Nishinomiya-Yukawa Symposium, Kyoto
December 6th 2012

Interacting vacuum energy

David Wands
Institute of Cosmology and Gravitation, University of Portsmouth

ICG Portsmouth

DW, Josue De-Santiago & Yuting Wang, arXiv:1203.6776
Josue De De-Santiago, DW & Yuting Wang, arXiv:1209.0563

How I first met Sasaki-san…

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Cosmological Perturbation Theory

Hideo KODAMA and Misao SASAKI*

Department of Physics, University of Tokyo, Tokyo 113
**Department of Physics, Kyoto University, Kyoto 606*

(Received September 5, 1984)

The linear perturbation theory of spatially homogeneous and isotropic universes is reviewed and reformulated extensively. In the first half of the article, a gauge-invariant formulation of the theory is carried out with special attention paid to the geometrical meaning of the perturbation. In the second half of the article, the application of the theory to some important cosmological models is discussed.

Contents

summary:

- **vacuum energy is simplest model for acceleration**
  - no new degrees of freedom

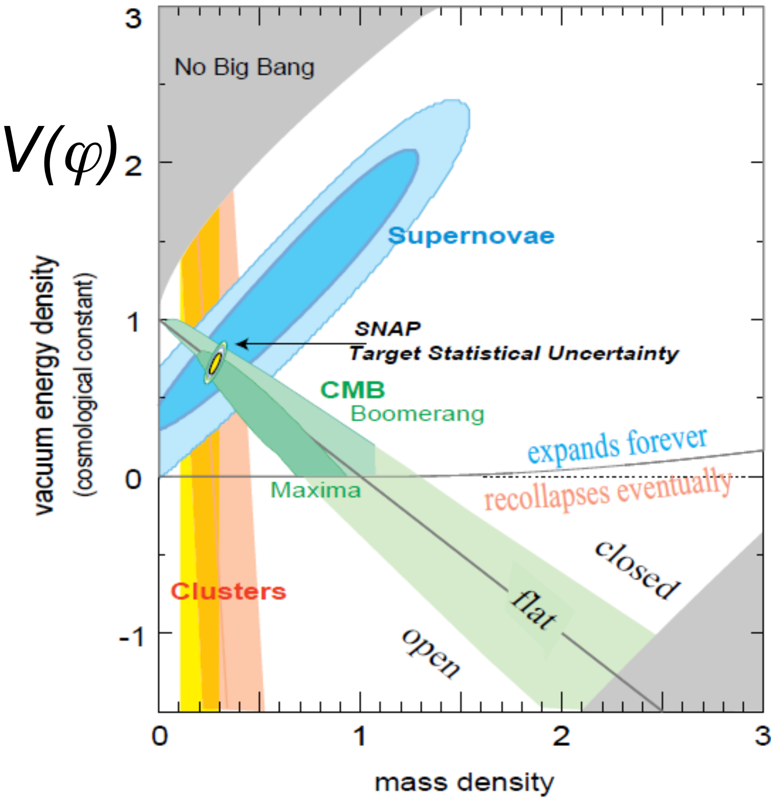
- **inhomogeneous** (space-time dependent) **vacuum implies energy transfer**

- **any dark energy fluid** can be decomposed into **interacting vacuum+fluid** (like scalar field quintessence)

- **inhomogeneous** (space dependent) **perturbations** require **physical model for interactions**
  - distinguish by observational data
  - worked example: decomposed Chaplygin gas

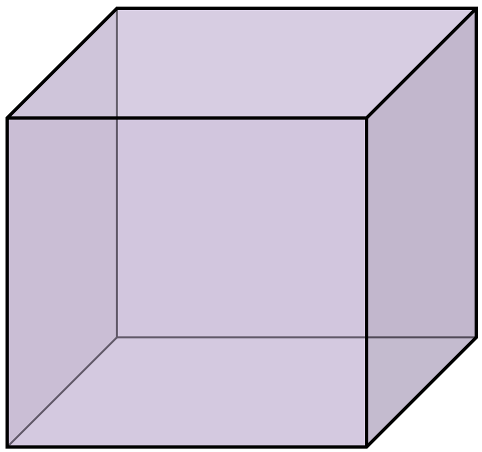
Dark energy models

- **quintessence**
  - self-interacting scalar fields, *V(φ)*
- **barotropic fluid**
  - exotic equation of state, *P(ρ)*
  - unified dark matter + energy
- **interacting dark energy, *Γ(t)***
  - coupled quintessence



motivated by astronomical observations, but lacking persuasive physical model

Simplest model



- **vacuum energy**
  - energy of empty space
  - undiluted by expansion
  - ***no new degrees of freedom***

