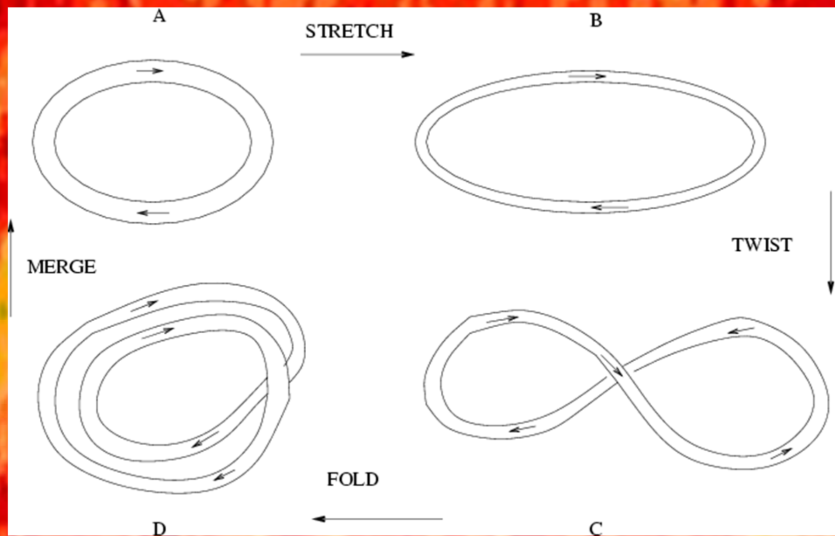
Dynamos in astrophysics

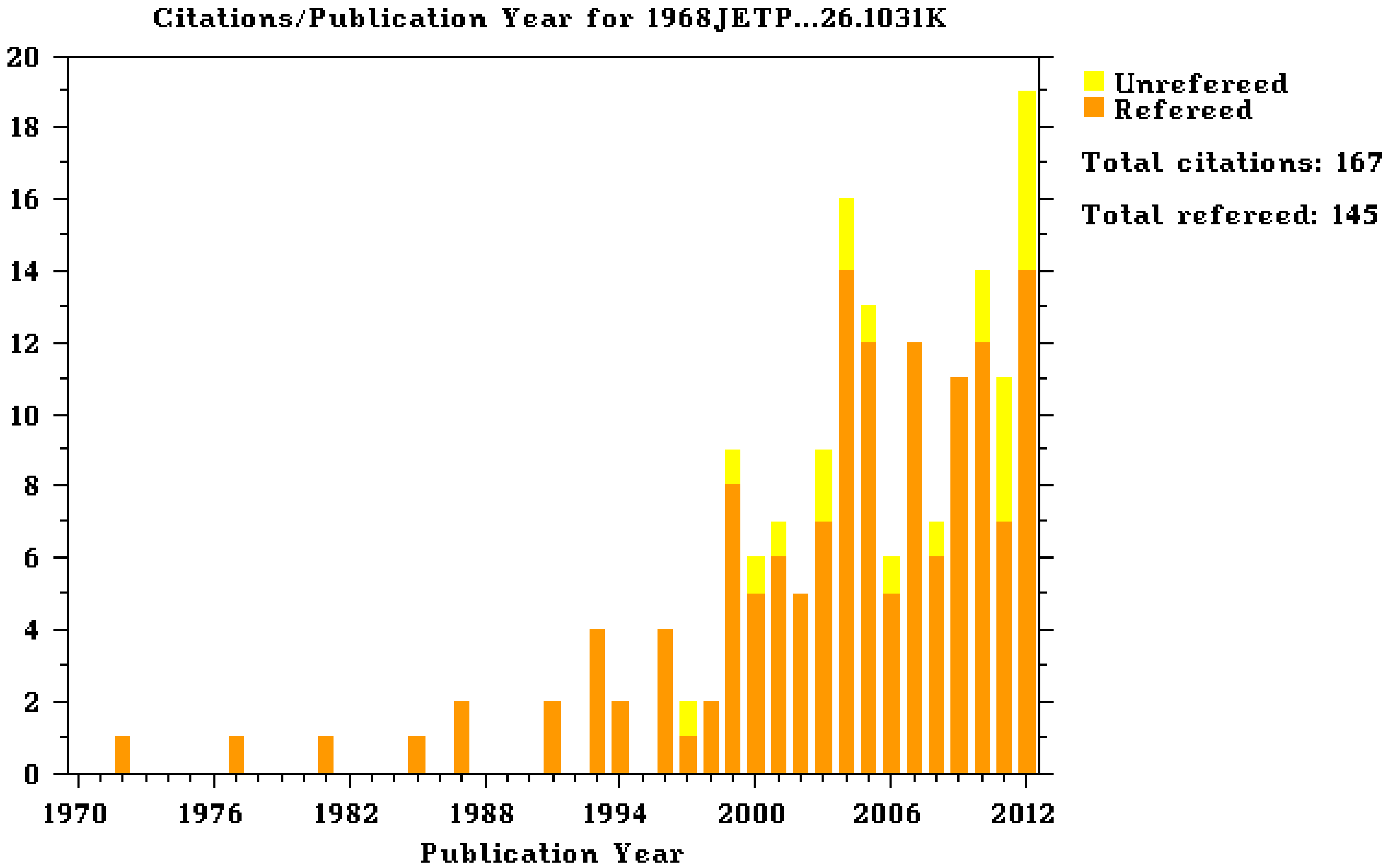
a) Motions from
   i. Convection instability
   ii. Magnetorotational inst.
   iii. Supernova forcing

b) Dynamo instability
   i. Stretch-twist-fold
   ii. Turbulent dynamo



Dynamo applications & issues

- (i) Solar dynamo
  - Rayleigh-Benard
  - *Why 22 years*
  - *Equatorward migr*
- (ii) Accretion discs
  - Magnetorotational
  - *Small mag Prandtl?*
  - *Why cycles?*
- (iii) Galactic
  - Supernova-driven
  - *Fast enough?*
- (iv) SN remnants
  - Bell instability
  - *Current sustained?*



- Kazantsev (1968): small-scale dynamo
  - Essentially unnoticed, simulations 1981, 2000-now

Mile stones in dynamo research

- 1970ies: mean-field models of Sun/galaxies
- 1980ies: direct simulations
- Gilman/Glatzmaier: *poleward* migration
- 1990ies: compressible simulations, MRI
  – Magnetic buoyancy overwhelmed by pumping
  – Successful geodynamo simulations
- 2000- magnetic helicity, catastr. quenching
  – Dynamos and MRI at low Pr_M=■📫♒

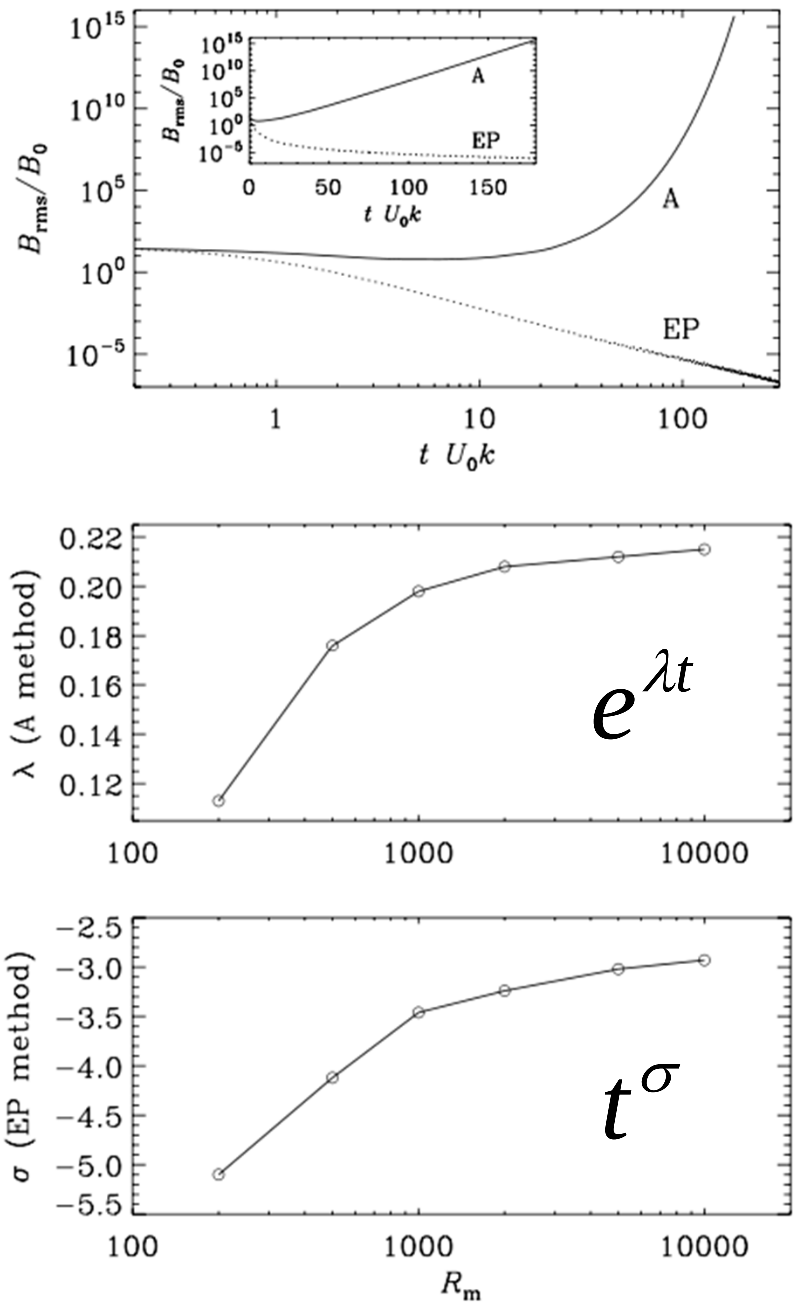
Easy to simulate?

- Yes, but it can also go wrong
- *2 examples: manipulation with diffusion*
- Large-scale dynamo in periodic box
  - With hyper-diffusion $\mathrm{curl}^{2n}\boldsymbol{B}$
  - amplitude by $(k/k_\mathrm{f})^{2n-1}$
- Euler potentials with artificial diffusion
  - Dα/Dt=♒👎α, Dβ/Dt=♒👎♌📪 👌 💾
    grad♋ x grad♌

Dynamos with Euler Potentials

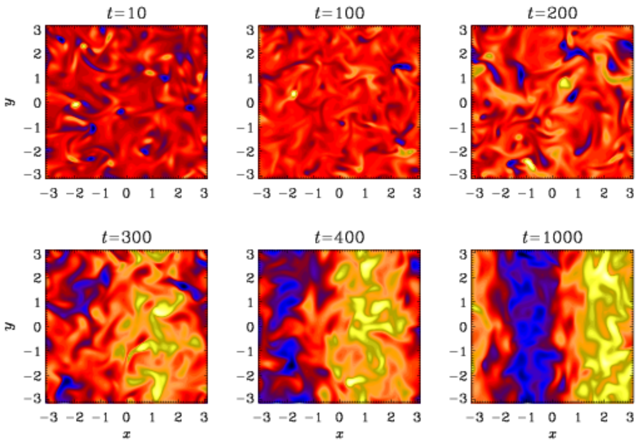
- ***B*** = grad♋ x grad♌
- ***A*** = ♋ grad♌, so ***A.B***=0
- Take non-helical flows
- Agreement for early *t*
  - For smooth fields, not for ♎-correlated initial fields
- Exponential growth (A)
- Algebraic decay (EP)

Brandenburg (2010, MNRAS 401, 347)

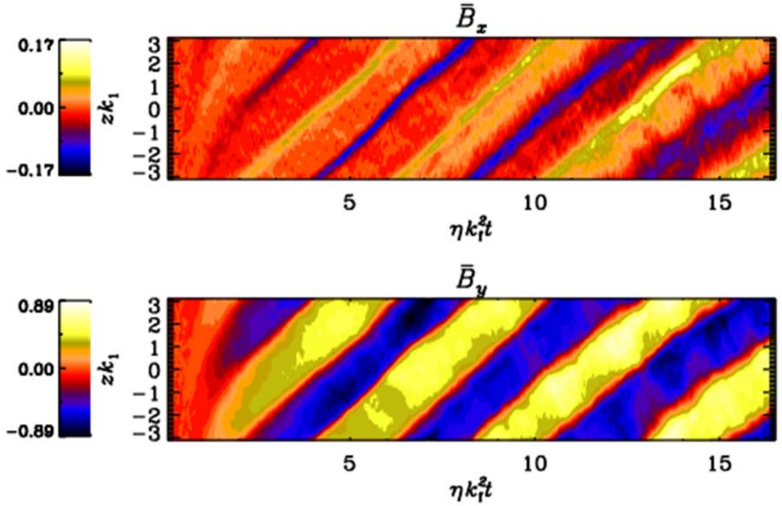


Other good examples of dynamos

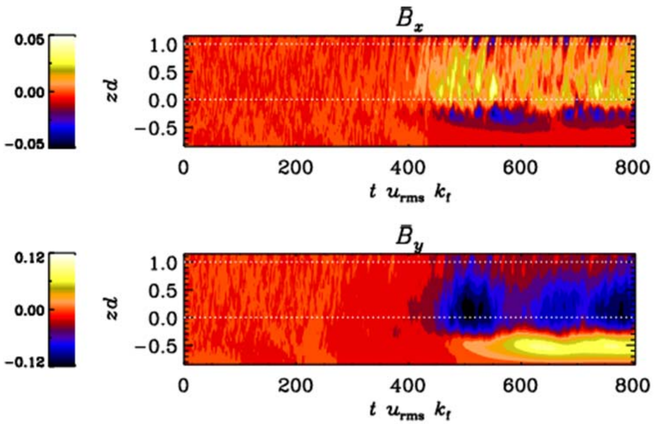
Helical turbulence (B_y)



Helical shear flow turb.

