New Cosmological Solutions in Massive Gravity

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$c = \hbar = 1$

Contents

Introduction

Basics of massive gravity (Notation)

de Rham & Gabadadze 2010
de Rham, Gabadadze, Tolley 2011

$$S = \frac{M_{\rm pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left(R + m^2\mathcal{U}\right) + S_{\rm m}.$$

Potential for the graviton: $\mathcal{U} := \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4.$

$$\begin{cases} \mathcal{U}_2 := [\mathcal{K}]^2 - [\mathcal{K}^2], \\ \mathcal{U}_3 := [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \\ \mathcal{U}_4 := [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \end{cases}$$

$[\mathcal{K}] = {\rm Tr}\mathcal{K}\ldots$

$$\begin{cases} \mathcal{K}_\mu^{\ \nu} := \delta_\mu^{\ \nu} - (\sqrt{g^{-1}\Sigma})_\mu^{\ \nu}, \\ \Sigma_{\mu\nu} := \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}. \end{cases}$$

$$\mathcal{K}_\nu^\mu = \partial^\mu\partial_\nu\pi$$

⇧

$$\phi^a = \delta_\mu^a x^\mu - \eta^{a\mu}\partial_\mu\pi.$$

ϕ^a : Stuckelberg fields

Unitary gauge: $\phi^a = \delta_\mu^a x^\mu$

Absence of Flat Friedmann Universe

D'Amico et al. 2011

Homogeneous & isotropic solution:

- **real metric:** $ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2$
- **imposes the same symmetry on Stuckelberg fields**

$$\phi^0 = f(t), \quad \phi^i = x^i.$$

$$\Rightarrow \quad \Sigma_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} = \mathrm{diag}\left(-\dot{f}^2, 1, 1, 1\right)$$

$$\Rightarrow \quad \left(g^{-1}\Sigma\right)^\mu_\nu = \mathrm{diag}\left(\dot{f}^2, a^{-2}, a^{-2}, a^{-2}\right)$$

$$\Rightarrow \quad \mathcal{K}^\nu_\mu = \delta^\nu_\mu - (\sqrt{g^{-1}\Sigma})^\nu_\mu = \mathrm{diag}\left(1 - |\dot{f}|, 1 - a^{-1}, 1 - a^{-1}, 1 - a^{-1}\right)$$

Absence of Flat Friedmann Universe II

$$\begin{cases} \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] = -6(1-a^{-1})\left(a^{-1} - 2 + |\dot{f}|\right), \\ \mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] = -6(1-a^{-1})^2\left(a^{-1} - 4 + 3|\dot{f}|\right), \\ \mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \\ \quad = -24(1-a^{-1})^3\left(-1 + |\dot{f}|\right). \end{cases}$$

⇨ **dot{f} appears only linearly !!**

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} a^3 \left[6(\dot{H} + 2H^2) + m^2\mathcal{U}\right] = \frac{M_{\rm pl}^2}{2} a^3 \left[-6H^2 + m^2\mathcal{U}\right].$$

Integration by part

⇨ $$\frac{\delta\mathcal{L}}{\delta f} = 3M_{\rm pl}^2 m^2 \frac{\mathrm{d}}{\mathrm{d}t}\left[a^3 - a^2 + 3\alpha_3 a(a-1)^2 + 4\alpha_4(a-1)^3\right] = 0.$$

⇨ $\dot{a} = 0$ **The scale factor cannot evolve !!**

⇨ **There is no homogeneous and isotropic flat Universe.**

What can we do ?

- Consider open Friedmann Universe instead of flat one.

  → Gumrukcuoglu, Lin and Mukohyama 2011.

- Abandon imposing the same symmetry on the Stuckelberg field.

  → Our work by use of Painleve-Gullstrand meric

Note that the same idea was done in D'Amico et al. 2011 & Gratia, Hu, and Wyman 2012 as well.

Painleve-Gullstrand meric

Spherically symmetric vacuum solution in (massless) GR

Schwarzschild metric:

$$ds^2 = -f(r)dt_s^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2.$$

$$f(r) = 1 - 2M/r.$$

This metric has a coordinate singularity at the horizon r = 2M.

