





# Cutoffs, Stretched Horizons and Black Hole Radiance

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## Outline

- Brief review of RS2 story
- Review of Hawking radiance calculation
- Stretched horizons: and then there were many
- A "paradox" and its resolution
- Summary

## QUANTUM BLACK HOLES AS HOLOGRAMS

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#### DUAL INTER PRETATION

BEFORE CUTTING : AdS/CFT MALDACENA CLASSICAL WEAK SUPERGRAVITY IN AdSD+1 IS DUAL TO QUANTUM SYM IN MD IN THE STRONG COUPLING REGIME TO LEADING

CLASSICAL BULK CONFIGURATIONS ENCODE QUANTUM EFFECTS TRUNCATED TO PLANAR PLAGRAMS

AFTER CUTTING:  $AdS/CFT @ CUTOFF M_{UV}$ + D-DIM GRAVITY (+ LOCALIZED GAUGE THEORY...) PUT IN  $M_{UV} \sim \frac{\pi}{L} & INTEGRATE OUT CFT ABOVE IT$ PUT IN D-PIM GRAVITY :  $M_D^{D-2} \simeq M_{D+1}^{D-1} L_{ADS}$ PUT IN D-PIM GRAVITY :  $M_D^{D-2} \simeq M_{D+1}^{D-1} L_{ADS}$ PUT IN S-FUNCTION MATTER MALDAGENA WITTEN GUBSER VERLINDE AREANI-HAMED ET AL...

CUTOFF CFT DEFINED BY THE  
BULK DUAL: SMALL FLUCTUATIONS  
OF BULK FIELDS IN THE VOLCAND  
  
LOCALISED  
D-DIMENSIONAL  
GRAVITON  
D-DIMENSIONAL DYNAMICAL GRAVITY  
(D>3) 
$$\rightarrow$$
 THERE MUST EXIST  
D-DIMENSIONAL BLACK HOLES:  
THEY SHOLLD BE LOCALISED TO THE BRANE  
BLACK HOLES  
THEY SHOLLD BE LOCALISED TO THE BRANE  
BRANE  
AdS HORIZON  
MASSIVE KK MORE  $\simeq$  CFT STATE AT ENERGY ME  
BRANE MUST EXIST  
D-DIMENSIONAL BLACK HOLES:  
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BRANE MUST EXIST  
D-DIMENSIONAL BLACK HOLES:  
ND =3  $\overrightarrow{P}$  DYNAMICAL GRAVITON  $\rightarrow$  M  
ASYMPTOTICALLY FLAT SPACE  $\overrightarrow{P}$  BLACK HOLES!  
DEFER, JACKING & THEOREM

COMPLETE FAILURE !?

# IN D>3 CASES TO DATE NO-ONE HAS FOUND ANY EXACT LOCALIZED BLACK HOLE SOLUTIONS!

TO MAKE THINGS EVEN WORSE, THERE IS A "THEOREM" STATING THAT A COLLAPSE "ON THE BRANE" WILL NOT PRODUCE A STATIC SPHERICALLY SYMMETRIC BLACK HOLE BRUNI, GERMANI & MADRTENS

IN D=3 CASES EXACT STATIC BLACK HOLES IN ASYMPTOTICALLY FLAT SPACE, LOCALIZED ON THE BRANE, WERE FOUND !

EMPARAN, HOROWITZ & MYERS

IS THIS COMPLETELY THE OPPOSITE TO WHAT SHOULD BE EXPECTED ?!

... MUCH CONFUSION IN THE LITERATURE ...

### Q M TO THE RESCUE

RESOLUTION: OUR EXPECTATIONS WERE UNREALISTIC SINCE WE IGNORED QUANTUM MECHANICS



## CONJECTURE:

BLACK HOLES WHICH SOLVE CLASSICAL BULK EQUATIONS WITH BRANE BOUNDARY CONDITIONS CORRESPOND TO QUANTUM-CORRECTED BLACK HOLE SOLUTIONS IN THE DUAL CFT+GRAVITY THEORY

