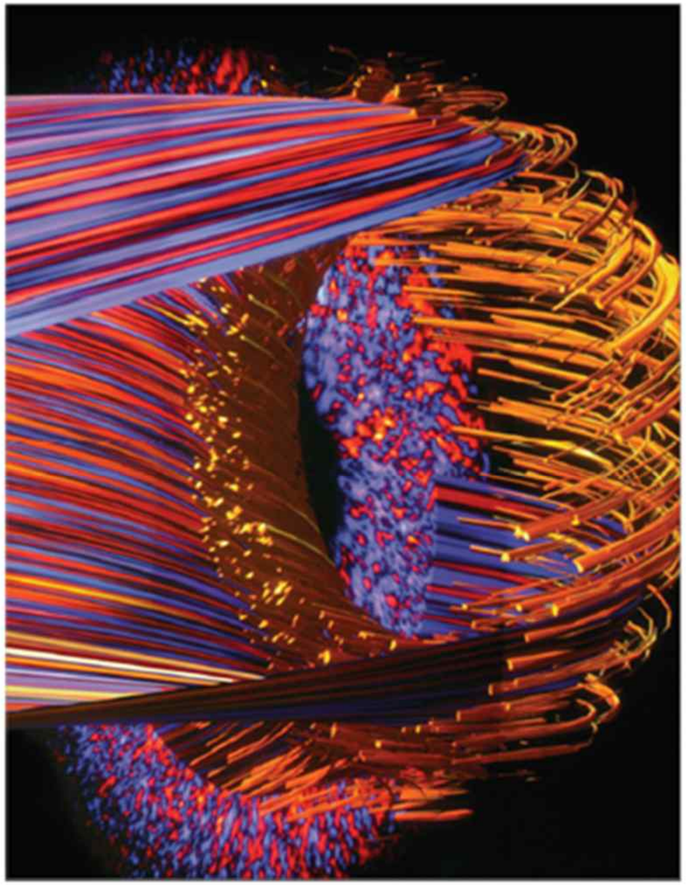
Gyrokinetic Simulation: Introduction and Prospect for Astrophysics

J.M. Kwon

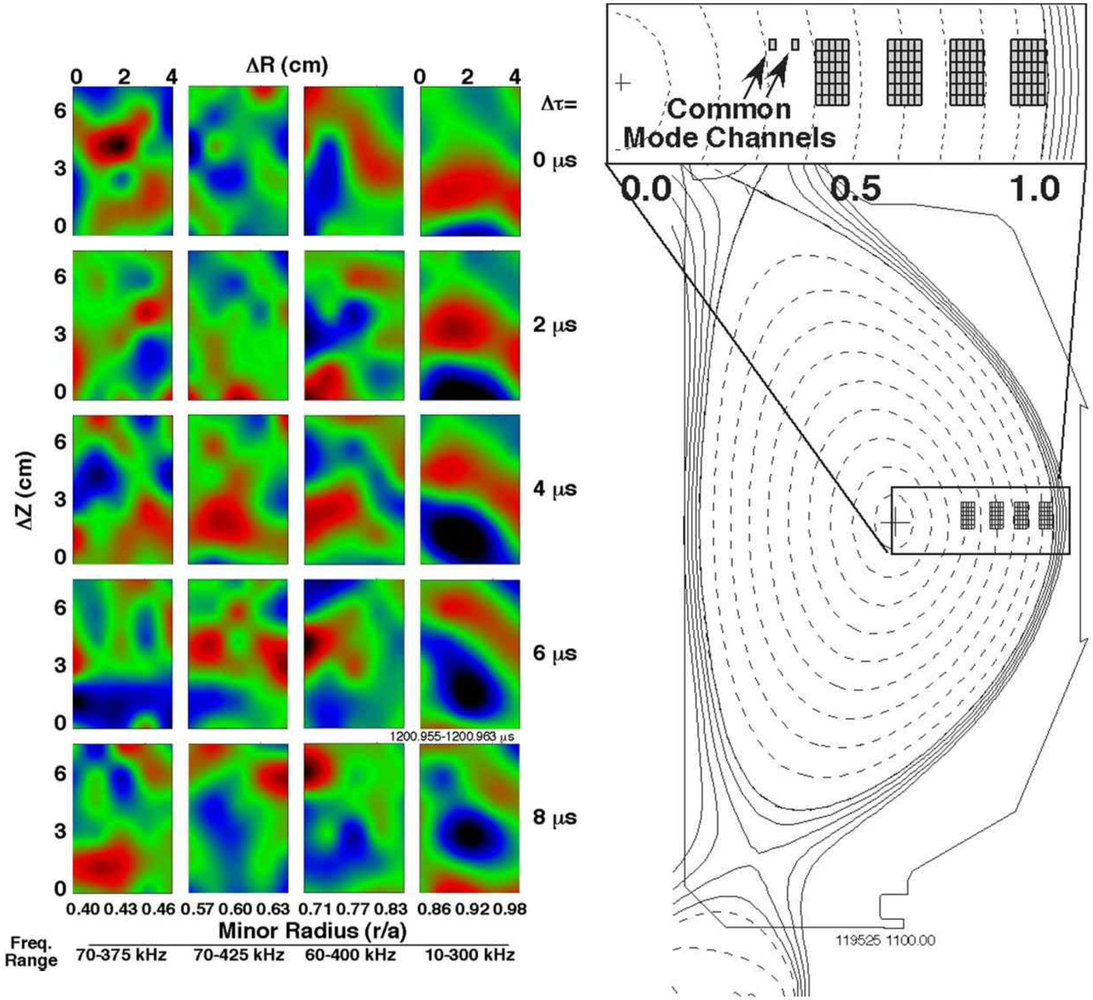
WCI Center for Fusion Theory, NFRI, Korea

East Asia Numerical Astrophysics Meeting
YITP, Kyoto, Japan
Oct. 29 ~ Nov. 2, 2012

Micro Turbulence in Fusion Device



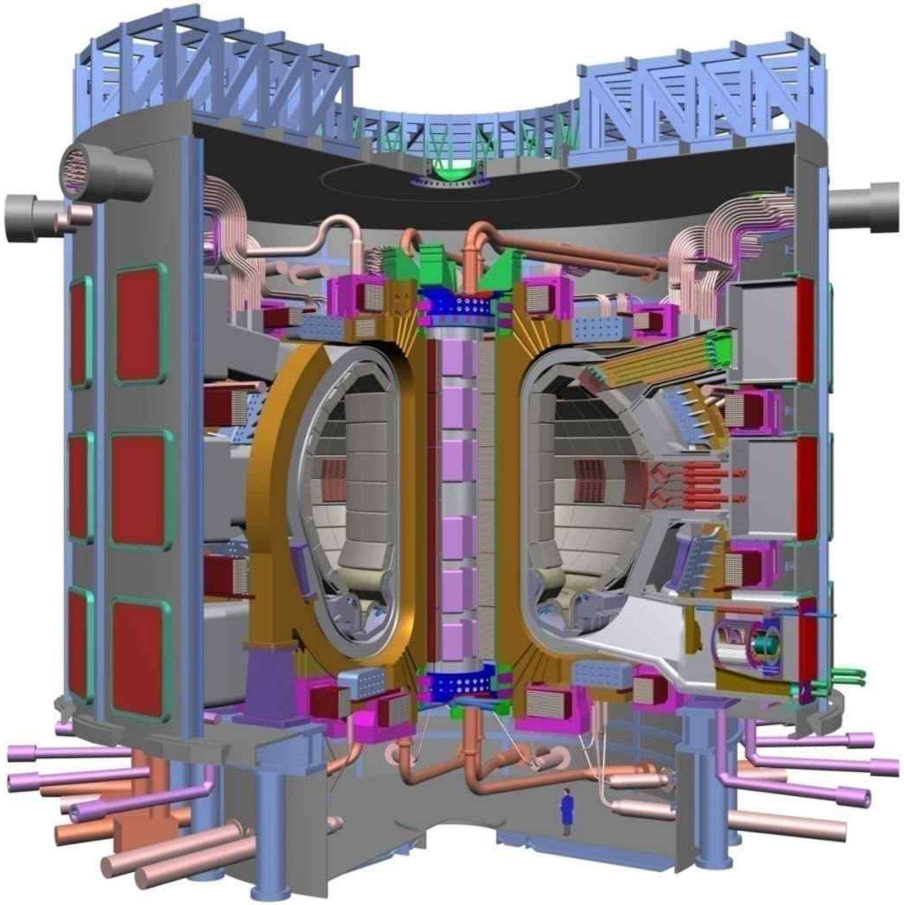
[Klasky, ORNL; Ethier, Wang, PPPL]