In GR, this is not a real singularity and can be removed by coordinate transformation.

Danger of coordinate singularity in Massive Gravity

Gruzinov & Mirbabayi 2011
Berezhiani et al. 2012

New invariant in Massive Gravity:

$$I^{ab} = g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b.$$

This quantity is invariant under coordinate transformation, namely, a scalar quantity and should have the same position as $R, R_{\mu\nu}R^{\mu\nu}, \cdots.$

In the unitary gauge: $\phi^a = \delta^a_\mu x^\mu$

$$\Rightarrow \quad I^{ab} = g^{\mu\nu}\delta^a_\mu\delta^b_\nu.$$

Any inverse metric with divergence leads to singularity in this invariant.

(Though such a singularity does not affect the geodesic motion, it would cause a problem for perturbations around classical solutions because inverse metric could change its sign across the singularity.)

Danger of coordinate singularity in Massive Gravity II

Gruzinov & Mirbabayi 2011
Berezhiani et al. 2012

The Schwarzschild like metric in Massive Gravity
(Schwarzschild, Schwarzschild-De Sitter, Reissner-Nordstrom…)
can be dangerous.

→ **Needs the metric without coordinate singularity**

Painleve-Gullstrand meric !!

- **BH solutions in PG metric:**

  Berezhiani, Chkareuli, de Rham, Gabadadze, Tolley 2011

- **Cosmological solutions in PG metric:**

  our work

Painleve-Gullstrand metric

Painleve 1922, Gullstrand 1922,
Kanai, Siino, Hosoya 2011

Merit of PG metric:

- includes an off-diagonal and spatially flat elements, which leads to no coordinate singularity (except real singularity at the origin)
- can cover both inside and outside the horizon by a single coordinate patch.
- Time coordinate as measured by an observer who is at rest at infinity and freely falls into the BH.
- The space described by the PG metric can be regarded as a river whose speed of current is the Newtonian escape velocity at each point.
- Generalized PG metric can also describe the FRLW universe.

Derivation of Painleve-Gullstrand metric

Painleve 1922, Gullstrand 1922,
Kanai, Siino, Hosoya 2011

Schwarzschild metric: $ds^2 = -f(r)dt_s^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2.$

$$f(r) = 1 - 2M/r.$$

Four velocity of an observer : $u_s^\mu = dx_s^\mu / d\tau = \dot{x}_s^\mu.$

normalization condition : $-1 = g_{\mu\nu} u_s^\mu u_s^\nu = -f\dot{t}_s^2 + f^{-1}\dot{r}^2.$

(conserved) energy per rest mass : $\epsilon = -g_{\mu\nu}\xi^\mu u_s^\nu = f\dot{t}_s.$

$\xi^\mu = (\partial/\partial t_s)^\mu$: timelike Killing vector

$$\Rightarrow \begin{cases} u_s^\mu = \left(\dot{t}_s, \dot{r}, \dot{\theta}, \dot{\phi}\right) = \left(\frac{\epsilon}{f}, -\sqrt{\epsilon^2 - f}, 0, 0\right), \\ u_{s\,\mu} = g_{\mu\nu} u_s^\nu = \left(-\epsilon, -\frac{\sqrt{\epsilon^2 - f}}{f}, 0, 0\right). \end{cases}$$

Derivation of Painleve-Gullstrand metric II

t_P : the proper time of the free-falling observer

⟺ **The geodesic is orthogonal to the surface t_P = const.**

⟺ **The geodesic tangent vector $u_{s\mu}$ is equal to the gradient of t_P.**

$$u_{s\mu} = -\frac{\partial}{\partial x_s^\mu} t_P(x_s). \qquad u_{s\mu} = \left(-\epsilon, -\sqrt{\epsilon^2 - f}/f, 0, 0\right).$$

⟺ $$dt_p = \epsilon dt_s + \frac{\sqrt{\epsilon^2 - f}}{f} dr.$$

$$\begin{aligned} ds^2 &= -f(r)dt_s^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \\ &= -dt_p^2 + \frac{1}{\epsilon^2}(dr + v(r)dt_p)^2 + r^2 d\Omega^2. \end{aligned}$$

$v(r) = \sqrt{\epsilon^2 - f(r)}$: radially free-falling velocity

At the horizon f(r)=0 ⬄ r =2M, the metric is non-singular.