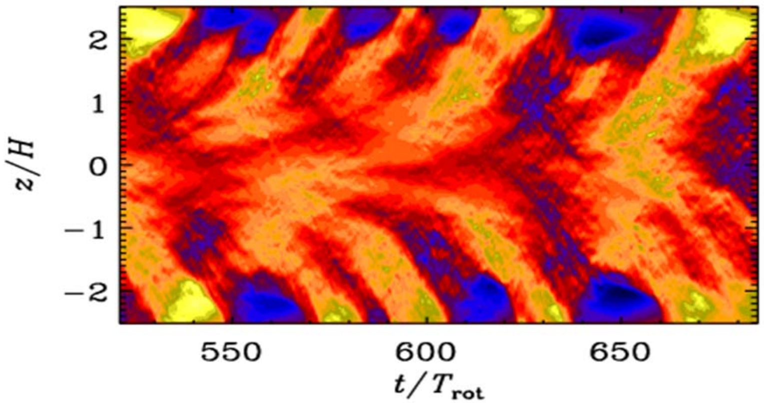


Convection with shear



Käpyla et al (2008)

Magneto-rotational Inst.

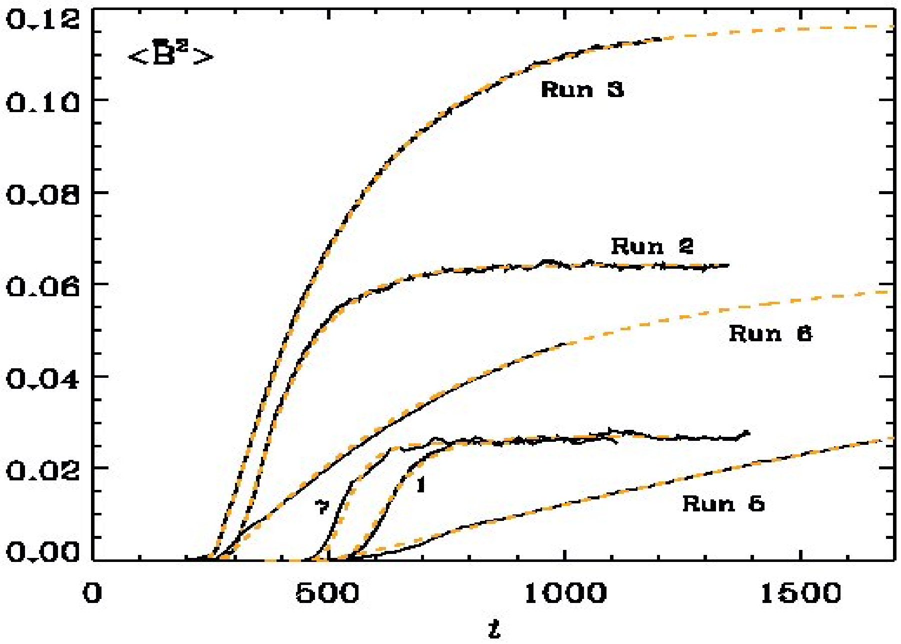


$\omega = \eta_t k_1^2$

$c = \eta_t k_1$

One big flaw: slow saturation

(explained by magnetic helicity conservation)



$$\frac{d}{dt}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

$$\langle \mathbf{J}\cdot\mathbf{B}\rangle = \langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle + \langle \mathbf{j}\cdot\mathbf{b}\rangle$$

$$\langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle \approx -k_1\langle \overline{\mathbf{B}}^2\rangle, \quad \langle \mathbf{j}\cdot\mathbf{b}\rangle \approx k_\mathrm{f}\langle \mathbf{b}^2\rangle$$

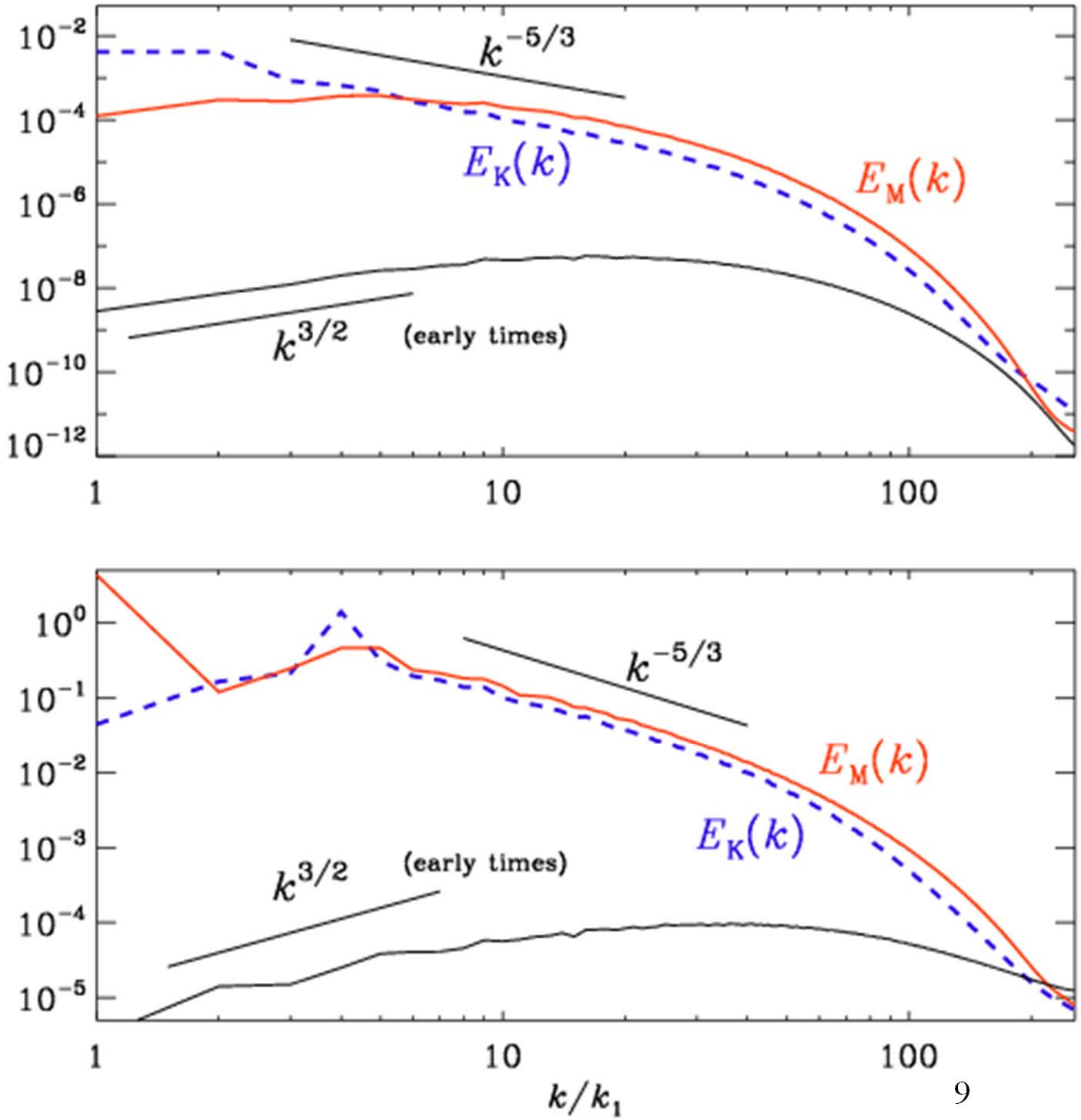
$$k_1^{-1}\frac{d}{dt}\langle \overline{\mathbf{B}}^2\rangle = -2\eta k_1\langle \overline{\mathbf{B}}^2\rangle + 2\eta k_\mathrm{f}\langle \mathbf{b}^2\rangle$$

molecular value!!

$$\langle \overline{\mathbf{B}}^2\rangle = \langle \mathbf{b}^2\rangle \frac{k_f}{k_1}\left[1 - e^{-2\eta k_1^2 (t-t_\mathrm{s})}\right]$$

Non-helical vs helical

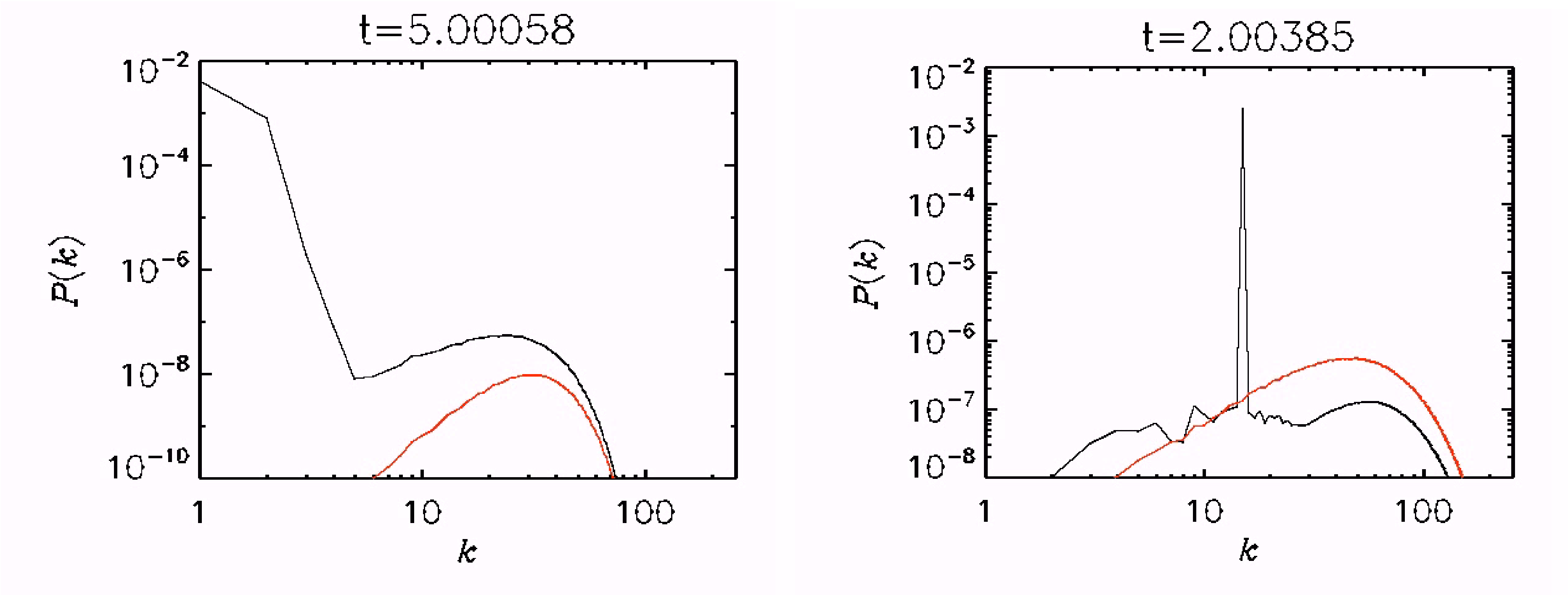
- Similar at intermediate k
- $k^{3/2}$ in both cases
- Super-equip.
- Difference: LS field at k=1
- Here Pr_M=1



