Vainshtein mechanism in a cosmological background in the most general second-order scalar-tensor theory

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Based on:
work with **Rampei Kimura** and **Kazuhiro Yamamoto** (Hiroshima University)
Phys. Rev. D 85, 024023 (2012) [**arXiv:1111.6749**]

Talk Plan

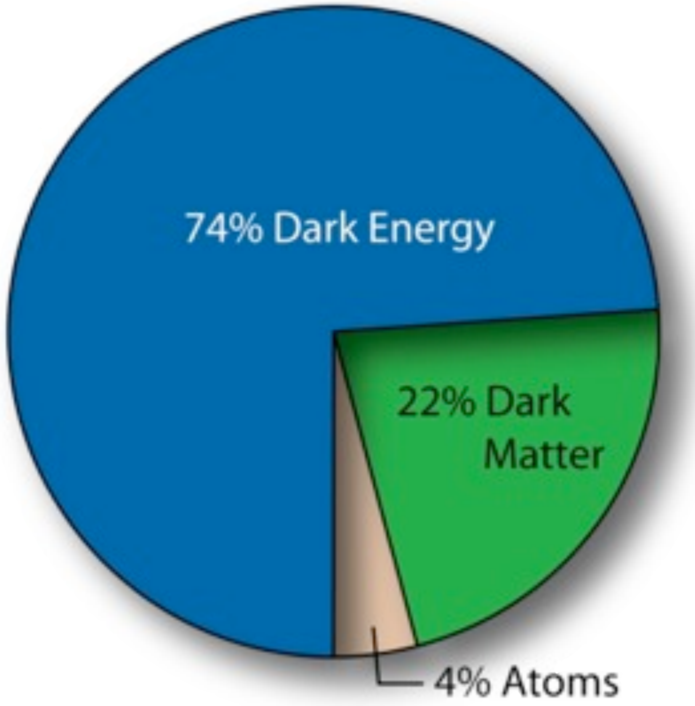
1. Motivation
2. Brief review
    - Vainshtein mechanism
    - The most general second-order scalar-tensor (ST) theory
3. **Vainshtein screening in the most general ST theory**
4. Summary

1. Motivation

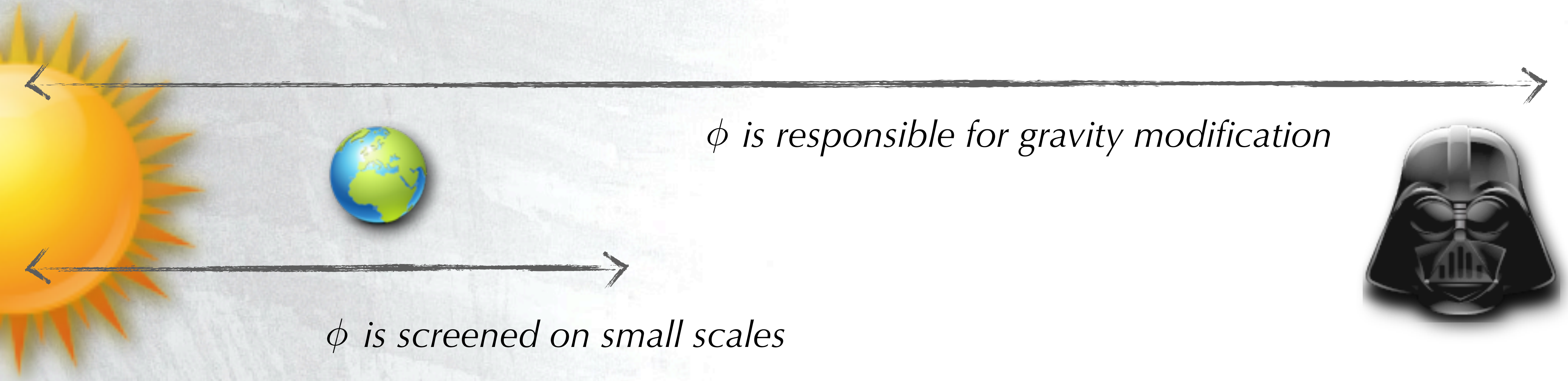
Modified gravity

DGP, Galileons, Massive gravity…

- A new scalar degree of freedom, ϕ , participates in long-range gravitational interaction
- Modification would persist down to small scales...
- Need screening mechanism in order to recover GR on small scales and to pass solar-system tests



Screening mechanisms



ϕ *is responsible for gravity modification*

ϕ *is screened on small scales*

This is made possible by the *Vainshtein mechanism*

Vainshtein 1972

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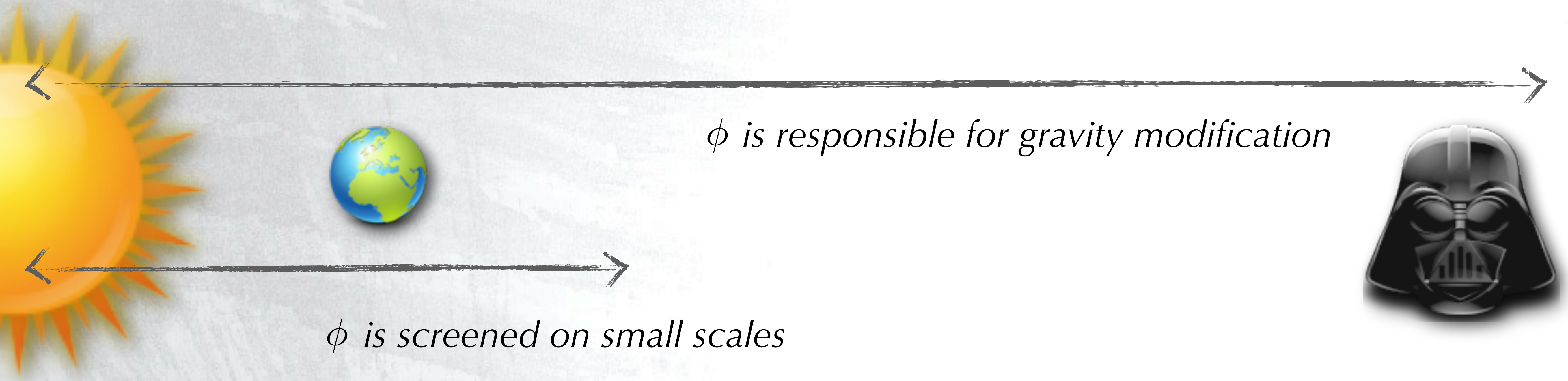

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Let’s analyze how the Vainshtein mechanism operates based on the most general second-order scalar-tensor theory!

