Revisiting perturbations of a scalar field in an anisotropic universe

Masato Minamitsuji (Kyoto University)

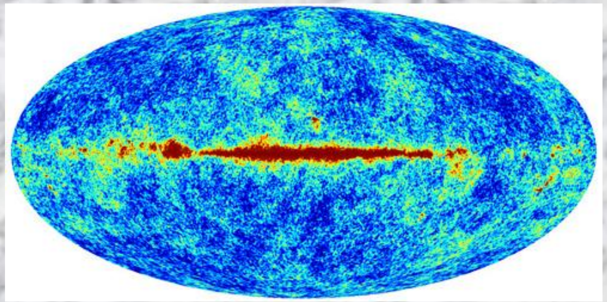
with Hyeong-Chan Kim
(Korea National University of Transportation)

Phys.Rev. D81 (2010) 083517, JCAP 1103 (2011) 038, arXiv: 1211.1132[gr-qc]

Introduction

Inflation and observations

- The predictions from inflation are consistent with observations, such as CMB



- The leading order spectral features of quantum fluctuations, such as
  scale-invariance, Gaussianity and statistical isotropy,
  respect the symmetries of the de Sitter spacetime.

In reality, the inflationary universe should not be exactly de Sitter.

How the de Sitter symmetries are brokne affects the spectral features as
spectral tilt, non-Gaussianities, statistical anisotropy,
which will be crucial to test models with observations.

implying a rotational symmetry breaking effects
in the very early universe.

Anomalies indicating the statistical anisotropy have been reported, though statistical significance is still uncertain.

See .e.g., Copi, Huterer, Schwarz and Starkman , arXiv: 1004.5602 (10)

⇒ However, many models suffer from either instabilities, fine-tuning, or naturalness problems.

Himmetoglu, Contaldi and Peloso (09), Esposito-Farese, Pitrou and Uzan (10)

- The goal of our study is to present more accurate formula for the spectra of the cosmological perturbations in a cosmic isotropization model.

Gumrukcuoglu, Contaldi and Peloso (07), Pitrou, Pereira and Uzan (08), Kim and Minamitsuji (10,11), Dey and Paban (11)

$$ds^2 = -d\tau^2 + \sinh^{\frac{2}{3}}(3H_i\tau)\left[\tanh^{-\frac{2}{3}}\left(\frac{3H_i\tau}{2}\right)(dx_1^2+dx_2^2) + \tanh^{\frac{4}{3}}\left(\frac{3H_i\tau}{2}\right)dx_3^2\right]$$

This simple model may provide a good test on an pre-inflationary anisotropy.

1) The initial geometry is the singularity-free Kasner spacetime.

$$ds^2 \sim -d\tau^2 + \tau^2 dx_a^2 + (dx_1^2 + dx_2^2).$$

→The adiabatic vacuum (≠ Bunch-Davies vacuum) can be defined *only* in this branch.

c.f., All the other Kasner branches are initially singular, e.g.

$$ds^2 \sim -d\tau^2 + \tau^{-\frac{2}{3}} dx_a^2 + \tau^{\frac{4}{3}}(dx_1^2 + dx_2^2).$$

⟹ No adiabatic vacuum state can be defined.

2) Nevertheless, it contains a large initial anisotropy.

Cosmic isotropization

We consider a model of the cosmic isotropization,
due to almost constant potential energy of a scalar field.

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) \qquad V(\phi) = \frac{1}{2}m^2\phi^2$$

$$ds^2 = -d\tau^2 + a^2(\tau)dx_a^2 + b^2(\tau)dx_b^2 = -d\tau^2 + e^{2\alpha}\left[e^{-4\beta}dx_a^2 + e^{2\beta}(dx_1^2 + dx_2^2)\right] ,$$

The isotropization due to the potential term requires $m \ll H_i$.

$\Rightarrow$ Scalar field is almost unchanged during isotropization $\phi \simeq \phi_0$.

$$\tau_\phi = 1/m \gg \tau_{iso} = 1/H_i$$

✓ The initial (anisotropic) geometry is regular.

$$ds^2 \sim -d\tau^2 + \tau^2 dx_a^2 + (dx_1^2 + dx_2^2).$$

The background geometry can be approximated by the Kasner-de Sitter universe

$$\boxed{\begin{aligned} e^\alpha &= \sinh^{\frac{1}{3}}\left(3H_i\tau\right), \\ e^\beta &= \coth^{\frac{1}{3}}\left(\frac{3H_i\tau}{2}\right) \end{aligned}} \qquad H_i = \frac{m\phi_0}{\sqrt{6}M_p}$$

