# Gravitational wave signal from Massive gravity

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with
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# Gravitational wave signal from Massive gravity

- We assume EoM of GW is modified by time-dependent graviton mass:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left( \frac{k^2}{a(t)^2} + M_{GW}^2(t) \right) \gamma_k = 0$$

→ Argue how to detect $M_{GW}(t)$ from observational signals

# Contents

1. Motivation, model description
2. Evolution of gravitational wave
3. Observed spectrum
4. Summary

# Gravitational wave signal from Massive gravity

(Initial) motivation:

- Cosmological solutions & perturbations in the Nonlinear Massive Gravity model
- In this model, the tensor mode EoM is modified by a time-dependent mass term.

# Massive Gravity

- Motivation: IR modification of gravity
- Pauli-Fierz massive gravity (1939)

$$S = \frac{M_{Pl}^2}{2} \int d^4x \left[ R - \frac{1}{4} m^2 \left( h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

Suffers from

- Ghost at non-linear level
- vDVZ discontinuity

# Non-linear Massive Gravity

- Non-linear extension of FP massive gravity

(de Rham, Gabadadze & Tolley 2011)

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} + m_g^2 \left( \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 \right) \right]$$

$$\mathcal{L}_2 = \frac{1}{2} \left( [\mathcal{K}]^2 - [\mathcal{K}^2] \right) , \quad \mathcal{L}_3 = \frac{1}{6} \left( [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \right) ,$$

$$\mathcal{L}_4 = \frac{1}{24} \left( [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4] \right) ,$$

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - \left( \sqrt{g^{-1}\hat{f}} \right)^\mu{}_\nu , \quad [\mathcal{K}] = \mathrm{tr}\mathcal{K} , \quad \hat{f}_{\mu\nu} = f_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

# Non-linear Massive Gravity

- Non-linear extension of FP massive gravity

(de Rham, Gabadadze & Tolley 2011)

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} + m_g^2 \left( \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 \right) \right]$$

  - No BD ghost even at non-linear level (Hassan & Rosen 2011)
  - Cosmological solutions, exact solutions
  - Cosmological perturbations

# Non-linear Massive Gravity

- Cosmological perturbations

(Gümrükçüoğlu, Lin & Mukohyama 2011)

  - Background solution: FRW with $\Lambda$

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho, \quad -\frac{2\dot{H}}{N} + \frac{2K}{a^2} = \frac{1}{M_{Pl}^2}(\rho + P)$$

$$\Lambda_\pm = -m_g^2(1 - X_\pm)[3 - X_\pm + \alpha_3(1 - X_\pm)]$$

$$X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

$$\left( S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right] \right)$$

# Non-linear Massive Gravity

- Cosmological perturbations

(Gümrükçüoğlu, Lin & Mukohyama 2011)

  - Background solution: FRW with $\Lambda$
  - 2 scalar + 2 vector + 2 tensor modes
    - Scalar & Vector:

      Vanishing kinetic terms + Finite mass terms
    - Tensor:

      GR + Mass term

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

# Non-linear Massive Gravity

- Tensor: GR + Mass term

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

$$\propto \left(1 + \frac{H(t)}{H_f(t)}\right)$$

- Modification only for tensor modes by $M_{GW}(t)$

  → Probe by **gravitational wave observations**

  - Direct observations of gravitational wave
  - CMB polarizations

| | | |
|---|---|---|
| 14:00–15:00 | Kimura(Hiroshima) | Observational constraints on galileon gravity |
| **1 August (K202)** | **seminar/discussion** | |
| 10:30–11:30 | Tanaka(Yukawa) | TBA |
| 14:00–15:00 | Tanahashi(UC Davis) | Gravitational wave signal from massive gravity |
| **2 August (K202)** | **seminar/discussion** | |
| 10:30–11:30 | de Rham/Tolley(CWR) | TBA |
| 14:00–15:00 | Gumrukcuoglu(KIPMU) | Fate of homogeneous and isotropic solutions in massive gravity |
| 15:30–16:00 | Lin(KIPMU) | Anisotropic Friedmann–Robertson–Walker universe from nonlinear massive gravity |
| 19:00– | *dinner* | |
| **3 August (K202)** | **seminar/discussion** | |
| 10:30–11:30 | Heisenberg(Geneve) | TBA |
| 14:00–14:30 | Yamaguchi(Titech) | New cosmological solutions in massive gravity |
| 14:30–15:00 | Zhang(Yukawa) | Tunneling fields in massive gravity |
| **6–8 August** | **discussion/collaboration** | |

# FRW solution is unstable!!

| | | |
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# Non-linear Massive Gravity

(Initial) motivation:

- Cosmology in the Non-linear Massive Gravity model

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- Assume a general quadratic action with modifications (only) in tensor sector:

$$I = \frac{M_{Pl}^2}{8} \int dt dx^3 N a^3 \sqrt{\Omega} \left[ \frac{1}{N^2} \dot{\gamma}^{ij} \dot{\gamma}_{ij} + \gamma^{ij} \left( \sum_{n=0}^{\infty} c_n(t) \frac{\triangle^n}{a^{2n}} \right) \gamma_{ij} \right]$$

# Gravitational wave signal from Massive gravity

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$$\left( ds^2 = -N(t)^2 dt^2 + a(t)^2 \left[ \Omega_{ij}(x^k) + \gamma_{ij} \right] dx^i dx^j \right)$$

# Gravitational wave signal from Massive gravity

- Assume a general quadratic action
  with modifications (only) in tensor sector:

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$$\left[ \frac{1}{N^2} \dot{\gamma}^{ij} \dot{\gamma}_{ij} + \frac{c_g^2(t)}{a^2} \gamma^{ij} \left( \triangle - 2K \right) \gamma_{ij} - M_{GW}^2(t) \gamma^{ij} \gamma_{ij} \right]$$

# Gravitational wave signal from Massive gravity

- Assume a general quadratic action
  with modifications (only) in tensor sector:

$$I = \frac{M_{Pl}^2}{8}\int dt d^3x\, N a^3 \sqrt{\Omega}\left[\frac{1}{N^2}\dot{\gamma}^{ij}\dot{\gamma}_{ij} + \frac{c_g^2(t)}{a^2}\gamma^{ij}\left(\triangle - 2K\right)\gamma_{ij} - M_{GW}^2(t)\gamma^{ij}\gamma_{ij}\right]$$

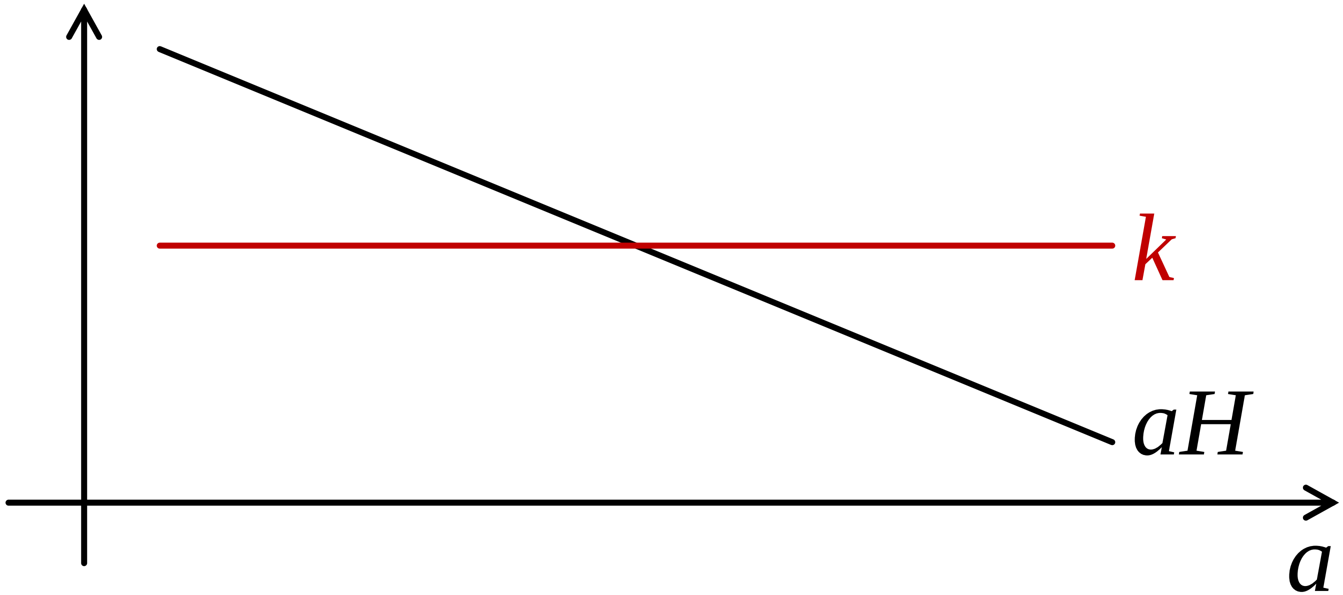
$$\Rightarrow \quad \bar{\gamma}_k'' + \left(c_g^2(t)\left(k^2 + 2K\right) - \frac{a''}{a} + a^2 M_{GW}^2(t)\right)\bar{\gamma}_k = 0$$

$$\left(\bar{\gamma}_k \equiv a\gamma_k\right)$$

# Evolution of gravitational wave

- Pure GR:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left( \frac{k^2}{a(t)^2} + M_{GW}^2(t) \right) \gamma_k = 0$$



