15 years on: a review of self force progress since MiSaTa(QuWa)

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Tribute to Misao Sasaki

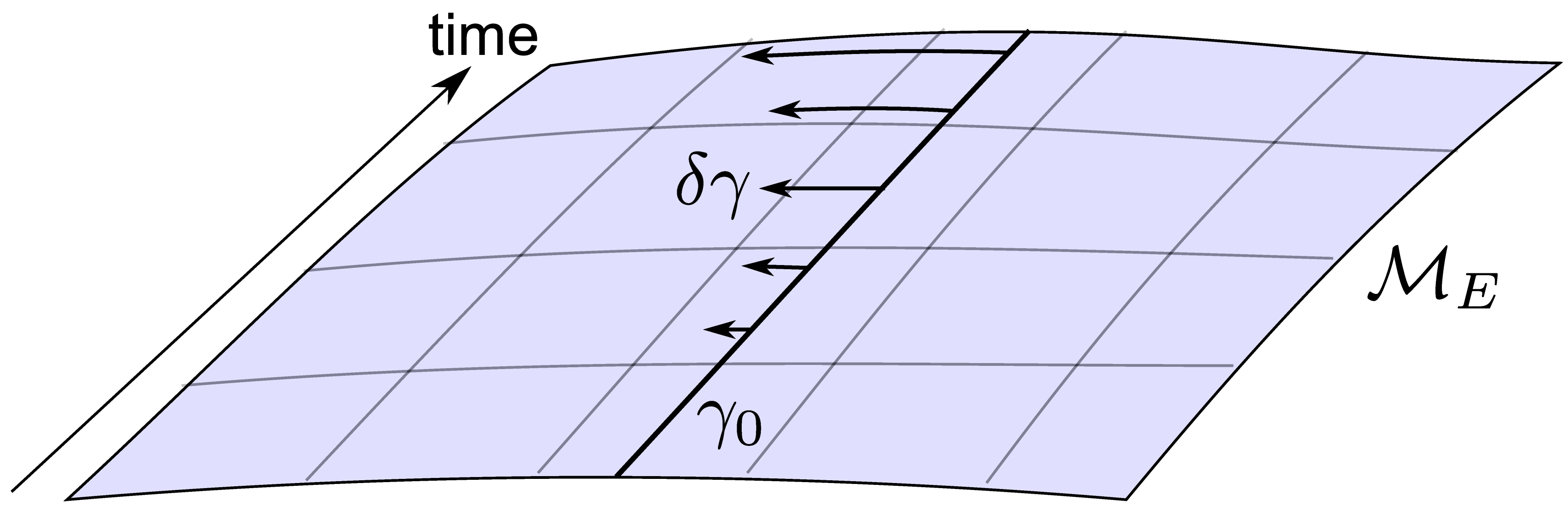
- Big impact on the self force problem
- Many students trained into it
- Introduced techniques; opened up the field
- Inspired many subsequent developments
- Prompted initiation of Capra Meetings
- Still having an impact through his students

Outline

- Self force?
- MiSaTa(QuWa)?
- Matched asymptotic expansions
- Understanding the singular filed
- Overlapping expansions
- Post-Newtonian impact
- Scaling limit
- Other contributions
- Regularization parameters
- Second Order
- Misao’s legacy

What is the self force problem?

- A small particle moves in the gravitational influence of a much larger mass
- At zeroth order the path is a geodesic
- At next order the path is altered
- There are both conservative (orbital) and dissipative (radiation reaction) effects occurring
- What is the new path of the particle?
- What is a computational scheme for progress?