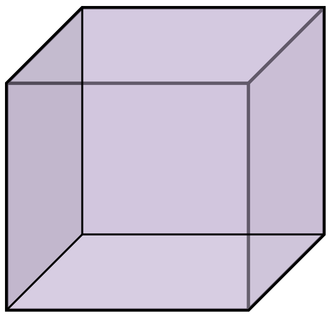
$$\check{T}^{\mu}_{\nu} = -V g^{\mu}_{\nu}$$

perfect fluid $$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) u^{\mu} u_{\nu}$$

with $$\check{\rho} = -\check{P} = V$$

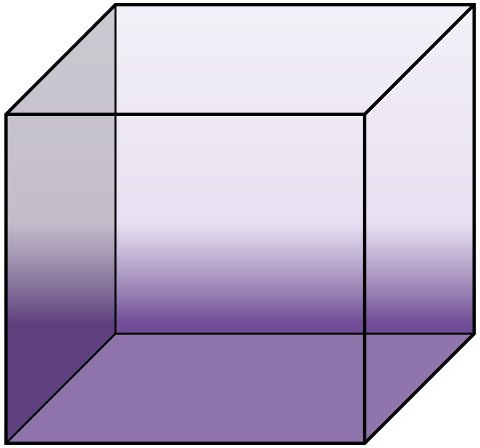
but no particle flow, hence 4-velocity, u, undefined

Homogeneous vacuum



- **8πG V = Λ = constant**
  - empirical value is cosmological constant problem

Inhomogeneous vacuum



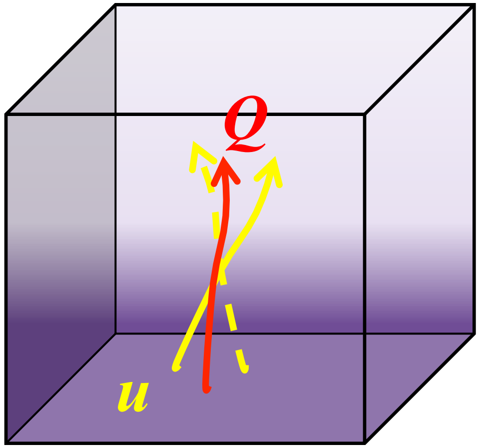
- **interacting vacuum:**

$$\nabla_\mu \check{T}^\mu_\nu = \nabla_\mu \left(-V g^\mu_\nu\right)$$
$$= -\nabla_\nu V$$
$$\equiv Q_\nu \qquad \textit{= energy flow}$$

- **conservation of total (matter + vacuum) energy:**

$$\nabla_\mu G_{\mu\nu} = 8\pi G_N \nabla_\mu \left(T^\mu_\nu + \check{T}^\mu_\nu\right) = 0 \quad \Rightarrow \quad \nabla_\mu T^\mu_\nu = -Q_\nu$$

4-velocity



- **perfect fluid**

$$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) u^{\mu} u_{\nu}$$

$$\Rightarrow \quad T^{\mu}_{\nu} u^{\nu} = -\rho u^{\mu}$$

- **vacuum**

$$\check{T}^{\mu}_{\nu} = -V g^{\mu}_{\nu}$$

$$\Rightarrow \quad \check{T}^{\mu}_{\nu} u^{\nu} = -V u^{\mu} \quad \forall\ u^{\mu}$$

  - all observers see same vacuum energy
    so 4-velocity undefined
  - **but *energy flow* defines irrotational potential flow**

$$\boxed{Q_{\mu} = -\nabla_{\mu} V}$$

FLRW vacuum cosmology:

homogeneous 3D space ⇒ $V=V(t)$

- Friedmann equation

$$H^2 = \frac{8\pi G_N}{3}(\rho + V) - \frac{K}{a^2}$$

- Continuity equations for matter + vacuum

$$\begin{aligned} \dot{\rho} + 3H(\rho + P) &= -Q\,, \\ \dot{V} &= Q\,. \end{aligned}$$

- e.g.,

  Freese et al (1987); Berman (1991); Pavon (1991); Chen & Wu (1992); Carvalho et al (1992); Al-Rawaf & Taha (1996); Shapiro & Sola (2002); Sola (2011); …

- Freedom to choose any $V(t)$
  - *more a description than an explanation? like $V(\varphi)$?*

Linear perturbations

inhomogeneous 3D space

$$ds^2 = -(1+2\phi)dt^2 + 2a\partial_i B dt dx^i + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right] dx^i dx^j$$

- Matter:

$$\rho \to \rho(t) + \delta\rho(t,x^i)\,, \quad P \to P(t) + \delta P(t,x^i)\,, \quad u_\mu \to u_\mu = [-1-\phi\,,\partial_i\theta]$$

matter 3-momentum

- Vacuum: $V \to V(t) + \delta V(t,x^i)$

- Interaction: (see Kodama & Sasaki 1984)

$$\boxed{Q_\nu \to (Q+\delta Q)u_\nu + f_\nu = [-Q(1+\phi) - \delta Q\,, \partial_i(f+Q\theta)]}$$

vacuum-matter momentum transfer

- need physical (covariant) interaction to determine energy-momentum transfer 4-vector Q_ν
  - *same FRW cosmologies may have different perturbations*

perturbed equations of motion

inhomogeneous 3D space $\Rightarrow$ $V(t,x^i) = V(t) + \delta V(t,x^i)$

- matter+vacuum energy conservation:

$$\begin{aligned} \dot{\delta\rho} + 3H(\delta\rho + \delta P) - 3(\rho + P)\dot{\psi} + (\rho + P)\frac{\nabla^2}{a^2}\left(\theta + a^2\dot{E} - aB\right) &= -\delta Q - Q\phi\,, \\ \delta\dot{V} &= \delta Q + Q\phi\,. \end{aligned}$$

