The Gravitational Trillion-Body Problem on K computer

Tomoaki Ishiyama
(Tsukuba Univ. HPCI Strategic Field Program 5)

co-developer
Keigo Nitadori (RIKEN, Advanced Institute for Computational Science)

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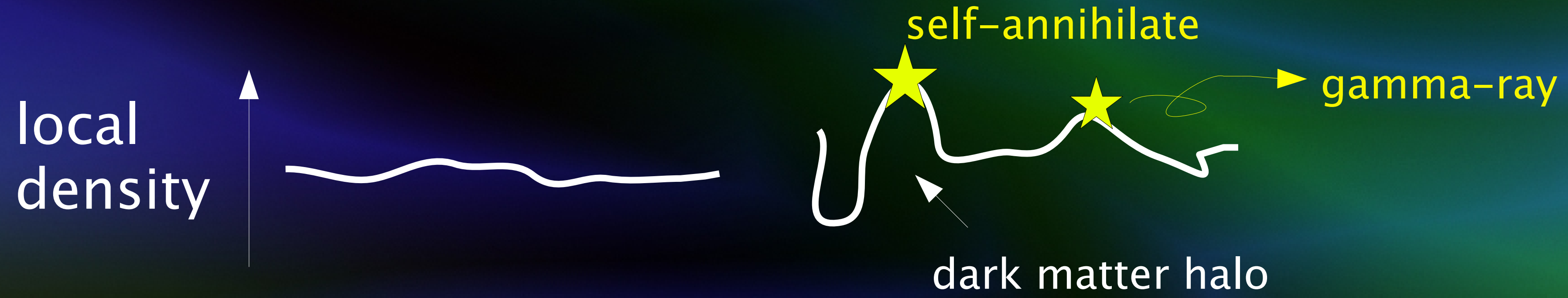
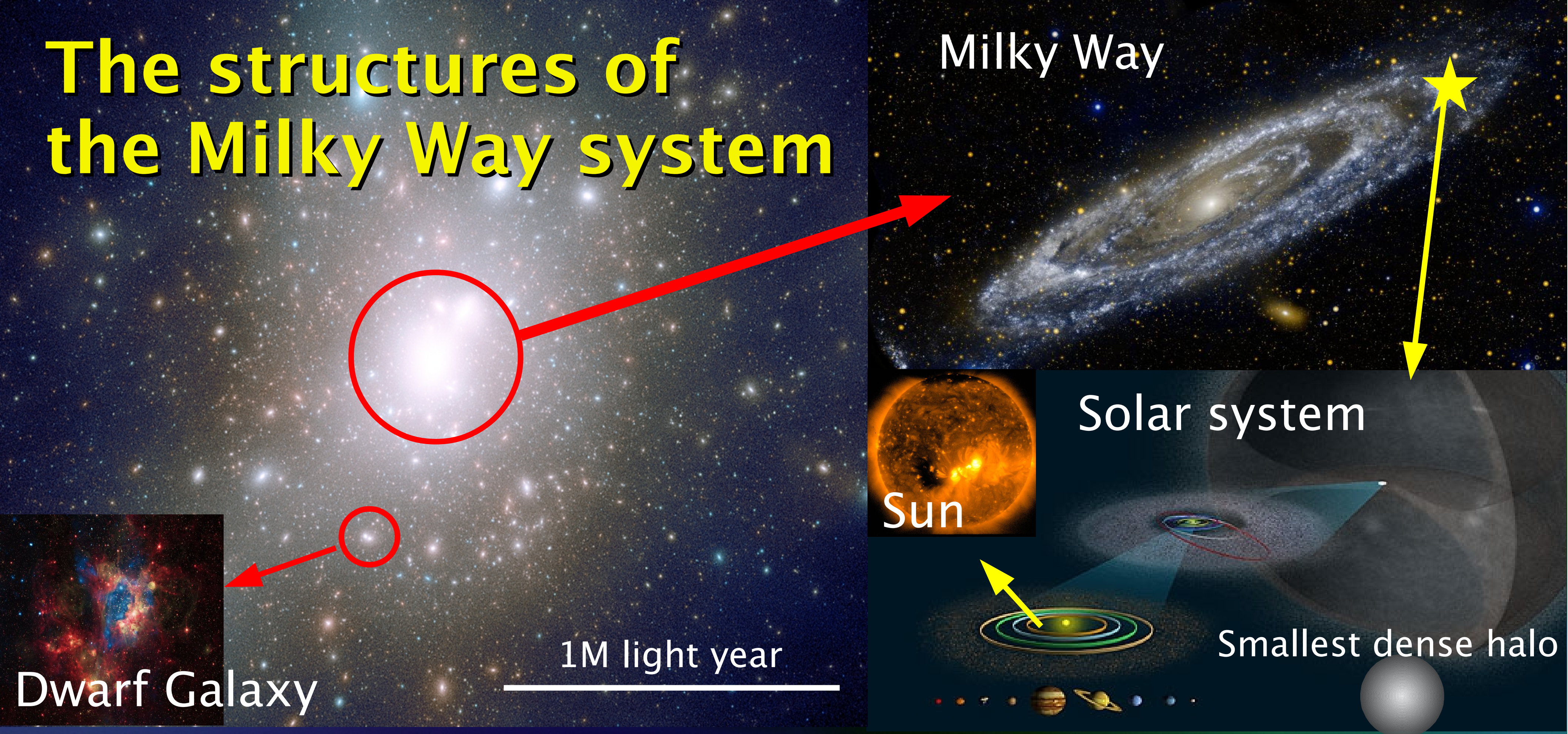
Nature of dark matter particle