Derivation of Painleve-Gullstrand metric III

$$ds^2 = -dt_p^2 + \frac{1}{\epsilon^2}(dr + v(r)dt_p)^2 + r^2 d\Omega^2.$$

$$\Rightarrow \begin{cases} u_p^\mu = \left(\dot{t}_p, \dot{r}, \dot{\theta}, \dot{\phi}\right) = (1, -v, 0, 0), \\ u_{p\,\mu} = g_{\mu\nu} u_p^\nu = (-1, 0, 0, 0). \end{cases}$$

$$\frac{dr}{dt_p} = \frac{\dot{r}}{\dot{t}_p} = -v(r) = -\sqrt{\epsilon^2 - f(r)}. \quad \Rightarrow \quad E = \frac{1}{2}\left(\frac{dr}{dt_p}\right)^2 + \Phi(r).$$

$$\begin{cases} E = (\epsilon^2 - 1)/2 & \text{: conserved energy} \\ \Phi(r) = -M/r & \text{: gravitational potential} \end{cases}$$

For a particle freely falling from infinity at rest (ε =1 ⇔ E=0),

$$ds^2 = -dt_p^2 + \left(dr + \sqrt{2M/r}\,dt_p\right)^2 + r^2 d\Omega^2.$$

Standard form given by PG.

This is a vacuum solution, so we want a solution including matter.

Kanai, Siino, Hosoya 2011

$$\Rightarrow \quad ds^2 = -dt_p^2 + \frac{1}{1 + 2E(t_p, r)}(dr + v(t_p, r)dt_p)^2 + r^2 d\Omega^2, \quad v(t_p, r) = \sqrt{2E(t_p, r) + 2m(t_p, r)/r}.$$

Spherical gravitational collapse – from infinity

Kanai, Siino, Hosoya 2011

Spherical gravitational collapse of matter with E = 0 :

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2m(r,t)}{r}}dt\right)^2 + r^2 d\Omega^2.$$

Einstein Eq.

$(8\pi T^\mu_\nu = R^\mu_\nu - \delta^\mu_\nu R/2)$

$$\begin{cases} 8\pi T^t_t = -\frac{2m'}{r^2}, \\ 8\pi T^r_t = \frac{2\dot{m}}{r^2}, \\ 8\pi T^r_r = -\frac{2m'}{r^2} + \frac{2\dot{m}}{r^2}\left(\frac{2m}{r}\right)^{-1/2}, \\ 8\pi T^\theta_\theta (T^\phi_\phi) = -\frac{m''}{r} + \left(\frac{\dot{m}}{2r^2} + \frac{\dot{m}'}{r}\right)\left(\frac{2m}{r}\right)^{-1/2} - \frac{\dot{m}m'}{r^2}\left(\frac{2m}{r}\right)^{-3/2}. \end{cases}$$

$\dot{} = \partial/\partial t, \ ' = \partial/\partial r.$

Only three are independent. ← $T^r_r = T^t_t + T^r_t (2m/r)^{-1/2}.$

Perfect fluid : $T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$

$u^\mu = (1, -v(t,r), 0, 0)$ ⇒ $T^t_t = -\rho, \ T^r_r = P, \ T^r_t = (\rho + P)v.$

$v(t,r) = \sqrt{\frac{2m(t,r)}{r}}$ **is equal to the escape velocity.**

Spherical gravitational collapse – from infinity II

Kanai, Siino, Hosoya 2011

- $8\pi T^t_t = -8\pi\rho = -2m'/r^2$:

$$\Rightarrow \quad m(t,r) = 4\pi\int_0^r \rho(t,r) r^2 dr \left(= \frac{2r^3}{9\gamma^2 t^2}\right)$$

- $8\pi T^r_t = 8\pi(\rho + P) = \frac{2\dot{m}}{r^2}, \quad P = (\gamma - 1)\rho, \quad \rho(t,r) = f(r)h(t)$:

$$\Rightarrow \quad \rho(t,r) = \frac{1}{6\pi\gamma^2 t^2}, \quad P(t,r) = \frac{\gamma - 1}{6\pi\gamma^2 t^2}.$$

$$\text{B.C.} \quad m|_{r=0} = 0, \quad \rho|_{t=0} = \infty.$$

(t : -∞ → 0)

Matter density (of the star) is uniform.

