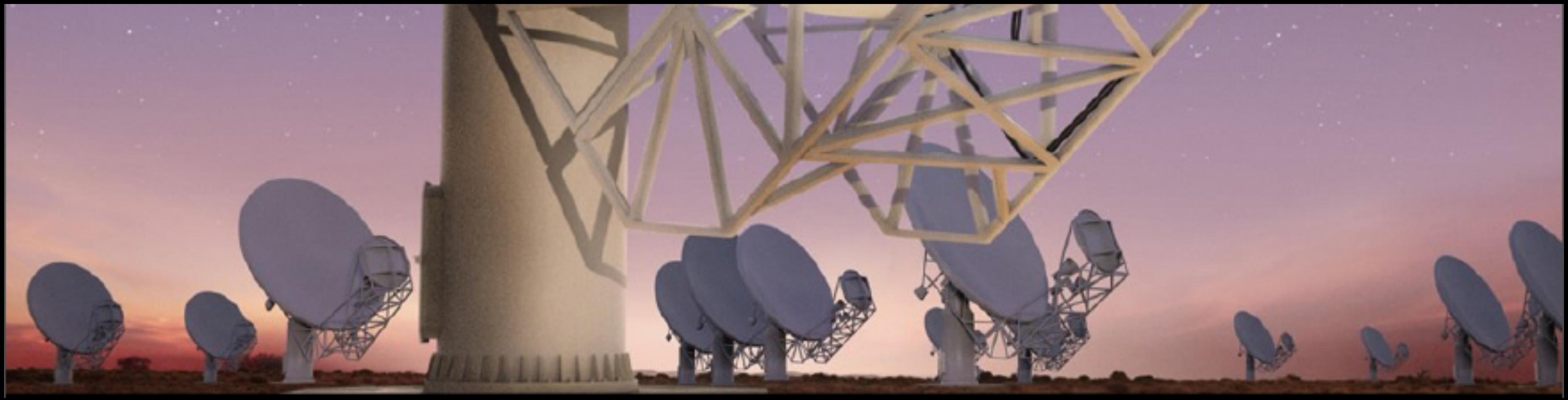
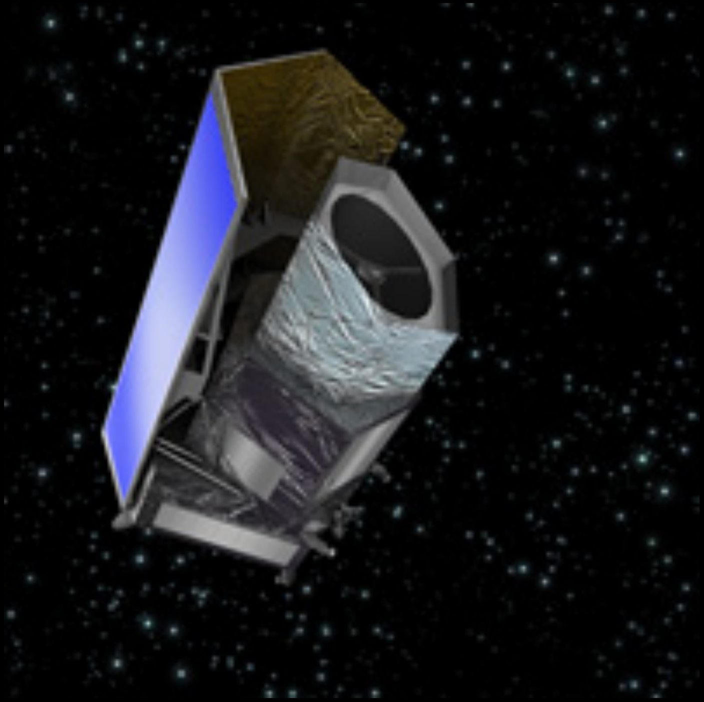


GENERAL RELATIVITY IN GALAXY SURVEYS



Kyoto, December 2012

Roy Maartens

Pioneering paper:

Mon. Not. R. astr. Soc. (1987) **228,** 653–669

The magnitude–redshift relation in a perturbed Friedmann universe

Misao Sasaki *Research Institute for Theoretical Physics, Hiroshima University, Takehara, Hiroshima 725, Japan*

$$\frac{\Delta d_{\mathrm{L}}(z,\gamma^i)}{d_{\mathrm{L}}(z)} = \frac{1}{\lambda_{\mathrm{s}}}\int_0^{\lambda_{\mathrm{s}}} d\lambda\left[(\lambda-\lambda_{\mathrm{s}})\lambda\overset{(3)}{\Delta}\Psi+2(\Psi-\Psi_0)\right]+\frac{1}{2}\left(\frac{\eta_0}{\lambda_{\mathrm{s}}}-3\right)(\Psi_{\mathrm{s}}-\Psi_0)$$
$$+\frac{1}{6}\left(\frac{\eta_0}{\lambda_{\mathrm{s}}}-3\right)[\eta_{\mathrm{s}}(\psi_{|i}\gamma^i)_{\mathrm{s}}-\eta_0(\psi_{|i}\gamma^i)_0]-\frac{1}{3}\eta_0(\psi_{|i}\gamma^i)_0+\Psi_0+\frac{3}{\eta_0}\delta_*\eta_0,$$

1106.3999 (PRD)

Disentangling non-Gaussianity, bias and GR effects in the galaxy distribution

Marco Bruni[1], Robert Crittenden[1], Kazuya Koyama[1], Roy Maartens[2,1], Cyril Pitrou[1], David Wands[1]

1205.5221 (JCAP)

Beyond the plane-parallel and Newtonian approach: Wide-angle redshift distortions and convergence in general relativity

Daniele Bertacca[a], Roy Maartens[a,b], Alvise Raccanelli[c,d], Chris Clarkson[e]

1206.0732

Relativistic corrections and non-Gaussianity in radio continuum surveys

Roy Maartens[1,2], Gong-Bo Zhao[2,3], David Bacon[2], Kazuya Koyama[2], Alvise Raccanelli[4,5]

1209.3142

Anti-lensing: the bright side of voids

Krzysztof Bolejko[1], Chris Clarkson[2], Roy Maartens[3,4], David Bacon[3], Nikolai Meures[4] and Emma Beynon[4]

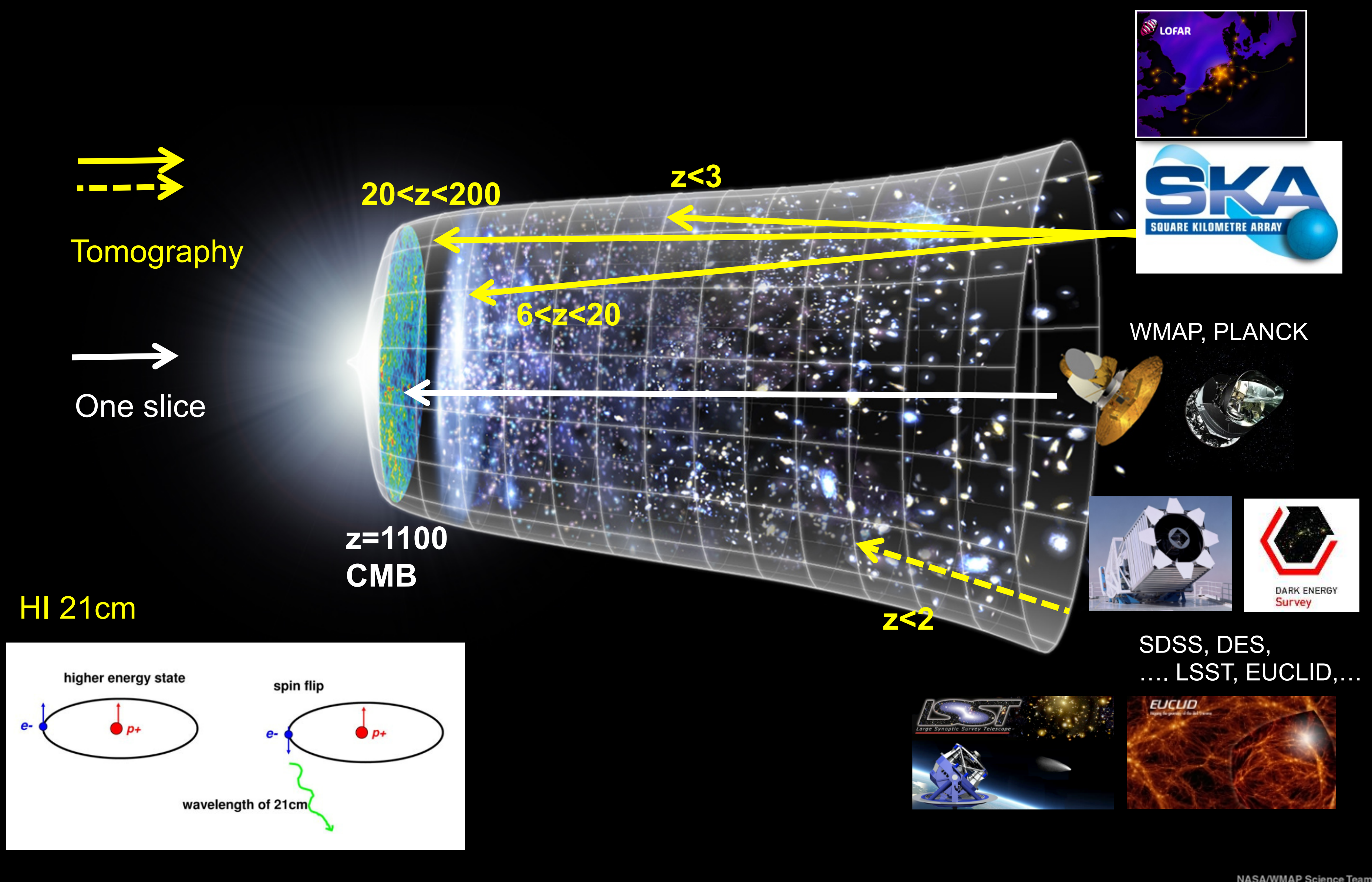
The next frontier of cosmology

Large-volume surveys of the matter distribution

* to maintain 'precision cosmology' on the largest scales (which are linear):
  - precision constraints on model parameters
  - tests of Dark Energy/ Modified Gravity models