- matter+vacuum momentum conservation:

$$\begin{aligned} (\rho + P)\dot{\theta} - 3c_s^2 H(\rho + P)\theta + (\rho + P)\phi + \delta P &= -f + c_s^2 Q\theta\,, \\ -\delta V &= f + Q\theta\,. \end{aligned}$$

vanishing vacuum momentum requires vacuum pressure gradient balanced by force on vacuum

$$\nabla_i\left(-V\right) = \nabla_i\left(f + Q\theta\right)$$

Gauge-invariant perturbations

- vacuum perturbation on hypersurfaces orthogonal to energy transfer vanishes identically:

$$\Delta\check{\rho}_{\rm com} = \delta V + \left(f + \dot{V}\theta\right) = 0$$

  - *∃ hypersurfaces on which vacuum is homogeneous*

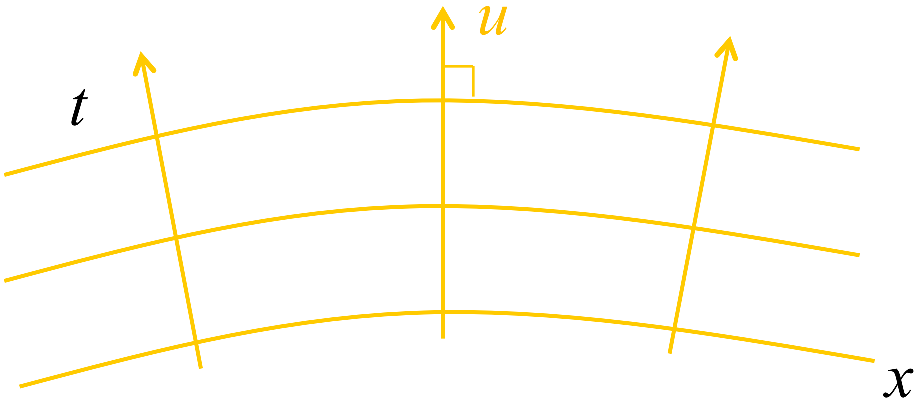
- comoving matter density: $$\delta\rho_{\rm com} = \delta\rho + \dot{\rho}\theta$$

- comoving vacuum density may be non-zero

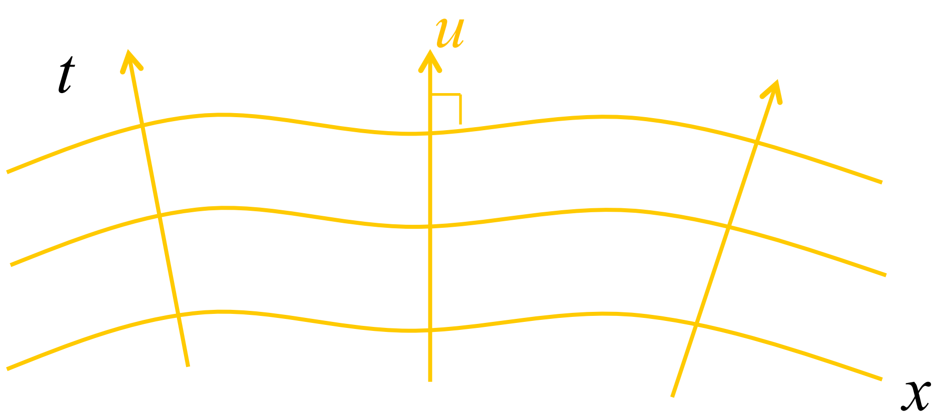
$$\delta\check{\rho}_{\rm com} = \delta V + \dot{V}\theta = -f$$

- *e.g., Poisson equation:*

$$\nabla^2\Phi = 4\pi G_N \left(\delta\rho_{\rm com} + \delta\check{\rho}_{\rm com}\right)$$

