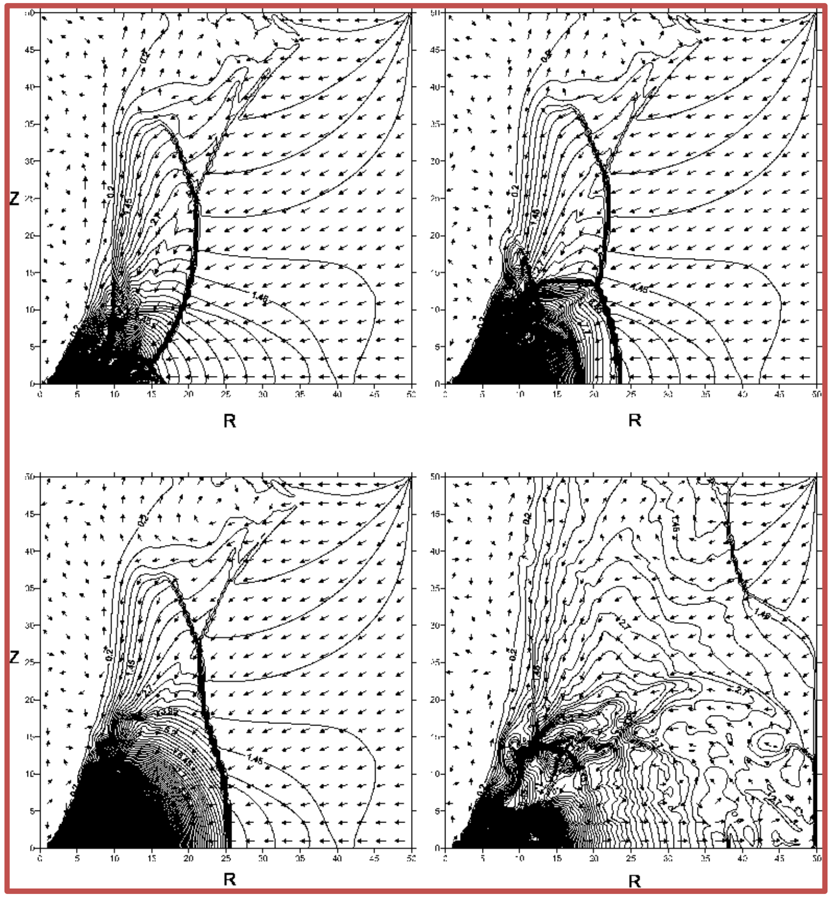
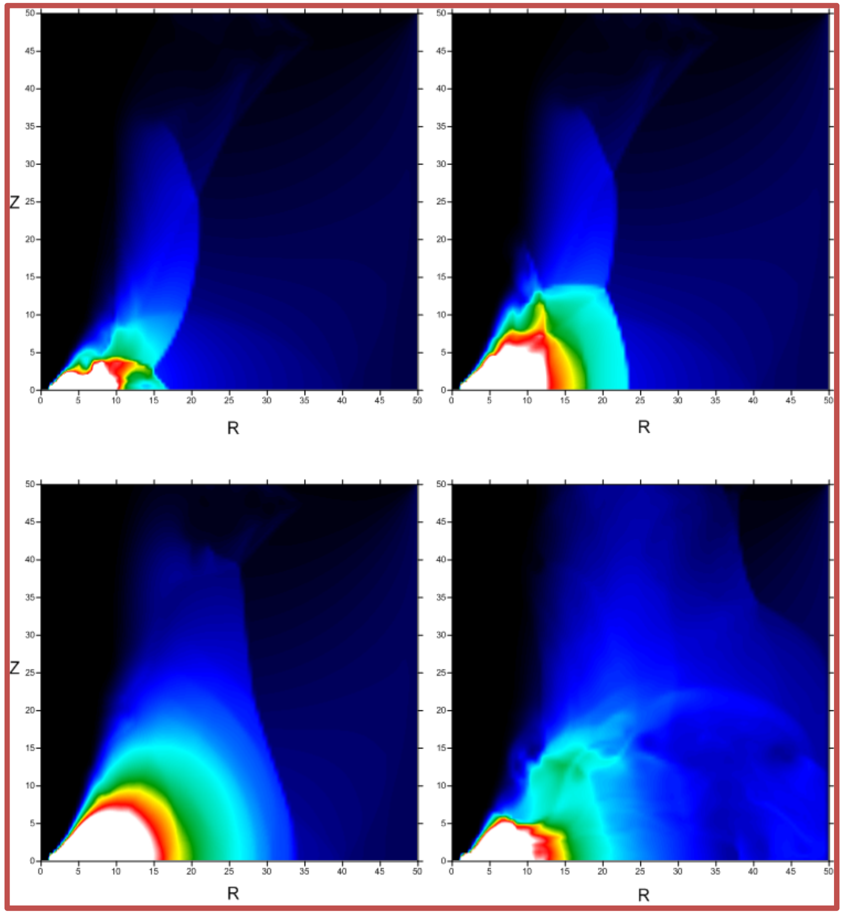
Changes in the density and velocity distribution with the change of viscous parameter at t = 24.75s. Here, densities are in normalized unit, radius and velocity are in Schwarzschild unit. Here, α = 0.0 (top left), 0.0525 (top right), 0.0735 (bottom left) 0.074 (bottom right) respectively.