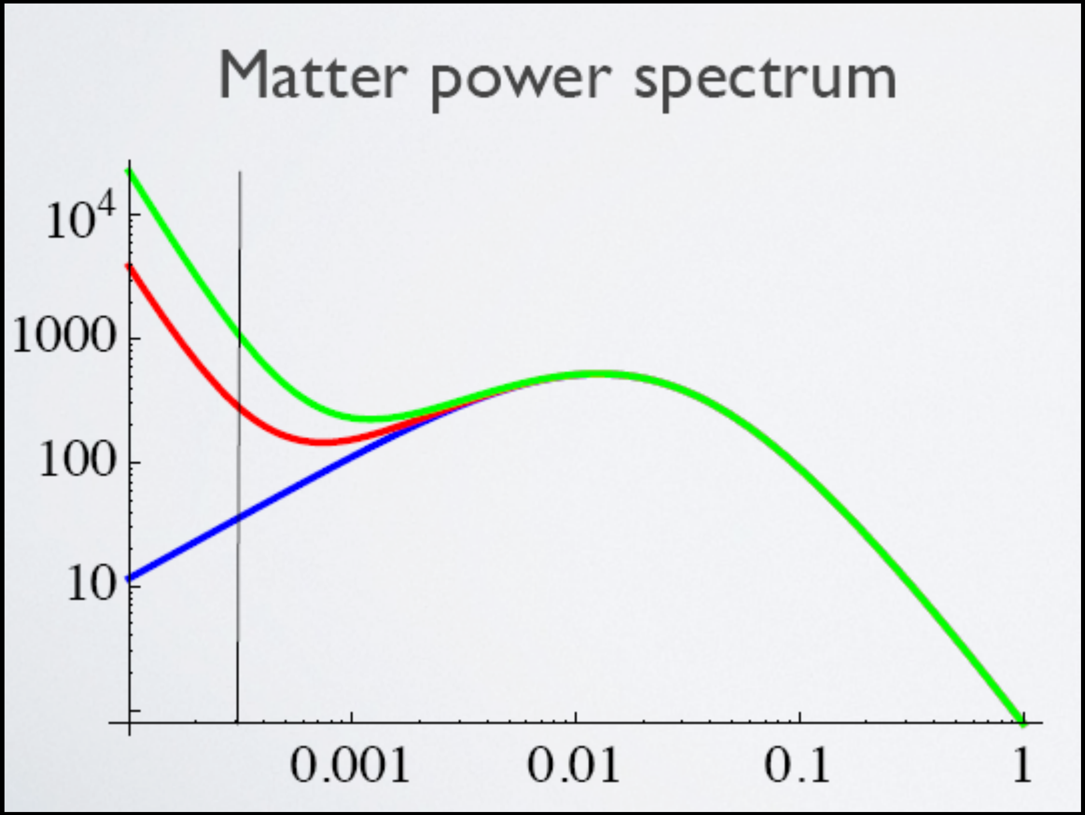
* to extract the primordial non-Gaussianity (better than CMB?)

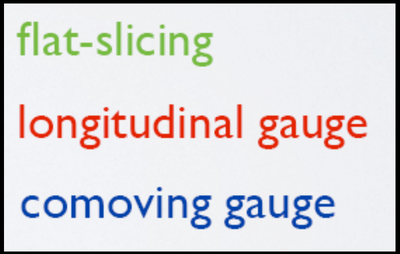
* to test General Relativity itself on the largest scales


Tomography
One slice
HI 21cm
higher energy state
spin flip
e-
p+
e-
p+
wavelength of 21cm
20<z<200
z<3
6<z<20
z=1100
CMB
z<2
LOFAR
SKA
SQUARE KILOMETRE ARRAY
WMAP, PLANCK
DARK ENERGY
Survey
SDSS, DES,
…. LSST, EUCLID,…
LSST
Large Synoptic Survey Telescope
EUCLID
NASA/WMAP Science Team

GR effects on the power spectrum

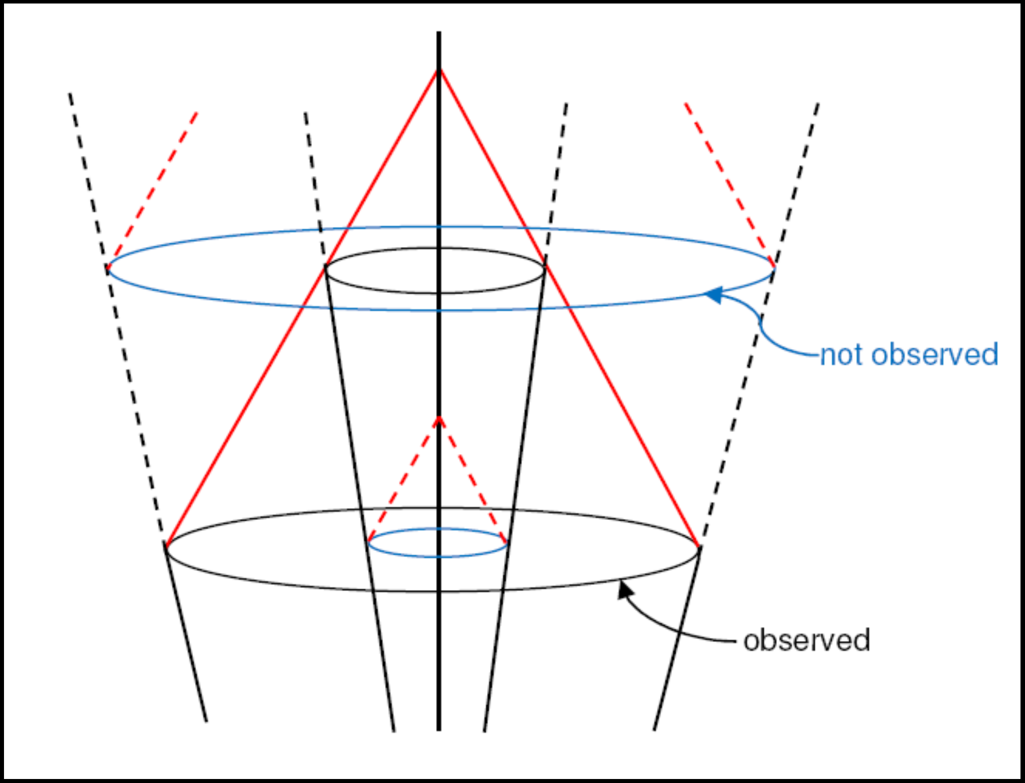
- On scales near and above the Hubble scale, H^{-1}, different gauges for δ give different answers.
- Gauge-invariance does not solve the problem – δ^{GI} defined on constant time-slices is not unique.





$$H^{-1}(z) = H_0\left[\Omega_m(1+z)^3 + \Omega_\Lambda\right]^{-1/2}$$

- We observe on the past lightcone, not t =const.
- We must use the observed Δ:
  - * it is unique
  - * it is automatically gauge-invariant
- GR effects: redshift, lensing and volume distortions



δ, $P(k)$ defined on t = const

Δ, C_ℓ defined on past lightcone

Observed number $N(\boldsymbol{n},z)$ – in observed direction $\boldsymbol{n}$ and at observed redshift z.