Though, in case of gravitational collapse, we need to match this inner solution with the outer solution given before, we are now interested in only the inner solution because…

Relation between this solution and Friedmann Universe

$$ds^2 = -dt^2 + (dr + v_-(t,r)dt)^2 + r^2 d\Omega^2.$$

$$v_-(t,r) = \sqrt{\frac{2m(t,r)}{r}} = \frac{2r}{3\gamma(-t)}.$$ **(t : -∞ ➔ 0)**

- $P = (\gamma - 1)\rho$ ➔ **FLRW** ➔ $H = \frac{2}{3\gamma t}$ ➔ $v_-(t,r) = -\frac{\dot{a}(t)}{a(t)} r.$

- **In fact,** $r = a(t)\tilde{r}$ ➔ $dr = \dot{a}(t)\frac{r}{a(t)}dt + a(t)d\tilde{r}$

➔
$$\begin{aligned} ds^2 &= -dt^2 + (dr + v_-(t,r)dt)^2 + r^2 d\Omega^2 \\ &= -dt^2 + a^2(t)\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right). \end{aligned}$$

The generalized Painleve-Gullstrand metric includes flat Friedmann Universe. (Expanding phase: v_- ➔ - v_-)

Spherical gravitational collapse – from a finite radius

Kanai, Siino, Hosoya 2011

$$ds^2 = -dt^2 + \frac{1}{1+2E(t,r)}(dr + v_-(t,r)dt)^2 + r^2 d\Omega^2, \ \ v_-(t,r) = \sqrt{2E(t,r) + 2m(t,r)/r}.$$

The boundary surface r=a(t) that freely falls from a radius a_0 at rest :

Solving Einstein Eq. inside the boundary $\left(M = m(t,r)|_{r=a(t)} = \frac{4\pi}{3}a^3(t)\rho(t)\right)$

$$\Rightarrow \quad E(t,r) = -\frac{M}{a_0}\left(\frac{r}{a(t)}\right)^2 < 0, \quad v_-(t,r) = \sqrt{\frac{2M}{a(t)} - \frac{2M}{a_0}\frac{r}{a(t)}}.$$

- **In this case also,** $r = a(t)\tilde{r}$ $\Rightarrow$ $v_-(t,r) = -\frac{\dot{a}(t)}{a(t)}r.$

$$\Rightarrow \quad \begin{aligned} ds^2 &= -dt^2 + \frac{1}{1 - \frac{2M}{r_0}\left(\frac{r}{a(t)}\right)^2}(dr + v_-(t,r)dt)^2 + r^2 d\Omega^2 \\ &= -dt^2 + a^2(t)\left(\frac{d\tilde{r}^2}{1 - \frac{2M}{a_0}\tilde{r}^2} + \tilde{r}^2 d\Omega^2\right). \end{aligned}$$

The generalized Painleve-Gullstrand metric includes closed (open ⇔ E>0) Friedmann Universe as well.