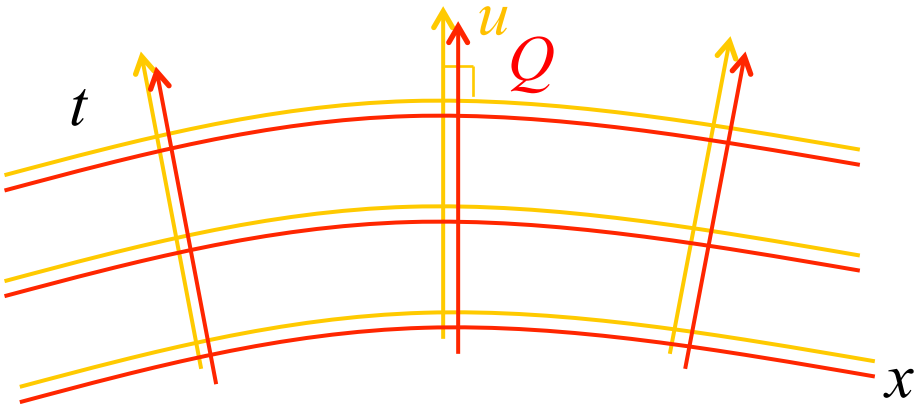


FRW cosmology

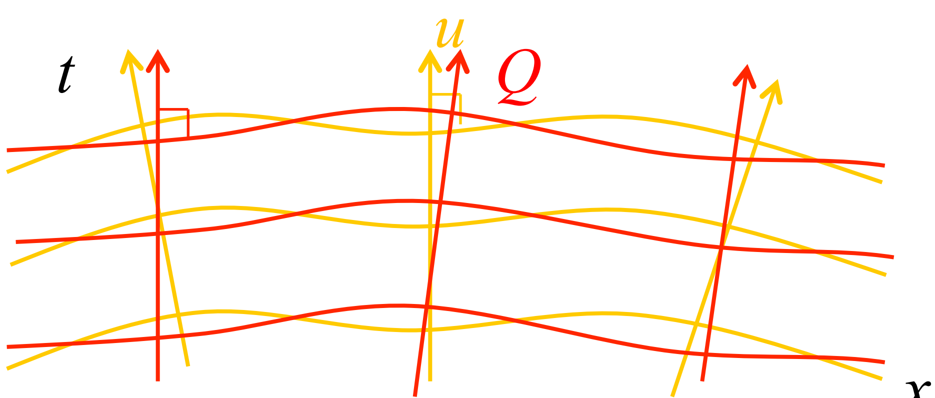


FRW cosmology
+ linear perturbations

comoving-orthogonal
coordinates *(t,x)*



FRW cosmology



FRW cosmology
+ linear perturbations

comoving-orthogonal coordinates *(t,x)*

vacuum energy uniform on spaces orthogonal to energy flow

Gauge-invariant perturbations (II)

- curvature perturbation on uniform-matter hypersurfaces:

$$\zeta = -\psi - \frac{H}{\dot\rho}\delta\rho$$

- curvature perturbation on uniform-*vacuum* hypersurfaces:

$$\check\zeta = -\psi - \frac{H}{\dot V}\delta V$$

- relative (entropy) vacuum perturbation:

$$\check S = 3\left(\check\zeta - \zeta\right)$$

- *e.g., non-adiabatic vacuum pressure perturbation:*

$$\delta\check P_{\rm nad} = \frac{(1+c_s^2)Q[Q+3H(\rho+P)]}{9H^2(\rho+P)}\check S$$

Dark energy cosmology

- any dark energy cosmology (with $\rho_{de}+P_{de}\geq 0$) can be decomposed into interacting vacuum + fluid

for example:

- interacting vacuum + scalar field (quintessence):

$$\rho_{de} = \frac{1}{2}\dot{\varphi}^2 + V, \quad P_{de} = \frac{1}{2}\dot{\varphi}^2 - V, \quad Q = V'(\varphi)\dot{\varphi}$$

$$\boxed{V(\varphi) \quad \Rightarrow \quad Q_\mu = V'(\varphi)\nabla_\mu \varphi}$$

- interacting vacuum + matter: Wands, De-Santiago & Wang (2012)

$$\rho_{de} = \rho_m + V, \quad P_{de} = -V, \quad Q = \dot{V}$$

- identical at background level
  distinguish by evolution of perturbations
  need physical (covariant) model for interaction

Generalised Chaplygin gas

Kamenshchik, Moschella and Pasquier (2001); Bento Bertolami & Sen (2002)

- exotic dark energy with barotropic equation of state:

$$P_{\rm gCg} = -A\rho_{\rm gCg}^{-\alpha}$$

  - *two constants (dimensionless, α, and A)*

  - unified dark matter + dark energy model

$$\begin{aligned}\rho_{\rm gCg} &= \left(A + Ba^{-3(1+\alpha)}\right)^{1/(1+\alpha)} \\ &\to B^{1/(1+\alpha)}a^{-3} \quad \text{as } a \to 0 \\ &\to A^{1/(1+\alpha)} \quad \text{as } a \to +\infty\end{aligned}$$

  - can be related to generalised higher-dimensional DBI scalar field

Decomposed Chaplygin gas

Bento, Bertolami and Sen (2004)

$$u_m^\mu = u^\mu \ , \quad \rho_m = \rho_{\rm de} + P_{\rm de} \ , \quad V = -P_{\rm de}$$

- FRW interaction can be written as

$$Q = 3\alpha H \left(\frac{\rho_m V}{\rho_m + V} \right)$$

  - *model has one dimensionless parameter,* α
  - *A appears as an integration constant*

$$A = (\rho_m + V)^\alpha V$$

- decomposed model allows two independent perturbations
  - *matter perturbations:* $$\zeta_m = -\psi - \frac{H}{\dot\rho_m}\delta\rho_m$$
  - *vacuum perturbations:* $$\check\zeta = -\psi - \frac{H}{\dot V}\delta V$$

Two different perturbed models

- Barotropic (adiabatic) model, $V=V(\rho_m)$:

$$\check{\zeta} = \zeta_m \quad \Rightarrow \quad \check{S} = 0$$

  - *adiabatic sound speed* $$c_s^2 = \frac{\dot{P}_{\rm gCg}}{\dot{\rho}_{\rm gCg}} = \frac{\alpha V}{\rho_m + V}$$
  - *comoving vacuum perturbation* $$f = -\delta\check{\rho}_{\rm com} = -\frac{\dot{V}}{\dot{\rho}_m}\delta\rho_{\rm com}$$

- Non-adiabatic: e.g., energy transfer along matter 4-velocity, $Q_\nu = Q\, u_\nu$

