Second order entropic transfer and non-Gaussianity in multi-field inflation

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PRD 85, 103505 (2012), [arXiv:1110.2462];
PRD 85, 083504 (2012), [arXiv:1201.3163]
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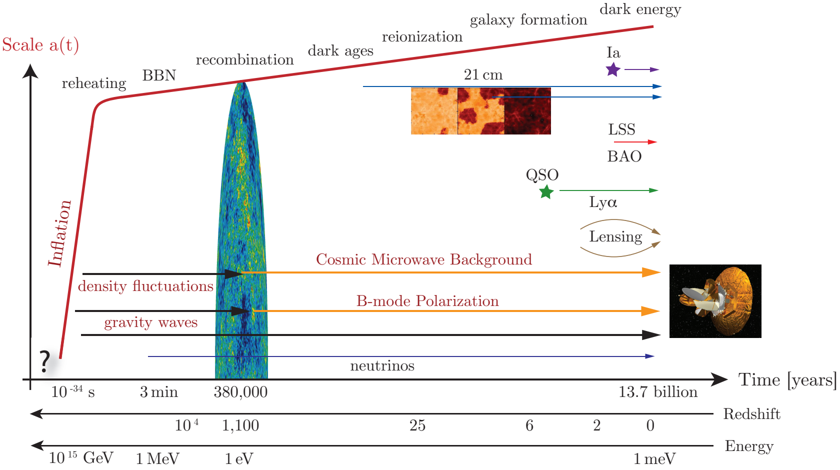
Nishinomiya Yukawa Symposium: New Waves in Gravity and Cosmology
4th December 2012

How do we test inflation?

How were primordial fluctuations generated?

Fig. from Baumann arXiv:0907.5424



Power Spectrum

A very successful explanation is:

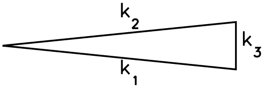
Mukhanov & Chibisov 1981; Guth & Pi 1982; Hawking 1982; Starobinsky 1982; Bardeen, Steinhardt & Turner 1983

- *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.* $\Longrightarrow$ almost Gaussian
- The prediction: a nearly scale-invariant power spectrum in the curvature perturbations, ζ:
  - $P_\zeta(k) = A/k^{4-n_s} \sim A/k^3$
  - where $n_s \sim 1$ and A is a normalization.
  - Two-point function $\langle\hat{\zeta}(\tau, \mathbf{k})\hat{\zeta}(\tau, \mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')P_\zeta(k)$
- The latest results from CMB [WMAP 7-year Komatsu et al. 2011], BAO [SDSS DR7 Percival et al 2010], and SNe Ia [SHOES Riess et al 2009]:
  - $\mathbf{n_s = 0.968 \pm 0.012}$ (68% CL)
  - $n_s \neq 1$: another line of evidence for inflation

Bispectrum

- Is there any information one can obtain, beyond the power spectrum?
  $\Rightarrow$ Three-point function!
- $\langle \hat{\zeta}(\mathbf{k_1})\hat{\zeta}(\mathbf{k_2})\hat{\zeta}(\mathbf{k_3})\rangle = (2\pi)^3\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_\zeta(k_1,k_2,k_3)$
  - $B_\zeta(k_1,k_2,k_3) = (\text{amplitude}) \times \underbrace{b(k_1,k_2,k_3)}_{\text{shape of triangle}}$
- Focus on the squeezed shape for today's talk.

(a) squeezed triangle
$(k_1 \simeq k_2 >> k_3)$



A non-linear correction to temperature anisotropy

- The CMB temperature anisotropy, $\Delta T/T$, is given by the curvature perturbation in the matter-dominated era, Φ.
  - On large scales (the Sachs-Wolfe limit), $\Delta T/T = -\Phi/3 = -\zeta/5$.
- Add a non-linear correction to Φ:
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ [Komatsu & Spergel 2001]
  - f_{NL} was predicted to be small (~ 0.01) for slow-roll inflation. [Salopek & Bond 1990; Gangui et al. 1994]
- For a scale-invariant spectrum, $P_\zeta(k) = A/k^3$, $B_\zeta(k_1, k_2, k_3) = (6A^2/5)f_{NL} \times [1/(k_1k_2)^3 + 1/(k_2k_3)^3 + 1/(k_3k_1)^3]$
  - $B_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$ peaks when $k_3 \ll k_2 \sim k_1$.
  - Therefore, the shape of f_{NL} bispectrum is the **squeezed triangle**! [Babich et al. 2004]