Cosmological perturbations

Gumrukcuoglu, Contaldi & Peloso (07)

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Phi) & \partial_3\chi & \partial_1 B & \frac{b^2}{a}B_2 \\ & a^2(1-2\Psi) & b^2\partial_1\partial_3\tilde{B} & b^2\partial_3\tilde{B}_2 \\ & & b^2(1-2\Sigma+2\partial_1^2 E_2) & b^2\partial_1 E_2 \\ & & & b^2(1-2\Sigma) \end{pmatrix} \begin{matrix} \tau \\ x^3 \\ x^1 \\ x^2 \end{matrix}$$

$\phi + \delta\phi$

$E_2, B_2, \widetilde{B_2}$ 3 vector variables (2D) $\partial_i V_i = 0$

The remainings 8 Scalar perturbations (2D)

Gauge fixing: -3 in scalar -1 in vector

$E = \Sigma = \tilde{B} = 0$ $E_2 = 0$

Constraints: -3 in scalar -1 in vector
(- non-dynamical modes)

Φ, χ, B B_2

Physical d.o.f.s in terms of 2D symmetry

2 scalar modes 1 vector mode

$\delta\phi, \Psi$ $\tilde{B}_2$

Cosmological perturbations

Gumrukcuoglu, Contaldi & Peloso (07)

$$g_{\mu\nu} = \begin{array}{cc} \begin{pmatrix} -(1+2\Phi) & \partial_3\chi & \partial_1 B & \frac{b^2}{a} B_2 \\ & a^2(1-2\Psi) & b^2\partial_1\partial_3\tilde{B} & b^2\partial_3\tilde{B}_2 \\ & & \boxed{\begin{matrix} b^2(1-2\Sigma+2\partial_1^2 E_2) & b^2\partial_1 E_2 \\ & b^2(1-2\Sigma) \end{matrix}} & \end{pmatrix} & \begin{matrix} \tau \\ x^3 \\ x^1 \\ x^2 \end{matrix} \end{array}$$

2D planar directions

$\phi + \delta\phi$

$E_2, B_2, \widetilde{B_2}$ 3 vector variables (2D) $\partial_i V_i = 0$

The remainings 8 Scalar perturbations (2D)

Gauge fixing: -3 in scalar -1 in vector

$E = \Sigma = \tilde{B} = 0$ $E_2 = 0$

Constraints: -3 in scalar -1 in vector

(- non-dynamical modes) Φ, χ, B B_2

Physical d.o.f.s in terms of 2D symmetry

2 scalar modes $\delta\phi, \Psi$

1 vector mode $\tilde{B}_2$

Correspondence with the isotropic limit $b \to a$

Scalar modes (2D)

$$Q = \delta\phi + \frac{p_b^2 \dot{\phi}^2}{2H_b p_a^2 + (H_a + H_b) p_b^2} \Psi,$$

$$H_+ = \frac{\sqrt{2} M_p p_b^2 H_b}{2H_b p_a^2 + (H_a + H_b) p_b^2} \Psi.$$

Vector mode (2D)

$$H_\times := \frac{M_p}{\sqrt{2}} \frac{b\, p_a p_b}{\sqrt{p_a^2 + p_b^2}} \tilde{B}_2$$

$$p_a := \frac{k_a}{a}, \quad p_b := \frac{k_b}{b}.$$

$$k^2 = k_a^2 + k_b^2 = k_a^2 + (k_1^2 + k_2^2), \quad p^2 := p_a^2 + p_b^2 = p_a^2 + (p_1^2 + p_2^2).$$

Without loss of generality, we may set $k_2 = 0$.

Isotropic limit
$b \to a$

Scalar mode (3D) $\mathcal{R}$

(comoving) curvature perturbation

$$\frac{H_i}{\dot{\phi}} Q \to \mathcal{R}$$

2 tensor modes (3D) $h_+, h_\times$

$$H_+ \to \frac{M_p}{\sqrt{2}} h_+$$

$$H_\times \to \frac{M_p}{\sqrt{2}} h_\times$$

Firstly, we will focus on the quantization of the massless scalar field, as the *mimic* of the metric perturbations.

$$S_\phi = -\frac{1}{2}\int d^4x\sqrt{-g}\Big(g^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi\Big)$$

on the Kasner de Sitter background

$$ds^2 = -d\tau^2 + \sinh^{\frac{2}{3}}(3H_i\tau)\left[\tanh^{-\frac{2}{3}}\left(\frac{3H_i\tau}{2}\right)(dx_1^2+dx_2^2) + \tanh^{\frac{4}{3}}\left(\frac{3H_i\tau}{2}\right)dx_3^2\right]$$

We then derive the spectrum of the vector mode (2D).