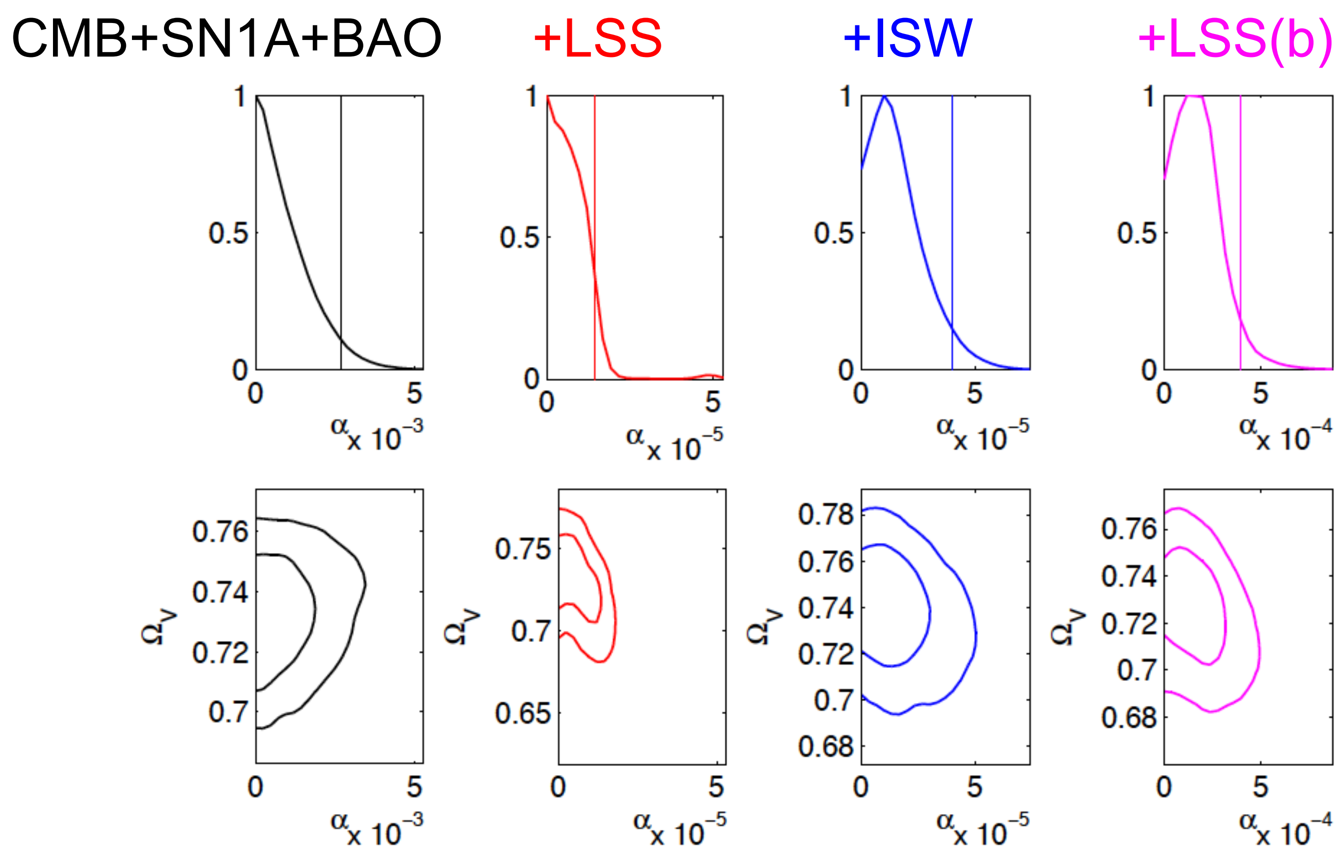
$$\check{\zeta} \neq \zeta_m \quad \Rightarrow \quad \check{S} \neq 0$$

  - *zero momentum transfer:* $f = -\delta\check{\rho}_{\rm com} = 0$
  - *zero sound speed:* $\delta\check{P}_{\rm com} = 0 \quad \Rightarrow \quad c_s^2 = \left(\frac{\delta P}{\delta\rho}\right)_{\rm com} = 0$

Barotropic GCG model (adiabatic perturbations):

- Oscillations (or blow-up) of GCG power spectrum if DE and DM combined in a single barotropic fluid with adiabatic sound speed $c^2 = -\alpha w$

- Allowed GCG model extremely close to the LCDM model ($\alpha \rightarrow 0$)



Effect on density power spectrum:

"The end of unified dark matter?"

