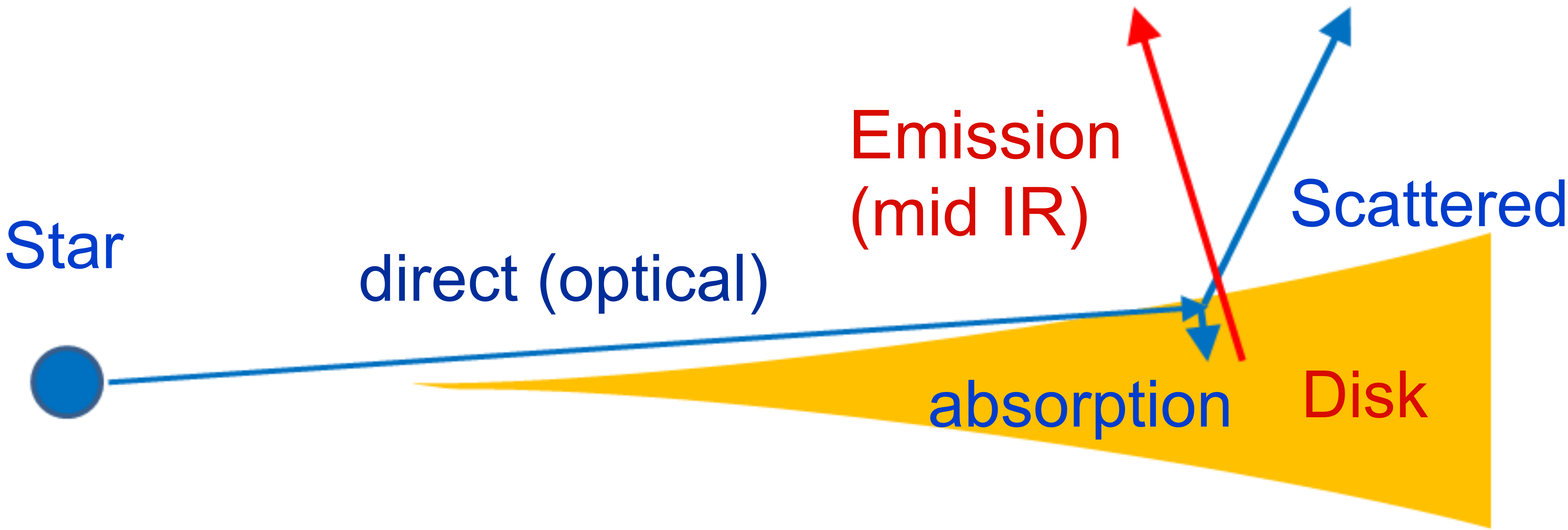
Reconstruction Method for M1 Equations of Radiative Transfer

Multi-color model for a Protoplanetary Disk



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- Requirements
  - Shadow by Disk
  - Scattering
  - Multi-Color (λ = 0.1 μ - 1mm)
  - Thermal and Hydrodynamical Balance

$I_\nu \ (\boldsymbol{r}, \ t, \ \boldsymbol{n})$ 6D

Full radiative transfer is hard to solve.

- Potential Solutions
  - Sampling: Monte-Carlo [noise]
  - Consider only dominant sources
  - Reduced Angular Dependence: FLD, M1

Reduce the Angular Resolution

Intensity $I_\nu\ (\boldsymbol{r},\ t,\ \boldsymbol{n})$

M1 model (*E*, ***F***)

↓

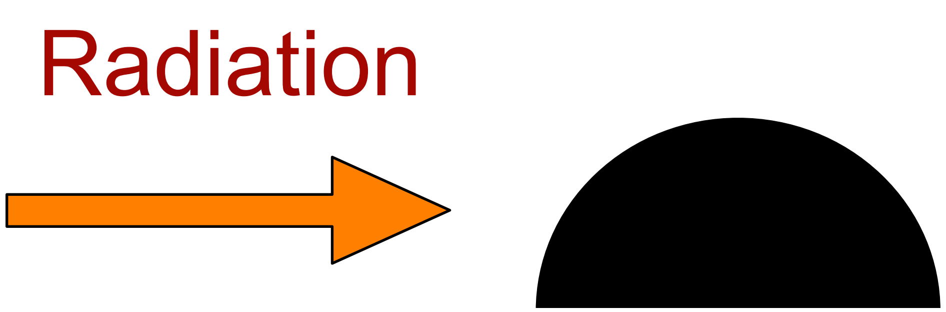
Moment

0th:
$$E_\nu\ (\boldsymbol{x},\ t)\ \equiv\ \frac{1}{c}\oint I\ (\boldsymbol{x},\ t.\ \boldsymbol{n}\, d\boldsymbol{n})\ d\boldsymbol{n}$$

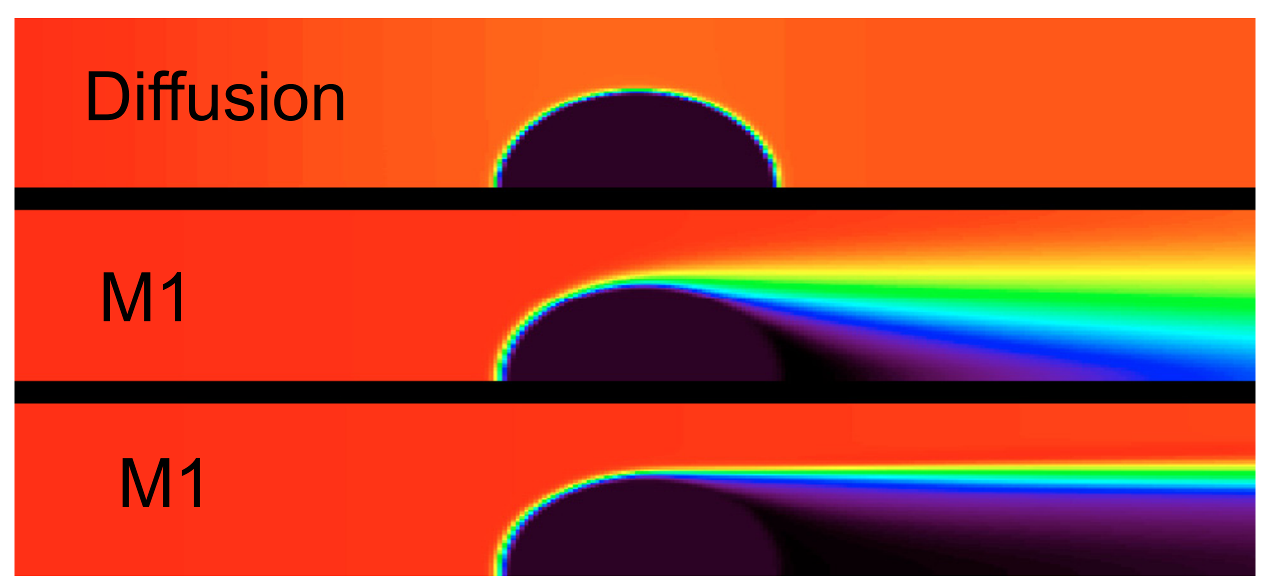
1st:
$$\boldsymbol{F}_\nu(\boldsymbol{x},\ t)\ \equiv\ \oint \boldsymbol{n}\, I(\boldsymbol{x},\ t,\ \boldsymbol{n})\, d\boldsymbol{n}$$

