A Proxy for Massive Gravity

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outline

Universe is accelerating, $\Omega_\Lambda \approx 0.72 \pm 0.08$

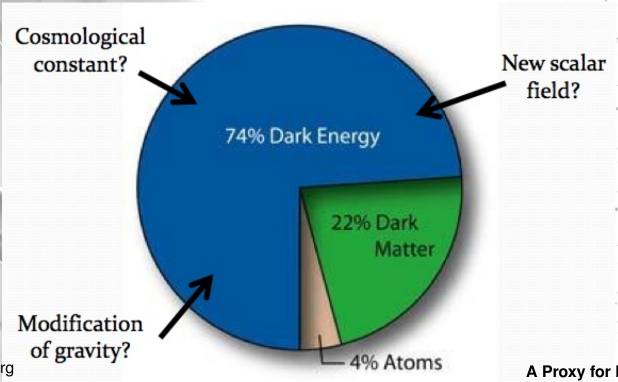
”The Universe never did make sense; I suspect it was built on government contract”. (Robert A. Heinlein)

An accelerating Universe

- **Maybe this is more illustrative**

What is Dark Energy?

- **Cosmological Constant** (Why is it so small?)
- **Dark Energy** (Why don't we see them? Similar fine-tuning problem?)
- **Dark Gravity** (Is there any viable model?)



Infra-red Modification of GR

Motivations for IR Modification of GR

- a very nice alternative to the CC or dark energy for explaining the recent acceleration of the Hubble expansion
- a way of attacking the Cosmological Constant problem
$\Lambda_{\text{phys}} = \Lambda_{\text{bare}} + \Delta\Lambda \sim (10^{-3}\text{eV})^4$ with $\Delta\Lambda \sim \text{TeV}^4$
- fun!

Dark Gravity

Lets concentrate on the third option: **Modifying gravity**

Maybe not modifying that much! only close to the horizon scale (~ 1Gpc$/h$) , corresponding to modifying gravity today.

New degrees of freedom (dof) in the infra-red (IR)

Modifying gravity in the IR typically requires new dof
usually: scalar field

$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\phi(\partial\delta\phi)^2 - \frac{1}{2}{m_\phi}^2(\delta\phi)^2 - g_\phi\delta\phi T$
where these quantities $\mathcal{Z}_\phi, m_\phi, g_\phi$ depend on the field.

Density dependent mass

- Chameleon
  m_ϕ depends on the environment
  (Khoury, Weltman 2004)

Density dependent coupling

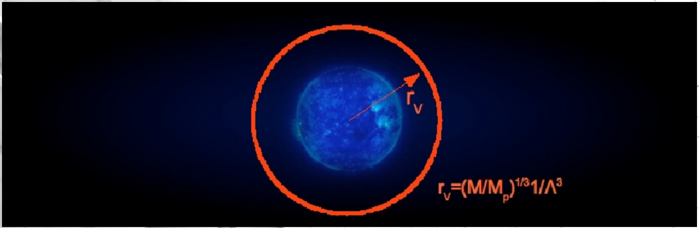
- **Vainshtein**
  $\mathcal{Z}_\phi$ depends on the environment
- Symmetron
  g_ϕ depends on the environment
  (Hinterbichler, Khoury 2010)

Vainshtein mechanism

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\phi(\partial\delta\phi)^2 - \frac{1}{2}m_\phi{}^2(\delta\phi)^2 - g_\phi\delta\phi T$$

Screening with

- effective coupling to matter depends on the self-interactions of these new dof
  $\Box\delta\phi \sim \frac{1}{M_p}\frac{1}{\sqrt{\mathcal{Z}}}T$
  $\rightarrow$ coupling small for properly canonically normalized field!
  ($\mathcal{Z} \gg 1 \rightarrow$ coupling small)
- non-linearities dominate within Vainshtein radius



Chameleon mechanism

important ingredients: a conformal coupling between the scalar and the matter fields $\tilde{g}_{\mu\nu} = g_{\mu\nu}A^2(\phi)$, and a potential for the scalar field $V(\phi)$ which includes relevant self-interaction terms.