Nonhelical & helical turbulence

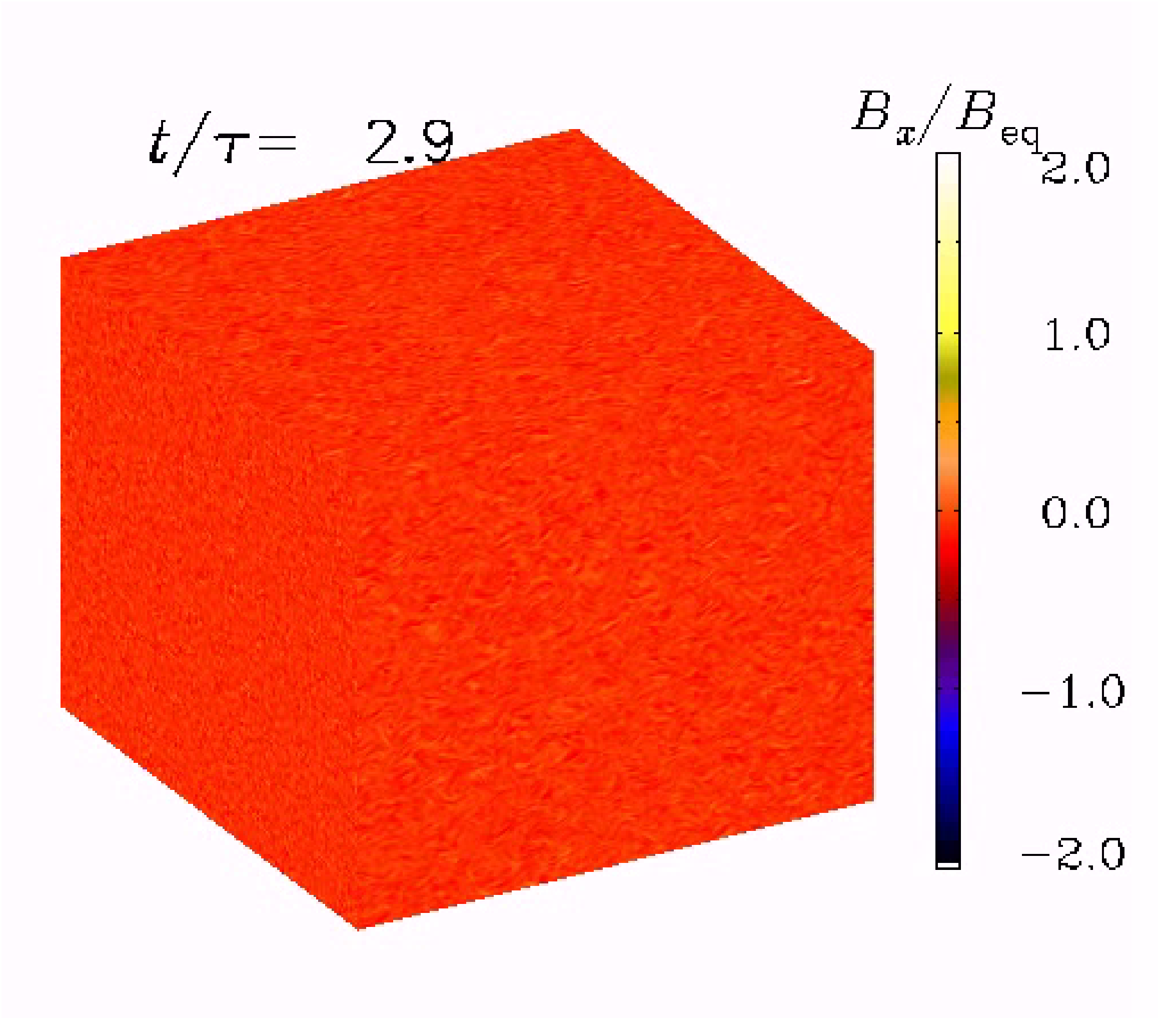
Dynamos in both cases: non-magnetic solutions do not exist

…when conductivity high enough

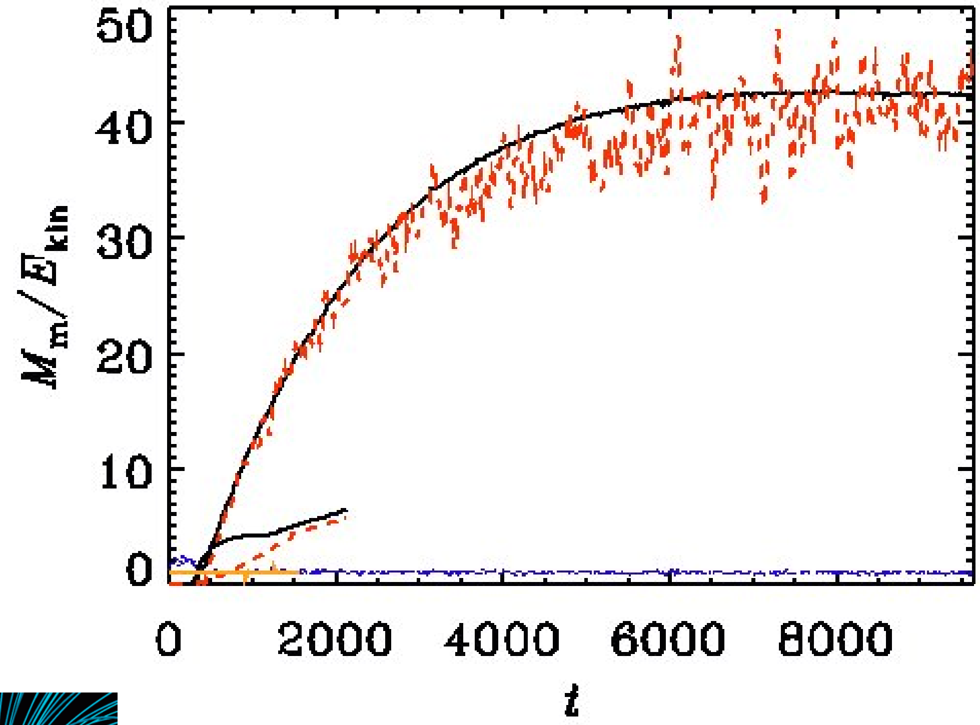


With helicity: gradual build-up of large-scale field

Inverse cascade



Helical dynamo saturation with hyperdiffusivity



PRL 88, 055003

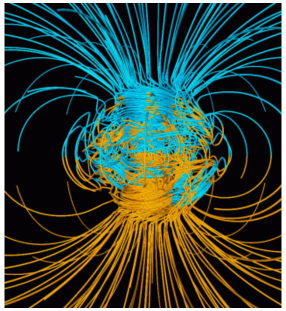
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

$$k_1^3\left\langle \overline{\mathbf{B}}^2 \right\rangle = k_\mathrm{f}^3\left\langle \mathbf{b}^2 \right\rangle$$

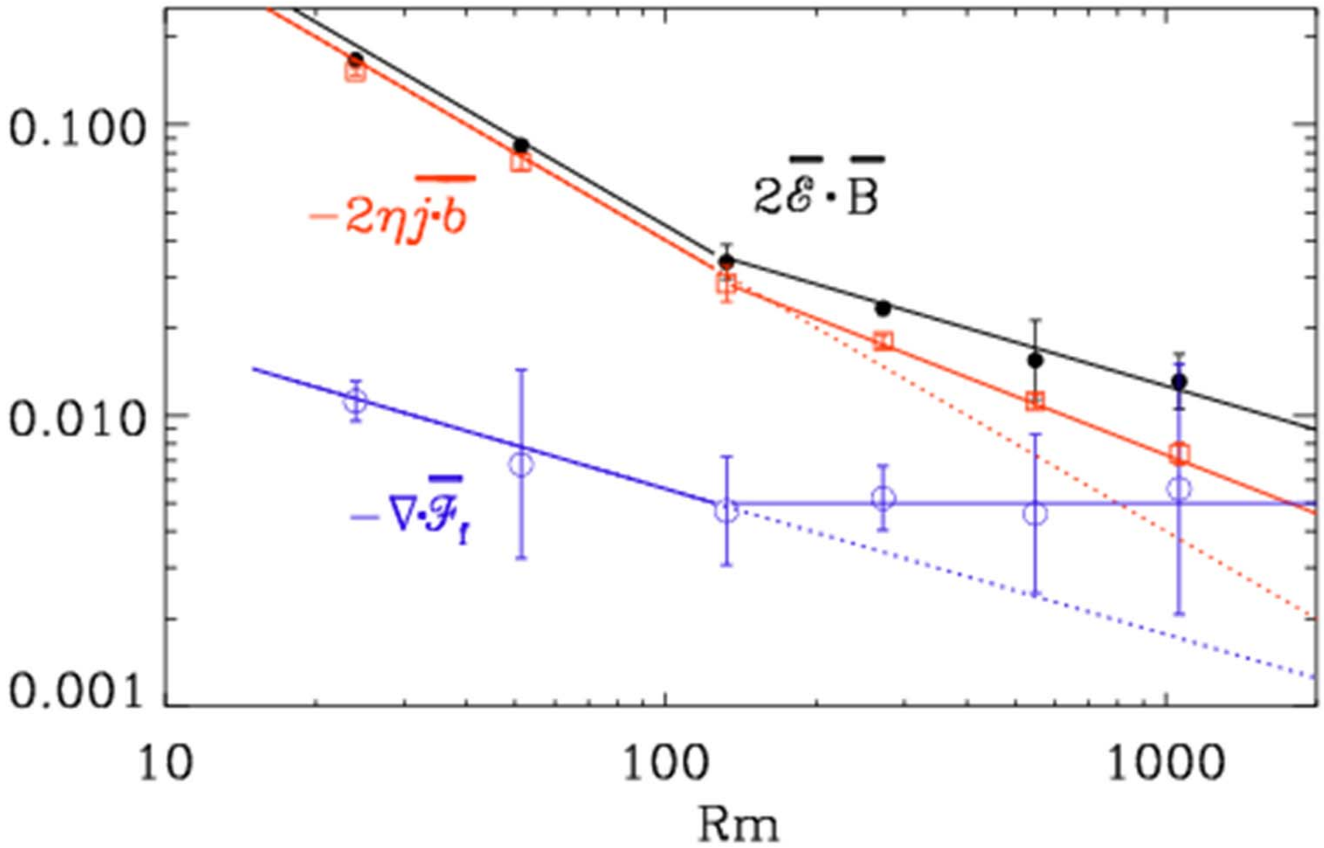
for ordinary hyperdiffusion $\propto \eta_2 k^4$

$$k_1\left\langle \overline{\mathbf{B}}^2 \right\rangle = k_\mathrm{f}\left\langle \mathbf{b}^2 \right\rangle$$

ratio 5^3=125 instead of 5



Boundaries instead of periodic



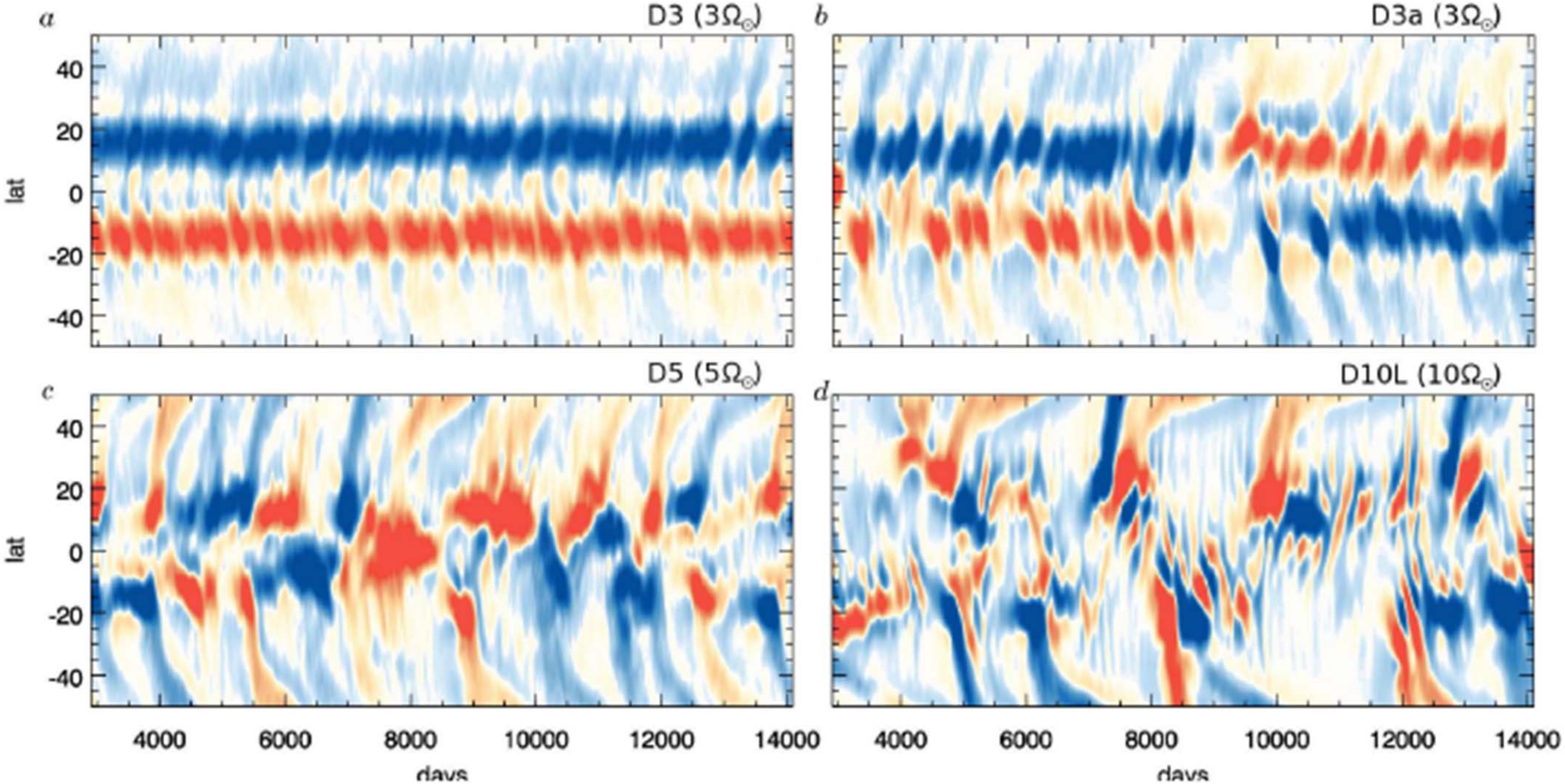
Del Sordo, Guerrero, & Brandenburg, 2012, MNRAS, subm'd

$$\frac{\partial}{\partial t}\overline{\mathbf{a}\cdot\mathbf{b}} = -2\overline{\mathcal{E}}\cdot\overline{\mathbf{B}} - 2\eta\overline{\mathbf{j}\cdot\mathbf{b}} - \nabla\cdot\overline{\mathbf{F}}$$

