Observational Constraints on galileon gravity

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Nonlinear massive gravity theory and its observational tests @Kyoto 07/31/2012

Based on
RK and Kazuhiro Yamamoto, JCAP 04 (2011) 025
RK, Tsutomu Kobayashi and Kazuhiro Yamamoto, Phys.Rev. D 85 123503 (2012)
RK and Kazuhiro Yamamoto, JCAP 07 (2012) 050

Contents

Introduction and motivation

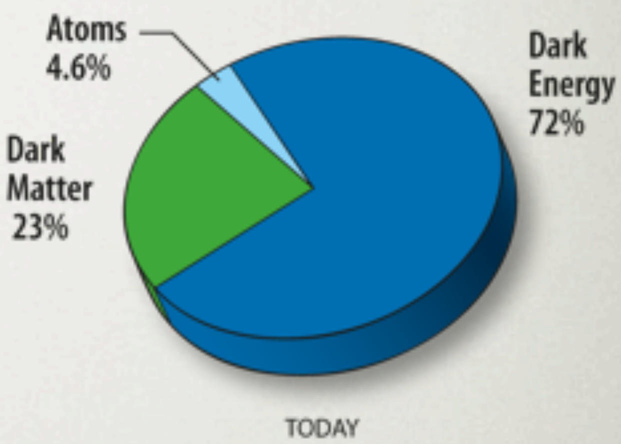
- The present cosmological observations indicate the accelerated expansion of the universe

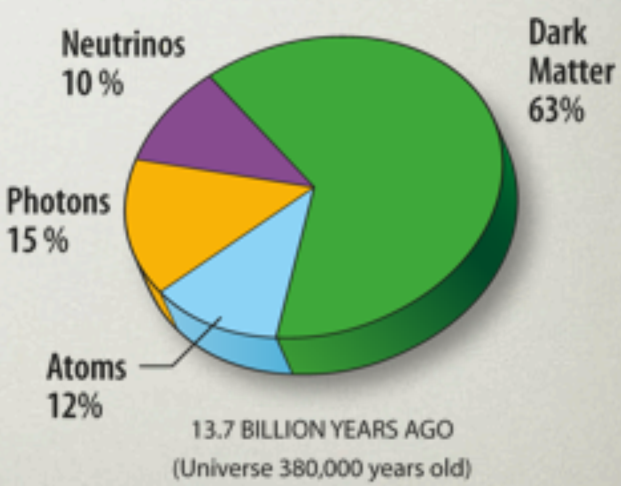
- What is the origin of an accelerated expansion of the universe?

  ✓ Dark energy ?

  ✓ Cosmological constant ?

- Alternative : Modification of gravity





(from WMAP website)

Introduction and motivation

- Modified gravity theories
  - ✓ *f(R)* gravity
  - ✓ DGP
  - ✓ (Nonlinear) massive gravity
  - ✓ Galileon
    - No ghost instabilities
- Need to check consistency with observations
  - We can test by using cosmological observations
- What is a powerful probe to distinguish modified gravity theories from ΛCDM?

Most general second-order scalar-tensor theory

✓ Horndeski found the most general Lagrangian whose EOM is second-order differential equation for ϕ and $g_{\mu\nu}$ (also known as Generalized galileon)

Deffayet, Gao, Steer (2011), Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011), Horndeski, Int. J. Theor. Phys. 10,363 (1974)

$$\mathcal{L}_2 = \boxed{K(\phi, X)}$$

K-essence term $\mathcal{L}_2 \supset (\partial\phi)^2,\ V(\phi)$

$$\mathcal{L}_3 = \boxed{-G_3(\phi, X)\Box\phi}$$

Cubic galileon term

$\mathcal{L}_3 \supset (\partial\phi)^2\Box\phi$

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)R} + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)]$$

Einstein-Hilbert term

$\mathcal{L}_4 \supset (M_{\mathrm{Pl}}^2/2)R$

$$\mathcal{L}_5 = \boxed{G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi)}$$
$$-\frac{1}{6}G_{5,X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)$$
$$+2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\Big]$$

Non-minimal derivative coupling

$\mathcal{L}_5 \supset G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

(Germani et al. 2011;
Gubitosi, Linder 2011)

$X = -(\partial\phi)^2/2, \quad G_{iX} = \partial G_i/\partial X$

Cosmology of Galileon

✓ Self-accelerating solution exists in various models

- F. P. Silva and K. Koyama, Phys. Rev. D 80, 121301 (2009)
- T. Kobayashi, H. Tashiro, and D. Suzuki, Phys. Rev. D 81, 063513 (2010)
- T. Kobayashi, Phys. Rev. D 81, 103533 (2010)
- A. De Felice and S. Tsujikawa, Phys. Rev. D 84, 124029 (2011)
- R. Gannouji and M. Sami, Phys. Rev. D 82, 024011 (2010)
- A. De Felice and S. Tsujikawa, Phys. Rev. Lett. 105, 111301 (2010)
- A. De Felice, R. Kase, and S. Tsujikawa, Phys. Rev. D 83, 043515 (2011)
- C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, J. Cosmol. Astropart. Phys. 10 (2010) 026
- R. Kimura and K. Yamamoto, J. Cosmol. Astropart. Phys. 04 (2011) 025

and many papers ...

Free of ghost instability and Vainshtein mechanism

Toy Model 1

The background evolution of Kinetic gravity braiding

RK and Kazuhiro Yamamoto, JCAP 04 (2011) 025

Kinetic Gravity Braiding

✔ KGB model (Deffayet et al. 2010, Kobayashi et al. 2010)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + K(X) - G(X)\Box\phi + \mathcal{L}_{\rm m}\right]$$

$\mathcal{L}_4 = M_{\rm Pl}^2 R/2$

$\mathcal{L}_5 = 0$

→ *Simple extension of the galileon model*

✔ Shift symmetry $\phi\to\phi+\mathrm{const}$ ensures the existence of the Nether current

$$\dot{J}_0 + 3HJ_0 = 0$$

$$J_0 = \dot\phi\left(3\dot\phi G_X H - K_X\right) \propto 1/a^3$$

✔ $J_0 = 0$ is the attractor solution, which leads to a self-accelerating solution if

$$K_X\Big|_{J_0=0} < 0$$

(Deffayet et al. 2010)

Toy model of KGB

✓ Example (RK and Yamamoto 2011)

$$K(X) = -X$$

$$G(X) = M_{\rm Pl}\left(\frac{r_c^2}{M_{\rm Pl}^2}X\right)^n$$

r_c : crossover scale ($\sim H_0^{-1}$)

n : model parameter ($n > 1/2$)

✓ Choosing the attractor solution (J_0=0), $\dot{\phi} = K_X/3G_X H$

$$\left(\frac{H}{H_0}\right)^2 = \underbrace{(1-\Omega_0)\left(\frac{H}{H_0}\right)^{-\frac{2}{2n-1}}}_{\text{behave like dark energy}} + \Omega_0 a^{-3}$$

Dvali-Turner's Model

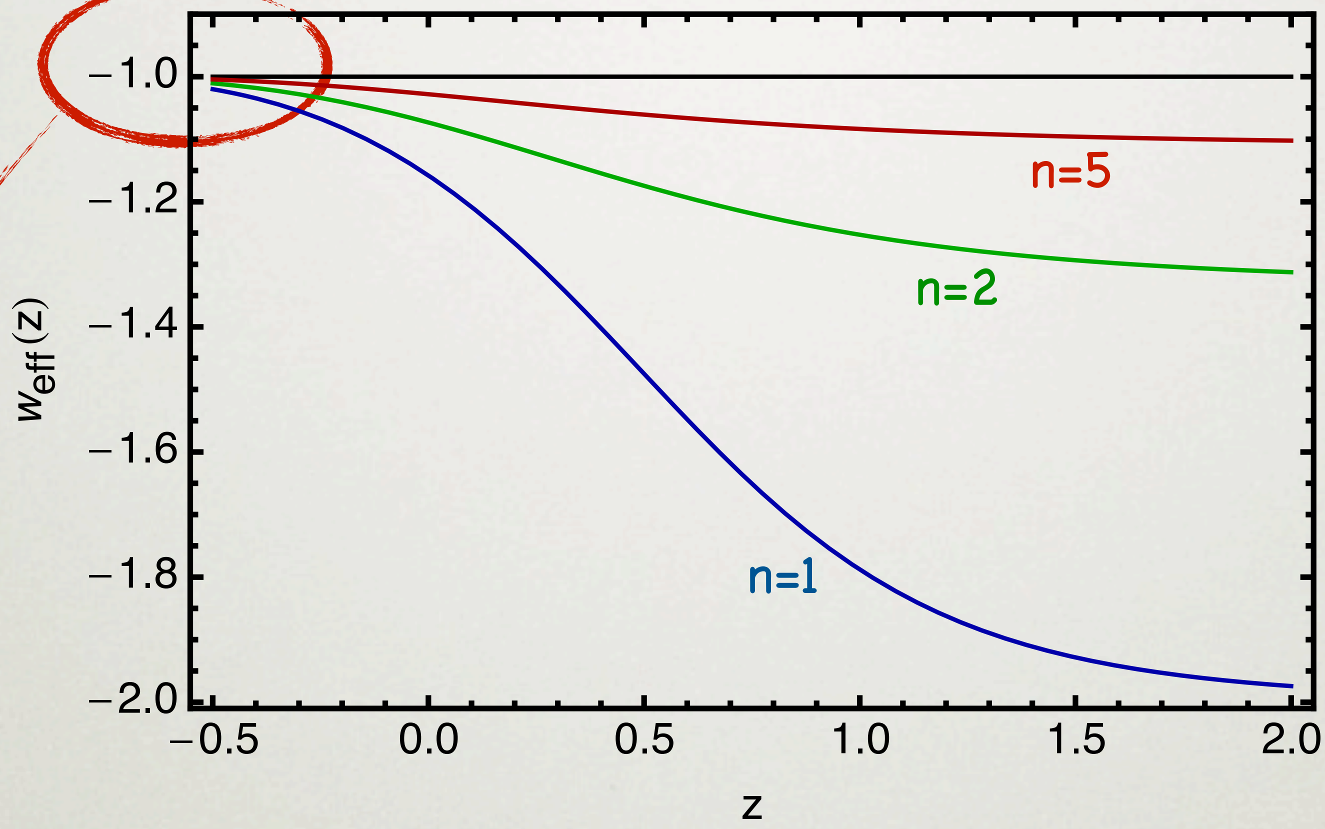
(Dvali, Turner 2003)