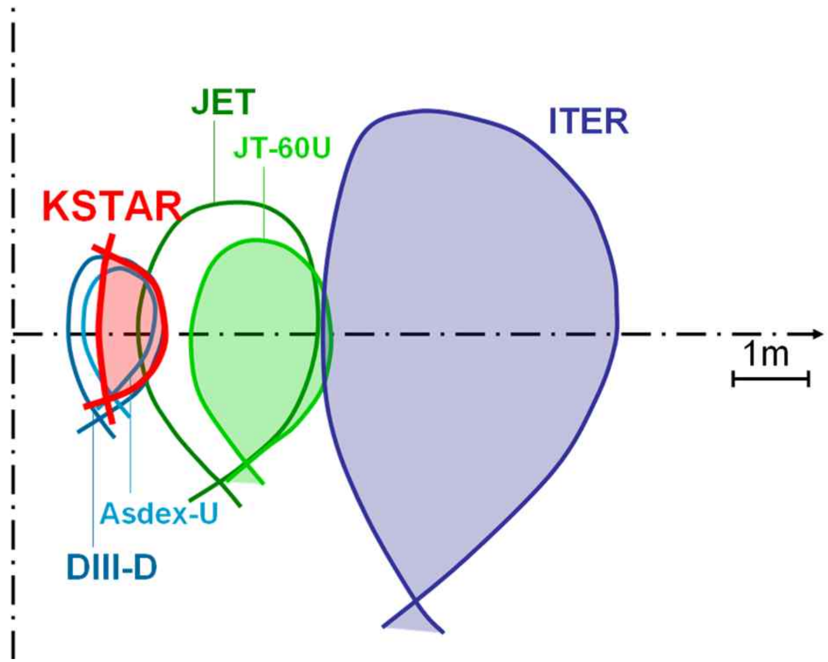


G. McKee et al., Plasma and Fusion Research 2007

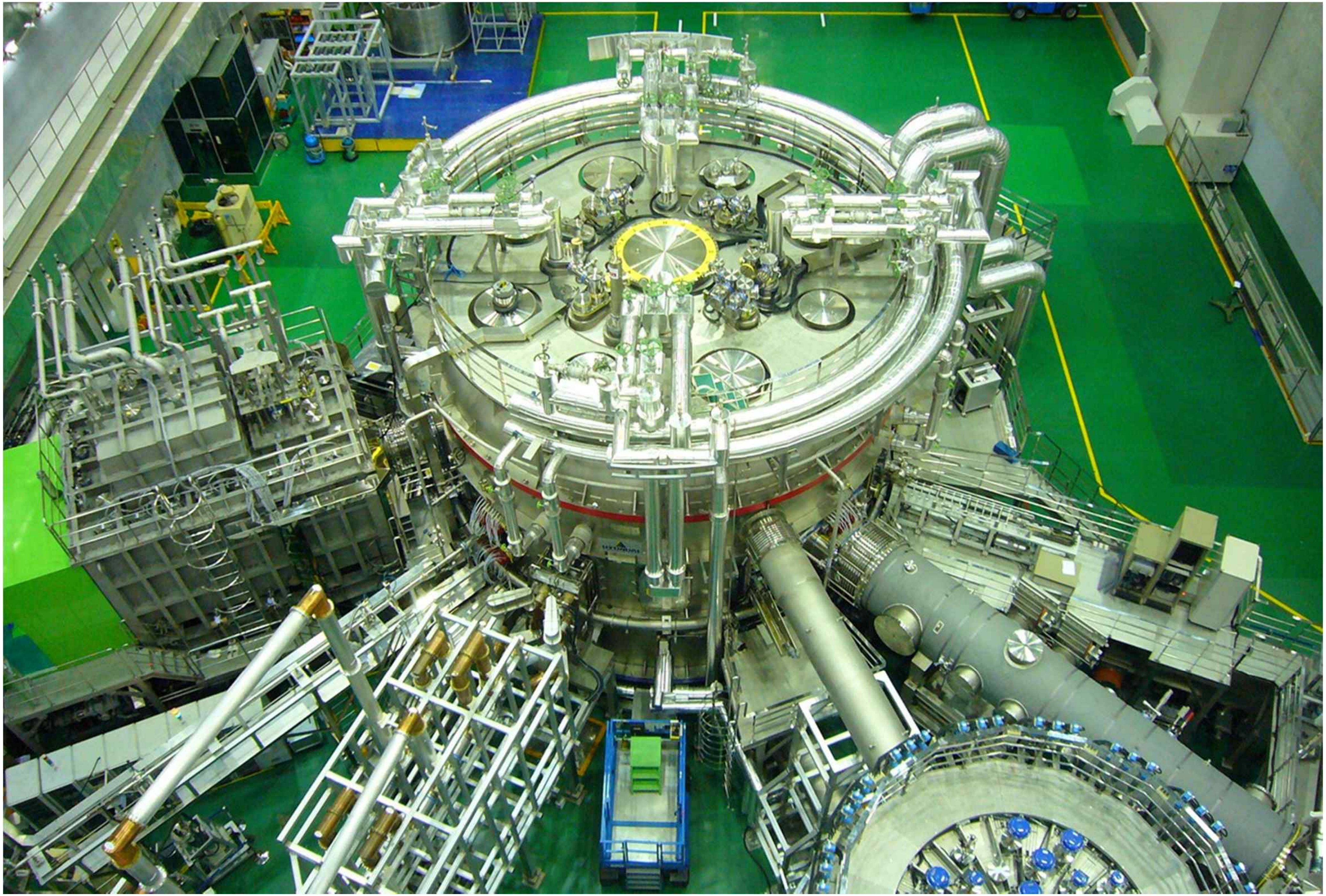
Micro Turbulence in Fusion Device

- Source of trouble and reason… why fusion people want to build ever larger machine





ITER



Outline

- Introduction to gyrokinetic theory
- Issues in gyrokinetic simulation
- Numerical methods for gyrokinetic simulation
- Summary

Introduction to Gyrokinetic Theory

Basic Idea of GK Theory

- GK orderings

T.S. Hahm, Phys. Fluids 31, 2670 (1988)
A. Brizard, T.S. Hahm, Rev. Mod. Phys. 79, 421(2007)

  – small fluctuation: $\frac{\delta f}{f_0} \sim \frac{e\delta\phi}{T} \sim \frac{\delta B}{B_0} \sim \epsilon$

  – low frequency: $\frac{\omega}{\Omega_i} \sim \epsilon$ ➡ Fast MHD waves and cyclotron waves are ruled out (high freq. GK, H. Qin '99)

  – anisotropic fluctuation: $\frac{k_\parallel}{k_\perp} \sim \epsilon,\ k_\perp \rho_i \sim 1$

  – mild non-uniformity in plasma profiles, background magnetic field etc.: $\frac{\rho_i}{L_0} \sim \epsilon$ ➡ Free energy to drive turbulence (GK with strong gradient, T.S. Hahm '09)

Basic Idea of GK Theory (cont'd)

- Guiding center transformation

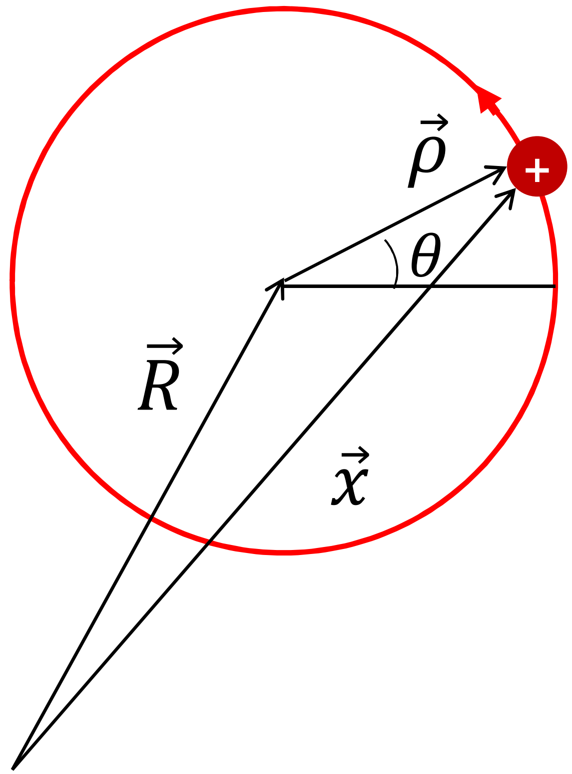
particle space $(\vec{x}, \vec{v}) \leftrightarrow$ guiding(gyro) center space $\left(\vec{R}, v_{\|}, \mu, \theta\right)$

$\boxed{\theta\text{: gyrophase-angle} \rightarrow \text{average out}}$

$$\vec{R} = \vec{x} - \vec{\rho}, \quad \vec{\rho} = \hat{b} \times \frac{\vec{v}}{\Omega}, \quad \Omega = \frac{eB_0}{m\,c}$$

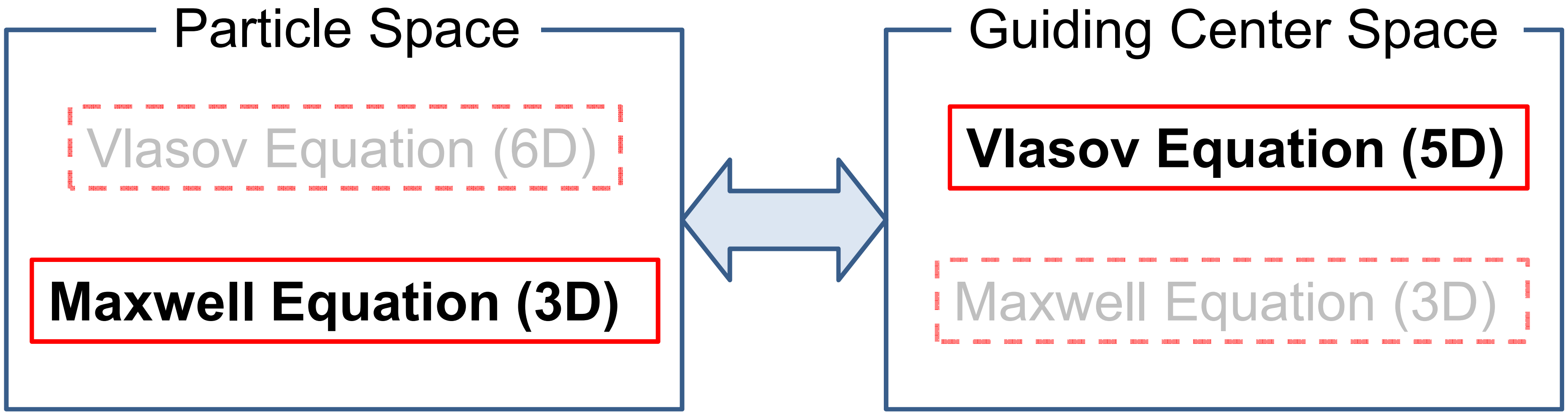
$$v_{\|} = \hat{b} \cdot \vec{v}, \quad \mu = \frac{v_{\perp}^2}{2B}$$

$$\vec{v} = v_{\|}\hat{b} + v_{\perp}\hat{e}_{\perp}$$



Basic Idea of GK Theory (cont'd)