(i) Simulations of the solar dynamo

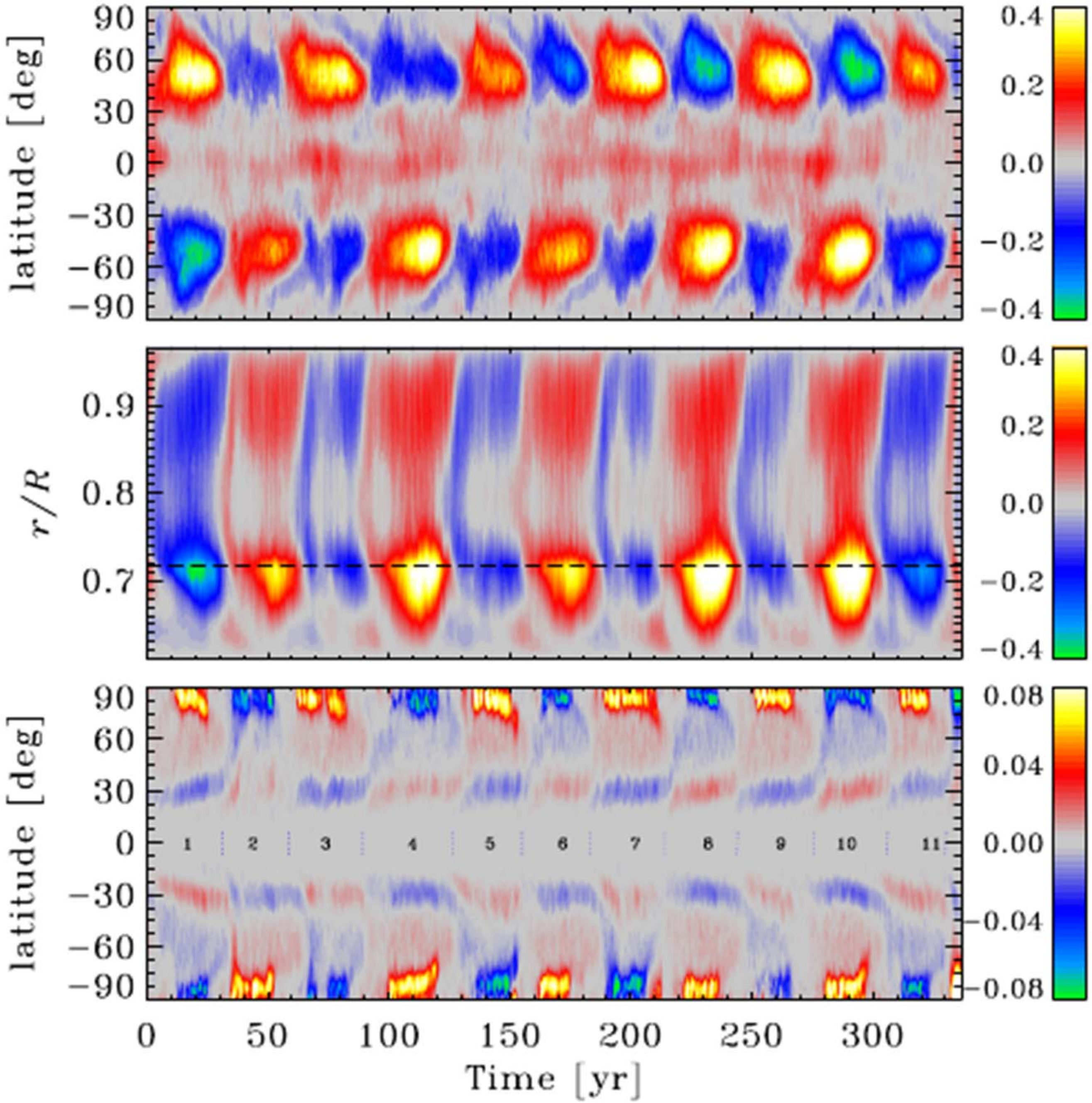
- Tremendous stratification
  - Not only density, also scale height change
- Near-surface shear layer (NSSL) not resolved
- Contours of Ω cylindrical, not spoke-like
- Rm dependence (catastrophic quenching)
  - Field is bi-helical: to confirm for solar wind
- Location: bottom of CZ or distributed
  - Shaped by NSSL (Brandenburg 2005, ApJ 625, 539)
  - Formation of active regions near surface

Brun, Brown, Browning, Miesch, Toomre

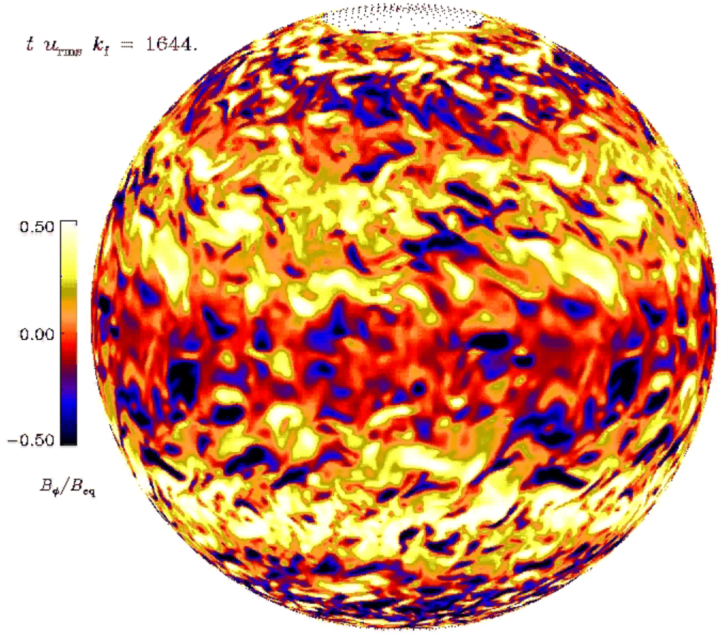


Ghizaru, Charbonneau, Racine, …

- Cycle now common!
- Activity from bottom of CZ
- but at high latitudes



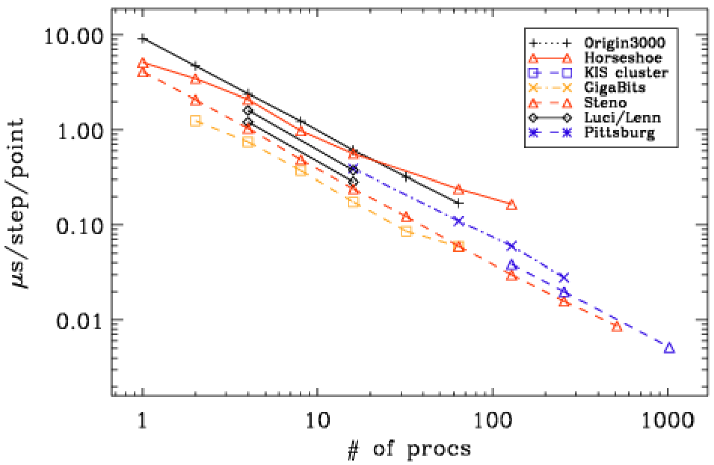
Pencil code

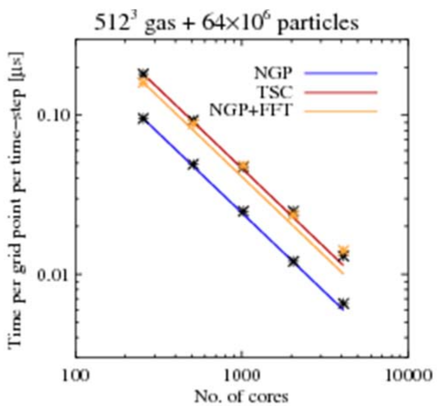


Kapyla et al (2012)

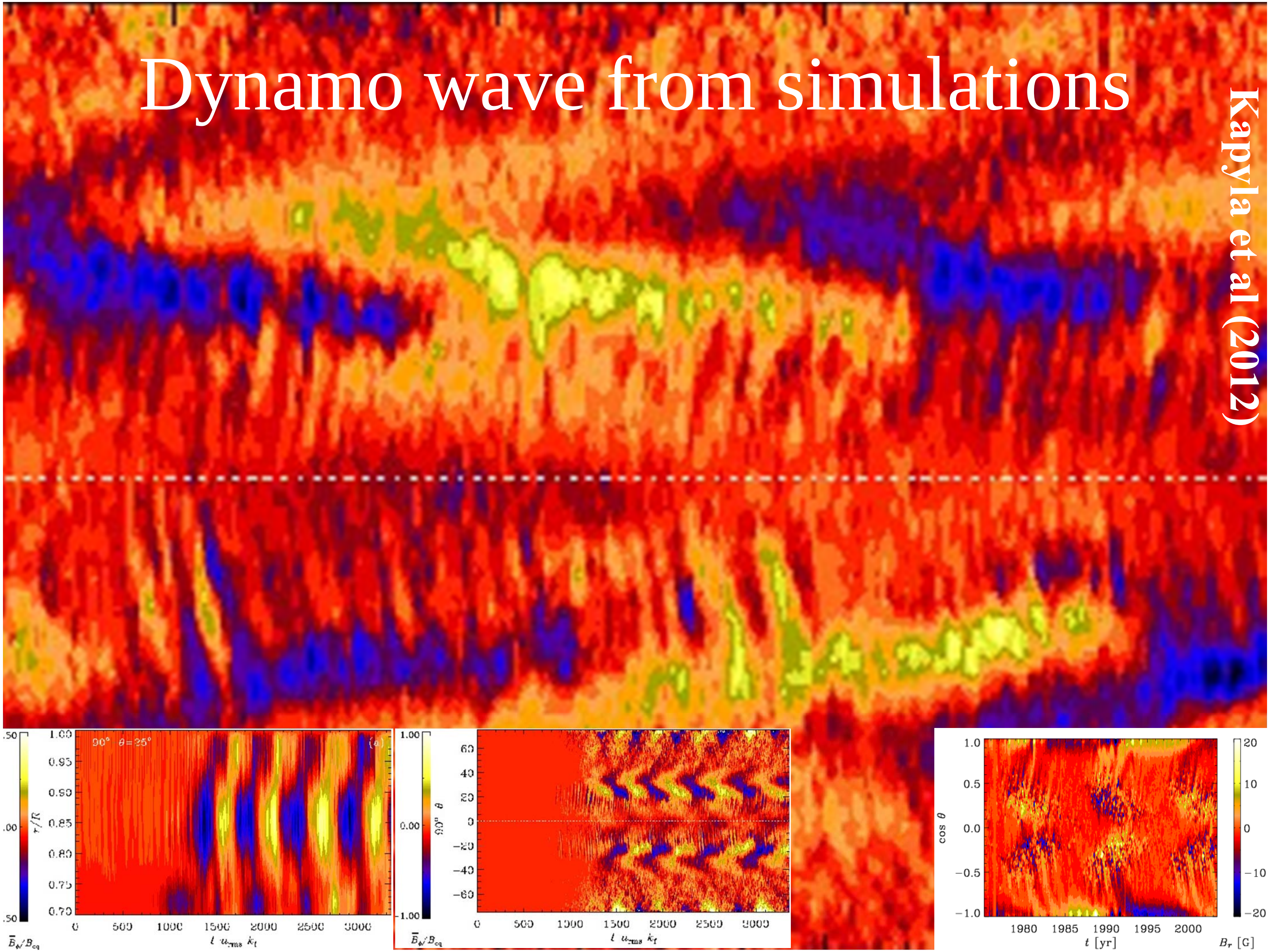
- Started in Sept. 2001 with Wolfgang Dobler
- High order (6th order in space, 3rd order in time)
- Cache & memory efficient
- MPI, can run PacxMPI (across countries!)
- Maintained/developed by ~80 people (SVN)
- Automatic validation (over night or any time)
- 0.0013 μs/pt/step at 1024^3 , 2048 procs
- http://pencil-code.googlecode.com

- Isotropic turbulence
  - MHD, passive scl, CR
- Stratified layers
  - Convection, radiation
- Shearing box
  - MRI, dust, interstellar
    - Self-gravity
- Sphere embedded in box
  - Fully convective stars
    - geodynamo
- Other applications
  - Chemistry, combustion
  - Spherical coordinates





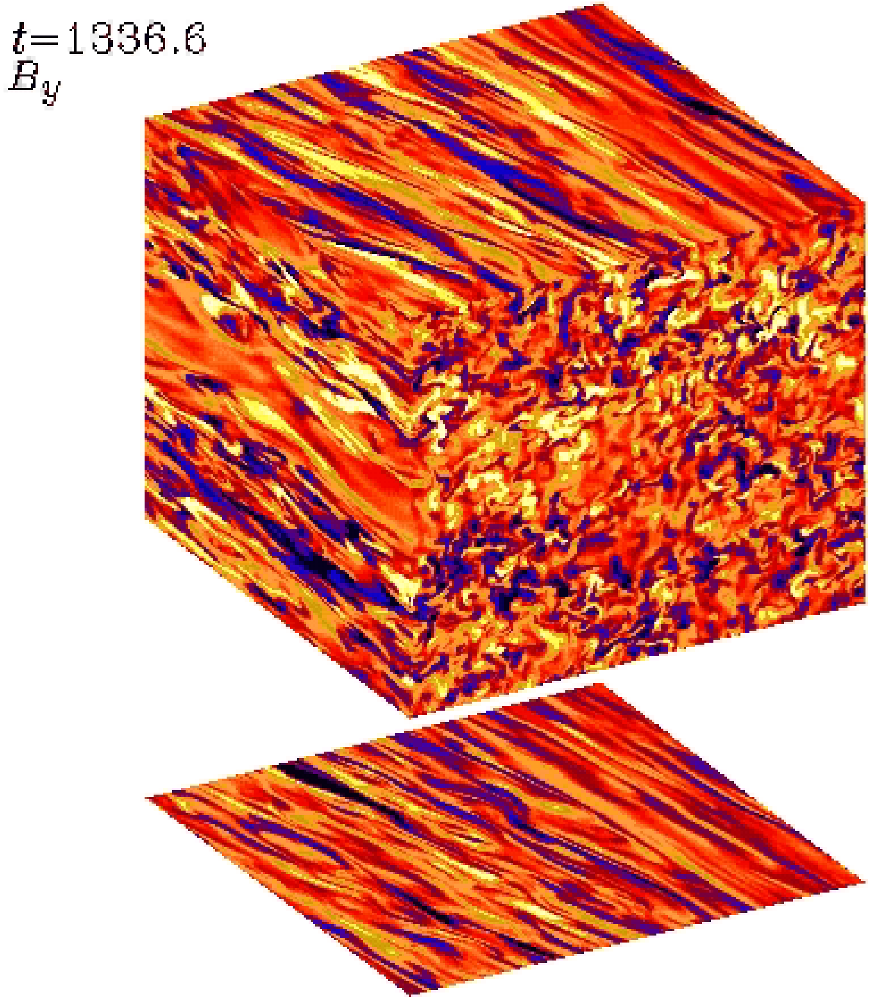
Dynamo wave from simulations



Kapyla et al (2012)