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Considering a thick flow, all viscous stress components are taken

V_r component :

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\phi^2}{r} + \frac{v_\phi}{r}\frac{\partial v_r}{\partial \phi} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial P}{\partial r} + \mu\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \phi^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\phi}{\partial \phi}\right] + F_r.$$

V_z Component :

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\phi}{r}\frac{\partial v_z}{\partial \phi} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + F_z$$

We have used the formula for μ as

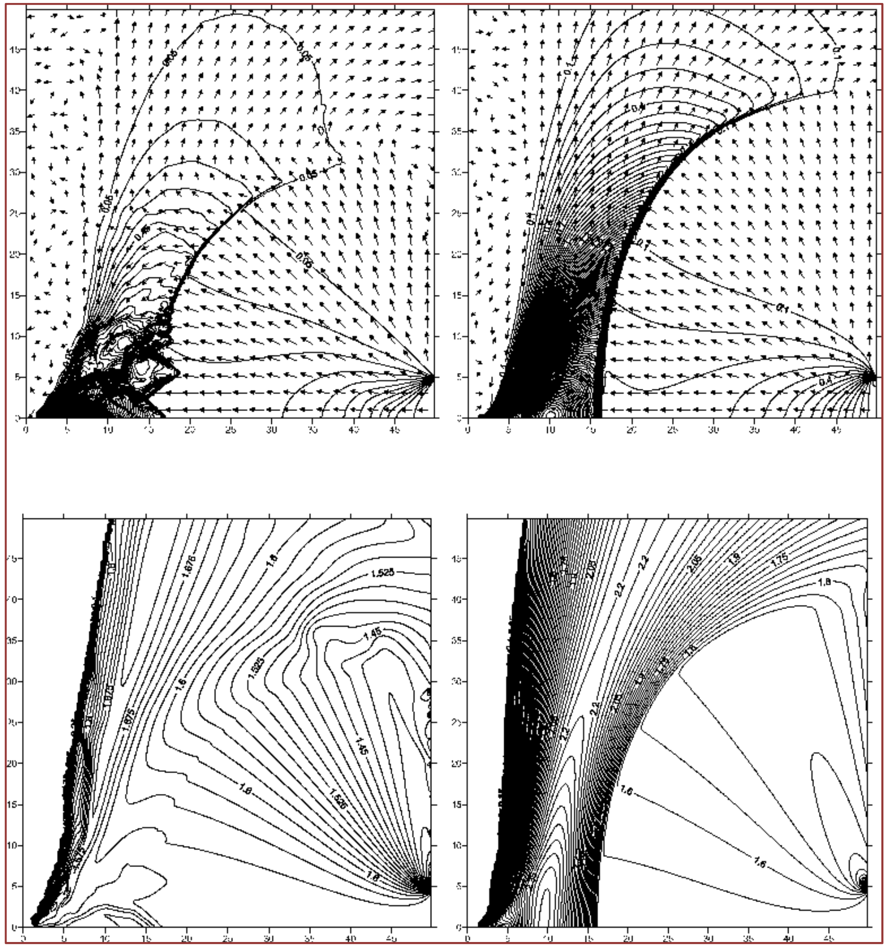
$$\mu = \alpha_s \rho \frac{a^2}{\Omega_k},$$