→

For n=1	Original galileon model (minimally coupled)
For large n $(n \gtrsim 100)$	Cosmological constant model

Effective Equation of State

$$w_{\text{eff}} \equiv p_\phi/\rho_\phi$$



de Sitter expansion (scalar dominated era)

$$w_{\text{eff}} \simeq -1$$

Absence of ghost and stability condition

✓ Quadratic action for the perturbed scalar field $\phi(t,\mathbf{x}) \to \phi(t) + \delta\phi(t,\mathbf{x})$

$$\delta S^{(2)} = \frac{1}{2}\int d^4x\sqrt{-g}\kappa(a)\left[\dot{\delta\phi}^2 - \frac{c_s^2(a)}{a^2}(\partial_i\delta\phi)^2\right]$$

$\kappa(a) > 0$ (No ghostlike behavior)

$c_s^2(a) > 0$ (Stable)

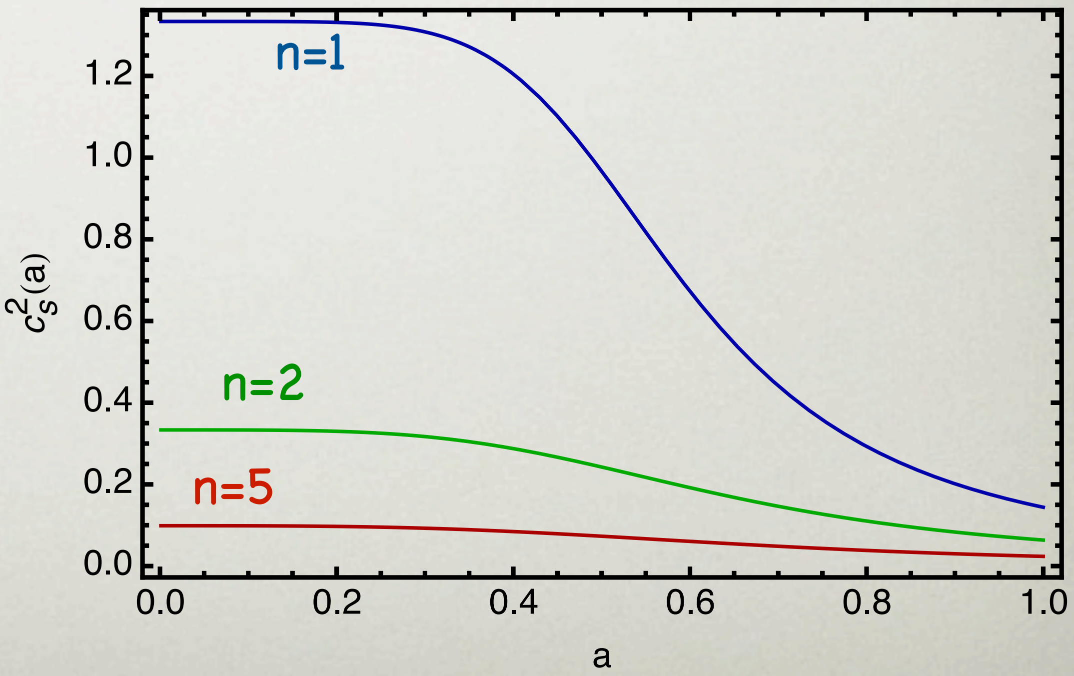
Parameter must satisfy

$n > 1/2$

✓ The sound speed

$$c_s^2 \propto 1/n$$

The sound speed of the perturbed scalar field becomes zero if n=∞ !



Toy Model 1

Cosmological perturbations in Kinetic gravity braiding

RK and Kazuhiro Yamamoto, JCAP 04 (2011) 025

Cosmological perturbations

✓ Setup

Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$$

Scalar and matter perturbations

$$\rho(x,t) = \rho(t)[1+\delta(x,t)]$$
$$\phi(x,t) = \phi(t) + \delta\phi(x,t)$$

For small n (well inside H_0^{-1} and r_s)	Quasi-static approximation is valid
For large n (c_s approaches zero)	Quasi-static approximation is no longer valid

Quasi-static approximation (small n)

✓ Quasi-static approximation is valid for $\mathcal{O}(k^2 c_s^2/a^2) \gg \mathcal{O}(H^2)$

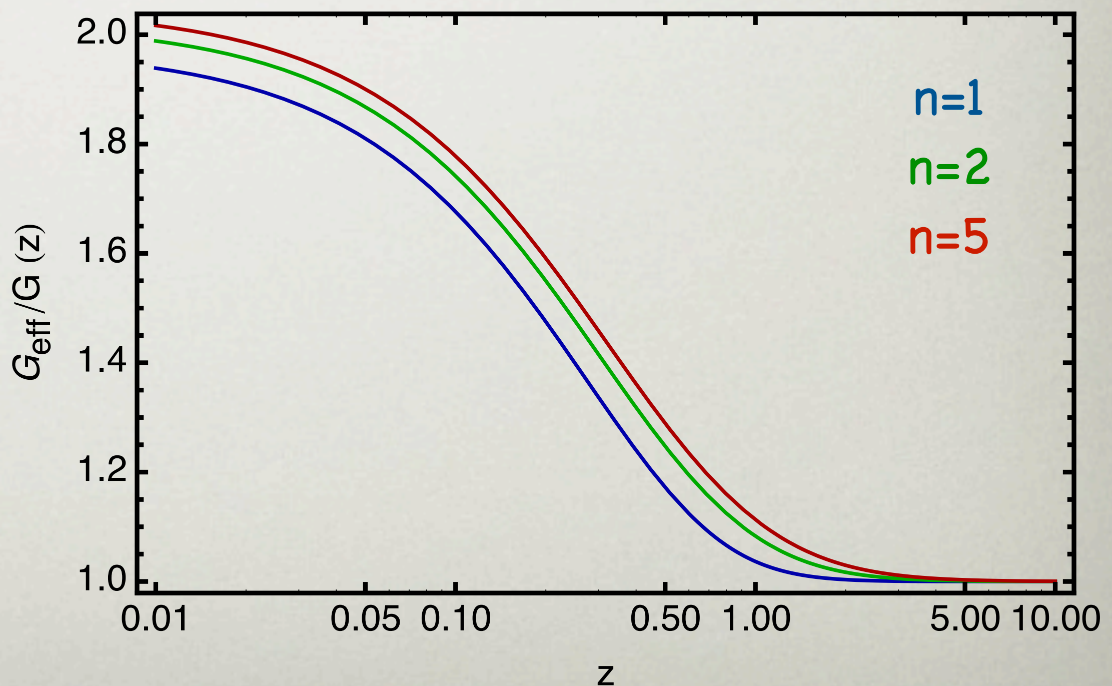
✓ Energy momentum conservation and modified Poisson equation

$$\ddot{\delta} + 2H\dot{\delta} \simeq \frac{\nabla^2}{a^2}\Psi \qquad \frac{\nabla^2}{a^2}\Psi \simeq 4\pi G_{\rm eff}\,\rho\delta$$

✓ Effective gravitational coupling

$$G_{\rm eff} = G\left[1 + 4\pi G \frac{G_X^2 \dot{\phi}^4}{\beta(a)}\right]$$

The growth is enhanced due to an additional scalar force



Full perturbation equation (large n)

$$2M_{\rm Pl}^2\left[-3H(\dot\Phi-H\Psi)+\frac{1}{a^2}\nabla^2\Phi\right]=-K_X\delta X-G_X\left(3\dot\phi^3\dot\Phi-12H\dot\phi^3\Psi+9H\dot\phi^2\dot{\delta\phi}-\frac{\dot\phi^2}{a^2}\nabla^2\delta\phi\right)$$
$$-3G_{XX}H\dot\phi^3\delta X-\delta\rho$$

$$2M_{\rm Pl}^2\left(\dot\Phi-H\Psi\right)=-K_X\dot\phi\delta\phi-G_X\dot\phi^2\left(\dot\phi\Psi-\dot{\delta\phi}+3H\delta\phi\right)+\delta q$$

$$2M_{\rm Pl}^2\left[(3H^2+2\dot H)\Psi+H\dot\Psi-\ddot\Phi-3H\dot\Phi\right]$$
$$=K_X\delta X+G_X\left(\dot\phi^3\dot\Psi-\dot\phi^2\ddot{\delta\phi}+4\dot\phi^2\ddot\phi\Psi-2\dot\phi\ddot\phi\dot{\delta\phi}\right)-G_{XX}\dot\phi^2\ddot\phi\delta X$$

$$\Psi+\Phi=0$$

Einstein equations

$$-K_X\left[3\dot\phi\dot\Phi-\dot\phi\dot\Psi-2(\ddot\phi+3H\dot\phi)\Psi+\ddot{\delta\phi}+3H\dot{\delta\phi}-\frac{1}{a^2}\nabla^2\delta\phi\right]$$
$$-3G_{XXX}H\dot\phi^3\ddot\phi\delta X-G_X\Big[3\dot\phi^2\ddot\Phi+6(\ddot\phi+3H\dot\phi)\dot\phi\dot\Phi-9H\dot\phi^2\dot\Psi$$
$$-12\left\{(\dot H+3H^2)\dot\phi^2+2H\dot\phi\ddot\phi\right\}\Psi-\frac{\dot\phi^2}{a^2}\nabla^2\Psi+6H\dot\phi\ddot{\delta\phi}$$
$$+6\left\{H\ddot\phi+(\dot H+3H^2)\dot\phi\right\}\dot{\delta\phi}-\frac{2}{a^2}(\ddot\phi+2H\dot\phi)\nabla^2\delta\phi\Big]$$
$$-G_{XX}\Big[3\dot\phi^3\ddot\phi\dot\Phi-3H\dot\phi^4\dot\Psi-3\left\{8H\dot\phi^3\ddot\phi+(\dot H+3H^2)\dot\phi^4\right\}\Psi$$
$$+3H\dot\phi^3\ddot{\delta\phi}+3\left\{5H\dot\phi^2\ddot\phi+(\dot H+3H^2)\dot\phi^3\right\}\dot{\delta\phi}-\frac{\dot\phi^2\ddot\phi}{a^2}\nabla^2\delta\phi\Big]=0$$

