Scale-dependent bias with primordial non-Gaussianities

Shuichiro Yokoyama (ICRR, Univ. of Tokyo)

collab. with Taka Matsubara (KMI, Nagoya Univ.), …

SY and Matsubara, arXiv:1210.2495

and also SY, JCAP 1111(2011)001
Gong and SY, MNRAS 417(2011)L79

Congratulations on 60th birthday, Sasaki-san!

Thanks for encouragement and useful discussion in my PhD days (my "delta N" life)!

Contents

- Local-type primordial non-Gaussianity
  - fNL, gNL, tauNL,…
- Formulation for the scale-dependent bias
- Stochastic bias (related with fNL and tauNL)
- Higher order corrections
- Summary

Simplest parameterization

- **Focusing on Local type non-Gaussianity**

(Komatsu & Spergel(2001), Byrnes, Sasaki & Wands(2006), ...)

primordial curvature fluctuations

$$\Phi = \underline{\Phi_{\rm G}} + \underline{f_{\rm NL}} \left(\Phi_{\rm G}^2 - \langle \Phi^2 \rangle\right) + \underline{g_{\rm NL}} \Phi_{\rm G}^3 + \cdots$$

Gaussian fluc. $\sim 10^{-5}$

non-linearity parameters

⇒ **Non-zero higher order spectra**

(higher order correlation functions)

leadingly, ...

- Bispectrum (3-point corr. func.) ⟺ $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3) \rangle$

$$B_\Phi(k_1, k_2, k_3) = 2 f_{\rm NL} \left[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_3)P_\Phi(k_1)\right]$$

- Trispectrum (4-point corr. func.) ⟺ $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4) \rangle_c$

Importance of trispcetrum

- **Trispectrum**

(Byrnes, Sasaki & Wands(2006), Boubekeur & Lyth(2006) …)

$$\langle \Phi_{\mathbf{k}_1} \Phi_{\mathbf{k}_2} \Phi_{\mathbf{k}_3} \Phi_{\mathbf{k}_4} \rangle_c = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) T_\Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

$$T_\Phi(k_1, k_2, k_3, k_4) = 6 g_{\rm NL} \left[P_\Phi(k_1) P_\Phi(k_2) + 2 \text{ perms.} \right] + \frac{25}{9} \tau_{\rm NL} \{ P_\Phi(k_1) \left[P_\Phi(k_{13}) + P_\Phi(k_{23}) \right] + 10 \text{ perms.} \}$$

need two terms (different momentum-dependence)

Suyama and Yamaguchi (2008), …

We can generalize …

e.g.)

$$\Phi = \phi_{\rm G} + \psi_{\rm G} + f_{\rm NL} \left(\phi_{\rm G}^2 - \langle \phi_{\rm G}^2 \rangle \right)$$

$$\langle \phi_{\rm G} \psi_{\rm G} \rangle = 0$$

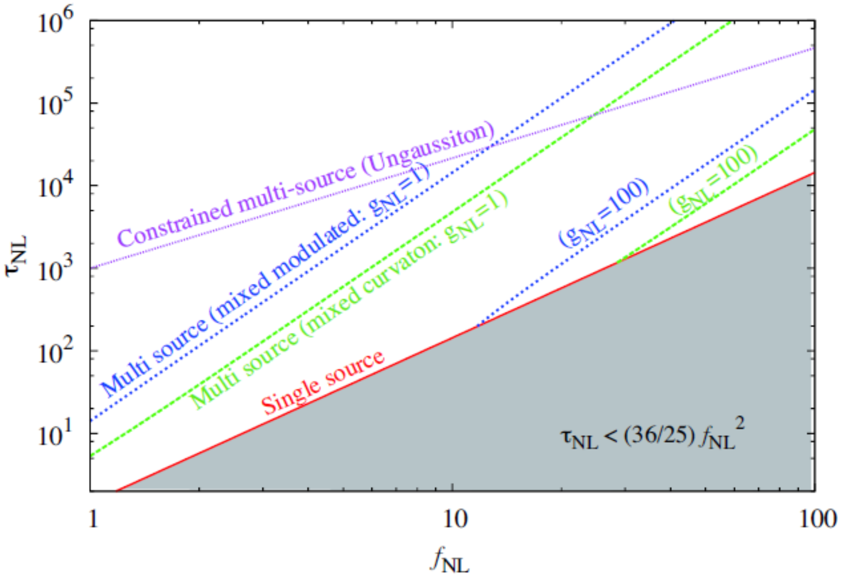
$$R \equiv P_\phi / P_\psi$$

$$\tau_{\rm NL} = \left(\frac{1+R}{R} \right) \left(\frac{6}{5} f_{\rm NL} \right)^2$$