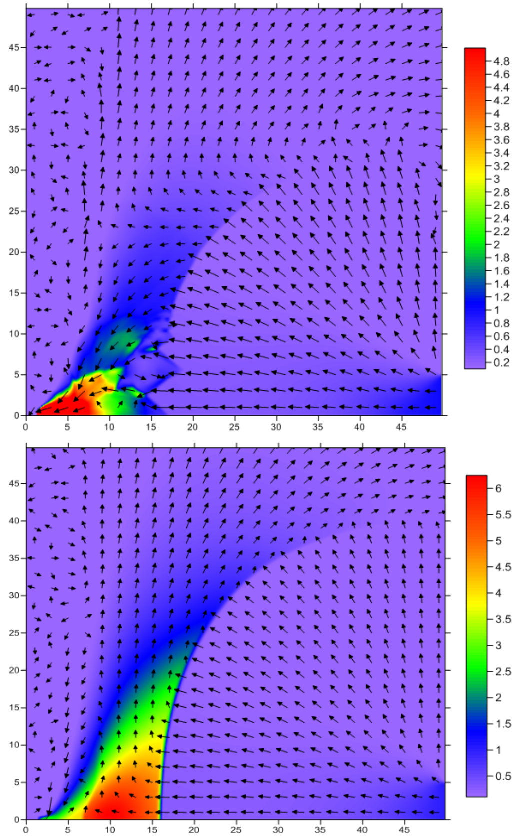
where, α_s is constant of order 1, a is adiabatic sound speed and

Keplerian angular velocity

$$\Omega_k = \left[\frac{1}{r}\frac{\partial \Phi}{\partial r}\right]^{\frac{1}{2}}$$

Distribution of density and velocity when only r component of the viscous stress is used (top left). Note that the raggedness of the shock goes away when all three components are included (top right). Density maximum and a consequent thick accretion disk like structure is formed in the post-shock region in the latter case. The corresponding specific angular momentum distributions are in bottom left and bottom right respectively.





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We now add Power-Law Cooling

Energy Equation

$$\frac{\partial(\rho\epsilon)}{\partial t} + \nabla.(\rho\epsilon\mathbf{v}) + \Lambda_{powcool} = 0$$

Volumetric power law cooling rate

$$\Lambda_{powcool} \propto \rho^2 T^\beta$$

β is cooling index

Energy Equation in Our System

$$\frac{\partial E}{\partial t} + \frac{1}{r}\frac{\partial(E+p)rv_r}{\partial r} + \frac{\partial(E+p)v_z}{\partial z} = -\frac{\rho\left(rv_r + zv_z\right)}{2\left(\sqrt{r^2+z^2}-1\right)^2\sqrt{r^2+z^2}} - \Lambda_{powcool}$$