(ii) Dynamos from MRI turbulence

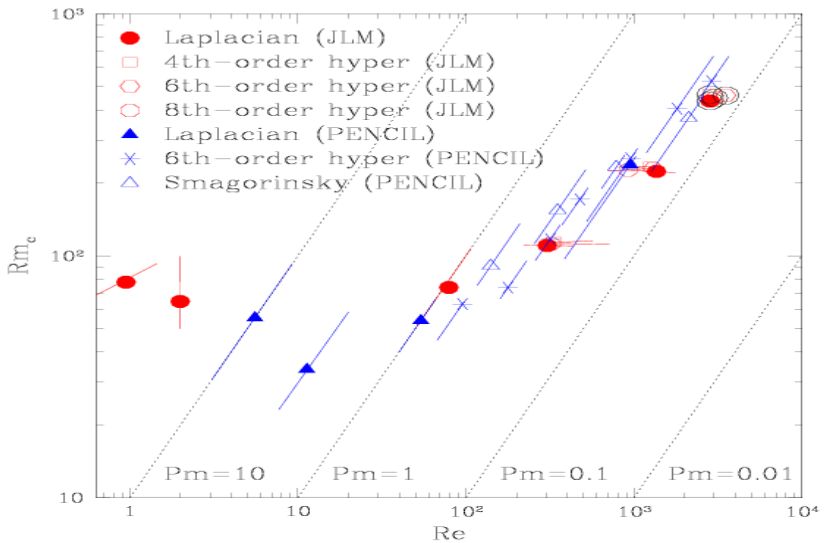
512^3
w/o hypervisc.
$\Delta t = 60 = 2$ orbits

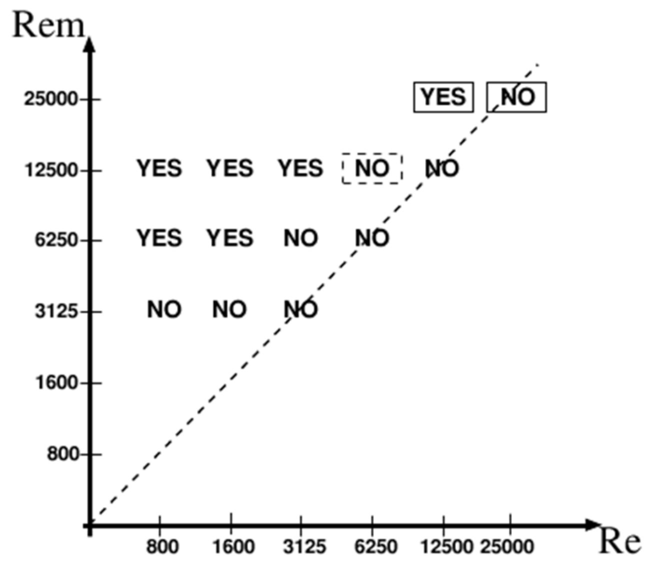


No large scale field
(i) Too short?
(ii) No stratification?

Low Pr_M issue

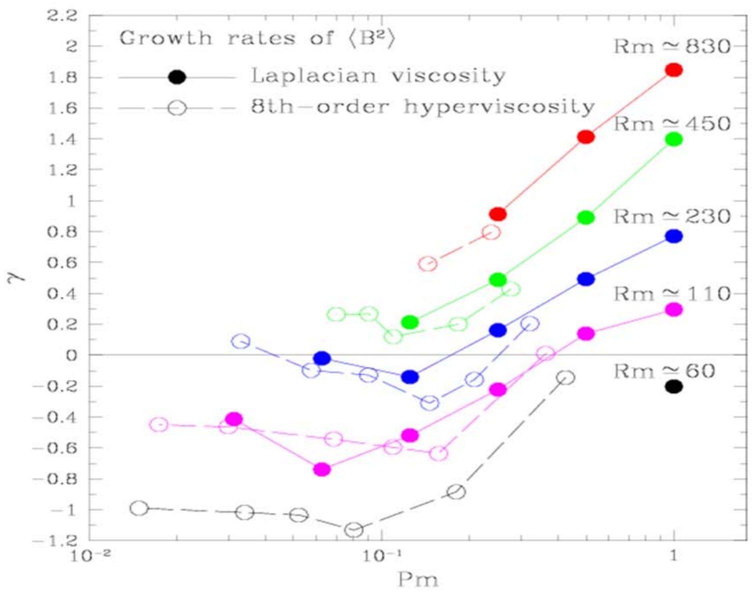
- Small-scale dynamo: $R_{m.crit}$=35-70 for Pr_M=1 (Novikov, Ruzmaikin, Sokoloff 1983)
- Leorat et al (1981): independent of Pr_M (EDQNM)
- Rogachevskii & Kleeorin (1997): $R_{m,crit}$=412
- Boldyrev & Cattaneo (2004): relation to roughness
- Ponty et al.: (2005): levels off at Pr_M=0.2



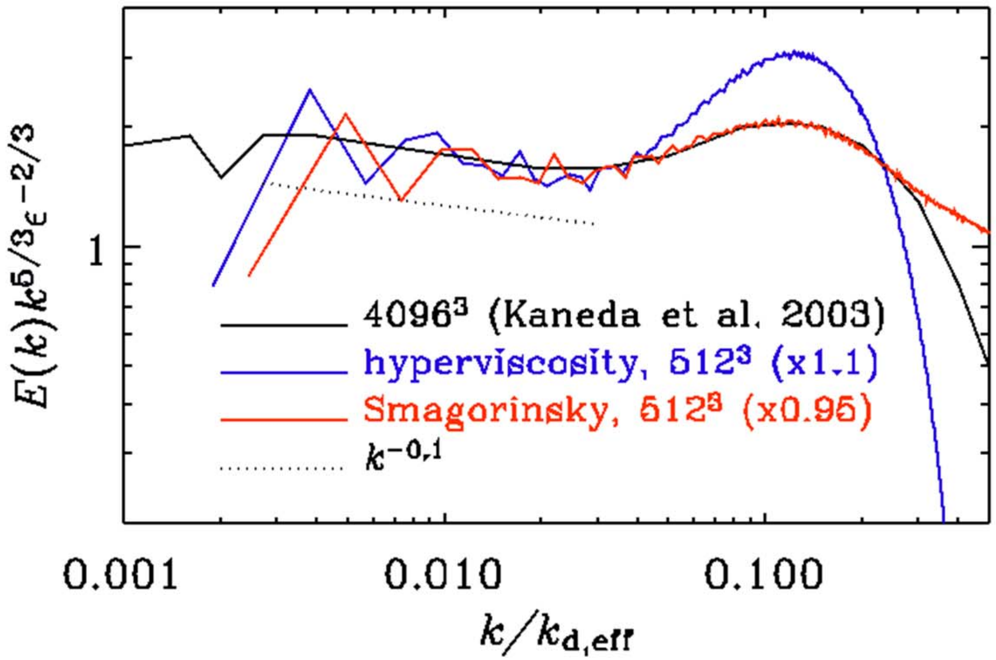


Re-appearence at low Pr_M

Gap between 0.05 and 0.2 ? Is just because of bottleneck effect



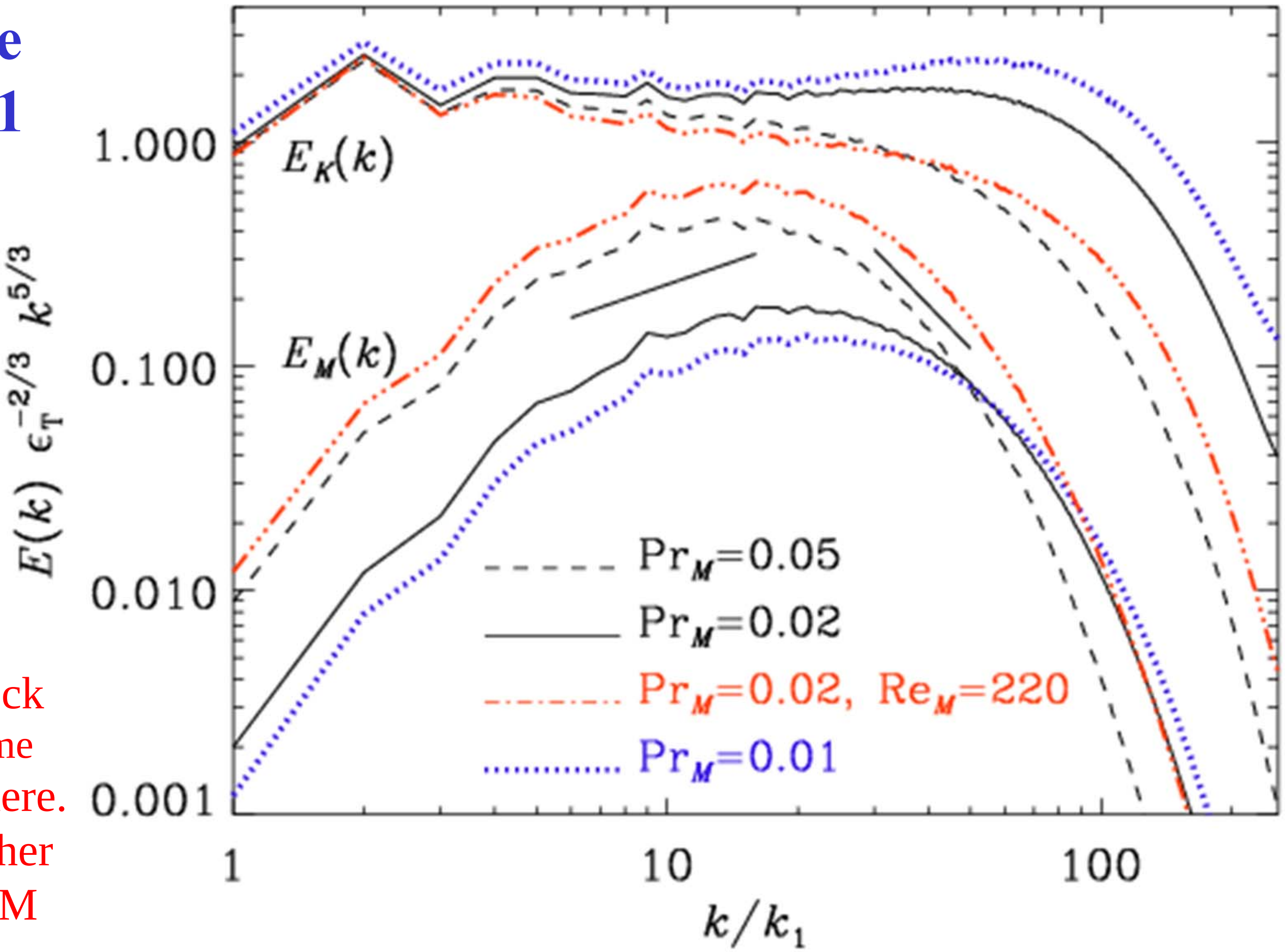
Iskakov et al (2007)



Haugen & Brandenburg (2006)

Nonlinear small-scale dynamo also works

Does survive for $Pr_M < 0.1$



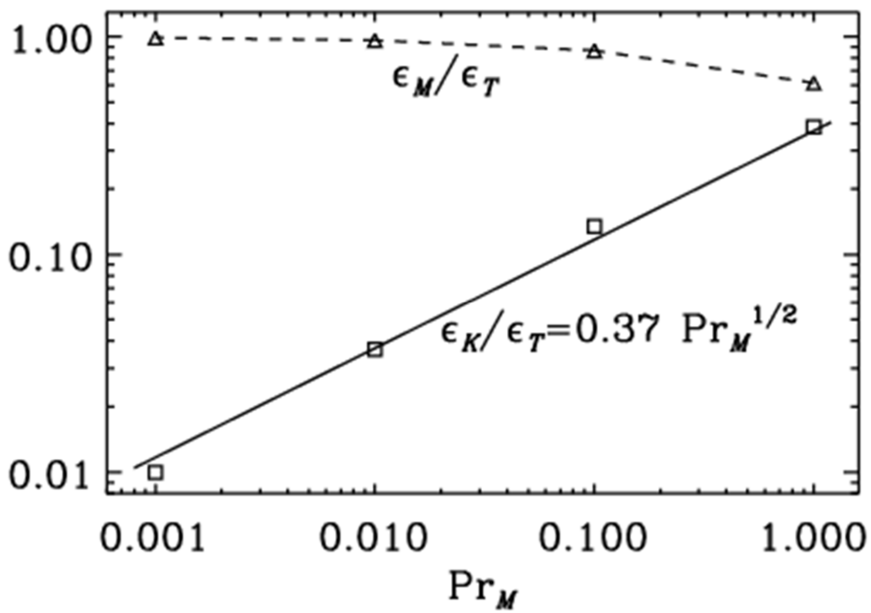
Related to bottleneck in kinematic regime
Velocity roughest there.
Magnetic peak further left for smaller PrM

