Happy 60th Birthday Dear Misao-san!

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Plan:

- Motivation
- Massive General Relativity
- Explicit vs. Dynamical mass; Quasi-Dilaton
- Quantum corrections and strong coupling
- Conclusions

Motivation:

To test General Relativity (GR), good to have an alternative theory to compare with, and test both against the data. The Brans-Dicke theory was introduced for that purpose long ago

Cosmic acceleration: a new physical scale of dark energy, 10^{-33} eV; this might be a scale where GR should be modified

Extension of GR by a mass term is arguably a well motivated IR modification. Yet, such an extension had been a problem up until some time ago. This problem – a good motivation for a field theorist.

GR extended with the mass term, evades S. Weinberg's no-go theorem for the old cosmological constant problem

GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '11*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M^2_{\rm pl}\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

Lagrangian Rewritten via Levi-Civita Symbols:

de Rham, GG, Heisenberg, Pirtskhalava '11 (decoupling limit)
Koyama, Niz, Tasinato; Th. Nieuwenhuizen; '11 (full theory)

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{g} \left(R + m^2 \left(\mathcal{U}_2 + \alpha_3\, \mathcal{U}_3 + \alpha_4\, \mathcal{U}_4 \right) \right)$$

$$\mathcal{U}_2 = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\alpha\beta}\mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma$$

$$\mathcal{U}_3 = \epsilon_{\mu\nu\alpha\gamma}\epsilon^{\rho\sigma\beta\gamma}\mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma \mathcal{K}^\alpha_\beta$$

$$\mathcal{U}_4 = \epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\mathcal{K}^\mu_\alpha \mathcal{K}^\nu_\beta \mathcal{K}^\rho_\gamma \mathcal{K}^\sigma_\delta$$

$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$ unitary gauge $\phi^a = x^a$

Hamiltonian construction: *Hassan, Rachel A. Rosen, '11,'12*
Another proof: *Mirbabayi, '12; Hinterbichler, R.A. Rosen, '12*

No flat FRW solution:

D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \phi^0(t) = f(t), \quad \phi^j(x) = x^j$$

Minisuperspace Lagrangian (for $\alpha_{3,4} = 0$):

$$L = 3M_{\rm pl}^2 \left(-a\dot{a}^2 + m^2(2a^3 - 3a^2 + a) - m^2\dot{f}(a^3 - a^2)\right)$$

$$\frac{d}{dt}(m^2(a^3 - a^2)) = 0$$

No cosmology if m is a constant [Exception: Open FRW selfaccelerated universe: *Gumrukcuoglu, Lin, Mykohyama, '11*].

Possible ways to proceed with the flat universe:

(1) Pseudo-homogeneous, or heterogeneous/anisotropic cosmologies

(2) Field dependent mass $m \to m(\sigma)$: FRW solutions re-emerge

Selfacceleration and pseudo-homogeneous solutions

In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*

Exact solution: *Koyama, Niz, Tasinato (1,2,3)*

Other solutions: *M. Volkov; L. Berezhiani, et al; Langlois, Naruko*

For instance, *Koyama-Niz-Tasinato* solution:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions:

$$\mathrm{arctanh}\left(\frac{\sinh(m\tau/2) + e^{m\tau/2} m^2 \rho^2/8}{\cosh(m\tau/2) - e^{m\tau/2} m^2 \rho^2/8}\right), \quad \rho e^{m\tau/2}$$

Selfacceleration with heterogeneous metric: *Gratia, Hu, Wyman*

Selfacceleration is a generic feature of this theory

However, vanishing of kinetic terms for extra modes seems to be a common feature of these solutions.

Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava, '12*

Invariance of the action to rescaling of the reference frame coordinates ϕ^a w.r.t. the physical space coordinates, x^a, requires a field σ. In the Einstein frame:

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - e^{\sigma/M_{\rm Pl}} \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2$$

In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Cosmology of Quasi-Dilaton: Flat FRW Solutions

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad \phi^0 = f(t), \quad \phi^i = x^i, \quad \sigma = \sigma(t)$$

Friedmann equation:

$$\begin{aligned} 3M_{\rm Pl}^2 H^2 &= \frac{\omega}{2}\dot{\sigma}^2 + \rho_m + \\ 3M_{\rm Pl}^2 m^2 &\left[c_0 + c_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + c_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + c_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 \right] \end{aligned}$$

Constraint equation:

$$q_0 + q_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + q_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + q_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 = \frac{k e^{-\sigma/M_{\rm Pl}}}{a^3} .$$

Particular Solutions for $k = 0$:

$$\left(\frac{e^{\sigma/M_{\rm Pl}}}{a}\right) = c, \quad \dot{\sigma} = M_{\rm Pl} H$$

Friedmann equation

$$(3 - \frac{\omega}{2}) M_{\rm Pl}^2 H^2 = \rho_m + 3 M_{\rm Pl}^2 m^2 \left[c_0 + c_1 c + c_2 c^2 + c_3 c^3\right]$$

Constraint equation

$$q_0 + q_1 c + q_2 c^2 + q_3 c^3 = 0$$

Determine $f(t)$ from the sigma equation:

$$a\dot{f} = 1 + \frac{\omega}{3\kappa m^3}(3H^2 + \dot{H})$$

Small Perturbations:

Unitary gauge, ϕ^a's are frozen to their background values

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}(t,x)), \quad \sigma = ln(ca) + \zeta(t,x)$$

No diff invariance for $h_{\mu\nu}$ as long as ϕ^a's are frozen

Lagrangian density in conformal coordinates

$$a^4 \quad \left(\frac{\omega}{a^2}((\zeta')^2 - (\partial_j\zeta)^2) + \frac{\omega H}{a}(h_{00} + h_{jj})\zeta' - \frac{2\omega H}{a} h_{0j}\partial_j\zeta\right) \;+$$
$$a^4 \quad ((\gamma_1 h_{00} + \gamma_2 h_{jj})\zeta + \gamma_3 h_{00}^2 + \gamma_4 h_{00} h_{jj} + \gamma_5 h_{0j} h_{0j} + \gamma_6 h_{ij} h_{ij} + \gamma_7 h_{jj}^2)$$

There is no BD ghost – should be absent by construction, selfconsistency check. $\gamma_6 = -\gamma_7$, and all modes have kinetic terms ($\omega = 0$ seems to be OK; more to appear)

Quantum consistency of massive gravity:

Exact Lagrangian in the decoupling Limit for helicity 2 and 0:

de Rham, GG, '10

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \left(X^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3} X^{(2)}_{\mu\nu} + \frac{\beta}{\Lambda_3^6} X^{(3)}_{\mu\nu} \right)$$

$$X^{(1)}_{\mu\nu} = \epsilon_{\mu\alpha}\epsilon_{\nu\beta}\Pi^{\alpha\beta}$$

$$X^{(2)}_{\mu\nu} = \epsilon_{\mu\alpha\rho}\epsilon_{\nu\beta\sigma}\Pi^{\alpha\beta}\Pi^{\rho\sigma}$$

$$X^{(3)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma}\epsilon_{\nu\beta\sigma\delta}\Pi^{\alpha\beta}\Pi^{\rho\sigma}\Pi^{\gamma\delta}$$

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

*Invariant, under linear diffs (up to a total derivative), under galilean transformations of π

*Can be diagonalized (for $\beta = 0$); gives rise to the galileons

$\beta = 0$: Classical theory

Partial diagonalization: $h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu}\pi$

$$\mathcal{L} = h\partial^2 h + \pi\partial^2\pi + \alpha\frac{(\partial\pi)^2\Box\pi}{\Lambda_3^3} + \alpha\frac{h^{\mu\nu}X^{(2)}_{\mu\nu}(\pi)}{\Lambda_3^3} + \pi T + h^{\mu\nu}T_{\mu\nu}$$

$\beta = 0$: Classical theory

Partial diagonalization: $h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu}\pi$

$$\mathcal{L} = h\partial^2 h + \pi\partial^2\pi + \alpha\frac{(\partial\pi)^2\Box\pi}{\Lambda_3^3} + \alpha\frac{h^{\mu\nu}X^{(2)}_{\mu\nu}(\pi)}{\Lambda_3^3} + \pi T + h^{\mu\nu}T_{\mu\nu}$$