Scalar field equation

Full perturbation equation (large n)

✔ Taking the limit n→∞, the scalar field equation becomes

$$\delta\dot{X} + 3H\delta X = 0$$

$$\delta X = \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\Psi$$

✔ One can find the decaying solution

$$\delta X = \mathrm{Const}/a^3$$

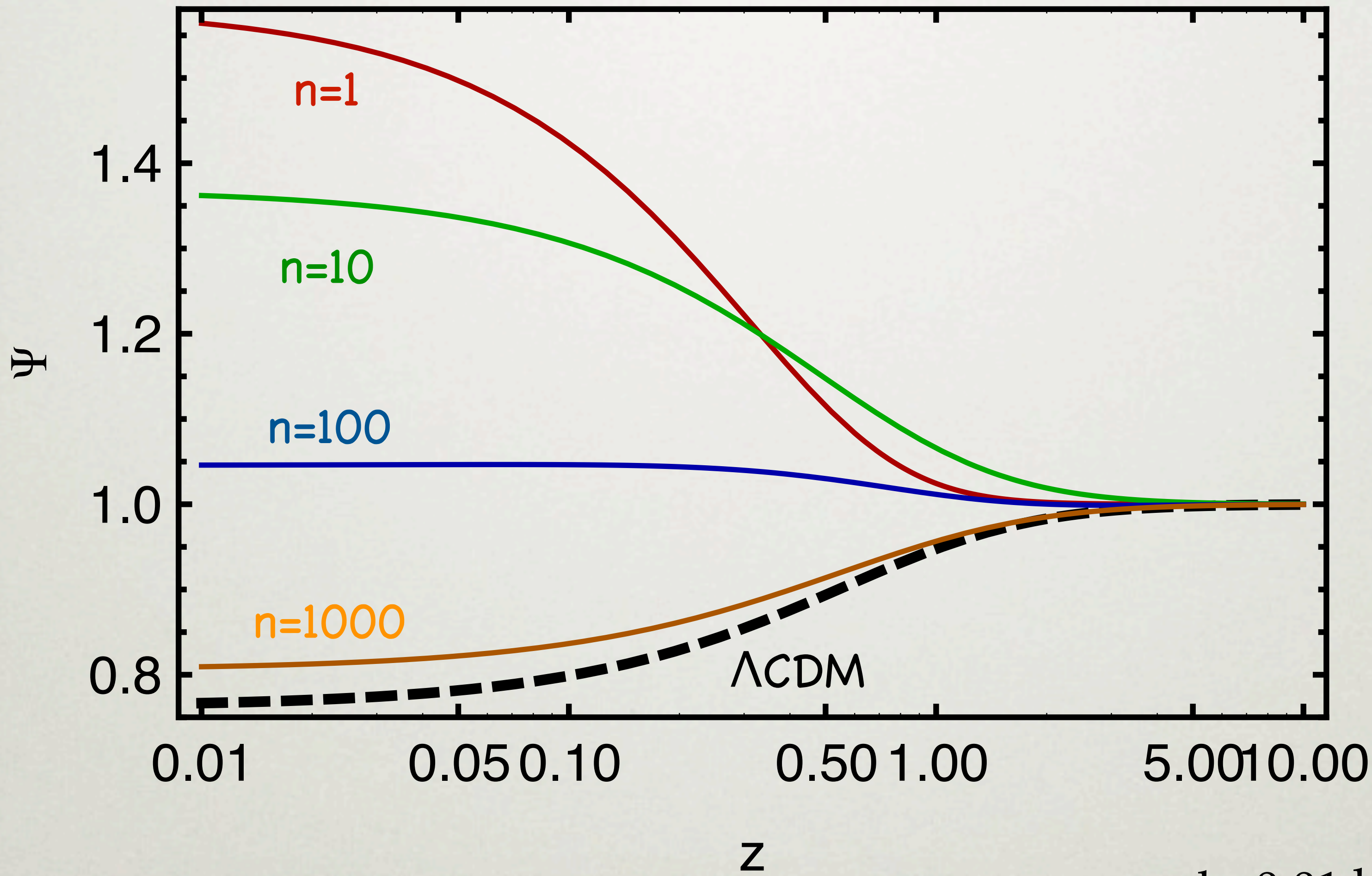
✔ As long as the initial fluctuation of the scalar field is small, one can set $\delta X = 0$

→ Scalar field does not contribute to the Einstein field equations

$$\delta\rho_\phi = \delta p_\phi = 0$$

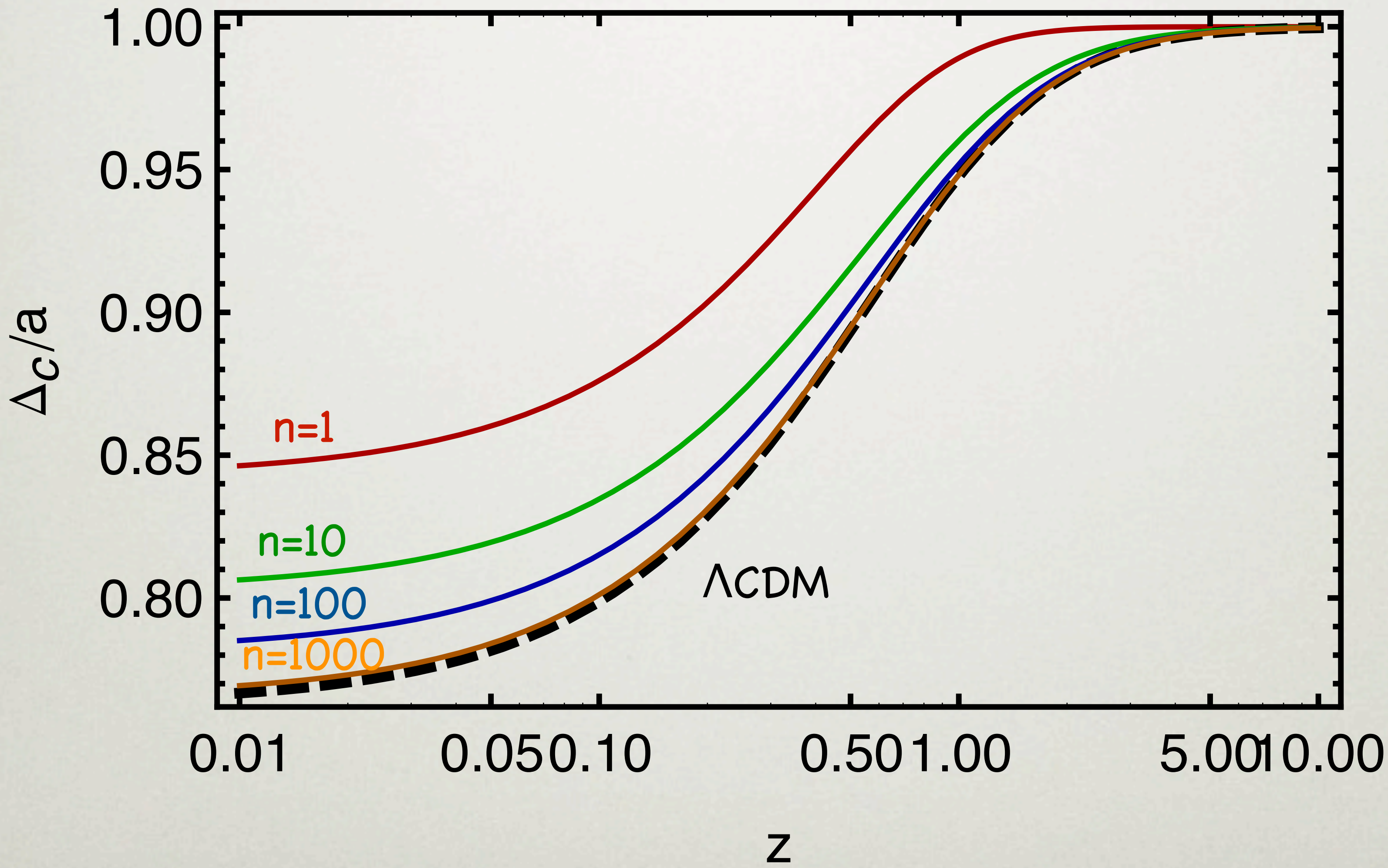
n→∞ limit corresponds to ΛCDM limit at both background and perturbations level

Numerical results of Gravitational potential



k=0.01 h /Mpc

Numerical results of growth factor



k=0.01 h /Mpc

How can we distinguish scalar-tensor theories from the ΛCDM model?