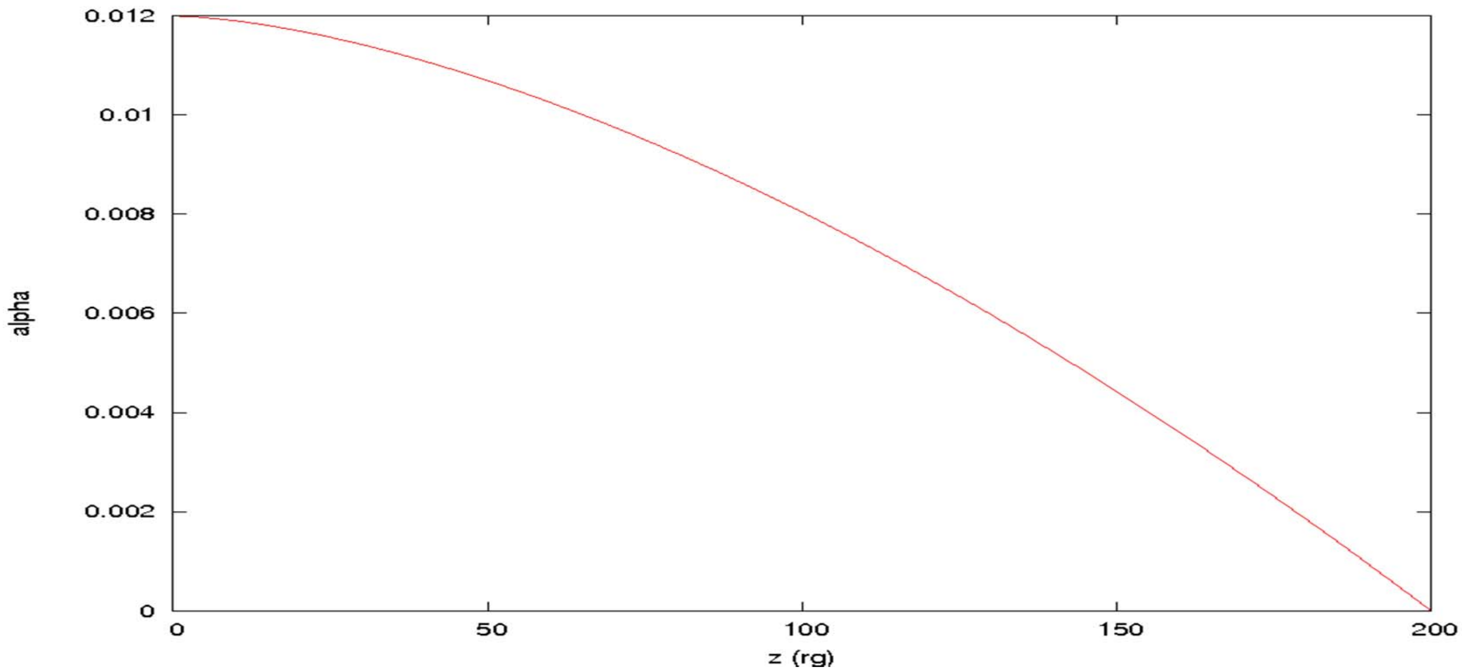
The Power-Law cooling expression is

$$\Lambda_{powcool} = 1.43 \times 10^{-27}\rho^2 T g_f$$

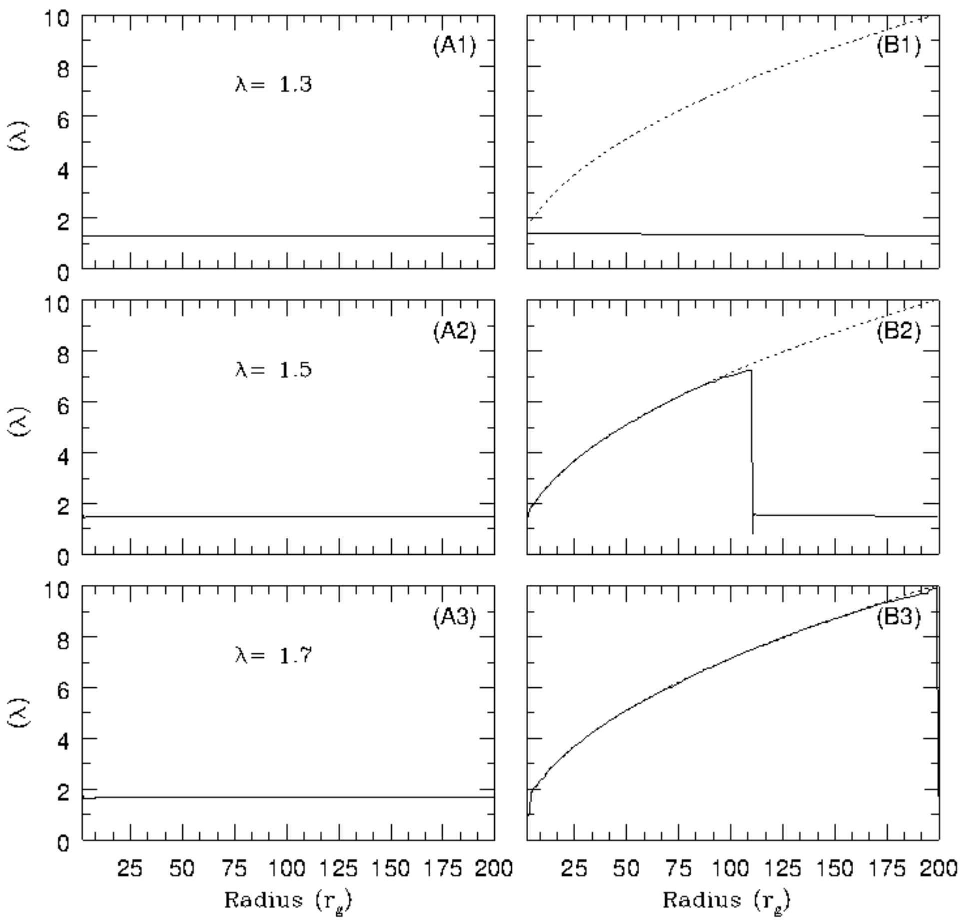
We express in the CGS units and g_f is Gaunt factor which is equal to 1. T is the temperature and ρ is the mass density.

To simulate a two component flow, we chose vertical variation of viscosity parameter α as conjectured by CT95

$$\alpha = \alpha_{max} - [\alpha_{max}(\frac{z}{r_{max}})^{\delta}]$$



where, $r_{max} = 200, 0 \leqslant z \leqslant 200$ and $\delta > 0$. In our cases, we have chosen $\delta = 1.6$. Clearly, when $z = 0$, i.e. at equatorial plane, $\alpha = \alpha_{max}$, while, $\alpha = 0$ for $z = z_{max} = r_{max}$.