Scalar field perturbations

Quantization of a scalar field

- The canonical quantization of the scalar field

$$\phi = \int d^3k \left(u_{\mathbf{k}} a_{\mathbf{k}} + u^*_{\mathbf{k}} a^\dagger_{\mathbf{k}} \right),$$

$$[a_{\mathbf{k_1}}, a^\dagger_{\mathbf{k_2}}] = \delta(\mathbf{k_1} - \mathbf{k_2})$$

$$u_{\mathbf{k}} = e^{i\mathbf{kx}} \phi_{\mathbf{k}} / (2\pi)^{3/2}$$

$$\phi_{\mathbf{k}} \partial_x \phi^*_{\mathbf{k}} - (\partial_x \phi_{\mathbf{k}}) \phi^*_{\mathbf{k}} = -\frac{i\varepsilon}{3H_i}$$

$$\sinh(\varepsilon x) = \frac{1}{\sinh(3H_i\tau)} = e^{-3\alpha},$$

- Equation of motion for the mode function

$$\left(\frac{d^2}{dx^2} + \Omega_{\mathbf{k}}(x)^2 \right) \phi_{\mathbf{k}} = 0$$

$$\Omega^2_{\mathbf{k}}(x) = \left(\frac{\varepsilon}{3H_i} \right)^2 \frac{2^{4/3}(k^2_\perp e^{-2\varepsilon x} + k^2_3)}{(1 - e^{-2\varepsilon x})^{4/3}} = \frac{2(\bar{k}\varepsilon^{2/3})^2}{9} \left(\frac{e^{\varepsilon x}}{\sinh \varepsilon x} \right)^{1/3} \left(\frac{1}{e^{2\varepsilon x} - 1} + r^2 \right)$$

= the dominant term of the vector mode

$$\bar{k} = \varepsilon^{1/3} \frac{k}{H_i}, \qquad r = \frac{k_3}{k}$$

$$k^2 := k^2_1 + k^2_2 + k^2_3 = k^2_\perp + k^2_3.$$

- The final spectrum of the scalar field contains the direction dependence

$$\boxed{\langle 0 | \phi^2 | 0 \rangle := \int d\ln k \int \frac{d\theta_{\mathbf{k}}}{2} P_\phi, \quad P_\phi = \frac{k^3}{2\pi^2} |\phi_{\mathbf{k}}|^2}$$

Non-planar modes $0 < r = \frac{k_3}{k} \lesssim 1$

- ✓ The adiabaticity condition is satisfied in the early time

$$\epsilon(x) := \left| \frac{\frac{d\Omega^2(x)}{dx}}{\Omega^3(x)} \right| = \frac{1}{\bar{k}} \left(\frac{2\varepsilon}{1 - e^{-2\varepsilon x}} \right)^{1/3} \frac{e^{-2\varepsilon x}}{\sqrt{1 + e^{-2\varepsilon x}}} \frac{1 + \frac{1+r^2}{2} \tanh \varepsilon x}{\left[\frac{1}{e^{2\varepsilon x}+1} + r^2 \tanh \varepsilon x \right]^{3/2}} \ll 1,$$

- ✓ The early time solution $\varepsilon x \gg 1$: WKB approximation

$$\phi_{\mathrm{WKB}} = \sqrt{\frac{\varepsilon}{3H_i}} \frac{1}{\sqrt{2\tilde{\Omega}(x)}} \left\{ \exp \left[i \int_{x_0}^{x} dx' \tilde{\Omega}(x') + i\psi \right] \right\}$$

$$\tilde{\Omega}(x)^2 = \Omega(x)^2 \, c(x)^2; \qquad c(x)^2 := 1 - \frac{\tilde{\Omega}_{,xx}}{2\Omega^2 \tilde{\Omega}} + \frac{3}{4} \left(\frac{\tilde{\Omega}_{,x}}{\Omega(x) \tilde{\Omega}(x)} \right)^2 .$$