Single-field Consistency Relation

Maldacena 2003; Creminelli & Zaldarriaga 2004; Seery & Lidsey 2005

For **ANY** single-field models*, the bispectrum in the squeezed limit is given by

- $B_\zeta(k_1, k_2, k_3) \approx (1 - n_s) \times P_\zeta(k_1) P_\zeta(k_3)$
- With the current limit $n_s = 0.968$, f_{NL} is predicted to be 0.013.
- Therefore, a convincing detection of $f_{NL} \gg O(1)$ would rule out **ALL** of the single-field inflation models, **regardless of**:
  - the form of potential [See, however, Chen, Easther & Lim 2007]
  - the form of kinetic term (or sound speed) [See, e.g., Seery & Lidsey 2005]
  - the form of gravitational coupling [See, e.g., Germani & YW 2011, 1106.0502]
  - **if** the initial state is BD vacuum [See, e.g., Agullo & Parker 2011; Ganc 2011]
- **Measurements**
  - $f_{NL} = 32 \pm 42$ (95% CL) from CMB [WMAP 7-year Komatsu et al 2011]
  - $f_{NL} = 27 \pm 32$ (95% CL) from CMB and LSS [Slosar et al 2008]
  - Planck's expected error bar is ~ 5 (68% CL)!
- A convincing detection of f_{NL} would be a breakthrough.

If f_{NL} is detected, in what kind of models?

- **Detection of f_{NL} = multi-field models**
- In multi-field inflation models, $\zeta(\mathbf{k})$ can evolve outside the horizon.
  - Curvaton mechanism [Linde & Mukhanov 1997; Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi 2001]
  - Inhomogeneous reheating [Dvali, Gruzinov & Zaldarriaga 2004]
- This evolution can give rise to non-Gaussianity; however, causality demands that the form of non-Gaussianity must be **local**!
  - $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + AS_g(\mathbf{x}) + B[S_g(\mathbf{x})]^2 + \cdots$

How to compute 2nd order ζ?

- Cosmological perturbation theory
  - Straightforward
  - Entropy modes source ζ: "**entropic transfer**" [García-Bellido & Wands 1995]
  - Very hard because 2nd order
- The δN formalism

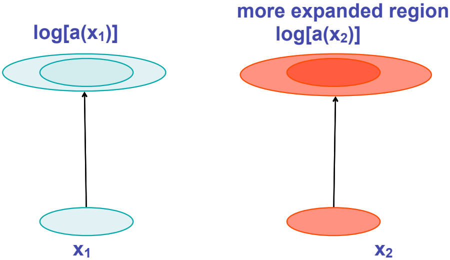
  [Starobinsky 1985; Salopek & Bond 1990; Sasaki & Stewart 1996]

  - $\zeta = \delta N$ on super-horizon scales
  - δN is powerful; it gives the statistics of perturbations *without* solving equations for perturbations!!

    "**It's just like magic.**"
  - Very popular in the literature

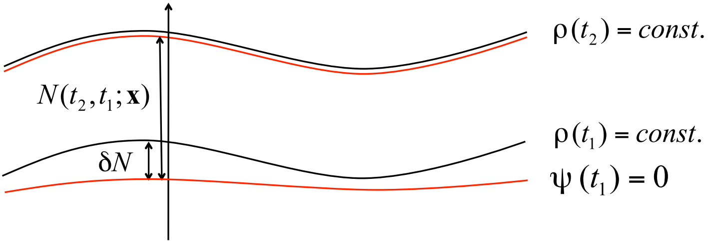
The δN formalism: intuitive picture

- N = number of e-folds counted backward in time (from the end of inflation) $\sim$ log[*expansion*]
  - $a(t_{end})/a(t) = \exp[N] \Rightarrow N(\varphi) = \int_{t(\varphi)}^{t_{end}} H dt = \ln[a(\varphi_{end})/a(\varphi)]$
- Difference in log[expansion] is ζ.