For each cases, Specific Energy ε = 0.001

For A1, A2 and A3 Viscosity Parameter α_{max} = 0 and no cooling are there

For B1, B2 and B3 Viscosity Parameter α_{max} = 0.012 and Cooling Index β = 1

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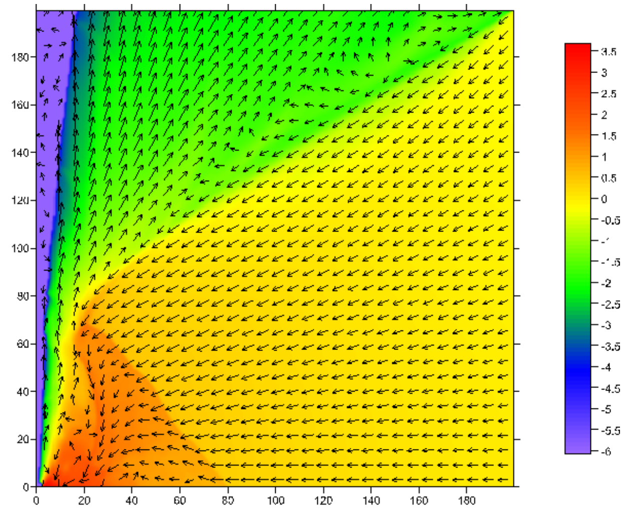
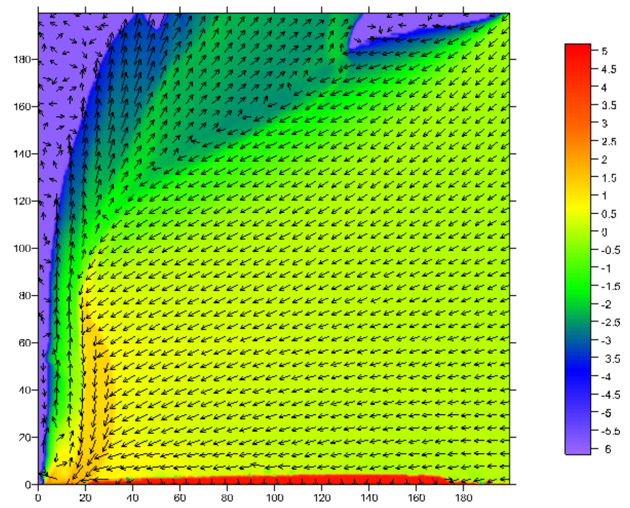
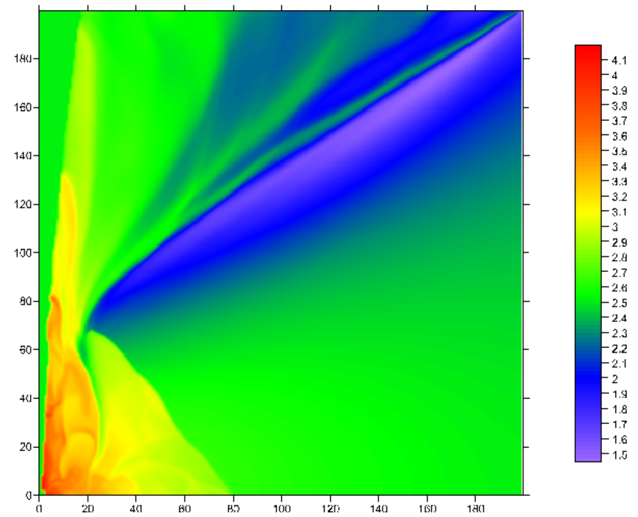
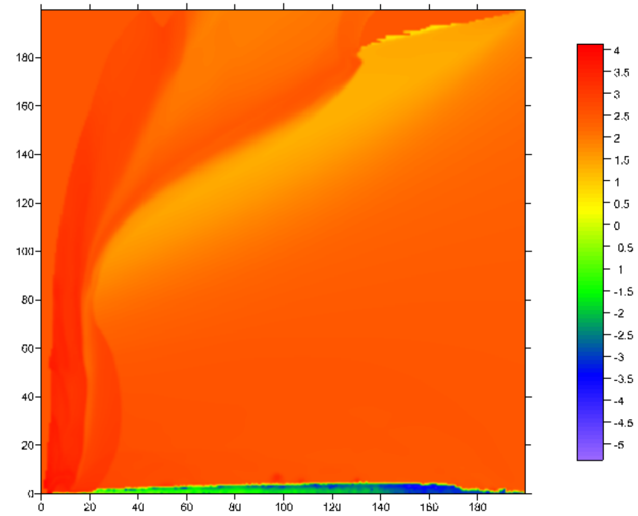
Comparison of the specific angular momentum distributions of an accretion flow at equatorial plane as λ is varied. For all the cases, the result is compared with the Keplerian angular momentum distribution (dotted curves).





Distribution of density and velocity (left) and temperature (right) has been shown. We found that a Keplerian disk is produced on the equatorial plane and the flow above and below remains sub-Keplerian. This is the two component advective flows as envisaged by Chakrabarti & Titurchuk (1995).