The accuracy is improved order by order, by

$$E_{\mathrm{WKB}}(x) \sim \epsilon(x)^2 \simeq \frac{1}{\bar{k}^2 \, x^{2/3}}$$

- The late time solution $\varepsilon x \ll 1$: deviation from the de Sitter mode functions

$$\left(\frac{d^2}{dx^2}+\sum_{n=0}^{\infty}\varepsilon^n V_n\right)\phi=0$$

$$V_0=\frac{\bar{k}^2}{9x^{4/3}}, \qquad V_1=-\frac{2\bar{k}^2q^2}{9x^{1/3}}, \qquad V_2=\frac{2\bar{k}^2r^2x^{2/3}}{27}, \qquad V_3=\frac{8\bar{k}^2x^{5/3}}{729}, \cdots$$

$$q^2=1/3-r^2$$

ε is regarded as an expansion parameter in the late-time limit

The mode functions for the positive and negative frequency modes

$$\phi=A_+u(x)+A_-v(x)$$

$$u=\sum_{n=0}^{\infty}\varepsilon^n u_n(x), \quad v=\sum_{n=0}^{\infty}\varepsilon^n v_n(x)$$

$$\left(\frac{d^2}{dx^2}+\frac{\bar{k}^2}{9x^{4/3}}\right)u_{n+1}=-\sum_{m=0}^{n}u_m(y)V_{n-m}(y)$$

$$\Longrightarrow \quad u_{n+1}(x)=-\int dy\, G(x,y)\left[\sum_{m=0}^{n}u_m(y)V_{n-m}(y)\right].$$

$$G(x,y)=-\frac{3}{2i\bar{k}^3}\left[u_0(x)v_0(y)\theta(y-x)+u_0(y)v_0(x)\theta(x-y)\right].$$

- 0th order solution: de Sitter mode functions

$$u_0(x) := \Big(-1+i\bar{k}x^{1/3}\Big)e^{i\bar{k}x^{1/3}}, \quad v_0(x) := \Big(-1-i\bar{k}x^{1/3}\Big)e^{-i\bar{k}x^{1/3}} = u_0^*(x)$$

- Higher corrections which follow the boundary conditions $u_i(0)=v_i(0)=0$

$$u_1(x) = e^{i\bar{k}\sqrt[3]{x}}\left(\frac{1}{4}q^2x^{5/3}\bar{k}^2+\frac{3}{4}iq^2x^{4/3}\bar{k}-\frac{3}{4}\left(q^2x\right)-\frac{9\left(q^2\sqrt[3]{x}\right)}{8\bar{k}^2}-\frac{9iq^2}{8\bar{k}^3}\right)+e^{-i\bar{k}\sqrt[3]{x}}\left(-\frac{9\left(q^2\sqrt[3]{x}\right)}{8\bar{k}^2}+\frac{9iq^2}{8\bar{k}^3}\right),$$

$$\begin{aligned}
u_2(x) &= e^{-i\bar{k}\sqrt[3]{x}}\left[-\frac{9iq^4x^{5/3}}{32\bar{k}}-\frac{27\left(q^4x^{4/3}\right)}{32\bar{k}^2}+\frac{27iq^4x}{32\bar{k}^3}+\frac{9i\left(99q^4+24q^2-8\right)\sqrt[3]{x}}{64\bar{k}^5}+\frac{9\left(99q^4+24q^2-8\right)}{64\bar{k}^6}\right]\\
&+ e^{i\bar{k}\sqrt[3]{x}}\Big[-\frac{1}{32}iq^4x^3\bar{k}^3+\frac{\left(459q^4+96q^2-32\right)x^{8/3}\bar{k}^2}{2016}+\frac{1}{112}i\left(99q^4+24q^2-8\right)x^{7/3}\bar{k}\\
&- \frac{1}{40}\left(99q^4+24q^2-8\right)x^2-\frac{9i\left(47q^4+12q^2-4\right)x^{5/3}}{80\bar{k}}+\frac{3\left(45q^4+12q^2-4\right)x^{4/3}}{16\bar{k}^2}+\frac{3i\left(45q^4+12q^2-4\right)x}{16\bar{k}^3}\\
&+ \frac{9i\left(99q^4+24q^2-8\right)\sqrt[3]{x}}{64\bar{k}^5}-\frac{9\left(99q^4+24q^2-8\right)}{64\bar{k}^6}+\cdots\Big],
\end{aligned} \tag{B3}$$

$$\begin{aligned}
u_3(x) &= e^{-i\bar{k}\sqrt[3]{x}}\left[\frac{9q^6x^3}{256}-\frac{iq^2\left(459q^4+96q^2-32\right)x^{8/3}}{1792\bar{k}}-\frac{9\left(q^2\left(99q^4+24q^2-8\right)x^{7/3}\right)}{896\bar{k}^2}+\frac{9iq^2\left(99q^4+24q^2-8\right)x^2}{320\bar{k}^3}\right]\\
&+ e^{i\bar{k}\sqrt[3]{x}}\Big[-\frac{1}{384}\left(q^6x^{13/3}\right)\bar{k}^4-\frac{iq^2\left(291q^4+96q^2-32\right)x^4\bar{k}^3}{8064}+\frac{\left(102789q^6+40176q^4-13392q^2-1792\right)x^{11/3}\bar{k}^2}{362880}\\
&+ \frac{i\left(50193q^6+19224q^4-6408q^2-896\right)x^{10/3}\bar{k}}{30240}+\frac{\left(-273807q^6-101952q^4+33984q^2+4480\right)x^3}{34560}\\
&- \frac{i\left(284607q^6+104352q^4-34784q^2-4480\right)x^{8/3}}{8960\bar{k}}+\frac{\left(288387q^6+105192q^4-35064q^2-4480\right)x^{7/3}}{2688\bar{k}^2}\\
&+ \frac{i\left(288387q^6+105192q^4-35064q^2-4480\right)x^2}{960\bar{k}^3}\Big]+\cdots,
\end{aligned} \tag{B4}$$