The δN formalism: more precise definition

- $\zeta = \delta N$ from an initial *flat* time slice to a final *uniform density* time slice on super-horizon scales.



$$\delta N = N(t_2, t_1; \mathbf{x}) - N_0(t_2, t_1), \qquad N_0(t_2, t_1) = \ln\left[\frac{a(t_2)}{a(t_1)}\right]$$

δN for slow-roll inflation

Sasaki & Tanaka 1998; Lyth & Rodriguez 2005

- In slow-roll inflation, the evolution, N, is determined only by the field value, φ.
- Non-linear δN for multi-field inflation:

$$\delta N = N(\varphi^I + \delta\varphi^I) - N(\varphi^I) \simeq \sum_I N_{,I}\delta\varphi^I_* + \frac{1}{2}\sum_{I,J} N_{,IJ}\delta\varphi^I_*\delta\varphi^J_*,$$

where derivatives are evaluated at the horizon exit: $N_{,I} \equiv \frac{\partial N}{\partial \varphi^I_*}$.

- Non-Gaussianity is given by

$$\frac{3}{5}f_{NL} = \frac{\sum_{I,J} N_{,I}N_{,J}N_{,IJ}}{2[\sum_I N_{,I}N_{,I}]^2}$$

Linear perturbation theory in multi-field inflation

Kodama & Sasaki 1984; Mukhanov, Feldman & Brandenberger 1992

$$ds^2 = -(1+2A)dt^2 + 2aB_{,i}dx^i dt + a^2[(1-2\psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j$$

- $\delta\varphi^I$'s determine how curvature perturbations, ψ, evolve.

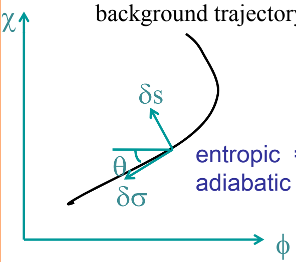
$$\delta\ddot{\varphi}^I + 3H\delta\dot{\varphi}^I + \frac{k^2}{a^2}\delta\varphi^I + \sum_J V_{,IJ}\delta\varphi^J = -2V_{,I}A + \dot{\varphi}^I\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB)\right]$$

$$\begin{aligned}
3H\left(\dot{\psi} + HA\right) + \frac{k^2}{a^2}\left[\psi + H(a^2\dot{E} - aB)\right] &= -4\pi G\delta\rho \\
\dot{\psi} + HA &= -4\pi G\delta q \\
\delta\rho &= \sum_I\left[\dot{\varphi}_I\left(\delta\dot{\varphi}_I - \dot{\varphi}_I A\right) + V_{\varphi_I}\delta\varphi_I\right] \\
\delta q_{,i} &= -\sum_I \dot{\varphi}_I\delta\varphi_{I,i}
\end{aligned}$$

Adiabatic and entropic perturbations

Gordon, Wands, Bassett & Maartens 2000; Nibbelink & van Tent 2001

$$\delta\sigma^{(1)} = \cos\theta\delta\phi + \sin\theta\delta\chi\ , \qquad \delta s^{(1)} = -\sin\theta\delta\phi + \cos\theta\delta\chi$$



entropic = orthogonal
adiabatic = parallel

$$\delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left(\cancel{\frac{k^2}{a^2}} + V_{\sigma\sigma} - \dot{\theta}^2\right)\delta\sigma$$
$$= -2V_\sigma A + \dot{\sigma}\left[\dot{A} + 3\dot{\psi} + \cancel{\frac{k^2}{a^2}(a^2\dot{E} - aB)}\right]$$
$$+ 2(\dot{\theta}\delta s)^{\cdot} - 2\frac{V_\sigma}{\dot{\sigma}}\dot{\theta}\delta s\,,$$

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\cancel{\frac{k^2}{a^2}} + V_{ss} + 3\dot{\theta}^2\right)\delta s = \cancel{\frac{\dot{\theta}}{\dot{\sigma}}\frac{k^2}{2\pi G a^2}}\Psi$$

$$\dot{\sigma} = (\cos\theta)\dot{\phi} + (\sin\theta)\dot{\chi} \qquad \dot{\theta} = -\frac{V_s}{\dot{\sigma}}$$