See **Sbisa, Niz, Koyama, Tasinato 1204.1193** for a similar analysis in the context of massive gravity

2. Brief review

—Vainshtein mechanism

Example

Scalar-field theory non-minimally coupled to matter:

$$\mathcal{L} = \frac{1}{8\pi G}\left[-\frac{1}{2}(\partial\varphi)^2 - \frac{r_c^2}{3}(\partial\varphi)^2\Box\varphi\right] + \varphi T_\mu^{\ \mu}$$

(φ : dimensionless)

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Non-linear derivative interaction

(cubic Galileon)

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$$\boxed{\mathcal{O}(H_0^{-1})}$$

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Key non-linearity

$r_c^2\Box\varphi$ **can be large** even if $\varphi \ll 1$

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Non-linear derivative interaction
(cubic Galileon)

Key non-linearity

$r_c^2\Box\varphi$ **can be large** even if $\varphi \ll 1$

→ $$\mathcal{L} \sim \frac{1}{8\pi G_{\rm eff}}\left[-\frac{1}{2}(\partial\varphi)^2\right] + \varphi T_\mu^{\ \mu}, \quad G_{\rm eff} \ll G$$

Effectively weakly coupled to matter

Spherically symmetric solution

EOM: $\Box\varphi + \frac{2r_c^2}{3}\left[(\Box\varphi)^2 - (\partial_\mu\partial_\nu\varphi)^2\right] = -8\pi G T_\mu^{\ \mu} \approx 8\pi G\rho$

Spherically symmetric solution:

$$\partial_r\varphi = \frac{3r}{8r_c^2}\left(-1 + \sqrt{1 + \frac{16}{3}\frac{r_s r_c^2}{r^3}}\right)$$

(r_s : Schwarzschild radius)

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$r_V := (r_s r_c^2)^{1/3}$: **Vainshtein radius**

(r_s : Schwarzschild radius)

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☑ $r \gg r_V \longrightarrow \partial_r\varphi \sim \frac{r_s}{r^2} \sim \partial_r\Phi$ (Φ : gravitational potential)

☑ $r \ll r_V \longrightarrow \partial_r\varphi \sim \frac{r_s}{r^2}\left(\frac{r}{r_V}\right)^{3/2} \ll \partial_r\Phi$ —— screened!

2. Brief review

—The most general second-order scalar-tensor theory

Horndeski's theory (Generalized Galileon)

In 1974, Horndeski determined **the most general** Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots ; g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots)$$

having **second-order** field equations both for ϕ and $g_{\mu\nu}$

Horndeski, Int. J. Theor. Phys. 10,363 (1974)

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Horndeski's theory is equivalent to **the generalized Galileons** [Deffayet *et al.* 2011] **in 4D**

TK, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)

Horndeski's theory (Generalized Galileon)

Lagrangian for *the most general second-order ST theory*:

$$\begin{aligned}\mathcal{L} \;=\;& K(\phi, X) - G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3\right. \\ & \qquad \left. -3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\end{aligned}$$

where $X := -\frac{1}{2}(\partial\phi)^2, \quad G_{iX} := \partial G_i/\partial X$

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& \qquad \left. - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
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- 4 arbitrary functions of ϕ and X

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where $X := -\frac{1}{2}(\partial\phi)^2, \quad G_{iX} := \partial G_i/\partial X$

- 4 arbitrary functions of ϕ and X
- Non-minimal coupling to gravity

Special cases

$$G_4 = \frac{M_{\rm Pl}^2}{2} \longrightarrow \mathcal{L} \supset \frac{M_{\rm Pl}^2}{2} R$$

Einstein-Hilbert

$$G_4 = f(\phi) \longrightarrow \mathcal{L} \supset f(\phi) R$$

Familiar non-minimal coupling

$$-G_3(\phi, X)\Box\phi \quad \supset \quad (\partial\phi)^2\Box\phi$$

Kinetic gravity braiding

Deffayet *et al*. 2010

DGP (brane bending mode)

Luty, Porrati, Rattazzi 2003;
Nicolis, Rattazzi 2004

$$G_5 = -\phi \longrightarrow \mathcal{L} \supset G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

Gravitationally enhanced friction/purely kinetic coupled gravity

Germani *et al*. 2011; Gubitosi, Linder 2011

$$
\begin{aligned}
\mathcal{L} \quad = \quad & K(\phi, X) - G_3(\phi, X)\Box\phi \\
& + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
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& \qquad\qquad - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\big]
\end{aligned}
$$

Galileon-like non-linear derivative interactions

$$
\begin{aligned}
\mathcal{L} \quad = \quad & K(\phi, X) - G_3(\phi, X)\Box\phi \\
& + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
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& \left. - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}
$$

$G_3 \propto X, \; G_4 \propto X^2, \; G_5 \propto X^2$ ⟶ covariant galileon

Vainshtein mechanism operates generically?

3. Vainshtein screening in the most general ST theory

Setup

- The most general ST theory, **minimally coupled to matter**

Horndeski's Lagrangian

$$S = \int \mathrm{d}^4x \sqrt{-g}\,[\mathcal{L} + \mathcal{L}_{\mathrm{m}}(\psi, g_{\mu\nu})]$$

- **Cosmological background**

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$$

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Non-relativistic matter

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- **Cosmological background**

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$$

Perturbations – *Consistent treatment for scalar-field and* ***metric*** *perturbations*

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\mathrm{d}\mathbf{x}^2$$

$$\phi \to \phi(t) + \delta\phi(t,\mathbf{x}), \quad \rho_\mathrm{m} \to \rho_\mathrm{m}(t)[1+\delta(t,\mathbf{x})]$$

Approximations

Weak gravitational field on **subhorizon** scales — Useful e.g. for the study of structure formation

De Felice, TK, Tsujikawa (2011)

☑ $$\Phi,\ \Psi,\ Q := H\frac{\delta\phi}{\dot{\phi}} \ll 1$$

☑ $\partial_t \ll \partial_i$ (Quasi-static approximation)