$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{matter}[g_{\mu\nu}A^2(\phi)]$

The equation of motion for ϕ:

$$\nabla^2\phi = V_{,\phi} - A^3\phi A_{,\phi}\tilde{T} = V_{,\phi} + \rho A_{,\phi}$$

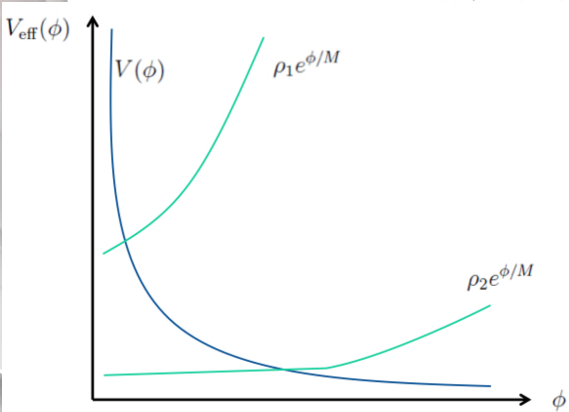
where $\tilde{T} \sim \rho/A^3(\phi)$

giving rise

to an effective potential

$V_{\text{eff}}(\phi) = V(\phi) + \rho A(\phi)$

Chameleon mechanism



- mass of the new dof depends on the local density (m_ϕ large in regions of high density)

Symmetron mechanism

important ingredients:

$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{matter}[g_{\mu\nu}A^2(\phi)]$

with a symmetry-breaking potential

$V(\phi) = \frac{-1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$

and a conformal coupling to matter of the form

$A(\phi) = 1 + \frac{\phi^2}{2M^2} + \mathcal{O}(\phi^4/M^4)$

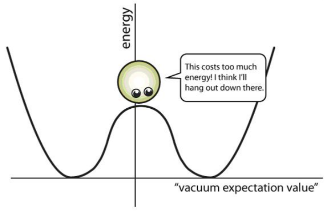
giving rise

to an effective potential

$V_{\text{eff}} = \left(\frac{-\rho}{2M^2} + \frac{\mu^2}{2}\right)\phi^2 + \frac{1}{4}\lambda\phi^4$

$\rho > \mu^2 M^2 \rightarrow$ the field sits in a minimum at the origin

Symmetron mechanism



- perturbations couple as $\frac{\bar{\phi}}{M^2}\delta\phi\rho$
- In high density symmetry-restoring environments, the scalar field vev $\sim 0 \rightarrow$ fluctuations of the field do not couple to matter
- As the local density drops the symmetry of the field is spontaneously broken and the field falls into one of the two new minima with a non-zero vev.
- $\rightarrow$ coupling to matter depends on the enviroment (g small in regions of high density)

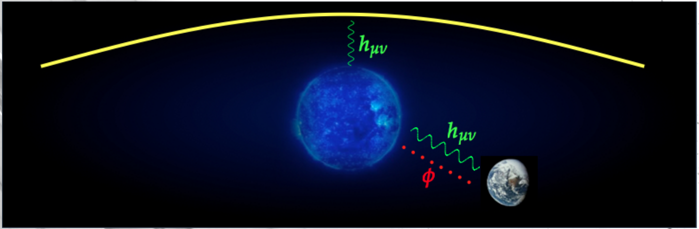
Massive Gravity

A general linear mass term for the graviton is

$$\mathcal{L}_{mass} = -\frac{1}{2}M_p^2(m_1^2 h^{\mu\nu}h_{\mu\nu} + m_2^2 h^2)$$

The only **ghost-free**: $m_1^2 = -m_2^2$ Fierz-Pauli tuning

$\rightarrow$ **vDVZ discontinuity**



Massive Gravity

Artifact:

The vDVZ discontinuity is just an artifact of the linear approximation
$\rightarrow$ non-linear extension

Issue:

The ghost we have cured by Fierz-Pauli tuning seems to come back at non-linear level (the sixth degree of freedom is associated to higher derivative terms)

Ghost

challenging task: non-linear extension of FP without ghost

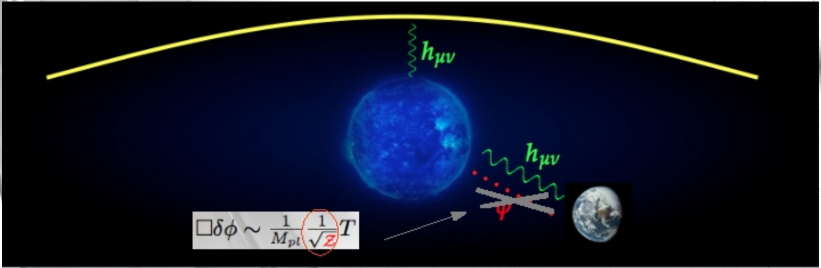
Massive Gravity

dRGT

Vainshtein mechanism at work in dRGT:
The effective coupling to matter depends on the self-interactions of the helicity-0 mode ϕ

$$\Box\delta\phi \sim \frac{1}{M_p}\frac{1}{\sqrt{Z}}T$$

$\rightarrow$ The non-linearities cure the vDVZ discontinuity



Ghost-free extension of FP = dRGT

a 4D covariant theory of a massive spin-2 field

$$\mathcal{L} = \frac{M_p^2}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,H)\right)$$

Defining the quantity $\mathcal{K}^{\mu}_{\nu}(g,H) = \delta^{\mu}_{\nu} - \sqrt{\delta^{\mu}_{\nu} - H^{\mu}_{\nu}}$ the most generic potential that bears no ghosts is $\mathcal{U}(g,H) = -4\,(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4)$ where the covariant tensor $H_{\mu\nu} = h_{\mu\nu} + 2\Phi_{\mu\nu} - \eta^{\alpha\beta}\Phi_{\mu\alpha}\Phi_{\beta\nu}$ and the potentials:

$$\begin{aligned}
\mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2] \\
\mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \\
\mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]
\end{aligned}$$

where $\Phi_{\mu\nu} = \partial_\mu\partial_\nu\phi$ and $[..]$ =trace.

Decoupling limit (DL)

Decoupling limit
($M_p \to \infty$, $m \to 0$ with $\Lambda_3^3 = m^2 M_p \to$ const)
and decomposition of $H_{\mu\nu}$ in terms of the canonically normalized helicity-2 and helicity-0 fields

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_p} + \frac{2\partial_\mu\partial_\nu\phi}{\Lambda_3^3} - \frac{\partial_\mu\partial^\alpha\phi\partial_\nu\partial_\alpha\phi}{\Lambda_3^6}$$

gives the following scalar-tensor interactions

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi]$$

where $a_1 = -\frac{1}{2}$ and $a_{2,3}$ are two arbitrary constants and $X^{(1,2,3)}_{\mu\nu}$ denote the interactions of the helicity-0 mode

$$X^{(1)}_{\mu\nu} = \Box\phi\eta_{\mu\nu} - \Phi_{\mu\nu}$$
$$X^{(2)}_{\mu\nu} = \Phi^2_{\mu\nu} - \Box\phi\Phi_{\mu\nu} - \frac{1}{2}([\Phi^2] - [\Phi]^2)\eta_{\mu\nu}$$
$$X^{(3)}_{\mu\nu} = 6\Phi^3_{\mu\nu} - 6[\Phi]\Phi^2_{\mu\nu} + 3([\Phi]^2 - [\Phi^2])\Phi_{\mu\nu} - \eta_{\mu\nu}([\Phi]^3 - 3[\Phi^2][\Phi] + 2[\Phi^3])$$