2nd:
$$\overleftrightarrow{\boldsymbol{P}}_\nu(\boldsymbol{x},\ t)\ \equiv\ \frac{1}{c}\oint \boldsymbol{n}\boldsymbol{n}\, I(\boldsymbol{x},\ t,\ \boldsymbol{n})\, d\boldsymbol{n}$$

Diffusion Approx. *E*



Gonzalez et al. ’07



M1 Model

transfer — absorption

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}\cdot\boldsymbol{\nabla}\right) I\left(\boldsymbol{r},\ t,\ \boldsymbol{n}\right) = -\sigma_\nu I_\nu(\boldsymbol{r},\ t,\ \boldsymbol{n})$$

$$+\sigma_\nu S_\nu\left(\boldsymbol{r},\ t,\ \boldsymbol{n}\right) + \sigma_{\nu,s}\oint g\left(\boldsymbol{n},\ \boldsymbol{n}'\right) I_\nu(\boldsymbol{r},\ t,\ \boldsymbol{n}')\, d\boldsymbol{n}'$$

emission — scattering

Exact

↑

M1

$$\frac{\partial E_\nu}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F}_\nu = \sigma_\nu\left(4\pi S_\nu - cE_\nu\right)$$

$$\frac{\partial \boldsymbol{F}_\nu}{\partial t} + \boldsymbol{\nabla}\cdot\overleftrightarrow{\boldsymbol{P}}_\nu = -c\left(\sigma_\nu + \sigma_{\nu,s}\right)\boldsymbol{F}_\nu$$

closure

$$\overleftrightarrow{\boldsymbol{P}}_\nu = \left(\frac{1-\chi}{2}\overleftrightarrow{\boldsymbol{I}} + \frac{3\chi-1}{2}\boldsymbol{n}\boldsymbol{n}\right)E_\nu,\quad \boldsymbol{n} = \frac{\boldsymbol{F}_\nu}{\boldsymbol{F}_\nu},\quad \chi = \frac{3+4f^2}{5+2\sqrt{4-3f^2}},\quad f = \frac{|\boldsymbol{F}_\nu|}{E_\nu}$$

Standard M1 model does not work when optically thick.

We propose an idea to fix this.

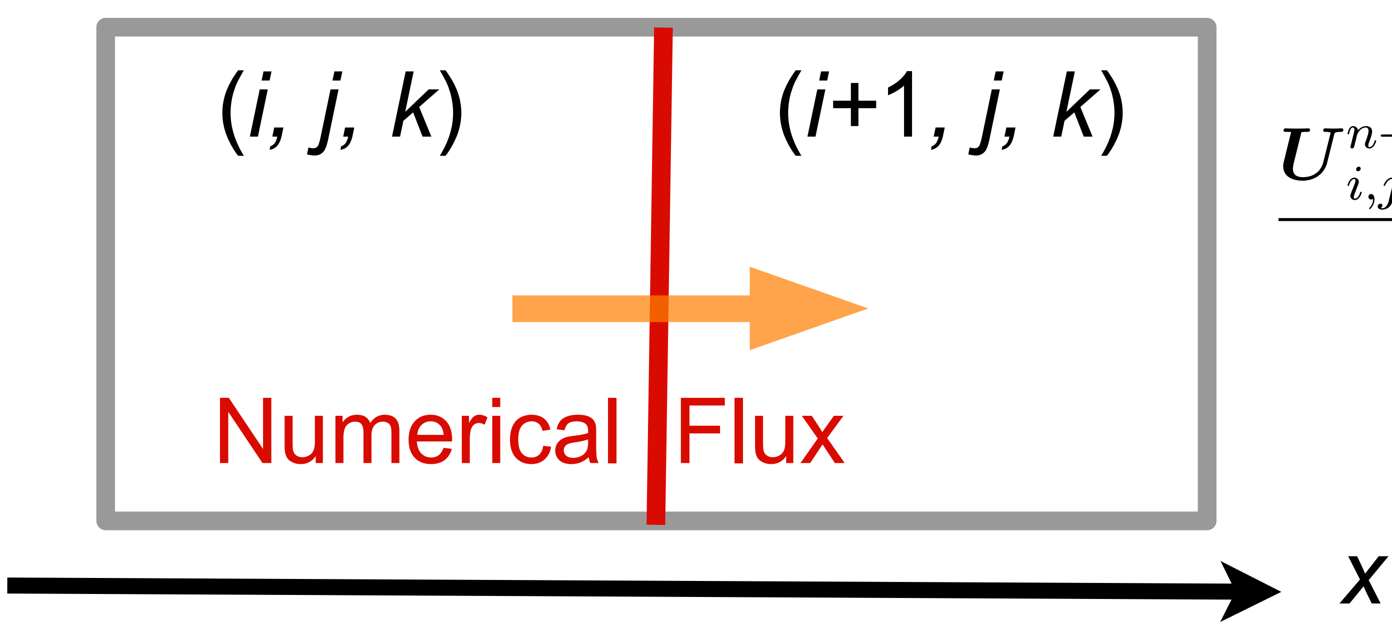
Numerical Integration of M1 model

Conservation Form

$$\frac{\partial}{\partial t}\boldsymbol{U} + \frac{\partial}{\partial x}\boldsymbol{F}_x + \frac{\partial}{\partial y}\boldsymbol{F}_y + \frac{\partial}{\partial z}\boldsymbol{F}_z = \boldsymbol{S}$$

$$\boldsymbol{U} = \begin{pmatrix} E_\nu \\ F_{x,\nu} \\ F_{y,\nu} \\ F_{z,\nu} \end{pmatrix}, \quad \boldsymbol{F}_x = \begin{pmatrix} F_{x,\nu} \\ c^2 P_{xx,\nu} \\ c^2 P_{xy,\nu} \\ c^2 P_{xz,\nu} \end{pmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} \sigma_\nu \left(4\pi S_\nu - cE_\nu\right) \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{x,\nu} \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{y,\nu} \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{z,\nu} \end{bmatrix}$$

$$\boldsymbol{F}_{x,i+1/2,j,k} = \boldsymbol{F}_{x,i+1/2,j,k}\left(\boldsymbol{U}_{x,i,j,k}, \boldsymbol{U}_{x,i+1,j,k}\right)$$



$$\frac{\boldsymbol{U}^{n+1}_{i,j,k} - \boldsymbol{U}^{n}_{i,j,k}}{\Delta t} + \frac{\boldsymbol{F}^{n}_{i+1/2,j,k} - \boldsymbol{F}^{n}_{i-1/2,j,k}}{\Delta x} = \boldsymbol{S}^{n}_{i,j,k}$$