- Gauge-invariant curvature perturb. is sourced only by entropy perturb. If the trajectory is curved, it can change on large scales.

$$-\zeta \equiv \psi + H\frac{\delta\rho}{\dot{\rho}} \qquad -\dot{\zeta} = \cancel{\frac{H}{\dot{H}}\frac{k^2}{a^2}}\Psi + \frac{2H}{\dot{\sigma}}\dot{\theta}\delta s$$

Two approaches to non-linear ζ

- Covariant formalism (valid on all scales) [Hawking 1966; Ellis, Hwang & Bruni 1989; Langlois & Vernizzi 2005; 2007]
  - Integrated expansion, $N \equiv \frac{1}{3}\int d\tau\Theta$, replaces ζ. ($\dot{N} \equiv \Theta/3$)
  - Non-perturbative generalization (covector) of ζ: $\zeta_\mu \equiv \partial_\mu N - \frac{\dot{N}}{\dot{\rho}}\partial_\mu \rho$
  - Promote perturbations to covectors, e.g., $\delta\sigma \to \sigma_\mu \equiv \cos\theta\partial_\mu\phi + \sin\theta\partial_\mu\chi$, $\delta s \to s_\mu \equiv -\sin\theta\partial_\mu\phi + \cos\theta\partial_\mu\chi$
  - Evolution of the covectors "mimics" linear perturbations:

$$\dot{\zeta}_\mu \equiv \mathcal{L}_u \zeta_\mu = -\frac{\dot{N}}{\rho + P}\left(\partial_\mu P - \frac{\dot{P}}{\dot{\rho}}\partial_\mu \rho\right) \sim s_\mu \text{(super-horizon)}$$

- δN formalism (valid on large scales) [Starobinsky 1985; Salopek & Bond 1990; Stewart & Sasaki 1996; Lyth, Malik & Sasaki 2005]

$$\zeta = \delta N - \int_{\bar{\rho}}^{\rho} \frac{N'}{\rho'} d\rho$$

Are they equivalent on large scales?
If yes, which approach has more advantages?

Fully non-linear equivalence between δN and covariant formalisms

Langlois, Vernizzi & Wands 2008; Suyama, YW & Yamaguchi 1201.3163; Naruko 2012

$$\zeta_\mu \equiv \partial_\mu N - \frac{\dot{N}}{\dot{\rho}}\partial_\mu \rho \qquad \dot{\zeta}_\mu = -\frac{\Theta}{3(\rho+P)}\left(\partial_\mu P - \frac{\dot{P}}{\dot{\rho}}\partial_\mu \rho\right)$$

$\Downarrow$ setting the ADM metric with $\beta_i = \mathcal{O}(\epsilon = k/aH)$ and on Σ_ρ

$$ds^2 = -\mathcal{N}^2 dt^2 + a^2 e^{2\psi}(e^h)_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\dot{\zeta}_i = \frac{1}{\mathcal{N}}\partial_i \psi' + \mathcal{O}(\epsilon^3), \quad -\frac{\Theta}{3(\rho+P)}\left(\partial_i P - \frac{\dot{P}}{\dot{\rho}}\partial_i \rho\right) = -\frac{\tilde{H}}{\bar{\rho}+P}\partial_i P + \mathcal{O}(\epsilon^3)$$

$\Downarrow$ integrating over x^i and choosing an integration constant

$$\psi' = -\frac{\rho'}{3(\rho+P)} - \frac{a'}{a} + \mathcal{O}(\epsilon^2)$$

2nd order ζ on large scales in two-field inflation

Rigopoulos et al 2004; Langlois & Vernizzi 2007

- Adiabatic: $\delta\sigma = \cos\theta\delta\phi + \sin\theta\delta\chi + \delta s\dot{\delta s}/(2\dot\sigma)$

$$\dot\zeta(t) \approx -\frac{2H}{\dot\sigma}\dot\theta(\delta s^{(1)} + \delta s^{(2)}) + \frac{H}{\dot\sigma^2}(V_{ss} + 4\dot\theta^2)\delta s^2 - \frac{H}{\dot\sigma^3}V_\sigma\delta s\dot{\delta s}$$