Diagonalized interactions

The transition to Einsteins frame is performed by the change of variable

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi$$

one recovers Galileon interactions for the helicity-0 mode of the graviton

$$\mathcal{L} = -\frac{1}{2}\bar{h}(\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2\phi\Box\phi - \frac{6a_2a_1}{\Lambda_3^3}(\partial\phi)^2[\Phi] + \frac{2a_2^2}{\Lambda_3^6}(\partial\phi)^2([\Phi^2] - [\Phi]^2) + \frac{a_3}{\Lambda_3^6}h^{\mu\nu}X^{(3)}_{\mu\nu}$$

with the coupling

$$\frac{1}{M_p}\left(\bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi\right)T^{\mu\nu}$$

Differences to Galileon interactions

Common

- IR modification of gravity as due to a light scalar field with non-linear derivative interactions
- respects the symmetry $\phi(x) \rightarrow \phi(x) + c + b_\mu x^\mu$
- Second order equations of motion, containing at most two time derivatives

Different

- undiagonazable interaction $+\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ $\rightarrow$ important for the self-accelerating solution
- extra coupling $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$ $\rightarrow$ important for the degravitating solution
- only 2 free-parameters
- **observational difference** due to $\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ and $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$

Two branches

The Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi] + \frac{1}{M_p}h^{\mu\nu}T_{\mu\nu}$$

Self-accelerating solution

- $T_{\mu\nu} = 0$
- $H \neq 0$

Degravitating solution

- $T_{\mu\nu} \neq 0$
- $H = 0$

Equation of motions

The equations of motion for the helicity-2 mode

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Phi] = -\frac{1}{M_p} T_{\mu\nu}$$

and for helicity-0 mode

$$\partial_\alpha \partial_\beta h^{\mu\nu} \left(a_1 \epsilon_\mu{}^{\alpha\rho\sigma} \epsilon_\nu{}^\beta{}_{\rho\sigma} + 2\frac{a_2}{\Lambda_3^3} \epsilon_\mu{}^{\alpha\rho\sigma} \epsilon_\nu{}^{\beta\gamma}{}_\sigma \Pi_{\rho\gamma} + 3\frac{a_3}{\Lambda_3^6} \epsilon_\mu{}^{\alpha\rho\sigma} \epsilon_\nu{}^{\beta\gamma\delta} \Pi_{\rho\gamma}\Pi_{\sigma\delta} \right) = 0$$

de Rham, Gabadadze, Heisenberg, Pirtskhalava (Phys.Rev.D 83,103516)

Self-accelerating solution

de Sitter as a small perturbation over Minkowski space-time

$$ds^2 = [1 - \frac{1}{2}H^2 x^\alpha x_\alpha]\eta_{\mu\nu}dx^\mu dx^\nu$$

For the helicity-0 field we look for the solution of the following isotropic form

$$\phi = \frac{1}{2}q\Lambda_3^3 x^a x_a + b\Lambda_3^2 t + c\Lambda_3$$

The equations of motion for the helicity-0 and helicity-2 fields are then given by

$$H^2\left(-\frac{1}{2} + 2a_2 q + 3a_3 q^2\right) = 0 \qquad H^2 \neq 0$$

$$M_p H^2 = 2q\Lambda_3^3\left[-\frac{1}{2} + a_2 q + a_3 q^2\right]$$

Self-accelerating solution

$H^2 = m^2\left(2a_2q^2 + 2a_3q^3 - q\right)$ and $q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$