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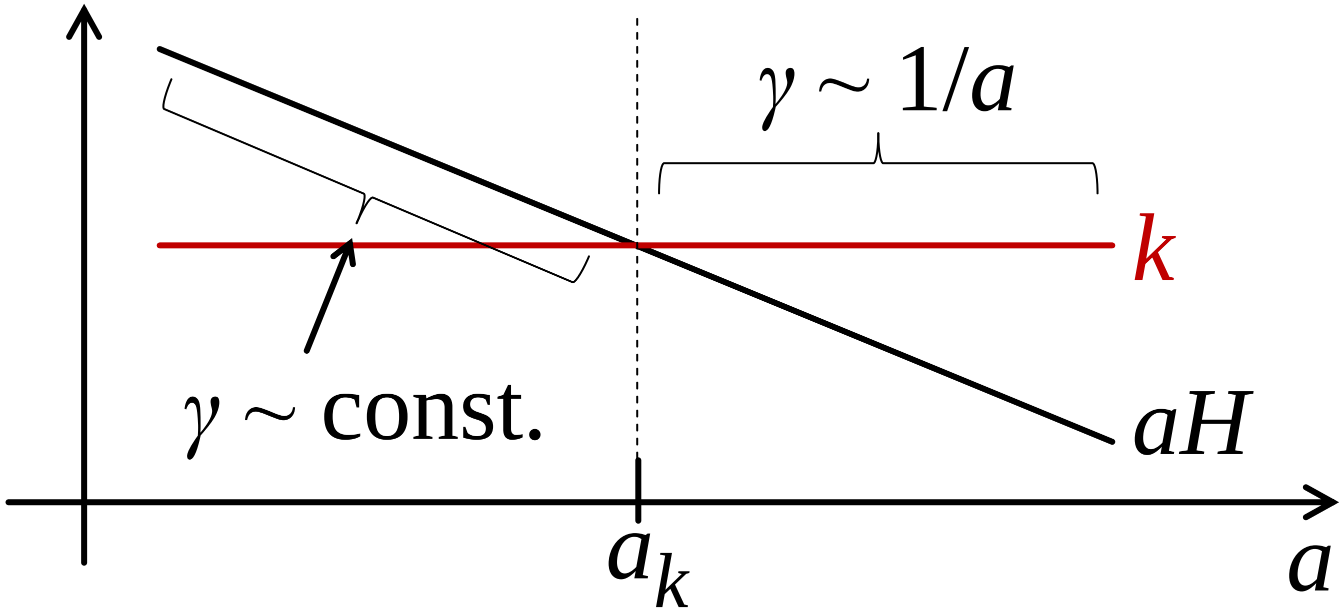


# Evolution of gravitational wave

- Pure GR:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

  - WKB solution with thin-horizon approximation

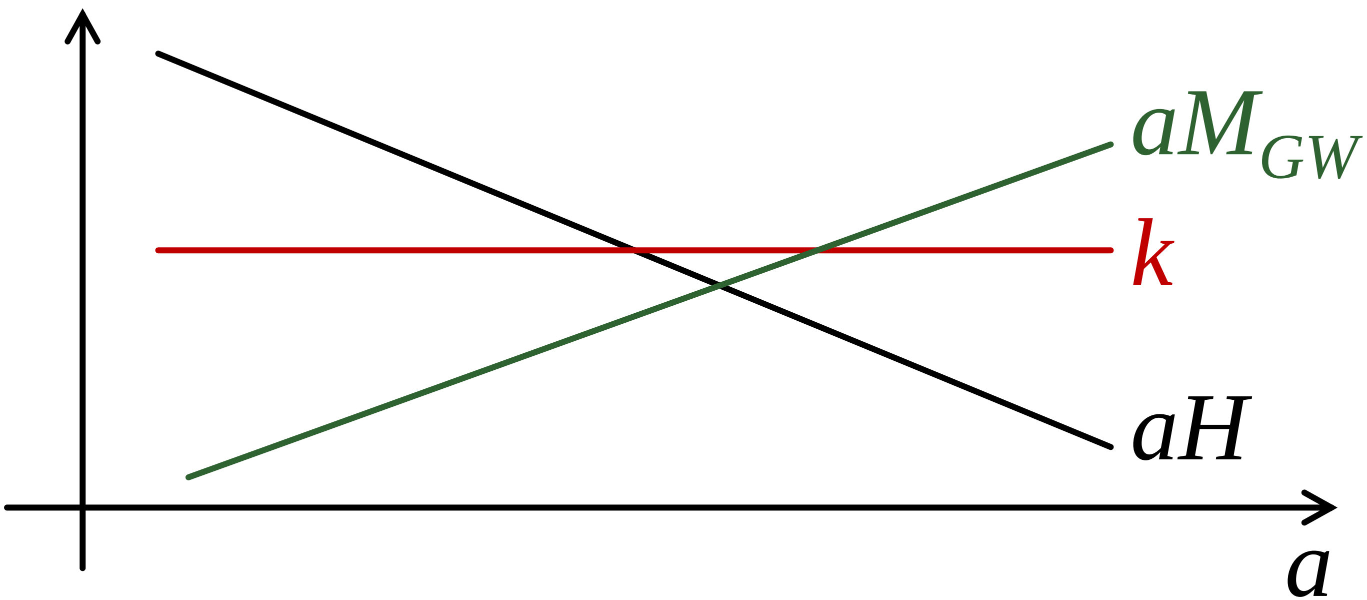
$$\gamma_k = A(k)\frac{a_k}{a(t)}\exp\left(i\int\frac{k}{a}dt\right)$$

$$\left[A(k) \equiv \frac{H_*}{M_{Pl}k^{3/2}} \ : \ \text{Primordial amplitude}\right]$$

# Evolution of gravitational wave

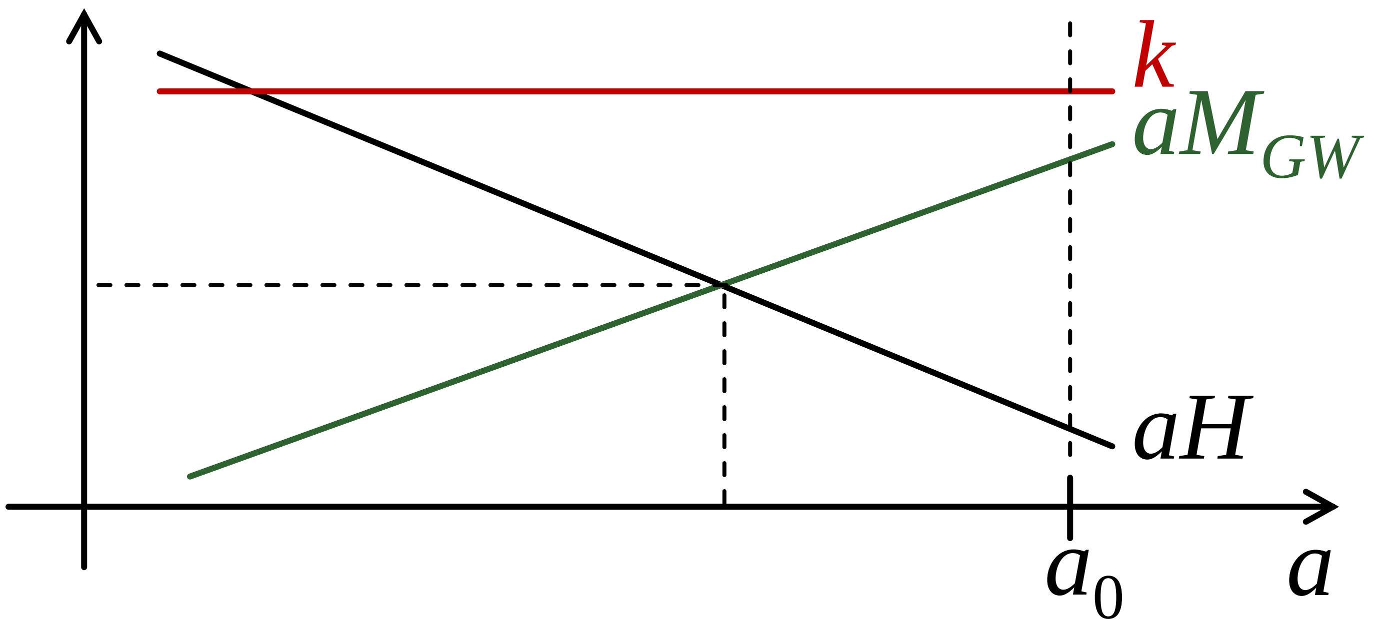
- Pure GR + Mass term:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$