Define the observed galaxy number overdensity:

$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \langle N \rangle(z)}{\langle N \rangle(z)}$$

Then $\rho(\mathbf{n}, z) = \dfrac{N(\mathbf{n}, z)}{V(\mathbf{n}, z)}$ and

$$z = \bar{z} + \delta z$$
$$V(z, \mathbf{n}) = \bar{V}(\bar{z}) + \delta V(z, \mathbf{n})$$

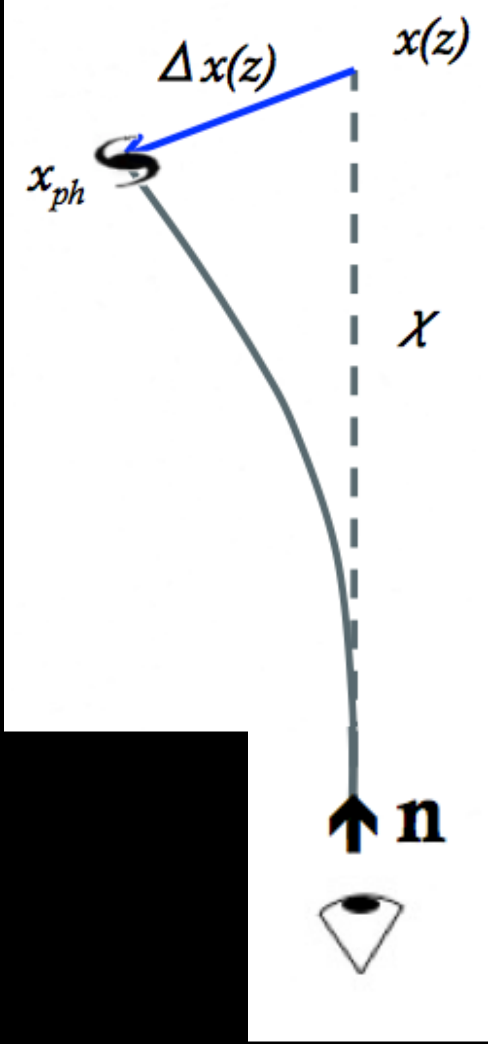
lead to

$$\Delta(\mathbf{n}, z) = \delta(\mathbf{n}, z) - 3\frac{\delta z}{1+\bar{z}} + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

Now compute perturbed geodesics and volume element:

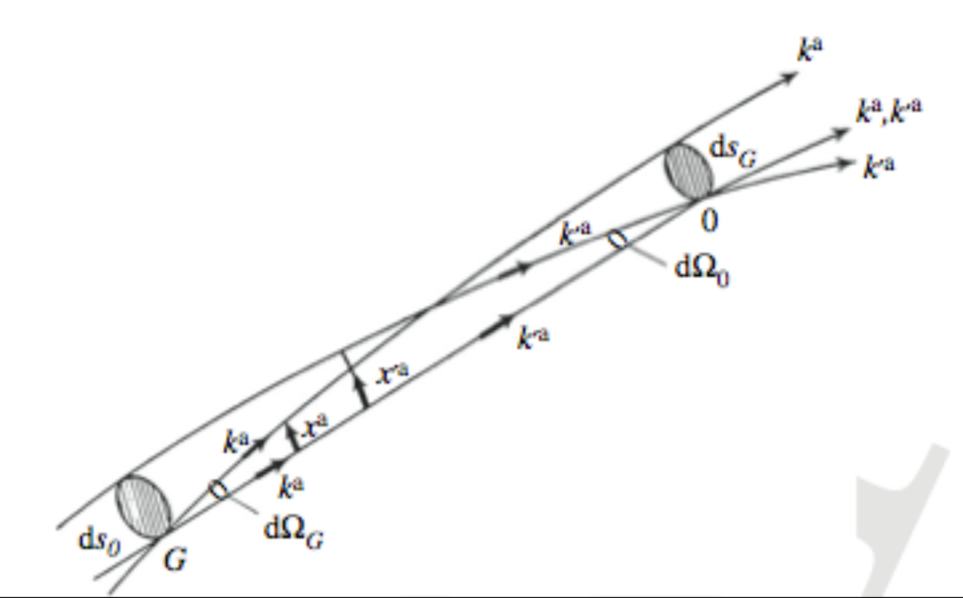
$$ds^2 = a(\eta)^2\Big[-(1+2\Phi)d\eta^2 + (1-2\Psi)d\boldsymbol{x}^2\Big].$$

longitudinal gauge



$$(\delta\theta,\ \delta\varphi) = (\theta_S - \theta_O,\ \varphi_S - \varphi_O)$$

$$\frac{\delta z}{1+\bar{z}} = \boldsymbol{n}\cdot\boldsymbol{v} - \Phi - \int^{\eta_o} d\eta(\Phi' + \Psi')$$

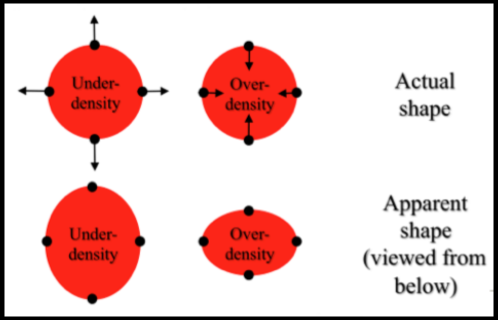


The result is $\Delta = \Delta^{\text{std}} + \Delta^{\text{GRcorr}}$

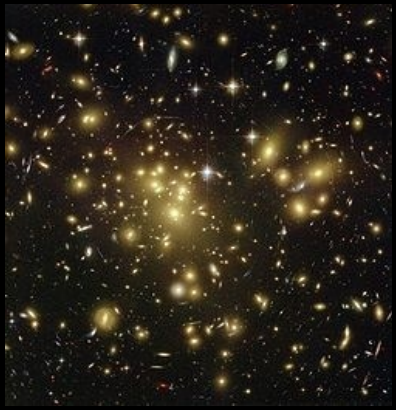
where

$$\Delta^{\text{std}} = \delta - \frac{1}{\mathcal{H}} \boldsymbol{n} \cdot \frac{\partial \boldsymbol{v}}{\partial \chi} + (5s-2)\kappa,$$

redshift space distortions



lensing convergence



$$\kappa = -\frac{1}{2}\nabla^2_{\boldsymbol{n}} \int_{\eta_o}^{\eta} d\tilde{\eta}\, \frac{(\tilde{\eta}-\eta)}{(\eta_o-\eta)(\eta_o-\tilde{\eta})}(\Phi+\Psi).$$

$$s \equiv \frac{\partial \log_{10} N(z, m < m_*)}{\partial m_*}$$

and....

The GR correction term is

$$\Delta^{\mathrm{GR\,corr}} = (A+1)\Phi + (5s-2)\Psi + \frac{1}{\mathcal{H}}\Psi' + A\int^{\eta_o} d\eta(\Phi'+\Psi') + \frac{(2-5s)}{\chi}\int^{\eta_o} d\eta(\Phi+\Psi) - A\,\boldsymbol{n}\cdot\boldsymbol{v},$$

potential term

Doppler term

with

$$A \equiv \frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2-5s}{\mathcal{H}\chi} + 5s.$$

(Yoo, Fitzpatrick, Zaldarriaga 2009; Yoo 2010)
(Bonvin, Durrer 2011; Challinor, Lewis 2011)

$$\Delta = \Delta^{\mathrm{std}} + \Delta^{\mathrm{GRcorr}}$$

The GR effects can become significant at higher *z* and then at smaller scales – since H^{-1} decreases with *z.*

Note that the redshift space distortion and lensing convergence terms are strictly also relativistic.

$$\Delta^{\rm std} = \delta - \frac{1}{\mathcal{H}} \boldsymbol{n} \cdot \frac{\partial \boldsymbol{v}}{\partial \chi} + (5s - 2)\kappa,$$

They are often omitted in the correlation function, but their effects can be significant.

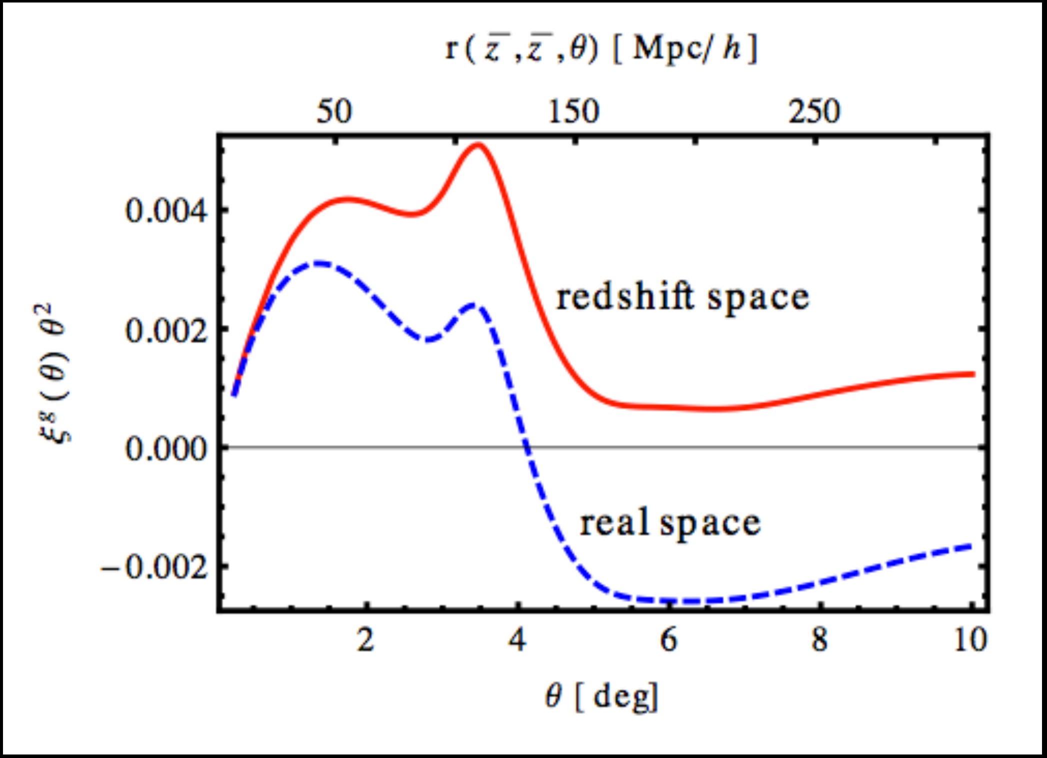
Example:

Transverse correlation of

$$\delta - \frac{1}{\mathcal{H}} \boldsymbol{n} \cdot \frac{\partial \boldsymbol{v}}{\partial \chi}$$

showing transverse BAO bump

(Montanari, Durrer 2012)



Power spectra and observations

P(k) is not observable – and we must assume a model (eg LCDM) in order to relate it to observations.