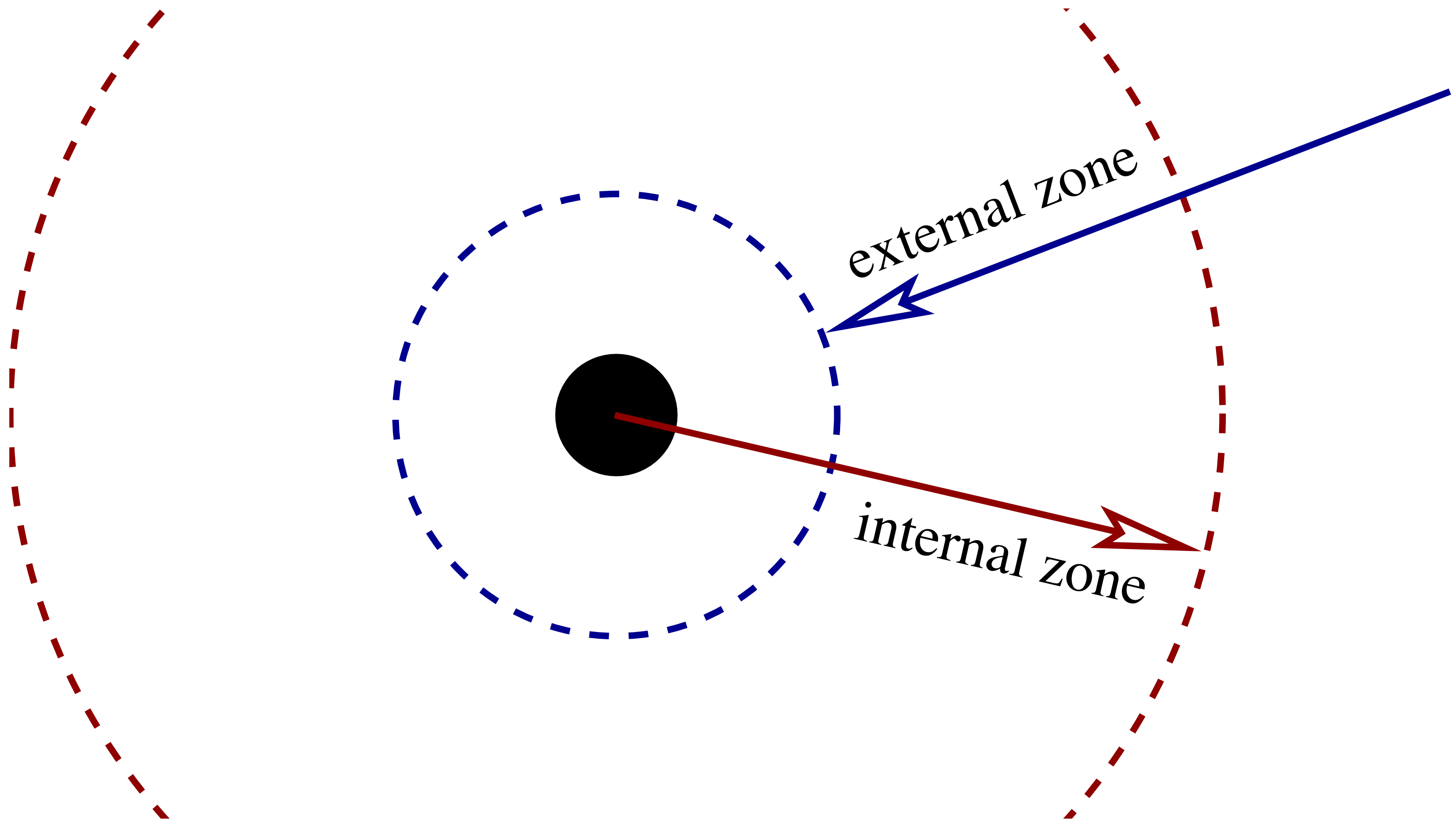
particle deviates from a fixed background geodesic

MiSaTa Paper

- A watershed paper: Used two separate approaches
- I. Linked with the past - integrate stress tensor over world tube surrounding the particle
- II. Asymptotic matching of two different expansion schemes - inner tidal distortion and outer perturbative treatment
- Lead to equation of motion for the small particle
- Inspired analysis with overlapping expansions
- Independently confirmed by QuWa paper
- Techniques used are a legacy to the field

Matched asymptotic expansions

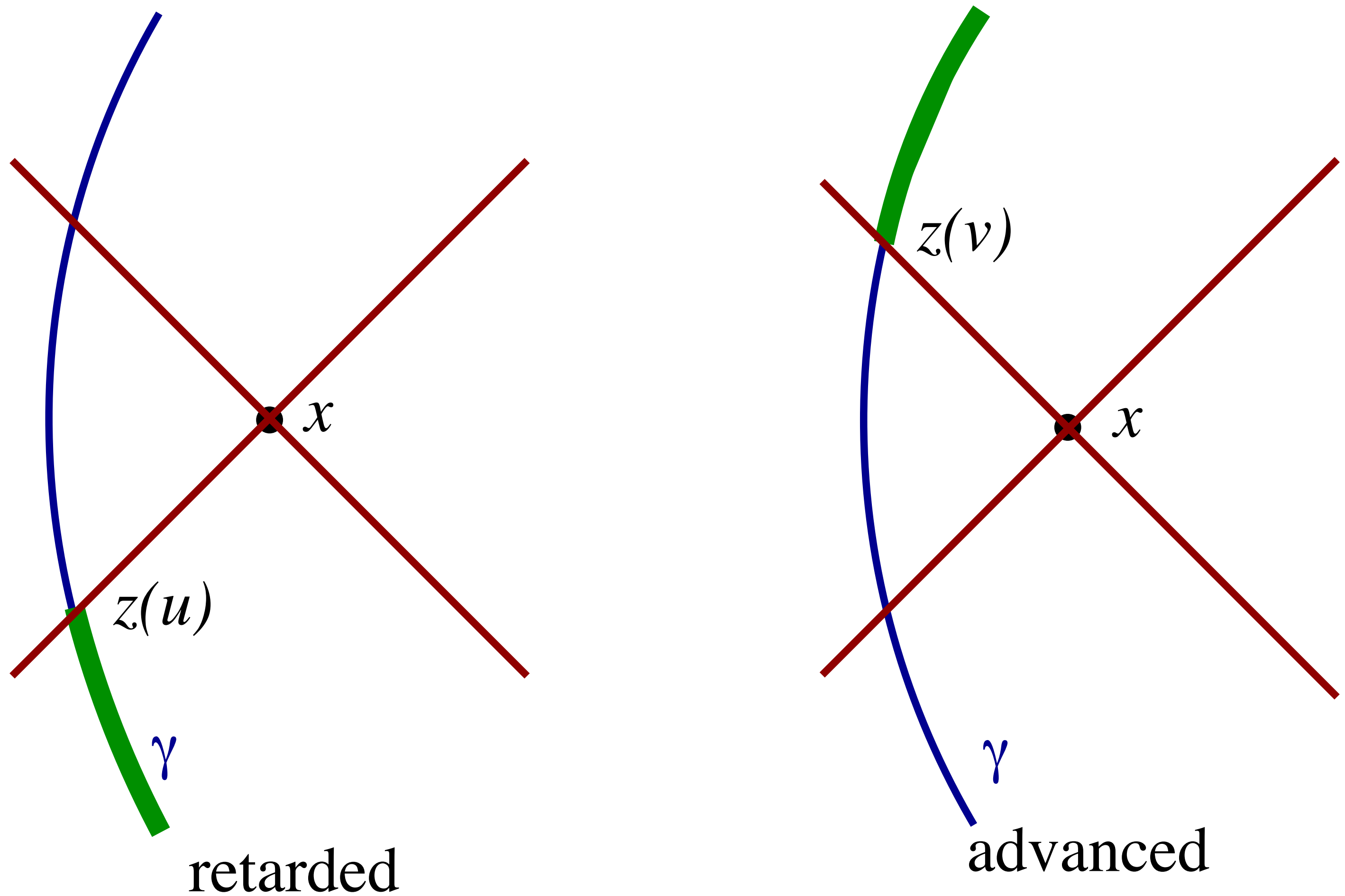
- Inner domain is described using multi-pole (far field) expansion around small source
- Outer domain uses perturbation around background of larger (rotating) mass or BH
- Expansions are matched in overlap region, relating inner and outer coefficients
- Equation of motion from simple multipole (body centered) choice for small object