- Schematics of guiding center transformations in GK simulation
  - ✓ Solve Vlasov equation in guiding center space and evaluate (n_s, j_s)
  - ✓ Transform (n_s, j_s) to particle space
  - ✓ Solve Maxwell equations to obtain EM fields
  - ✓ Transform EM fields to guiding center space



Gyrokinetic Vlasov Equation

- Transform original 6D Vlasov equation in particle space into guiding center space
- Take gyro-angle average to remove θ

  $\rightarrow$ reduction to 5D $\bar{f}(\vec{R}, v_\parallel, \mu, t)$, large time step $\Delta t > 1/\Omega_i$

$$\boxed{\begin{aligned}&\frac{\partial \bar{f}}{\partial t} + \left(v_\parallel \hat{b}^* + \frac{\mu}{B}\,\hat{b} \times \nabla B + \frac{c}{B_0} \nabla_\perp \hat{b} \times \langle \delta\psi \rangle \right) \cdot \frac{\partial \bar{f}}{\partial \vec{R}} \\ &+ \frac{q}{m}\left(-\hat{b}^* \cdot \mu \nabla B - \hat{b}^* \cdot \nabla \langle \delta\phi \rangle - \frac{1}{c}\frac{\partial}{\partial t} \langle \delta A_\parallel \rangle \right) \frac{\partial \bar{f}}{\partial v_\parallel} = 0\end{aligned}}$$

$$\delta\psi = \delta\phi - \frac{v_\parallel}{c}\delta A_\parallel$$

$$\hat{b}^* = \hat{b} + \frac{v_\parallel}{B}\hat{b} \times \hat{b} \cdot \nabla \hat{b}$$

$\langle \cdot \rangle$ = gyro-phase averaged fluctuations

Gyrokinetic Maxwell Equation

- Solved in particle space i.e. no guiding center transformation

$$-\nabla^2\delta\phi(\vec{x},t) = 4\pi\sum_s q_s n_s(\vec{x},t)$$

$$-\nabla^2\delta A_\parallel(\vec{x},t) = \frac{4\pi}{c}\sum_s j_s(\vec{x},t)$$

- Evaluation of (n_s, j_s) in particle space ➔ pull-back transformation of sources (i.e. from guiding center space to particle space) is needed: $\bar{f}_s(\vec{R}, v_\parallel, \mu, t) \rightarrow f_s(\vec{x}, \vec{v}, t)$

$$n_s(\vec{x},t) = \int B^* d^3R dv_\parallel d\mu d\theta\ \ \delta(\vec{R}+\vec{\rho}-\vec{x})\left\{\bar{f}_s + \frac{q_s}{m_s B_0}(\delta\psi - \langle\delta\psi\rangle)\frac{\partial \bar{f}_s}{\partial\mu}\right\}$$

$$j_s(\vec{x},t) = \int B^* d^3R dv_\parallel d\mu d\theta\ \ \delta(\vec{R}+\vec{\rho}-\vec{x}) v_\parallel q_s\left\{\bar{f}_s + \frac{q_s}{m_s B_0}(\delta\psi - \langle\delta\psi\rangle)\frac{\partial \bar{f}_s}{\partial\mu}\right\}$$

Gyrokinetic Maxwell Equation (cont'd)

- We need to solve to the following forms:

$$\nabla^2 \delta\phi(\vec{x},t) = -4\pi \sum_s \left\{ q_s \left[\bar{N}_s - \sum_k e^{i\vec{k}\cdot\vec{x}} \frac{1-\Gamma_0(b_s)}{T_s} \left(q_s \delta\phi_k N_s - \frac{1}{c} J_s \delta A_{\parallel k} \right) \right] \right\}$$

$$\nabla^2 \delta A_\parallel(\vec{x},t) = -\frac{4\pi}{c} \sum_s \left\{ \left[\bar{J}_s - \sum_k e^{i\vec{k}\cdot\vec{x}} \frac{q_s\left(1-\Gamma_0(b_s)\right)}{T_s} \left(\delta\phi_k J_s - \frac{q_s}{c} \Pi_s \delta A_{\parallel k} \right) \right] \right\}$$

- If we take long wave length limit, we can simplify these as

(long wave length limit $\rightarrow \mathrm{b}_i = (k_\perp \rho_i)^2 \ll 1, \Gamma_0(b_i) \approx 1 - b_i\,, \Gamma_0(b_e) \approx 1$)

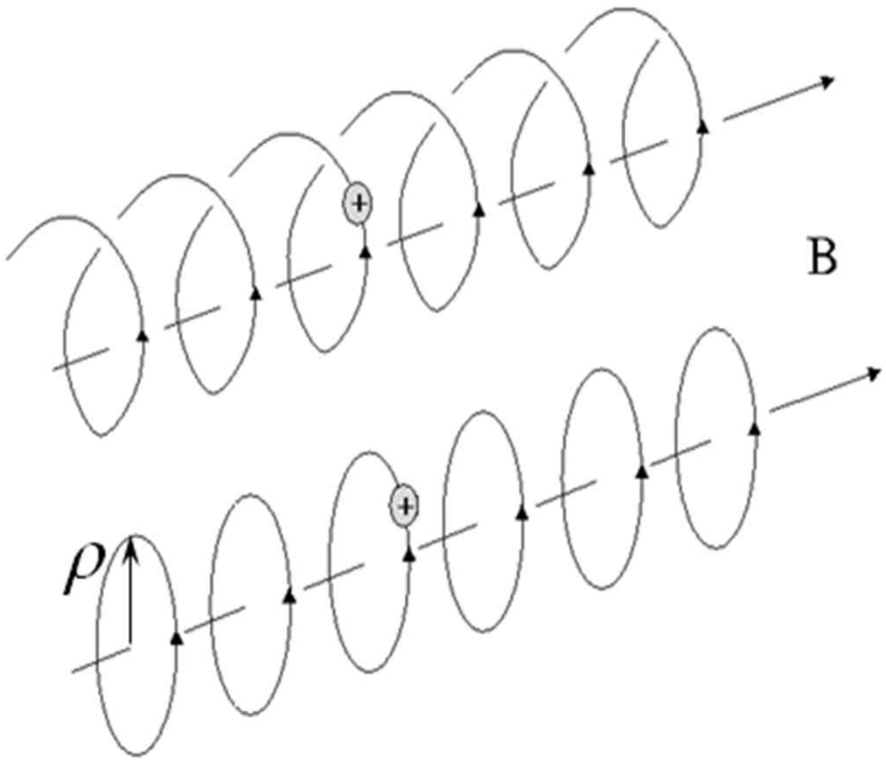
$$-\left(1 + \frac{\rho_i^2}{\lambda_{Di}^2}\right) \nabla^2 \delta\phi(\vec{x},t) = 4\pi \sum_s q_s \bar{N}_s \qquad \bar{N}_s \equiv \int B^* d^3R\, dv_\parallel d\mu d\theta \;\; \delta\left(\vec{R} + \vec{\rho} - \vec{x}\right) \bar{f}_s$$

$$-\nabla_\perp^2 \delta A_\parallel = \frac{4\pi}{c} \sum_s \bar{J}_s \qquad \bar{J}_s \equiv \int B^* d^3R\, dv_\parallel d\mu d\theta \;\; v_\parallel q_s \delta\left(\vec{R} + \vec{\rho} - \vec{x}\right) \bar{f}_s$$

(A. Brizard, T.S. Hahm, Rev. Mod. Phys. 2007)

Simple Minded View on GK Theory

- Gyrokinetic description of magnetized plasmas



Motion of charged particle

$$\frac{d}{dt}\vec{x} = \vec{v}$$

$$\frac{d}{dt}\vec{v} = -\frac{q}{m}\nabla\delta\phi + \frac{q}{mc}\vec{v}\times\left(\vec{B}_0 + \delta\vec{B}\right)$$

Motion of charged ring centered at $\vec{R}$

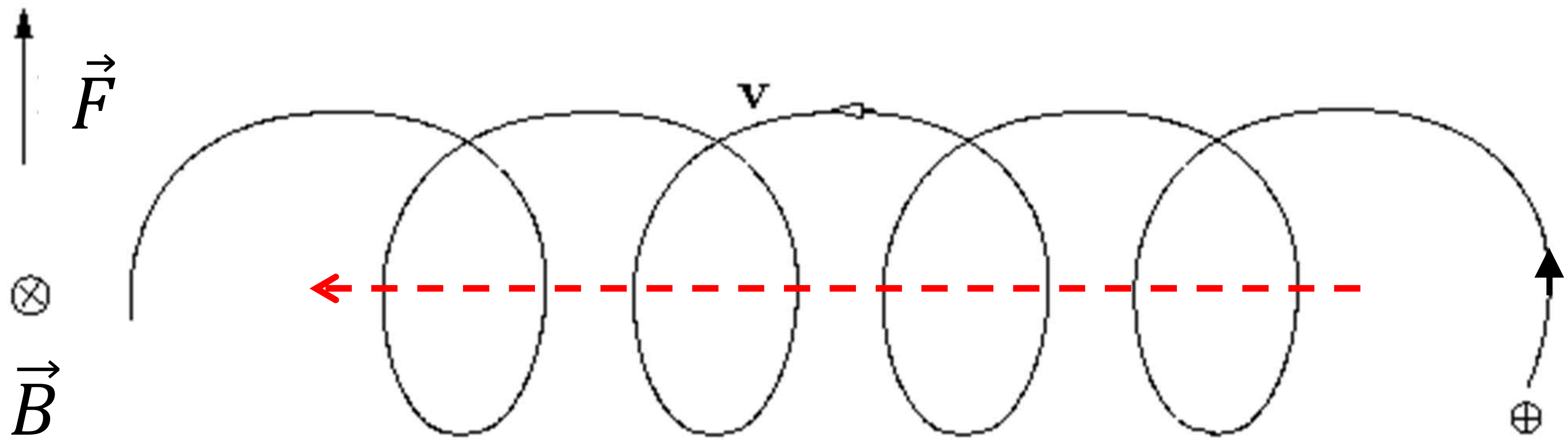
$$\frac{d}{dt}\vec{R} = v_{\|}\hat{b}^* + \frac{\mu}{B}\hat{b}\times\nabla B + c\frac{\hat{b}}{B}\times\nabla\langle\delta\psi\rangle$$

$$\frac{d}{dt}v_{\|} = -\hat{b}^*\cdot\mu\nabla B - \hat{b}^*\cdot\nabla\langle\delta\phi\rangle - \frac{1}{c}\frac{\partial}{\partial t}\langle\delta A_{\|}\rangle$$