Keep relevant **non-linear** terms written schematically as

$$(\nabla^2\epsilon)^2,\ (\nabla^2\epsilon)^3, \cdots \qquad \text{where} \qquad \epsilon = \Phi, \Psi, Q$$

(Quartic terms $(\nabla^2\epsilon)^4$ do not appear)

Neglect "mass" terms: $K_{\phi\phi} \ll \nabla^2, \cdots$

[1] Scalar-field EOM

$$
\begin{aligned}
& A_0 \nabla^2 Q - A_1 \nabla^2 \Psi - A_2 \nabla^2 \Phi + \frac{B_0}{a^2 H^2} \mathcal{Q}^{(2)} \\
& - \frac{B_1}{a^2 H^2} \left(\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q \right) \\
& - \frac{B_2}{a^2 H^2} \left(\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q \right) \\
& - \frac{B_3}{a^2 H^2} \left(\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi \right) \\
& - \frac{C_0}{a^4 H^4} \left[\left(\nabla^2 Q \right)^3 - 3 \nabla^2 Q \left(\partial_i \partial_j Q \right)^2 + 2 \left(\partial_i \partial_j Q \right)^3 \right] \\
& - \frac{C_1}{a^4 H^4} \left[\mathcal{Q}^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi + 2 \partial_i \partial_j Q \partial^j \partial^k Q \partial_k \partial^i \Phi \right] = 0,
\end{aligned}
$$

where $\mathcal{Q}^{(2)} := \left(\nabla^2 Q \right)^2 - \left(\partial_i \partial_j Q \right)^2$

[1] Scalar-field EOM

Linear terms

$$
\begin{aligned}
&\underline{A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi} + \frac{B_0}{a^2H^2}\mathcal{Q}^{(2)} \\
&-\frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right) \\
&-\frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right) \\
&-\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2 \Psi - \partial_i\partial_j\Phi\partial^i\partial^j \Psi\right) \\
&-\frac{C_0}{a^4H^4}\left[\left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3\right] \\
&-\frac{C_1}{a^4H^4}\left[\mathcal{Q}^{(2)}\nabla^2\Phi - 2\nabla^2 Q\partial_i\partial_j Q\partial^i\partial^j\Phi + 2\partial_i\partial_j Q\partial^j\partial^k Q\partial_k\partial^i\Phi\right] = 0,
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[1] Scalar-field EOM

Linear terms

$(\nabla^2\epsilon)^2$ terms

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&-\frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial^i\partial^j Q\right) \\
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&-\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2 \Psi - \partial_i\partial_j\Phi\partial^i\partial^j \Psi\right) \\
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$(\nabla^2\epsilon)^2$ *terms*

$(\nabla^2\epsilon)^3$ *terms*

where $\mathcal{Q}^{(2)} := \left(\nabla^2 Q\right)^2 - \left(\partial_i\partial_j Q\right)^2$

[1] Scalar-field EOM

Linear terms

$$A_0\nabla^2 Q - A_1\nabla^2\Psi - A_2\nabla^2\Phi + \frac{B_0}{a^2H^2}$$

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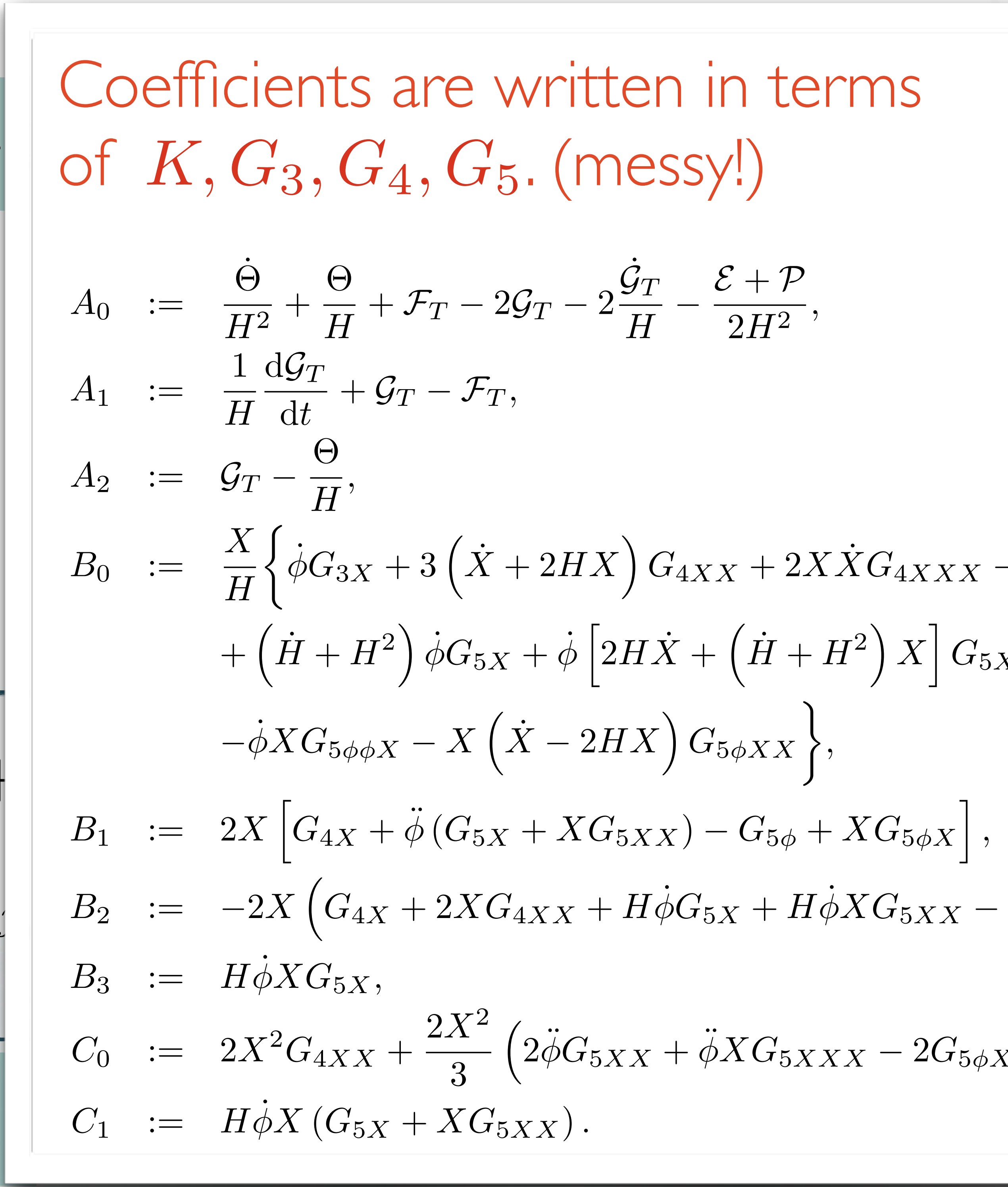
$$-\frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2 Q - \partial_i\partial_j\Phi\partial^i\partial^j Q\right)$$