α_{max} = 0
ε = 0.001
λ = 1.7

α_{max} = 0.012
β = 1
ε = 0.001
λ = 1.7

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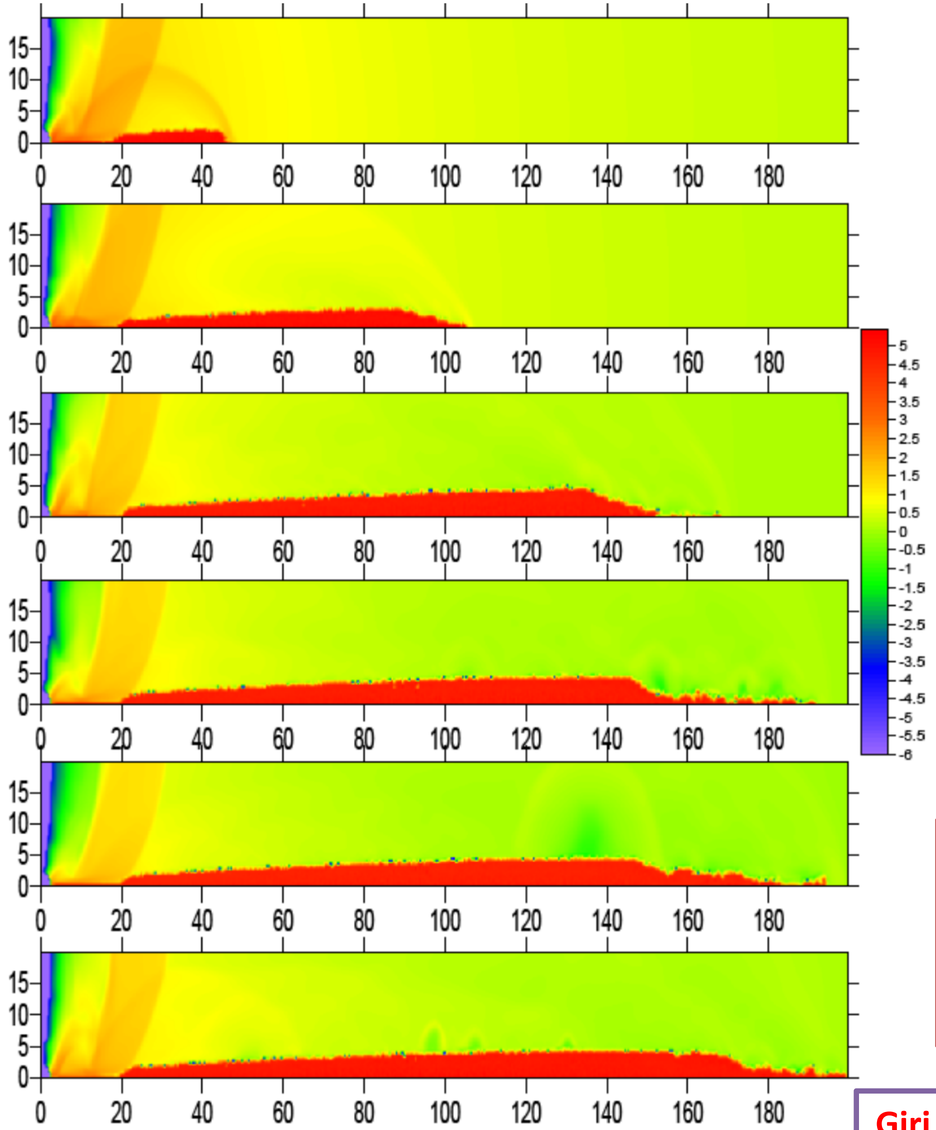
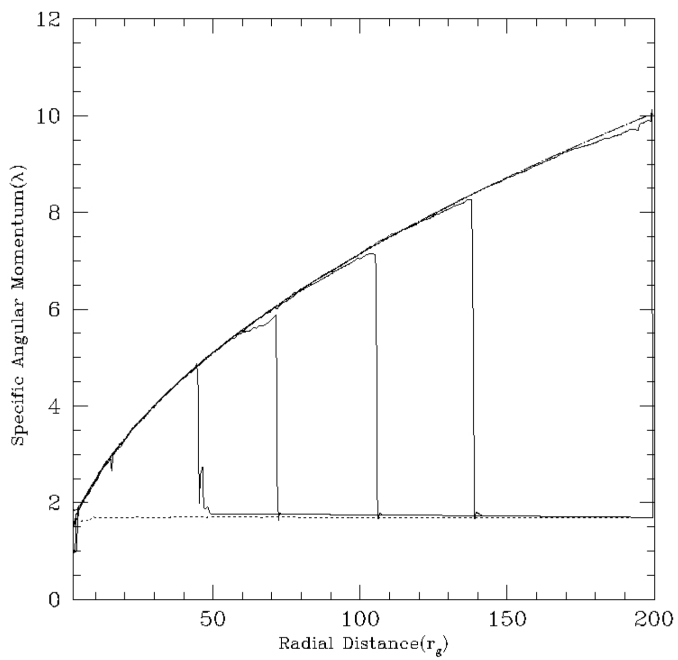
Time variation of the density distribution of equatorial zone with logarithmic scale for viscous flow with cooling. For all cases, the density ranges from log10T = −6 to 5.2