Elementary Plasma Physics

- $\vec{F} \times \vec{B}$ drift motion of charged particle

  ➔ drift motion of gyration center in $\vec{F} \times \vec{B}$ direction



Closer look at GK equations

Grad-B + Curvature drift

$$\frac{d\vec{R}}{dt} \cong v_{||}\left[\hat{b} + \boxed{\frac{\nabla\langle\delta A_{||}\rangle \times \hat{b}}{cB}}\right] + \boxed{\left(m_s v_{||}^2 + \mu_s B\right)\frac{\hat{b}\times\nabla B}{q_s B^2}} + \boxed{c\,\frac{\vec{B}\times\nabla\langle\delta\phi\rangle}{B^2}}$$

ExB drift

Parallel motion along perturbed magnetic field

$$\frac{d}{dt}v_{||} \cong \boxed{-\hat{b}\cdot\mu\nabla B}\,-\boxed{\hat{b}\cdot\nabla\langle\delta\phi\rangle - \frac{1}{c}\frac{\partial}{\partial t}\langle\delta A_{||}\rangle}$$

Parallel E-field

mirror force

Low beta

$$\hat{b}^* = \hat{b} + \frac{v_{\|}}{B}\hat{b}\times\hat{b}\cdot\nabla\hat{b}$$

$$\approx \hat{b} + \frac{v_{\|}}{B}\,\hat{b}\times\nabla B/B$$

➔ GK equations of motion are nothing but a combination of familiar drift motions ensuring phase space volume conservation and making them Hamiltonian flows

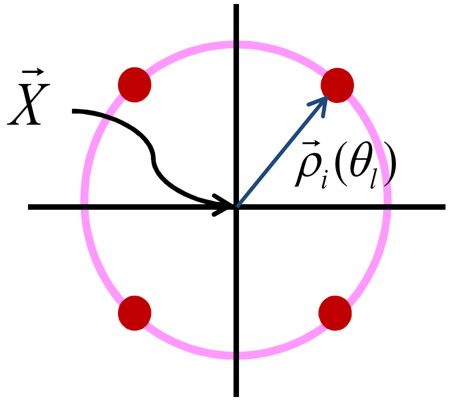
$$\boxed{\frac{\partial}{\partial t}(B^*\bar{f}) + \frac{\partial}{\partial\vec{R}}\left(\frac{d\vec{R}}{dt}B^*\bar{f}\right) + \frac{\partial}{\partial v_{||}}\left(\frac{dv_{||}}{dt}B^*\bar{f}\right) = 0}$$

Simple Minded View on GK Theory

- What is gyro-average $\langle\ \rangle$ and how to compute it?

  → Gyro-averaged field is nothing but field felt by "charged ring"

Integration can be approximated by a few points sum



$$\left\langle \delta\phi \right\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta \int d\vec{x}\,\delta(\vec{X}+\vec{\rho}_i(\theta)-\vec{x})\delta\phi(\vec{x})$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \delta\phi(\vec{X}+\vec{\rho}_i(\theta))d\theta \cong \frac{1}{N}\sum_{l=1}^{N}\delta\phi(\vec{X}+\vec{\rho}_i(\theta_l))$$

or in Fourier space (as is often done in continuum codes)

$$\left\langle \delta\phi \right\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta \int d\vec{x}\,\delta(\vec{X}+\vec{\rho}_i(\theta)-\vec{x})\delta\phi(\vec{x})$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \delta\phi(\vec{X}+\vec{\rho}_i(\theta))d\theta = \frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{1}{(2\pi)^3}\int \delta\hat{\phi}(\vec{k})e^{i\vec{k}\cdot\left(\vec{X}+\vec{\rho}_i(\theta)\right)}d\vec{k}\right\}d\theta$$

$$= \frac{1}{(2\pi)^3}\int\left\{\frac{1}{2\pi}\int_0^{2\pi}\delta\hat{\phi}(\vec{k})e^{ik_\perp\rho_i\cos\theta}d\theta\right\}e^{i\vec{k}\cdot\vec{X}}d\vec{k} = \frac{1}{(2\pi)^3}\int \delta\hat{\phi}(\vec{k})J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right)e^{i\vec{k}\cdot\vec{X}}d\vec{k}$$

Simple Minded View on GK Theory

- Charged rings have stronger shielding effect than point particles

$$-\left(1+\boxed{\frac{\rho_i^2}{\lambda_{Di}^2}}\right)\nabla^2\delta\phi(\vec{x},t) = 4\pi\sum_s q_s\boxed{\bar{N}_s} \leftarrow \text{Charge density from charged rings}$$

✓ Additional shielding by polarization charges carried by charged rings
✓ Significantly enhanced as compared to Debye shielding

- Charged rings have no response to parallel direction i.e. no such thing as polarization current in parallel direction

$$-\nabla_\perp^2\delta A_\parallel = \frac{4\pi}{c}\sum_s\boxed{\bar{J}_s} \leftarrow \text{Parallel current carried by charged rings}$$

Simple Minded View on GK Theory

- Gyrokinetic equation for guiding center distribution

$$\frac{\partial}{\partial t}\bar{f} + \frac{d\vec{R}}{dt}\cdot\frac{\partial}{\partial\vec{R}}\bar{f} + \frac{dv_{\|}}{dt}\frac{\partial}{\partial v_{\|}}\bar{f} = 0$$

$$\frac{d}{dt}\vec{R} = v_{\|}\hat{b}^{*} + \frac{\mu}{B}\hat{b}\times\nabla B + c\frac{\hat{b}}{B}\times\nabla\langle\delta\psi\rangle$$

$$\frac{d}{dt}v_{\|} = -\hat{b}^{*}\cdot\mu\nabla B - \hat{b}^{*}\cdot\nabla\langle\delta\phi\rangle - \frac{1}{c}\frac{\partial}{\partial t}\langle\delta A_{\|}\rangle$$

- Gyrokinetic Maxwell Equation

$$-(1+\frac{\rho_i^2}{\lambda_{Di}^2})\nabla^2\delta\phi = 4\pi\sum_s q_s\overline{N}_s$$

$$-\nabla_{\perp}^2\delta A_{\|} = \frac{4\pi}{c}\sum_s \overline{J}_{\|s}$$

Hyperbolic + Elliptic PDEs

- Standard numerical techniques can be employed
- Issue is mainly problem size and computational cost
- Blind choice of scheme can easily end up with practically unsolvable one

Where dose it stand?

- Where does it stand?

Boltzmann

- Strongly magnetized plasma (e.g. tokamak), low frequency fluctuations $\omega << \Omega_i$
- Reduction from 6D to 5D kinetic equation

Gyrokinetic

- Reduction of GK eqs to fluid moments (3D)
- Sophisticated closure to model wave-particle interaction and FLR effects

Gyrofluid

- $k_\perp \rho_i \ll 1$, reduction to single fluid model
- Simple closures

MHD

Where dose it stand?

- Alfvenic turbulence:

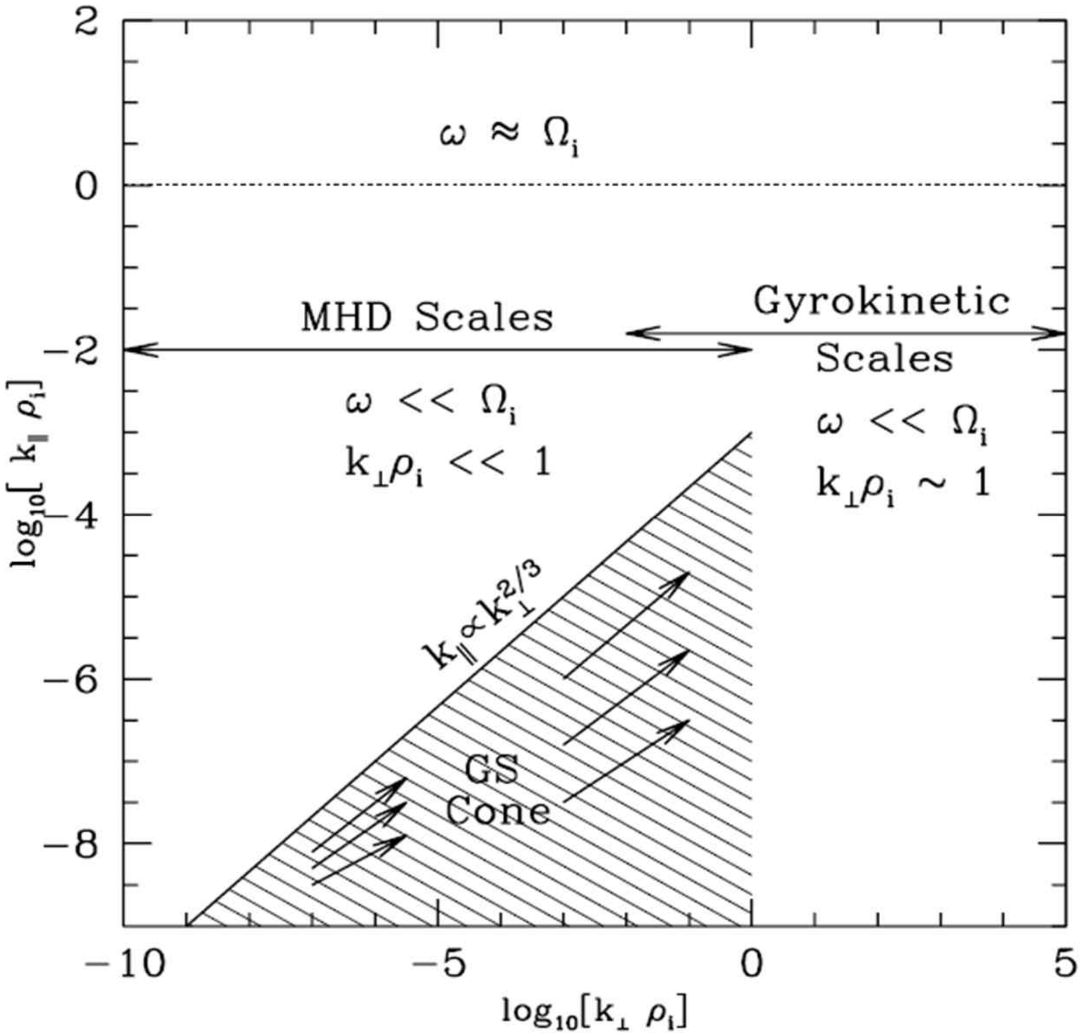
  $\omega \sim k_{\parallel} v_A$

- GS scaling of

  anisotropy: $k_{\parallel} \propto k_{\perp}^{2/3}$

➔ $\omega < \Omega_i$ for $k_{\perp}\rho_i \to 1$

➔ regime of gyrokinetic



G.G. Howes et al., ApJ'06, PRL'08

Basic plasma physics for the intracluster medium

mean free-path for electron-electron & proton-proton collisions

$$l_{p-p} \sim l_{e-e} \sim \frac{10^5}{\ln \Lambda} \frac{T^2(\mathrm{K})}{n_e(\mathrm{cm}^{-3})} \mathrm{cm} \sim \text{a few kpc}$$

1 kpc = 3,261 light-years
= 3.1 x 10^{19} m

mean free-path for electron-proton relaxation

$$l_{e-p} \sim l_{p-p} \times \left(\frac{m_p}{m_e} \right)^{\frac{1}{2}} \sim 100 \text{ kpc}$$

gyro-radius of protons

$$r_{\mathrm{gyro,p}} \sim \frac{\sqrt{T(\mathrm{K})}}{B(\mathrm{G})} \mathrm{cm} \sim 10^4 \text{ km}$$

gyro-radius of elections

$$r_{\mathrm{gyro,e}} = r_{\mathrm{gyro,p}} \times \frac{m_e}{m_p} \sim 10 \text{ km}$$

$L > l_{e-p} \Longrightarrow$ collisional regime

$l_{p-p} < L < l_{e-p} \Longrightarrow \Delta T_e \neq \Delta T_p$

$L < l_{p-p} \Longrightarrow$ viscous regime?

D.S. Ryu, WCI-CNU workshop 2010

Merits and Limitations

- Applicable to small-scale (gyro-radius) turbulence in strongly magnetized plasmas ➔ solar winds, corona, ISM, intracluster etc.
- Describe kinetic cascade in 5D phase space (both space and velocity) ➔ collisionless Landau damping
- Compressional component can be included
- Recover various MHD results for some appropriate limiting cases
- Limitation: fast MHD waves, cyclotron resonance are ruled out (extension to high frequency gyrokinetic theory, H. Qin, PoP'99)

Issues in Gyrokinetic Simulation

Problem Size and Parallelization

- Though reduced to 5D, problem size is still challenging!

  Number of grids: $N_x \times N_y \times N_z \times N_{v\|} \times N_\mu$
  $\geq 256 \times 256 \times 64 \times 128 \times 32 \sim 10^{10}$

  Electron-proton mass ratio ~ 1:2000
  ➔ time scale disparity ~ 45

- Complexity of equations ➔ careful choice of numerical scheme is required i.e. well balanced between good numerical property and simplicity for easy implementation and parallelization

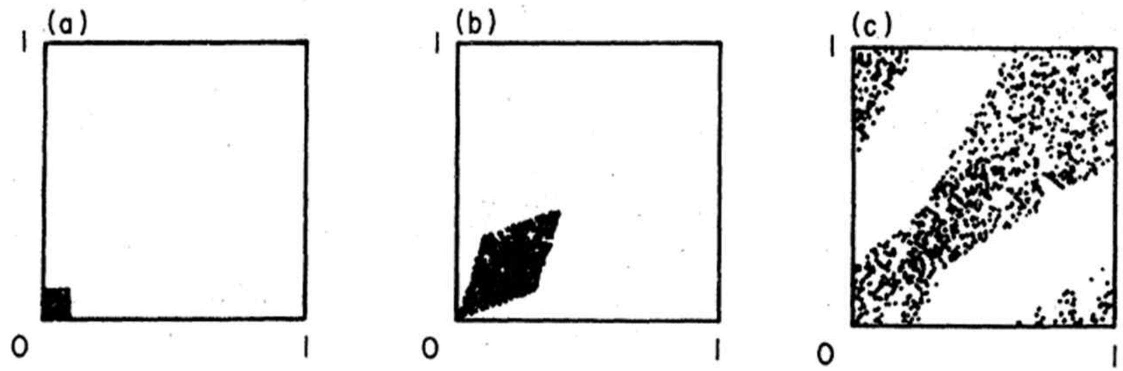
Phase Space Filamentation

- Collisionless Vlasov equation

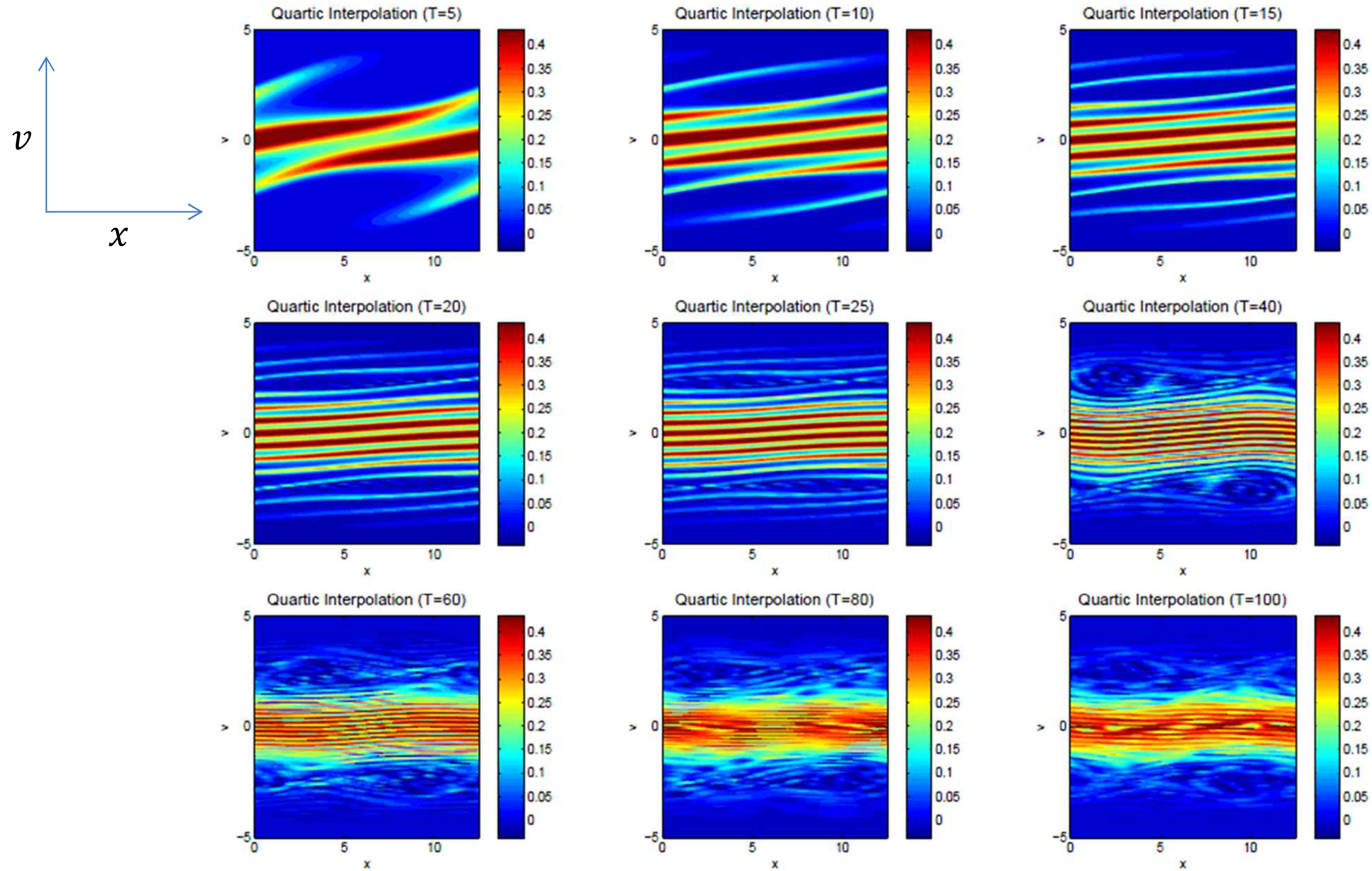
$$\frac{\partial f}{\partial t} + \frac{d\vec{R}}{dt}\frac{\partial f}{\partial \vec{R}} + \frac{dv_{\parallel}}{dt}\frac{\partial f}{\partial v_{\parallel}} = \frac{\partial f}{\partial t} - \{H, f\} = 0$$