Consider perturbations

$$h_{\mu\nu} = h^b_{\mu\nu} + \chi_{\mu\nu} \quad \text{and} \quad \phi = \phi^b + \pi$$

the Lagrangian for the perturbations

$$\mathcal{L} = -\frac{\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta}}{2} + 6(a_2 + 3a_3q)\frac{H^2M_p}{\Lambda_3^3}\pi\Box\pi - 3a_3\frac{H^2M_p}{\Lambda_3^6}(\partial_\mu\pi)^2\Box\pi$$
$$+\frac{a_2+3a_3q}{\Lambda_3^3}\chi^{\mu\nu}X^{(2)}_{\mu\nu}[\Pi] + \frac{a_3}{\Lambda_3^6}\chi^{\mu\nu}X^{(3)}_{\mu\nu}[\Pi] + \frac{\chi^{\mu\nu}T_{\mu\nu}}{M_p}$$

- abscence of ghost implies $a_2 + 3a_3q > 0$
- the perturbation of the helicity-0 mode keeps a kinetic term in this decoupling limit $\rightarrow$ no strong coupling issues

Self-accelerating solution

$$H^2 = m^2\left(2a_2q^2 + 2a_3q^3 - q\right) \text{ and } q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$$

stability

- $H^2 > 0$ and $a_2 + 3a_3q > 0$
- stable self-accelerating solution: $a_2 < 0$ and $\frac{-2a_2^2}{3} < a_3 < \frac{-a_2^2}{2}$
- **interaction $h^{\mu\nu}X^{(3)}_{\mu\nu}$ plays a crucial role for the stability since $a_3 = 0 \to$ ghost**
- there is no quadratic mixing term between χ and π
- cosmological evolution very similar to ΛCDM

Degravitating solution

We now focus on a pure cosmological constant source $T_{\mu\nu} = -\lambda\eta_{\mu\nu}$ and make the following ansatz

$$h_{\mu\nu} = -\frac{1}{2}H^2x^2M_p\eta_{\mu\nu}$$
$$\phi = \frac{1}{2}qx^2\Lambda_3^3$$

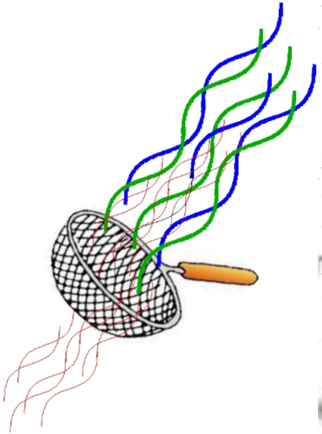
The equations of motion then simplify to

$$\left(-\frac{1}{2}M_pH^2 + \sum_{n=1}^{3} a_nq^n\Lambda_3^3\right)\eta_{\mu\nu} = -\frac{\lambda}{6M_p}\eta_{\mu\nu}$$
$$H^2\left(a_1 + 2a_2q + 3a_3q^2\right) = 0$$

$$\rightarrow a_1q + a_2q^2 + a^3q^3 = \frac{-\lambda}{\Lambda_3^3M_p}$$

Degravitating solution

- degravitating solution: high pass filter modifying the effect of long wavelength sources such as a CC $\rightarrow$ vacuum energy gravitates very weakly
- $H = 0 \quad \rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu}$
- $a_1 q + a_2 q^2 + a^3 q^3 = \frac{-\lambda}{\Lambda_3^3 M_p}$
  as long as the parameter a_3 is present, this equation has always at least one real root
- this static solution is stable for any region of the parameter space for which $2(a_1 + 2a_2 q + 3a_3 q^2) \neq 0$ and real



Proxy theory

We had the following Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{a_2}{\Lambda^3}h^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{a_3}{\Lambda^6}h^{\mu\nu}X^{(3)}_{\mu\nu} + \frac{1}{2M_p}h^{\mu\nu}T_{\mu\nu}$$

lets integrate by part the first interaction $h^{\mu\nu}X^{(1)}_{\mu\nu}$:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} = h^{\mu\nu}(\Box\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) &= h^{\mu\nu}(\partial_\alpha\partial^\alpha\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) \\ &= (\Box h - \partial_\mu\partial_\nu h^{\mu\nu})\phi \\ &= -R\phi \end{aligned}$$

so covariantization of the first interaction: $h^{\mu\nu}X^{(1)}_{\mu\nu} \longleftrightarrow -R\phi$