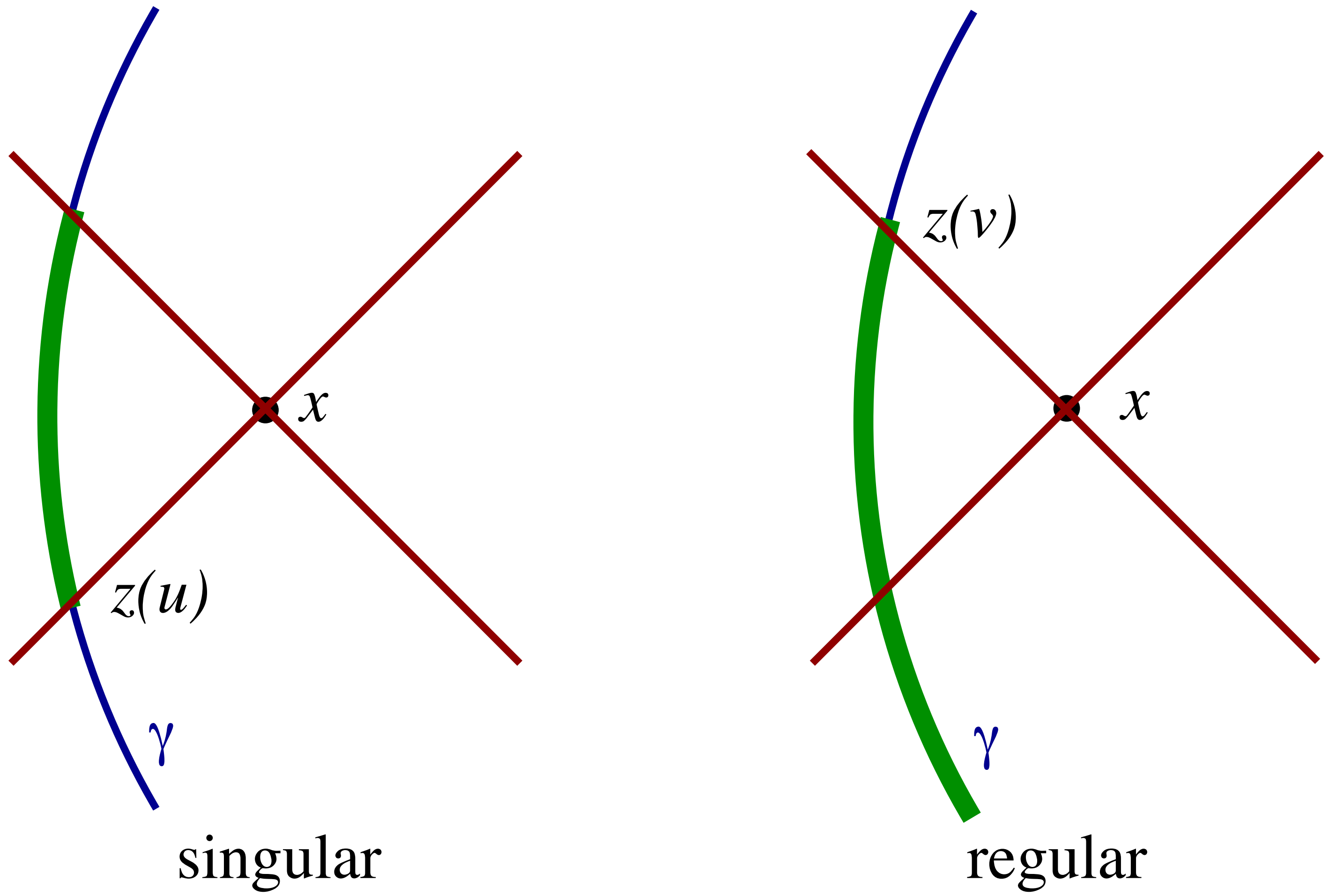
internal and external zones;
matching done in overlap region