> R. ENPARAN, A. FABBRI, N.K. T. TANAKA



THIS MUST BE INTERPRETED AS ARISING  
COMPLETELY FROM QUANTUM CORRECTIONS  
SINCE THERE ARE NO CLASSICAL BH'S!  
INDEED: TAKE A LUMP OF CFT WITH MASS M  
AND IGNORE QUANTUM CORRECTIONS - IN D=3  
IT PRODUCES THE CONICAL GEOMETRY:  

$$PESER_{i} JACKIW, 'THOPT$$
  
M  
BUT NOW: SINCE THE BACKGROUND GEOMETRY  
IS A CONE, CAN COMPUTE THE QUANTUM  
CORRECTIONS TO TAV - CASIMIR ENERGY!  
FOR A WEAKLY COUPLED CFT WITH 9x POF'S  
 $\langle T^{n}_{Y} \rangle = 9 \propto (M) (1, 1, -2) 
FOR A WEAKLY COUPLED CFT WITH 9x POF'S
 $\langle T^{n}_{Y} \rangle = 9 \propto (M) (1, 1, -2) 
SOURADEEPR SAHNI
NOW PLUG BACK INTO THE EINSTEIN'S
EQUATIONS AND SOLVE ANEW;$$ 

LO AND BEHOLD !  $ds^{2} = -(1 - \frac{r_{o}}{r})dt^{2} + \frac{dr^{2}}{1 - \frac{r_{o}}{r}} + r^{2}d\psi^{2}$ H. Soleng, 1993 SAME AS EHM  $r_o = \pi g_* \frac{\alpha(M)}{M_*}$  $M \ll M_3$   $r_o \sim t g_{\#} \frac{M}{M_2^2}$  $M \lesssim \frac{M_3}{4} \qquad r_o \sim \frac{k \, 9 *}{M_3} \frac{1}{\left(1 - 4 M_{M_s}\right)^3} \rightarrow \infty$ UP TO O(1) COEFFICIENTS THE SAME RESULT IS OBTAINED BOTH FROM THE BULK AND FROM THE CALINGS IN THE DUAL CFT+GRAVITY THEORY QUANTUM CORRECTIONS DRESS UP THE NAKED SINGULARITY!

THE GREATER 9 + , THE BETTER ...

\* IN 4D, J DYNAMICAL GRAVITY => JBLACE HOLE: SCHWARZSCHILD SOLN CLARGE HOLE: MY >L -> CFT+GRAVITY MALID!)

\* BUT: IN ASYMPTOTICALLY Ads SPACE IN THE BULK, THERE'S NO STATIC SPHERICALLY SYMMETRIC BH SOLNS ?

#### DUAL CFT+ GRAVITY:

IF BULK Ad S IN THE IR (REGULAR HORIZON THE CFT IS CONFORMAL IN THE IR AND THERE IS NO MASS GAP!

SO ANY BLACK HOLE, HOWEVER HEAVY AND COLD (TBH ~ My) CAN EMIT HERMAL CFT STATES WHICH ARE ACCESSIBLE TO IT IN THE DEEP IR!

BULK SOLUTIONS TAKE THIS AUTOMATICALLY INTO ACCOUNT : THUS THEY DESCRIBE BLACK HOLE CORRECTED BY THE BACKREACTION FROM THE CFT HAWEING RADIATION !

ESCAPING HAWKING RAPIATION

JA CO

CFT CLOUP INTERIOR : FRW ALA OPPENHEIMER - SHYPER  $\frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = \frac{M}{R^3} + \frac{h}{g_{\#}} \frac{M^2}{R^6 M_g^2} \frac{Shilowithelal}{Shilowithelal}$ EXTERIOR : FAR FIELD REGION OUTGOLS VAIDYA ds= - (1- 2M/r) due + 2dudr + 12 dS22 MATCHING : FLUX OF RADIATION THE = 2404  $\mathcal{X} = \frac{L^2}{G_4} (G_4 M)^2 = \frac{\pi g_{\pi}}{(G_4 M)^2}$ THERMAL HAWKING FLUX OF 9+ FLAVORS! But now I think this is wrong... but not for any of the reasons used to criticize the idea... Why do the numerical solutions found by

Wiseman et al actually exist?



$$\nabla^2 \Phi = 0$$
  

$$\phi = e^{\mp i\omega t} Y_{lm}(\phi, \theta) \varphi_{\omega}(r) / r$$
  

$$r_* = r + r_0 \ln(r/r_0 - 1)$$

$$\begin{split} \varphi_{\omega}'' + \left(\omega^2 - \frac{l(l+1)}{r^2}\right) \varphi_{\omega} &= 0, & \text{in Minkowski}, \\ \ddot{\varphi}_{\omega} + \left(\omega^2 - \frac{l(l+1)}{r^2} (1 - \frac{r_0}{r}) - \frac{r_0}{r^3} (1 - \frac{r_0}{r})\right) \varphi_{\omega} &= 0, & \text{in Schwarzschild}. \end{split}$$