Proxy theory

Similarly, we can covariantize the other interaction terms. One finds the following correspondences:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} &\longleftrightarrow -\phi R \\ h^{\mu\nu}X^{(2)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} \\ h^{\mu\nu}X^{(3)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta} \end{aligned}$$

such that the Lagrangian becomes

$$\mathcal{L}^\phi = M_p\left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right).$$

with the dual Riemann tensor

$$\begin{aligned} L^{\mu\alpha\nu\beta} = 2R^{\mu\alpha\nu\beta} + 2(R^{\mu\beta}g^{\nu\alpha} + R^{\nu\alpha}g^{\mu\beta} - R^{\mu\nu}g^{\alpha\beta} - R^{\alpha\beta}g^{\mu\nu}) \\ +R(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}) \end{aligned}$$

Proxy theory

Instead of focusing on the entire complicated model, study a proxy theory:

$$\mathcal{L} = \sqrt{-g} M_p (M_p R + -\phi R - \frac{a_2}{\Lambda^3} \partial_\mu \phi \partial_\nu \phi G^{\mu\nu} - \frac{a_3}{\Lambda^6} \partial_\mu \phi \partial_\nu \phi \Phi_{\alpha\beta} L^{\mu\alpha\nu\beta})$$

- in 4D $G_{\mu\nu}$ and $L^{\mu\alpha\nu\beta}$ are the only divergenceless tensors $\rightarrow \nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu L^{\mu\alpha\nu\beta} = 0$
- All eom are 2^{nd} order $\rightarrow$ No instabilities
- Reproduces the decoupling limit $\rightarrow$ Exhibits the Vainsthein mechanism

de Rham, Heisenberg (PRD84 (2011) 043503)

Proxy theory

The Einstein equation is given by

$$G_{\mu\nu} = M_p T^{\phi}_{\mu\nu} + T^{\rm matter}_{\mu\nu}$$

with

$$T^{\phi}_{\mu\nu} = T^{\phi(1)}_{\mu\nu} - \frac{a_2}{\Lambda^3} T^{\phi(2)}_{\mu\nu} - \frac{a_3}{\Lambda^6} T^{\phi(3)}_{\mu\nu}$$

with the shortcut stress-energy tensors

$$\begin{aligned} T^{\phi(1)}_{\mu\nu} &= X^{(1)}_{\mu\nu} + \phi G_{\mu\nu} \\ T^{\phi(2)}_{\mu\nu} &= X^{(2)}_{\mu\nu} + \frac{1}{2} L_{\mu\alpha\nu\beta} \partial^{\alpha}\phi \partial^{\beta}\phi + \frac{1}{2} G_{\mu\nu} (\partial\phi)^2 \\ T^{\phi(3)}_{\mu\nu} &= X^{(3)}_{\mu\nu} + \frac{3}{2} L_{\mu\alpha\nu\beta} \Phi^{\alpha\beta} (\partial\phi)^2 \end{aligned}$$

Proxy theory

- In the Einstein frame $D_\mu T^\mu_\nu = \partial_\nu \phi \mathcal{E}_\phi$ where $\mathcal{E}_\phi$ is the equation of motion with respect to ϕ.
- Since $\nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu L^{\mu\nu\alpha\beta} = 0$, $\mathcal{E}_\phi$ is also at most second order in derivative

$$\begin{aligned} \mathcal{E}_\phi &= \frac{\delta \mathcal{L}^\phi}{\delta \phi} \\ &= -R - \frac{2a_2}{\Lambda^3} G^{\mu\nu}\Phi_{\mu\nu} - \frac{3a_3}{\Lambda^6} L^{\mu\alpha\nu\beta}(\Phi_{\mu\nu}\Phi_{\alpha\beta} + R^\gamma{}_{\beta\alpha\nu}\partial_\gamma\phi\partial_\mu\phi) = 0 \end{aligned}$$

Equations of motion for both, ϕ and $g_{\mu\nu}$ are at most second order.