$$-\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2 \Psi - \partial_i\partial_j\Phi\partial^i\partial^j \Psi\right)$$

$$-\frac{C_0}{a^4H^4}\Big[\left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 +$$

$$-\frac{C_1}{a^4H^4}\Big[\mathcal{Q}^{(2)}\nabla^2\Phi - 2\nabla^2 Q\partial_i\partial_j Q\partial^i\partial$$

where $\mathcal{Q}^{(2)} := \left(\nabla^2 Q\right)^2 - \left(\partial_i\partial_j Q\right)^2$



Coefficients are written in terms of K, G_3, G_4, G_5. (messy!)

$$A_0 := \frac{\dot\Theta}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\mathcal{G}_T - 2\frac{\dot{\mathcal{G}}_T}{H} - \frac{\mathcal{E}+\mathcal{P}}{2H^2},$$

$$A_1 := \frac{1}{H}\frac{\mathrm{d}\mathcal{G}_T}{\mathrm{d}t} + \mathcal{G}_T - \mathcal{F}_T,$$

$$A_2 := \mathcal{G}_T - \frac{\Theta}{H},$$

$$B_0 := \frac{X}{H}\Big\{\dot\phi G_{3X} + 3\left(\dot X + 2HX\right)G_{4XX} + 2X\dot X G_{4XXX} -$$

$$+\left(\dot H + H^2\right)\dot\phi G_{5X} + \dot\phi\left[2H\dot X + \left(\dot H + H^2\right)X\right]G_{5X}$$

$$-\dot\phi X G_{5\phi\phi X} - X\left(\dot X - 2HX\right)G_{5\phi XX}\Big\},$$

$$B_1 := 2X\left[G_{4X} + \ddot\phi\left(G_{5X} + XG_{5XX}\right) - G_{5\phi} + XG_{5\phi X}\right],$$

$$B_2 := -2X\Big(G_{4X} + 2XG_{4XX} + H\dot\phi G_{5X} + H\dot\phi X G_{5XX} -$$

$$B_3 := H\dot\phi X G_{5X},$$

$$C_0 := 2X^2G_{4XX} + \frac{2X^2}{3}\Big(2\ddot\phi G_{5XX} + \ddot\phi X G_{5XXX} - 2G_{5\phi X}$$

$$C_1 := H\dot\phi X\left(G_{5X} + XG_{5XX}\right).$$

[2] (00) equation

$$
\begin{aligned}
& \mathcal{G}_T \nabla^2 \Psi + A_2 \nabla^2 Q && \longleftarrow \quad \nabla^2 \epsilon \\
& + \frac{B_2}{2a^2 H^2} \mathcal{Q}^{(2)} + \frac{B_3}{a^2 H^2} \left(\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial_i \partial_j Q \right) && \longleftarrow \quad (\nabla^2 \epsilon)^2 \\
& + \frac{C_1}{3a^4 H^4} \left[\left(\nabla^2 Q \right)^3 - 3 \nabla^2 Q \left(\partial_i \partial_j Q \right)^2 + 2 \left(\partial_i \partial_j Q \right)^3 \right] && \longleftarrow \quad (\nabla^2 \epsilon)^3 \\
= \quad & \frac{a^2}{2} \rho_{\mathrm{m}} \delta
\end{aligned}
$$

[2] (00) equation

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

$$\begin{aligned}
&\mathcal{G}_T\nabla^2\Psi + A_2\nabla^2 Q && \longleftarrow \quad \nabla^2\epsilon \\
&+\frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial_i\partial_j Q\right) && \longleftarrow \quad (\nabla^2\epsilon)^2 \\
&+\frac{C_1}{3a^4H^4}\left[\left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3\right] && \longleftarrow \quad (\nabla^2\epsilon)^3 \\
= \quad &\frac{a^2}{2}\rho_{\mathrm{m}}\delta
\end{aligned}$$

[2] (00) equation

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

$$\begin{aligned}
&\mathcal{G}_T\nabla^2\Psi + A_2\nabla^2 Q && \longleftarrow \quad \nabla^2\epsilon \\
&+\frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}\left(\nabla^2\Psi\nabla^2 Q - \partial_i\partial_j\Psi\partial_i\partial_j Q\right) && \longleftarrow \quad (\nabla^2\epsilon)^2 \\
&+\frac{C_1}{3a^4H^4}\left[\left(\nabla^2 Q\right)^3 - 3\nabla^2 Q\left(\partial_i\partial_j Q\right)^2 + 2\left(\partial_i\partial_j Q\right)^3\right] && \longleftarrow \quad (\nabla^2\epsilon)^3 \\
=\ &\boxed{\frac{a^2}{2}\rho_{\mathrm{m}}\delta}
\end{aligned}$$

[3] Traceless part

$$\begin{aligned}\nabla^2\left(\mathcal{F}_T\Psi-\mathcal{G}_T\Phi-A_1Q\right) &= \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} \\ &+\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q-\partial_i\partial_j\Phi\partial^i\partial^jQ\right)\end{aligned}$$

[3] Traceless part

$$\begin{aligned}\nabla^2\left(\mathcal{F}_T\Psi-\mathcal{G}_T\Phi-A_1Q\right) &= \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)}\\ &+\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q-\partial_i\partial_j\Phi\partial^i\partial^jQ\right)\end{aligned}$$