 $\begin{array}{ll} \mbox{Near the horizon:} & \varphi_{\omega}'' + \omega^2 \varphi_{\omega} = 0 \,, & \mbox{in Minkowski} \,, \\ & \ddot{\varphi}_{\omega} + \omega^2 \varphi_{\omega} = 0 \,, & \mbox{in Schwarzschild} \,. \end{array}$ 

$$\Phi_{\omega}^{"out"}{}_{outgoing} = \frac{\bar{A}}{r} \Big[ \Big( \frac{v_H - u_M}{2r_0} \Big)^{2ir_0\omega} e^{-i\omega u_M} - \Big( \frac{v_H - v}{2r_0} \Big)^{2ir_0\omega} e^{-i\omega v} \theta(v_H - v) \Big]$$
$$\bar{A} = 1/[2\omega(2\pi)^3]^{-1/2}$$

$$\hat{\Phi}^{"out"}_{\omega}{}_{a} = \int_{\omega',b} \left\{ \alpha_{ab}^{*}(\omega,\omega') \hat{\Phi}^{"in"}_{\omega'}{}_{b} - \beta_{ab}^{*}(\omega,\omega') \hat{\Phi}^{"in"}_{\omega'}{}_{b}^{\dagger} \right\}$$
$$\langle N^{"out"}_{a} \rangle = \int_{\omega} ||\Phi^{"out"}_{\omega}{}_{a}|0_{in}\rangle||^{2} \qquad N^{"out"}_{a}(\omega) = \int_{\omega',b} |\beta_{ab}(\omega,\omega')|^{2}$$

$$\int_{\omega,\omega',b} \left( |\alpha_{ab}(\omega,\omega')|^2 - |\beta_{ab}(\omega,\omega')|^2 \right) = 1$$

$$\alpha_{o,i}(\omega,\omega') = -\frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\frac{e^{-i(\omega-\omega')v_H}}{(2r_0)^{2ir_0\omega'}}\frac{\Gamma(1+2ir_0\omega')}{(\epsilon+i\omega)^{1+2ir_0\omega'}},$$
  
$$\beta_{o,i}(\omega,\omega') = -\frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\frac{e^{-i(\omega+\omega')v_H}}{(2r_0)^{2ir_0\omega'}}\frac{\Gamma(1+2ir_0\omega')}{(\epsilon-i\omega)^{1+2ir_0\omega'}}.$$

$$N_{outgoing}^{"out"}(\omega) = \frac{1}{e^{\omega/T_{BH}} - 1}$$
$$T_{BH} = (4\pi r_0)^{-1}$$

Not all of the near horizon heat goes out; only the stuff which can get out/over the barrier does.

$$V_B = \frac{r_0}{r^3} (1 - r_0/r) + \frac{l(l+1)}{r^2} (1 - \frac{r_0}{r})$$

The nonzero angular momenta extremely strongly suppressed by the centrifugal barrier; s-wave is not! Starobinsky & Churilov; D. Page

That is what accounts for all the BH energy that goes off to infinity.

All the (infinite) rest of the near horizon "excitations" are vacuum fluctuations in thermal equilibrium, forming the black hole in the thermal state |0in>

$$\begin{aligned} \mathcal{L} &= \sum_{l=0}^{\infty} \frac{2l+1}{2\pi} \Gamma_{\omega,l} \int_{0}^{\infty} d\omega \frac{\hbar \omega}{e^{\omega/T_{BH}} - 1} \\ \dot{S} &\sim T_{BH}^{3} A \\ \tau_{BH} &\sim M/(T_{BH}^{4} A) \end{aligned}$$

 $s\mbox{-wave}$  is around 90% of black hole radiance.