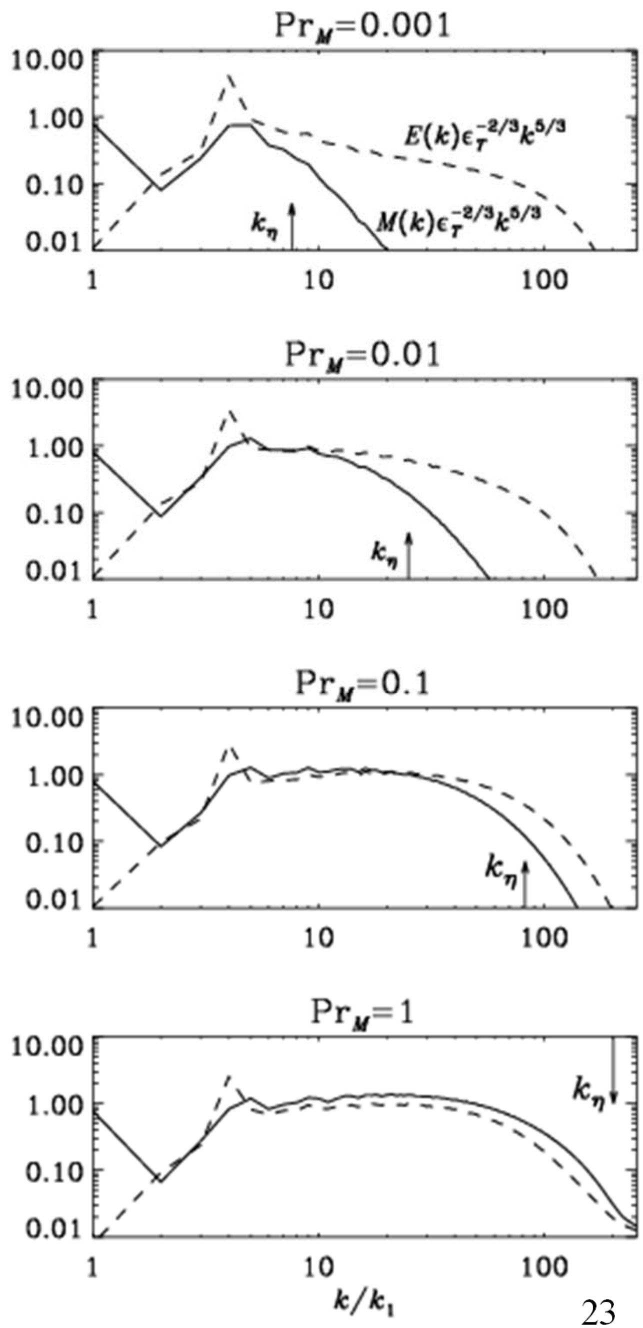
Brandenburg (2011, ApJ 741:92)

Helical dynamos at low Pr_M always work

- Energy dissipation via Joule
- Viscous dissipation weak
- Can increase Re substantially!

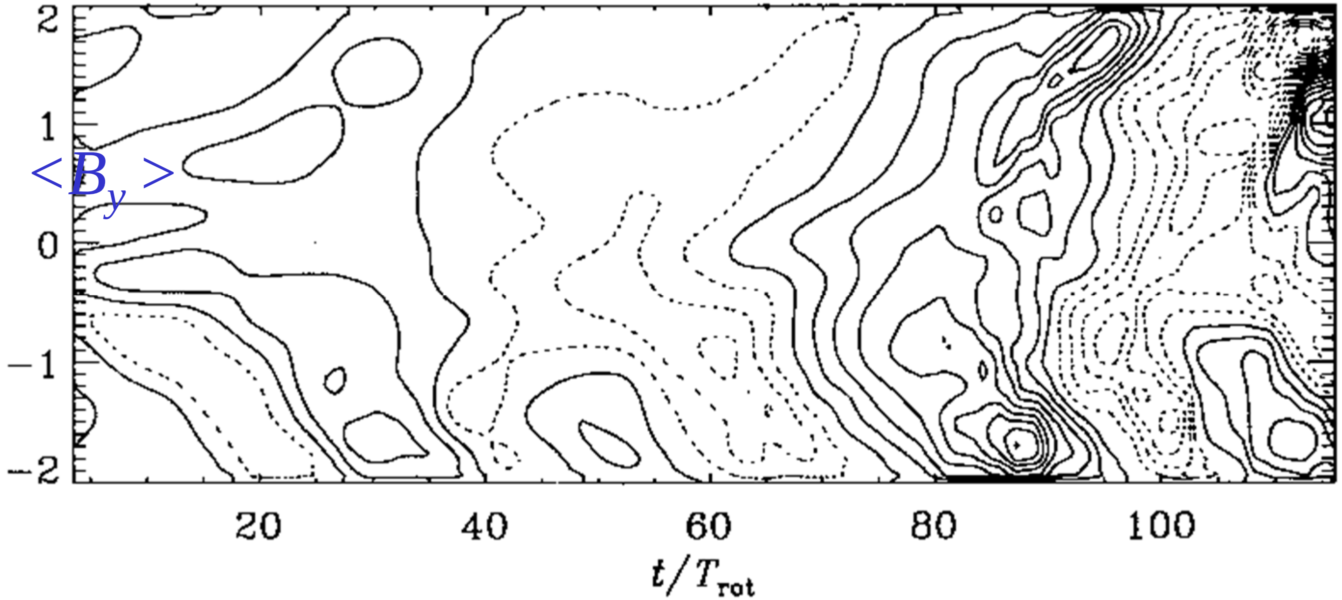
Brandenburg (2009, ApJ)

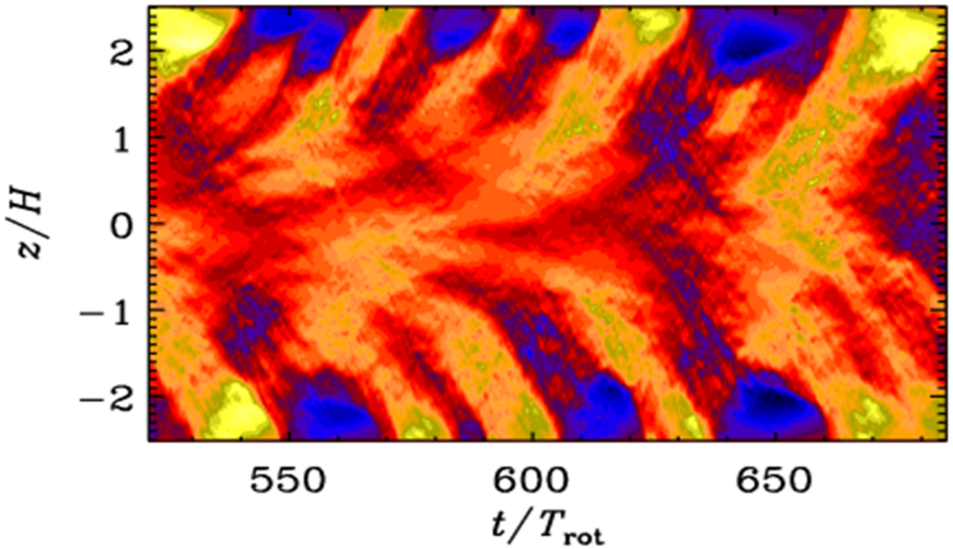




Large scale dynamo in *stratified* discs

Brandenburg et al. (1995)



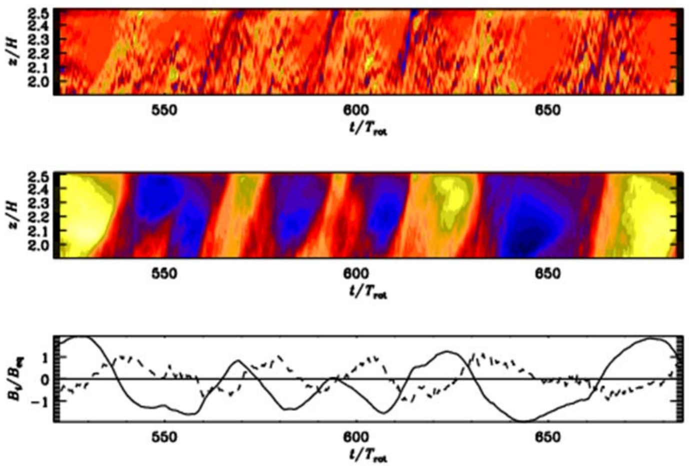


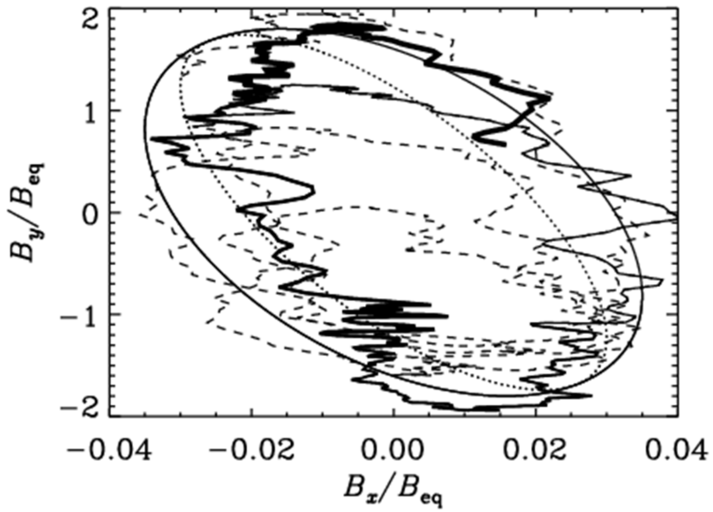
Dynamo waves

versus

Buoyant escape

Phase relation in dynamos





Mean-field equations:

$$\frac{\partial \overline{B}_x}{\partial t} = -\alpha \frac{\partial \overline{B}_y}{\partial z} + \eta_T \frac{\partial^2 \overline{B}_x}{\partial z^2}$$

$$\frac{\partial \overline{B}_y}{\partial t} = -\tfrac{3}{2}\Omega \overline{B}_x + \eta_T \frac{\partial^2 \overline{B}_y}{\partial z^2}$$

Solution:

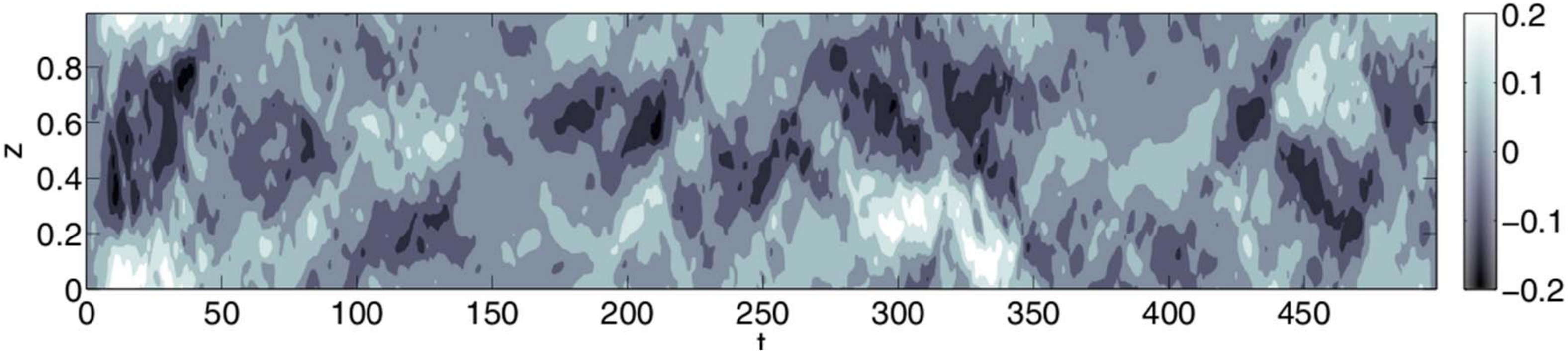
$$\overline{B}_x = \sin k(z - ct)$$

$$\overline{B}_x = \sqrt{2}\,\tfrac{c}{\alpha} \sin\left[k(z - ct) \pm \tfrac{3}{4}\pi\right]$$