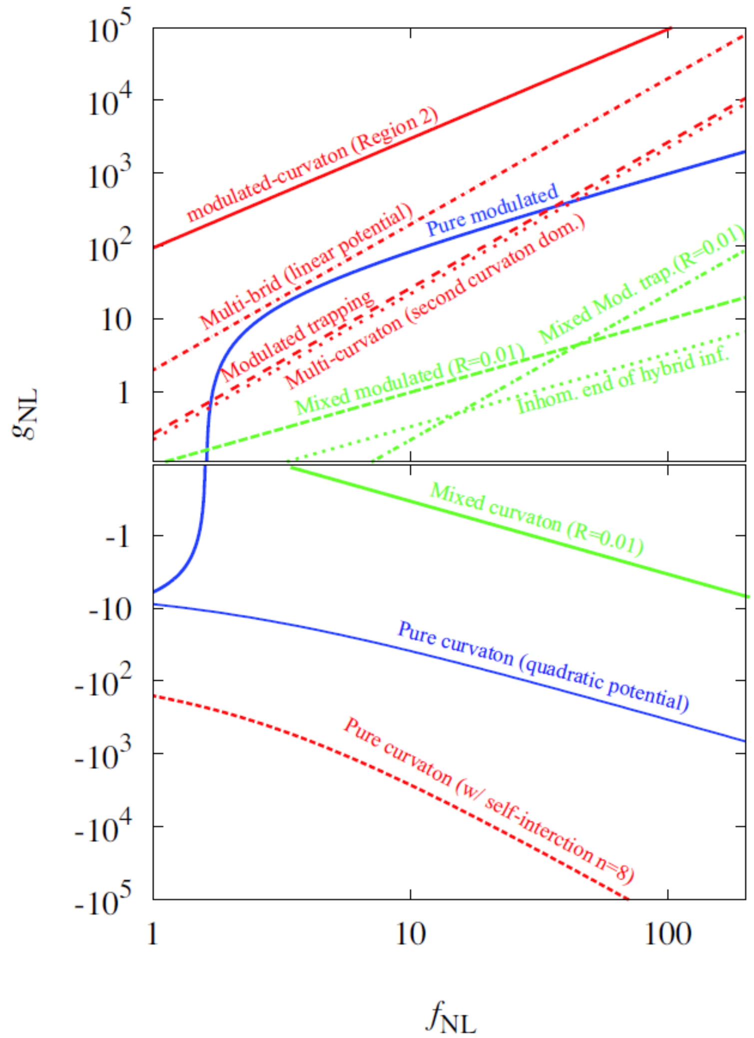
$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL} \right)^2$$

(Suyama, Takahashi, Yamguchi and SY (2010))

fNL vs tauNL



fNL vs gNL



Different lines represent different models

distinguishing models !!!

How accuracy can we measure these parameters with using cosmological observations?

Observational constraints

- From CMB observations (CMB trispectrum)

Smidt et al.(2010), Fergusson et al.(2010)

$$-7.4 \times 10^5 < g_{\rm NL} < 8.2 \times 10^5$$

$$-0.6 \times 10^4 < \tau_{\rm NL} < 3.3 \times 10^4$$

95%CL. with WMAP data

- Large scale structure?

How can we see these non-Gaussian effects ?

Structure formation
in non-Gaussian universe

- scale-dependent bias
with primordial non-Gaussianity -

Simple picture

- Halo bias **bias = P_h / P_m**

•matter density fluctuations (matter power spectrum; P_m(k))

⇩ ***biased***

•Halo number density fluctuations (halo power spectrum; P_h(k))

(peaky objects)

Gaussian + linear ➔ linear (constant) bias

Kaiser (1984), Bardeen, Bond, Kaiser and Szalay (1986), Sheth, Mo and Thormen(2001), ...

⇕

non-Gaussian case ? ➔ scale-dependent bias

Dalal et al.(2008), Slosar et al.(2008), ...

Simple picture

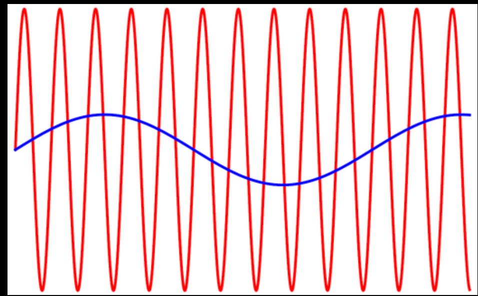
$$\Phi = \Phi_G + f_{NL}\Phi_G^2$$

decomposition;

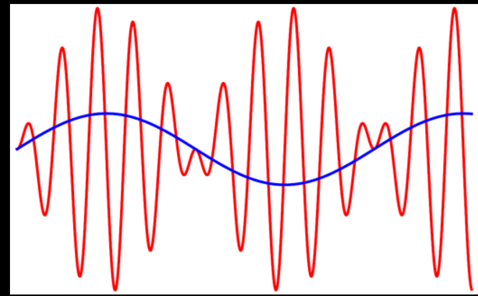
$$\Phi_G = \Phi_{long} + \Phi_{short}$$

due to the non-Gaussianity

➔ long-short mode coupling !



Gaussian

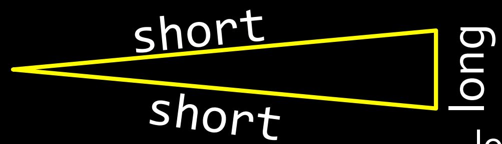


non-Gaussian

Blue; long mode (background)

Red; short mode (peak)

• collapsed objects ⇔ short mode fluctuations



local-type (in k-space)

$$\delta_{short} = \delta_G(1 + 2f_{NL}\Phi_{long}(x))$$

$$\sigma_{short}^2 \equiv \langle \delta_{short}^2 \rangle$$;variance

$\Phi_{long} \propto \delta_{long}/k^2$

$\Phi \rightarrow \delta$

Poisson eq. $\Delta\Phi = 4\pi G\rho\delta$

➔ modulation of the variance of the short mode !!

