

# **New Cosmological Solutions in Massive Gravity**

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$$c = \hbar = 1$$

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# Introduction

# Basics of massive gravity (Notation)

de Rham & Gabadadze 2010  
de Rham, Gabadadze, Tolley 2011

$$S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} (R + m^2 \mathcal{U}) + S_m.$$

Potential for the graviton:  $\mathcal{U} := \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4.$

$$\left\{ \begin{array}{l} \mathcal{U}_2 := [\mathcal{K}]^2 - [\mathcal{K}^2], \\ \mathcal{U}_3 := [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \\ \mathcal{U}_4 := [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \end{array} \right. \quad [\mathcal{K}] = \text{Tr} \mathcal{K} \dots$$

$$\left\{ \begin{array}{l} \mathcal{K}_\mu^\nu := \delta_\mu^\nu - (\sqrt{g^{-1}} \Sigma)_\mu^\nu, \\ \Sigma_{\mu\nu} := \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}. \end{array} \right. \quad \left\{ \begin{array}{l} \mathcal{K}_\nu^\mu = \partial^\mu \partial_\nu \pi \\ \uparrow \\ \phi^a = \delta_\mu^a x^\mu - \eta^{a\mu} \partial_\mu \pi. \end{array} \right.$$

$\phi^a$  : Stuckelberg fields

Unitary gauge:  $\phi^a = \delta_\mu^a x^\mu$

# Absence of Flat Friedmann Universe

D'Amico et al. 2011

**Homogeneous & isotropic solution:**

- **real metric:**  $ds^2 = -dt^2 + a^2(t)dx^2$
- **imposes the same symmetry on Stuckelberg fields**

$$\phi^0 = f(t), \quad \phi^i = x^i.$$


$$\Rightarrow \Sigma_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} = \text{diag}(-\dot{f}^2, 1, 1, 1)$$

$$\Rightarrow (g^{-1}\Sigma)^\mu{}_\nu = \text{diag}(\dot{f}^2, a^{-2}, a^{-2}, a^{-2})$$

$$\Rightarrow \kappa_\mu{}^\nu = \delta_\mu{}^\nu - (\sqrt{g^{-1}\Sigma})_\mu{}^\nu = \text{diag}(1 - |\dot{f}|, 1 - a^{-1}, 1 - a^{-1}, 1 - a^{-1})$$

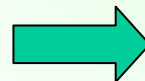
# Absence of Flat Friedmann Universe II

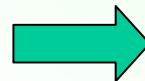
$$\begin{cases} \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] = -6(1 - a^{-1})(a^{-1} - 2 + |f|), \\ \mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] = -6(1 - a^{-1})^2(a^{-1} - 4 + 3|f|), \\ \mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \\ = -24(1 - a^{-1})^3(-1 + |f|). \end{cases}$$

  **$\dot{f}$  appears only linearly !!**

$$\mathcal{L} = \frac{M_{\text{pl}}^2}{2} a^3 [6(\dot{H} + 2H^2) + m^2 \mathcal{U}] = \frac{M_{\text{pl}}^2}{2} a^3 [-6H^2 + m^2 \mathcal{U}].$$

Integration by part

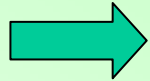
  $\frac{\delta \mathcal{L}}{\delta f} = 3M_{\text{pl}}^2 m^2 \frac{d}{dt} [a^3 - a^2 + 3\alpha_3 a(a-1)^2 + 4\alpha_4 (a-1)^3] = 0.$

  **$\dot{a} = 0$       The scale factor cannot evolve !!**

 **There is no homogeneous and isotropic flat Universe.**

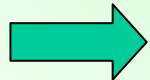
# What can we do ?

- **Consider open Friedmann Universe instead of flat one.**



**Gumrukcuoglu, Lin and Mukohyama 2011.**

- **Abandon imposing the same symmetry on the Stuckelberg field.**



**Our work by use of Painleve-Gullstrand metric**

**Note that the same idea was done in D'Amico et al. 2011 & Gratia, Hu, and Wyman 2012 as well.**

**Painleve-Gullstrand metric**



# Spherically symmetric vacuum solution in (massless) GR

Schwarzschild metric:

$$ds^2 = -f(r)dt_s^2 + f^{-1}(r)dr^2 + r^2d\Omega^2.$$

$$f(r) = 1 - 2M/r.$$

This metric has a coordinate singularity at the horizon  $r = 2M$ .

In GR, this is **not a real singularity** and  
can be removed by coordinate transformation.