- Entropic: $\delta s = -\sin\theta\delta\phi + \cos\theta\delta\chi + \delta\sigma(2\dot{\delta s} + \dot\theta\delta\sigma)/(2\dot\sigma)$

$$\ddot{\delta s} + 3H\dot{\delta s} + (V_{ss} + 3\dot\theta^2)\delta s \approx$$
$$-\frac{\dot\theta}{\dot\sigma}(\dot{\delta s})^2 - \frac{2}{\dot\sigma}\left(\ddot\theta + \dot\theta\frac{V_\sigma}{\dot\sigma} - \frac{3}{2}H\dot\theta\right)\delta s\dot{\delta s} - \left(\frac{1}{2}V_{sss} - 5\frac{\dot\theta}{\dot\sigma}V_{ss} - 9\frac{\dot\theta^3}{\dot\sigma}\right)\delta s^2$$

- Solutions:

$$\zeta(t) = \zeta_* + \delta s_* T_\zeta^{(1)}(t) + \delta s_*^2 T_\zeta^{(2)}(t)$$
$$\delta s(t) = \delta s_* T_{\delta s}^{(1)}(t) + \delta s_*^2 T_{\delta s}^{(2)}(t)$$

f_{NL} in two-field inflation

- Covariant (long wavelength) formalism [YW 2012, 1110.2462]

$$f_{NL} = f_{NL}^{\text{horizon}} + f_{NL}^{\text{transfer}} \sim T_\zeta^{(2)} / \left[T_\zeta^{(1)}\right]^2$$

$$\frac{3}{5} f_{NL}^{\text{transfer}} = \frac{4\epsilon_*^2 \left[T_\zeta^{(1)}\right]^2 T_\zeta^{(2)}}{\left[1 + 2\epsilon_* \left(T_\zeta^{(1)}\right)^2\right]^2},$$

$$\frac{3}{5} f_{NL}^{\text{horizon}} = \frac{-(\epsilon\eta_{ss})_* \left[T_\zeta^{(1)}\right]^2 + 3\sqrt{\epsilon_*/2}\,\eta_{\sigma s *} T_\zeta^{(1)} + (\epsilon - \eta_{\sigma\sigma}/2)_*}{\left[1 + 2\epsilon_* \left(T_\zeta^{(1)}\right)^2\right]^2}$$

- δN formalism [Lyth & Rodriguez 2005]

$$\frac{3}{5} f_{NL} = \frac{\sum_{I,J} N_I N_J N_{IJ}}{2[\sum_I N_I N_I]^2}$$

"Large" f_{NL} in two-field inflation

- $f_{NL} \sim \mathcal{O}(1-10)$ for $m_1/m_2 = 1/9$ [Rigopoulos et al 2005];
  $f_{NL} \sim \mathcal{O}(0.01)$ [Vernizzi & Wands 2006; Rigopoulos et al 2006; S. Yokoyama et al 2007]

$$V(\phi,\chi) = \frac{m_1^2}{2}\phi^2 + \frac{m_2^2}{2}\chi^2, \quad m_1 < m_2 \ll H_* \tag{1}$$

- $-f_{NL} \sim \mathcal{O}(1-10)$ [Byrnes et al 2008; Mulryne et al 2009]

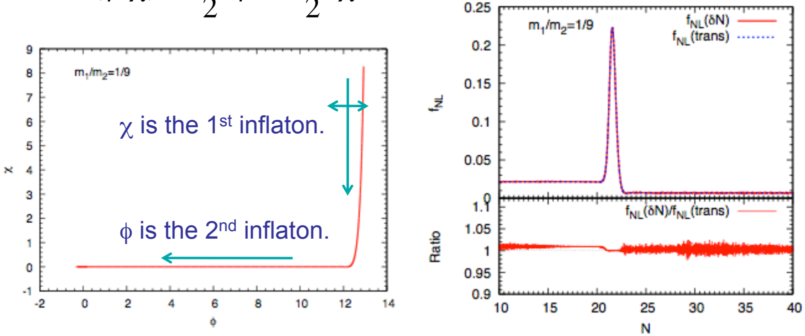
$$V(\phi,\chi) = \frac{m_\chi^2}{2}\chi^2 e^{-\lambda\phi^2} \tag{2}$$