Through the local-type non-Gaussianity, the formation process of the objects can be modified ➔ scale-dependence of the bias

Toward precision cosmology

need more precise theoretical prediction

- not so high-peak objects
- long- and short- decomposition is right?
- GR effects? (Roy Maartens-san's talk)

➡

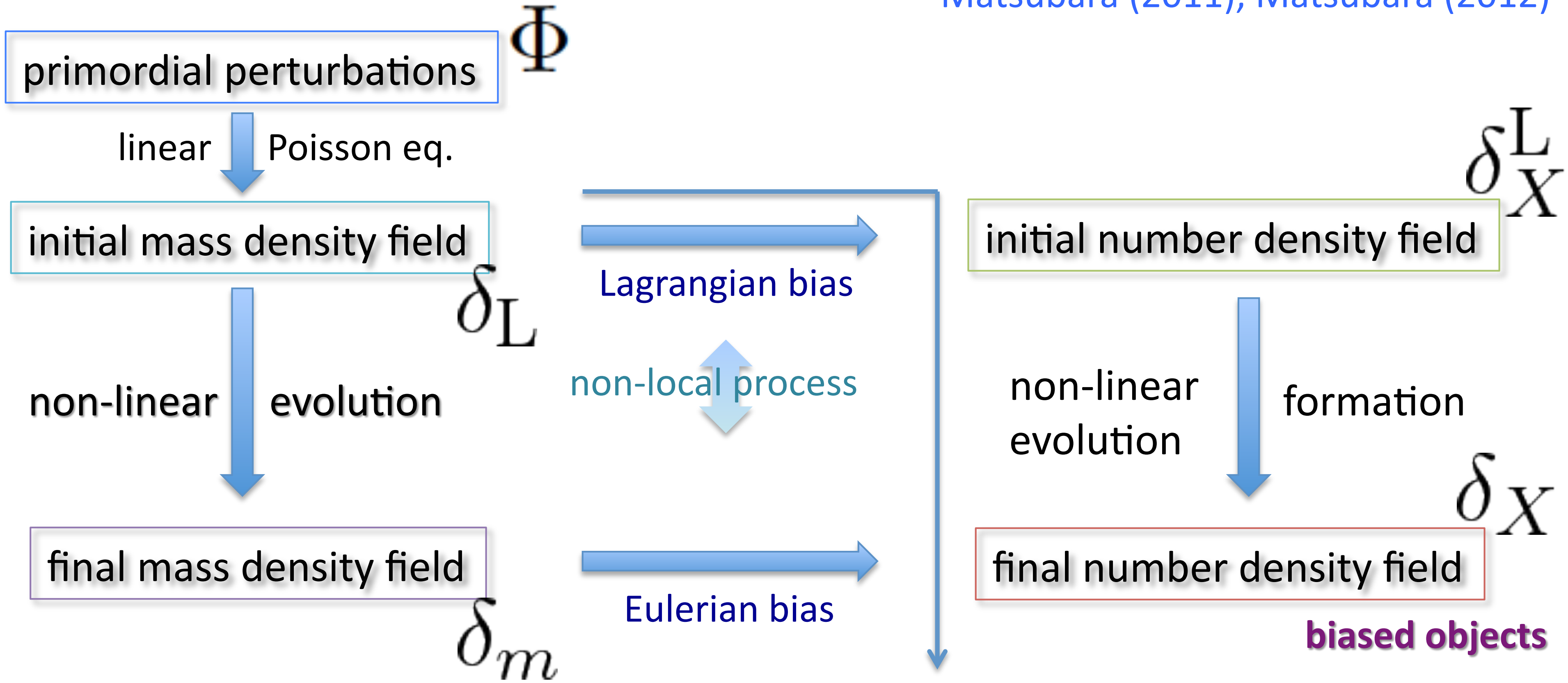
- using N-body simulation
- deriving accurate analytic formula

e.g., Desjacques + (2011) ➔ mass dependence, scale-independent corrections

Scoccimarro + (2012)

Integrated Perturbation Theory (iPT)

Matsubara (2011), Matsubara (2012)



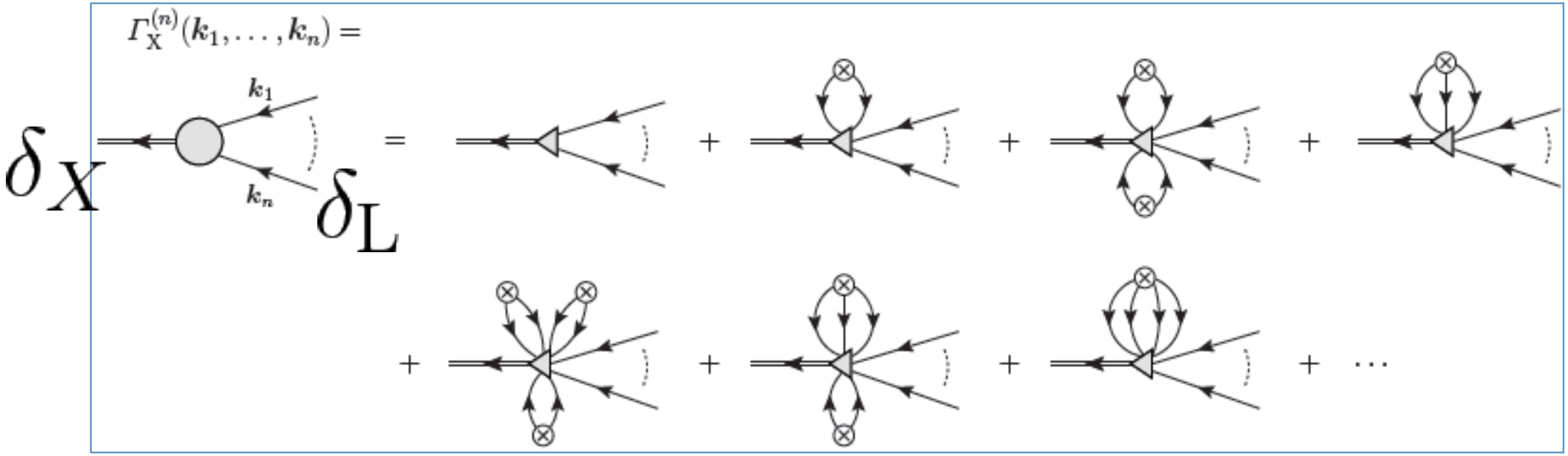
Non-linear perturbation theory integrated with non-local bias, redshift-distortions, and primordial non-Gaussianity

without high peak limit and peak-background split picture

Integrated Perturbation Theory (iPT)