Upwind Scheme

(simple) HLL

$$\boldsymbol{F}^{(\mathrm{HLL})}_{i+1/2,j,k} = \frac{\lambda_{\mathrm{R}}\boldsymbol{F}_{i,j,k} - \lambda_{\mathrm{L}}\boldsymbol{F}_{i+1,j,k} + \lambda_{\mathrm{R}}\lambda_{\mathrm{L}}\left(\boldsymbol{U}_{i+1,j,k} - \boldsymbol{U}_{i,j,k}\right)}{\lambda_{\mathrm{R}} - \lambda_{\mathrm{L}}}$$

$\lambda_{\mathrm{R}} = c, \quad \lambda_{\mathrm{L}} = -c$ Maximal Speed, safe but diffusive

Godunov (eigenvalues & eigenvectors)
less diffusive but difficult to calculate

Reconstruction (this work)

$$\boldsymbol{U} = \begin{pmatrix} E_\nu \\ F_{x,\nu} \\ F_{y,\nu} \\ F_{z,\nu} \end{pmatrix} \longrightarrow$$

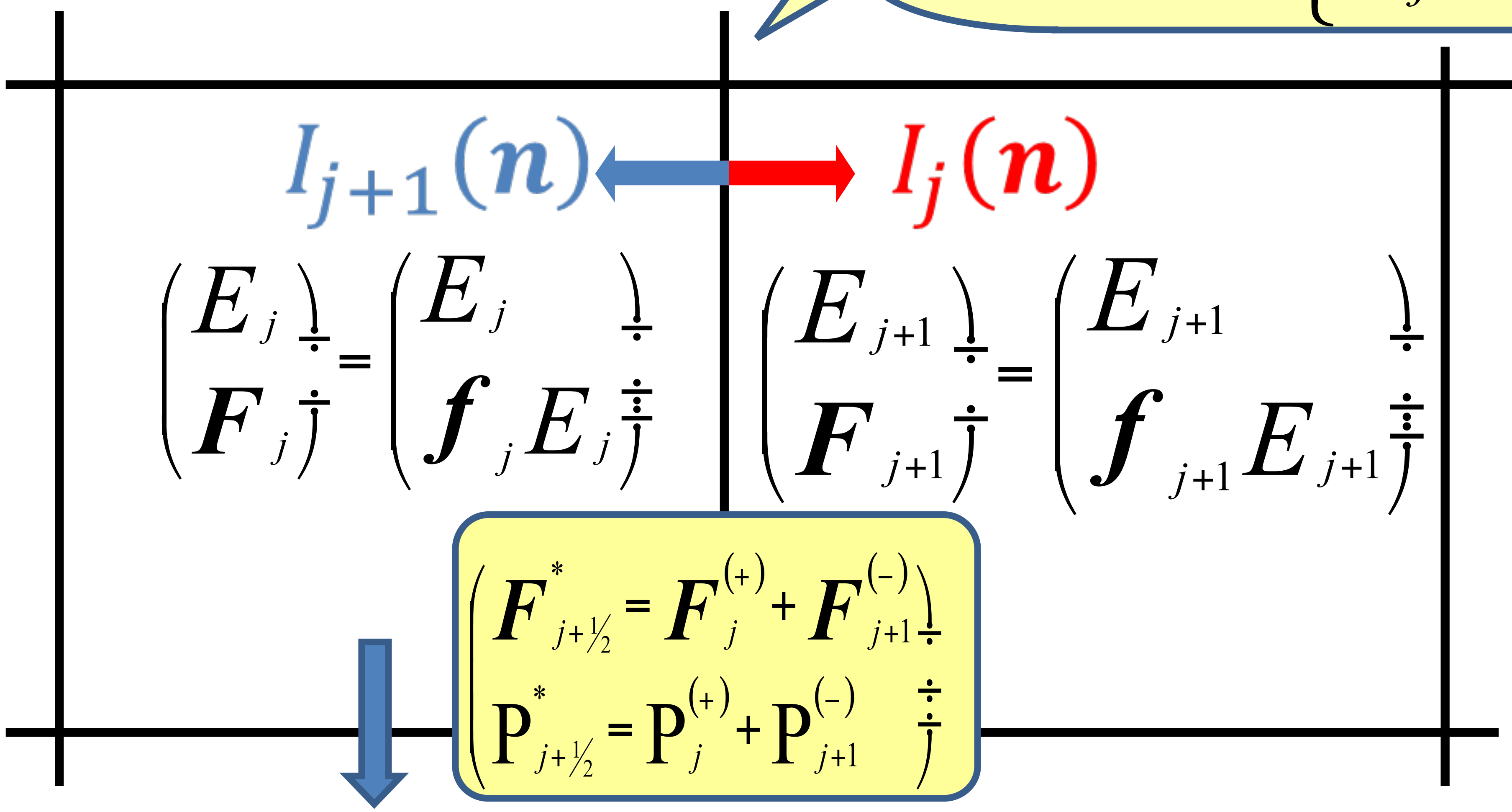
$$I_\nu(\boldsymbol{n}) = \frac{3E_\nu}{8\pi}\frac{(1-\beta^2)^3}{3+\beta^2}\left(1-\boldsymbol{\beta}\cdot\boldsymbol{n}\right)^{-4}$$

$$\beta = \frac{3f}{2+\sqrt{4-3f^2}}, \quad \boldsymbol{\beta} = \beta\frac{\boldsymbol{F}}{|\boldsymbol{F}|}$$

Consistent with the closure relation

Upwind Reconstruction of the Radiation Field

$$I^{*}_{j+1/2}(\boldsymbol{n}) = \begin{cases} I_{j}(\boldsymbol{n}) & (\mu > 0) \\ I_{j+1}(\boldsymbol{n}) & (\mu < 0) \end{cases}$$



$$I_{j+1}(\boldsymbol{n}) \qquad I_{j}(\boldsymbol{n})$$

$$\begin{pmatrix} E_j \\ \boldsymbol{F}_j \end{pmatrix} = \begin{pmatrix} E_j \\ \boldsymbol{f}_j E_j \end{pmatrix} \qquad \begin{pmatrix} E_{j+1} \\ \boldsymbol{F}_{j+1} \end{pmatrix} = \begin{pmatrix} E_{j+1} \\ \boldsymbol{f}_{j+1} E_{j+1} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{F}^{*}_{j+1/2} = \boldsymbol{F}^{(+)}_{j} + \boldsymbol{F}^{(-)}_{j+1} \\ \mathrm{P}^{*}_{j+1/2} = \mathrm{P}^{(+)}_{j} + \mathrm{P}^{(-)}_{j+1} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{F}^{*}_{j+1/2} \\ \mathrm{P}^{*}_{j+1/2} \end{pmatrix}$$

$$I_\nu(\boldsymbol{n}) = \frac{3E_\nu}{8\pi}\frac{(1-\beta^2)^3}{3+\beta^2}(1-\boldsymbol{\beta}\cdot\boldsymbol{n})^{-4}$$

$$\beta = \frac{3f}{2+\sqrt{4-3f^2}}, \qquad \boldsymbol{\beta} = \beta\frac{\boldsymbol{F}}{|\boldsymbol{F}|}$$

Radiation Field consistent with the M1 closure

Reconstructed Flux is Analytic.