Self-accelerating solution

- self-acceleration solution: H =const and $\dot{H} = 0$.
- make the ansatz $\dot{\phi} = q\frac{\Lambda^3}{H}$.
- assume that we are in a regime where $H\phi \ll \dot{\phi}$

The Friedmann and field equations can be recast in

$$H^2 = \frac{m^2}{3}(6q - 9a_2q^2 - 30a_3q^3)$$
$$H^2(18a_2q + 54a_3q^2 - 12) = 0$$

Assuming $H \neq 0$, the field equation then imposes,

$$q = \frac{-a_2 \pm \sqrt{a_2^2 + 8a_3}}{6a_3}$$

$\rightarrow$ similar to DL our proxy theory admits a self-accelerated solution, with the Hubble parameter set by the graviton mass.

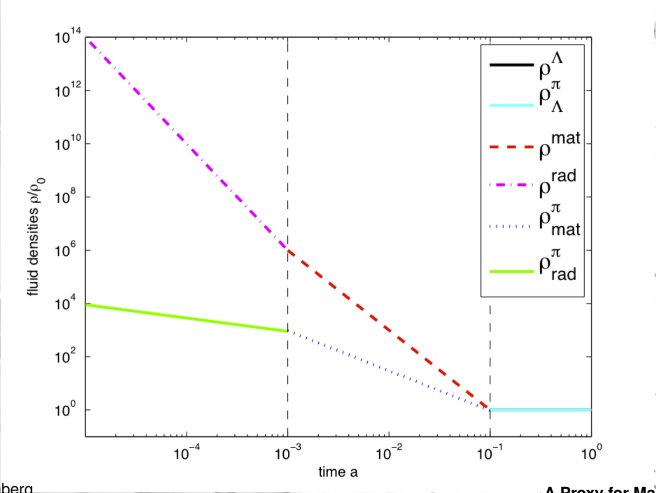
Proxy theory

$$\mathcal{L}^{\phi} = M_p \left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$

- We recover some decoupling limit results:
  - stable self-accelerating solutions within the space parameter space
- During the radiation domination the energy density for ϕ goes as $\rho^{\phi}_{\rm rad} \sim a^{-1/2}$ and during matter dominations as $\rho^{\phi}_{\rm mat} \sim a^{-3/2}$ and is constant for later times $\rho^{\phi}_{\Lambda} = \rm const$
- At early time, interactions for scalar mode are important $\rightarrow$ cosmological screening effect
- Below a critical energy density, screening stop being efficient $\rightarrow$ scalar contribute significantly to the cosmological evolution
- But still the cosmological evolution different than in ΛCDM

Densities

$$\mathcal{L}^{\phi} = M_p \left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$



Degravitation solution

The effective energy density of the field ϕ is

$$\rho^{\phi} = M_p(6H\dot{\phi} + 6H^2\phi - \frac{9a_2}{\Lambda^3}H^2\dot{\phi}^2 - \frac{30a_3}{\Lambda^6}H^3\dot{\phi}^3)$$

- If one takes $\phi = \phi(t)$ and $H = 0 \to \rho^{\phi} = 0$
  $\to$ so the field has absolutely no effect and cannot help the background to degravitate.
- Charmousis et al. has similar interactions, they find degravitation solution!
  BUT they rely strongly on spatial curvature
- in the absence of spatial curvature $\kappa = 0$, the contribution from the scalar field vanishes if $H = 0$.
- $\to$ BUT relying on spatial curvature brings concerns over instabilities

Different frames

Jordan frame

$\mathcal{L}^\phi = M_p^2 R + M_p \left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu}\right)$ with $a_3 = 0$

Do conformal transformation

$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = \left(1 - \frac{\pi}{M_p}\right)$