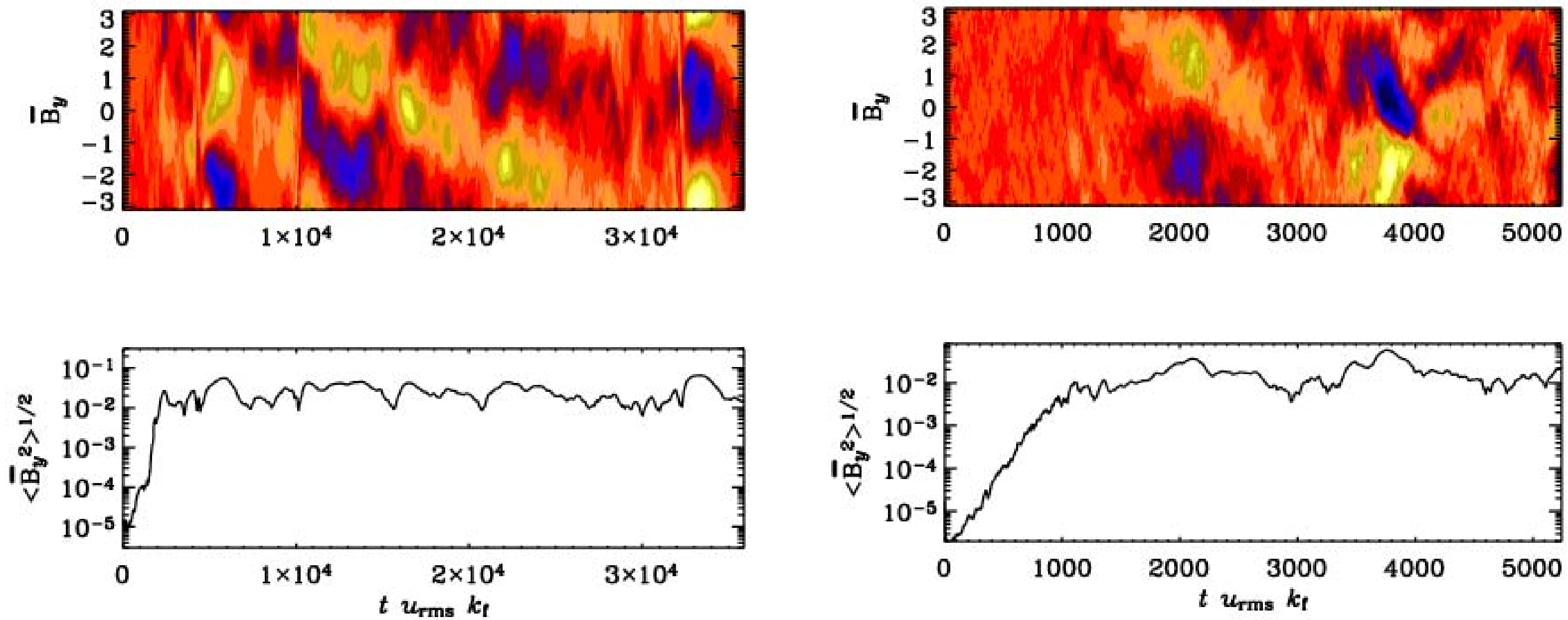
$$c = \tfrac{\omega}{k} = -\alpha\sqrt{3\Omega / 4\alpha k} = \mp \eta_T k$$

Unstratified: also LS fields?

Lesur & Ogilvie (2008)

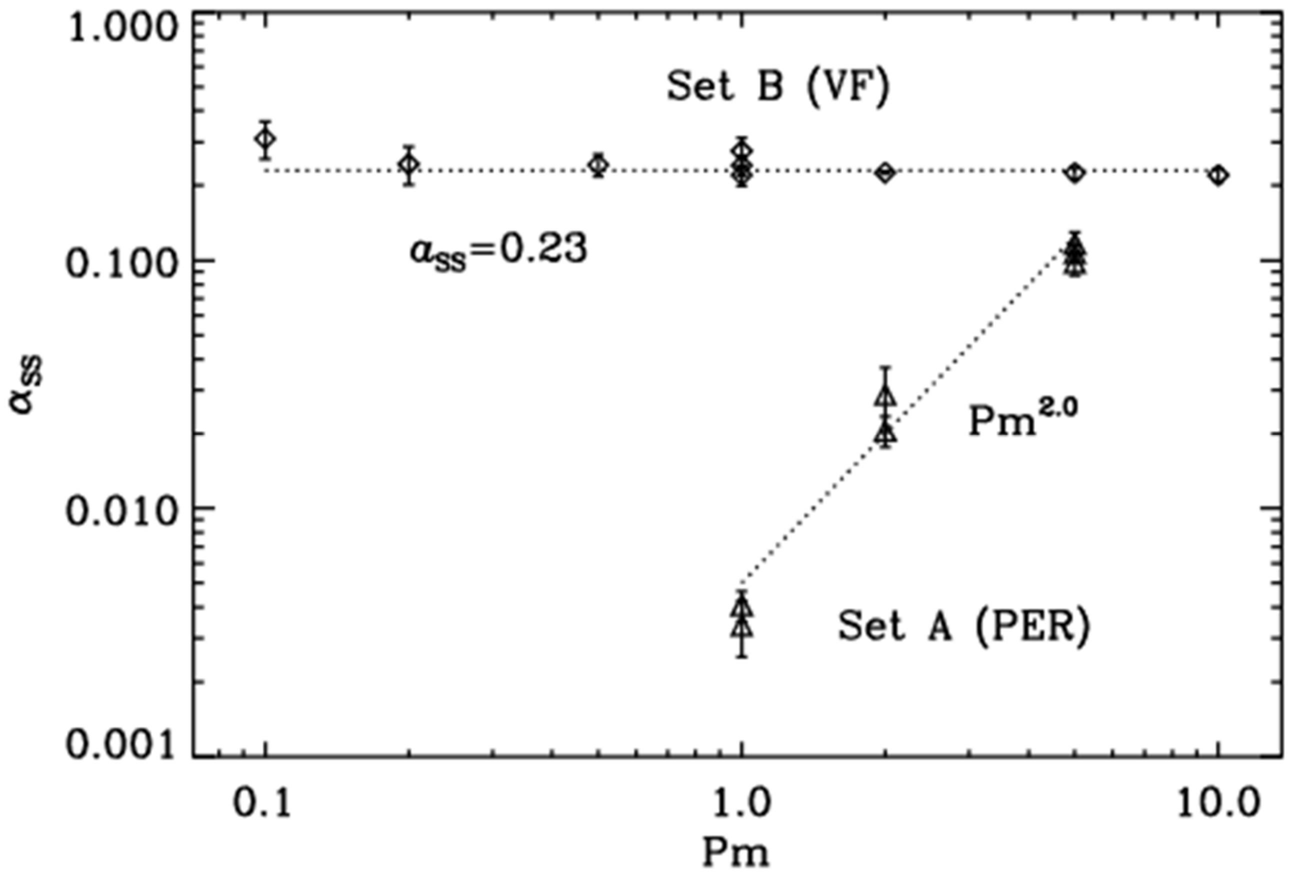


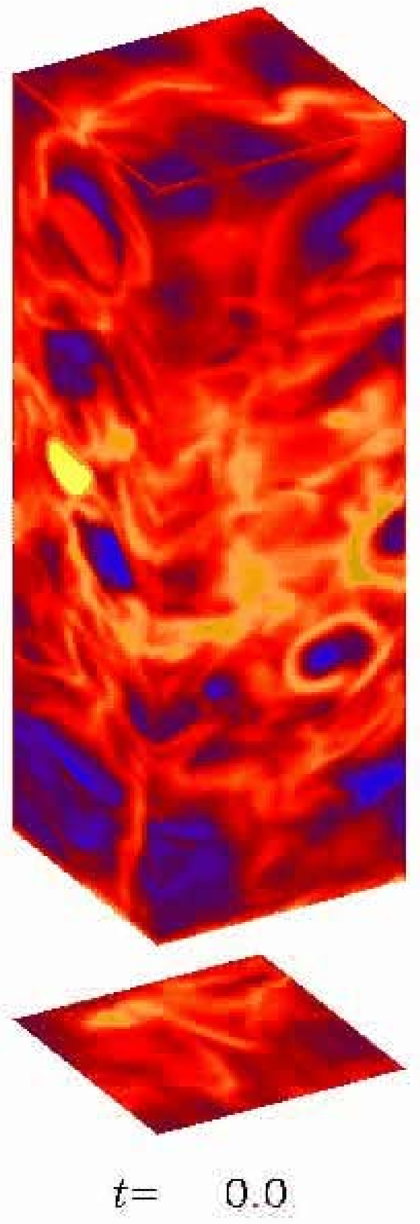
Brandenurg et al. (2008)



Low Pr_M issue in unstratified MRI

Käpylä & Korpi (2010, MNRAS): vertical field condition





(iii) Galactic dynamo

Based on temperature:

- 20-30% near mid plane
- 50-60% at 300 pc height

Based on field strength (dynamos):

- Small (1/Rm) in kinematic stage
- O(1) during saturated stage

(iv) SNR: MHD plasma with CRs

$$\rho \frac{\partial \mathbf{U}}{\partial t} = -\nabla P + \tfrac{1}{c}\mathbf{J} \times \mathbf{B} + e\left(n_i - n_e\right)\mathbf{E} + \mathbf{F}_\nu$$

$$\nabla \times \mathbf{B} = \tfrac{4\pi}{c}\left(\mathbf{J} + \mathbf{J}_{\mathrm{cr}}\right) \qquad\qquad n_i + n_{cr} = n_e$$

$$\rho \frac{\partial \mathbf{U}}{\partial t} = -\nabla P + \left(\tfrac{1}{4\pi}\nabla \times \mathbf{B} - \tfrac{1}{c}\mathbf{J}_{\mathrm{cr}}\right) \times \mathbf{B} + e n_{\mathrm{cr}} \mathbf{U} \times \mathbf{B} + \mathbf{F}_\nu$$

To be solved with induction equation
and continuity equation, isothermal EOS

Introduces pseudoscalar

$\boldsymbol{\Omega} \cdot \mathbf{g} \rightarrow \alpha$ effect in stars

$$\mathbf{J}_{\mathrm{cr}} \cdot \mathbf{B}_0 \rightarrow \alpha \text{ effect}$$

α effect important for large-scale field in the Sun

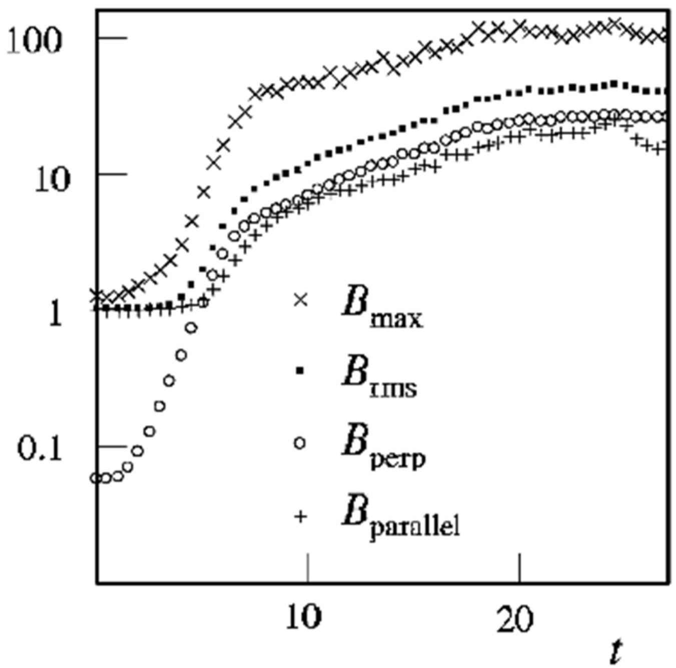
$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}} - \overline{\mathbf{J}} / \sigma \right)$$

$$\overline{\mathcal{E}} \equiv \overline{\mathbf{u} \times \mathbf{b}} = \alpha \overline{\mathbf{B}} + \ldots$$

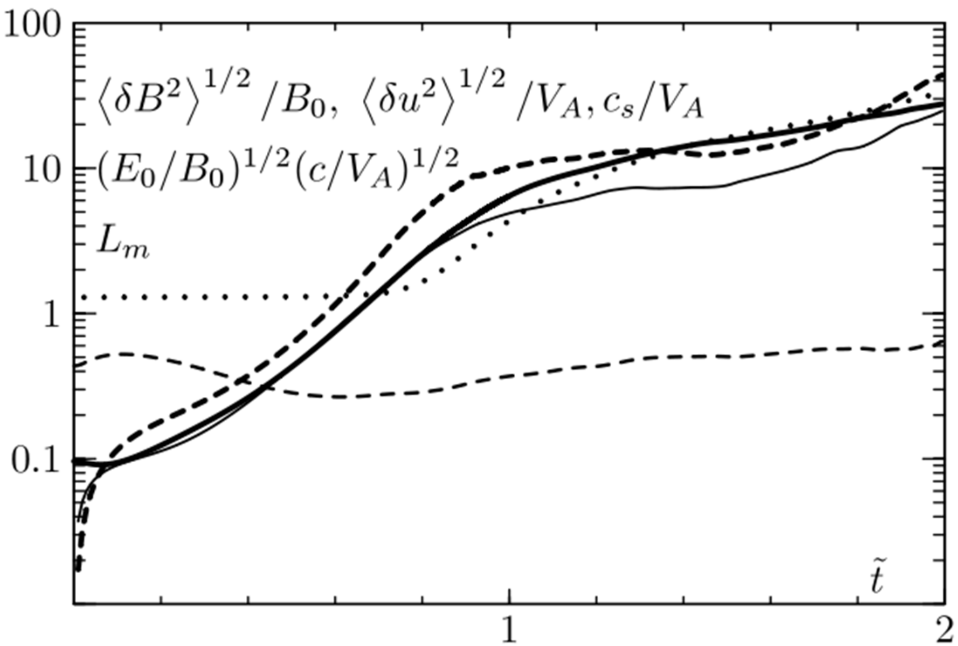
$$\overline{\mathcal{E}}_i = \alpha_{ij} \overline{B}_j - \eta_{ij} \overline{J}_j + \ldots$$