Comparison of the specific angular momentum distributions (solid curves) at equatorial plane at different times of run

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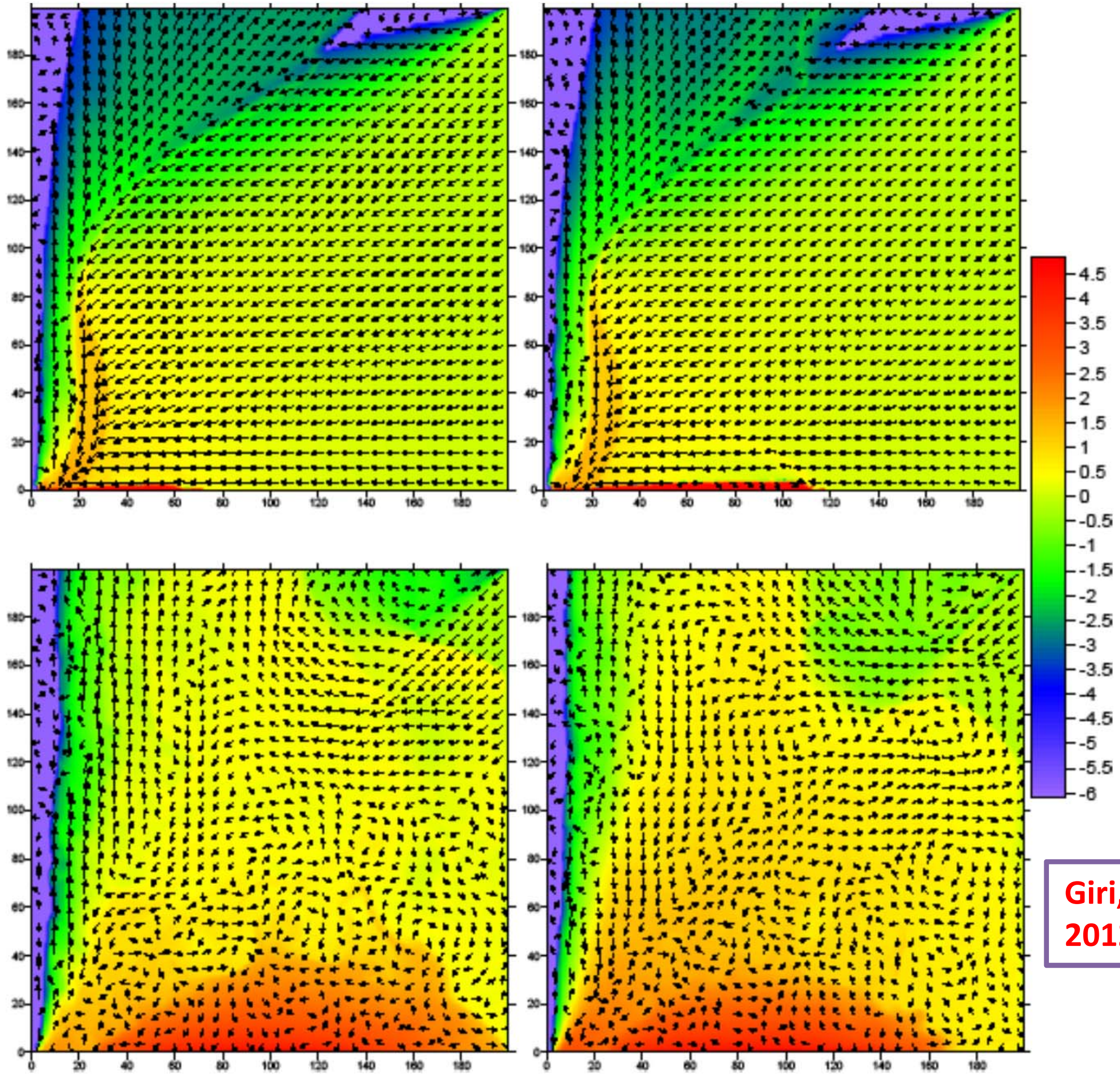
TCAF is stable