[3] Traceless part

$$\begin{aligned}\nabla^2\left(\mathcal{F}_T\Psi-\mathcal{G}_T\Phi-A_1Q\right) \quad &= \quad \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} \\ &\quad +\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q-\partial_i\partial_j\Phi\partial^i\partial^jQ\right)\end{aligned}$$

$$\mathcal{F}_T := 2\left[G_4-X\left(\ddot{\phi}G_{5X}+G_{5\phi}\right)\right]$$

[3] Traceless part

$$\nabla^2\left(\mathcal{F}_T\Psi - \mathcal{G}_T\Phi - A_1Q\right) = \frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)} + \frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q - \partial_i\partial_j\Phi\partial^i\partial^jQ\right)$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right]$$

Related to **propagation speed of gravitational waves**:

$$c_h^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

TK, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)

Spherically symmetric configurations

$$r = a\sqrt{\delta_{ij}x^i x^j}, \quad rH \ll 1$$

The 3 equations can be integrated once to give

algebraic equations for Φ', Ψ', Q'

$$\begin{aligned}
c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} &= \frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2 + 2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}, \\
\frac{\Psi'}{r} + \alpha_2 \frac{Q'}{r} &= \frac{1}{8\pi\mathcal{G}_T}\frac{\delta M(t,r)}{r^3} - \frac{\beta_2}{H^2}\left(\frac{Q'}{r}\right)^2 - 2\frac{\beta_3}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} - \frac{2}{3}\frac{\gamma_1}{H^4}\left(\frac{Q'}{r}\right)^3, \\
\alpha_0 \frac{Q'}{r} - \alpha_1 \frac{\Psi'}{r} - \alpha_2 \frac{\Phi'}{r} &= 2\left[-\frac{\beta_0}{H^2}\left(\frac{Q'}{r}\right)^2 + \frac{\beta_1}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} + \frac{\beta_2}{H^2}\frac{\Phi'}{r}\frac{Q'}{r} + \frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{\Psi'}{r} \right. \\
&\quad \left. + \frac{\gamma_0}{H^4}\left(\frac{Q'}{r}\right)^3 + \frac{\gamma_1}{H^4}\frac{\Phi'}{r}\left(\frac{Q'}{r}\right)^2\right]
\end{aligned}$$

Coefficients are dimensionless and written in terms of K, G_3, G_4, G_5

(TIme-dependent)

Spherically symmetric configurations

$$r = a\sqrt{\delta_{ij}x^i x^j}, \quad rH \ll 1$$

The 3 equations can be integrated once to give algebraic equations for Φ', Ψ', Q'

Enclosed mass:

$$\delta M(t,r) = 4\pi\rho_{\mathrm{m}}(t)\int^r \delta(t,r)\, r'^2 \mathrm{d}r'$$

$$\begin{aligned}
c_h^2\frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1\frac{Q'}{r} &= \frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2 + 2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}, \\
\frac{\Psi'}{r} + \alpha_2\frac{Q'}{r} &= \frac{1}{8\pi\mathcal{G}_T}\boxed{\frac{\delta M(t,r)}{r^3}} - \frac{\beta_2}{H^2}\left(\frac{Q'}{r}\right)^2 - 2\frac{\beta_3}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} - \frac{2}{3}\frac{\gamma_1}{H^4}\left(\frac{Q'}{r}\right)^3, \\
\alpha_0\frac{Q'}{r} - \alpha_1\frac{\Psi'}{r} - \alpha_2\frac{\Phi'}{r} &= 2\left[-\frac{\beta_0}{H^2}\left(\frac{Q'}{r}\right)^2 + \frac{\beta_1}{H^2}\frac{\Psi'}{r}\frac{Q'}{r} + \frac{\beta_2}{H^2}\frac{\Phi'}{r}\frac{Q'}{r} + \frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{\Psi'}{r}\right. \\
&\qquad \left. + \frac{\gamma_0}{H^4}\left(\frac{Q'}{r}\right)^3 + \frac{\gamma_1}{H^4}\frac{\Phi'}{r}\left(\frac{Q'}{r}\right)^2\right]
\end{aligned}$$

Coefficients are dimensionless and written in terms of K, G_3, G_4, G_5

(TIme-dependent)

Let's see whether or not usual gravity is reproduced in the vicinity of the source:

$$\Phi' \simeq \Psi' \simeq \frac{G_N \delta M}{r^2} \qquad \text{— ???}$$

Linear solution at large r

At sufficiently large *r* all the non-linear terms may be neglected

$$\begin{aligned}
\Phi' &= \frac{c_h^2\alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2} \\
\Psi' &= \frac{\alpha_0 + \alpha_1\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2} \\
Q' &= \frac{\alpha_1 + c_h^2\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2}
\end{aligned}$$

where $\mu := \dfrac{\delta M}{8\pi\mathcal{G}_T}$

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At sufficiently large *r* all the non-linear terms may be neglected

$$\boxed{\begin{aligned} \Phi' &= \frac{c_h^2\alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2} \\ \Psi' &= \frac{\alpha_0 + \alpha_1\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2} \end{aligned}}$$

$$Q' = \frac{\alpha_1 + c_h^2\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\mu}{r^2}$$

where $\mu := \dfrac{\delta M}{8\pi\mathcal{G}_T}$

In general, $\Phi' \neq \Psi'$

(as expected)

Linear solution at large *r*
(but $rH \ll 1$)

Large non-linearity
(even for weak field)

Linear solution at large *r*
(but $rH \ll 1$)

Large non-linearity
(even for weak field)

Linear solution at large *r*
(but $rH \ll 1$)

Solve the 3 algebraic equations for Φ', Ψ', Q':

Case 1 $G_{4X} = 0, G_5 = 0$ → Single quadratic equation for Q'

$$\Rightarrow \quad \mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

Case 2 $G_{5X} = 0$ → Single cubic equation for Q'

Case 3 the most general case, $G_{5X} \neq 0$

→ Difficult to solve, but can draw some conclusion

Case I: $G_{4X} = 0 = G_5$

(Non-minimally coupled version of)
Kinetic gravity braiding

Deffayet, Pujolas, Sawicki, Vikman 2010

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In order for the solution to be real, $G_{3X}\left(XG_{3X} + G_{4\phi}\right) > 0$

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$$\mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

In order for the solution to be real, $G_{3X}\left(XG_{3X} + G_{4\phi}\right) > 0$

Short-distance solution: $\Phi' \simeq \Psi' \simeq \dfrac{G_N \delta M}{r^2}$

Two potentials coincide!

where $8\pi G_N = \dfrac{1}{2G_4} = \dfrac{1}{2G_4(\phi(t))}$

Time-dependent G in cosmological background

(Consequences of time dependence will be discussed later)