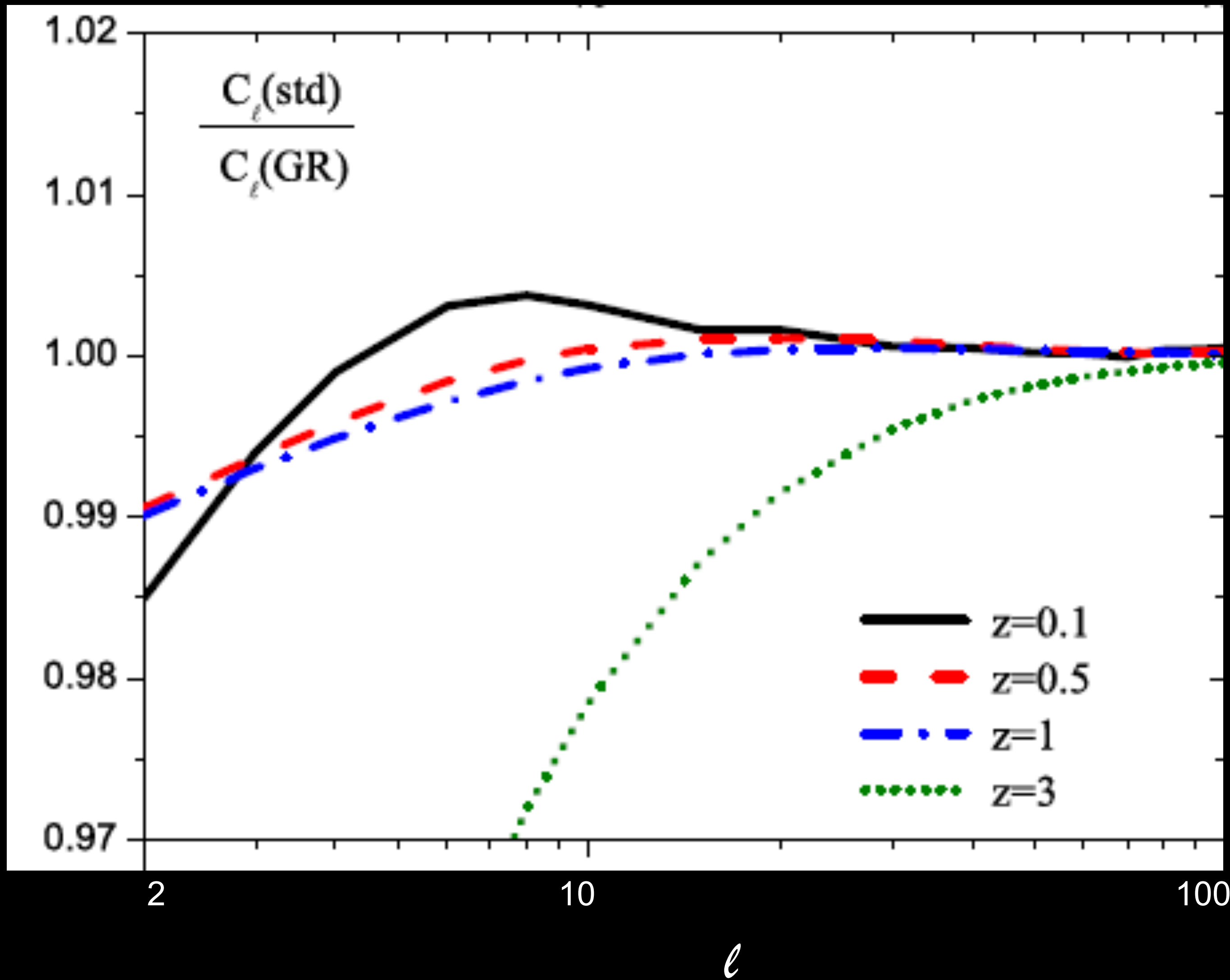
The lightcone correlation functions / angular power spectra are observable and do not require an assumed model to test against observations.

$$\xi(\mathbf{n}_1, \mathbf{n}_2, z_1, z_2) = \langle \Delta(\mathbf{n}_1, z_1)\Delta(\mathbf{n}_2, z_2)\rangle = \sum_{\ell,m} C_\ell(z_1, z_2) Y_{\ell m}(\mathbf{n}_1) Y^*_{\ell m}(\mathbf{n}_2)$$

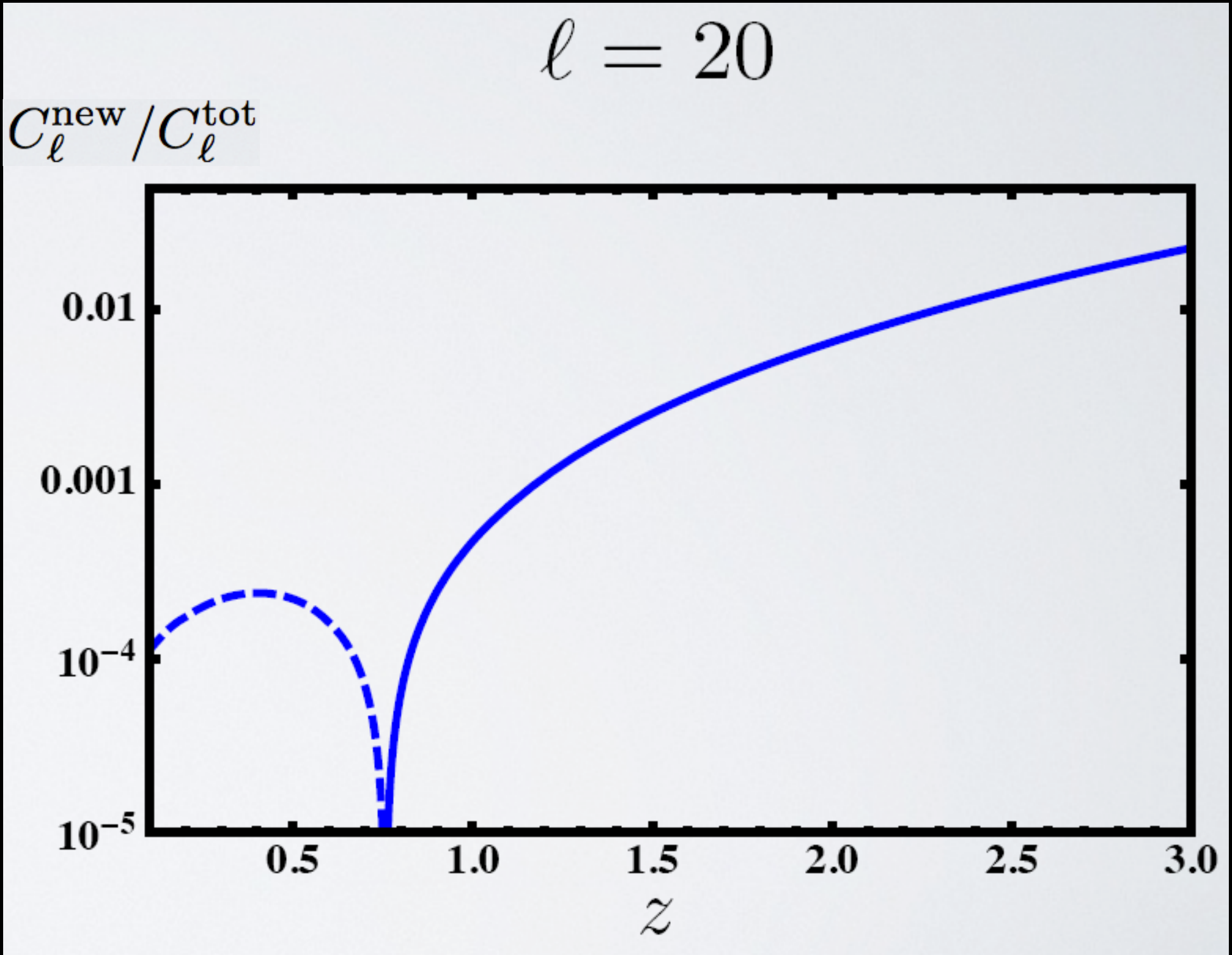
Transverse case $z_1 = z_2 = z$:

$$\begin{aligned} \Delta(\boldsymbol{n}, z) &= \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\boldsymbol{n}), \\ a_{\ell m}(z) &= \int d\Omega_{\boldsymbol{n}} Y^*_{\ell m}(\boldsymbol{n})\Delta(\boldsymbol{n}, z), \quad C_\ell(z) = \langle |a_{\ell m}(z)|^2 \rangle \end{aligned}$$

$C_\ell(z)$



(Bruni et al 2011)