- $f_{NL} \sim \mathcal{O}(1)$ [Tzavara & van Tent 2011]

$$V(\phi,\chi) = a_2\chi^2 + b_0 - b_2\phi^2 + b_4\phi^4 \tag{3}$$

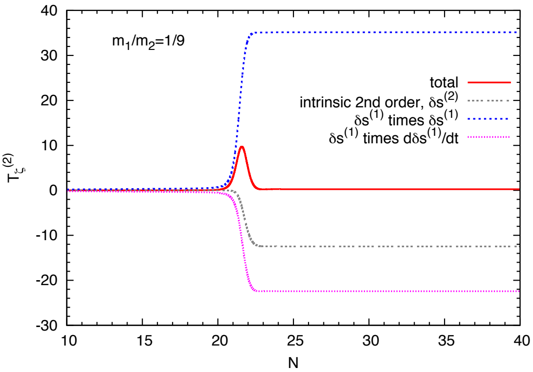
Numerical estimate: f_{NL} in two-field inflation

$$V(\phi,\chi) = \frac{m_1^2}{2}\phi^2 + \frac{m_2^2}{2}\chi^2$$



- A *peak* in NG shows up at the turn. It is sourced by entropy modes.
- The plateau contribution of NG is from the horizon exit *~O(ε) ~0.01*.
- δN and covariant formalisms match within ~ 1%.
- Slow–roll approx. has been used only for the initial condition (at horizon exit).

How did the peak in f_{NL} show up at the turn?



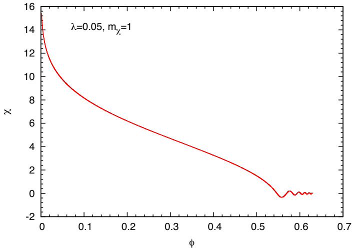
- Each term in 2nd order entropic transfer becomes large but almost cancels out! Only the small, net effect remains due to symmetry of the potential.
- The difference in growths of terms makes the peak shape.

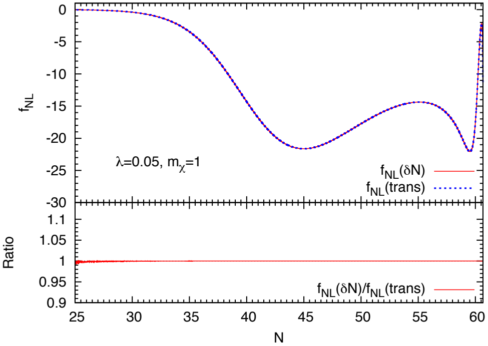
Numerical estimate: f_{NL} in two-field inflation

$$V(\phi, \chi) = \frac{m_\chi^2}{2}\chi^2 e^{-\lambda\phi^2}$$

[Byrnes et al 2008; Mulryne et al 2009; YW 2012]

- Large negative NG shows up during the turn. But $f_{NL} \sim -2.1$ after inflaiton.



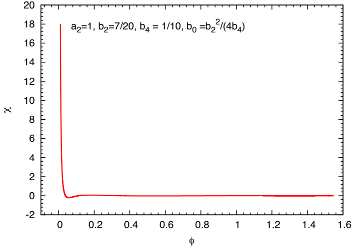


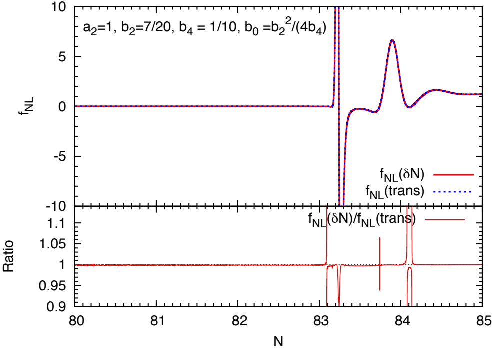
Numerical estimate: f_{NL} in two-field inflation

$$V(\phi,\chi) = a_2\chi^2 + b_0 - b_2\phi^2 + b_4\phi^4$$

[Tzavara & van Tent 2011; YW 2012]

- $f_{NL} \sim 1.2$. The ratio is ill-defined when f_{NL} crosses zeros.