- The nature of dark matter is still unknown
- One candidate is ~100GeV **neutrarlino**
  - Self-annihilates and produces gamma-ray (indirect dark matter search)
  - Gamma-ray flux is proportial to (local density)^2



- The mass of the smallest halo is comparable to earth-mass (10^{-6} solar mass, e.g. Berezinsky+ 2003)
- The smallest halo is **denser** than larger ones and many of them can **survive near Sun** (Ishiyama+ 2010)

The structures of the Milky Way system



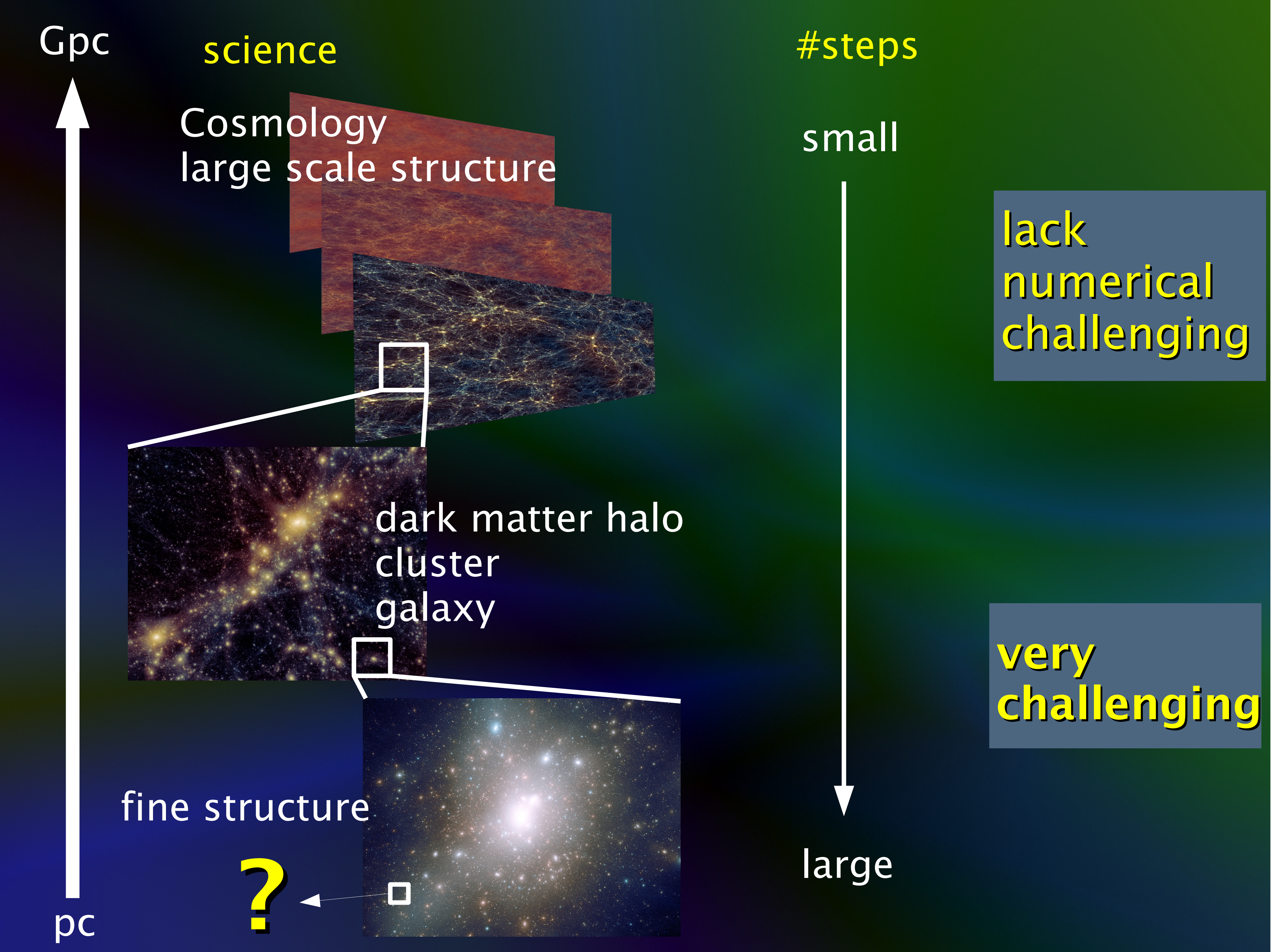
- Baryon (stars, gas, etc) is located in the center of ~10^{12} solar mass halo.
- Myriad subhalos (10^{-6} ~ 10^{10} solar mass)
- Where can we observe gamma-ray flux from dark matter ?
  - The center of the Milky Way halo ?
  - Dwarf Galaxy ?
  - Smallest dense halos near Sun ?