- Introducing multi-point propagators

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta\delta_{\mathrm{L}}(\mathbf{k}_1)\cdots\delta\delta_{\mathrm{L}}(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta^{(3)}(\mathbf{k}_1+\cdots+\mathbf{k}_n-\mathbf{k})\Gamma_X^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n),$$

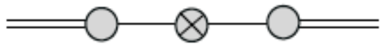


➔ response of the final number density field to the initial mass density field

Power spectrum of the biased objects

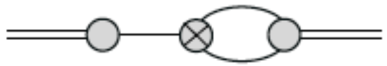
Up to the leading order contributions from fNL, gNL and tauNL

$$P_X(k) = \left[\Gamma_X^{(1)}(\mathbf{k})\right]^2 P_{\rm L}(k)$$



Gaussian case

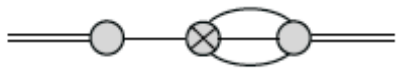
$$+\Gamma_X^{(1)}(\mathbf{k}) \int \frac{d^3p}{(2\pi)^3} \Gamma_X^{(2)}(\mathbf{p}, \mathbf{k}-\mathbf{p}) B_{\rm L}(\mathbf{k}, -\mathbf{p}, -\mathbf{k}+\mathbf{p})$$



←→ fNL

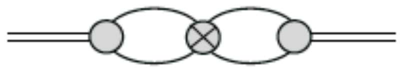
$$+\frac{1}{3}\Gamma_X^{(1)}(\mathbf{k}) \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \Gamma_X^{(3)}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}-\mathbf{p}_1-\mathbf{p}_2) T_{\rm L}(\mathbf{k}, -\mathbf{p}_1, -\mathbf{p}_2, -\mathbf{k}+\mathbf{p}_1+\mathbf{p}_2)$$

←→ gNL



$$+\frac{1}{4} \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \Gamma_X^{(2)}(\mathbf{p}_1, \mathbf{k}-\mathbf{p}_1) \Gamma_X^{(2)}(-\mathbf{p}_2, -\mathbf{k}+\mathbf{p}_2) T_{\rm L}(\mathbf{p}_1, \mathbf{k}-\mathbf{p}_1, -\mathbf{p}_2, -\mathbf{k}+\mathbf{p}_2)$$

←→ tauNL



We can also consider the higher order effects order by order.

$$\Gamma_X^{(n)}(\mathbf{k}_1, \cdots, \mathbf{k}_n)$$

- Multi-point propagators depend on
  - non-linear evolution of the matter density field (initial field ←→ final field)
  - non-linear bias function (matter density field ←→ number density field)
- Introducing renormalized bias functions

$$c_n^{\mathrm{L}}(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n) = (2\pi)^{3n} \int \frac{d^3p}{(2\pi)^3} \langle \frac{\delta^n \delta_X^{\mathrm{L}}(\mathbf{p})}{\delta\delta_{\mathrm{L}}(\mathbf{k}_1) \cdots \delta\delta_{\mathrm{L}}(\mathbf{k}_n)} \rangle$$

← can be written in terms of the standard Lagrangian local bias; b_n^{L}
(obtained from the mass function)

$$\Gamma_{\mathrm{X}}^{(1)}(\boldsymbol{k}) = 1 + c_1^{\mathrm{L}}(k)$$

$$\Gamma_{\mathrm{X}}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{{k_1}^2}\right) c_1^{\mathrm{L}}(k_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{{k_2}^2}\right) c_1^{\mathrm{L}}(k_1) + c_2^{\mathrm{L}}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

non-linear evolution of the matter density field;

$$F_n(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n) \to 0$$

$$\mathbf{k}_1 + \mathbf{k}_2 + \cdots + \mathbf{k}_n \to 0$$ (on large scales)

Scale-dependent bias with fNL, gNL and tauNL

- Bias parameter

$$P_X(k) \equiv b_X(k) P_{\rm L}(k)$$

on large scales $(k \to 0)$, $b_X(k) \approx b_1(M)^2 + \Delta b(k)$

Scale-dependent part;

$$\begin{aligned}\Delta b(k) \approx\ & 4 f_{\rm NL} \frac{b_1(M)}{\mathcal{M}(k)} \left[b_2^{\rm L}(M) + 2 \frac{1+\delta_c(b_1(M)-1)}{\delta_c^2} \right] \sigma_M^2 \\ & + \left(g_{\rm NL} + \frac{25}{27}\tau_{\rm NL} \right) \frac{b_1(M)}{\mathcal{M}(k)} \\ & \quad \times \left[b_3^{\rm L}(M) + \frac{2+2\delta_c(b_1(M)-1)+\delta_c^2 b_2^{\rm L}(M)}{\delta_c^3} \left(4 + \frac{d\ln S_3(M)}{\ln \sigma_M} \right) \right] \sigma_M^4 S_3(M) \\ & + \frac{25}{9}\tau_{\rm NL} \frac{1}{\mathcal{M}(k)^2} \left[b_2^{\rm L}(M) + 2\frac{1+\delta_c(b_1(M)-1)}{\delta_c^2} \right]^2 \sigma_M^4,\end{aligned}$$

$$S_3(M) = \frac{\langle \delta_M^3 \rangle}{\langle \delta_M^2 \rangle^2} \quad \delta_c = 1.68 \quad b_1(M) = 1 + b_1^{\rm L}(M)$$