Understanding the Singular Field

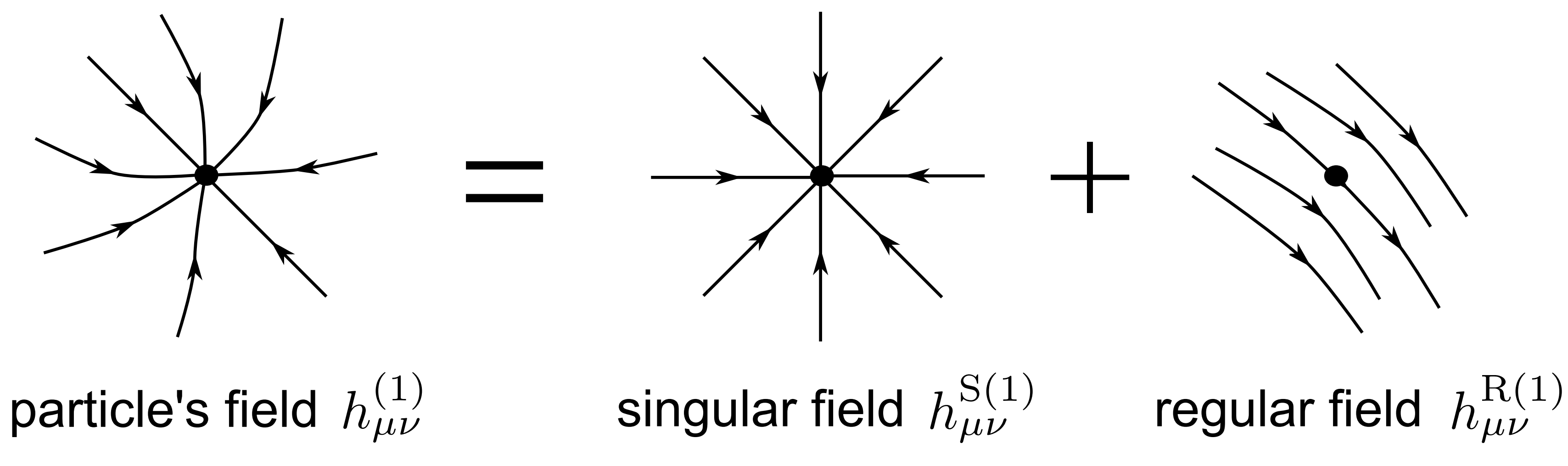
- Inspired by MiSaTa paper
- Purely local; solves perturbation equation
- Leaves a regular (vacuum perturbation) to describe all self force effects
- Can interpret motion as geodesic in a modified (vacuum) background
- Enabled effective regularization calculations



retarded and advanced fields, with their causal dependence



singular and regular fields, with their (a)causal dependence



decomposing the singular field

Overlapping Expansions

- Sasaki-Nakamura basis for solutions
- Can replace numerical integration
  - Sago and Fujita's poster on PN expansions
- Can be exact, though not yet for self force
  - Shah now has 4PN coefficient to 100 places
- Starting to be the way of the future

$$
\begin{aligned}
\left\langle \frac{d\mathcal{E}}{dt} \right\rangle &= -\frac{32}{5}\left(\frac{\mu}{M^2}\right) v^{10}(1-e^2)^{3/2} \\
&\times \left\{ 1 + \frac{73}{24}\,e^2 + \frac{37}{96}\,e^4 - \left(\frac{1247}{336} + \frac{9181}{672}\,e^2 - \frac{809}{128}\,e^4\right) v^2 + \left(-\frac{949}{32}\,Y e^4 - \frac{73}{12}\,Y - \frac{823}{24}\,e^2 Y\right) q v^3 \right. \\
&\quad + \left(\frac{3935}{192}\,e^4 + 4 + \frac{1375}{48}\,e^2\right) \pi v^3 + \left(-\frac{44711}{9072} - \frac{172157}{2592}\,e^2 - \frac{2764345}{24192}\,e^4\right) v^4 \\
&\quad + \left(\frac{527}{96}\,Y^2 - \frac{329}{96} + \frac{6533}{192}\,e^2 Y^2 - \frac{4379}{192}\,e^2 + \frac{6753}{256}\,e^4 Y^2 - \frac{3823}{256}\,e^4\right) q^2 v^4 \\
&\quad + \left(-\frac{4311389}{43008}\,e^4 - \frac{8191}{672} - \frac{44531}{336}\,e^2\right) \pi v^5 + \left(-\frac{113093}{1344}\,Y e^4 + \frac{3665}{336}\,Y + \frac{827}{28}\,e^2 Y\right) q v^5 \\
&\quad + \left(-\frac{675}{256}\,e^4 Y^3 - \frac{405}{256}\,Y e^4 - \frac{225}{64}\,e^2 Y^3 - \frac{135}{64}\,e^2 Y - \frac{15}{32}\,Y^3 - \frac{9}{32}\,Y\right) q^3 v^5 \\
&\quad + \left[-\frac{1712}{105}\,\ln(v) + \frac{135}{8}\,q^2 + \frac{73}{21}\,q^2 Y^2 + \frac{6643739519}{69854400} - \frac{169}{6}\,q \pi Y - \frac{3424}{105}\,\ln(2) - \frac{1712}{105}\,\gamma + \frac{16}{3}\pi^2 \right. \\
&\quad + \left(\frac{205747}{1344}\,q^2 + \frac{43072561991}{27941760} + \frac{13697}{192}\,q^2 Y^2 - \frac{13696}{315}\,\ln(2) - \frac{4339}{16}\,q \pi Y - \frac{14552}{63}\,\ln(v) \right. \\
&\quad \left. + \frac{680}{9}\,\pi^2 - \frac{14552}{63}\,\gamma - \frac{234009}{560}\,\ln(3)\right) e^2 \\
&\quad + \left(\frac{919773569303}{279417600} + \frac{2106081}{448}\,\ln(3) + \frac{208571}{1792}\,q^2 - \frac{553297}{1260}\,\ln(v) - \frac{12295049}{1260}\,\ln(2) \right. \\
&\quad \left. \left. \left. - \frac{42271}{96}\,q \pi Y + \frac{5171}{36}\,\pi^2 + \frac{471711}{1792}\,q^2 Y^2 - \frac{553297}{1260}\,\gamma \right) e^4 \right] v^6 \right\}
\end{aligned}
$$