This study

- In order to detect annihilation gamma-ray, we have to know the fine structures of the Milky Way halo
  - The nonlinear evolution of smallest halos
  - How many smallest halos survive at present near Sun?
- Follow the merging, growth of smallest halos
  - The mass of the smallest halo is 18 orders of smaller than the Milky Way
  - The number of particles required is very huge

The Gravitational Trillion-Body Problem

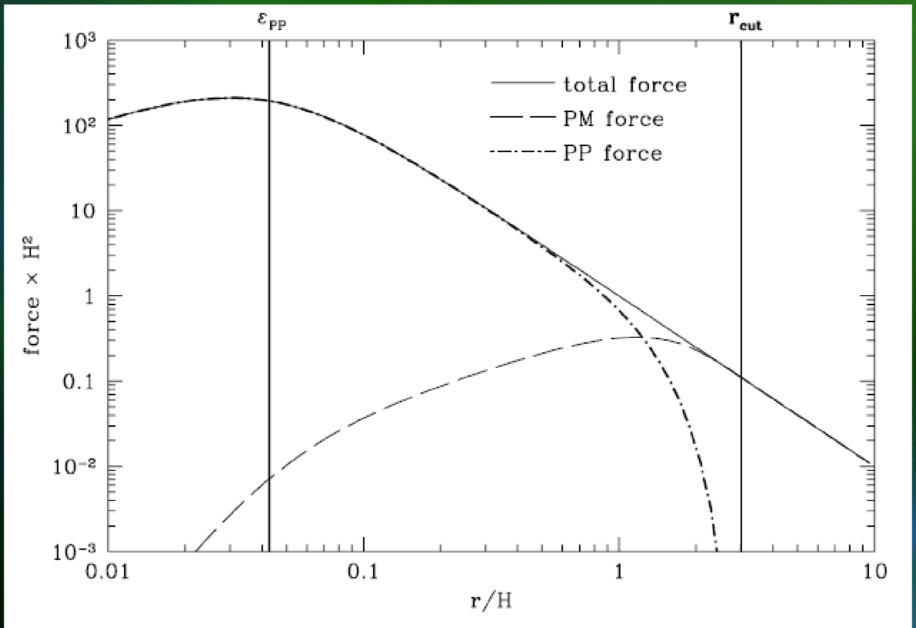
High performance code


Gpc
science
#steps
Cosmology
large scale structure
small
lack
numerical
challenging
dark matter halo
cluster
galaxy
very
challenging
fine structure
?
large
pc