- We find that Keplerian disk surface inside TCAF does not show any instability in presence of supersonic sub-Keplerian flow
- Once formed, it is expected to maintain that way
- The changes of rates of matter in Keplerian and sub-Keplerian components depend on α_{max}

Formation and Evaporation of the Keplerian disk



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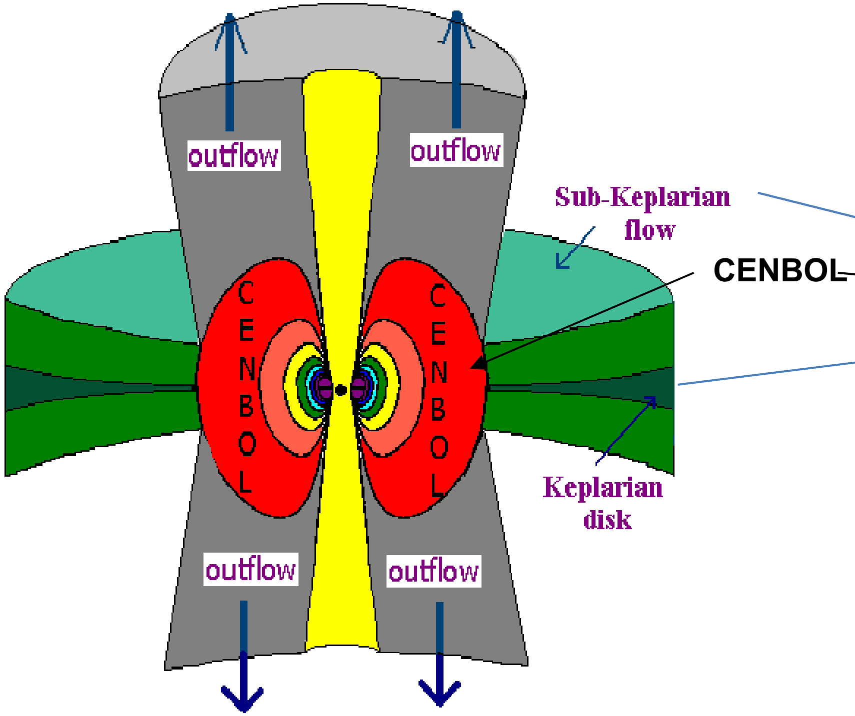
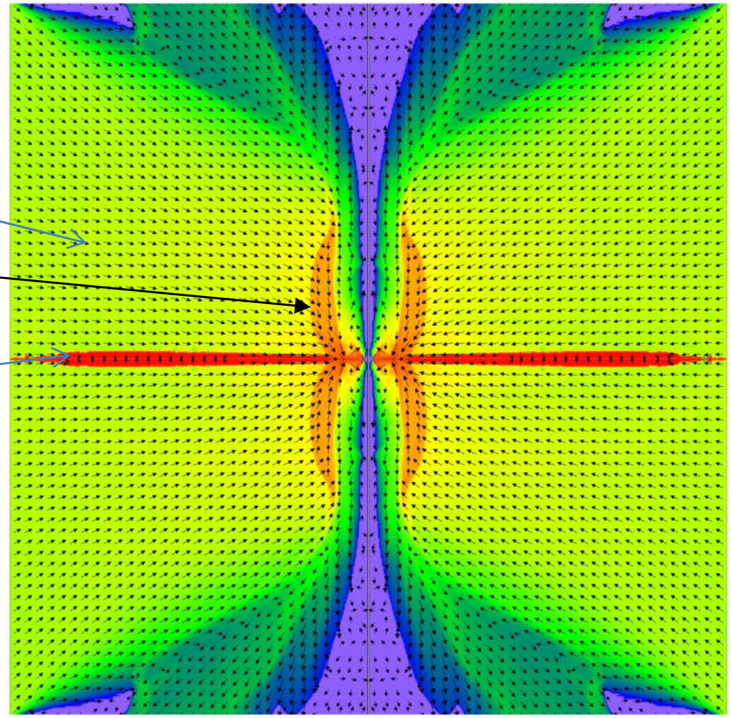
Conclusions





- **The shocks appear to move outward when viscosity is enhanced and the post-shock region roughly attains a Keplerian distribution.**
- **When the viscosity parameter is very high, the shock moves to a large distance and the whole disc becomes a Keplerian disc.**
- **The distribution of viscosity parameter controls the entire accretion flow**
- **Even with a simple power-law cooling, and a vertical viscosity variation, we found that a two component advective flow (TCAF) as conjectured by Chakrabarti and Titarchuk (1995) can form. TCAF is found to be stable as well. This gives a complete picture of the formation of TCAF.**

Thank You!