Full diagonalization: $h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu}\pi - \alpha\frac{\partial_\mu\pi\partial_\nu\pi}{\Lambda_3^3}$

$$\begin{aligned}\mathcal{L} &= h\partial^2 h + \pi\partial^2\pi + \alpha\frac{(\partial\pi)^2\Box\pi}{\Lambda_3^3} + \alpha^2\frac{(\partial\pi)^2((\Box\pi)^2 - (\partial\partial\pi)^2)}{\Lambda_3^6} \\ &+ \pi T - \alpha\frac{\partial_\mu\pi\partial_\nu\pi}{\Lambda_3^3}T^{\mu\nu} + h^{\mu\nu}T_{\mu\nu}\end{aligned} \tag{1}$$

Importance of the new coupling:

$$-\partial^\mu \pi \partial^\nu \pi (\eta_{\mu\nu} + \alpha \frac{T_{\mu\nu}}{\Lambda_3^3})$$

An additional classical renormalization of the π quadratic term, e.g., due to Earth's atmosphere

$$(1 + \alpha 10^{26})(\partial_0 \pi)^2 - (1 + \alpha 10^{14})(\partial_j \pi)^2$$

Similar effect is due to any local source with density/pressure greater than the critical density ρ_c, (e.g., a measuring device); this effect makes π weakly coupled, even before the global Vainshtein mechanism is invoked. Different from DGP and Galileons.
This extra suppression should be taken into account while imposing bounds on graviton mass (more to appear).

Quantum corrections:

The nonlinear terms do not get renormalized by quantum loops

de Rham, GG, Heisenberg, Pirtskhalava, to appear

$$\mathcal{L} \;=\; -\frac{1}{2}h^{\mu\nu}\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\left(aX^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3}X^{(2)}_{\mu\nu} + \frac{\beta}{\Lambda_3^6}X^{(3)}_{\mu\nu}\right)$$

Quantum loop calculations: due to specific structure of the vertices loops do no renormalize a, α, β.

For the full theory, this implies that a choice of the value of m, and the two parameters α and β, is technically natural.

What about other terms that are induced by quantum loops?

Quantum theory with $\beta = 0$:

Partial diagonalization: Scale invariance in the UV, similar to cubic galileon, Rattazzi and Nicolis, '04

$$\mathcal{L}_{UV} \to \alpha(\partial\bar{\pi})^2\Box\bar{\pi} + \alpha\bar{h}^{\mu\nu}X^{(2)}_{\mu\nu}(\bar{\pi}) \to \alpha(\bar{h}^{\mu\nu} + \eta^{\mu\nu}\bar{\pi})X^{(2)}_{\mu\nu}(\bar{\pi})$$

$\bar{\pi}$ is a dimensionless field.

Full diagonalization: quartic Galileon dominates in the UV; the latter is not scale invariant

$$\mathcal{L}_{UV} = \alpha^2\frac{(\partial\bar{\pi})^2((\Box\bar{\pi})^2 - (\partial\partial\bar{\pi})^2)}{\Lambda_3^2}$$

Hence, the counterterms could be naturally organized into scale invariant terms (*a la* Rattazzi and Nicolis) only for the partial diagonalization. Massive gravity differs from quartic Galileon!

Conclusions:

- A classical theory that extends GR by the mass and potential term is available now
- Many questions of classical gravity can be studied and comparisons can be made with GR, and perhaps with data; e.g., generic cosmological solutions have no FRW symmetries, but can approximate well FRW cosmologies in the early universe
- Selfaccelerated solutions emerge as a generic feature; but some fluctuations loose kinetic terms
- Dynamical mass theories differ – FRW solutions re-emerge. An example: Quasi-dilaton with selfacceleration exhibits nonvanishing kinetic terms for all perturbations
- Quantum aspects not yet well understood: good effective theory below a certain scale, but needs UV extension up to $M_{\rm pl}$ – dynamical mass theories may be easier to complete.