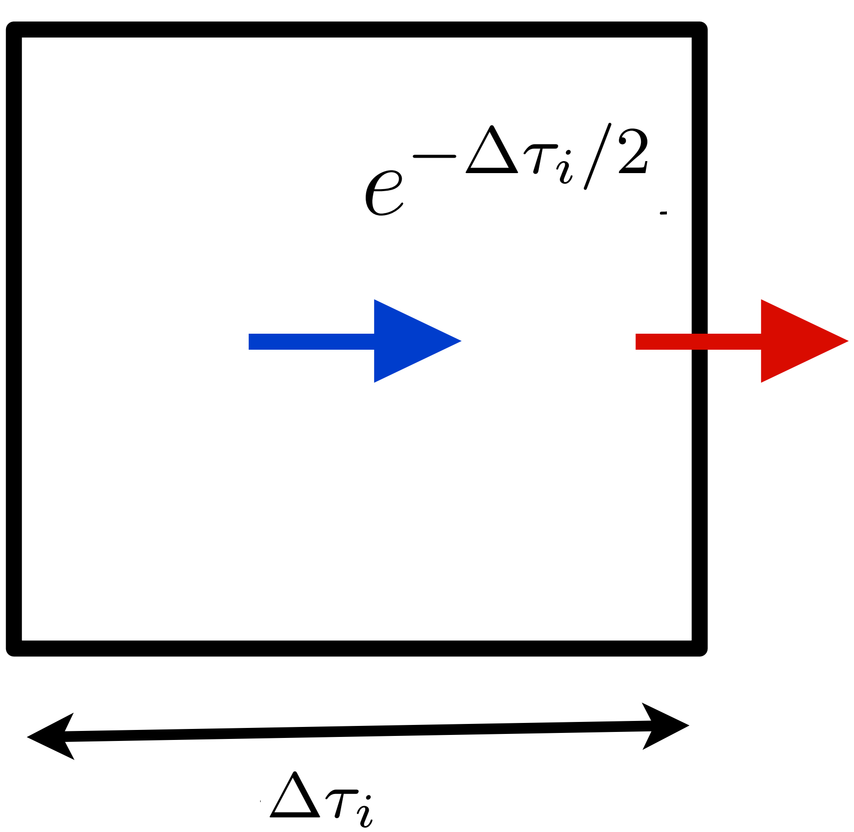
$$
\begin{aligned}
F^*_{\nu,i} &= F^{(+)}_{\nu,i,\mathrm{L}} + F^{(-)}_{\nu,i,\mathrm{R}} \,, \\
F^{(+)}_{\nu,i,\mathrm{L}} &= \left[\frac{3\, q_{\nu,\mathrm{L}} + 6\, \beta^2_{\nu,\mathrm{L}} \cos^2\psi_{\nu,\mathrm{L}}\, q^{-1}_{\nu,\mathrm{L}} - \beta^4_{\nu,\mathrm{L}} \cos^4\psi_{\nu,\mathrm{L}} q^{-3}_{\nu,\mathrm{L}} + 8\beta_{\nu,\mathrm{L}} \cos\psi_{\nu,\mathrm{L}}}{4\,(\beta^2_{\nu,\mathrm{L}} + 3)} \right] cE_{\nu,\mathrm{L}} \,, \\
F^{(-)}_{\nu,i,\mathrm{R}} &= -\left[\frac{3\, q_{\nu,\mathrm{R}} + 6\, \beta^2_{\nu,\mathrm{R}} \cos^2\psi_{\nu,\mathrm{R}} q^{-1}_{\nu,\mathrm{R}} - \beta^4_{\nu,\mathrm{R}} \cos^4\psi_{\nu,\mathrm{R}} q^{-3}_{\nu,\mathrm{R}} - 8\beta_{\nu,\mathrm{R}} \cos\psi_{\nu,\mathrm{R}}}{4\,(\beta^2_{\nu,\mathrm{R}} + 3)} \right] cE_{\nu,\mathrm{R}} \,. \\
q_{\nu,\mathrm{L}} &= \left(1 - \beta^2_{\nu,\mathrm{L}} \sin^2\psi_{\nu,\mathrm{L}}\right)^{1/2} , \quad (\boldsymbol{e}_i \cdot \boldsymbol{\beta}_\mathrm{L}) = \beta_{\nu,\mathrm{L}} \cos\ \psi_{\nu,\mathrm{L}} \,. \\
q_{\nu,\mathrm{R}} &= \left(1 - \beta^2_{\nu,\mathrm{R}} \sin^2\psi_{\nu,\mathrm{R}}\right)^{1/2} , \quad (\boldsymbol{e}_i \cdot \boldsymbol{\beta}_\mathrm{R}) = \beta_{\nu,\mathrm{R}} \cos\ \psi_{\nu,\mathrm{R}} \,.
\end{aligned}
$$

$$
\begin{aligned}
P^*_{\nu,ii} &= P^{(+)}_{\nu,ii,\mathrm{L}} + P^{(-)}_{\nu,ii,\mathrm{R}} \,, \\
P^{(+)}_{\nu,ii.\mathrm{L}} &= \left[\frac{\beta^3_{\nu,\mathrm{L}} \cos^3\psi q^{-1}_{\nu,\mathrm{L}} + 3\beta_{\nu,\mathrm{L}} \cos\psi_{\nu,\mathrm{L}} q_{\nu,\mathrm{L}} + 4\beta^2_{\nu,\mathrm{L}} \cos^2\psi_{\nu,\mathrm{L}} + 1 - \beta^2_{\nu,\mathrm{L}}}{2\,(\beta^2_{\nu,\mathrm{L}} + 3)} \right] , \\
P^{(-)}_{\nu,ii.\mathrm{R}} &= \left[\frac{-\beta^3_{\nu,\mathrm{R}} \cos^3\psi q^{-1}_{\nu,\mathrm{R}} - 3\beta_{\nu,\mathrm{R}} \cos\psi_{\nu,\mathrm{R}} q_{\nu,\mathrm{R}} + 4\beta^2_{\nu,\mathrm{R}} \cos^2\psi_{\nu,\mathrm{R}} + 1 - \beta^2_{\nu,\mathrm{R}}}{2\,(\beta^2_{\nu,\mathrm{R}} + 3)} \right] , \\
P^*_{\nu,ij} &= \beta_{\nu,j,\mathrm{L}}\, F^{(+)}_{\nu,i,\mathrm{L}} + \beta_{\nu,j,\mathrm{R}}\, F^{(-)}_{\nu,i,\mathrm{R}} \,.
\end{aligned}
$$