$$
\begin{aligned}
u_4(x) &= e^{-i\bar{k}\sqrt[3]{x}}\left[\frac{3iq^8x^{13/3}\bar{k}}{1024}+\frac{q^4\left(291q^4+96q^2-32\right)x^4}{7168}\right]+e^{i\bar{k}\sqrt[3]{x}}\left[\frac{iq^8x^{17/3}\bar{k}^5}{6144}-\frac{\left(q^4\left(477q^4+192q^2-64\right)x^{16/3}\right)\bar{k}^4}{129024}\right.\\
&- \frac{i\left(484623q^8+247752q^6-71064q^4-20224q^2+1280\right)x^5\bar{k}^3}{10160640}\\
&+ \frac{\left(30240459q^8+15875352q^6-4410504q^4-1553408q^2+141824\right)x^{14/3}\bar{k}^2}{66044160}\\
&+ \frac{i\left(5658255q^8+2920077q^6-812511q^4-278256q^2+25712\right)x^{13/3}\bar{k}}{1572480}\\
&- \left.\frac{\left(64266939q^8+32444658q^6-9097542q^4-2976096q^2+269216\right)x^4}{2661120}\right]+\cdots, \qquad \text{(B5)}
\end{aligned}
$$

$$
\begin{aligned}
u_5(x) &= e^{i\bar{k}\sqrt[3]{x}}\left[\frac{q^{10}x^7\bar{k}^6}{122880}+\frac{iq^6\left(2133q^4+960q^2-320\right)x^{20/3}\bar{k}^5}{7741440}\right.\\
&- \frac{\left(q^2\left(853821q^8+519264q^6-127008q^4-55808q^2+5120\right)x^{19/3}\right)\bar{k}^4}{162570240}\\
&- \left.\frac{i\left(175688271q^{10}+114725160q^8-25188408q^6-16032384q^4+1286400q^2+186368\right)x^6\bar{k}^3}{2377589760}\right]+\cdots, \qquad \text{(B6)}
\end{aligned}
$$

$$
u_6(x) = e^{i\bar{k}\sqrt[3]{x}}\left[-\frac{iq^{12}x^{25/3}\bar{k}^7}{2949120}+\frac{q^8\left(993q^4+480q^2-160\right)x^8\bar{k}^6}{61931520}\right]+\cdots,
$$

The other solution is as the complex conjugate $v_i(x)=(u_i)^*(x)$

The accuracy is improved order by order, by the factor $\boxed{E_{\mathrm{asym}}(x) \sim \varepsilon\bar{k}\,x^{4/3}.}$

- Matching of the WKB and perturbative solutions at the time which $E_{\mathrm{WKB}} = E_{\mathrm{asym}}$

$$\boxed{x_* = (\varepsilon \bar{k}^3)^{-1}}$$

For simplicity, we choose $\varepsilon = \bar{k}^{-3} = \left(\frac{H_i}{k}\right)^{3/2}$ so that $x_* = 1$.