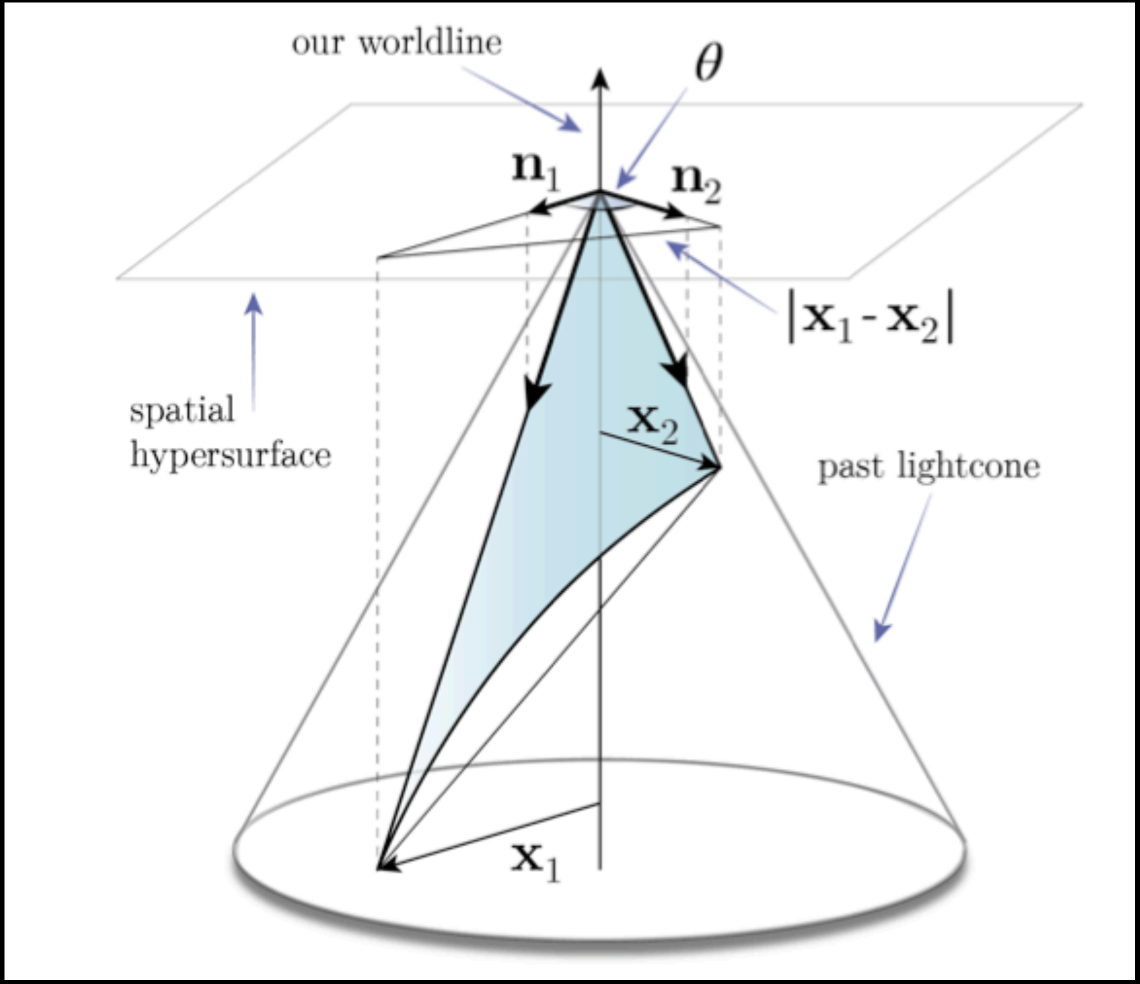


(Bonvin, Durrer 2011)

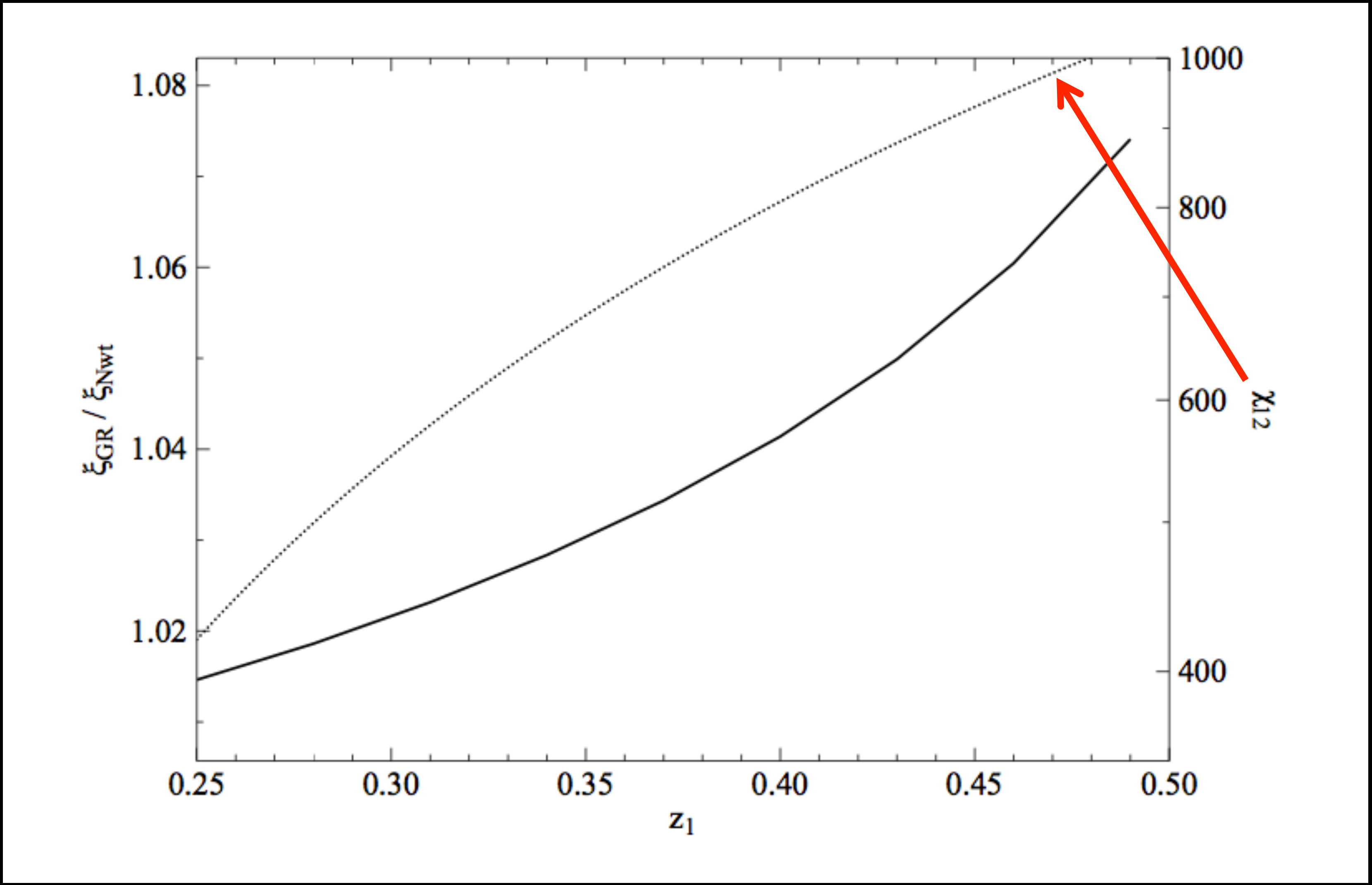
Correlation functions

$$\xi(\mathbf{n}_1, \mathbf{n}_2, z_1, z_2) = \langle \Delta(\mathbf{n}_1, z_1)\Delta(\mathbf{n}_2, z_2)\rangle$$