Fate of f_{NL} in the adiabatic limit

Meyers & Sivanandam 2011; YW 2012

- It is possible to have large f_{NL} in two-field inflation; the asymptotic values are model-dependent.
- After all entropy modes decay, the inflationary trajectory approaches to **the adiabatic limit** in which ζ is conserved and one can make predictions for observations.
- In this case, there are two regimes:
  - Initially, entropy modes are light and source ζ.
  - Eventually, they get heavy and damps away.

$\Downarrow$

How fast f_{NL} approaches to its final value?

Analytic estimate: f_{NL} in the adiabatic limit

YW 2012, 1110.2462

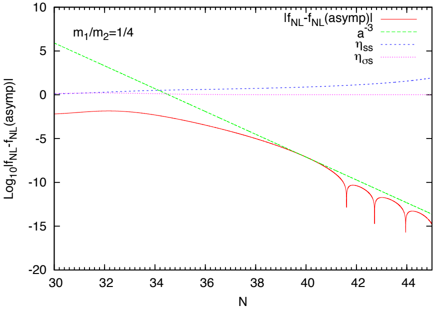
$$
\begin{aligned}
|\delta s^{(1)}| &\approx \frac{2^{\mathrm{Re}(\nu)-3/2}}{\sqrt{2H}} \frac{|\Gamma(\nu)|}{\Gamma(3/2)} a^{-3/2} \left(\frac{k}{aH}\right)^{-\mathrm{Re}(\nu)} \\
\nu &= \sqrt{\frac{9}{4} - \left(3\eta_{ss} + \frac{3\dot{\theta}^2}{H^2}\right)}
\end{aligned}
$$

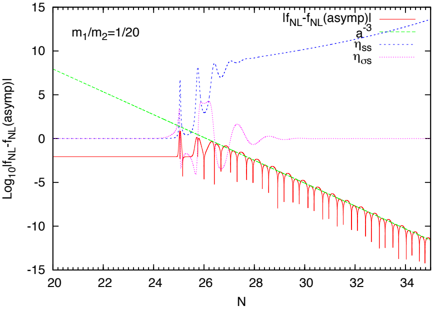
To answer the fate of f_{NL}, we solve the super-Hubble evolution of ζ in 3 cases:

- (A) Overdamped (light) $\delta s \sim a^{-\eta_{ss}}$: $\eta_{ss} \ll 3/4$ and $(\dot{\theta}/H)^2 \ll 3/4$ (slow-roll & slow-turn $\sim$ const.) $\Rightarrow f_{NL} \sim \zeta^{(2)}/\zeta^2_{\mathrm{asymp}} \sim a^{-2\eta_{ss}}$
- (B) Underdamped (heavy) $\delta s \sim a^{-3/2}$: $\eta_{ss} \gg 3/4$ and $(\dot{\theta}/H)^2 \ll 3/4$ (slow-roll & slow-turn $\dot{\theta}/H \sim \eta_{\sigma s} \sim a^{-\eta_{ss}}$) $\Rightarrow f_{NL} \sim \zeta^{(2)}/\zeta^2_{\mathrm{asymp}} \sim a^{-3}$
- (C) Underdamped (heavy) $\delta s \sim a^{-3/2}$: $\eta_{ss} \gg 3/4$ and $(\dot{\theta}/H)^2 \gg 3/4$ (fast-turn $\dot{\theta}/H \sim a^{-3/2}$) $\Rightarrow f_{NL} \sim \zeta^{(2)}/\zeta^2_{\mathrm{asymp}} \sim a^{-3}$

Comparison of f_{NL} in the adiabatic limit

YW 2012, 1110.2462





- The analytic estimate of f_{NL} matches with numerical results with various potentials.

Conclusions

- We have showed the super-Hubble evolution of the primordial NG in two-field inflation by taking two approaches: the δN and the covariant perturbative formalisms.
- The numerical results agree each other within 1% accuracy.
- The peak feature appears on f_{NL} at the turn in the field space, which can be understood as the cancellation between terms in the entropic transfer.
- It is possible but difficult to have persistently large NG in two-field inflation.
- f_{NL} approaches to its asymptotic value as a^{-3} in the adiabatic limit for generic potentials.