# Evolution of gravitational wave

- Pure GR + Mass term:
  - Large $k$ : Same as pure GR
  - Medium $k$ : Suppression of $\gamma$ near today
  - Small $k$ : Dominated by $M_{GW}(t)$



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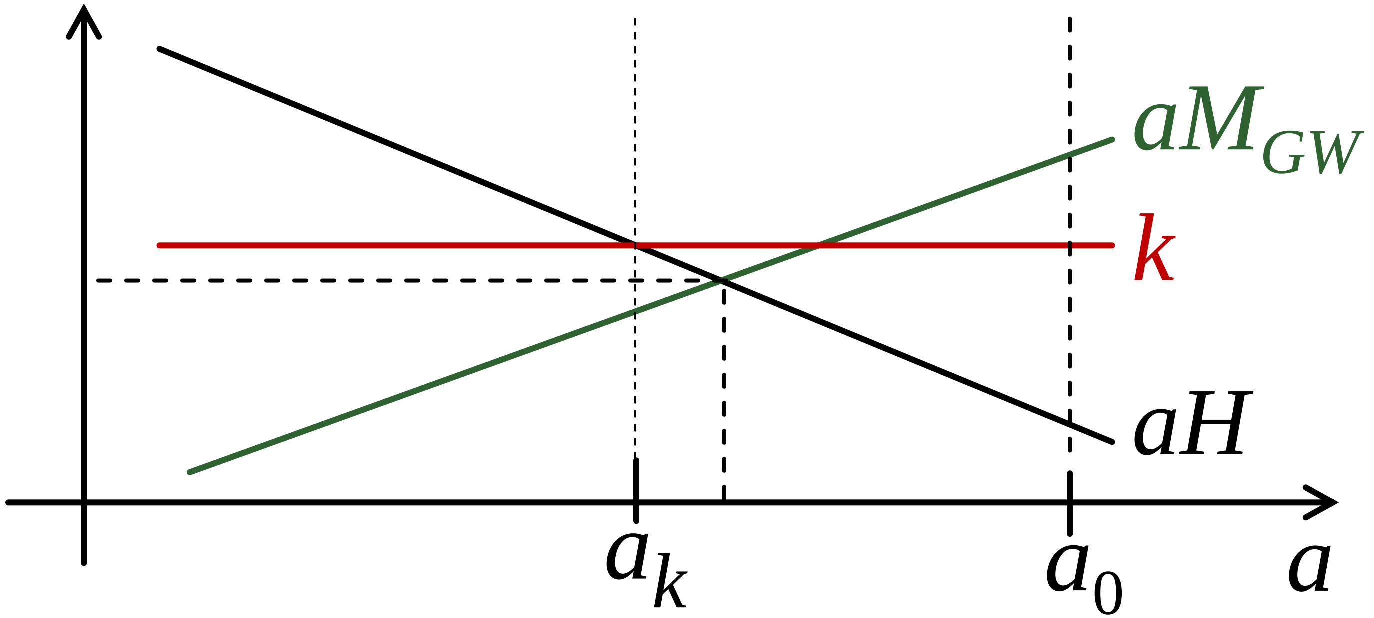

# Evolution of gravitational wave

- Pure GR + Mass term:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \boxed{\left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)}\gamma_k = 0$$

$$\equiv \omega^2(t)$$

  - WKB solution with thin-horizon approximation

$$\gamma_k = A(k)\sqrt{\frac{a_k^3\ \omega_k}{a(t)^3\,\omega(t)}}\exp\left(i\int \omega(t)dt\right)$$

$$\left[\text{GR:}\quad \gamma_k = A(k)\frac{a_k}{a(t)}\exp\left(i\int \frac{k}{a}dt\right)\right]$$

# Evolution of gravitational wave

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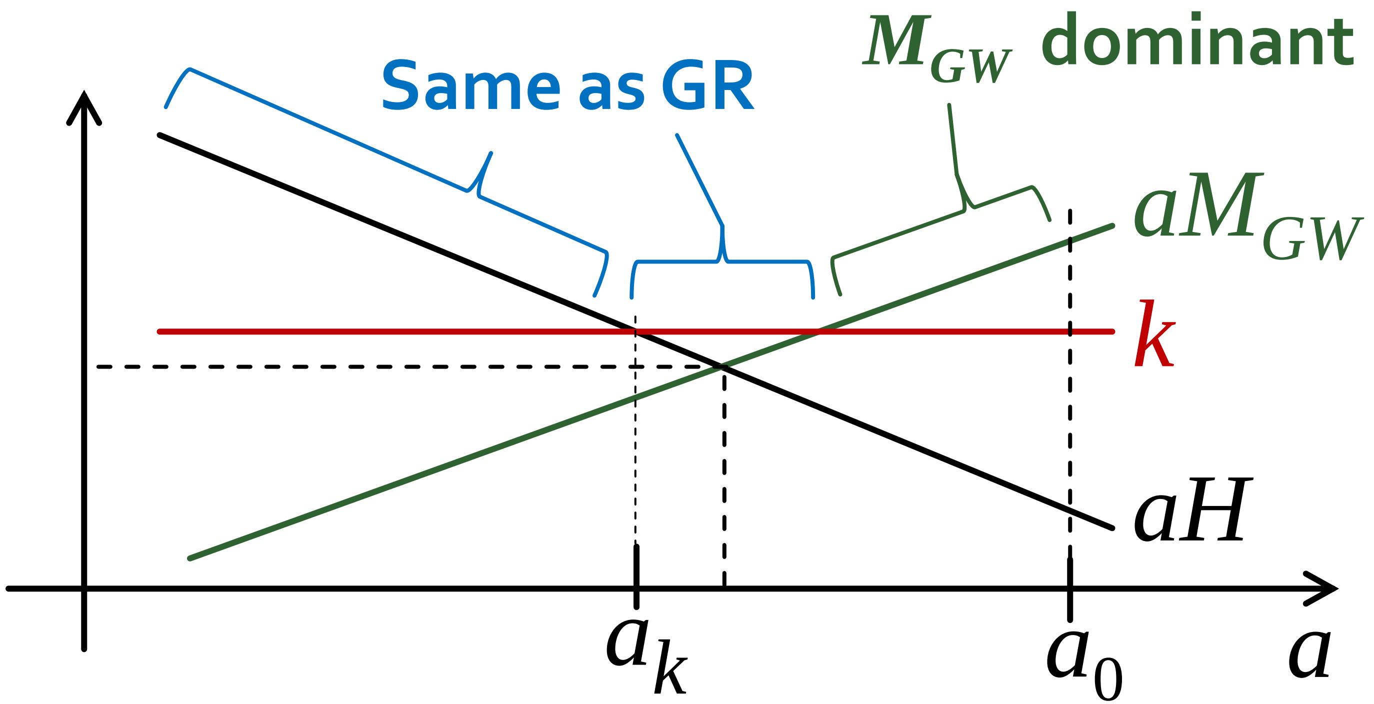