# Danger of coordinate singularity in Massive Gravity


Gruzinov & Mirbabayi 2011  
Berezhiani et al. 2012

**New invariant in Massive Gravity:**

$$I^{ab} = g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b.$$

This quantity is invariant under coordinate transformation, namely, a scalar quantity and should have the same position as  $R, R_{\mu\nu}R^{\mu\nu}, \dots$ .

**In the unitary gauge:**  $\phi^a = \delta_\mu^a x^\mu$

  $I^{ab} = g^{\mu\nu} \delta_\mu^a \delta_\nu^b.$

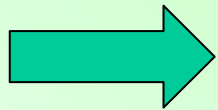
**Any inverse metric with divergence leads to singularity in this invariant.**

(Though such a singularity does not affect the geodesic motion, it would cause a problem for perturbations around classical solutions because inverse metric could change its sign across the singularity.)

# Danger of coordinate singularity in Massive Gravity II

Gruzinov & Mirbabayi 2011  
Berezhiani et al. 2012

**The Schwarzschild like metric in Massive Gravity  
(Schwarzschild, Schwarzschild-De Sitter, Reissner-Nordstrom...)  
can be dangerous.**



**Needs the metric without coordinate singularity**

**Painleve-Gullstrand metric !!**

- **BH solutions in PG metric:**

**Berezhiani, Chkareuli, de Rham, Gabadadze, Tolley 2011**

- **Cosmological solutions in PG metric:**

**our work**

# Painleve-Gullstrand metric

Painleve 1922, Gullstrand 1922,  
Kanai, Siino, Hosoya 2011

## Merit of PG metric:

- includes **an off-diagonal and spatially flat elements**, which leads to **no coordinate singularity** (except real singularity at the origin)
- can cover both inside and outside the horizon by a **single** coordinate patch.
- Time coordinate **as measured by an observer** who is at rest at infinity and freely falls into the BH.
- The space described by the PG metric can be regarded as a river whose **speed of current is the Newtonian escape velocity at each point**.
- **Generalized PG metric can also describe the FRLW universe.**

# Derivation of Painleve-Gullstrand metric


Painleve 1922, Gullstrand 1922,  
Kanai, Siino, Hosoya 2011

**Schwarzschild metric:**  $ds^2 = -f(r)dt_s^2 + f^{-1}(r)dr^2 + r^2d\Omega^2.$   
 $f(r) = 1 - 2M/r.$

**Four velocity of an observer :**  $u_s^\mu = dx_s^\mu/d\tau = \dot{x}_s^\mu.$

**normalization condition :**  $-1 = g_{\mu\nu}u_s^\mu u_s^\nu = -f\dot{t}_s^2 + f^{-1}\dot{r}^2.$   
**(conserved) energy per rest mass :**  $\epsilon = -g_{\mu\nu}\xi^\mu u_s^\nu = f\dot{t}_s.$

$\xi^\mu = (\partial/\partial t_s)^\mu$  : timelike Killing vector

  $\left\{ \begin{array}{l} u_s^\mu = (\dot{t}_s, \dot{r}, \dot{\theta}, \dot{\phi}) = \left( \frac{\epsilon}{f}, -\sqrt{\epsilon^2 - f}, 0, 0 \right), \\ u_{s\mu} = g_{\mu\nu}u_s^\nu = \left( -\epsilon, -\frac{\sqrt{\epsilon^2 - f}}{f}, 0, 0 \right). \end{array} \right.$

# Derivation of Painleve-Gullstrand metric II

$t_P$  : the proper time of the free-falling observer

↔ The geodesic is orthogonal to the surface  $t_P = \text{const.}$

↔ The geodesic tangent vector  $u_{s\mu}$  is equal to the gradient of  $t_P$ .

$$u_{s\mu} = -\frac{\partial}{\partial x_s^\mu} t_P(x_s). \quad u_{s\mu} = \left(-\epsilon, -\sqrt{\epsilon^2 - f}/f, 0, 0\right).$$

↔  $dt_P = \epsilon dt_s + \frac{\sqrt{\epsilon^2 - f}}{f} dr.$

$$\begin{aligned} ds^2 &= -f(r) dt_s^2 + f^{-1}(r) dr^2 + r^2 d\Omega^2, \\ &= -dt_P^2 + \frac{1}{\epsilon^2} (dr + v(r) dt_P)^2 + r^2 d\Omega^2. \end{aligned}$$

$$v(r) = \sqrt{\epsilon^2 - f(r)} : \text{radially free-falling velocity}$$

At the horizon  $f(r)=0 \Leftrightarrow r=2M$ , the metric is non-singular.