Approach 1

Supernovae and CMB shift parameter

RK and Kazuhiro Yamamoto, JCAP 04 (2011) 025

Constraints from supernovae and CMB

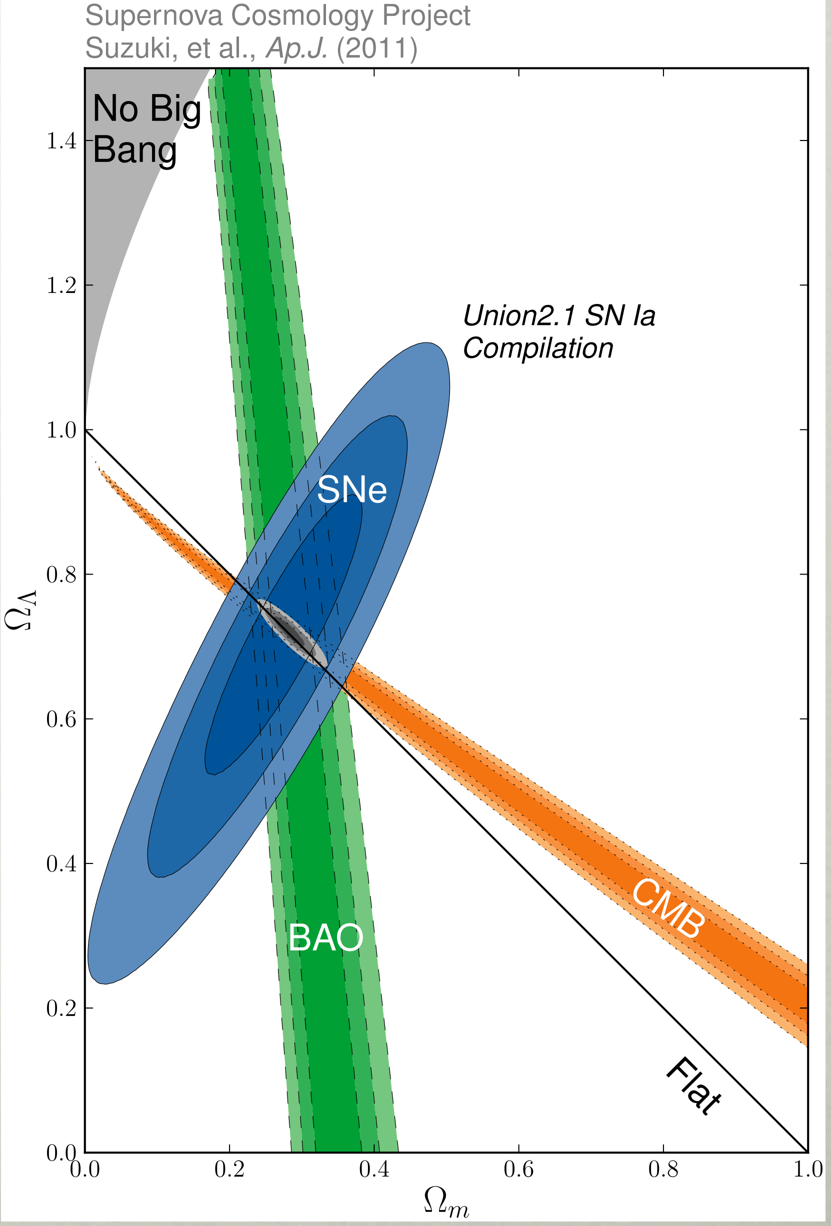
✓ Supernovae (SCP Union 2)

- 557 supernovae
- Redshift $z < 1.4$
- Distance modulus

$$\mu = 5\log\left(\frac{d_L(z)}{Mpc}\right) + 25$$

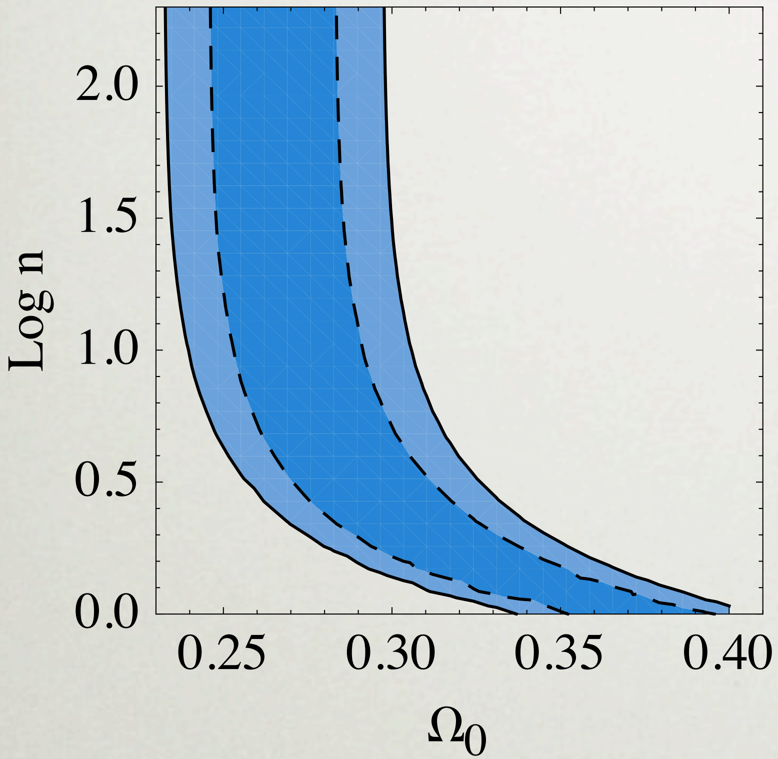
✓ CMB shift parameter (WMAP 7 year)

$$\mathrm{R} = \sqrt{\Omega_{m0}}\int_0^{z_{\mathrm{CMB}}} \frac{dz'}{H(z')/H_0}$$



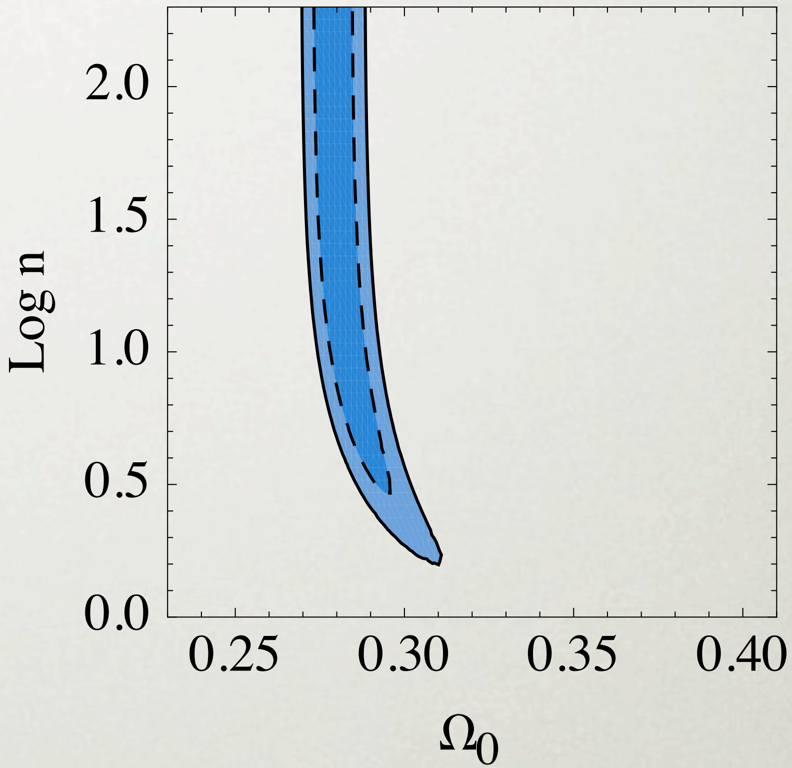
Constraints from supernovae and CMB

Type Ia supernovae



(SCP Union 2)

CMB shift parameter



(WMAP 7year)

$n \gtrsim 3$ (95% C.L.)