Havard Sandvik, Max Tegmark , Matias Zaldarriaga, Ioav Waga astro-ph/0212114

$$c^2 = - \alpha w$$

$$\boxed{-0.00000081 < \alpha < 0.0000079}$$

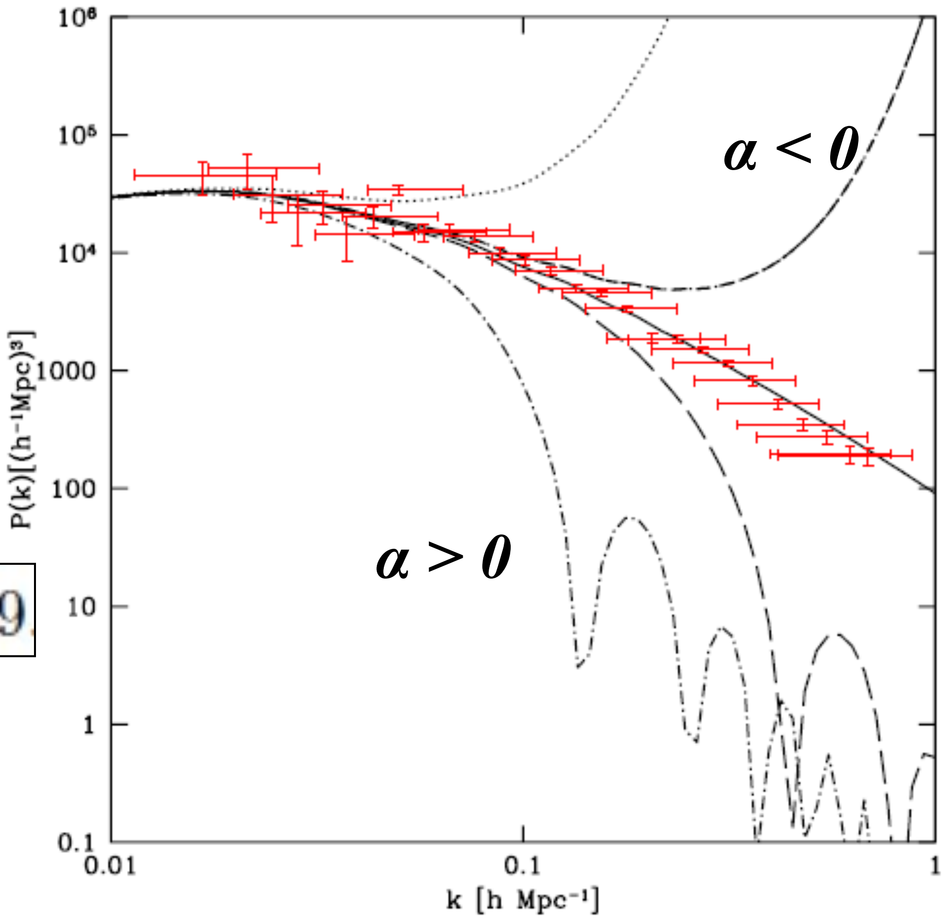
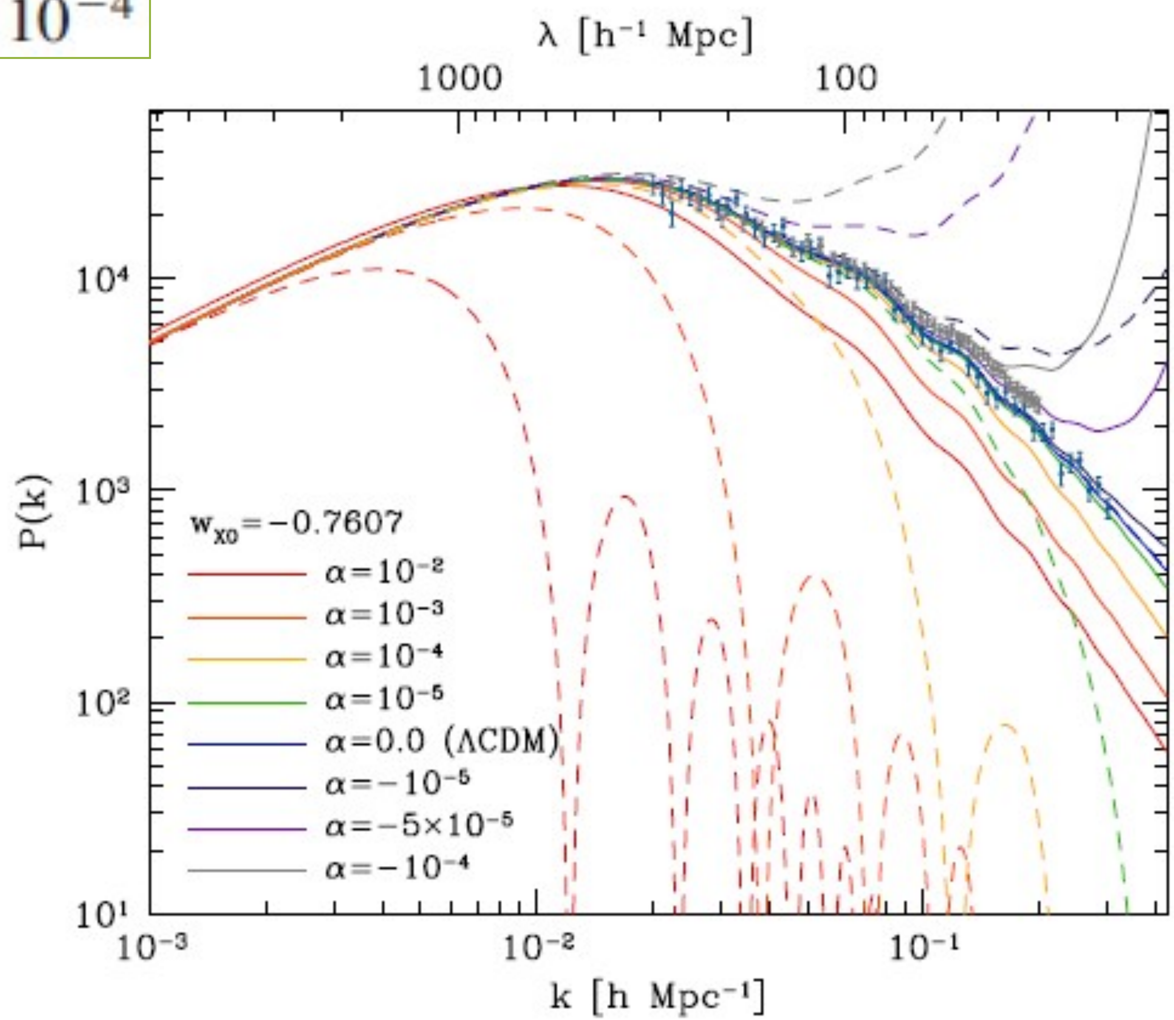