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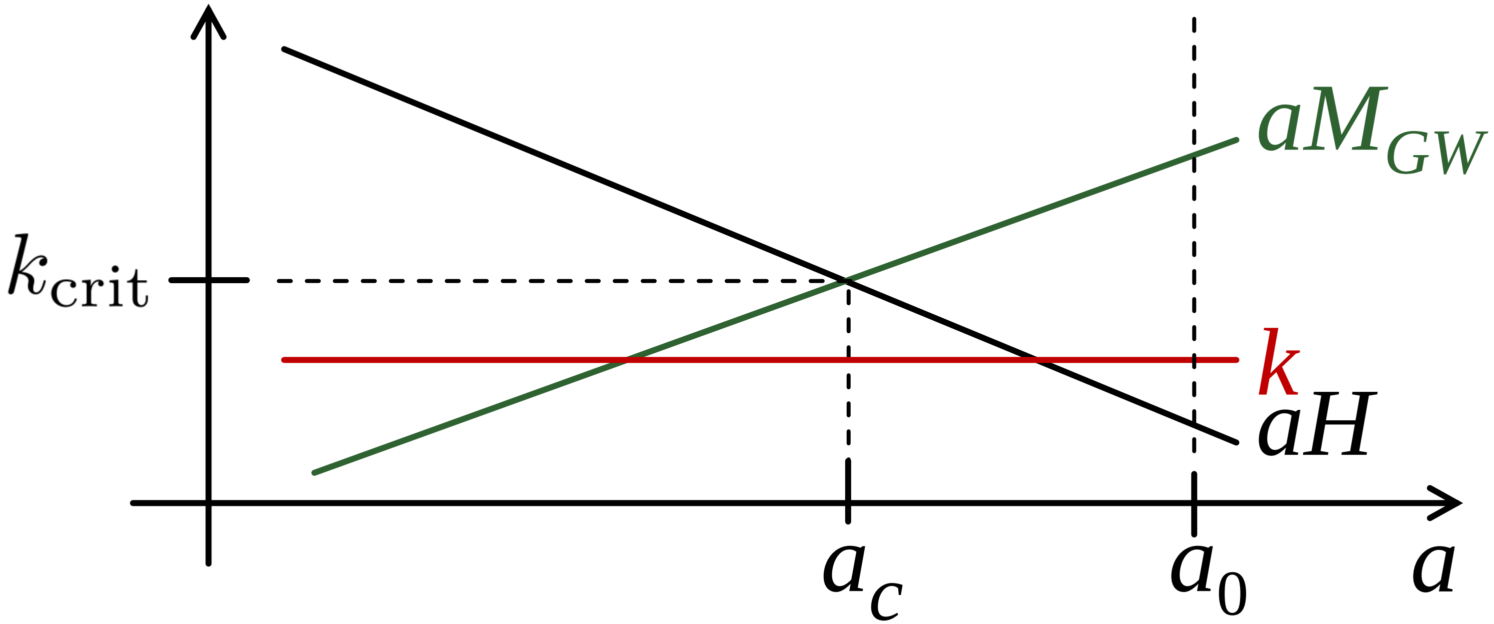


# Evolution of gravitational wave

- Pure GR + Mass term:

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right)\gamma_k = 0$$

$$\left(\frac{k^2}{a(t)^2} + M_{GW}^2(t)\right) \equiv \omega^2(t)$$

  - WKB solution with thin-horizon approximation

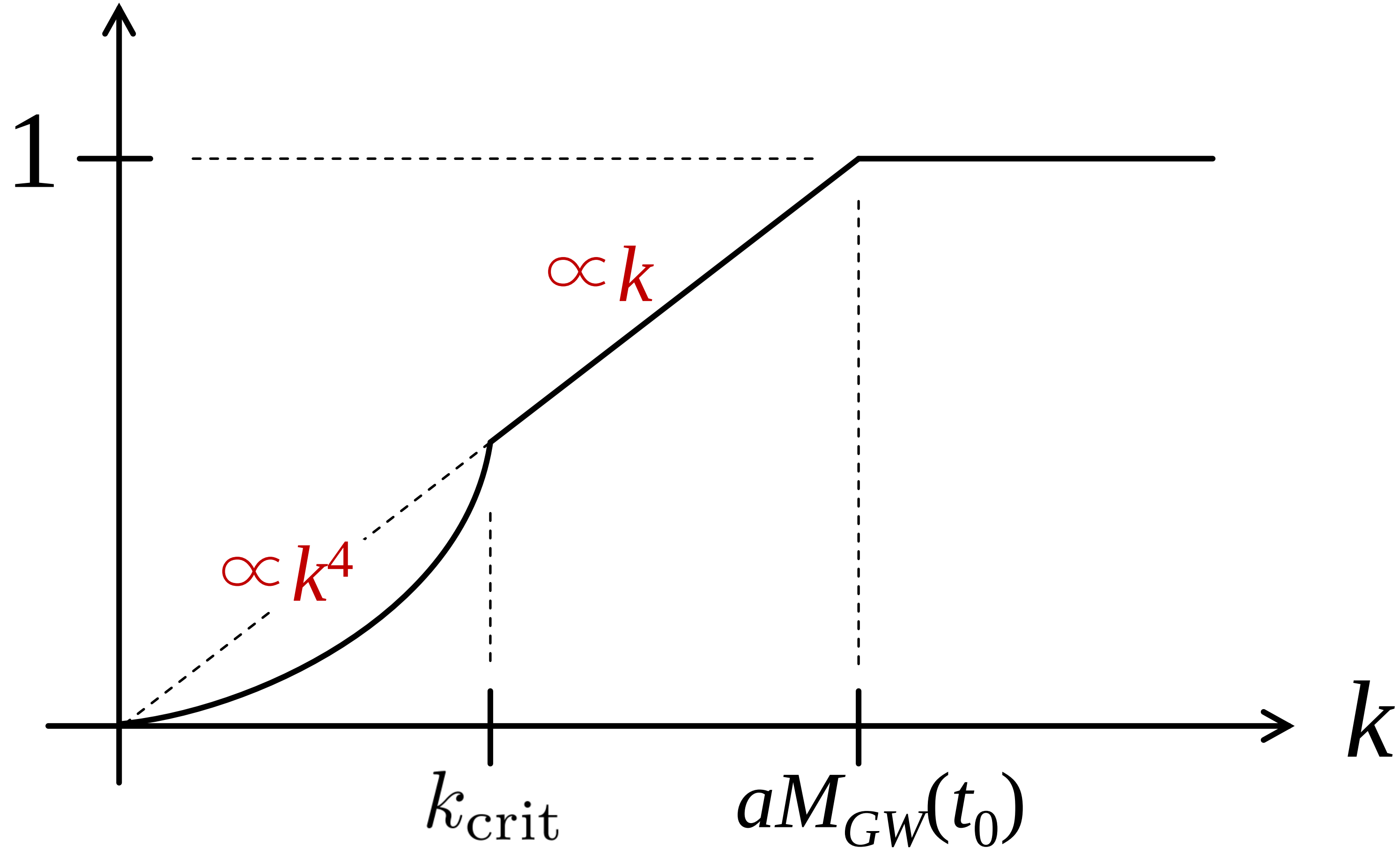
$$\gamma_k = A(k)\sqrt{\frac{a_k^3\ \omega_k}{a(t)^3\,\omega(t)}}\exp\left(i\int\omega(t)dt\right)$$

$$\Rightarrow \quad |\gamma_k(t_0)| = A(k)\sqrt{\frac{a_c^3 M_{GW}(t_c)}{a_0^3 M_{GW}(t_0)}}$$

$$\left(\text{GR:} \quad \gamma_k = A(k)\frac{a_k}{a(t)}\exp\left(i\int\frac{k}{a}dt\right)\right)$$

# Evolution of gravitational wave

- ( $k^3|\gamma_k|^2$ in MG) / ( $k^3|\gamma_k|^2$ in GR) for the same $\boldsymbol{k}$



# Observed spectrum

- We've discussed ***power spectrum w.r.t.*** $k$:

$$\mathcal{P}(k) \equiv \frac{d}{d\ln k}\langle\gamma_{ij}\gamma^{ij}\rangle\Big|_{t=t_0} = \frac{2k^3}{\pi^2}\left|\gamma_k(t_0)\right|^2$$

- What we really observe is ***power spectrum w.r.t.*** $\omega$:

$$\mathcal{P}(\omega_0) \equiv \frac{d}{d\ln\omega_0}\langle\gamma_{ij}\gamma^{ij}\rangle\Big|_{t=t_0} = \frac{d\ln k}{d\ln\omega_0}\mathcal{P}(k(\omega))$$

$$\left(\omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0)\right)$$

# Observed spectrum

- We've discussed ***power spectrum* w.r.t.** $\boldsymbol{k}$:

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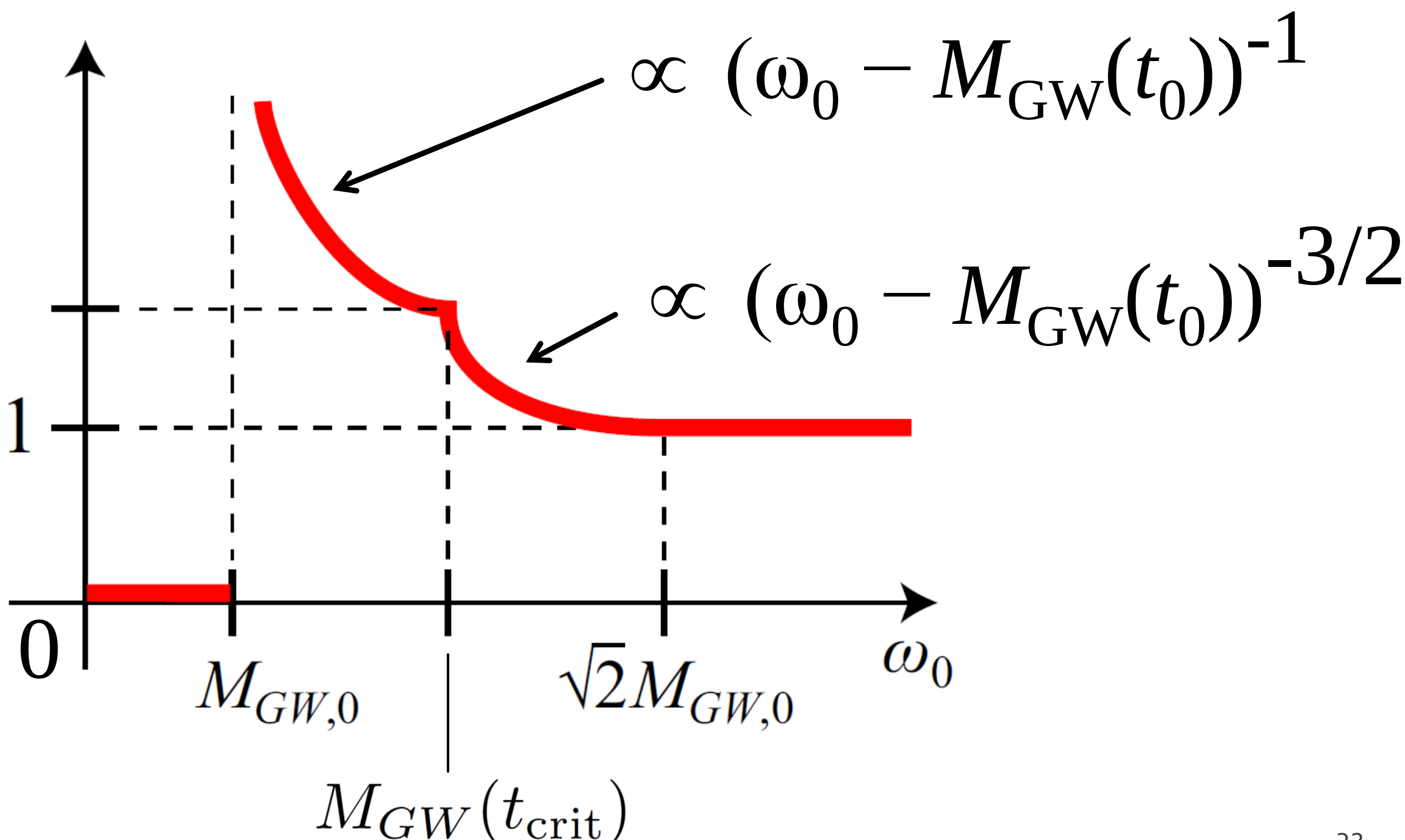
$$\left(\omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0)\right)$$

$$\frac{\omega_0^2}{\omega_0^2 - M_{GW}^2(t_0)}$$

# Observed spectrum

- ( $\mathcal{P}(\omega)$ in MG) / ( $\mathcal{P}(\omega)$ in GR) for the same $\boldsymbol{\omega}$



# Observed spectrum

- Divergence of $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\mathrm{prim}}(k)$:

$$\mathcal{P}(\omega_0) = \frac{d\ln k}{d\ln \omega_0}\mathcal{P}\left(k(\omega_0)\right) \propto k^{-2}\mathcal{P}_{\mathrm{prim}}(k)\Big|_{k=k(\omega_0)}$$

$$\left(\omega_0^2 = \frac{k^2}{a_0^2} + M_{GW}^2(t_0) \quad \Leftrightarrow \quad k(\omega_0) = a_0\sqrt{\omega_0^2 - M_{GW}^2(t_0)}\right)$$

- If $\mathcal{P}_{\mathrm{prim}}(k)$ has IR cutoff,

$$\lim_{k\to+0} k^{-2}\mathcal{P}_{\mathrm{prim}}(k) < +\infty$$

$$\therefore \text{ Peak height } \propto \lim_{k\to+0} k^{-2}\mathcal{P}_{\mathrm{prim}}(k)$$

# Observed spectrum

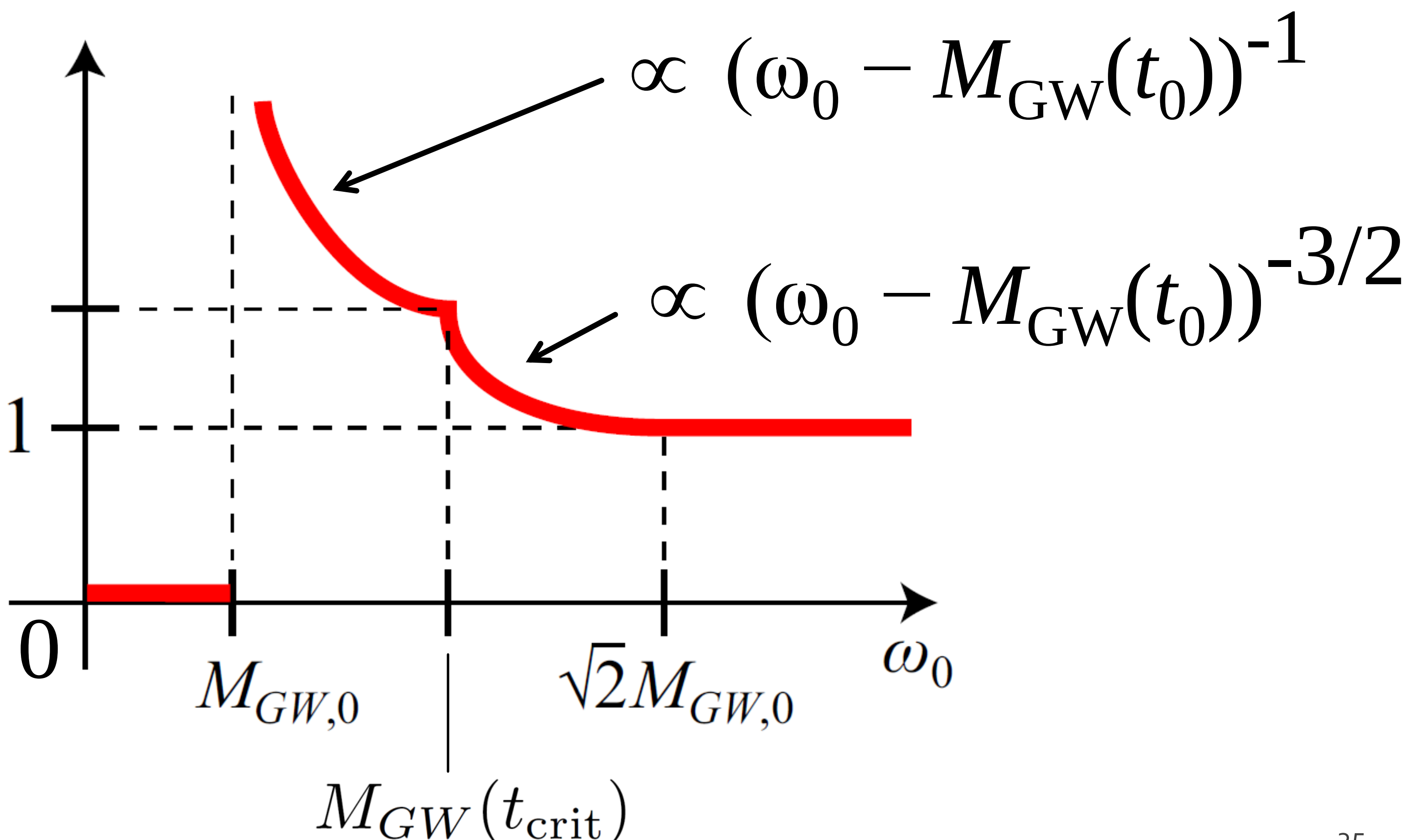
- Divergence of $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\rm prim}(k)$
- Frequency resolution ~ $1/T_{\rm obs}$
  → possible suppression for $|\omega_0 - M_{\rm GW}(t_0)| < 1/T_{\rm obs}$