- The final amplitude of the scalar field

$$
\begin{aligned}
\sqrt{\frac{2H_i\bar{k}^3}{\varepsilon}}\frac{e^{i\bar{k}}}{\Phi_*}\phi_{\mathrm{fin}} &= i + \frac{1}{\bar{k}} + \frac{-\frac{q^2}{4} - \frac{i}{2}}{\bar{k}^2} + \frac{-\frac{1}{2} + \frac{iq^2}{4}}{\bar{k}^3} - \frac{i\left(q^4 + 12iq^2 - 12\right)}{32\bar{k}^4} \\
&+ \frac{-207q^4 - (96+756i)q^2 + 788}{2016\bar{k}^5} + \frac{21q^6 + 1206iq^4 - (3780-384i)q^2 - 2648i}{8064\bar{k}^6} \\
&- \frac{i\left(825q^6 + (480+74142i)q^4 - (19060-18432i)q^2 - 18744i\right)}{40320\bar{k}^7} \\
&+ \frac{i\left(945q^8 + 446904iq^6 - (11016648-262656i)q^4 - (2654208+3262752i)q^2 + (2472336-28672i)\right)}{5806080\bar{k}^8} \\
&+ \frac{13905q^8 + (8640+3002616i)q^6 - (37893384-926208i)q^4 - (9103104+3483936i)q^2 + (4621968-28672i)}{5806080\bar{k}^9} \\
&+ \frac{1}{162570240\bar{k}^{10}}\Big[-1323q^{10} - 2928798iq^8 + (831208392-2149632i)q^6 + (338204160+1072579536i)q^4 \\
&\quad -(212753520-255210496i)q^2 - (17461248+124990304i)\Big] \\
&+ \frac{i}{2113413120\bar{k}^{11}}\Big[410319q^{10} + (262080+260683110i)q^8 - (14181596520-142566912i)q^6 \\
&\quad -(5225036544-163571184i)q^4 + (3041923248+133211136i)q^2 + (226996224+471614432i)\Big] \\
&\quad \frac{i}{16907304960\bar{k}^{12}}\Big[5733q^{12} + 41102100iq^{10} - (23757489540-34085376i)q^8 - (11283840000+1401723182496i)q^6 \\
&\quad -(232347906000+516373012480i)q^4 - (57447653376-183565086016i)q^2 + (15556834880+22417833984i)\Big]
\end{aligned}
$$

• The power spectrum

$$P = \frac{{H_i}^2}{4\pi^2}\left\{1 + \frac{9(11 - 90r^2 + 99r^4)}{32}\left(\frac{H_i}{k}\right)^6 + O\left(\left(\frac{H_i}{k}\right)^7\right)\right\}$$

$$r = \cos\theta = k_3/k$$

✓ The quartic order direction-dependence is as large as the quadratic one

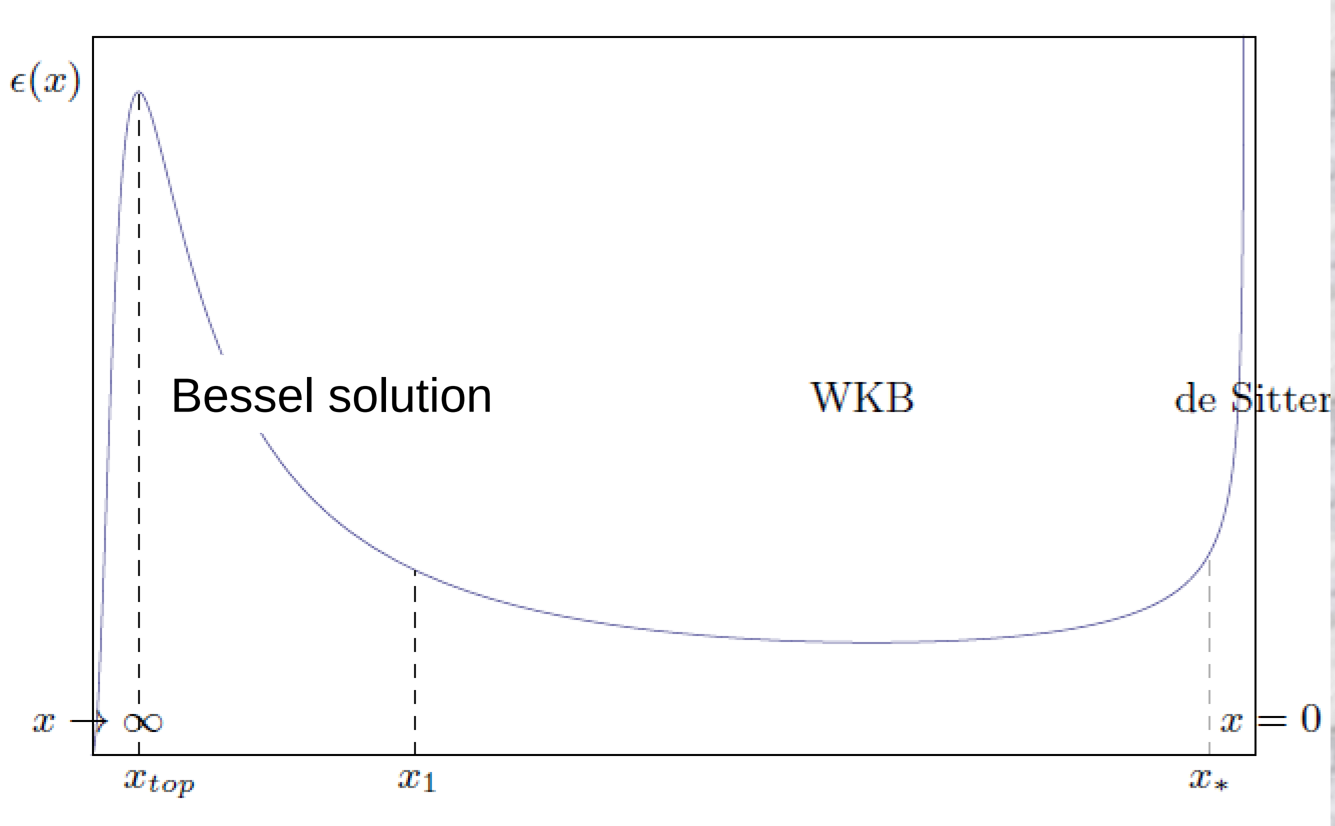
c.f. Ackerman, Carroll and Wise (07)