Field theory knows this: Stretched Horizon!

$$S \sim \int dV S \qquad S \sim T^3 \qquad T = T_{BH} / \sqrt{g_{00}(r)}$$
  
$$dV = \sqrt{g_{rr}} dr r^2 d\Omega = 4\pi \sqrt{g_{rr}} dr r^2 \qquad g_{rr} = g_{00}^{-1} = (1 - r_0/r)^{-1}$$
  
$$S \sim \int dV S \sim T_{BH}^3 \int_{r_{UV}} dr \frac{\dot{r^4}}{(r - r_0)^2} \sim \frac{r_0}{r_{UV} - r_0}$$
  
$$r_{UV} \simeq r_0 + (M_{Pl}^2 r_0)^{-1}$$
  
proper distance  $\ell \simeq \int_{r_0}^{r_{UV}} \frac{dr}{\sqrt{1 - r_0/r}} \simeq \ell_{Pl}$   
$$T_{SH} = T_{BH} / \sqrt{1 - r_0 / r_{UV}} \simeq M_{Pl}$$

Susskind & Uglum

Completely consistent with EFT calculations of BH entropy; eg 't Hooft's brick wall story yields

$$S = -\partial F / \partial T_{BH}$$

$$F = -\frac{1}{\pi} \int_0^\infty d\omega \int_{r_{UV}}^{r_{IR}} \frac{dr}{1 - r_0/r} \int_0^{l_{max}(\omega)} \frac{dl(2l+1)}{e^{\omega/T_{BH}} - 1} \sqrt{\omega^2 - (1 - \frac{r_0}{r})} \frac{l(l+1)}{r^2}$$

Change to  $y = l(l+1)(r-r_0)/r^3$  to find leading piece!

$$F \simeq -T_{BH}^4 \int_{r_{UV}}^{r_{IR}} dr \frac{r^4}{(r-r_0)^2}$$

This is just the stretched horizon formula, fully consistent with the full quantum theory calculation (ie covariant regularization, renormalization of Planck scale etc)!



The key ingredient is to ensure that the quantum state of the system out of the black hole is the ingoing vacuum,  $|0_{in}\rangle$ 

No problem with an EFT with few dofs; Lorentz symmetry and gravity decoupling limit ensure it is in the state  $|0_{in}\rangle$ 

This is why there is NO "transplanckian problem"

Plot thickens with many species; by equivalence principle, in weak (EFT) coupling,

$$S \sim N \frac{r_0}{r_{UV} - r_0} \sim M_{Pl}^2 A$$

$$r_{UV} \simeq r_0 + N(M_{Pl}^2 r_0)^{-1}$$

$$\ell_{SH} \simeq \sqrt{N} \ell_{Pl}$$



### Sorkin et al; Jacobson

Now we suddenly seem to have a problem: we need to cut off the low energy EFT much below the Planck scale in order to NOT overshoot the Bekenstein-Hawking BH entropy. The cutoff is given by the Tolman temperature

$$\mu \sim M_{Pl}/\sqrt{N}$$

at the multispecies stretched horizon,

$$\ell_{SH} \simeq \sqrt{N} \ell_{Pl}$$

Insisting that the IR modes are all in thermal equilibrium and weakly coupled near the BH, and that this description goes on till the naive UV cutoff, will overshoot the entropy. Examples: induced gravity, theories with many species, etc. Generally, one expects that the weak coupling assumption is bad since at high boosts (ie energies) gravity is strong, and induces strong field theory couplings The crux of the problem: cannot decide how to evolve the quantum state out of the black hole; description fails much below the Planck scale



Could open all kinds of questions... bottomline: the naive weakly coupled multispecies theory in the IR can't describe BH radiance

To decide what happens with BH radiance we do need a UV completion; RS2 is an example of this! In RS2, in the IR, we have 4D gravity coupled to SU(N) strongly coupled YM, with UV cutoff

 $\mu = L^{-1}$ 

and where 4D gravity is induced,  $M_{PL}^2 \sim N \mu^2$ 

This description valid down to L; below L we need UV completion; in RS2 it is known: bulk gravity on  $AdS_5 \times S^5$ At distances below the string scale,  $\ell_5 = M_5^{-1}$ , where

$$M_{5}^{3} = M_{Pl}^{2}/L$$

this transitions to the full IOD string theory

In the IR, one sees all the N CFT dofs; for example one can calculate the correction to the Newton's potential between two particles with large 4D separation; both bulk and 4D dual pictures yield

$$\Delta V \sim -G_N m M L^2 / r^3 \simeq \frac{N}{M_{Pl}^2 r^2} V_N$$

So there is all N modes; the theory is gapless so no confinement; what does a black hole radiate in this case? Garriga & Tanaka; Katz, Giddings, Randall