$$\rightarrow \quad \frac{\mathcal{P}^{\rm MG}(\omega_0)}{\mathcal{P}^{\rm GR}(\omega_0)} \sim \frac{a_c^2 k_c}{a_{k_0}^{GR^2} k_0} \times \min\left(\left(\frac{\omega_{\rm cutoff}^2}{M_{GW,0}^2} - 1\right)^{-1},\ M_{GW,0} T_{obs}\right)$$

$$\left(\omega_{\rm cutoff} = \omega(k_{\rm cutoff}),\ \ k_{\rm cutoff} < H_0\right)$$

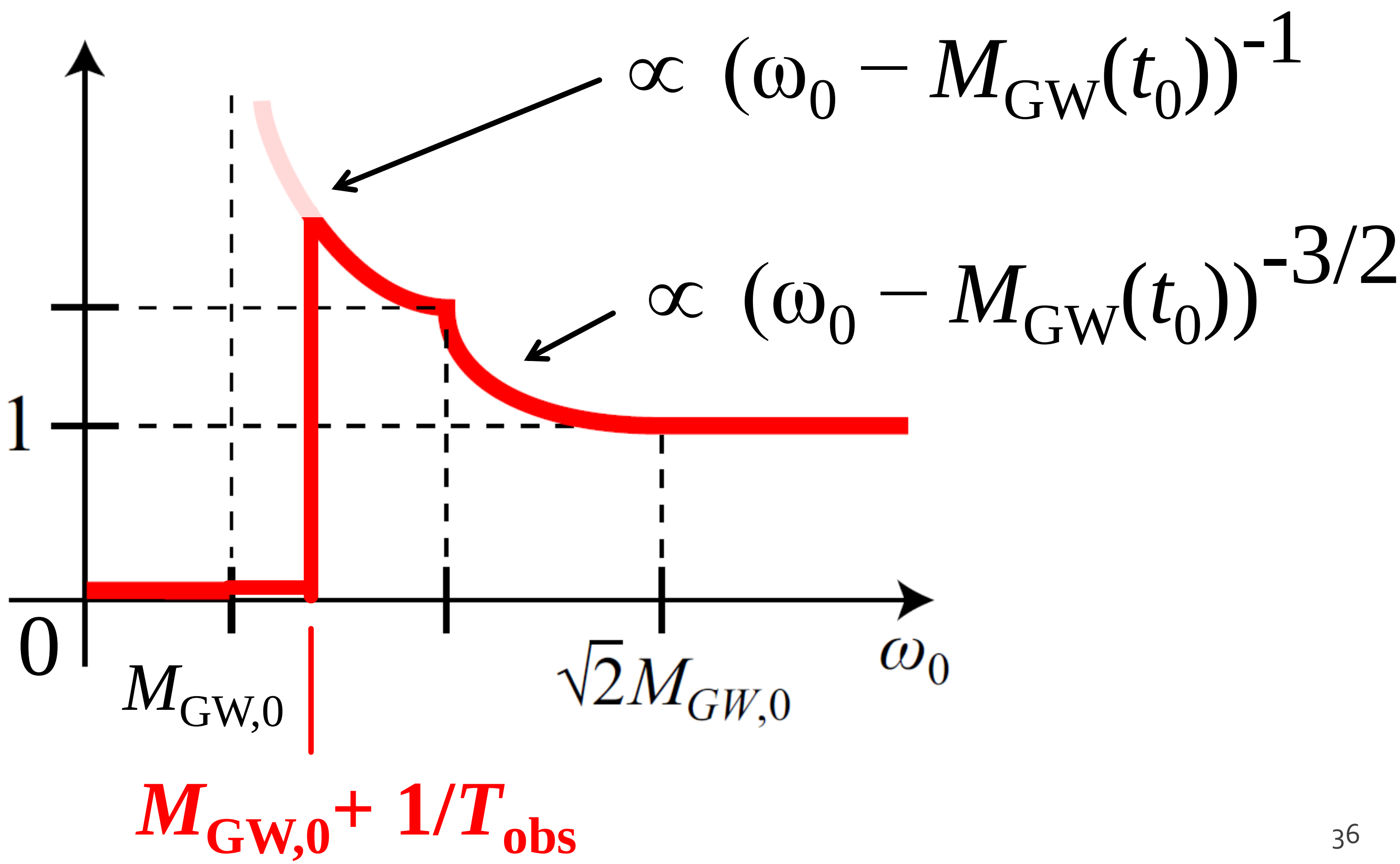
# Observed spectrum

- ( $\mathcal{P}(\omega)$ in MG) / ( $\mathcal{P}(\omega)$ in GR) for the same $\boldsymbol{\omega}$



# Observed spectrum

- ( $\mathcal{P}(\omega)$ in MG) / ( $\mathcal{P}(\omega)$ in GR) for the same $\boldsymbol{\omega}$



# Observed spectrum

- Divergence of $\mathcal{P}(\omega)$ is sensitive to $\mathcal{P}_{\rm prim}(k)$
- Frequency resolution ~ $1/T_{\rm obs}$
  → possible suppression for $|\omega_0 - M_{\rm GW}(t_0)| < 1/T_{\rm obs}$

$\therefore$

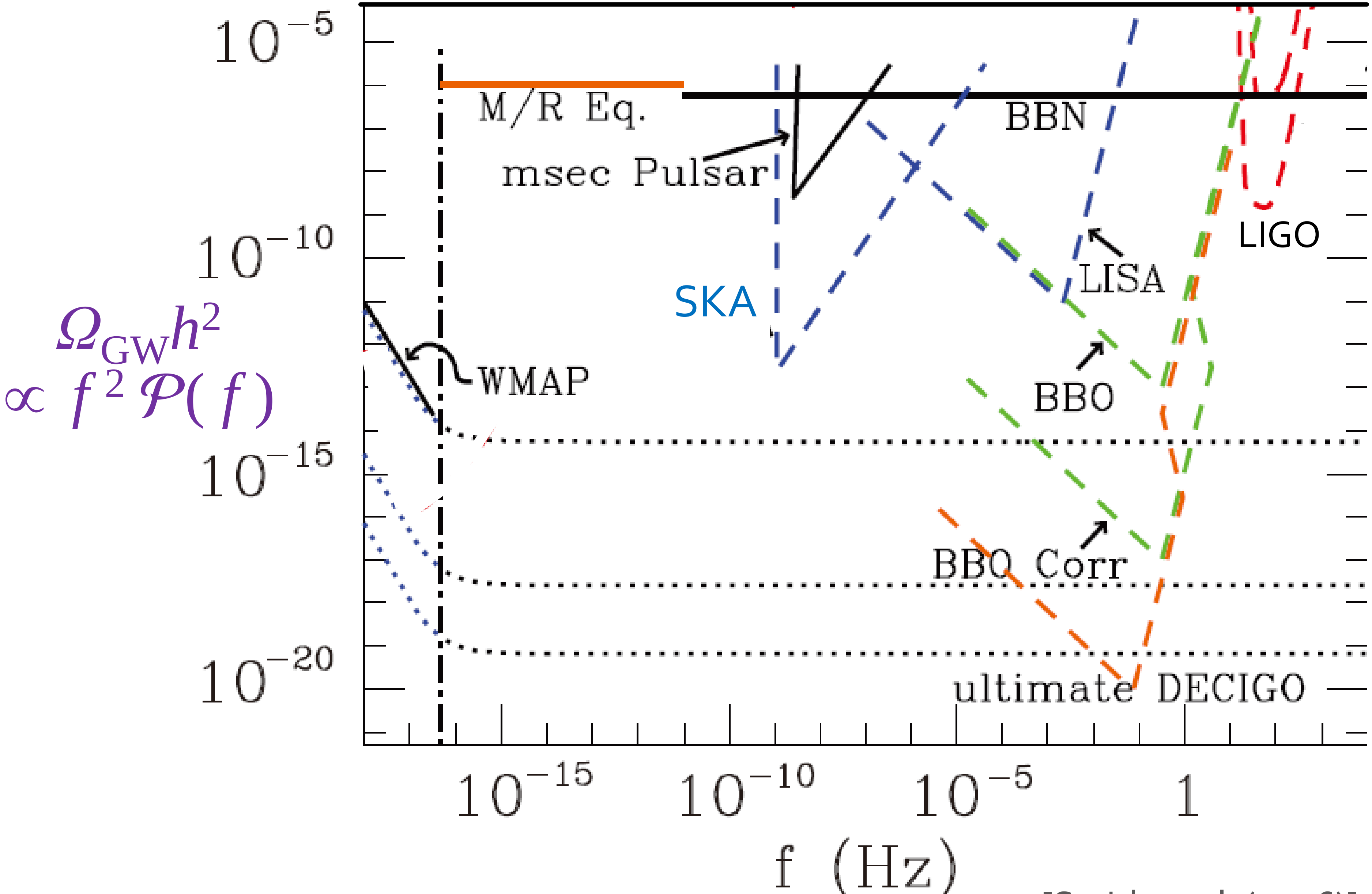
- Peak height → $\lim_{k \to +0} k^{-2} \mathcal{P}_{\rm prim}(k)$
  → IR cutoff of $\mathcal{P}_{\rm prim}(k)$?
- Peak location → $M_{GW}(t_0)$
- Peak shape → $M_{GW}(t_{\rm crit})$