FIG. 1. UDM solution for perturbations as function of wavenumber, k. From top to bottom, the curves are GCG models with $\alpha = -10^{-4}$, -10^{-5}, 0 (ΛCDM), 10^{-5} and 10^{-4}, respectively. The data points are the power spectrum of the 2df galaxy redshift survey.

Park, Hwang, Park & Noh, arXiv:0910.4202

——baryon power spectra behave relatively smoothly, compared with the GCG power spectra which shows large oscillations

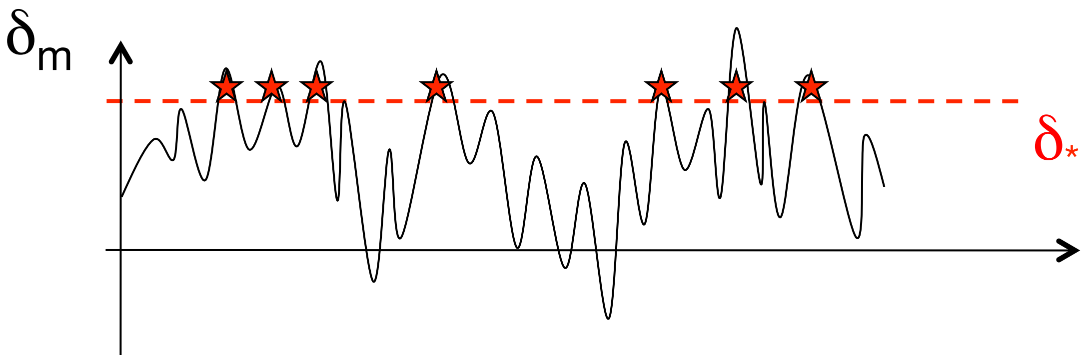
$|\alpha| \lesssim 10^{-4}$



Issues for unified dark matter models:

Galaxy power spectrum: $\delta_g = b\,\delta_m$

identify galaxies with collapsed halos in linear density field



should we include inhomogeneous dark energy in threshold for collapse?

what about unified dark matter models?

$$\delta_m = \frac{\delta\rho_c + \delta\rho_b}{\rho_c + \rho_b}\ ?$$

or interacting matter + vacuum?

$$\delta_m = \frac{\delta\rho_c + \delta\rho_b + \delta V}{\rho_c + \rho_b + V}\ ??$$

Geodesic (non-adiabatic) model:

Energy flow is along dark matter velocity, $Q_\nu = Q\, u_\nu$

- No momentum exchange in the dark matter rest frame

  $\Rightarrow$ matter follows geodesics $\quad f = -\delta\check{\rho}_{\rm com} = 0$

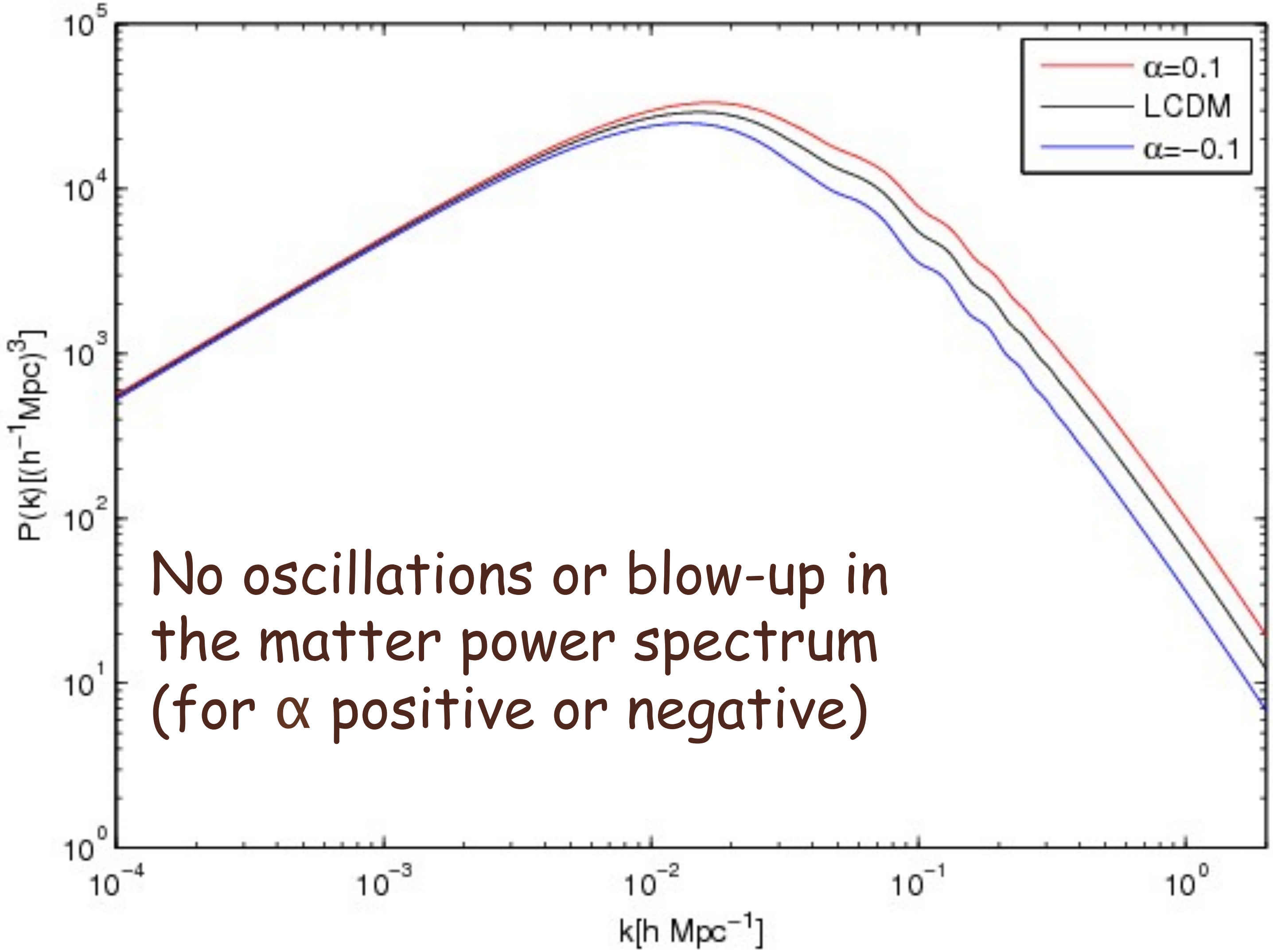
  $\Rightarrow$ zero sound speed $\quad \delta\check{P}_{\rm com} = 0$