Approach 2

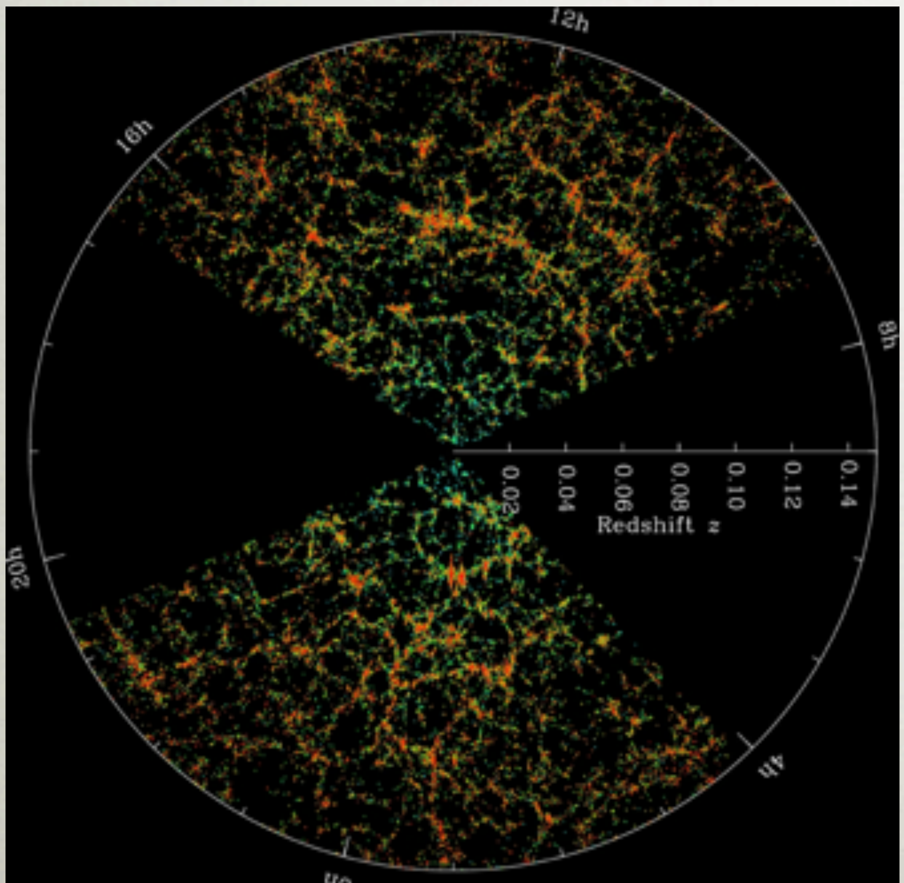
Matter power spectrum

RK and Kazuhiro Yamamoto, JCAP 04 (2011) 025

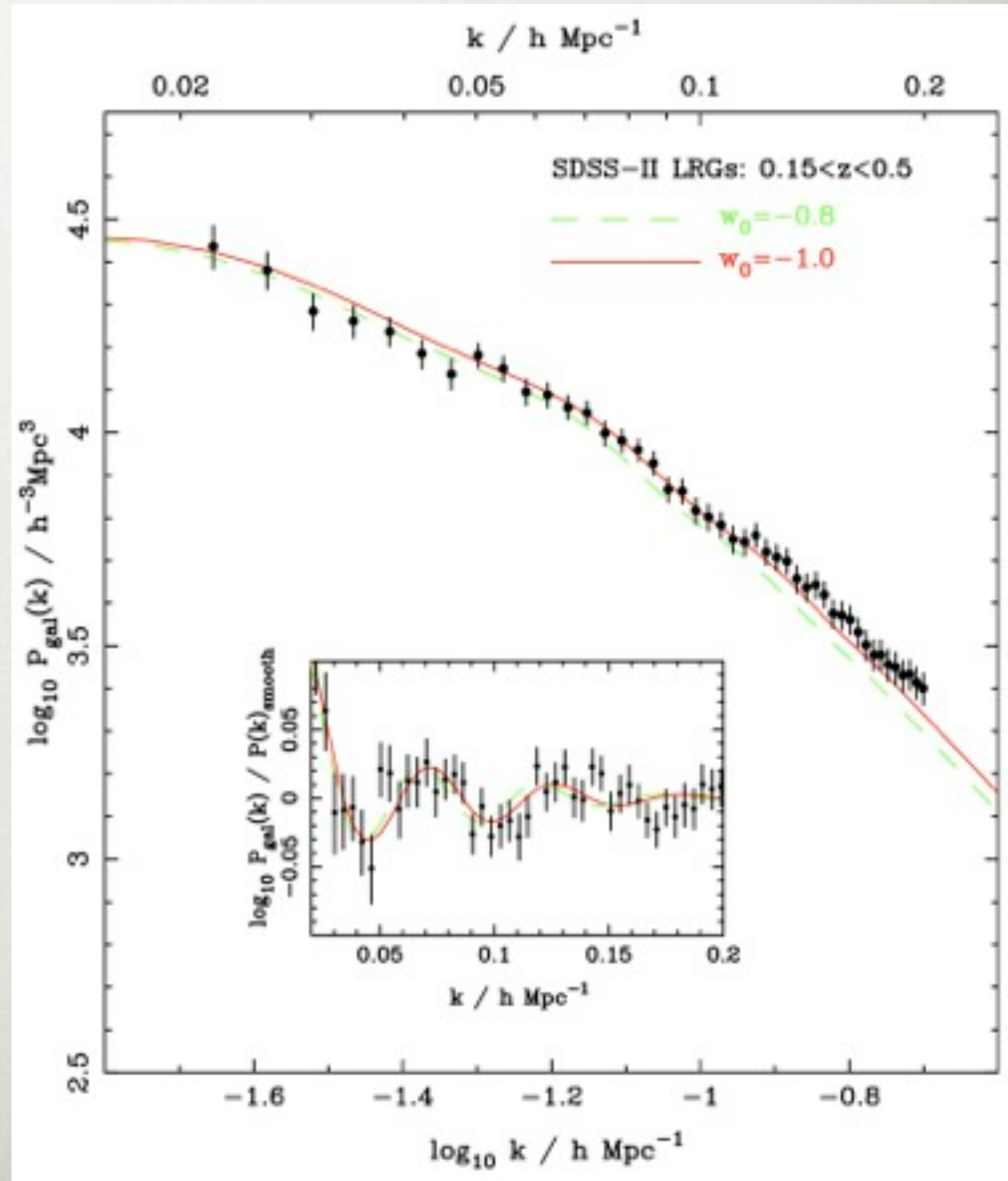
Galaxy distribution

✓ Matter power spectrum

$$\langle \delta(t,\mathbf{k})\delta(t,\mathbf{k}')^*\rangle = P_m(k,a)\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$



(from SDSS website)



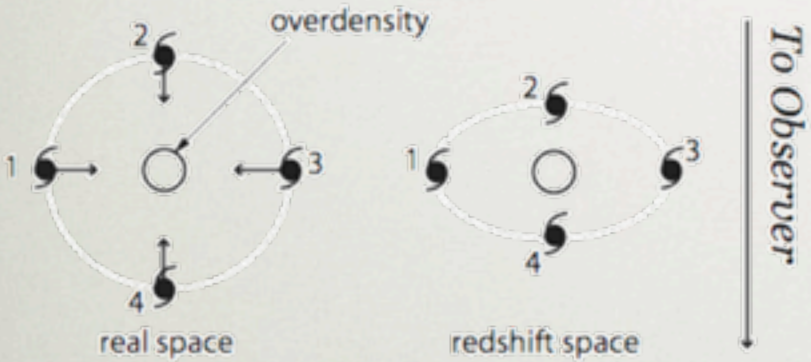
(Percival et al. 2010)

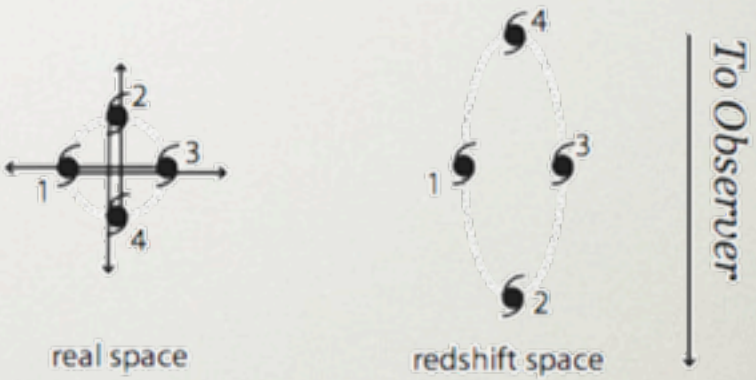
Redshift space distortion

✓ Observed matter power spectrum is distorted by these effects in redshift space

Kaiser effect

Finger of God effect





Anisotropy of matter power spectra could be powerful tool to constrain the model parameter as well as the cosmological parameters...

Redshift space distortion

✓ Power spectrum in redshift space

$$P_\mathrm{s}(k) = (1+\beta\mu^2)^2 P(k)$$

$$\beta = f/b$$

$$f(a) = \frac{d\ln D_1}{d\ln a}$$

✓ Multipole expansion

$$P_s(k,\mu) = \sum_{l=0,2,4,\dots} P_l(k)\mathcal{L}_l(\mu)(2l+1)$$

$$\mu = \frac{\vec{k}\cdot\hat{z}}{|\vec{k}|}$$

✓ The ratio of the quadrupole to the monopole

$$\frac{P_2(k)}{P_0(k)} = \frac{\frac{4}{3}\beta + \frac{4}{7}\beta}{1+\frac{2}{3}\beta+\frac{1}{5}\beta^2}$$

Degeneracy of bias and growth can be broken by using both P_0 and P_2

Constraints from matter Power Spectrum

✓ SDSS LRG sample data release 7

Survey area	7150 deg^2
Number of galaxies	100157 LRG
Redshift range	$0.16 < z < 0.47$

✓ Power spectrum in redshift space with non-linearity

$$P_{\rm gal}(k,\mu) = \left(b^2(k)P_{\delta\delta}(k) + 2fb(k)P_{\delta\theta}(k)\mu^2 + f^2P_{\theta\theta}(k)\mu^4\right)e^{-(fk\mu\sigma_v)^2}$$

✓ Unknown parameters $\quad n,\ \Omega_0,\ A,\ b,\ \sigma_{\rm v}$

✓ Marginalizing parameters $\quad \Omega_0,\ A,\ b,\ \sigma_{\rm v}$

Constraints from matter Power Spectrum

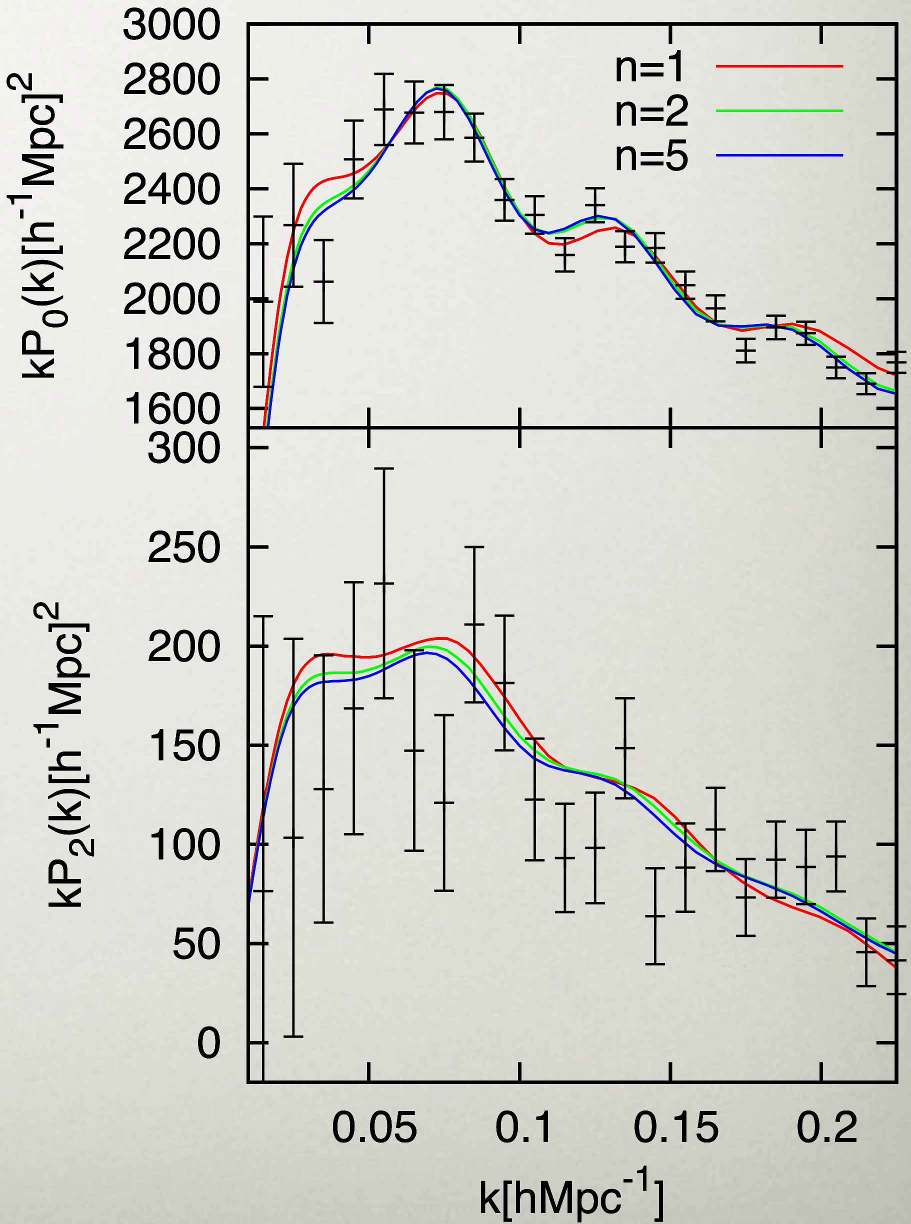
✓ In general,

- Ω_0 can be determined by the baryon acoustic oscillation
- P_0 determines the combination of bias and growth
- P_2 determines growth