$$P(k) = P_0(k)\left\{1 + g(k)\cos^2\theta\right\}$$

✓ The leading order nontrivial corrections were obtained.

The nonvanishing corrections appear at the 6th order of WKB approximation, because of the de Sitter expansion and the high-momentum effects.

Planar modes $r = \frac{k_3}{k} \sim 0$



The adiabaticity condition is violated around the peak $x \simeq x_{top}$

and is recovered for $x_1 > x > x_*$

- The 0th order solution in the early time

$$\Omega^2 = \varepsilon^2 \left(\bar{r}^2 + \frac{\bar{q}^2}{e^{2\varepsilon x}} \right) + \underline{V_1(x)}, \quad \text{for } \varepsilon x \gg 1$$

sub-leading corrections

$$\phi_0 = \sqrt{\frac{\pi}{6H_i \sinh(\pi \bar{r})}} J_{-i\bar{r}}(\bar{q}e^{-\varepsilon x}), \quad \text{for } \varepsilon x \gg 1$$

$$\boxed{\bar{r} = \frac{2^{2/3} k r}{3H_i}, \quad \bar{q} = \frac{2^{2/3} k}{3H_i}}$$

The 1st order correction improves the accuracy, by the factor $E_{\rm ini} = \left| \frac{\phi_1}{\phi_0} \right| \sim \frac{k}{H_i} e^{-3\varepsilon x_1}$

- The WKB approximation is valid around $x = x_1$

$$\psi = \frac{B_+}{\sqrt{2\Omega}} \exp\left\{ i \int_{x_1}^{x} dx' \Omega(x') \right\} + \frac{B_-}{\sqrt{2\Omega}} \exp\left\{ -i \int_{x_1}^{x} dx' \Omega(x') \right\}$$

The matching point determined by $E_{\rm ini} = E_{WKB}$ $\longrightarrow$ $\boxed{e^{\varepsilon x_1} = c_1 \left(\frac{k}{H_i} \right)^{3/5}}$

$$\boxed{B_+ \simeq \sqrt{\frac{\varepsilon}{3H_i}} \frac{e^{i(\frac{\pi}{4} - \bar{q}e^{-\varepsilon x_1})}}{\sqrt{1 - e^{-2\pi \bar{r}}}}, \quad B_- \simeq \sqrt{\frac{\varepsilon}{3H_i}} \frac{e^{-\pi \bar{r}} e^{-i(\frac{\pi}{4} - \bar{q}e^{-\varepsilon x_1})}}{\sqrt{1 - e^{-2\pi \bar{r}}}}}$$

- At the later time x_*, the WKB solution is matched to the late-time solution

  In the late time limit, $$\phi(x \to 0) = -(A_+ + A_-) \simeq \left(\frac{H_i^2}{2k^3}\right)^{\frac{1}{2}} \left(e^{-i\Psi} - e^{-\pi\bar{r}} e^{i\Psi}\right)$$

$$\Psi(k) = \bar{k} x_*^{1/3} - \int_{x_1}^{x_*} \Omega(x) dx + \bar{q} e^{-\varepsilon x_1} - \frac{\pi}{4}$$ $\Rightarrow$ independent of the matching points

- The power spectrum of the scalar field

$$\boxed{P = \left(\frac{H_i}{2\pi}\right)^2 \left(\coth \pi\bar{r} - \frac{\cos(2\Psi)}{\sinh \pi\bar{r}}\right)} \qquad \boxed{\bar{r} = \frac{2^{2/3} k r}{3 H_i}}$$

Kim and Minamitsuji (10), Dey and Paban (11)

✓ The power of a scalar fluctuation has a sharp peak around the plane $P \propto \frac{1}{r}$.