$M_p^2 R - \frac{3}{2}\Omega^{-4}(\tilde{\partial}\pi)^2 -$

$\frac{a_2 M_p}{\Lambda_3^3}\left(\tilde{\partial}_\mu\phi\tilde{\partial}_\nu\phi\tilde{G}^{\mu\nu} + \frac{3\Omega^{-2}}{2M_p}(\tilde{\partial}\pi)^2\tilde{\Box}\pi + \frac{5\Omega^{-4}}{4M_p^2}(\tilde{\partial}\pi)^4\right)$

Einstein frame

$h_{\mu\nu} \to h_{\mu\nu} + \phi\eta_{\mu\nu}$

covariantize

$\to M_p^2 R + \frac{3}{2}\pi\Box\pi - \frac{a_2 M_p}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{3}{2}\frac{a_2}{\Lambda^3}\Box\pi(\partial\pi)^2$

So within the regime of validity of our results $\pi \ll M_p$, our conclusions are independent of the choice of frame.

Conclusion

- decoupling limit of dRGT
  - stable self-accelerating solution similar to ΛCDM
  - degravitating solution
- Proxy theory
  - stable self accelerating solution
  - no degravitating solution
  - the scalar mode does not decouple around the self-accelerating background
  - leads to an extra force during the history of the Universe
  - would influence the time sequence of gravitational clustering and the evolution of peculiar velocities, as well as the number density of collapsed objects.

Future work

Quantum corrections

- the mass needs to be tuned $m \lesssim H_0$
  same tuning as Cosmological Constant
  $\frac{\Lambda}{M_p^4} \sim \frac{H_0^2}{M_p^2} \sim \frac{m^2}{M_p^2} \sim 10^{-120}$
- But the graviton mass is expected to remain stable against quantum corrections
- Check quantum corrections beyond the decoupling limit
  $\delta m^2 \sim m^2 \rightarrow$ the theory would be tuned but technically natural

constraining dRGT through observations

- put observational constraint on the free parameters of dRGT and test it against ΛCDM.

observations are always a challenging task!

challenging task: observation!

we would like to study

- the distance-redshift relation of supernovae
- the angular diameter distance as a function of redshift
  - CMB
  - BAO
- Weak Lensing
- integrated Sachs-Wolfe Effect
- Gravitational Clustering and Number density of collapsed objects

for massive gravity!

cosmological observations

two categories:

geometrical probes

measurement of the Hubble function

- distance-redshift relation of supernovae
- measurements of the angular diameter distance as a function of redshift (CMB+BAO)

structure formation probes

measurement of the Growth function

- homogeneous growth of the cosmic structure
  $\rightarrow$ integrated Sachs-Wolfe effects
- non-linear growth
  $\rightarrow$ gravitational lensing
  $\rightarrow$ formation of galaxies
  $\rightarrow$ clusters of galaxies by gravitational collapse

(e.g Kimura et al.)

cosmological observations

In Proxy theory:

- modified Hubble function:
  $H^2 = \frac{m^2}{3}(6q - 9a_2q^2 - 30a_3q^3)$ with $q = \frac{-a_2 \pm \sqrt{a_2^2+8a_3}}{6a_3}$
- The scalar mode does not decouple around the self-accelerating background (screened at high energy though)
  $\rightarrow$structure formation different
  Growth function:

$$\ddot{\delta}_m + 2H\dot{\delta}_m = \frac{\rho_m \delta_m}{M_p^2}\left(1 + \frac{1}{3\mathcal{Q}}\right)$$

where $\mathcal{Q}$ stands for

$$\mathcal{Q} \equiv 1 - \frac{2a_2}{\Lambda^3}(2H\dot{\pi}_0 + \ddot{\pi}_0 + M_p(2\dot{H} + 3H^2)) - \frac{a_3}{\Lambda^6}\dots$$

with $\dot{\pi}_0 = q\frac{\Lambda^3}{H}$