✓ However,

The error bar is still large to distinguish the model from the LCDM model

Even if we combine the constrains from the standard ruler and power spectrum, small n is still allowed



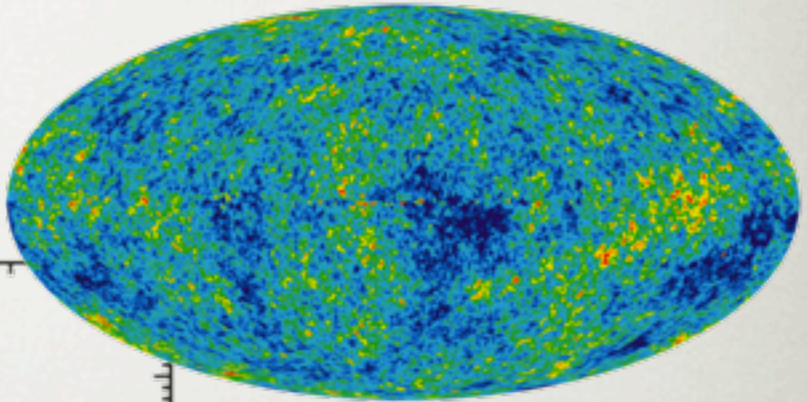
Approach 3

LSS-ISW cross-correlation

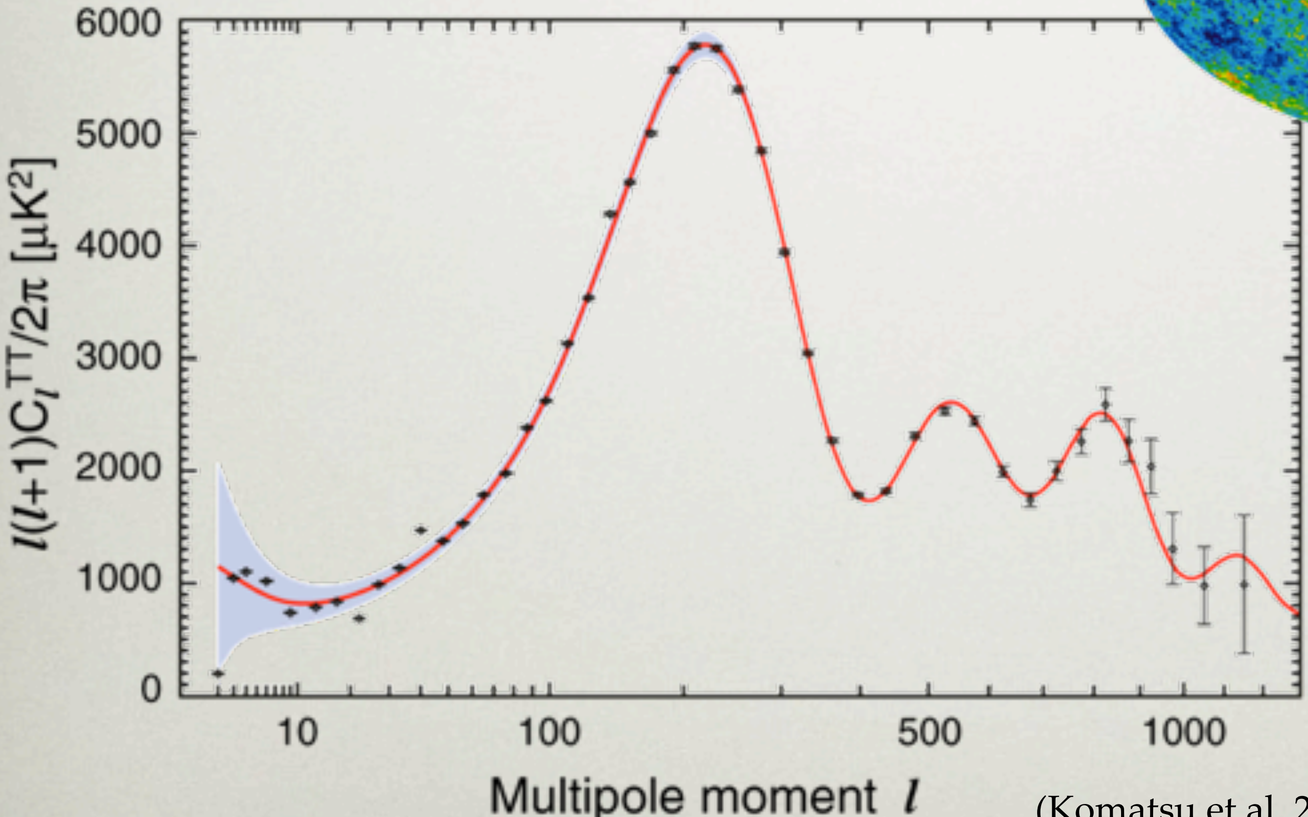
RK, Tsutomu Kobayashi and Kazuhiro Yamamoto, Phys.Rev. D 85 123503 (2012)

Integrated Sachs-Wolfe effect

✓ CMB contains lots of information and useful to determine cosmological parameters



(from WMAP website)



(Komatsu et al. 2010)

INTEGRATED SACHS-WOLFE EFFECT

✓ ISW term in CMB anisotropy

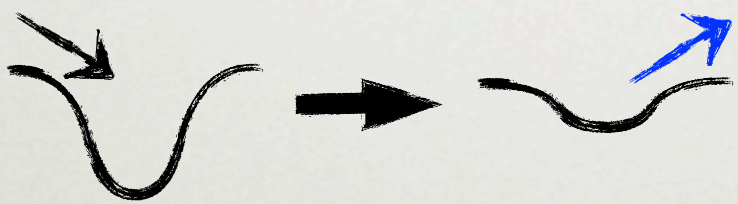
$$\left(\frac{\Delta T(\vec{\gamma})}{T}\right)_{\rm ISW} = \int_{\eta_d}^{\eta_0} d\eta[\Psi'(\eta, \mathbf{x}) - \Phi'(\eta, \mathbf{x})]$$

Einstein de-Sitter universe

Potential is constant

NO ISW

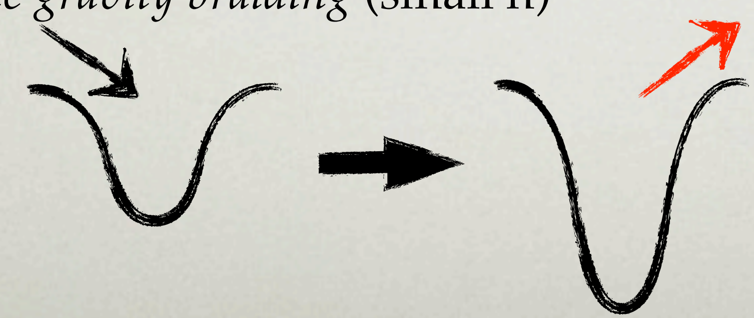
ΛCDM model



Potential decays

Blueshift

Kinetic gravity braiding (small n)



Potential grows

Redshift

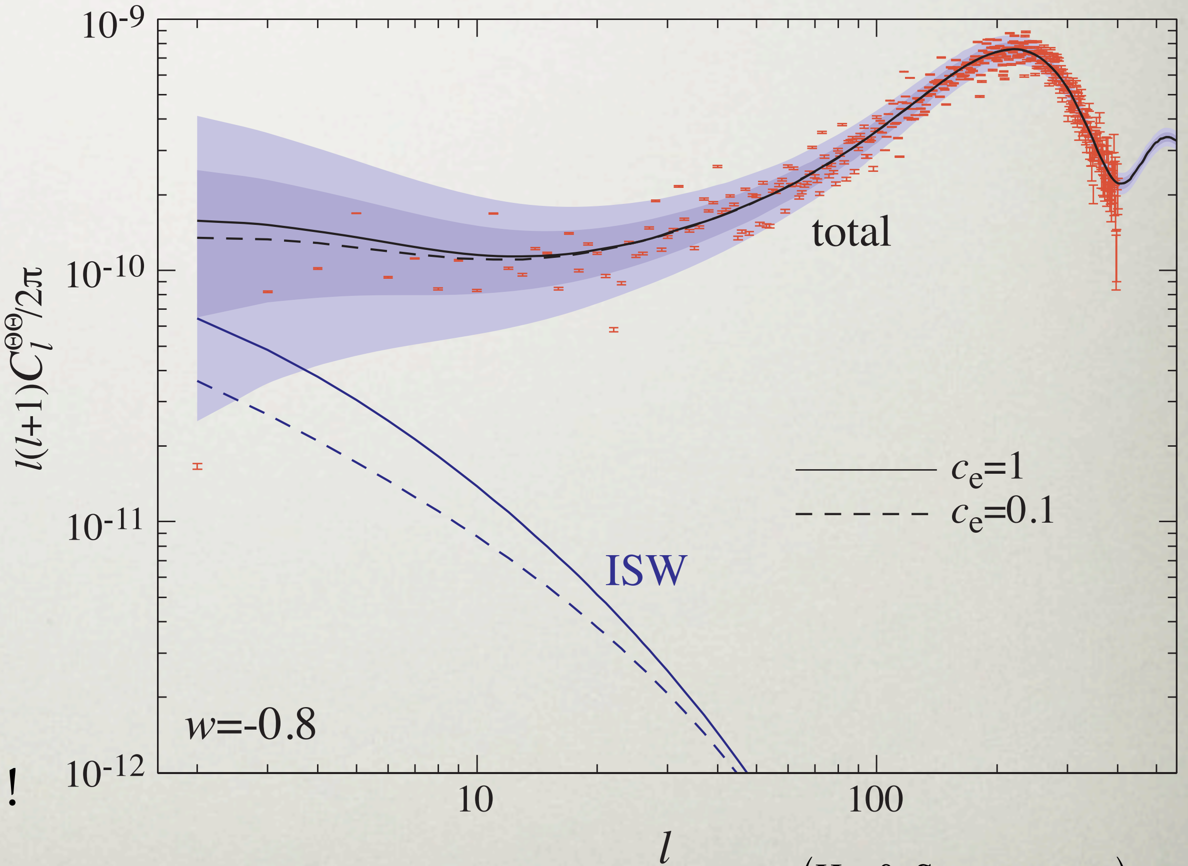
Integrated Sachs-Wolfe effect

- ✓ ISW information ~10% of the primary CMB anisotropy
- ✓ Cosmic variance

$$\left(\frac{\Delta C_l}{C_l}\right) = \sqrt{\frac{2}{2l+1}}$$

ISW in CMB power spectrum is dominated by **cosmic variance** !