Absorption & Emission within Cell

$$F'^{(+)}_{\nu,x,i+1/2,j,k} = e^{-\Delta\tau_i/2} F^{(+)}_{\nu,x,i+1/2,j,k} + \left(1 - e^{-\Delta\tau_i/2}\right) \frac{S_\nu}{4}$$

Flux at boundary absorption Flux at center Emission

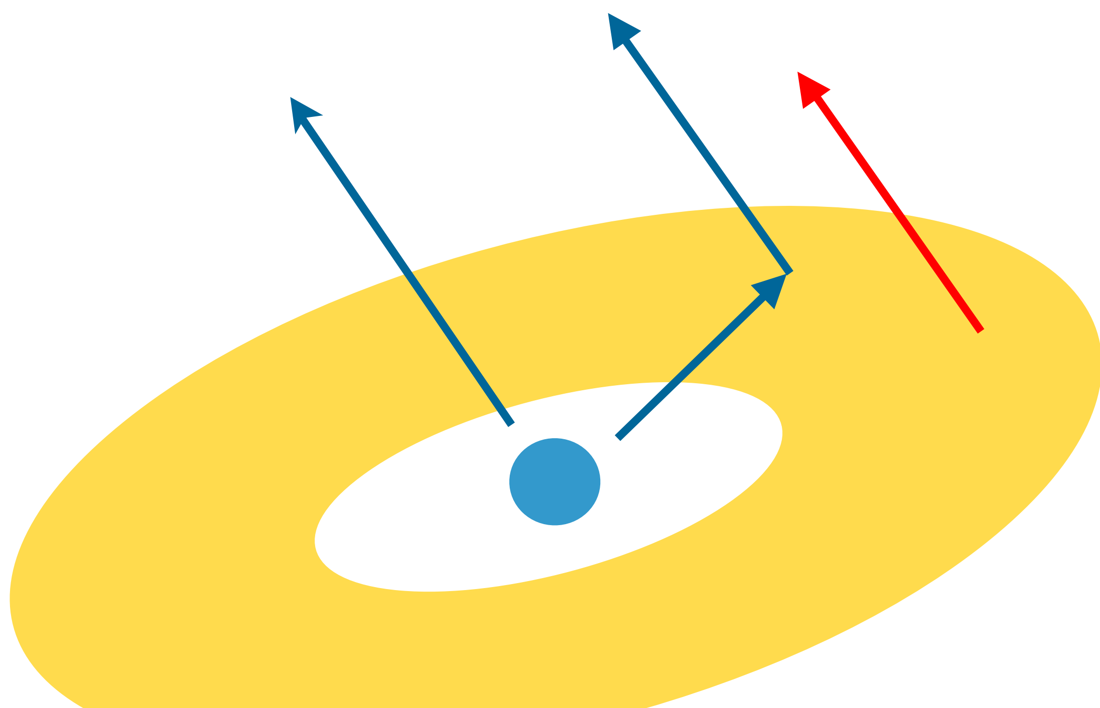


$\Delta\tau_i$ optical depth

approaching to diffusion limit when $\Delta\tau_i$ is large

$$P'^{(+)}_{\nu,x,i+1/2,j,k} = e^{-\Delta\tau_i/2} P^{(+)}_{\nu,x,i+1/2,j,k} + \left(1 - e^{-\Delta\tau_i/2}\right) \frac{S_\nu}{6}$$

Irradiated Protoplanetary Disk



stellar, scattered, and
reemitted photons

Two sets of M1 models

$$E_\nu = E'_\nu + E''_\nu\,,$$
$$\boldsymbol{F}_\nu = \boldsymbol{F}'_\nu + \boldsymbol{F}''_\nu\,,$$

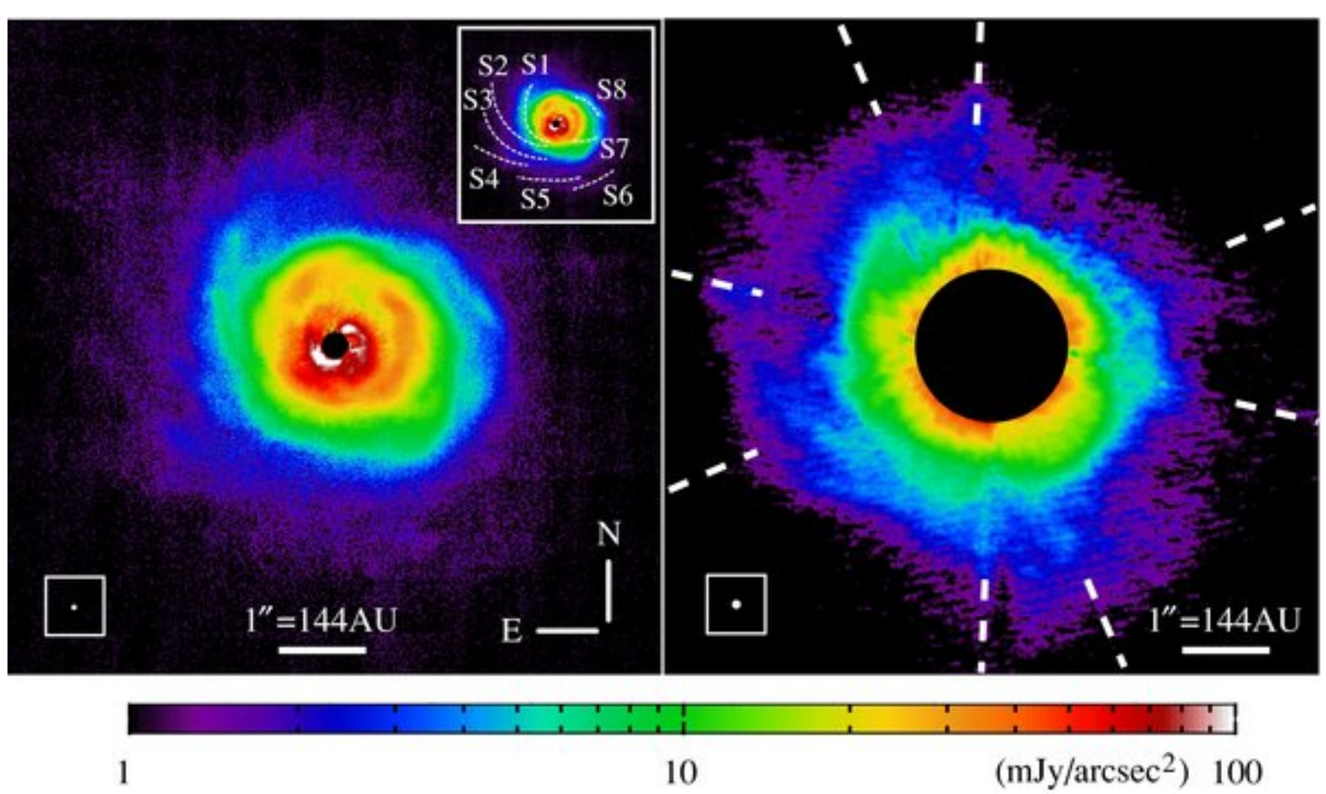
stellar — scattered +emission

Thermal & Hydrostatic Equilibrium

$$0.1\ \mu\mathrm{m} \le \lambda \le 1\ \mathrm{mm}$$

$$\Delta \log \lambda = 0.02$$

AB Aur
Hashimoto+ 2011
H-band
(1.6 μm)