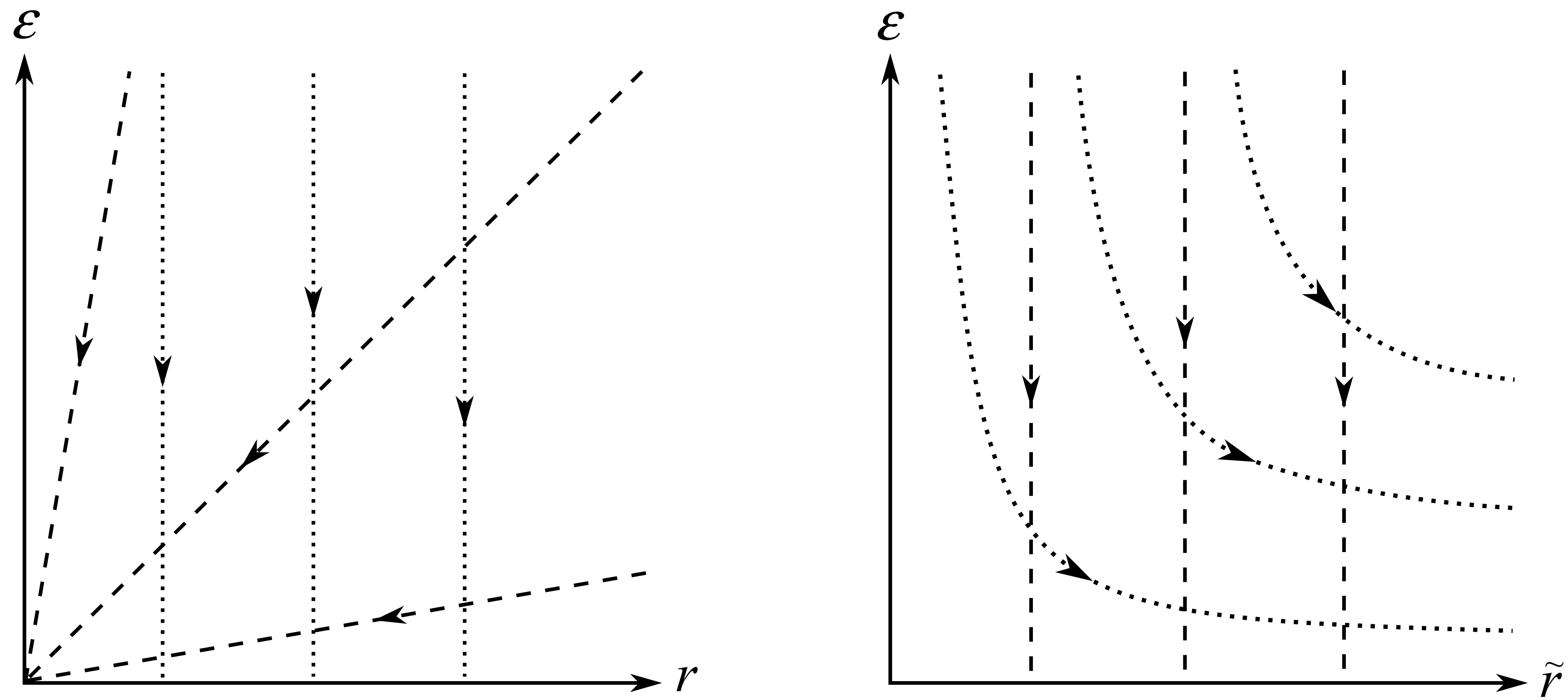
3PN, $\mathcal{O}(e^4)$, BH flux included

Gauge invariant PN red-shift coefficients

- $168\alpha_4-79\alpha_5$=0.344512957073003453473117696417377851149455276905113885124009299746219438821199770108 3 (85 digits)
- Needed nearly 500 digits to obtains this
- Calculated with $r=10^{30}M$ to achieve this
- Involved numerical regularization (to ℓ =85)