GR corrections for wide-angles and large redshift difference

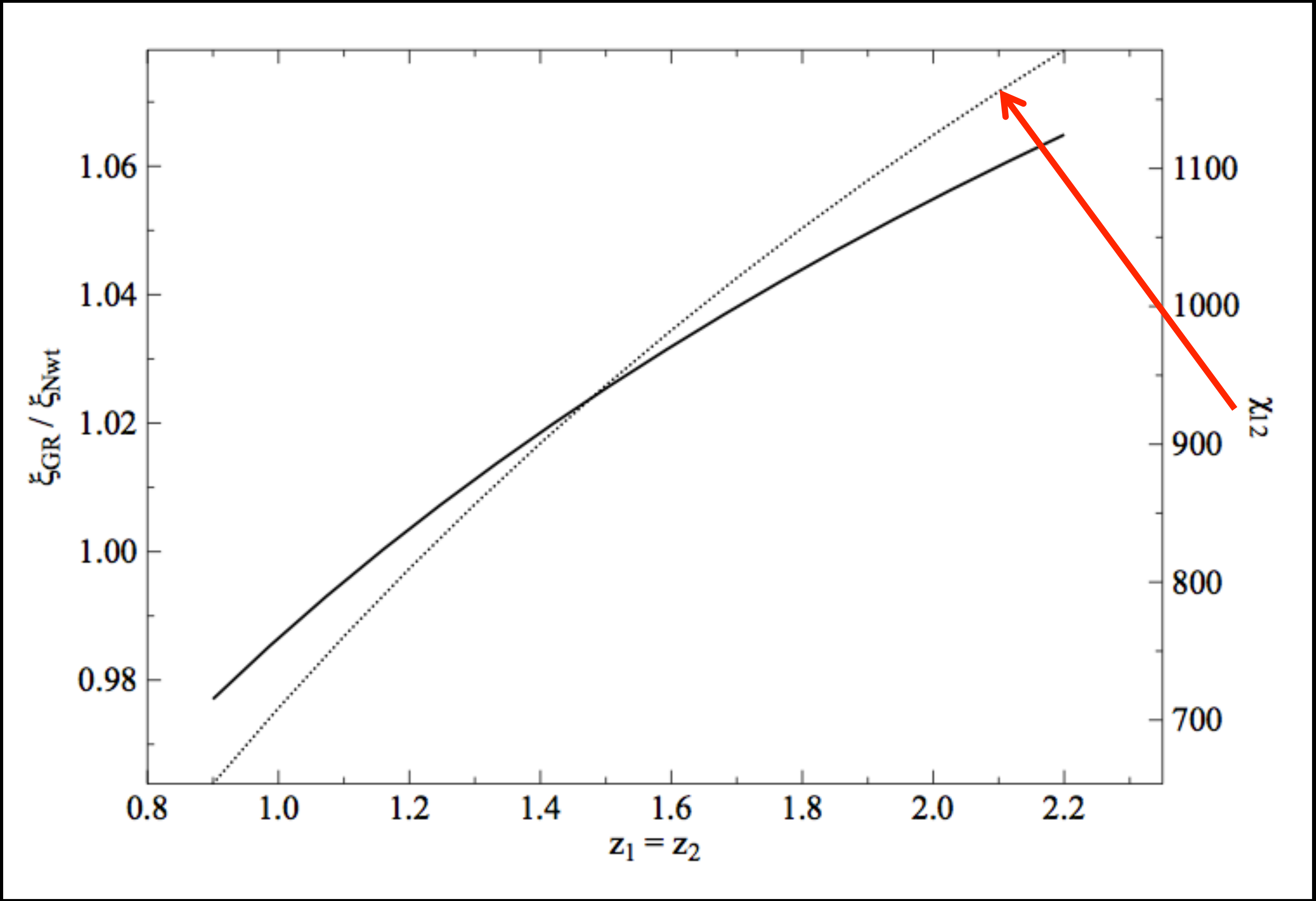


Radial $\xi(z_1, z_2 = 0.1, \theta = 0.1 \text{ rad})$ (local terms)



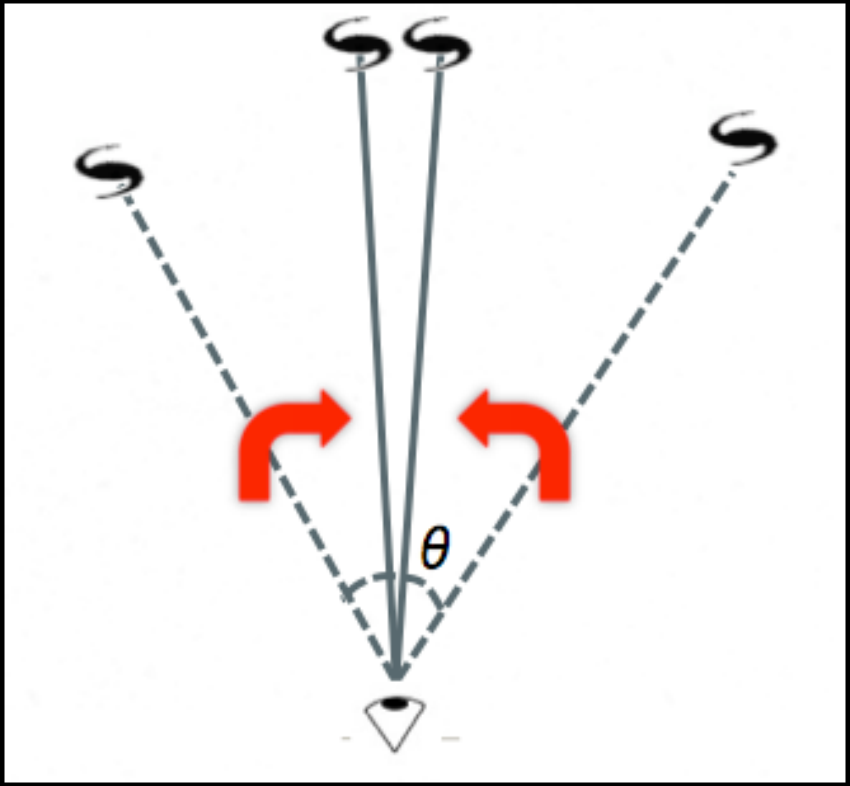
(Bertacca, RM et al 2012)

Transverse $\xi(z_1 = z_2, \theta = 0.3 \text{ rad})$ (local terms)

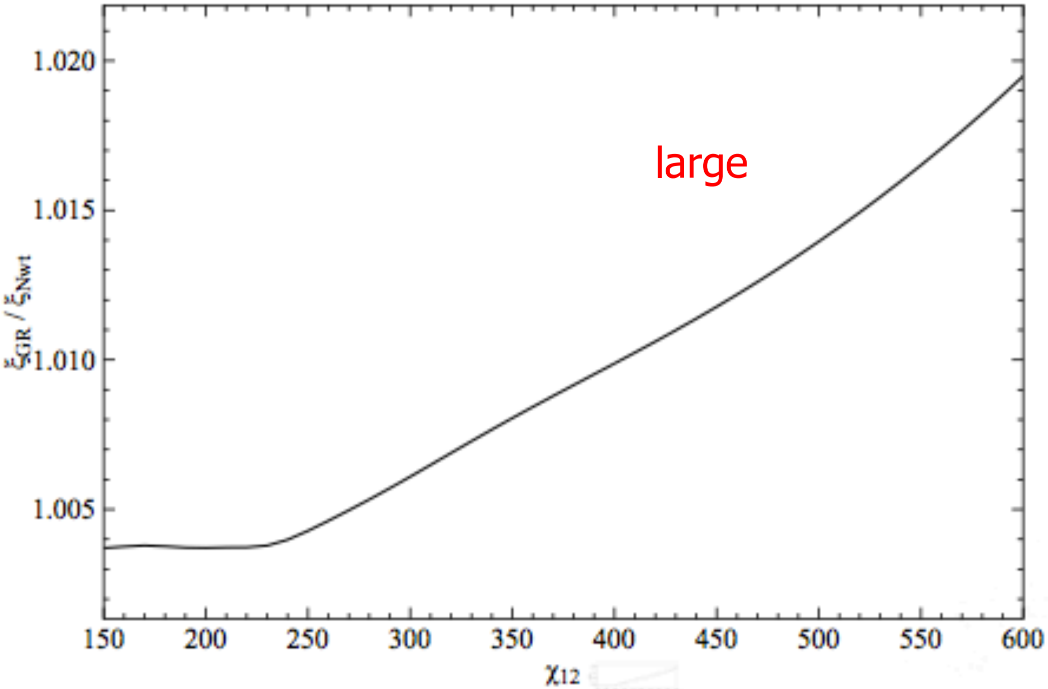


Flat-sky (plane-parallel) limit:

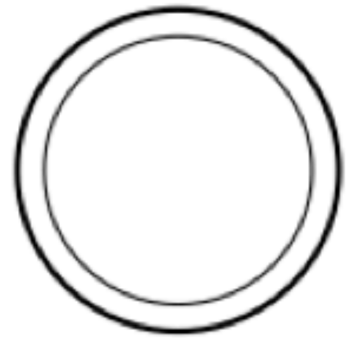
as in the standard Kaiser formulation.



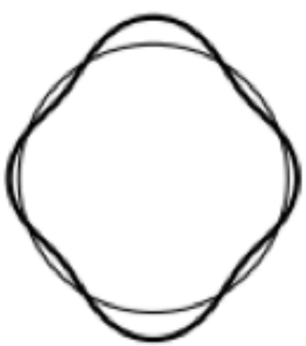
What are the GR corrections to the Newtonian Kaiser case?