Cosmological solution

Generalized Painleve-Gullstrand metric as a cosmological solution

- Our strategy is **to find generalized PG metric in Massive Gravity** instead of the standard FLRW metric.

$$\mathrm{d}s^2 = -V^2(t,r)\mathrm{d}t^2 + U^2(t,r)\left(\mathrm{d}r + \epsilon\sqrt{f(t,r)}\mathrm{d}t\right)^2 + W^2(t,r)r^2\mathrm{d}\Omega^2.$$

$$(\epsilon = \pm 1)$$

- Stuckelberg fields in the unitary gauge:

$$\phi^0 = t, \qquad \phi^i = r\hat{n}^i. \qquad \hat{n} = (\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta)$$

- One parameter family:

$$\alpha_3 = \frac{1}{3}(\alpha - 1), \quad \alpha_4 = \frac{1}{12}\left(\alpha^2 - \alpha + 1\right).$$

$$W(t,r) = \tilde{\alpha} := \frac{\alpha}{\alpha+1}. \quad \Longrightarrow \quad X_{\mu\nu} = \frac{1}{\alpha}g_{\mu\nu}.$$

Effective C.C.

$$M_{\mathrm{pl}}^2\left(G_{\mu\nu} + m^2 X_{\mu\nu}\right) = T_{\mu\nu}$$

Any PG-type metric in GR (with a cosmological constant) is also a solution to Massive Gravity.

Friedmann Universe in Massive Garvity

- **The FLRW metric can be rewritten in a general PG form :**

$$\mathrm{d}s^2 = -\kappa^2\mathrm{d}t^2 + \frac{\tilde{\alpha}^2}{1 - K\tilde{\alpha}^2 r^2/a^2(t)}\left(\mathrm{d}r - \frac{\dot{a}}{a}r\mathrm{d}t\right)^2 + \tilde{\alpha}^2 r^2 \mathrm{d}\Omega^2.$$

$K = 0, \pm 1.$ ⟺ **All types of Friedmann Universe**

- **Perfect fluid :** $T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$

$$\rho = \rho(t), \ \ P = P(t). \qquad u_\mu = (-\kappa, 0, 0, 0)$$

EOM ➡

$$\begin{cases} \dfrac{3}{\kappa^2}\tilde{H}^2 = \dfrac{\rho}{M_{\mathrm{pl}}^2} + \dfrac{m^2}{\alpha} - \dfrac{3K}{a^2}, \\ -\dfrac{1}{\kappa^2}\left(3\tilde{H}^2 + 2\dot{\tilde{H}}\right) = \dfrac{p}{M_{\mathrm{pl}}^2} - \dfrac{m^2}{\alpha} + \dfrac{K}{a^2}. \end{cases} \qquad \left(\tilde{H}(t) := \mathrm{d}\ln a/\mathrm{d}t\right)$$

Rescaling time coordinate t ➔ $\tau = \kappa$ t with H:=d lna/d τ gives
the standard cosmological equation with effective C.C. $\Lambda_{\mathrm{eff}} = m^2/\alpha.$

Thus, our solution can accommodate spatially flat, open, and closed models.

More familiar form

$$ds^2 = -d\tau^2 + \frac{\tilde{\alpha}^2}{1 - K\tilde{\alpha}^2 r^2/a^2(\tau)}\left(dr - \frac{\dot{a}}{\kappa a} r d\tau\right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2.$$

Coordinate transformation: $r \to \tilde{r} = \dfrac{\tilde{\alpha} r}{a(\tau)}.$

$$\Longrightarrow \quad ds^2 = -d\tau^2 + a^2\left(\frac{d\tilde{r}^2}{1 - K\tilde{r}^2} + \tilde{r}^2 d\Omega^2\right).$$

with Stuckelberg fields, which do not respect the same symmetry,

$$\phi^0 = \frac{\tau}{\kappa}, \qquad \phi^i = \frac{a(\tau)\tilde{r}}{\tilde{\alpha}}\hat{n}^i.$$

$$\Longleftrightarrow \quad \Sigma_{\mu\nu} dx^\mu dx^\nu = -\left(\frac{1}{\kappa^2} - \frac{a^2 H^2 \tilde{r}^2}{\tilde{\alpha}^2}\right) d\tau^2 + 2\frac{a^2 H \tilde{r}}{\tilde{\alpha}^2} d\tau d\tilde{r} + \frac{a^2}{\tilde{\alpha}^2}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right).$$