TreePM Poisson solver

- **Tree** for short-range forces
- **PM** for long-range forces

- Drastically reduces the cost of evaluating pairwise forces
- Periodic boundary
- Becomes popular from 21th
  - GADGET-2 (Springel 2005)
  - GOTPM (Dubinski+ 2004)
  - GreeM (Ishiyama+ 2009)

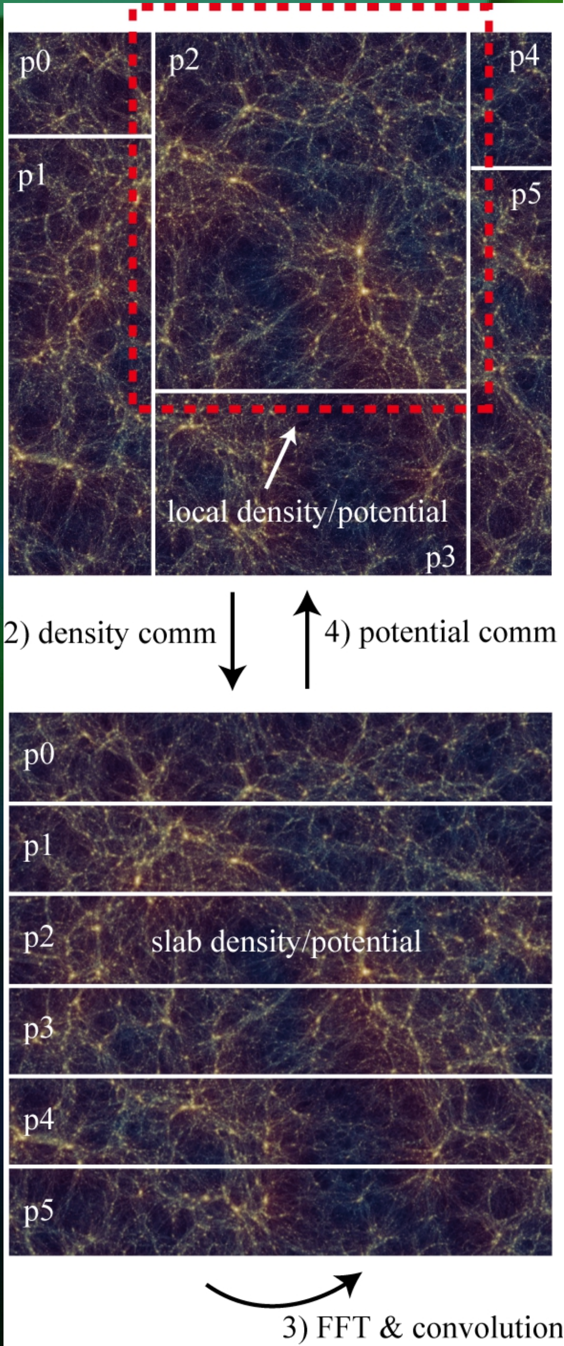


$$\boldsymbol{a}_i = \sum_{j \neq i} \frac{m_j(\boldsymbol{r}_j - \boldsymbol{r}_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^3} g_{\mathrm{P3M}}(|\boldsymbol{r}_j - \boldsymbol{r}_i|)/\eta),$$