Matsubara(2012), SY and Matsubara(2012)

Scale-dependent bias with fNL, gNL and tauNL

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Scale-dependent part;

$$\Delta b(k) \approx 4 f_{\rm NL} \frac{b_1(M)}{\mathcal{M}(k)} \left[b_2^{\rm L}(M) + 2 \frac{1+\delta_c(b_1(M)-1)}{\delta_c^2} \right] \sigma_M^2$$

→ 1/k^2 - dependence

$$\delta_{\rm L} = \mathcal{M}(k)\Phi(k) \ , \ \mathcal{M}(k) \propto k^2$$

from Poisson eq.

Large enhancement on large scales (k → 0)

Scale-dependent bias with fNL, gNL and tauNL

- Bias parameter

$$P_X(k) \equiv b_X(k) P_{\rm L}(k)$$

on large scales $(k \to 0)$, $\boxed{b_X(k) \approx b_1(M)^2 + \Delta b(k)}$

Scale-dependent part;

$$\Delta b(k) \approx$$

$$+\left(g_{\rm NL} + \frac{25}{27}\tau_{\rm NL}\right)\frac{b_1(M)}{\mathcal{M}(k)}$$

$$\times\left[b_3^{\rm L}(M) + \frac{2 + 2\delta_c(b_1(M)-1) + \delta_c^2 b_2^{\rm L}(M)}{\delta_c^3}\left(4 + \frac{d\ln S_3(M)}{\ln\sigma_M}\right)\right]\sigma_M^4 S_3(M)$$

→ 1/k^2 - dependence

Scale-dependent bias with fNL, gNL and tauNL

- Bias parameter

$$P_X(k) \equiv b_X(k) P_{\rm L}(k)$$

on large scales $(k \to 0)$, $\boxed{b_X(k) \;\approx\; b_1(M)^2 + \Delta b(k)}$

Scale-dependent part;

$$\Delta b(k) \;\approx$$

$$+\frac{25}{9}\tau_{\rm NL}\frac{1}{\mathcal{M}(k)^2}\left[b_2^{\rm L}(M) + 2\frac{1+\delta_c(b_1(M)-1)}{\delta_c^2}\right]^2 \sigma_M^4,$$

→ 1/k^4 - dependence

Higher order local bias parameters-dependence

- Bias parameter

$$P_X(k) \equiv b_X(k) P_{\rm L}(k)$$

on large scales $(k \to 0)$, $\boxed{b_X(k) \;\approx\; b_1(M)^2 + \Delta b(k)}$

Scale-dependent part;

$$\Delta b(k) \;\approx\; 4 f_{\rm NL} \frac{b_1(M)}{\mathcal{M}(k)} \left[b_2^{\rm L}(M) + 2 \frac{1 + \delta_c (b_1(M) - 1)}{\delta_c^2} \right] \sigma_M^2$$

→ k^-2-dependence

$$+ \left(g_{\rm NL} + \frac{25}{27} \tau_{\rm NL} \right) \frac{b_1(M)}{\mathcal{M}(\ldots}$$

$$\times \left[b_3^{\rm L}(M) + \frac{2+2\ldots}{\ldots} \right.$$

$$+ \frac{25}{9} \tau_{\rm NL} \frac{1}{\mathcal{M}(k)^2} \left[b_2^{\rm L}(M \ldots \right.$$

In general, we need to know higher order Lagrangian local bias parameters !!
(c.f. for Press-Schechter mass function, b_n can be written in b_n-1;

well-known result; $\Delta b = 4 f_{\rm NL} \dfrac{(b_1^{\rm L} + 1) b_1^{\rm L}}{\mathcal{M}(k)}$

$$S_3(M) = \frac{\langle \delta_M^3 \rangle}{\langle \delta_M^2 \rangle^2} \qquad \delta_c = 1.68 \qquad b_1(\ldots$$

Matsubara(2012), SY and Matsubara(2012)

Stochastic bias

S-Y inequality in the structure formation

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$$

- fNL vs tauNL ; inequality $\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$

Introducing a stochasticity parameter;