Formal scaling limit

- Two-space as opposed to two time formulation (cf Flanagan and Hinderer)
- Genuine spacetime with two distinct limits (al la Wald's talk yesterday)
- Used for "rigorous" treatment
- Exploited for second order (see Pound)
- Incredibly versatile analytical tool



formal scaling: inner (dashed) and outer (dotted) limits

Other contributions

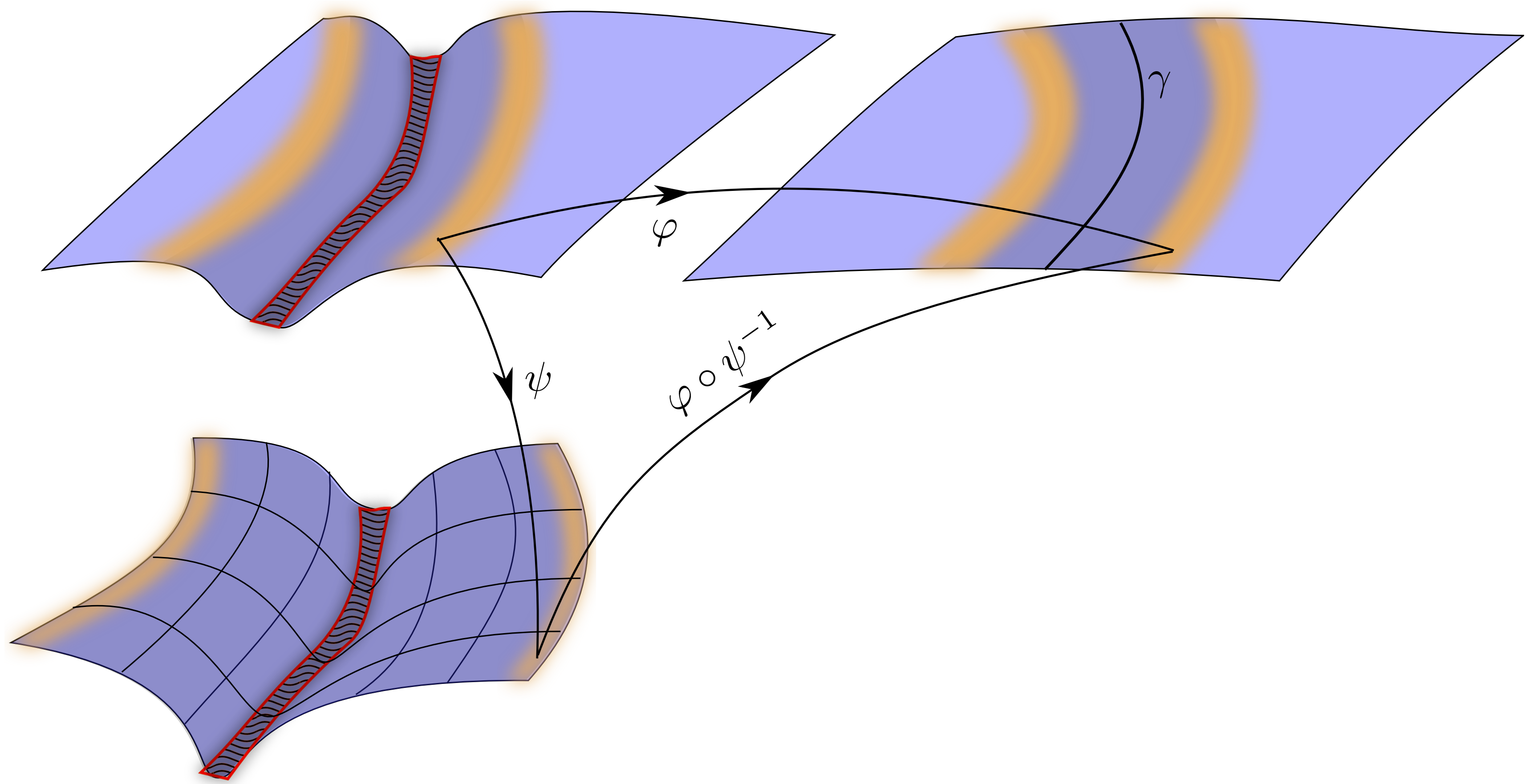
- Role of ℓ =0 and ℓ =1 in reliable regularization
- Resonance occurrence & effects
- Gauge invariant results
- Higher order PN coefficients
- Precession and ISCO changes
- Periastron advance at equal mass ratio
- Effective source formulation
- Time domain integration
- Rotating (Kerr) black hole backgrounds
- Higher order regularization parameters
- Progress to second order
- EOB extensions