LSS-ISW **cross-correlation** can extract information of ISW effect !
(Crittenden &Turok '95)



(Hu & Scaranton 04)

LSS-ISW CROSS-CORRELATION

✓ LSS-ISW cross-correlation function (CCF)

$$\left\langle \frac{\Delta T(\vec{\gamma})}{T} \frac{\Delta N_g(\vec{\gamma}')}{N} \right\rangle = \frac{1}{4\pi} \sum_\ell (2\ell+1) C_\ell P_\ell(\mu),$$

Temperature anisotropy *Galaxy distribution*

Selection function *Growth factor* *Bias* *Power spectrum*

$$C_\ell = \frac{3\Omega_0 H_0^2}{(\ell+1/2)^2} \int dz H(z) \mathcal{W}(z) \frac{D_1(z)}{D_1^2(z_0)} \underline{\frac{dU_k(z)}{dz}} b(z,k) P(k) \Big|_{k=(\ell+1/2)/\chi}$$

determine the sign of CCF

$$U_k(a) = \frac{G_{\text{eff}}(a)}{G} \frac{D_1(a)}{a}$$

Fixed cosmological parameters

✓ Cross-correlation data Giannantonio et al. 2008

✓ Scale independent bias for the ΛCDM model (Giannantonio et al. 2008)

	2MASS	SDSS galaxies	SDSS LGR	NVSS	HEAO	QSO
b	1.4	1.0	1.8	1.5	1.06	2.3

determined by the auto-correlation function of galaxy distribution

✓ Rescaled bias

$$b = b^{(\Lambda CDM)} \frac{D_1^{(\Lambda CDM)}}{D_1}$$

✓ Fixed cosmological parameters (WMAP 7year value)

$$h = 0.702$$

$$\Omega_b = 0.0451$$

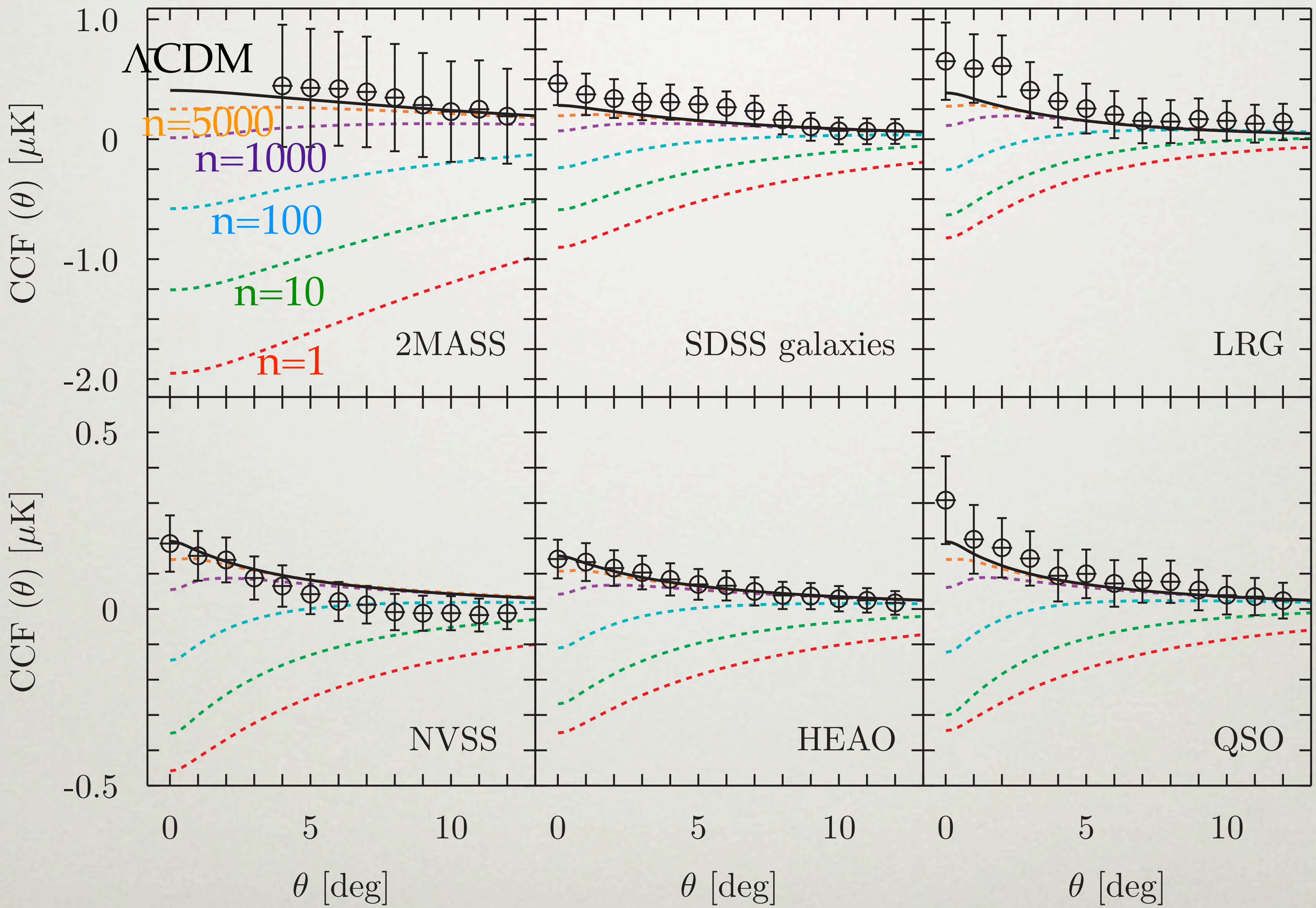
$$\Omega_0 h^2 = 0.1338$$

$$n_s = 0.966$$

$$\Delta_{\mathcal{R}}^2 = 2.42 \times 10^{-9} \text{ at } k_0 = 0.002\mathrm{Mpc}^{-1}$$

Constraints from Integrated Sachs-Wolfe effect

Data from Giannantonio et al. '08



$n \gtrsim 10^3$ (95% C.L.)

Approach 4

Gravitational Cherenkov radiation

RK and Kazuhiro Yamamoto, JCAP 07 (2012) 050

Sound speed of graviton in MGST

✓ Most general second-order scalar-tensor theory

$$S = \int d^4x\sqrt{-g}\left[\sum_{i=2}^{4}\mathcal{L}_i + \mathcal{L}_m[g_{\mu\nu},\psi]\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

✓ Quadratic action for a tensor mode

Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011),

$$S_T^{(2)} = \frac{1}{8}\int dtd^3xa^3\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla}h_{ij})^2\right]$$

✓ Sound speed of graviton

$$c_T^2 \equiv \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

For KGB model, $c_T^2 = 1$

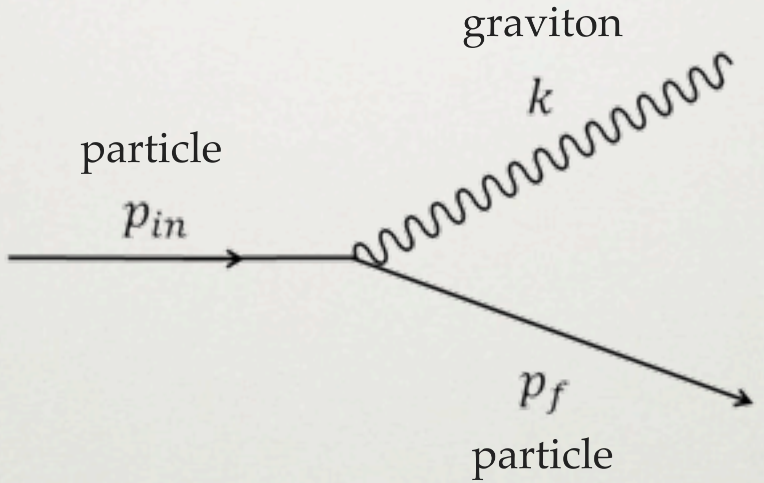
$$\mathcal{F}_T \equiv 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right]$$

$$\mathcal{G}_T \equiv 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

Gravitational Cherenkov Radiation

✓ If the sound speed of graviton is smaller than the speed of light, particle should emit graviton through the similar process to Cherenkov radiation

Moore and Nelson (2001)



✓ Highest energy cosmic ray ($p \sim 3\times10^{11}$ GeV) can provide us the lower bound on the sound speed of graviton

Gravitational Cherenkov Radiation

- ✓ Consider the complex scalar in a FRW background

$$S_m = \int d^4x \sqrt{-g}\left[-g^{\mu\nu}\partial_\mu\Psi^*\partial_\nu\Psi - m^2\Psi^*\Psi - \xi R\Psi^*\Psi\right]$$

- ✓ Quantize the complex scalar and tensor field as

$$\hat{\Psi}(\eta,\mathbf{x}) = \frac{1}{a}\int \frac{d^3p}{(2\pi)^{3/2}}\left[\hat{b}_{\mathbf{p}}\psi_p(\eta)e^{i\mathbf{p}\cdot\mathbf{x}} + \hat{c}^\dagger_{\mathbf{p}}\psi^*_p(\eta)e^{-i\mathbf{p}\cdot\mathbf{x}}\right]$$

$$\hat{h}_{\mu\nu} = \frac{1}{a}\sqrt{\frac{2}{\mathcal{G}_T}}\sum_\lambda\int\frac{d^3k}{(2\pi)^{3/2}}\left[\varepsilon^{(\lambda)}_{\mu\nu}\hat{a}_{\mathbf{k}}h_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \varepsilon^{(\lambda)}_{\mu\nu}\hat{a}^\dagger_{\mathbf{k}}h^*_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$[\hat{b}_{\mathbf{p}}, \hat{b}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$$

$$[\hat{c}_{\mathbf{p}}, \hat{c}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$$