The trick: IF WE ASSUME all of the N modes are equilibrated, we can only trust the IR theory down to

$$\ell_{SH} \sim \sqrt{N} \ell_{Pl} = L = \mu^{-1}$$

But this is just the UV cutoff! So the IR theory does NOT extend all the way to the black hole! Multispecies stretched horizon is really sitting on the UV cutoff! So use the UV completion. We cannot guarantee that the CFT state outside of the BH is really |0in>, which we would need to do to have the system under control. But: now at least we DO have the UV completion: bulk gravity on  $AdS_5 \times S^5$ 

So: we have a BIG black hole, with 4D horizon >> L. In the bulk it is warped, a "pancake" with

$$r_{bulk} \simeq L \ln(r_0/L) > L$$

Chamblin, Hawking & Reall

The bulk, on the other hand, is at these scales like one flat extra D of size L - warping is negligible once we consider the distances less than L from the black hole horizon, in any direction; and we need to get that close to CHECK what the state of the BH is close to the horizon.



So we have a 4D theory with many - gapped - continua of states. BH is big - so cold:

$$T_{BH} \sim 1/r_0 \ll L^{-1}$$

The modes which wrap on 5-sphere are gapped by  $L^{-1}$  so they cannot be emitted

The zero modes on the sphere can wiggle in the bulk. If they wiggle a lot in 4D they look massive. If their mass is above  $T_{BH}$  they cannot be emitted.

That leaves the modes gapped by less than  $T_{BH}$ ; to understand their contribution, put in an IR regulator brane: but a a distance R>>L.



With the IR regulator the gapped continua turn into gapped KK towers. The interstate gap is

$$m_{gap} \sim L^{-1} e^{-R/L}$$

and the 4D state masses are m  $\sim$  n m<sub>gap</sub>

Only the states lighter than  $T_{BH}$  can contribute to radiance; so

$$N_{light} \simeq \frac{T_{BH}}{m_{gap}} \simeq T_{BH} L e^{R/L}$$

But these modes are supressed by a LARGE gray body factor: the brane volcano! Their wavefunctions obey

$$\frac{d^2\psi}{dz^2} + (m^2 - \frac{\kappa}{(z+L)^2})\psi = \sigma\delta(z)\psi$$

So for light modes close to the brane (and BH!),

$$\begin{split} \psi &= \{ [m(z+L)]^{p_+} + (mL)^{\gamma} [m(z+L)]^{p_-} \} / \sqrt{z_{IR}} \\ z &= Le^{R/L} \qquad p_{\pm} = (1 \pm \sqrt{1+4\kappa}) / 2 \\ \gamma &= p_+ - p_- \quad \text{or} \quad \gamma = 2 \quad \text{(for spin 2)} \end{split}$$

The probability to find these modes close to the BH is

$$\sim L|\psi|^2 \lesssim (mL)^{2p}L/z_{IR}$$

where  $p = p_+$  or  $p_+ -2$ ; so they cannot be emitted efficiently:

$$N_{light} \times L |\psi|^2 \lesssim (T_{BH}L)^{2p+1}$$

because 2p+1 is greater or equal to 2; this is just the Breitenlohner-Freedman bound!

$$\mathcal{L}_{BH} \sim (T_{BH}L)^2 \mathcal{L}_{hawking} \sim (\frac{T_{BH}}{M_{Pl}})^2 N \mathcal{L}_{hawking}$$

When we use the consistent UV description valid close to the BH horizon we see that most of the `partons' of the IR CFT states DO NOT escape thanks to the bulk gray body factors. They are like Unruh modes of higher angular momenta - a `near-horizon' heat that defines the BH vacuum and builds its entropy but cannot escape away. These modes do not carry energy away. This is very much like in the case of ADD black holes, which do not radiate extensively into the bulk because of the centrifugal barrier. Here there is an additional barrier: the RS2 volcano.

The suppression is even stronger than O(1/N). In itself it is directly a consequence of strong coupling per se. After all below the cutoff CFT is conformal, so the coupling is fixed (big - but it allows Newtonian corrections etc). Strong coupling merely facilitates the resolution of the problem by providing a bulk dual, along with large L. The suppression, from IR, ought to follow from locality - once partons "condense" into CFT modes they are too big and fluffy to fit on the BH - as higher angular momentum waves in plain vanilla 4D BHs

An (interesting) musings: any way to change this? Or to ask deeper questions about gravity? I.e. from the 4D EFT perspective, gravity need not be induced. So cutoff and Planck scale do not need to be linked by the # of IR modes. Can one play games where the radiance rate can be enhanced in a calculable manner?

# HAPPY BIRTHDAY MISAO!