$$r(k) \equiv \frac{P_{\rm m}(k) P_X(k)}{P_{{\rm m}X}(k)^2}.$$

$P_{{\rm m}X}(k)$; matter density field – biased objects cross power spectrum

$P_{\rm m}(k)$; matter density field power spectrum

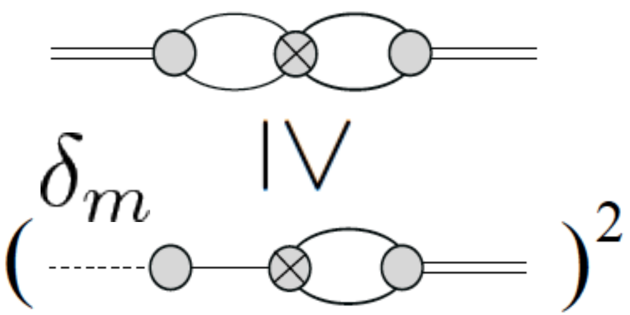
On large scales,

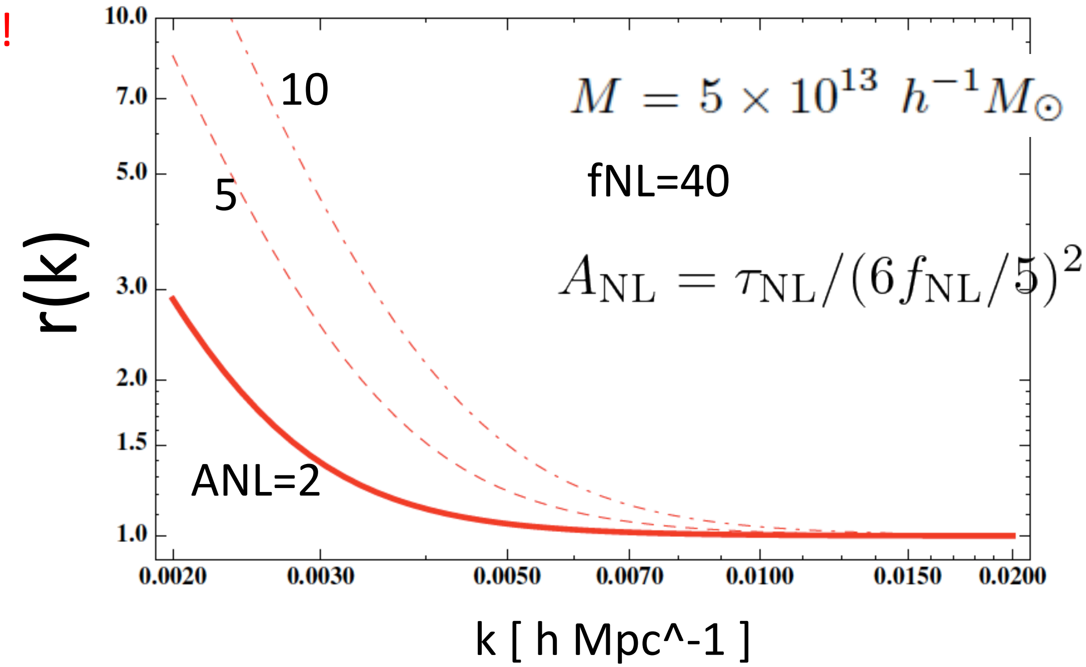
$$r(k) \simeq 1 + \left(\frac{25}{9}\tau_{\rm NL} - 4 f_{\rm NL}^2\right) \frac{1}{b_1(k)^2 \mathcal{M}(k)^2} \left[\int \frac{d^3p}{(2\pi)^3} c_2^{\rm L}(\mathbf{p}, -\mathbf{p}) P_{\rm L}(p)\right]^2$$

Directly dependent on the inequality !

r(k)-1 = 0, or < 0, or > 0 ??