Regularization Parameters

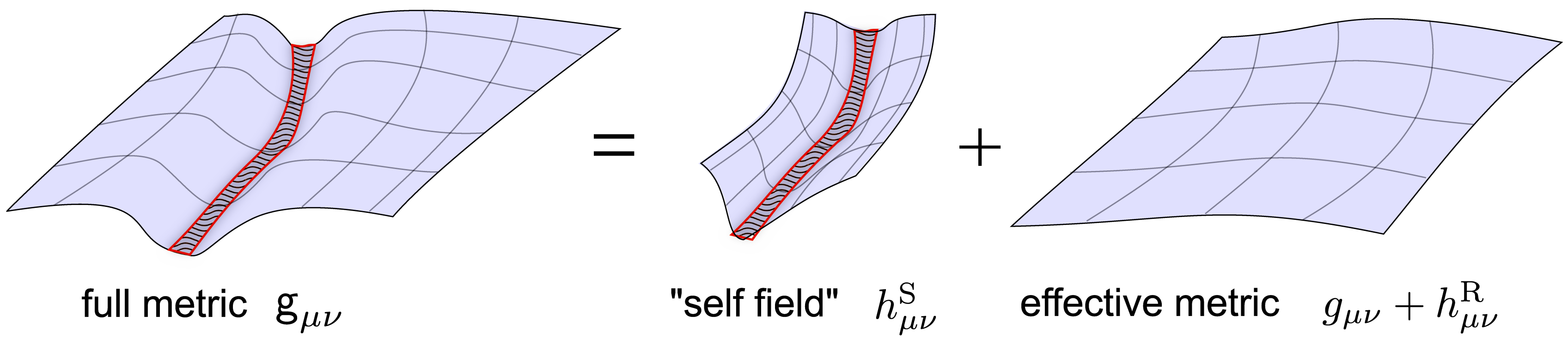
- Higher order coefficients now available
- Purely analytical results
- Based extensively on computer algebra
- Improve utility of numerical computations
- Applicable for Kerr geometries (see http://arXiv.org/abs/arXiv:1211.6446)
- Can be used in frequency or time domain

Second order progress

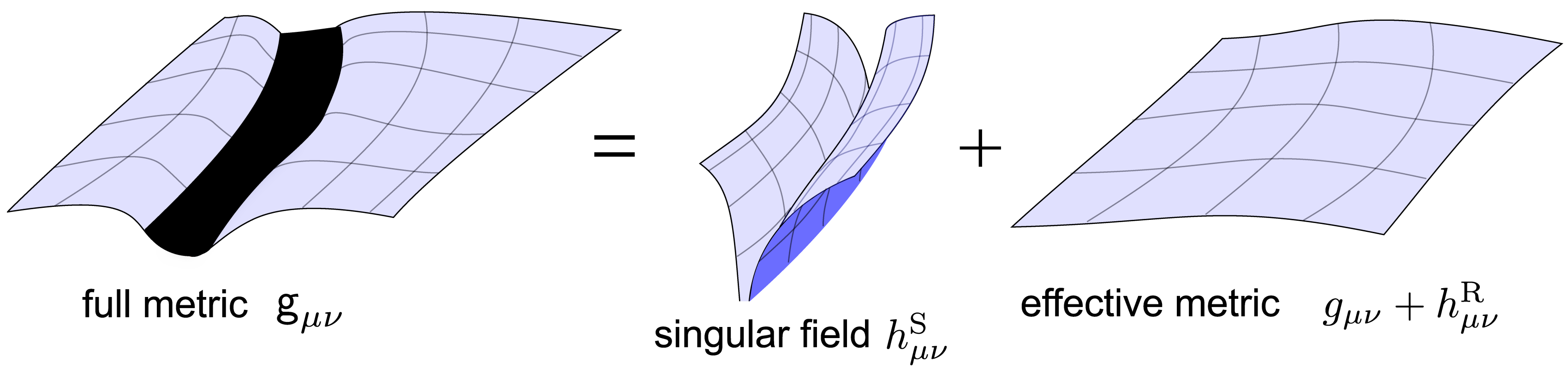
- Needed for LISA (or other space) data analysis
- Gralla has a formulation (family of gauges)
- Pound has a formulation (no runaway solutions)
- Detweiler has a formulation (fixed coordinates)
- They have inequivalent physical interpretations
- They do not agree on the equations of motion
- Singular field descriptions barely comparable
- Framework exists; need to work out the details
- Can formally avoid problem of runaway solutions