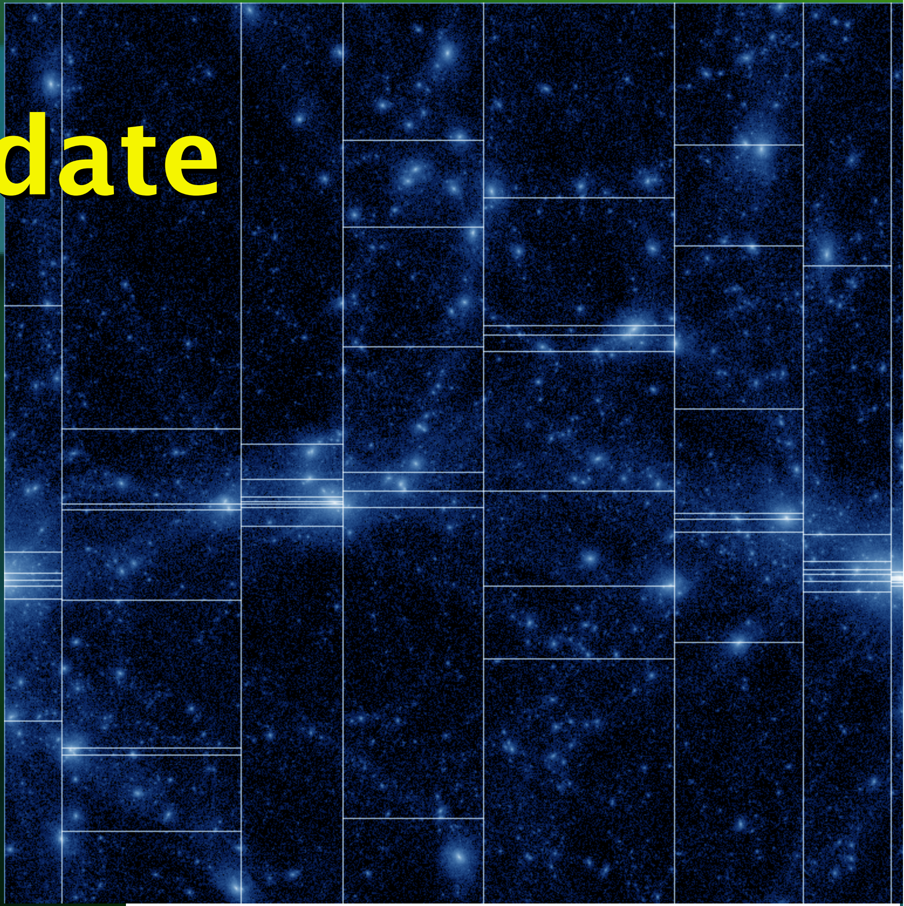
$$g_{\mathrm{P3M}}(R) = \begin{cases} 1 - \frac{1}{140}\left(224R^3 - 224R^5 + 70R^6 + 48R^7 - 21R^8\right) & (0 \le R \le 1) \\ 1 - \frac{1}{140}\left(12 - 224R^2 + 869R^3 - 840R^4 + 224R^5 + 70R^6 - 48R^7 + 7R^8\right) & (1 \le R \le 2) \\ 0 & (2 \le R) \end{cases},$$

Difficulties and Overcoming

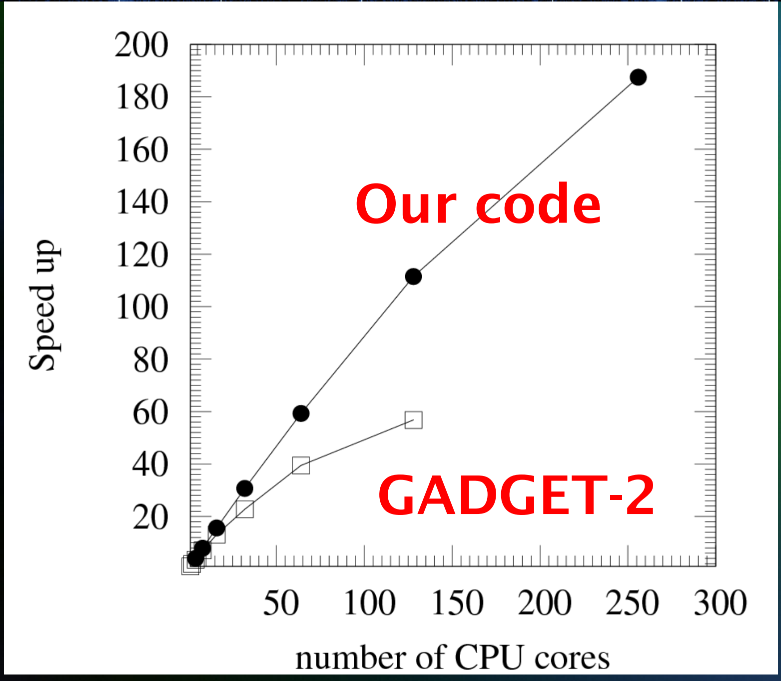
- Dense stuructures form everywhere via the gravitationaly instability
  - Dynamic domain update with a good load balancer
- Gravity is the long-range force
  - Novel communication algorithm for long communication of PM part
- O ($N \log N$) is stil expensive
  - SIMD
  - (GRAPEs, GPU)