  - Characteristic equations ⇒ Hamiltonian flow
  - Phase space volume is conserved along the characteristics
  - evolution of distribution function with streaming and interactions

    ⇒ finer and finer filamentation in phase space down to sub-grid scales



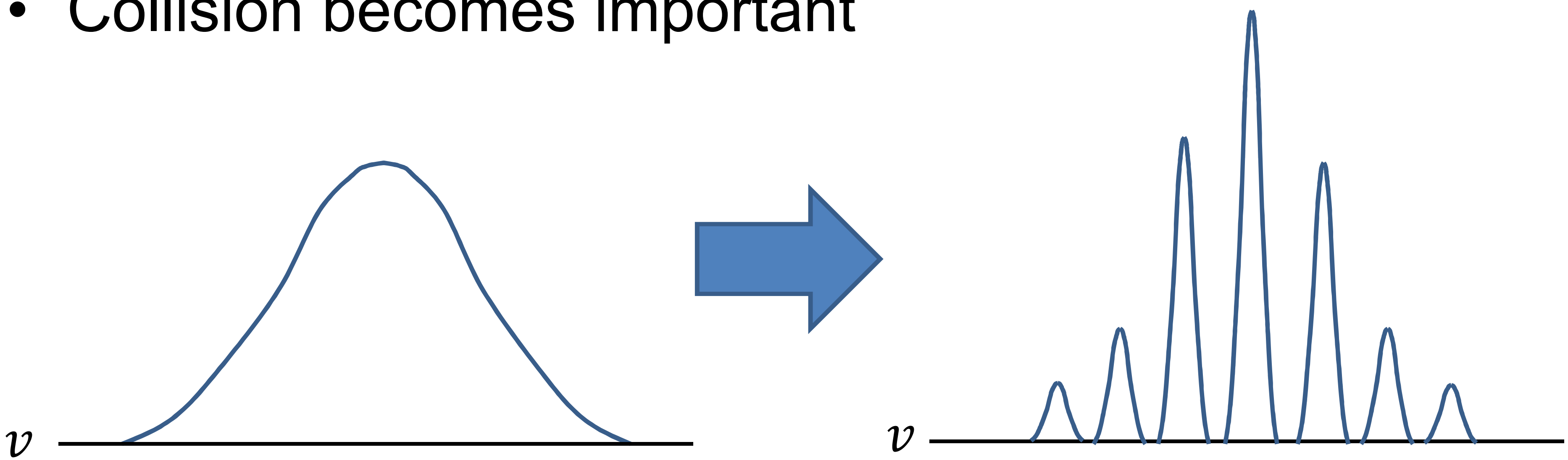
Phase Space Filamentation



D.K. Jang and D.K. Lee '12

Phase Space Filamentation

- Collision becomes important



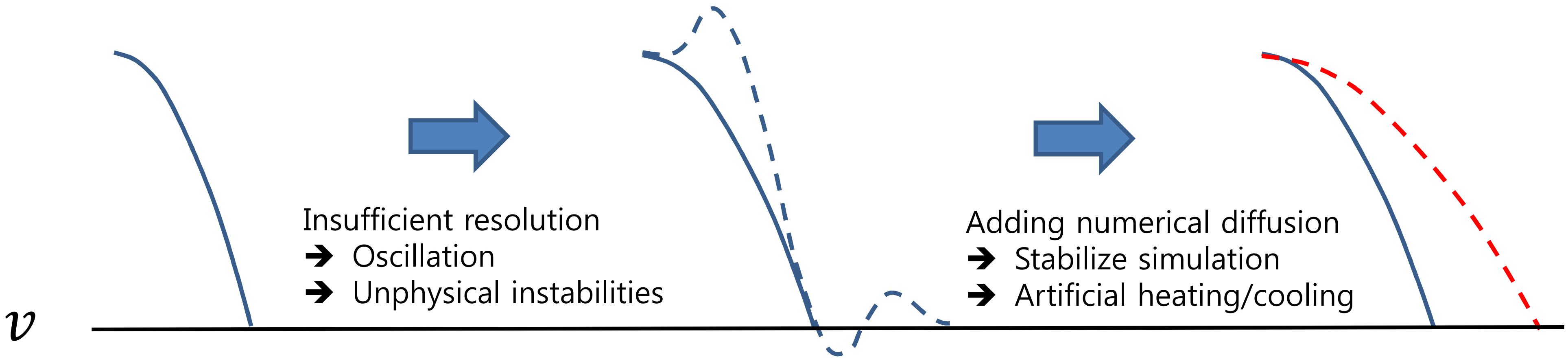
This process continues until (neglected) collision can catch up.

(Note that neglected collision term becomes important for smaller velocity space scales)

$$\frac{df}{dt} = \frac{\partial f}{\partial t} - \{H, f\} = \boxed{C(f,f)} \qquad C(f,f) \sim f\,\frac{\partial^2 f}{\partial v^2} \uparrow \text{ as } \Delta v \rightarrow 0$$

Phase Space Filamentation

- Issues in numerical simulation
  - But sufficient grid resolution all the way down to the collisional scale is practically impossible!
  - Strong gradients in phase space ➔ source of many numerical troubles
  - Careful choice of scheme and/or adding artificial dissipation is needed to minimize non-physical behaviors



Numerical Simulation of Gyrokinetic Equations

Particle in Cell (PIC) method (Lagrangian)

Phase space is sampled by a fewer number of particles carrying "weight"

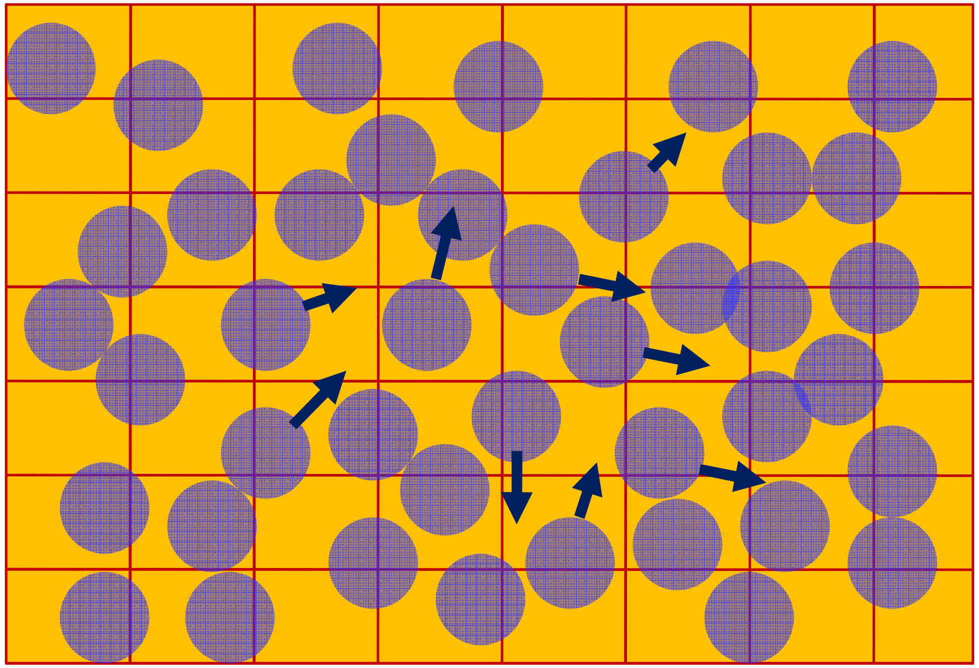
$$\bar{f}(\vec{R}, v_{\parallel}, \mu, t) = \sum_p \boxed{w_p} \delta(\vec{R} - \vec{R}_p(t))\delta(v_{\parallel} - v_{\parallel_p}(t))\delta(\mu - \mu_p(t))$$

$$w_p = \frac{\text{number of real particles}}{\text{number of simulation "markers"}}$$

$$\frac{d}{dt}\vec{R}_p = v_{\parallel}\hat{b}^* + \frac{\mu}{B}\hat{b} \times \nabla B + c\frac{\hat{b}}{B} \times \nabla\langle\delta\psi\rangle$$