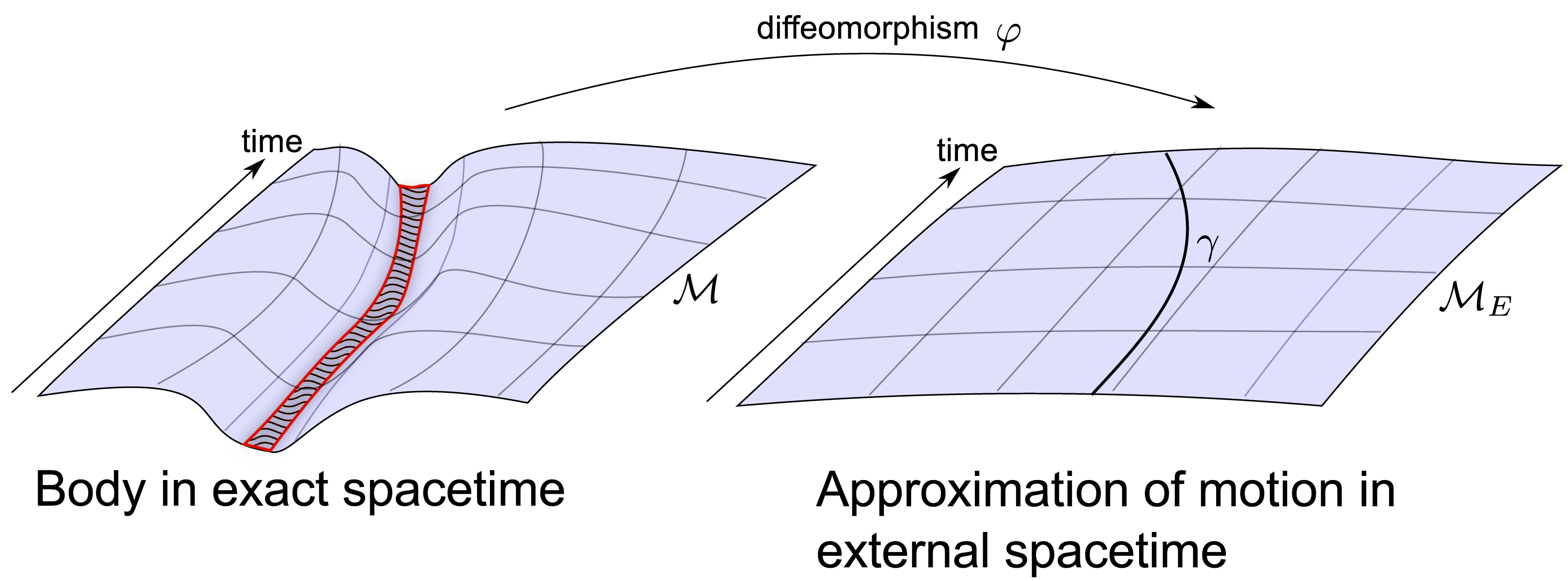
Pound follows MiSaTaQuWa approach



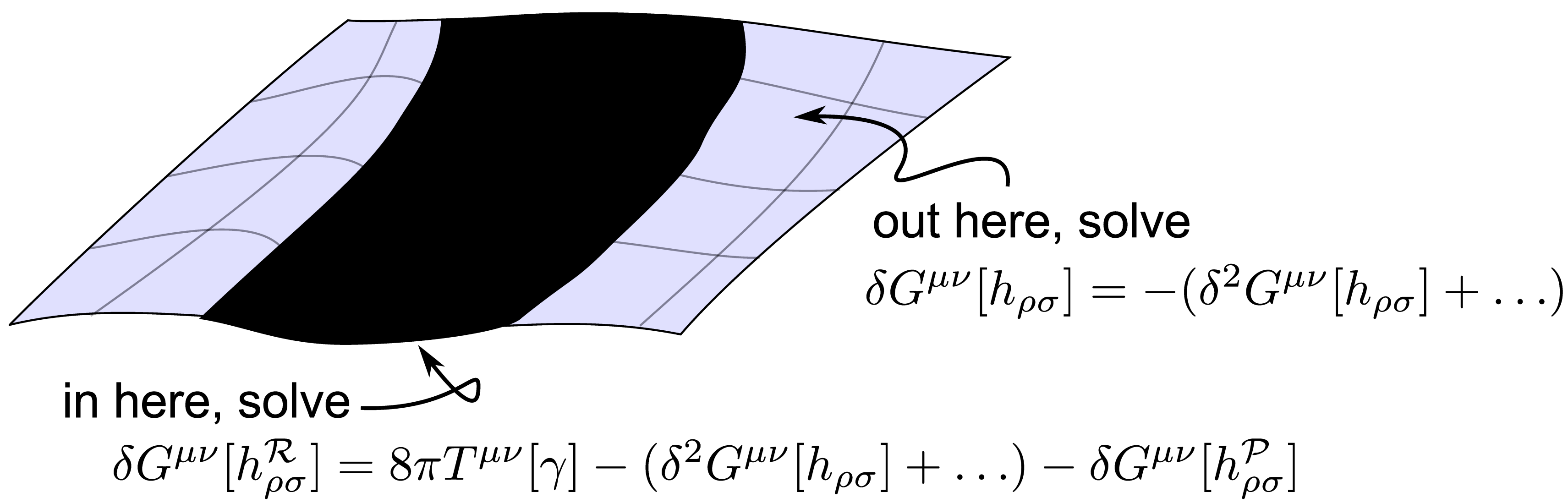
uses explicit field decomposition



scaling and matching combined in single formulation



effective orbit defined; avoids runaway solutions



clear computational scheme, with regular and singular pieces defined

Contributions from:

- Leor Barack
- Luc Blanchet
- Steve Detweiler
- Eanna Flanagan
- John Friedman
- Chad Galley
- Sam Gralla
- Abraham Harte
- Anna Heffernan
- Tanja Hinderer
- Alex Le Tiec
- Eric Poisson
- Adam Pound (including graphics)
- Adrian Ottewill
- Norichika Sago
- Abhay Shah
- Takahiro Tanaka
- Ian Vega
- Niels Warburton
- Barry Wardell
- Bob Wald
- 40+ Capra other attendants, 2012

Misao's Legacy

- A physical problem which is interesting
- A field which is still growing after 15 years
- Students who continue to contribute
- Methods which are being extended
- Let's reconvene in another 15 years to check on further progress
- Happy 60^{th} Birthday Misao-san.