Dynamic domain update

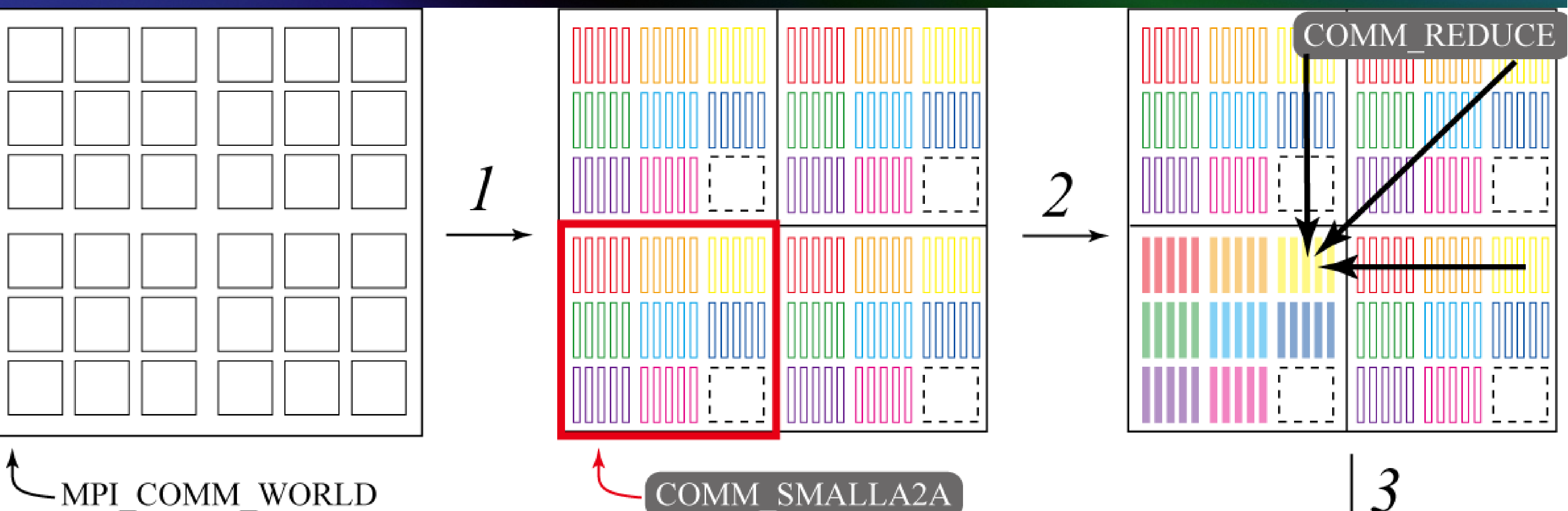


- Δ Space filling curve
- ○ multi section
  - Enables each node to know easily where to perform short communications
- x equal #particles
- Δ equal #interactions
- ○ equal #interactions + correction
  - equal calculation time



Novel communication algorithm for the PM part

- Hierarchical communication
- One MPI_Alltoallv(..., MPI_COMM_WORLD) ->
  - MPI_Alltoallv(..., COMM_SMALLA2A)
  - MPI_Reduce(..., COMM_REDUCE)



Gravity kernel

This implementation is done by **Keigo Nitadori** as an extension of Phantom-GRAPE library (Nitadori+ 2006, Tanikawa+ 2012a, 2012b)

$$g_{\mathrm{P3M}}(R) = \begin{cases} 1 - \frac{1}{140}\left(224R^3 - 224R^5 + 70R^6 + 48R^7 - 21R^8\right) & (0 \le R \le 1) \\ 1 - \frac{1}{140}\left(12 - 224R^2 + 869R^3 - 840R^4 + 224R^5 + 70R^6 - 48R^7 + 7R^8\right) & (1 \le R \le 2) \\ 0 & (2 \le R) \end{cases},$$