- ✓ Mode functions satisfy

$$\left(\frac{d^2}{d\eta^2} + p^2 + m^2a^2\right)\psi_p(\eta) = 0$$

$$\left(\frac{d^2}{d\eta^2} + c_T^2k^2 - \frac{a''}{a}\right)h_k(\eta) = 0$$

Gravitational Cherenkov Radiation

- ✓ In-in formalism

$$\langle Q(t)\rangle = i^2 \int_{t_{\rm in}}^{t} dt_2 \int_{t_{\rm in}}^{t_2} dt_1 \,\langle [H_I(t_1),[H_I(t_2),Q]]\rangle$$

- ✓ The total radiation energy from the complex scalar field

$$E = \sum_{\lambda}\sum_{\mathbf{k}} (\omega_k/a)\langle \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}}\rangle$$

where

$$\left\langle \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}}\right\rangle = 2\Re \int_{t_{\rm in}}^{t} dt_2 \int_{t_{\rm in}}^{t_2} dt_1 \left\langle H_I(t_1) \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}} H_I(t_2)\right\rangle$$

$$H_I = a\int d^3x h_{ij}\partial_i\Psi\partial_j\Psi^*$$

- ✓ Sub-horizon approximation k/a, p/a, m , $c_s\, k/a >> H$, and the particle is relativistic

$$\frac{dE}{dt} \simeq \frac{G_N\, p_{\rm in}^4}{a^4}\frac{4(1-c_T)^2}{3(1+c_T)^2}$$

Gravitational Cherenkov Radiation

✓ Graviton emission rate (using sub-horizon approximation)

$$\frac{dE}{dt} \simeq \frac{G_N \, p_{\rm in}^4}{a^4} \frac{4(1-c_T)^2}{3(1+c_T)^2}$$

✓ A particle with momentum p cannot possibly have been traveling for longer than

$$t \sim \frac{a^4}{G_N} \frac{(1+c_T)^2}{4(1-c_T)^2} \frac{1}{p_{\rm in}^3}$$

✓ The highest energy cosmic ray puts the constraint

$$1 - c_T \lesssim 2 \times 10^{-17} \left(\frac{10^{11}\mathrm{GeV}}{p} \right)^{3/2} \left(\frac{1\mathrm{Mpc}}{ct} \right)^{1/2}$$

✓ Observations tell us $E \sim p \sim 3 \times 10^{11}\mathrm{GeV}, \quad c\,t \sim 10\,\mathrm{kpc}$

$$1 - c_T \lesssim 2 \times 10^{-15}$$

Toy Model 2

$K = X$

$G_3 = 0$

$G_4 = M_{\rm Pl}^2/2$

$G_5 = -\lambda\phi/M_{\rm Pl}^2$

✓ Gubitosi and Linder model (Gubitosi and Linder 2011)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + X + \frac{\lambda}{M_{\rm Pl}^2}G^{\mu\nu}\nabla_\mu\phi\nabla_\mu\phi + \mathcal{L}_m[g_{\mu\nu},\psi]\right]$$

✓ Friedmann equation

$$1 = \Omega_m + \Omega_\phi, \qquad \text{where} \quad \Omega_\phi = \frac{X}{3M_{\rm Pl}^2H^2}(1+18C(z)) \quad \text{and} \quad C(z) \equiv \frac{\lambda H^2}{M_{\rm Pl}^2}$$

✓ Condition for existence of self-accelerating solution and avoiding the ghost-instability

$$-\frac{1}{18} < C(z=0) < -\frac{1}{30} \quad \longrightarrow \quad \lambda \text{ is always negative}$$

✓ Sound speed of graviton

$$c_T^2 = \frac{M_{\rm Pl}^2 + 2\lambda X/M_{\rm Pl}^2}{M_{\rm Pl}^2 - 2\lambda X/M_{\rm Pl}^2} < 1$$

Inconsistent with the constraint from the gravitational Cherenkov radiation...

Toy Model 3

✓ Extended galileon model (De Felice and Tsujikawa 2011)

$$\begin{aligned} K &= -c_2 M_2^{4(1-p_2)} X^{p_2}, \\ G_3 &= c_3 M_3^{1-4p_3} X^{p_3}, \\ G_4 &= \frac{1}{2} M_{\rm pl}^2 - c_4 M_4^{2-4p_4} X^{p_4}, \\ G_5 &= 3c_5 M_5^{-(1+4p_5)} X^{p_5}, \end{aligned}$$

✓ Assumption

$$H\dot{\phi}^{2q} = \text{const}$$

$$\rho_\phi \propto \dot{\phi}^{2p}$$

$$\longrightarrow$$

$$\begin{aligned} p_2 &= p \\ p_3 &= p + (2q - 1/2) \\ p_4 &= p + 2q \\ p_5 &= p + (6q - 1)/2 \end{aligned}$$

4 model parameters p, q, α, β

✓ Friedmann equation in the case of the attractor solution

$$s = p/2q$$

$$\left(\frac{H}{H_0}\right)^2 = (1 - \Omega_{m0} - \Omega_{r0}) \left(\frac{H}{H_0}\right)^{-2s} + \Omega_{m0} a^{-3} + \Omega_{r0} a^{-4}$$

Observational constraints shows $s \ll 1$

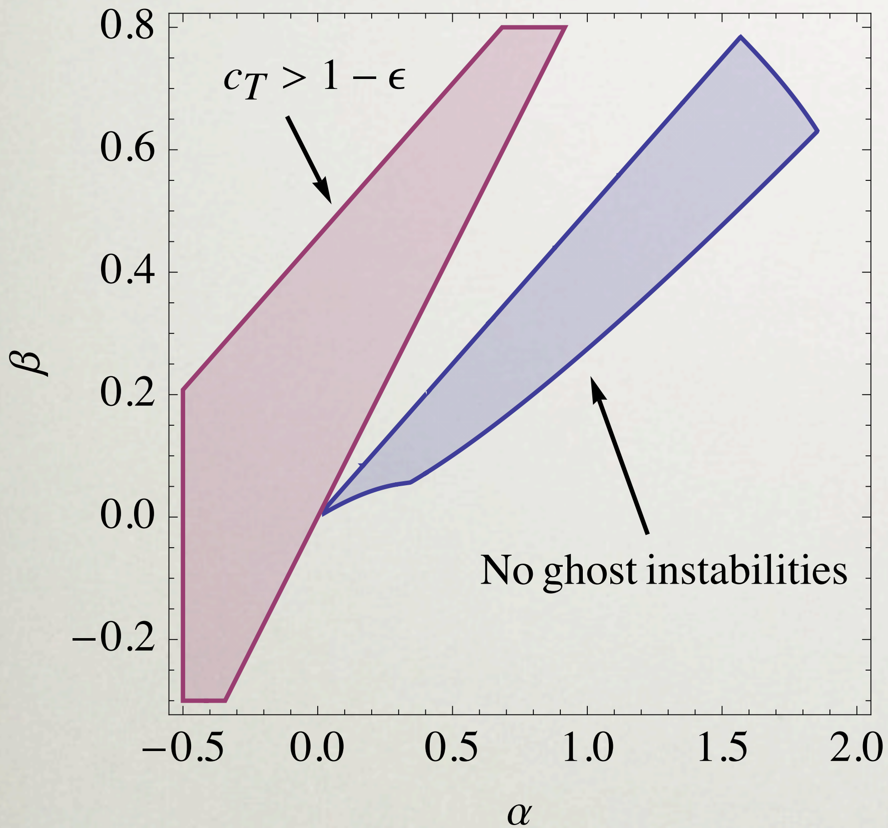
Dvali-Turner's Model
(Dvali, Turner 2003)

Constraints from GCR

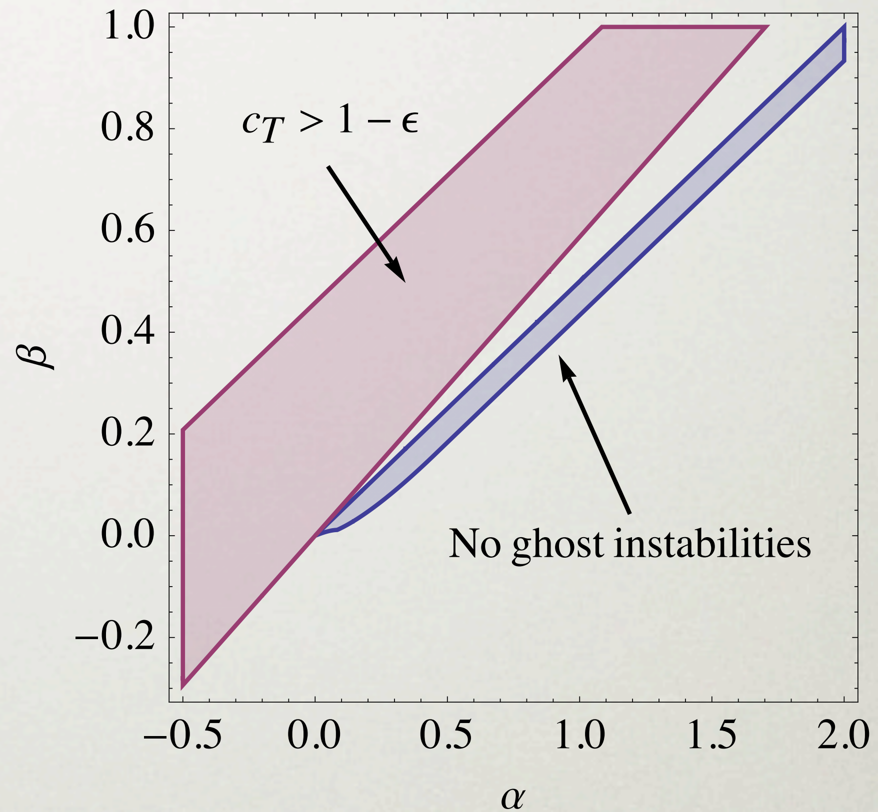
✓ Allowed parameter space

$c_T > 1 - \epsilon$

$\epsilon = 2 \times 10^{-15}$



p = 1 and q = 1/2



p = 1 and q = 3/2

Inconsistent with GCR for any model parameters α and β

Summary

- Constraints from the standard ruler is not a powerful tool to constrain some modified gravity model, because the background evolution is almost same as the ΛCDM model.

- Constraints from LSS-ISW cross correlation could be a powerful probe for the model, whose effective gravitational coupling G_{eff} is strongly enhanced.

- For modified gravity model, whose sound speed of graviton is smaller than the speed of light, the constraints from gravitational Cherenkov radiation would be a powerful probe.