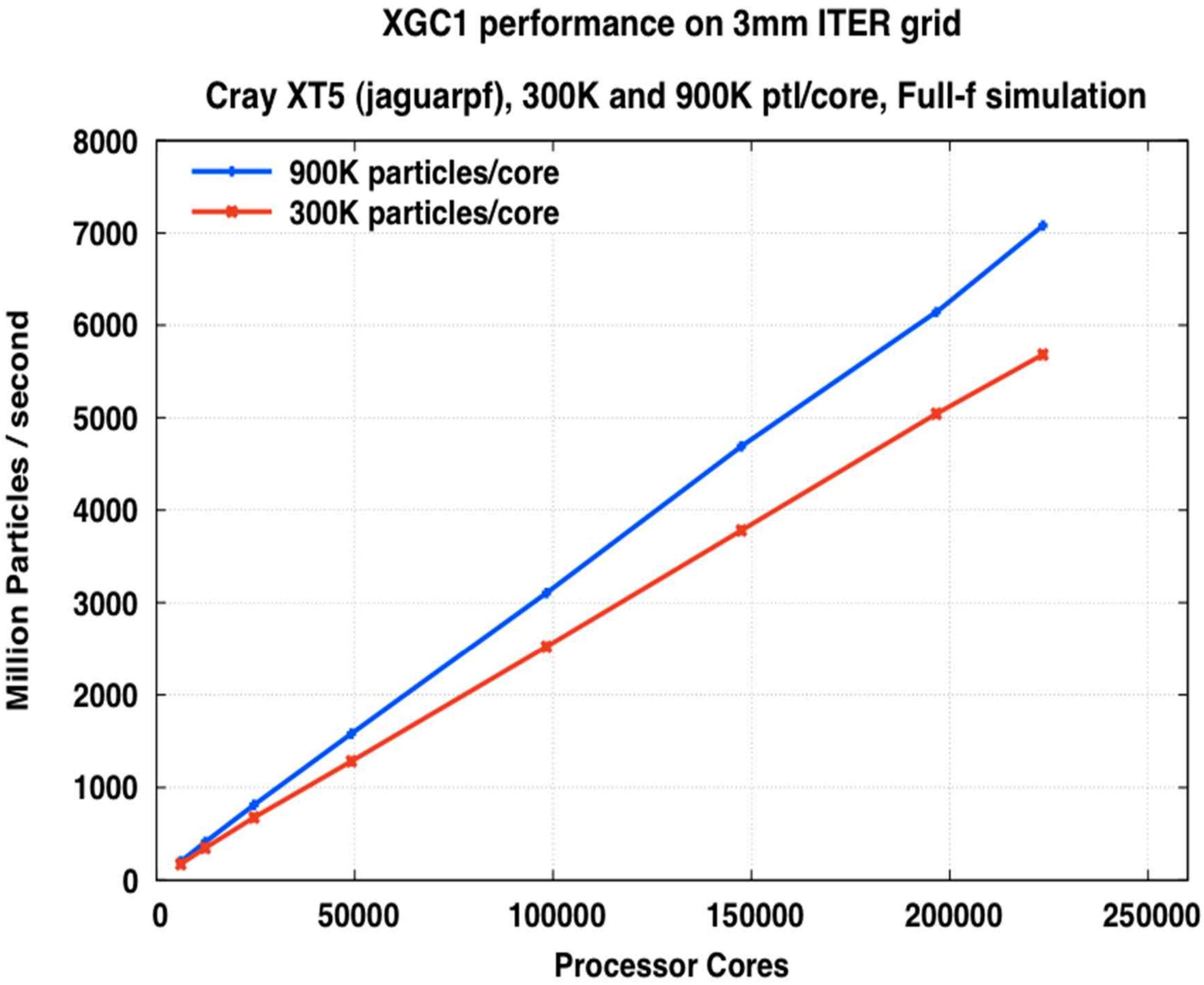
$$\frac{d}{dt}v_{\parallel p} = \hat{b}^* \cdot \mu\nabla B - \hat{b}^* \cdot \nabla\langle\delta\phi\rangle - \frac{1}{c}\frac{\partial}{\partial t}\langle\delta A_{\parallel}\rangle$$

$$\frac{d}{dt}\mu_{\parallel p} = 0$$



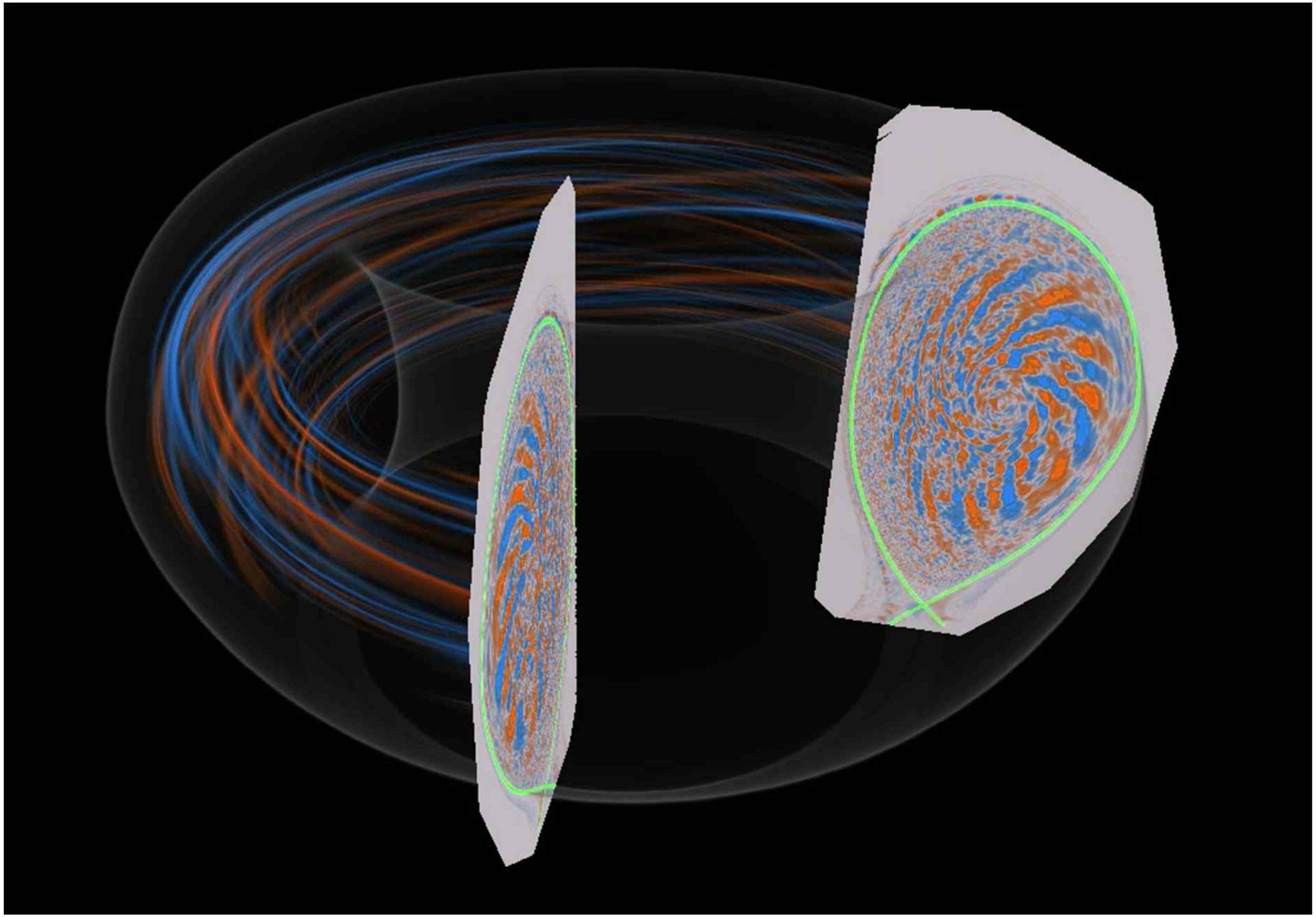
Particle in Cell (PIC) method (Lagrangian)

- Almost same with conventional PIC simulations ("particles" → "charged rings")
- All previous numerical methods developed for PIC can be employed
- Issue of small scale noise (~ $1/\sqrt{N}$)
- ~ 2000 particles per cell as rule of thumb
- Explicit scheme, larger time step etc. possible
- Easier to parallelize



S. Ku and CPES team, ICNSP'11

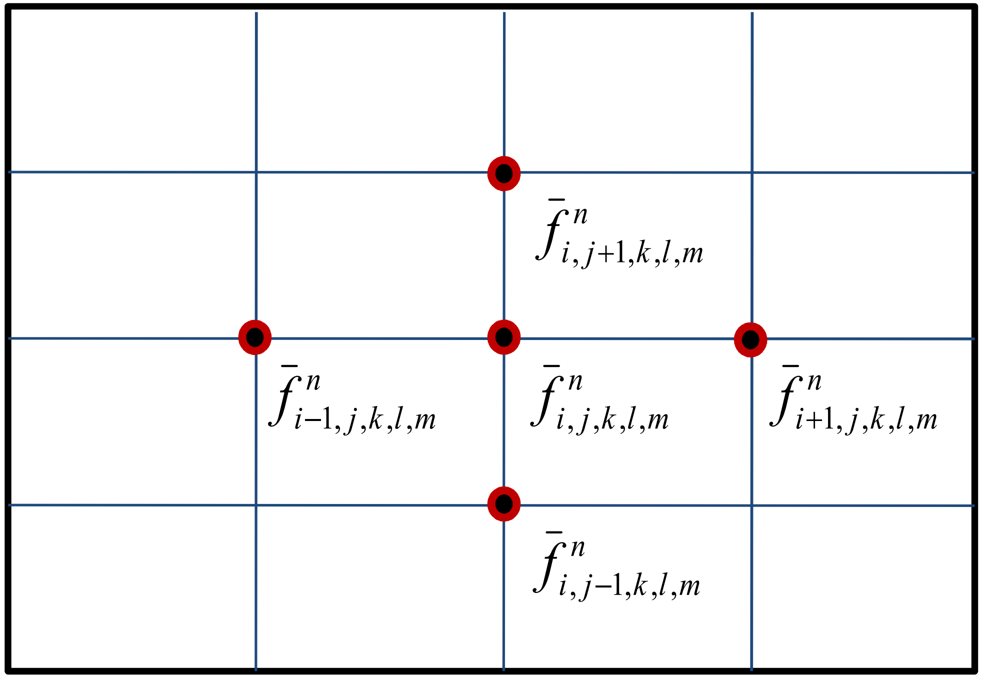
NFRI National Fusion Research Institute



XGC1 full torus simulation of ITG turbulence (S. Ku et al., EPS’12)

Continuum Method (Eulerian)

Setup grid system for entire phase space (5D) and apply standard method to solve hyperbolic PDE i.e. FDM, FVM, etc