Gumrukcuoglu, Contaldi & Peloso (07)

~ Giant rings? Kovetz, Ben-David and Itzhaki (10) , Rathaus and Itzhaki (12)

Since the purely planar mode $r = 0$ remains classical, there should be a cut-off on r.

→ Vector mode can be quantized almost in the same way.

Summary

- We have considered an anisotropic universe model, where isotropization is due to the potential term and the initial geometry is singularity-free.

$$ds^2 = -d\tau^2 + \sinh^{\frac{2}{3}}(3H_i\tau)\left[\tanh^{-\frac{2}{3}}\left(\frac{3H_i\tau}{2}\right)\left(dx_1^2 + dx_2^2\right) + \tanh^{\frac{4}{3}}\left(\frac{3H_i\tau}{2}\right)dx_3^2\right]$$

- The spectrum of the perturbations has a sharp peak around the plane.

- The analysis of the scalar modes is in progress.

Thank you.

Appendix

A bound from CMB anisotropy

The primordial power spectrum, around the plane, takes the form of

$$P(\mathrm{k}) = \underline{P_0(k)}\Big(1 + \mathcal{G}(k, \Omega_k)\Big)$$

isotropic part
(scale invariant)

corrections due to the anisotropy

$$\mathcal{G}(k, \Omega_k) = \coth\left(\frac{2^{2/3}\pi k}{3H_i}|\cos\theta_k|\right) - 1.$$

A scale-invariant power spectrum $k^3 P_0(k) = A_s,$

The angular power spectrum of the CMB anisotropy

$$a_{\ell m} = \int d\Omega Y_{\ell m}(\Omega)\frac{\delta T}{T}(\Omega),$$

$$\langle a_{\ell m} a^*_{\ell' m'}\rangle = \delta_{mm'}\Big\{C_\ell \delta_{\ell\ell'} + \Delta C_{\ell\ell' m}(r_0)\Big\},$$

Neglecting ISW corrections

$$\frac{\ell(\ell+1)}{2\pi}C_\ell \approx \frac{A_s}{18\pi^2},$$

Corrections to the angular power spectrum due to anisotropy localized on the plane

$$\Delta C_{\ell\ell' m}(r_0) := \frac{(-i)^{\ell-\ell'}}{9\pi}(1+(-1)^{\ell+\ell'})\sqrt{(2\ell+1)(2\ell'+1)\frac{(\ell-m)!(\ell'-m)!}{(\ell+m)!(\ell'+m)!}}$$
$$\times \int_{k_{ir}}^{\infty} dk k^2 P_0(k) j_\ell(k\eta_*) j_{\ell'}(k\eta_*) \int_{r_0}^{1} dr P_\ell^m(r) P_{\ell'}^m(r) f(k,r),$$

$$\frac{\Delta C_{\ell\ell' m}(r_0)}{C_\ell} = (-i)^{\ell-\ell'}\ell(\ell+1)(1+(-1)^{\ell+\ell'})\sqrt{\frac{(2\ell+1)(2\ell'+1)(\ell-m)!(\ell'-m)!}{(\ell+m)!(\ell'+m)!}}$$
$$\times \int_{k_{ir}/H_i}^{\infty} \frac{dy}{y} j_\ell(y\eta_* H_i) j_{\ell'}(y\eta_* H_i) \int_{r_0}^{1} dr P_\ell^m(r) P_{\ell'}^m(r)\left(\coth\left(\frac{2^{2/3}\pi y r}{3}\right)-1\right),$$

Non-zero contribution from the anisotropic part is obtained for the cases

1) $\ell = \text{even and } \ell' = \text{even}$

2) $\ell = \text{odd and } \ell' = \text{odd.}$

We focus on the case $\ell' = \ell$

The leading order contribution is given by

$$\frac{\Delta C_{\ell\ell m}}{C_\ell} \sim \tilde{C}(\ell, m)\left[(k_{ir}\eta_*)\ln r_0^{-1}\right],$$

- ✓ The anisotropic part is highly dependent on the cut-off of the planarity parameter r_0 .
- ✓ Avoiding too large anisotropic corrections put a bound

$$r_0 > \exp\left[-\frac{1}{\tilde{C}(\ell, m)(k_{ir}\eta_*)}\right].$$

$\ell = 2 \;\; m = 0$ $$r_0 > \exp\left[-\frac{3.70}{k_{ir}r_*}\right].$$

If $k_{ir}\eta_*$=1, $r_0 > 0.0246$

A bound is given for each ℓ,
but the right-hand-side becomes smaller as ℓ increases.