  $\Rightarrow$ note: matter velocity *irrotational* (like a scalar field)

$$u_\mu \propto \nabla_\mu V$$

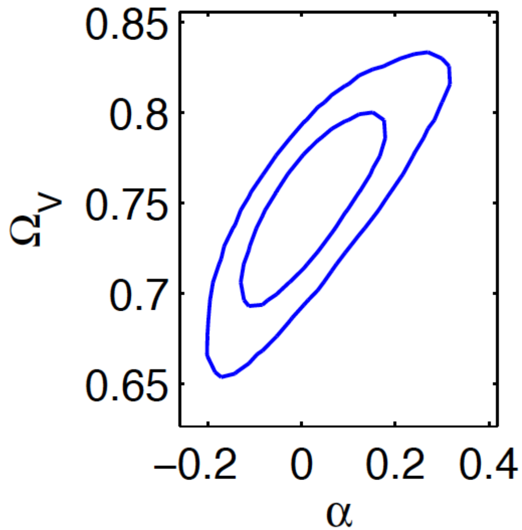
See also Dust of dark energy, Lim, Sawicki & Vikman, arXiv:1003.5751
+ Creminelli, d'Amico, Norena, Senatore & Vernizzi, arXiv:0911.2701

Results : Matter power spectrum, P(k)

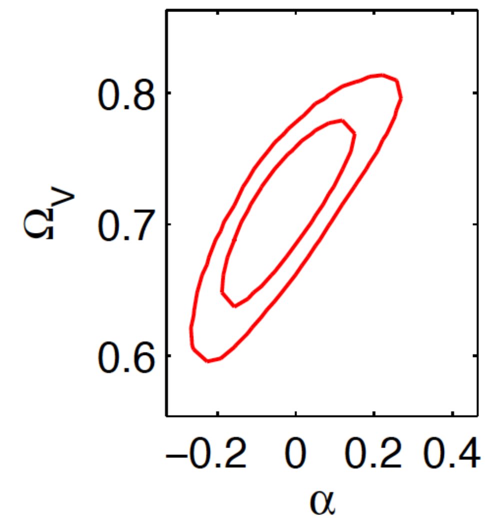


No oscillations or blow-up in the matter power spectrum (for α positive or negative)

Constraints on geodesic model



CMB+SNIa+BAO



CMB+SNIa+LSS

- Allowed region for α parameter becomes much larger than for barotropic GCG model:
  - barotropic model: $\alpha < 0.000014$
  - geodesic model: $-0.15 < \alpha < 0.26$

Dark energy sound speed

- ***any dark energy cosmology*** can be decomposed into interacting vacuum + barotropic fluid (ρ+P>0)

- fluid sound speed determines dark energy sound speed if energy-flow follows fluid flow, $Q_\mu \propto u_\mu$
(since vacuum then homogenous on comoving-orthogonal hypersurfaces)

- e.g., quintessence is interacting vacuum+stiff fluid (c=1)

- interacting vacuum + fluid provides an effective model for dark energy with ***arbitrary sound speed***

summary:

- **vacuum energy is simplest model for acceleration**
  - no new degrees of freedom

- **inhomogeneous** (space-time dependent) **vacuum implies energy transfer**

- **any dark energy fluid** can be decomposed into **interacting vacuum+fluid** (like scalar field quintessence)

- **inhomogeneous** (space dependent) **perturbations** require **physical model for interactions**
  - distinguish by observational data
  - worked example: decomposed Chaplygin gas


Happy 60th Birthday!