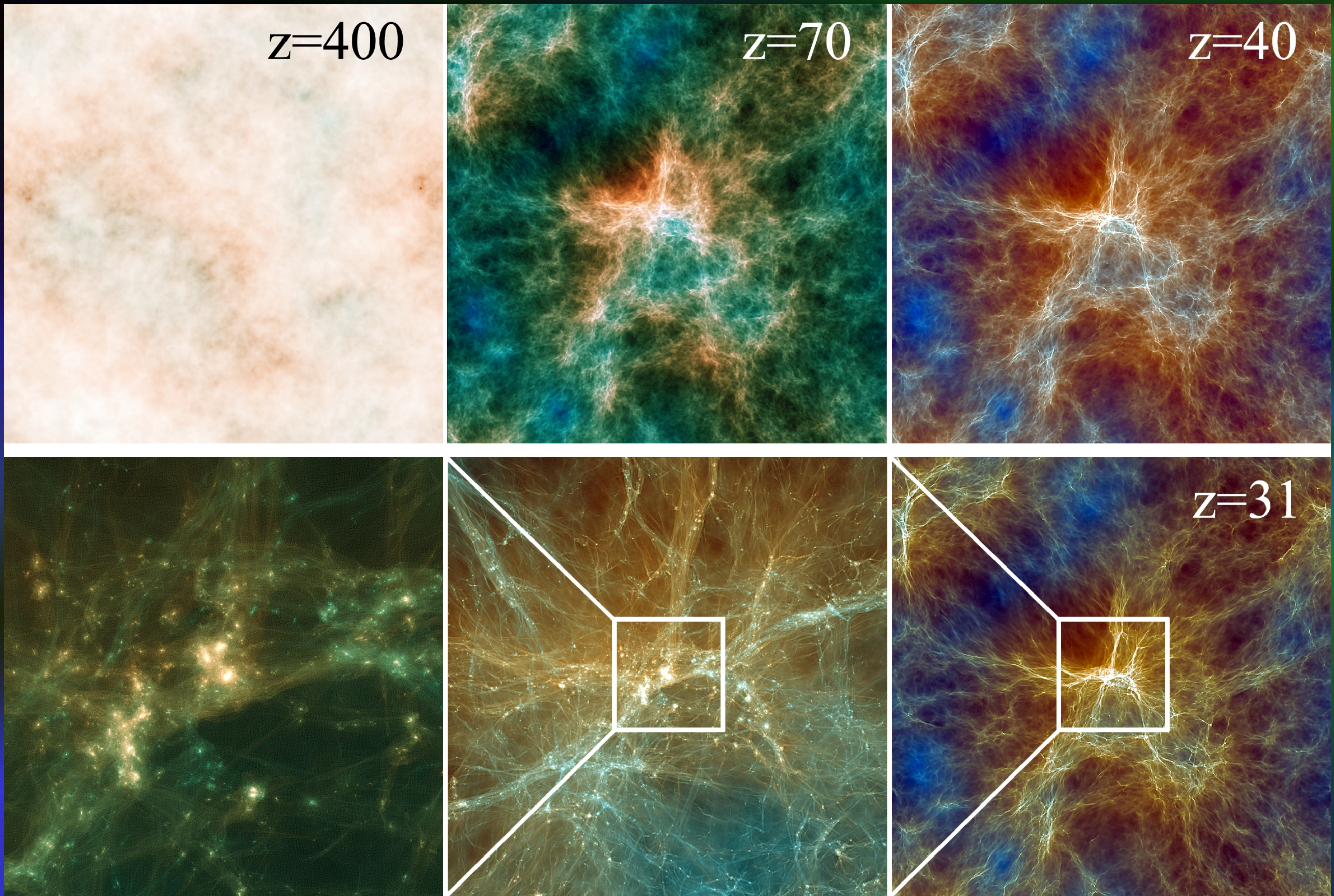
$$g_{\mathrm{P3M}}(R) = 1 + R^3\left(-\frac{8}{5} + R^2\left(\frac{8}{5} + R\left(-\frac{1}{2} + R\left(-\frac{12}{35} + R\frac{3}{20}\right)\right)\right)\right) - S^6\left(\frac{3}{35} + R\left(\frac{18}{35} + R\frac{1}{5}\right)\right) \quad (0 \le R \le 2)$$