# Observed spectrum

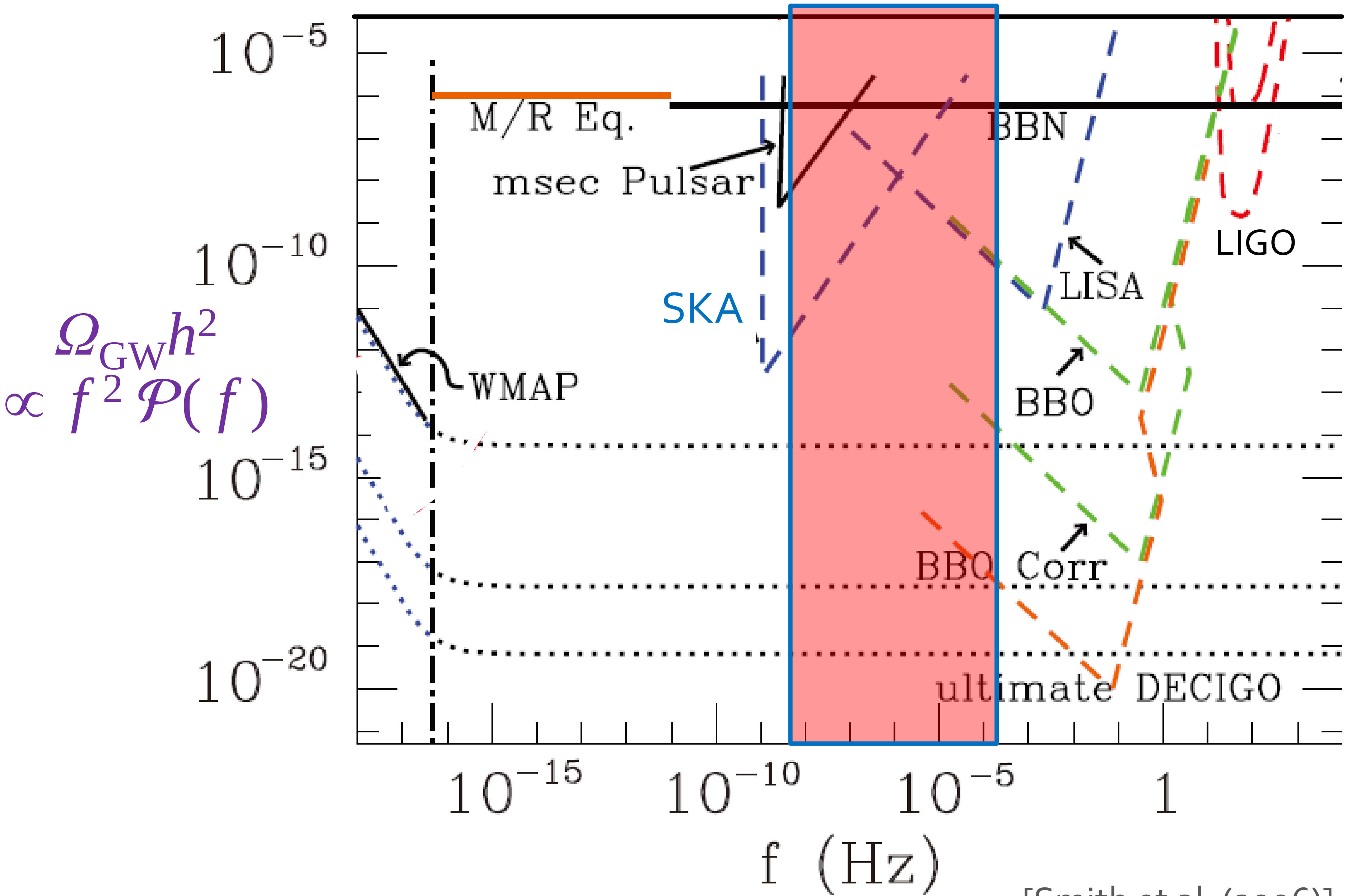
- Sensitivity range:
  - LISA: $10^{-4}\sim 1$ Hz
  - DECIGO: $10^{-1}\sim 1$ Hz
  - SKA, PPTA: $\sim 10^{-8}$ Hz
- Current bound:
  - $M_{\mathrm{GW}}(t_0) < 10^{-4}$ Hz from binary pulsar timing

[Finn & Sutton 2002]

$$\rightarrow \quad 10^{-8}\ \mathrm{Hz} < M_{\mathrm{GW}}(t_0) < 10^{-4}\ \mathrm{Hz}$$




[Smith et al. (2006)]