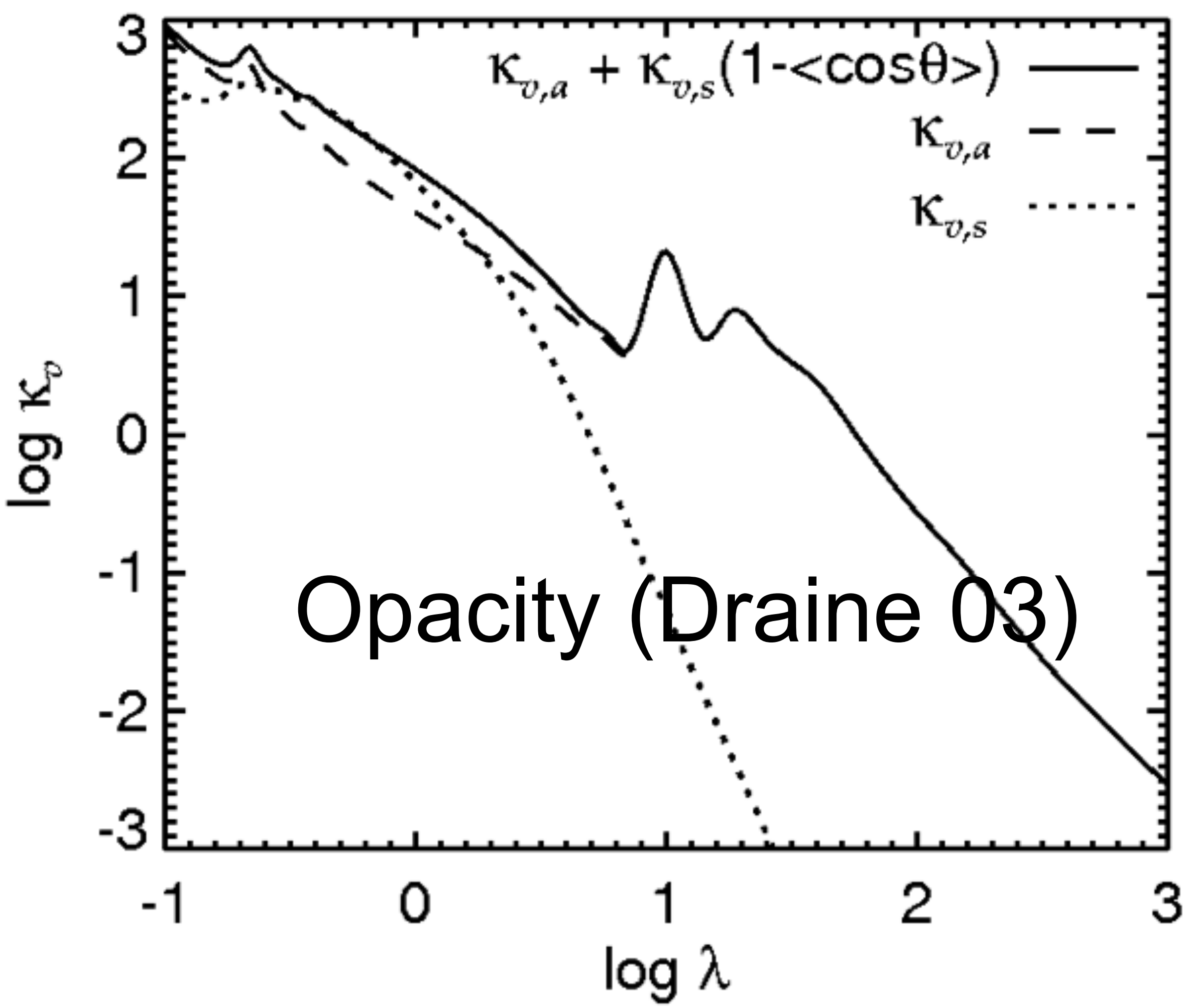


Model

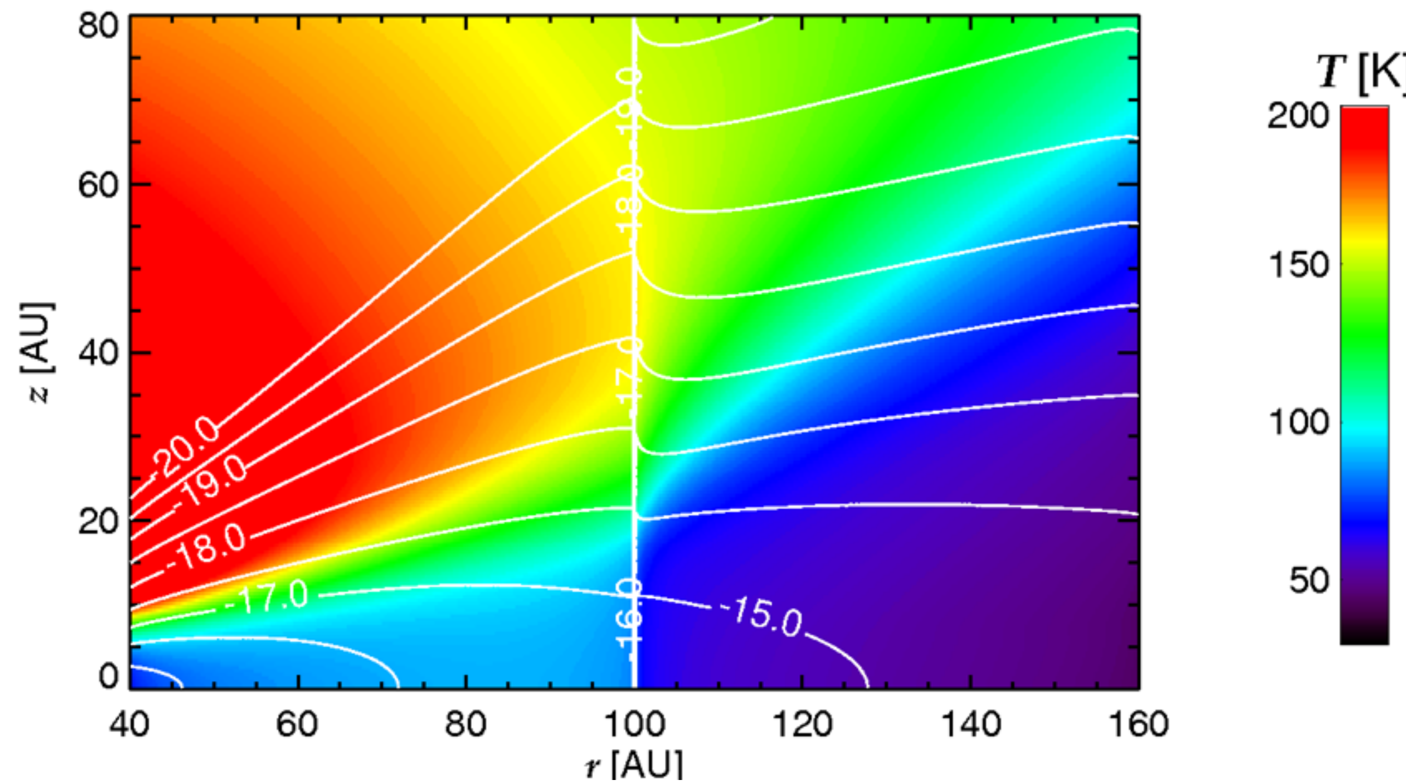
9,500 K
2 $M_\odot$, 2.5$R_\odot$

Disk with inner hole = Transition disk

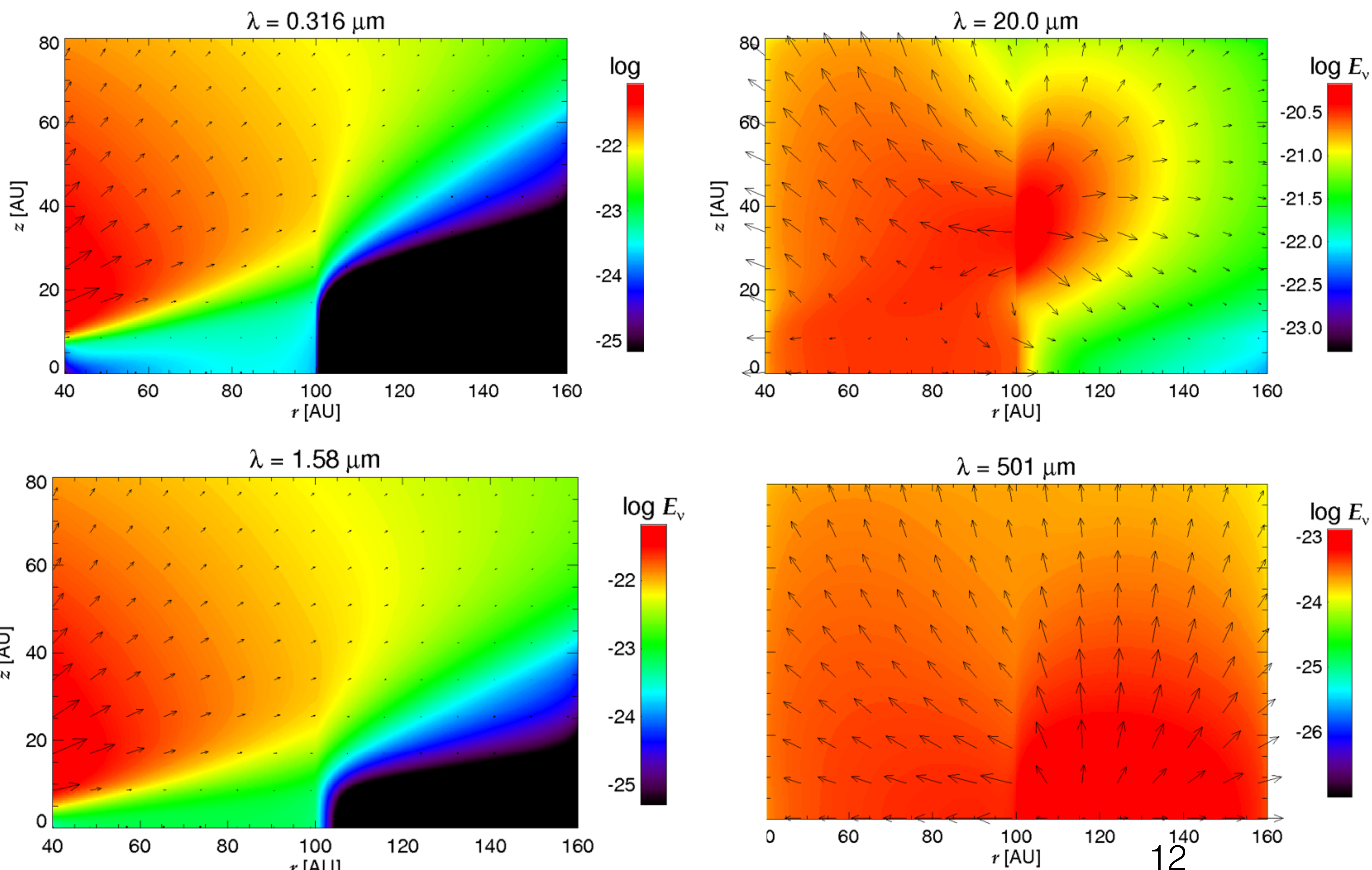
cf. Honda+ 12   HD169142