Relation to the work of Gratia, Hu, and Wyman

arXiv:1205.4241

Try to find spatially isotropic solution:

$$\mathrm{d}s^2 = -b^2(t,r)\mathrm{d}t^2 + a^2(r,t)\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right).$$

with Stuckelberg fields, which respects the same symmetry,

$$\phi^0 = f(t,r), \qquad \phi^i = g(t,r)\frac{x^i}{r}.$$

(N.B. b(t,r) = 1 & a(t,r) = a(t) ➔ Flat FRW metric)

➡ **Potential:** $-\mathcal{U} = P_0\left(\frac{g}{ar}\right) + \sqrt{X}P_1\left(\frac{g}{ar}\right) + WP_2\left(\frac{g}{ar}\right).$

$$\begin{cases} P_0(x) = -12 - 2x(x-6) - 12(x-1)(x-2)\alpha_3 - 24(x-1)^2\alpha_4, \\ P_1(x) = 2(3-2x) + 6(x-1)(x-3)\alpha_3 + 24(x-1)^2\alpha_4, \\ P_2(x) = -2 + 12(x-1)\alpha_3 - 24(x-1)^2\alpha_4. \end{cases}$$

$$X = \left(\frac{\dot{f}}{b} + \mu\frac{g'}{a}\right)^2 - \left(\frac{\dot{g}}{b} + \mu\frac{f'}{a}\right)^2, \quad W = \frac{\mu}{ab}\left(\dot{f}g' - \dot{g}f'\right), \quad \mu = \mathrm{sgn}\left(\dot{f}g' - \dot{g}f'\right), \quad \dot{} = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r}.$$

Relation to the work of Gratia, Hu, and Wyman II

EOMs for f & g :

$$\begin{cases} \partial_t\Big[\frac{a^3r^2}{\sqrt{X}}\big(\frac{\dot f}{b}+\mu\frac{g'}{a}\big)P_1+\mu a^2r^2g'P_2\Big]-\partial_r\Big[\frac{a^2br^2}{\sqrt{X}}\big(\mu\frac{\dot g}{b}+\frac{f'}{a}\big)P_1+\mu a^2r^2\dot g P_2\Big]=0, \\ -\partial_t\Big[\frac{a^3r^2}{\sqrt{X}}\big(\frac{\dot g}{b}+\mu\frac{f'}{a}\big)P_1+\mu a^2r^2f'P_2\Big]+\partial_r\Big[\frac{a^2br^2}{\sqrt{X}}\big(\mu\frac{\dot f}{b}+\frac{g'}{a}\big)P_1+\mu a^2r^2\dot f P_2\Big]=a^2br\left[P_0'+\sqrt{X}P_1'+WP_2'\right]. \end{cases}$$

The solution to the first EOM is given by $P_1(x_0) = 0$ & $g(t,r)=x_0\, a\, r$.

$$x_0=\frac{1+6\alpha_3+12\alpha_4\pm\sqrt{1+3\alpha_3+9\alpha_3^2-12\alpha_4}}{3(\alpha_3+4\alpha_4)}.$$

The second EOM reduces to

$$\sqrt{X}P_1'(x_0) = \left[\frac{2P_2(x_0)}{x_0}-P_2'(x_0)\right]W-P_0'(x_0).$$

Our parameter choice with $\alpha_3=\frac{1}{3}(\alpha-1),\ \alpha_4=\frac{1}{12}\left(\alpha^2-\alpha+1\right)$ **automatically satisfies this equation.**

$$\left(P_0'(x_0)=P_1'(x_0)=0,\quad P_2'(x_0)=\frac{2P_2(x_0)}{x_0}.\right)$$

Summary and comments

- We have presented a spatially flat, open, and closed Friedmann Universe in Massive Gravity, though the Stuckelberg fields are inhomogeneous.
- Our analysis is based on the observation that any PG metric with the Stuckelberg fields in the unitary gauge generates an effective cosmological constant for a choice of one parameter family.
- Our choice of parameter is special in that fluctuation modes become non-dynamical at quadratic order. However, recently, it is suggested that they may acquire kinetic term at cubic order, signaling the ghost instabilities, though they use a different fiducial metric. I am also not sure what happens if we take into account quantum corrections.