# Derivation of Painleve-Gullstrand metric III

$$ds^2 = -dt_p^2 + \frac{1}{\epsilon^2} (dr + v(r)dt_p)^2 + r^2 d\Omega^2.$$

$$\longrightarrow \begin{cases} u_p^\mu = (\dot{t}_p, \dot{r}, \dot{\theta}, \dot{\phi}) = (1, -v, 0, 0), \\ u_{p\mu} = g_{\mu\nu}u_p^\nu = (-1, 0, 0, 0). \end{cases}$$

$$\frac{dr}{dt_p} = \frac{\dot{r}}{\dot{t}_p} = -v(r) = -\sqrt{\epsilon^2 - f(r)}. \quad \longrightarrow \quad E = \frac{1}{2} \left( \frac{dr}{dt_p} \right)^2 + \Phi(r).$$

$$\begin{cases} E = (\epsilon^2 - 1)/2 & : \text{conserved energy} \\ \Phi(r) = -M/r & : \text{gravitational potential} \end{cases}$$

For a particle freely falling from infinity at rest ( $\epsilon = 1 \Leftrightarrow E=0$ ),

$$ds^2 = -dt_p^2 + \left( dr + \sqrt{2M/r} dt_p \right)^2 + r^2 d\Omega^2.$$

Standard form  
given by PG.

This is a vacuum solution, so we want a solution including matter.

Kanai, Siino, Hosoya 2011

$$\longrightarrow ds^2 = -dt_p^2 + \frac{1}{1 + 2E(t_p, r)} (dr + v(t_p, r)dt_p)^2 + r^2 d\Omega^2, \quad v(t_p, r) = \sqrt{2E(t_p, r) + 2m(t_p, r)/r}.$$

# Spherical gravitational collapse – from infinity

Kanai, Siino, HoSoya 2011

Spherical gravitational collapse of matter with  $E = 0$  :

$$ds^2 = -dt^2 + \left( dr + \sqrt{\frac{2m(r,t)}{r}} dt \right)^2 + r^2 d\Omega^2.$$

Einstein Eq.

$$(8\pi T_{\nu}^{\mu} = R_{\nu}^{\mu} - \delta_{\nu}^{\mu} R/2)$$



$$\left\{ \begin{array}{l} 8\pi T_t^t = -\frac{2m'}{r^2}, \\ 8\pi T_t^r = \frac{2\dot{m}}{r^2}, \\ 8\pi T_r^r = -\frac{2m'}{r^2} + \frac{2\dot{m}}{r^2} \left(\frac{2m}{r}\right)^{-1/2}, \\ 8\pi T_{\theta}^{\theta} (T_{\phi}^{\phi}) = -\frac{m''}{r} + \left(\frac{\dot{m}}{2r^2} + \frac{\dot{m}'}{r}\right) \left(\frac{2m}{r}\right)^{-1/2} - \frac{\dot{m}m'}{r^2} \left(\frac{2m}{r}\right)^{-3/2}. \end{array} \right. \quad \cdot = \partial/\partial t, \quad ' = \partial/\partial r.$$

Only three are independent.  $\leftarrow T_r^r = T_t^t + T_t^r (2m/r)^{-1/2}.$

Perfect fluid :  $T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} + Pg^{\mu\nu}$

$$u^{\mu} = (1, -v(t, r), 0, 0) \quad \rightarrow \quad T_t^t = -\rho, \quad T_r^r = P, \quad T_t^r = (\rho + P)v.$$


$v(t, r) = \sqrt{\frac{2m(t, r)}{r}}$  is equal to the escape velocity.




# Spherical gravitational collapse – from infinity II

Kanai, Siino, Hosoya 2011

●  $8\pi T_t^t = -8\pi\rho = -2m'/r^2 :$

  $m(t, r) = 4\pi \int_0^r \rho(t, r) r^2 dr \left( = \frac{2r^3}{9\gamma^2 t^2} \right)$

●  $8\pi T_t^r = 8\pi(\rho + P) = \frac{2\dot{m}}{r^2}, \quad P = (\gamma - 1)\rho, \quad \rho(t, r) = f(r)h(t) :$

  $\rho(t, r) = \frac{1}{6\pi\gamma^2 t^2}, \quad P(t, r) = \frac{\gamma - 1}{6\pi\gamma^2 t^2}.$