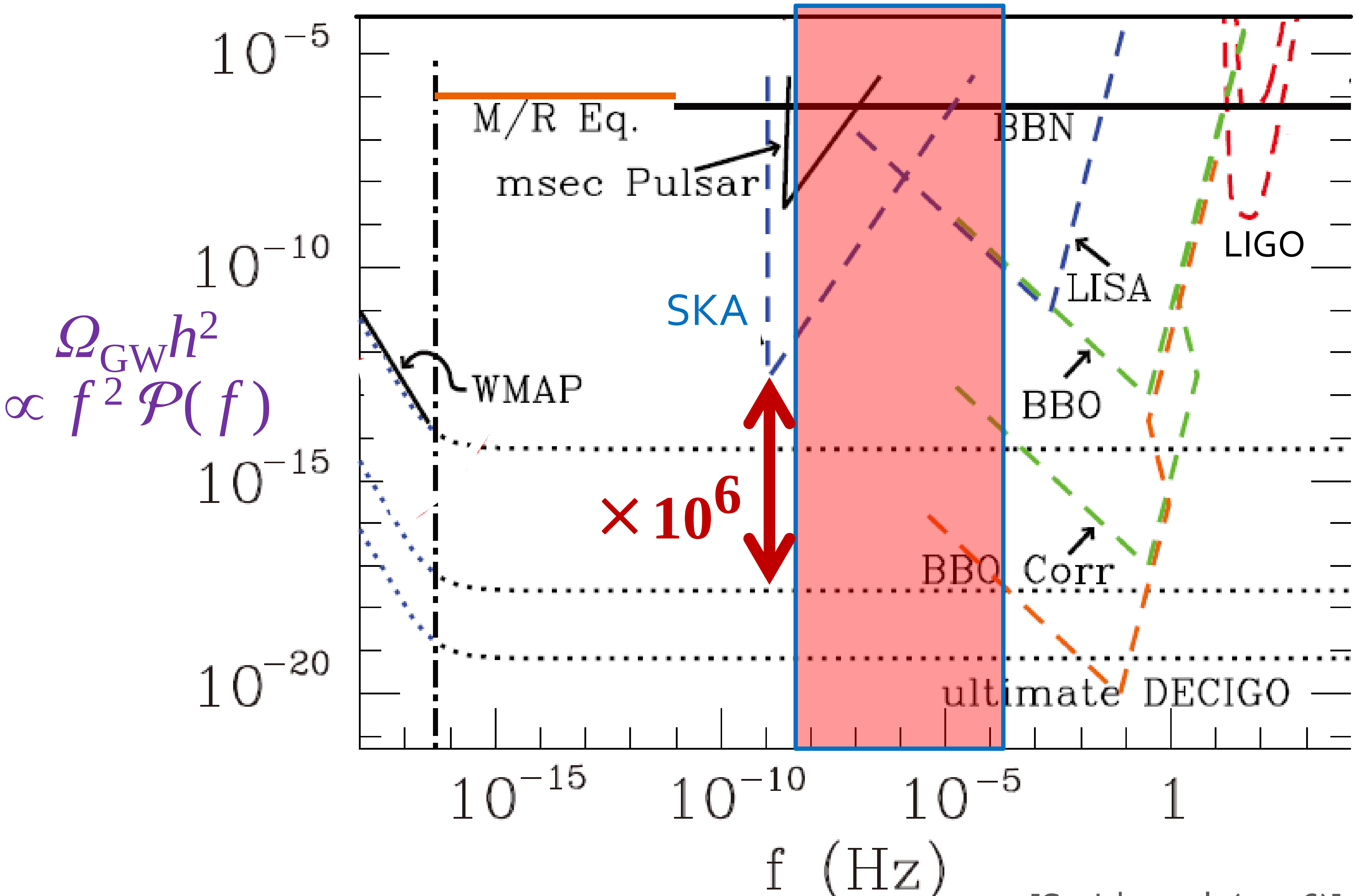

[Smith et al. (2006)]

# Observed spectrum

$$\frac{\mathcal{P}^{\rm MG}(\omega_0)}{\mathcal{P}^{\rm GR}(\omega_0)} \sim \frac{a_c^2 k_c}{a_{k_0}^{GR\,2} k_0} \times \min\left( \left( \frac{\omega_{\rm cutoff}^2}{M_{GW,0}^2} - 1 \right)^{-1} ,\; M_{GW,0} T_{obs} \right)$$

ex.)

| | | |
|---|---|---|
| • $M_{\rm GW}(t_0) =$ **$10^{-8}$ Hz** $\approx 10^9\ {\rm H}_0$<br>• $k_{\rm cutoff} = 1\ {\rm H}_0$<br>• $T_{\rm obs} = 5$ years | $\sim 10^{23}$ | $\sim 10^5$ |
| • $M_{\rm GW}(t_0) =$ **$10^{-4}$ Hz** $\approx 10^{13}\ {\rm H}_0$<br>• $k_{\rm cutoff} = 1\ {\rm H}_0$<br>• $T_{\rm obs} = 5$ years | $\sim 10^{35}$ | $\sim 10^{13}$ |




[Smith et al. (2006)]

# Grav. Wave → CMB Polarizations

- GW observations → $M_{GW}(t)$ at $t = t_0$ & $t_{\rm crit}$
  - $M_{GW}(t)$ at any other time?

- GW → CMB polarizations [Dubovsky et al. 2009]
  - Sensitive to $M_{GW}(t)$ at recombination
  - Suppression at lower multipoles:

$$\ell < 10^{-3} \times M_{GW}(t_{\rm rec})/H_0$$

→ $M_{GW}(t)$ at recombination

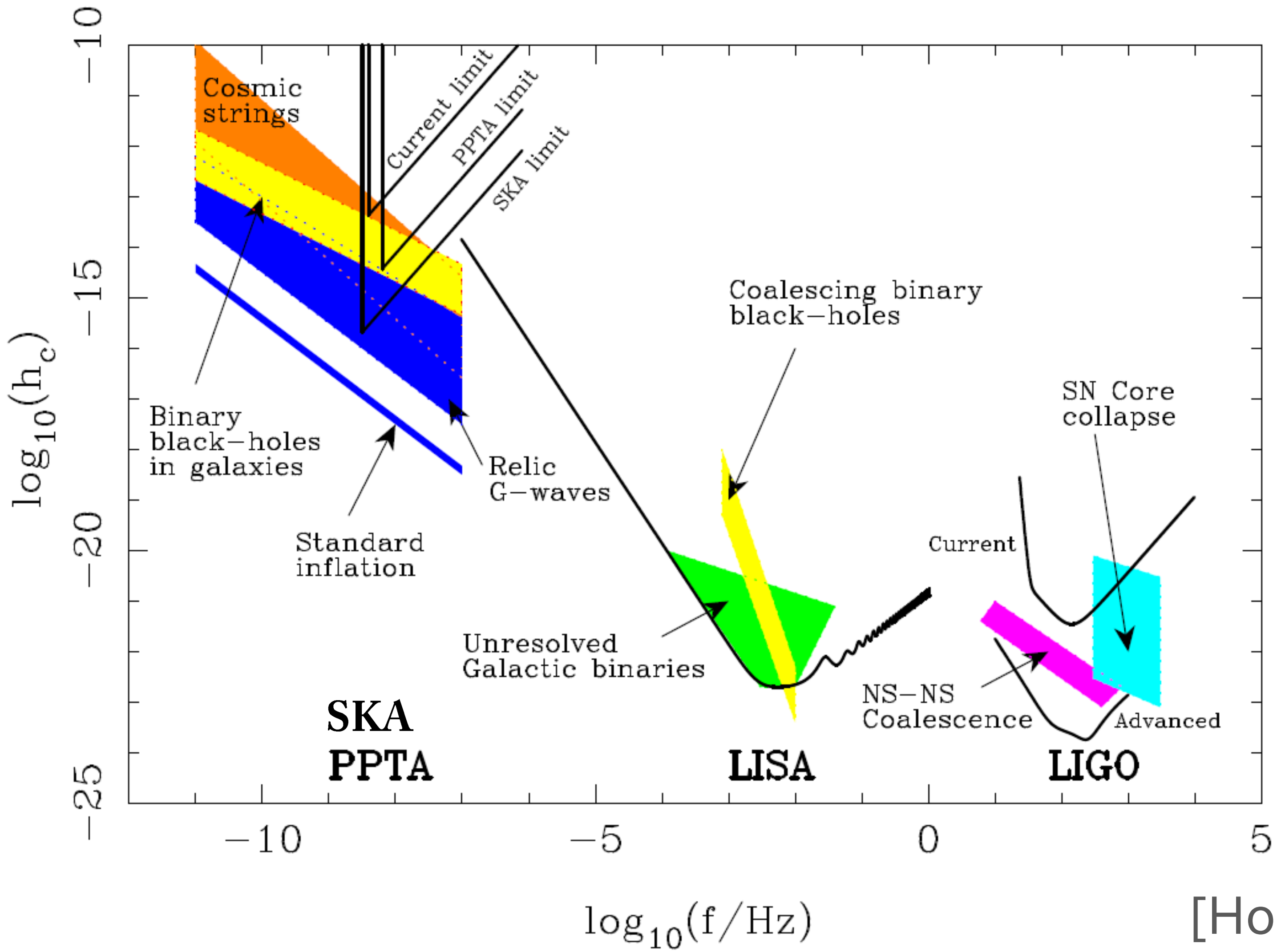
# Summary

- Probe time-dependent mass of general massive gravity theories by GW observations

- GW direct observations:
  - Sharp peak in $\mathcal{P}(\omega)$
  - $M_{GW}(t)$ at $t = t_0$ & $t_{\rm crit}$

- CMB polarizations
  - $M_{GW}(t)$ at recombination time

- Other probes for $M_{GW}(t)$ ?
  - $\Omega_{\rm GW}h^2 \propto \omega^2\,\mathcal{P}(\omega) \sim M_{\rm GW}(t)^2\,\mathcal{P}(\omega)$ → BBN constraint?



Cosmic strings
Current limit
PPTA limit
SKA limit
Coalescing binary black−holes
SN Core collapse
Binary black−holes in galaxies
Relic G−waves
Standard inflation
Current
Unresolved Galactic binaries
NS−NS Coalescence
Advanced
SKA PPTA
LISA
LIGO
log10(hc)
−10
−15
−20
−25
−10
−5
0
5
log10(f/Hz)


[Hobbs (2008)]