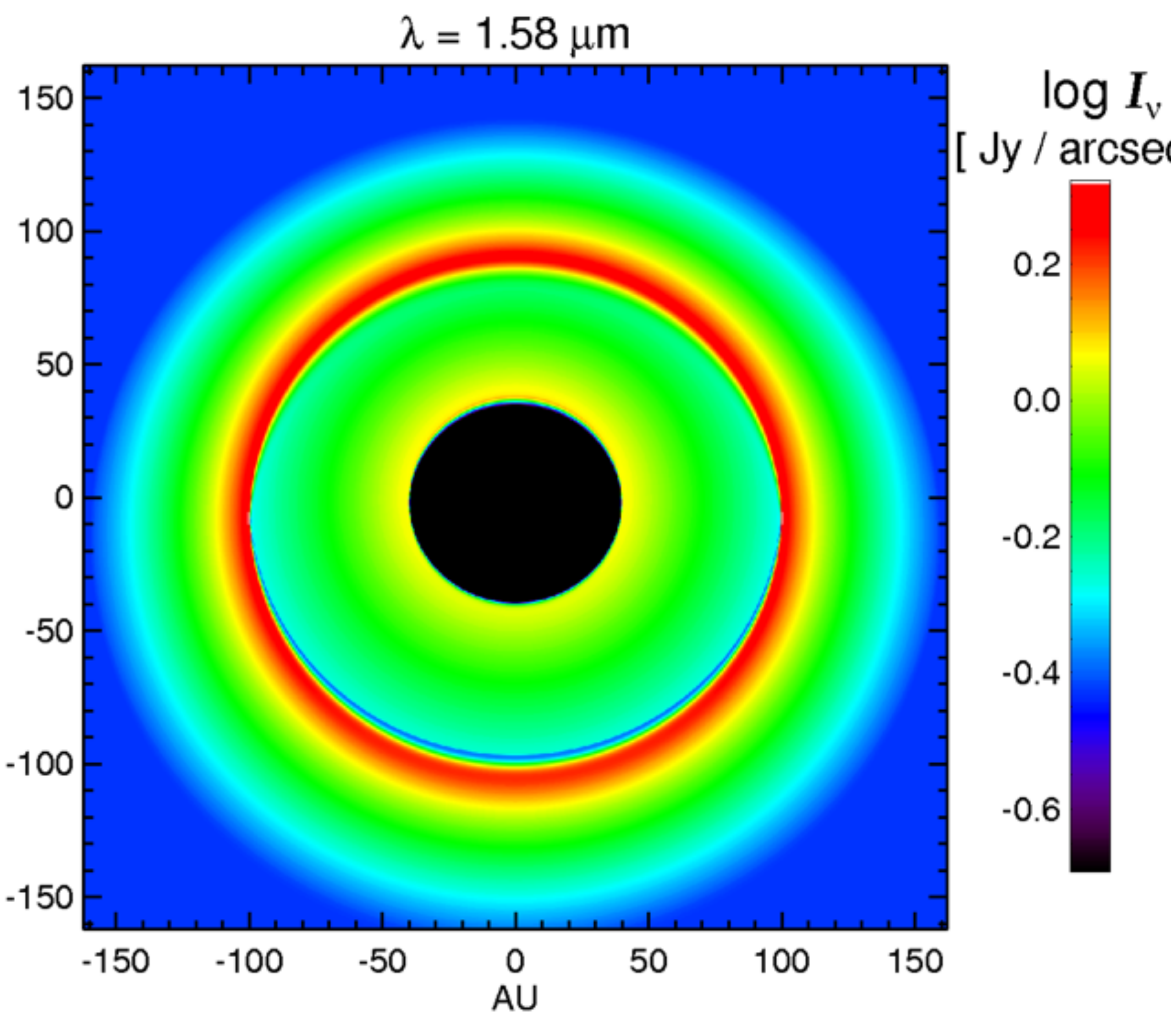
$\Delta r = 0.3$ AU, $\Delta z = 0.4$ AU

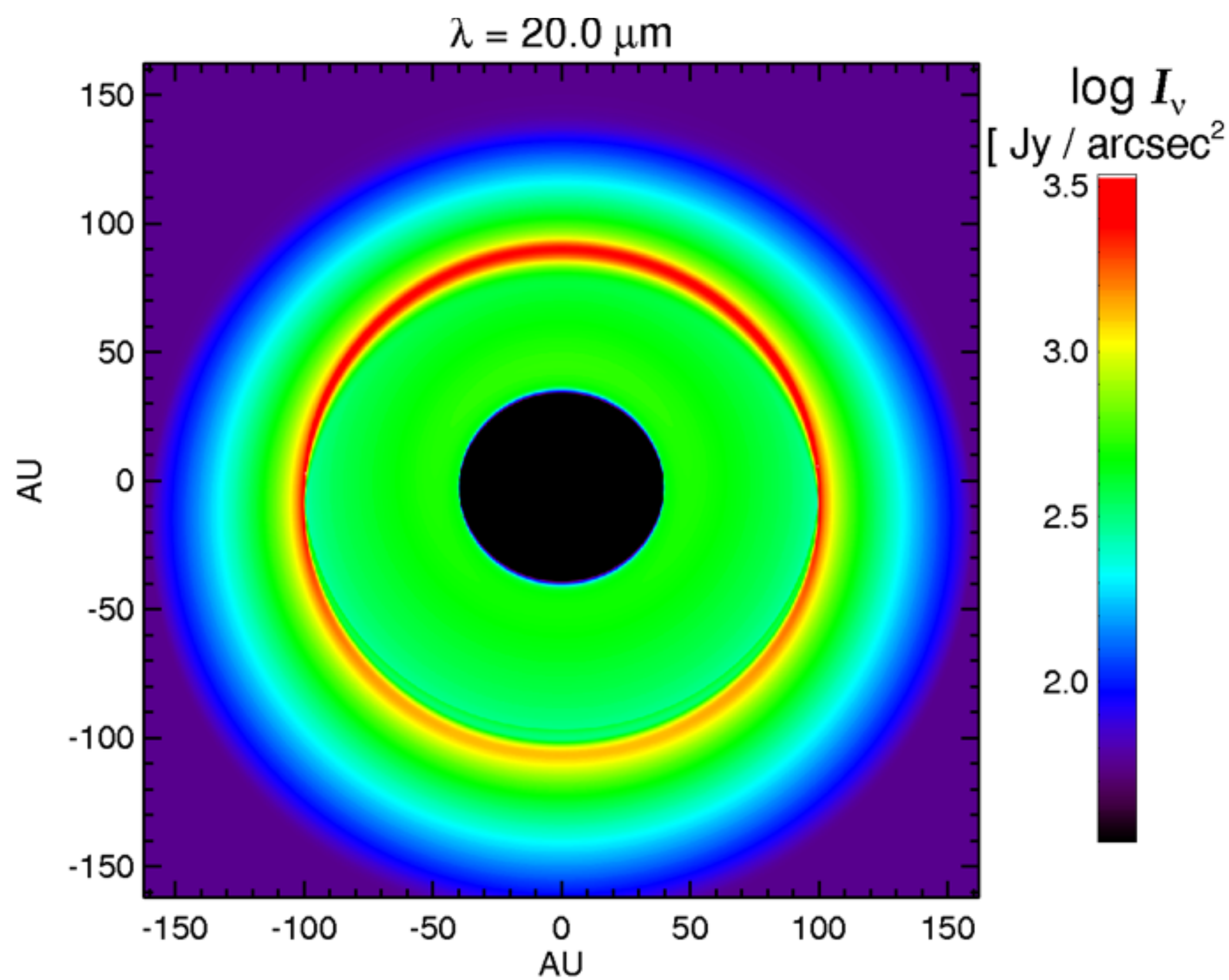


Radiation Field



Simulated Images





Ray tracing based on the source function obtained by M1.

Summary

- M1 model can be applied to optically thick media if our reconstructed flux is applied.
- M1 model can simulate a protoplanetary disk if two sets are used.
- M1 model can be used for hydrodynamics (Poster by Harada).
  - If we use a semi-implicit solution and reduced speed of light.