Case 2: $G_{5X} = 0$

The problem reduces to solving the following cubic equation:

$$(Q')^3 + \mathcal{C}_2 H^2 r (Q')^2 + \left(\frac{\mathcal{C}_1}{2} H^4 r^2 - H^2 \mathcal{C}_\beta \frac{\mu}{r} \right) Q' - \frac{H^4 \mathcal{C}_\alpha \mu}{2} = 0$$

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Time-dependent coefficients, written in terms of $K, G_3, G_4, G_5 = G_5(\phi)$

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$$\mu \propto \delta M$$

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Time-dependent coefficients, written in terms of $K, G_3, G_4, G_5 = G_5(\phi)$

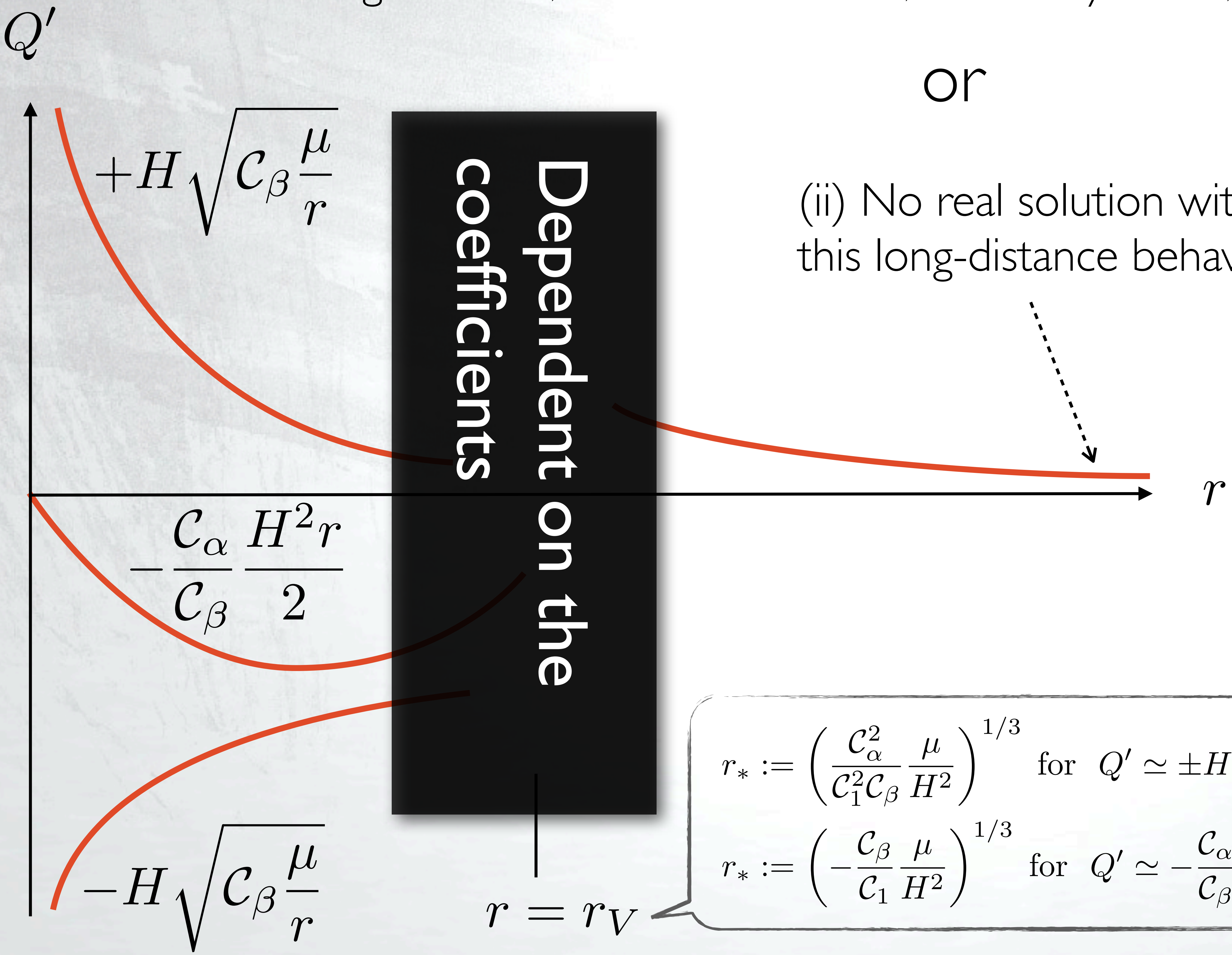
3 possible solutions at short distances:

$$Q' \simeq +H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta} \frac{H^2 r}{2}$$

(i) Either of 3 short-distance solutions is joined to the long-distance, linearized solution; real everywhere;