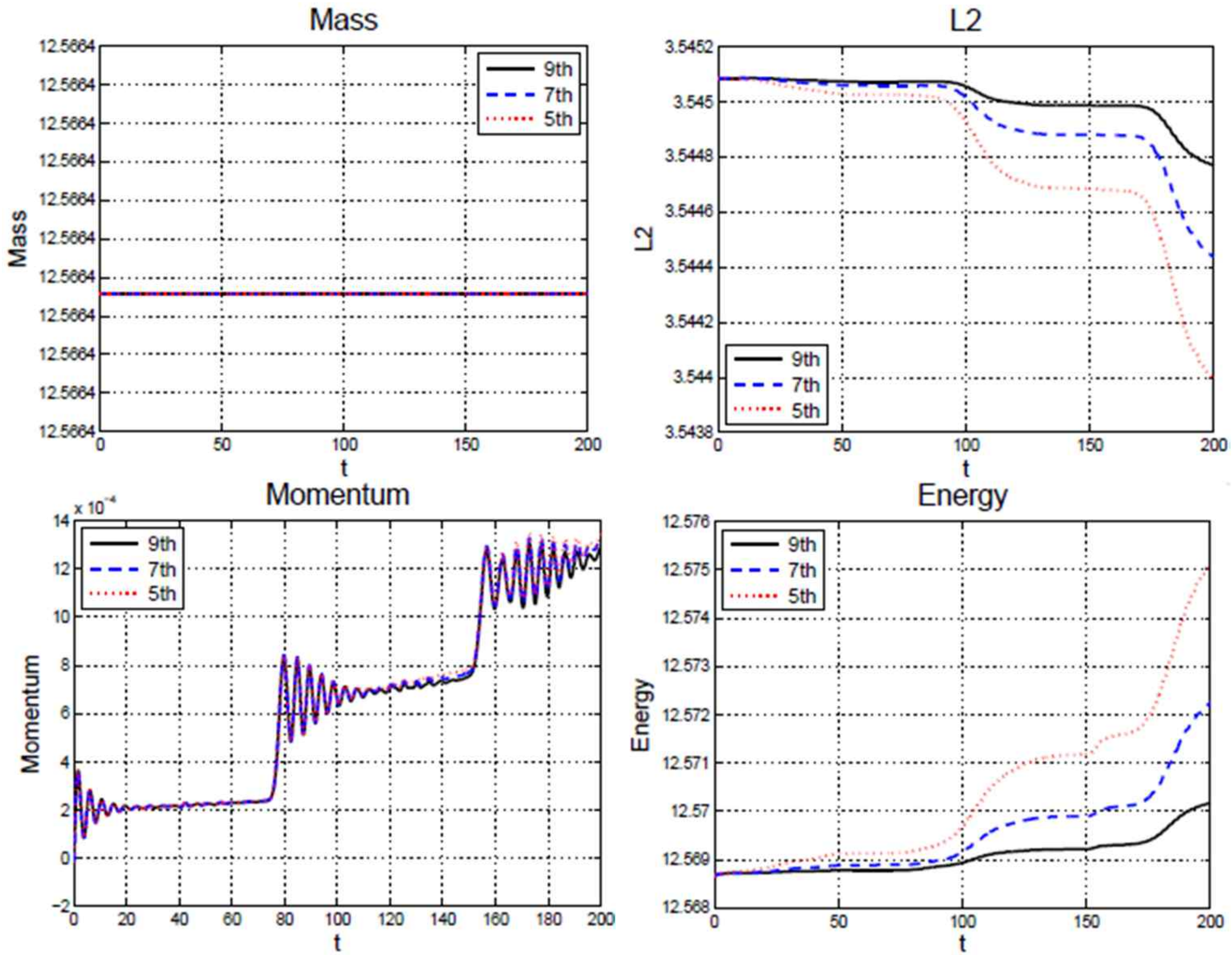


$$M_{IJ}\bar{f}^{n+1}_{J} = N_{IJ}\bar{f}^{n}_{J}$$

Inversion of "huge matrix" is usually required

$$\bar{f}^{n+1} = M^{-1}N\bar{f}^{n}$$

Continuum Method (Eulerian)



Vlasov simulation of two steam instability (D.K. Jang and D.K. Lee '12)

Continuum Method (Eulerian)

- All advanced schemes developed in hydro communities can be employed
- Some operations are very costly e.g. $\langle\phi\rangle \sim \int J_0(k\rho_i)\phi_k dk$
- CFL condition often dictates implicit time integration
- Phase space granulation poses high velocity space resolution

  ➔ grid scale dissipation is necessary ➔ poorer conservation than PIC
- Parallelization efficiency is often less than PIC
- Low noise high quality simulation is possible if sufficient resolution is provided

➔ development of massively parallel supercomputer makes this possible!

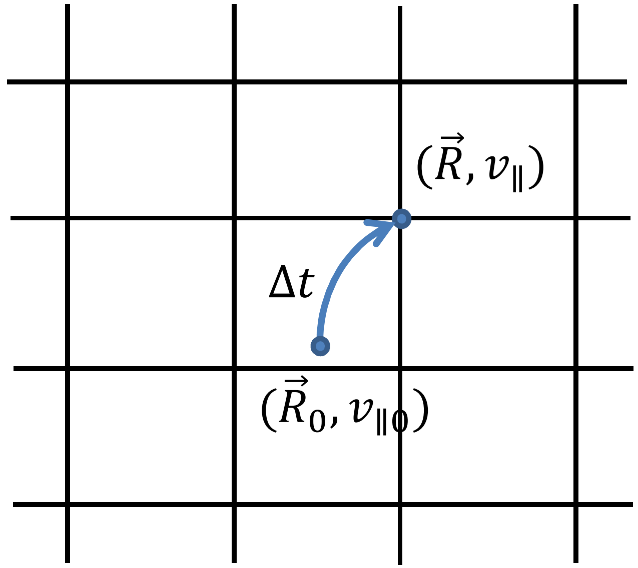
Semi-Lagrangian Scheme

- Invariance of distribution function along characteristics

$$\frac{\partial}{\partial t}\bar{f} + \frac{d\vec{R}}{dt}\cdot\frac{\partial}{\partial \vec{R}}\bar{f} + \frac{dv_{\|}}{dt}\frac{\partial}{\partial v_{\|}}\bar{f} = 0$$

$$\Rightarrow \frac{d}{dt}\bar{f} = 0$$

$$\Rightarrow \bar{f}(\vec{R}(\Delta t;\vec{R}_0), v_{\|}(\Delta t; v_{\|0}), t) = \bar{f}(\vec{R}_0, v_{\|0}, 0)$$



- Find f_{now} at grid point by tracing back characteristic line and interpolating f_{old}
- Relatively free from CFL constraint i.e. larger Δt
- Interpolation on Eulerian grid → greatly reduce discrete particle noise

Numerical Simulation of Gyrokinetic Equation

- What is δf scheme?
  - Write distribution function as a sum of known f_0 and δf
  - Solve only perturbed part δf only

$$\frac{\partial f}{\partial t} + \frac{d\vec{R}}{dt} \cdot \nabla f + \frac{dv_{\parallel}}{dt}\frac{\partial f}{\partial v_{\parallel}} = 0$$

Gyrokinetic equation in δf form (W.W. Lee CPC'87)

$$\Rightarrow \frac{\partial}{\partial t}\delta f + \frac{d\vec{R}}{dt} \cdot \nabla \delta f + \frac{dv_{\parallel}}{dt}\frac{\partial}{\partial v_{\parallel}}\delta f = -\frac{d\vec{R}}{dt} \cdot \nabla f_0 - \frac{dv_{\parallel}}{dt}\frac{\partial f_0}{\partial v_{\parallel}}$$

$$f(\vec{R}, v_{\parallel}, \mu, t) = \sum_p w_p \delta(\vec{R} - \vec{R}_p(t))\delta(v_{\parallel} - v_{\parallel_p}(t))\delta(\mu - \mu_p(t))$$

$$\Rightarrow \delta f(\vec{R}, v_{\parallel}, \mu, t) = \sum_p w_p(t) \delta(\vec{R} - \vec{R}_p(t))\delta(v_{\parallel} - v_{\parallel_p}(t))\delta(\mu - \mu_p(t))$$

$$\frac{d}{dt} w_p(t) = -(1 - w_p)\left[\frac{d\vec{R}}{dt} \cdot \frac{\nabla f_0}{f_0} + \frac{dv_{\parallel}}{dt}\frac{\partial f_0}{f_0 \partial v_{\parallel}}\right]$$

Numerical Simulation of Gyrokinetic Equation

- Why δf scheme?

For PIC method $\varepsilon \propto \frac{1}{\sqrt{N}}\sqrt{\langle w^2 \rangle} << \frac{1}{\sqrt{N}}$ for $\langle w^2 \rangle \equiv \frac{1}{N}\sum_{p=1}^{N} w_p^2 \sim (\delta f / f_0)^2 << 1$

Simulation noise is reduced by $\delta f/f_0$

- Why not δf scheme?

But… what if $\langle w^2 \rangle$ increases during simulation?

We lose the merit ⇒ actually, it can become large in

✓ Short time by phase space filamentation (granulation)

✓ Longer time by equilibrium evolution

$$\frac{d}{dt}\langle w^2 \rangle = D\left(\frac{\partial}{\partial x} f_0\right)^2 - \nu \langle w^2 \rangle \qquad \langle w^2 \rangle \sim \int \delta f^2 dx dv \ (\propto -H \ :\text{entropy})$$

In collisionless simulation ($\nu=0$), particle weights grow linearly in time and δf scheme stops to work

Summary

- Gyrokinetic simulation is a well developed tool to study low frequency micro-scale turbulence. Serious validation efforts are also ongoing in fusion community.
- With careful examination of parameter regime, it can be applied to astrophysical environments such as ISM, intracluster medium etc.
  - microscopic turbulence with kinetic processes e.g. collisionless Landau damping
  - sub-grid transport model for global simulation studies
  - stimulate the extension of gyrokinetic model beyond present limit
- Careful choice of numerical scheme is absolutely critical
  - beneficial for both communities to expand present simulation capabilities