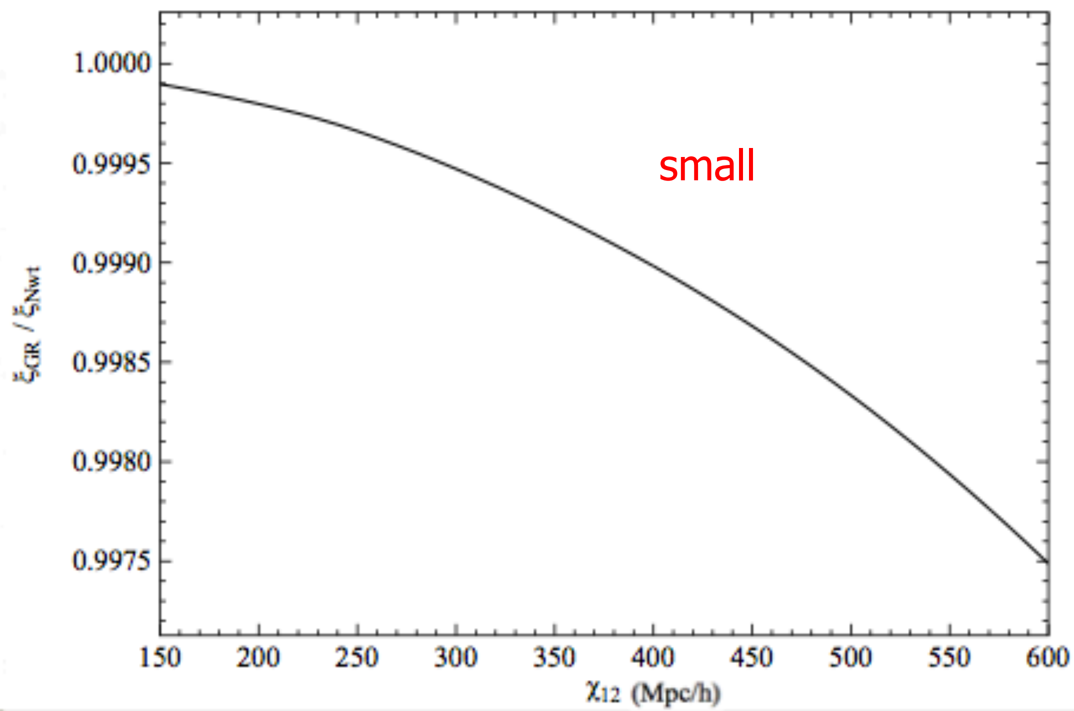


The GR corrections to the Newtonian plane-parallel limit of ξ monopole



The GR corrections to the Newtonian plane-parallel limit of ξ quadrupole





Bias and GR

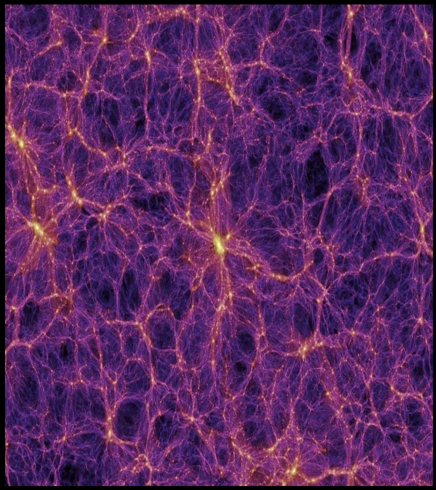
Simple scale-independent bias model for linear scales:

$$\delta(k,a) = b(a)\delta_m(k,a)$$

Gauge-dependent – only meaningful on sub-Hubble scales.
What is the physical frame that this law holds in?
The comoving (rest) frame:

(Wands, Slosar 2009)

$$\delta_{\rm com} = b\delta_{m,\rm com}$$
$$\delta_{\rm com} = \delta - 3aHv, \quad v = v_m$$
$$\delta = b\delta_m - 3(b-1)aHv$$



Note that GR Poisson is

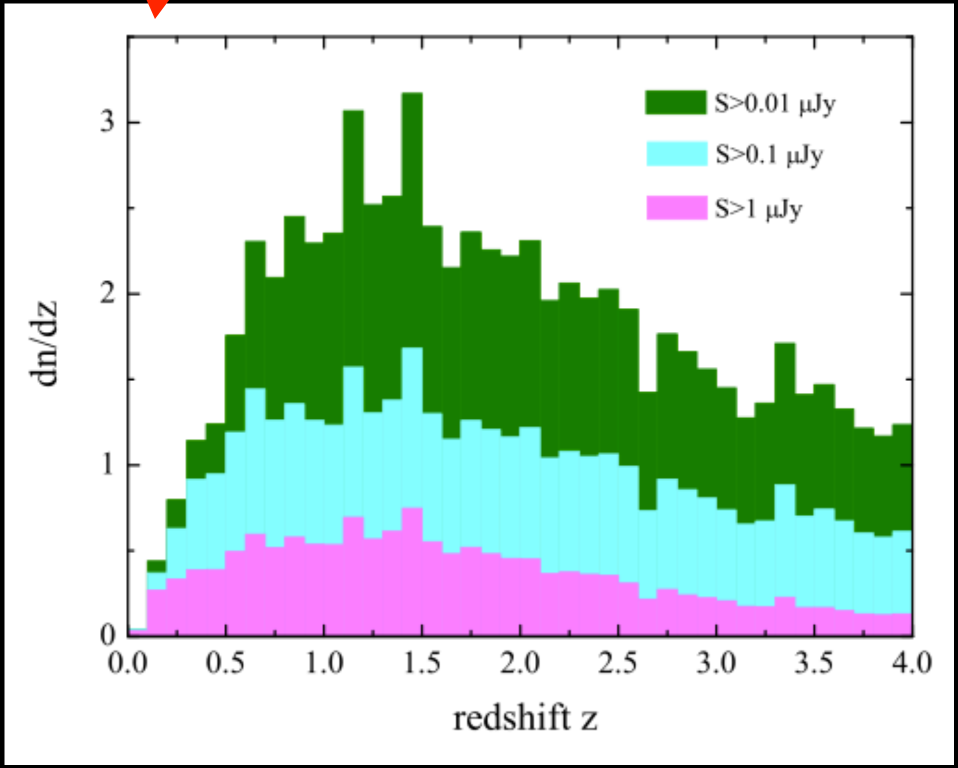
$$\nabla^2\Phi = 4\pi G a^2 \rho \delta_{\rm com}$$

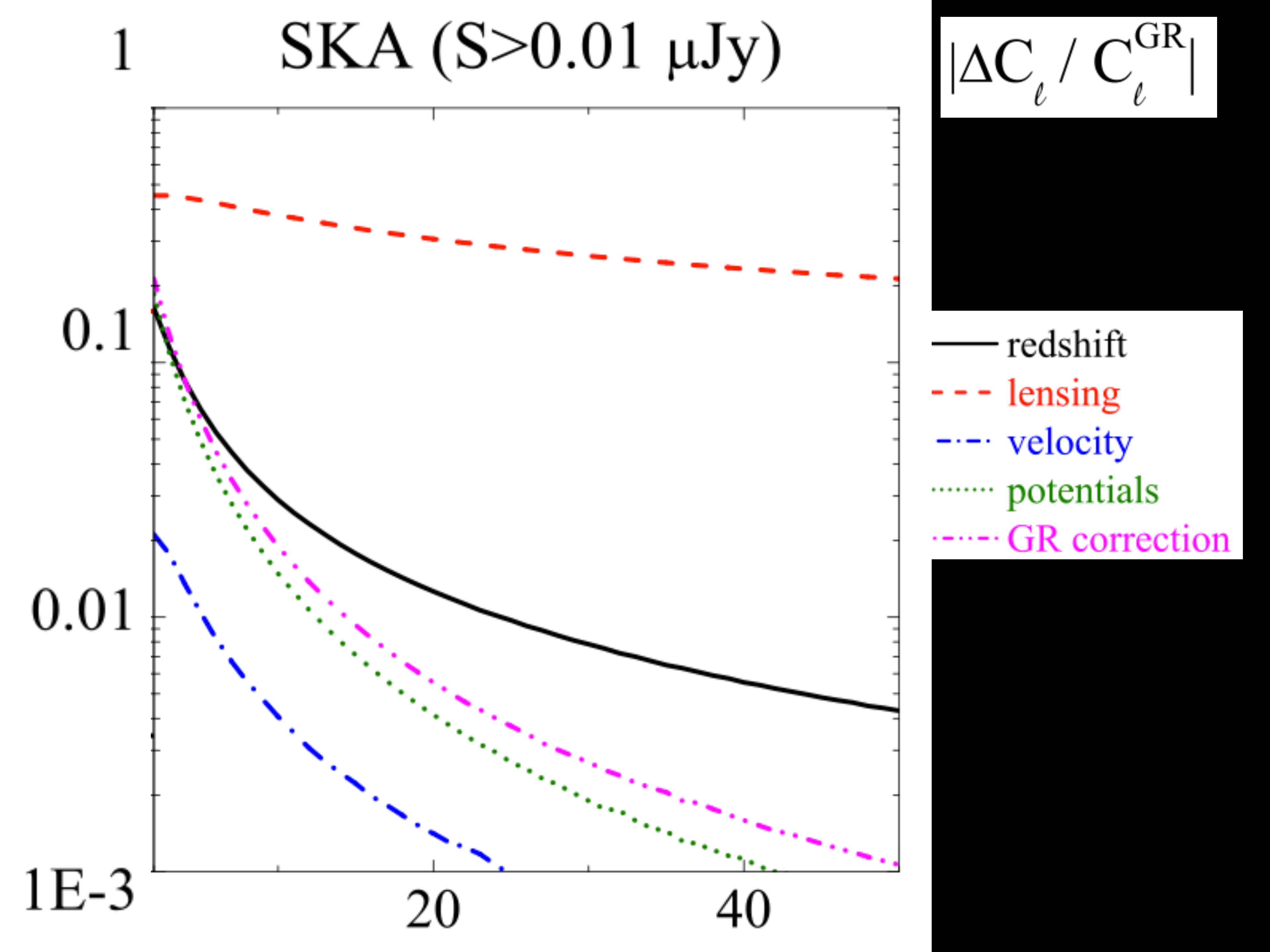
SKA radio continuum surveys

Integrated signal from all radio sources – no tomography

$$a_{\ell m} = \int d\Omega_{\boldsymbol{n}}\, dz\, Y^*_{\ell m}(\boldsymbol{n}) W(z) \Delta(\boldsymbol{n}, z)$$