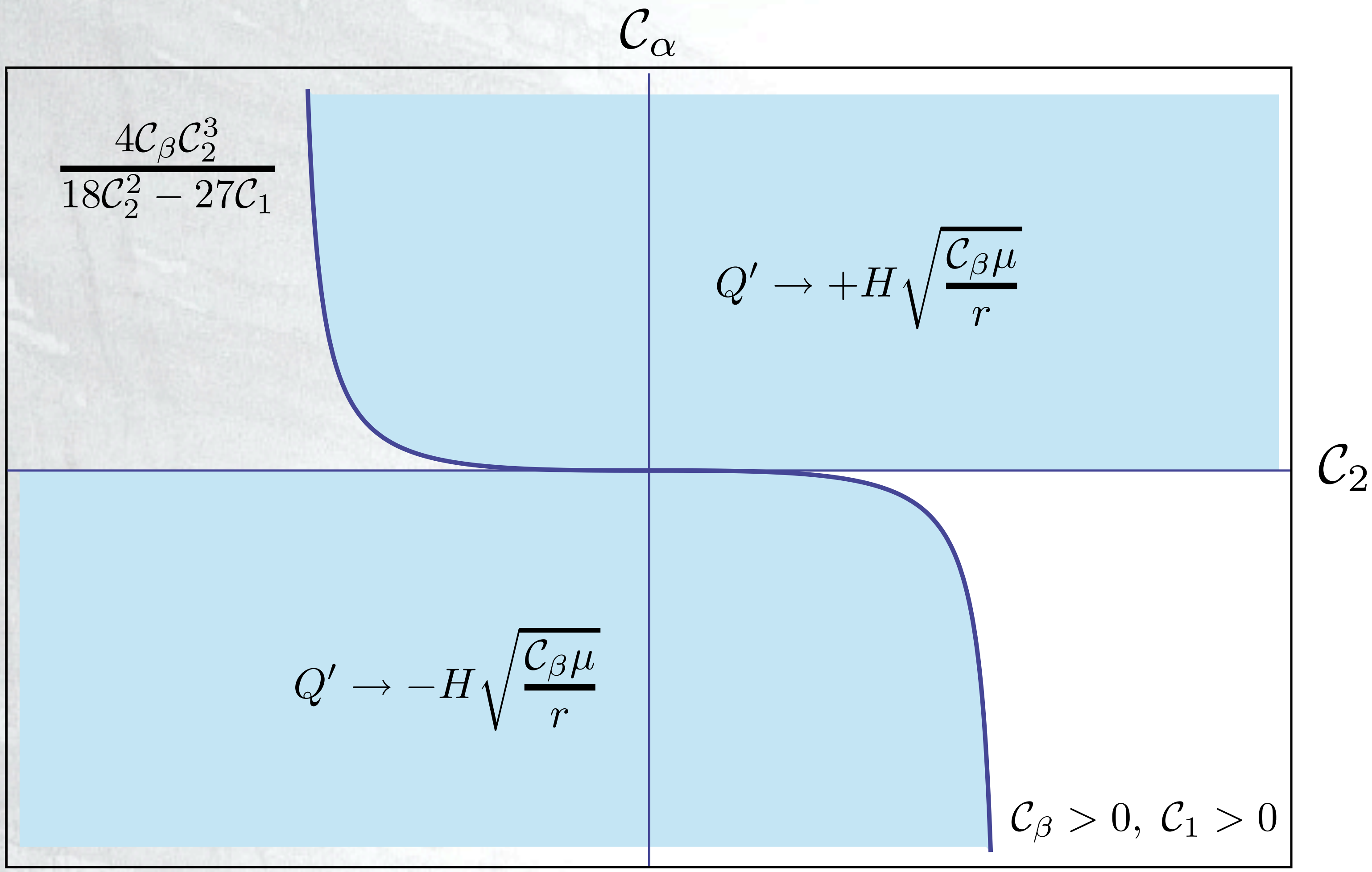
or

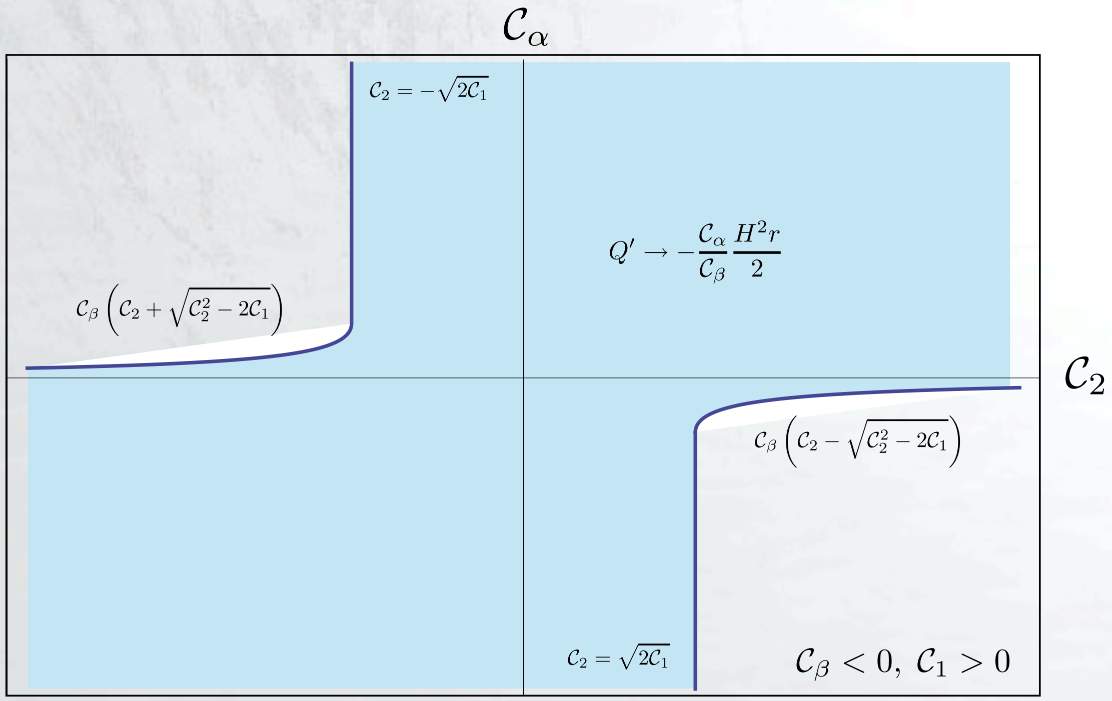
(ii) No real solution with this long-distance behavior