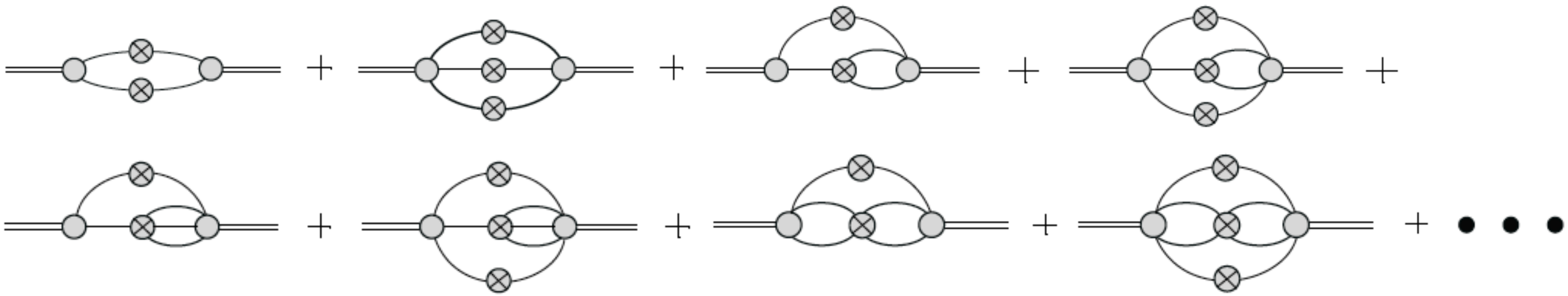
Diagrammatically,



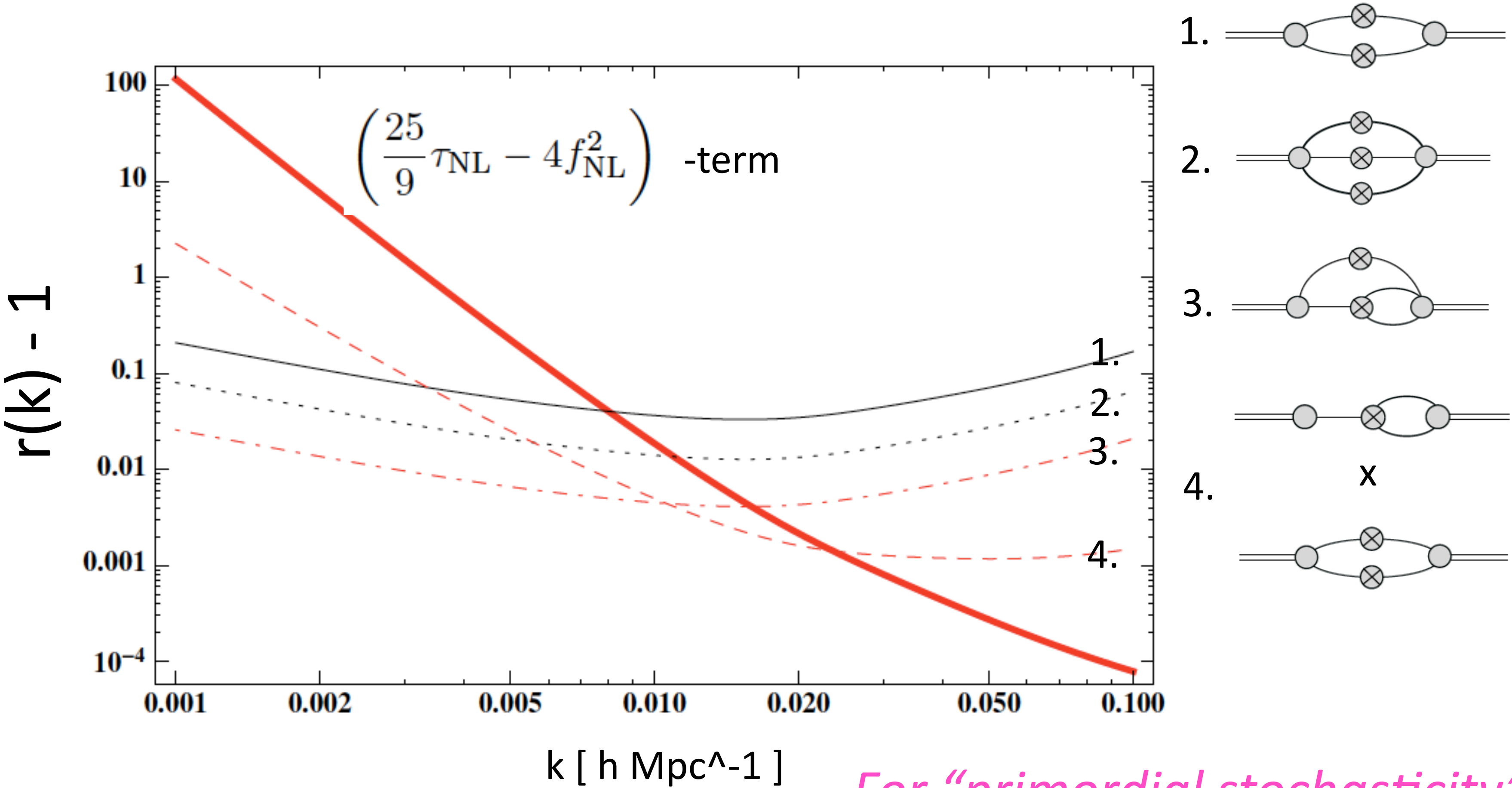


- Higher order contribution (from b2, b3, …)

$$
\begin{aligned}
P_X(k) \;=\; \sum_{n=1}^{\infty} \Bigg[& \frac{1}{n!} \int \frac{d^3p_1 \cdots d^3p_{n-1}}{(2\pi)^{3n}} \Gamma_X^{(n)}(\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{k} - \mathbf{P}_{n-1})^2 P_{\rm L}(p_1) P_{\rm L}(p_2) \cdots P_{\rm L}(|\mathbf{k} - \mathbf{P}_{n-1}|) \\
& + \frac{1}{(n-1)!} \int \frac{d^3p_1 \cdots d^3p_n}{(2\pi)^{(3n)}} \Gamma_X^{(n)}(\mathbf{p}_1, \cdots, \mathbf{p}_{n-1}, \mathbf{k} - \mathbf{P}_{n-1}) \\
& \quad \times \Gamma_X^{(n+1)}(-\mathbf{p}_1, \cdots, -\mathbf{p}_n, -\mathbf{k} + \mathbf{P}_n) P_{\rm L}(p_1) \cdots P_{\rm L}(p_{n-1}) B_{\rm L}(\mathbf{k} - \mathbf{P}_{n-1}, -\mathbf{p}_n, -\mathbf{k} + \mathbf{P}_n) \\
& + \frac{1}{3(n-1)!} \int \frac{d^3p_1 \cdots d^3p_{n+1}}{(2\pi)^{3n+3}} \Gamma_X^{(n)}(\mathbf{p}_1, \cdots, \mathbf{p}_{n-1}, \mathbf{k} - \mathbf{P}_{n-1}) \\
& \quad \times \Gamma_X^{(n+2)}(-\mathbf{p}_1, \cdots, -\mathbf{p}_{n+1}, -\mathbf{k} + \mathbf{P}_{n+1}) P_{\rm L}(p_1) \cdots P_{\rm L}(p_{n-1}) T_{\rm L}(\mathbf{k} - \mathbf{P}_{n-1}, -\mathbf{p}_n, -\mathbf{p}_{n+1}, -\mathbf{k} + \mathbf{P}_{n+1}) \Bigg] \\
& + \sum_{n=2}^{\infty} \frac{1}{4(n-2)!} \int \frac{d^3p_1 \cdots d^3p_{n-1} d^3q}{(2\pi)^{3n}} \Gamma_X^{(n)}(\mathbf{p}_1, \cdots, \mathbf{p}_{n-1}, \mathbf{k} - \mathbf{P}_{n-1}) \\
& \quad \times \Gamma_X^{(n)}(-\mathbf{p}_1, \cdots, -\mathbf{p}_{n-2}, -\mathbf{q}, -\mathbf{k} + \mathbf{P}_{n-2} + \mathbf{q}) \\
& \qquad \times P_{\rm L}(p_1) \cdots P_{\rm L}(p_{n-2}) T_{\rm L}(\mathbf{p}_{n-1}, \mathbf{k} - \mathbf{P}_{n-1}, -\mathbf{q}, -\mathbf{k} + \mathbf{P}_{n-2} + \mathbf{q}), \qquad (33)
\end{aligned}
$$