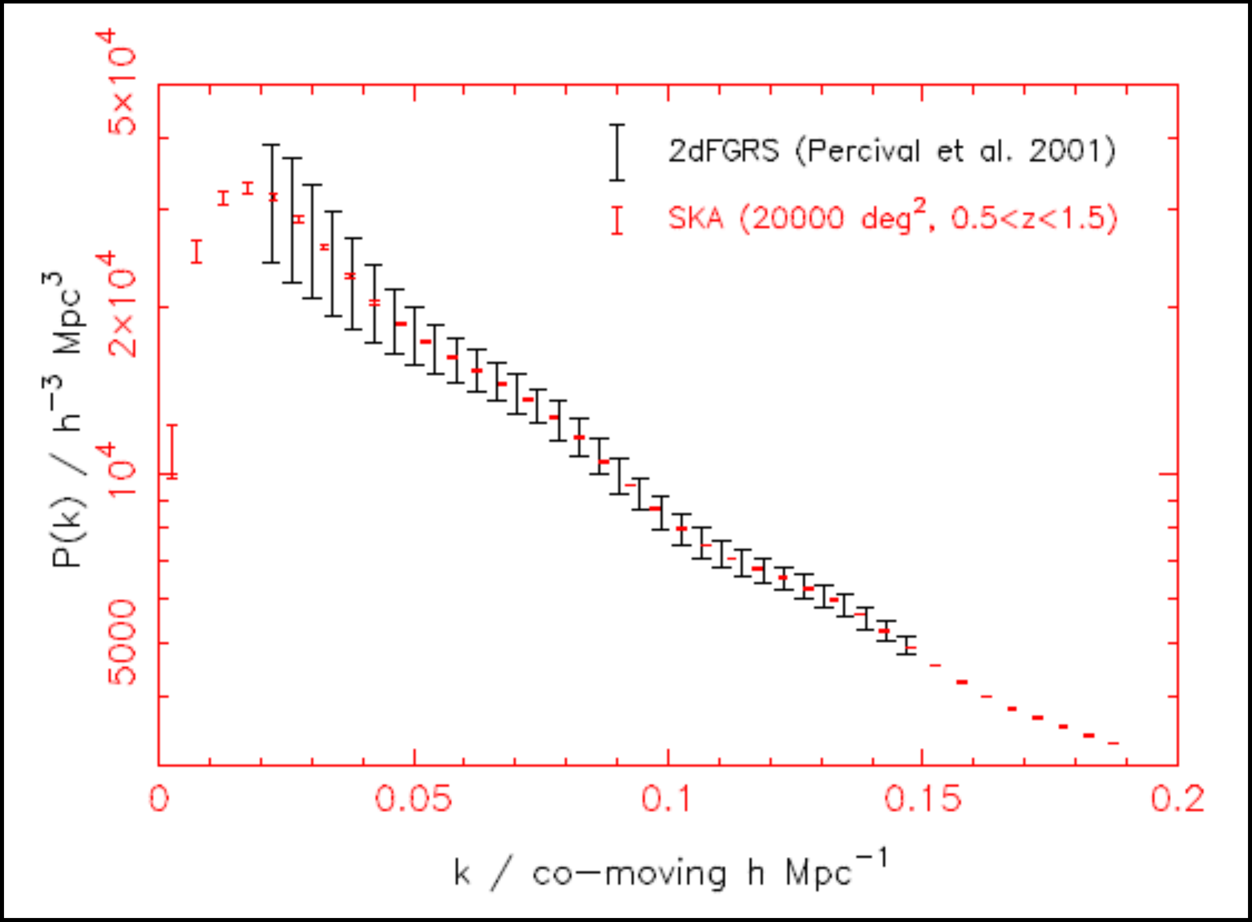
(RM et al 2012)




SKA (S>0.01 μJy)
|ΔC_ℓ / C_ℓ^GR|
1
0.1
0.01
1E-3
20
40
redshift
lensing
velocity
potentials
GR correction

SKA HI surveys: power spectrum

~ billion galaxies, z ~ 1-2, spectroscopic,huge volume.
Previous work on SKA HI surveys, using Newtonian analysis
(and no lensing or redshift space distortions)

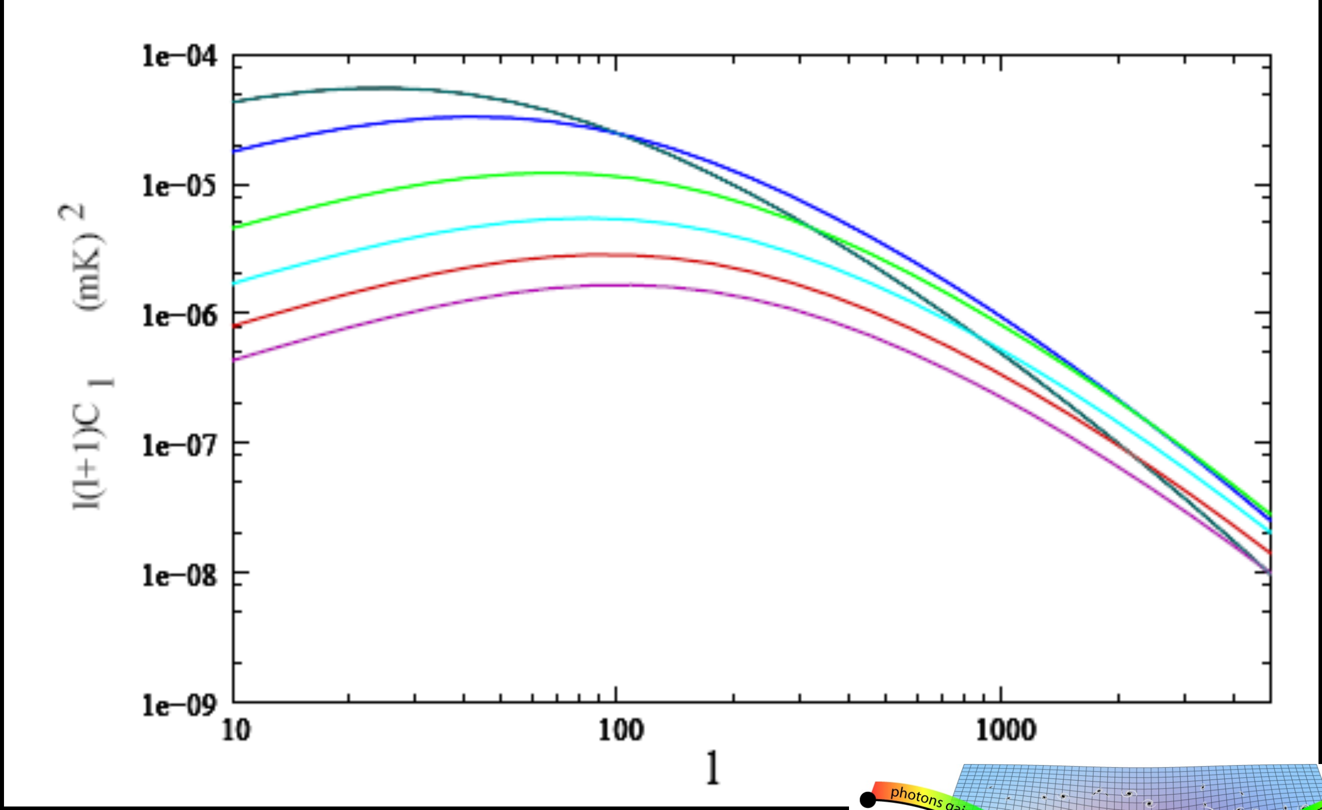


(Blake et al 2004)

Late ISW

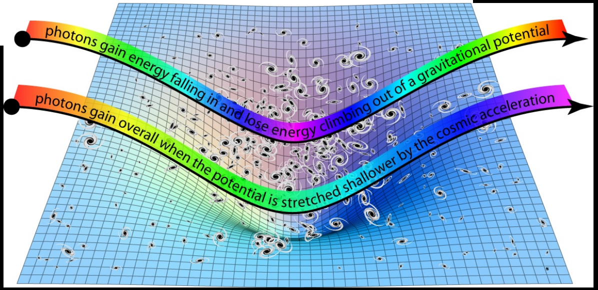
Previous work on SKA HI correlated with the CMB (WMAP):
Newtonian analysis of HI galaxy distribution

z=0.5
z=1
z=2
z=3
z=4
z=5



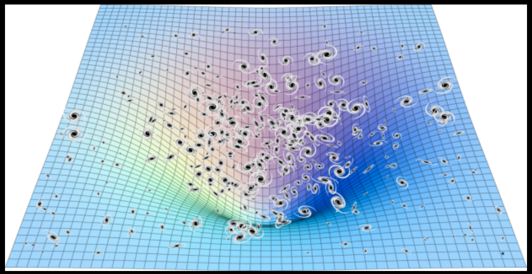
(Sarkar, Datta, Bharadwaj 2008)

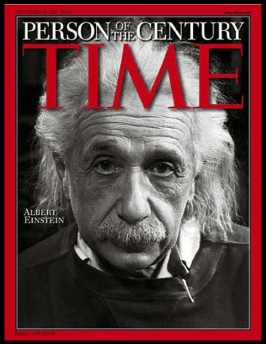
$$\frac{\Delta T}{T}(\vec{n}) = \int_{\mathrm{ray}\ \vec{n}} \left(\Phi' + \Psi'\right) d\eta$$



Message

Cosmology is not Newtonian gravity + expanding background
 - even for the galaxy distribution.





Proper GR analysis of the observed galaxy distribution shows corrections O(1 – 10%) on large scales.

However, cosmic variance also grows on large scales
 - threatens to submerge the GR corrections.

Multiple-tracer method can solve this

(Seljak 2009; Yoo et al 2012)

Future work

GR corrections are small but could affect precision cosmology.

1. Observability of GR corrections:

   Multiple tracers to beat down cosmic variance

2. Predictions for SKA HI surveys

   Tomography will improve GR signal

3. Synergy of SKA + EUCLID