$$S \equiv \max(0, R - 1),$$

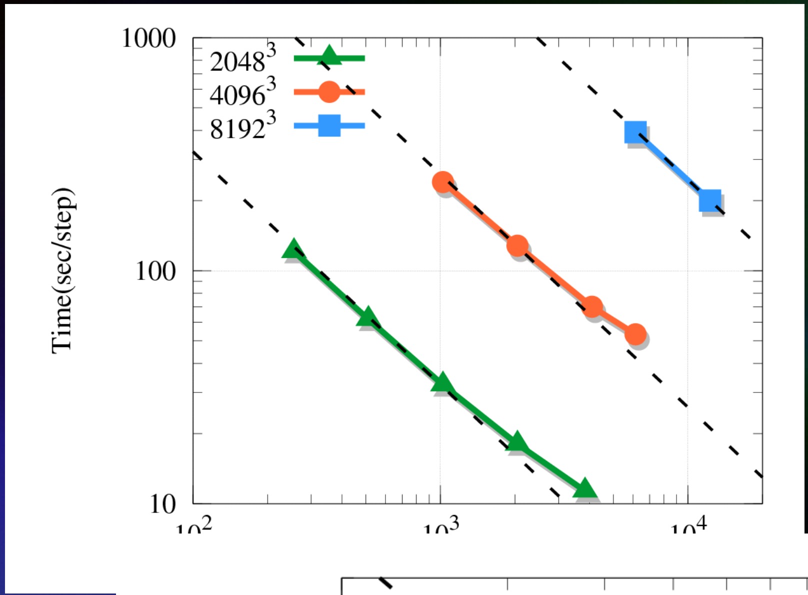
1. One if-branch is reduced
2. optimized for a SIMD hardware with FMA

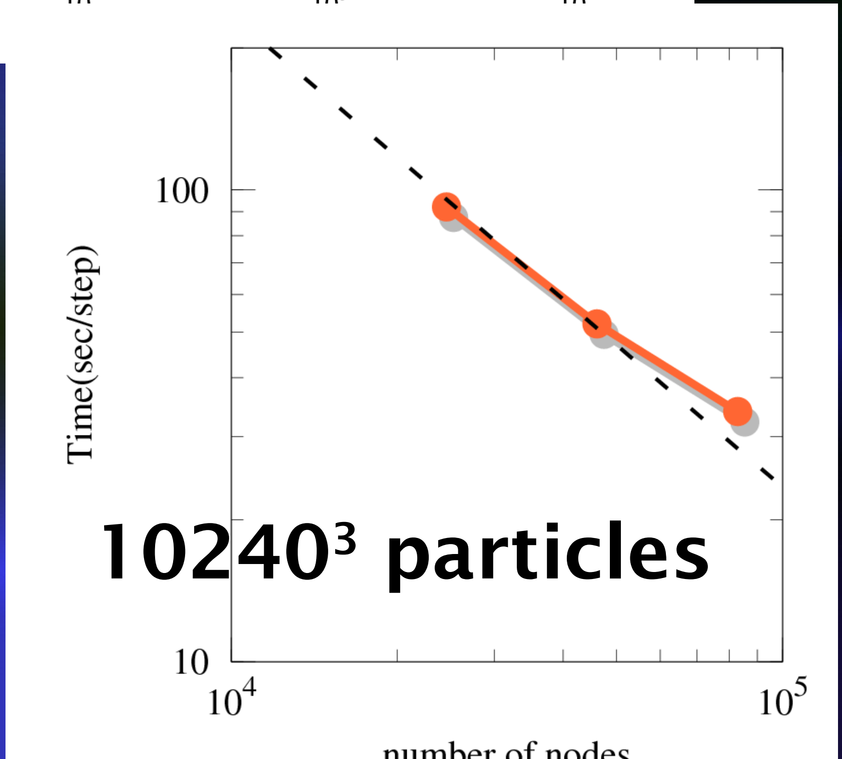
- Implemented with SIMD builtin functions (Fujitsu C++ compiler)
- Unrolled eight times by hand
- 17 FMA and 17 non-FMA operations for a interaction
- 11.65 Gflops on a simple kernel benchmark
  - **97**% of the theoretical limit (12Gflops)

Simulation setup



Performance results





- Scalability (2048^3 - 10240^3)
  - Excellent strong scaling
  - 10240^3 simulation is well scaled from 24576 to 82944 (full) nodes of K computer
- Performance (12600^3)
  - The average performance on fullsystem is ~**5.67Pflops**, which correspond to ~**55**% of the peak speed

Ishiyama, Nitadori and Makino SC12 Gordon Bell prize finalists

Summary and future prospects

- We developed a massively parallel TreePM code for large gravitationlal *N*-body simulations
  - Novel communication algorithm for long-range forces
  - Highly-tuned Gravity Kernel for short-range forces
  - ***SC12 Gordon Bell prize finalists***
- The average performance on fullsystem of K computer is ~**5.67Pflops**, which correspond to ~**55**% of the peak speed
- Our implementation enables to perform gravitational trillion-body problem within practical time