- Higher order in stochasticity parameter



$f_{\rm NL} = 40$ and $\tau_{\rm NL} = 5 \times 36 f_{\rm NL}^2/25.$

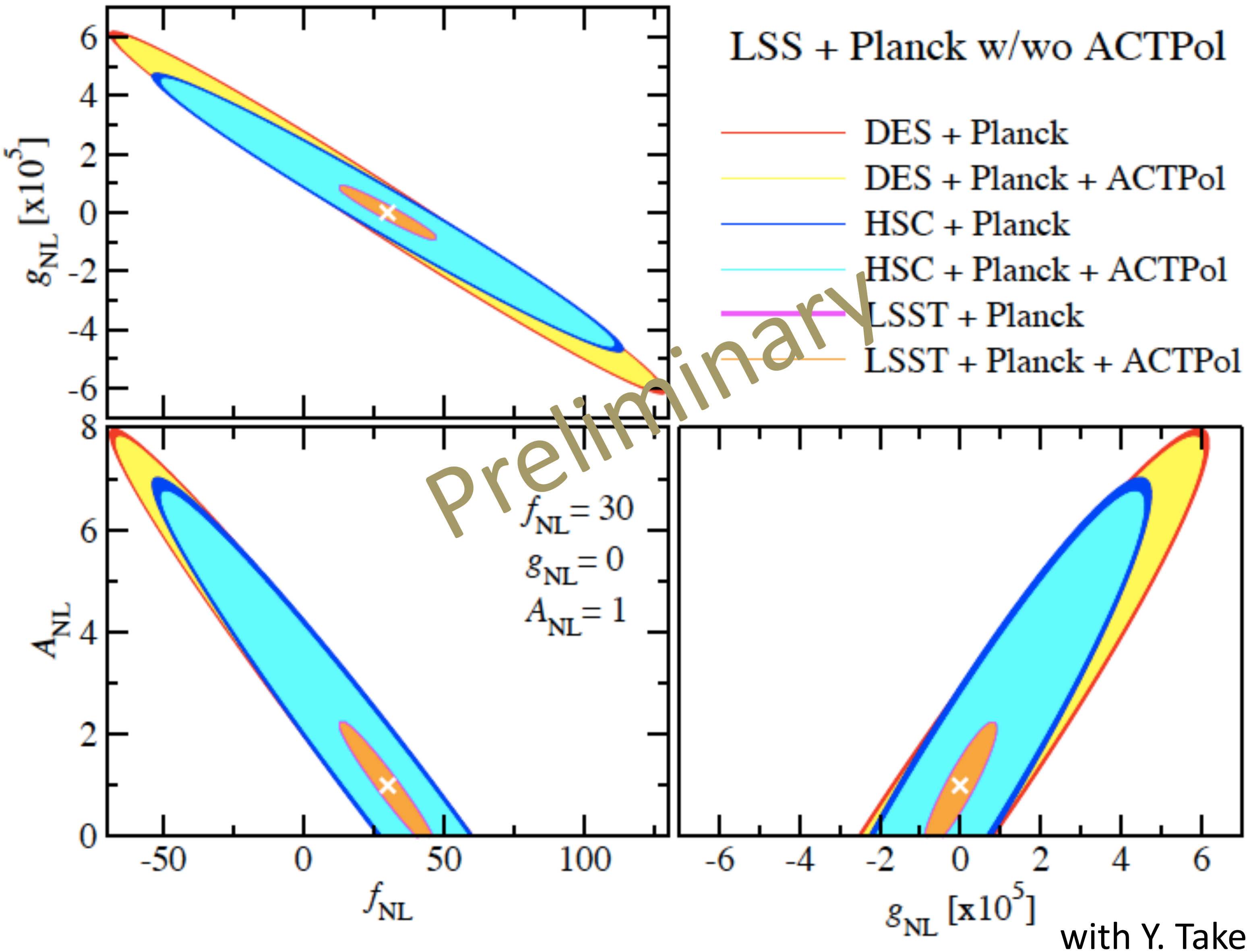
fixing $z = 1$ and $M = 5 \times 10^{13}\ h^{-1} M_\odot$

For "primordial stochasticity",

$k < O(10^{-2})\ h\text{Mpc}^{-1}$

⇔ $P_X(k)k^3 < 1$ is needed ??

- We are trying to obtain precise forecast…



with Y. Takeuchi in prep.

Summary

- Derive an accurate formula for the bias parameter with primordial non-Gaussianity by using integrated Perturbation Theory
- wide (large scales) and deep (redshift-dependence) surveys are needed.
- Stochasticity of bias ←→ primordial Suyama-Yamaguchi ineq.
- We find that higher order contributions can be safely neglected

  in the regime where the dimensionless power spectrum is less than order of unity..

- Galaxy bispectrum?
- Forecast for the constraints on fNL, gNL and tauNL? (HSC, …)