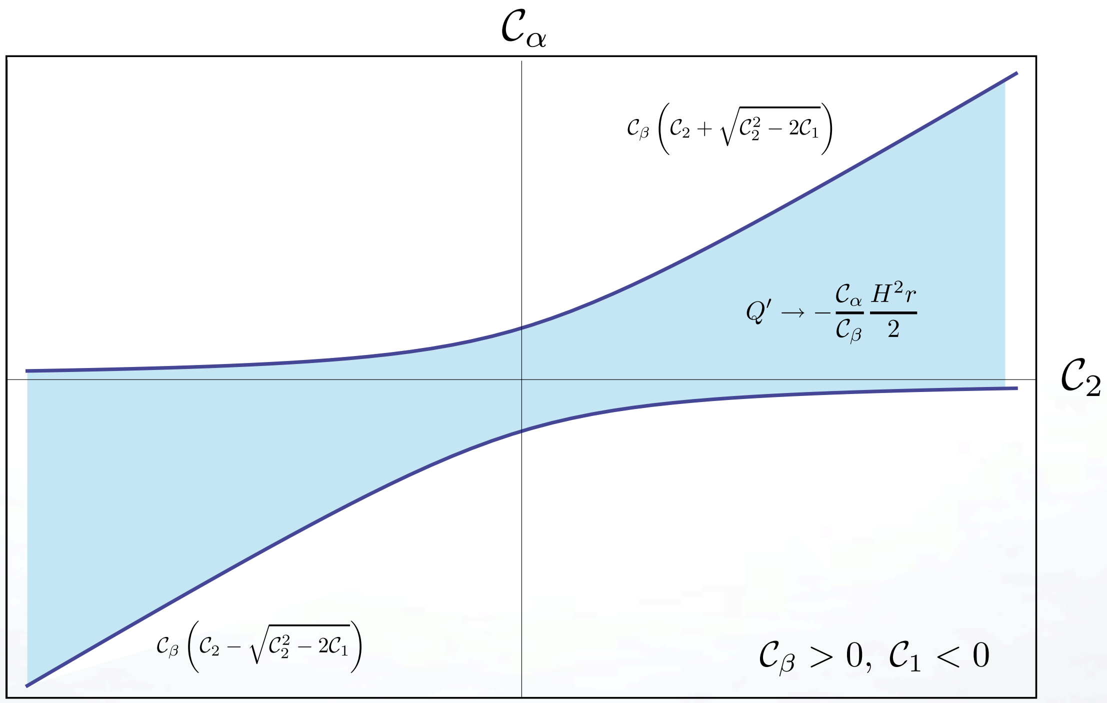


$$r_* := \left(\frac{\mathcal{C}_\alpha^2}{\mathcal{C}_1^2\mathcal{C}_\beta}\frac{\mu}{H^2}\right)^{1/3} \quad \text{for} \quad Q' \simeq \pm H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}$$

$$r_* := \left(-\frac{\mathcal{C}_\beta}{\mathcal{C}_1}\frac{\mu}{H^2}\right)^{1/3} \quad \text{for} \quad Q' \simeq -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$$


$\mathcal{C}_\alpha$
$\frac{4\mathcal{C}_\beta\mathcal{C}_2^3}{18\mathcal{C}_2^2 - 27\mathcal{C}_1}$
$Q' \to +H\sqrt{\frac{\mathcal{C}_\beta\mu}{r}}$
$\mathcal{C}_2$
$Q' \to -H\sqrt{\frac{\mathcal{C}_\beta\mu}{r}}$
$\mathcal{C}_\beta > 0,\ \mathcal{C}_1 > 0$


$\mathcal{C}_\alpha$
$\mathcal{C}_2 = -\sqrt{2\mathcal{C}_1}$
$Q' \to -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$
$\mathcal{C}_\beta\left(\mathcal{C}_2 + \sqrt{\mathcal{C}_2^2 - 2\mathcal{C}_1}\right)$
$\mathcal{C}_2$
$\mathcal{C}_\beta\left(\mathcal{C}_2 - \sqrt{\mathcal{C}_2^2 - 2\mathcal{C}_1}\right)$
$\mathcal{C}_2 = \sqrt{2\mathcal{C}_1}$
$\mathcal{C}_\beta < 0,\ \mathcal{C}_1 > 0$


$\mathcal{C}_\alpha$
$\mathcal{C}_\beta\left(\mathcal{C}_2 + \sqrt{\mathcal{C}_2^2 - 2\mathcal{C}_1}\right)$
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$\mathcal{C}_2$
$\mathcal{C}_\beta\left(\mathcal{C}_2 - \sqrt{\mathcal{C}_2^2 - 2\mathcal{C}_1}\right)$
$\mathcal{C}_\beta > 0,\ \mathcal{C}_1 < 0$

Gravity at short distances

☑ $Q' \simeq \pm H \sqrt{\mathcal{C}_\beta \frac{\mu}{r}}$ → Two potentials coincide!

$$\Phi' \simeq \Psi' \simeq \frac{G_N \delta M}{r^2}$$

Time-dependent *G*:

$$\boxed{8\pi G_N = \frac{1}{2\left(G_4 - 4XG_{4X} - 4X^2G_{4XX} + 3XG_{5\phi}\right)}}$$

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❗ Experimental constraints: $|\dot{G}_N / G_N| < 0.02 H_0$ (Lunar Laser Ranging)

— must be much slower than the cosmological time scale

Williams *et al.* 2004; Babichev *et al.* 2011

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Williams *et al.* 2004; Babichev *et al.* 2011

❗ $G_N(t) = G_{\cos}(t)$ (“*G*” in Friedmann equation)

Gravity at short distances

☑ $Q' \simeq -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$

→ Two potentials do not coincide...

$$\Phi' \simeq \frac{c_h^2}{8\pi\mathcal{G}_T}\frac{\delta M}{r^2}, \qquad \Psi' \simeq \frac{1}{8\pi\mathcal{G}_T}\frac{\delta M}{r^2}$$

$$\gamma_{\rm PPN} = \frac{1}{c_h^2}$$

Case 3: $G_{5X} \neq 0$

Difficult to analyze a variety of possible solutions in detail...

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Difficult to analyze a variety of possible solutions in detail...

But, can show that **inverse-square** potentials,

$$\Phi' \simeq \Psi' \propto \frac{1}{r^2}$$

cannot be a solution on the shortest scales

Evolution of density perturbations

ϕ is minimally coupled to matter
— matter equations are not modified

Poisson equation
is modified

$$\ddot{\delta}+2H\dot{\delta}-\frac{4}{3}\frac{\dot{\delta}^2}{1+\delta}=(1+\delta)\frac{\nabla^2}{a^2}\Phi \quad \longleftrightarrow \quad \frac{\nabla^2}{a^2}\Phi=4\pi G_{\rm eff}\rho_{\rm m}\delta$$

- On large scales (but well inside the horizon)

$$G_{\rm eff}\to\cdots\quad(\neq G_N)$$

De Felice, TK, Tsujikawa (2011)

(messy expression)

- On small scales

$$G_{\rm eff}\to G_N(t)$$

(For an appropriate model choice
with $G_{5X}=0$)

4. Summary

Summary

- Generic scalar-tensor theory contains Galileon-like non-linear derivative interaction
- **Vainshtein screening in the most general ST theory?**
  - **Time-dependent G** in cosmological background
    … Time dependence is *not* screened
    - constrained from observations and experiments
  - Inverse-square law cannot be reproduced on the smallest scales if $G_{5X} \neq 0$
  - Application to the study of structure formation

Thank you!