Bell instability

$$\gamma_B^2 = \left(\frac{4\pi}{c} \frac{J_{cr}}{B} k_z - k^2 \right) v_A^2 \qquad \mathcal{J} = \frac{4\pi}{c} J_{cr} / kB_0$$



Bell (2004): $\mathcal{J}$=2



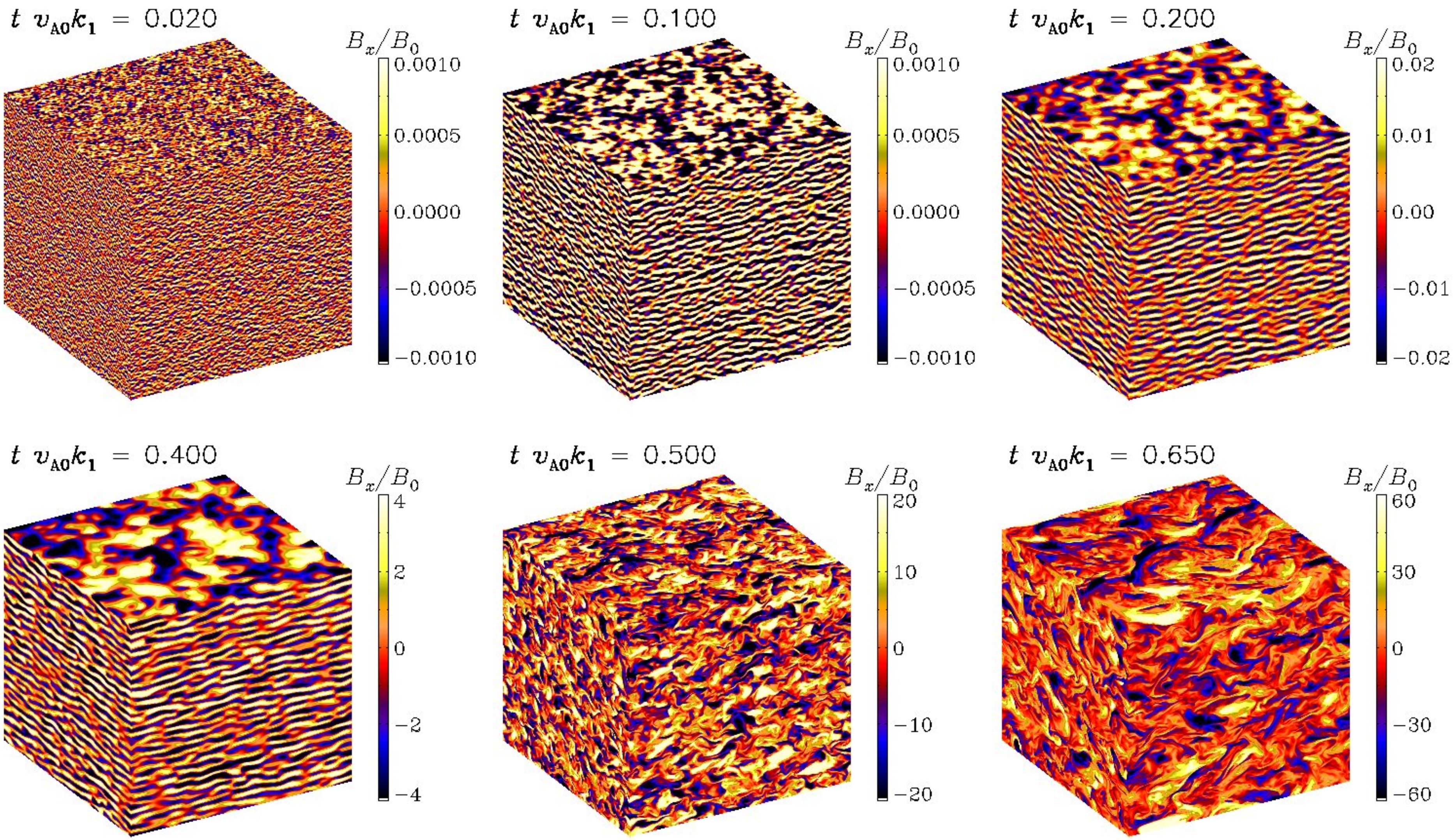
Zirakashvili et al (2008): $\mathcal{J}$=16

Continued growth in both cases! → α effect important?

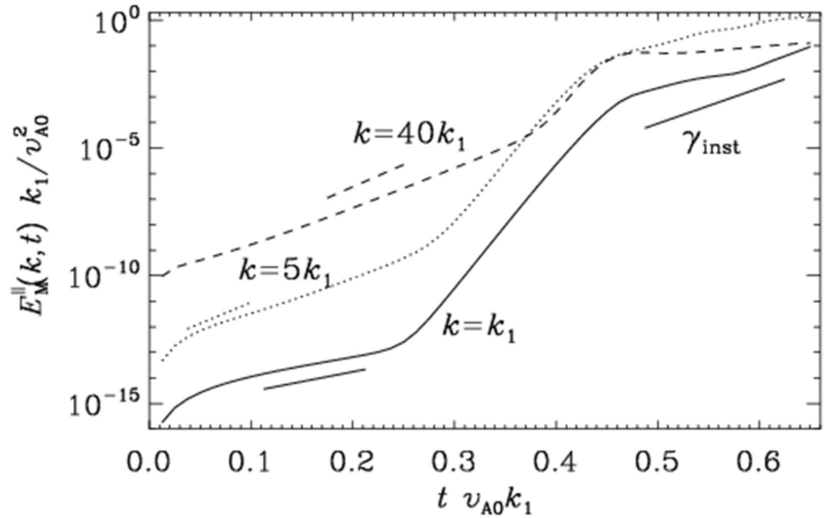
New simulations

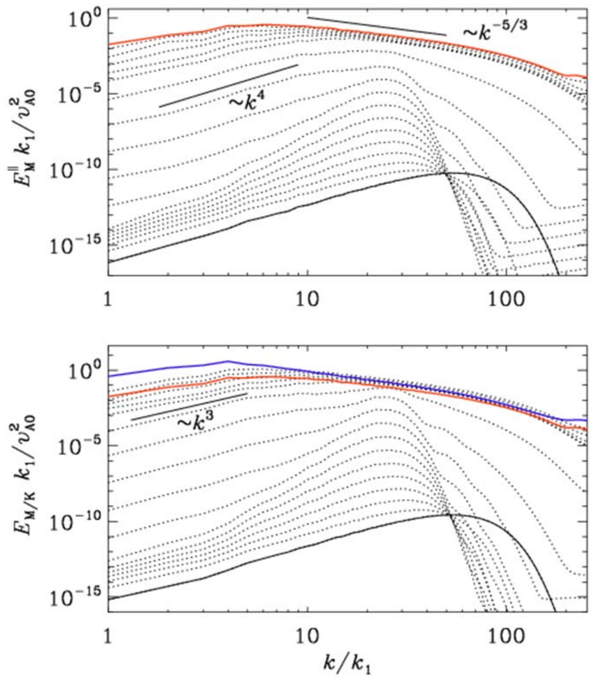
- 512^3 resolution, non-ideal (Re=Lu < 300)
- larger J parameter (80 and 800)
- most unstable k /k_1= 40 and 400 (unresolved)
- measure alpha and turbulent diff. tensor
- Related to earlier work by Bykov et al. (2011)

Bell instability → turbulence (σ=80)



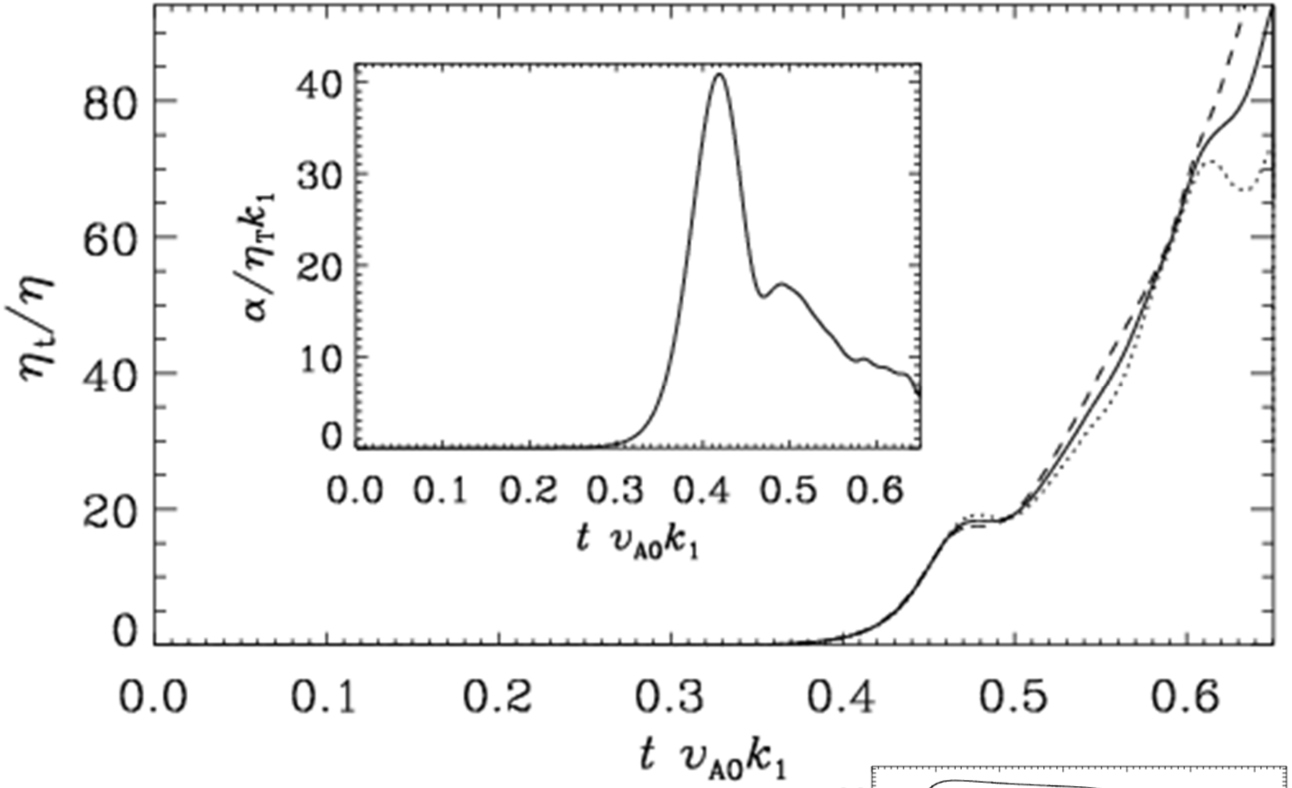
3 stages

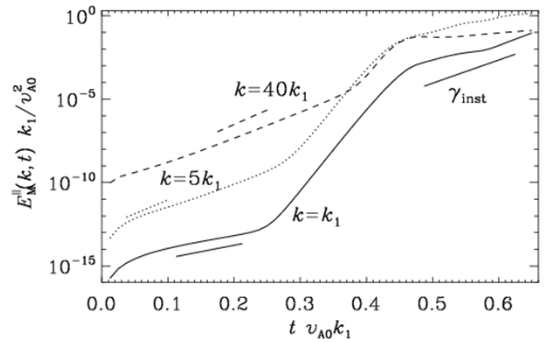


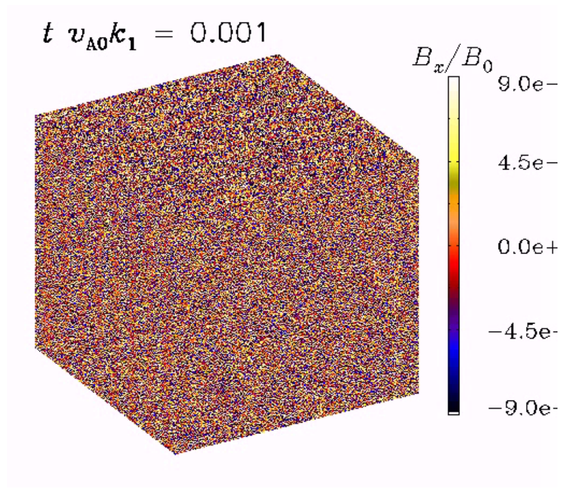


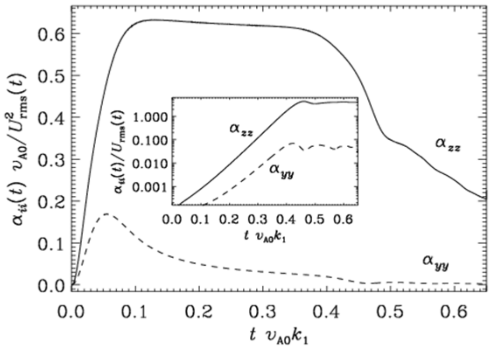
- Bell instability,small scale, k/k_1=40
- Accelerated large-scale growth
- Slow growth after initial saturation

Dynamo number, turb diff









Critical value 1, turb diff
>> microscopic value

Thanks to the Astrophysics group at Nordita

Conclusions

- Magnetic diffusion important in dynamos
- Slow growth avoided by helicity fluxes
- Outflows, coronal mass ejections
- Solar dynamo: equatorward migration
- MRI dynamos: Pr_M-independent
- Dynamos in SNR to explain strong fields