B.C.  $m|_{r=0} = 0, \quad \rho|_{t=0} = \infty. \quad (t : -\infty \rightarrow 0)$

**Matter density (of the star) is uniform.**

**Though, in case of gravitational collapse, we need to match this inner solution with the outer solution given before, we are now interested in only the inner solution because...**

# Relation between this solution and Friedmann Universe

$$ds^2 = -dt^2 + (dr + v_-(t, r)dt)^2 + r^2 d\Omega^2.$$

$$v_-(t, r) = \sqrt{\frac{2m(t, r)}{r}} = \frac{2r}{3\gamma(-t)}. \quad (t : -\infty \rightarrow 0)$$

●  $P = (\gamma - 1)\rho$  ➡  $H = \frac{2}{3\gamma t}$  ➡  $v_-(t, r) = -\frac{\dot{a}(t)}{a(t)}r.$   
**FLRW**

● **In fact,**  $r = a(t)\tilde{r}$  ➡  $dr = \dot{a}(t)\frac{r}{a(t)}dt + a(t)d\tilde{r}$

➡  $ds^2 = -dt^2 + (dr + v_-(t, r)dt)^2 + r^2 d\Omega^2$   
 $= -dt^2 + a^2(t) (d\tilde{r}^2 + \tilde{r}^2 d\Omega^2).$

**The generalized Painleve-Gullstrand metric includes flat Friedmann Universe. (Expanding phase:  $v_- \rightarrow -v_-$ )**

# Spherical gravitational collapse – from a finite radius

Kanai, Siino, Hosoya 2011

$$ds^2 = -dt^2 + \frac{1}{1 + 2E(t,r)} (dr + v_-(t,r)dt)^2 + r^2 d\Omega^2, \quad v_-(t,r) = \sqrt{2E(t,r) + 2m(t,r)/r}.$$

The boundary surface  $r=a(t)$  that **freely falls from a radius  $a_0$  at rest** :

Solving Einstein Eq. inside the boundary  $(M = m(t,r)|_{r=a(t)} = \frac{4\pi}{3} a^3(t) \rho(t))$

$$\Rightarrow E(t,r) = -\frac{M}{a_0} \left( \frac{r}{a(t)} \right)^2 < 0, \quad v_-(t,r) = \sqrt{\frac{2M}{a(t)} - \frac{2M}{a_0} \frac{r}{a(t)}}.$$

● In this case also,  $r = a(t) \tilde{r} \Rightarrow v_-(t,r) = -\frac{\dot{a}(t)}{a(t)} r.$

$$\begin{aligned} ds^2 &= -dt^2 + \frac{1}{1 - \frac{2M}{r_0} \left( \frac{r}{a(t)} \right)^2} (dr + v_-(t,r)dt)^2 + r^2 d\Omega^2 \\ &= -dt^2 + a^2(t) \left( \frac{d\tilde{r}^2}{1 - \frac{2M}{a_0} \tilde{r}^2} + \tilde{r}^2 d\Omega^2 \right). \end{aligned}$$

The generalized Painleve-Gullstrand metric includes closed (open  $\Leftrightarrow E > 0$ ) Friedmann Universe as well.

# Cosmological solution

# Generalized Painleve-Gullstrand metric as a cosmological solution

- Our strategy is to find generalized PG metric in Massive Gravity instead of the standard FLRW metric.

$$ds^2 = -V^2(t, r)dt^2 + U^2(t, r) \left( dr + \epsilon \sqrt{f(t, r)} dt \right)^2 + W^2(t, r) r^2 d\Omega^2. \quad (\epsilon = \pm 1)$$

- Stuckelberg fields in the unitary gauge:

$$\phi^0 = t, \quad \phi^i = r \hat{n}^i. \quad \hat{n} = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta)$$

- One parameter family:

$$\alpha_3 = \frac{1}{3}(\alpha - 1), \quad \alpha_4 = \frac{1}{12}(\alpha^2 - \alpha + 1).$$

$$W(t, r) = \tilde{\alpha} := \frac{\alpha}{\alpha + 1}. \quad \longrightarrow \quad X_{\mu\nu} = \frac{1}{\alpha} g_{\mu\nu}. \quad \text{Effective C.C.} \quad M_{\text{pl}}^2 (G_{\mu\nu} + m^2 X_{\mu\nu}) = T_{\mu\nu}$$

**Any PG-type metric in GR (with a cosmological constant)  
is also a solution to Massive Gravity.**

# Friedmann Universe in Massive Gravity

- The FLRW metric can be rewritten in a general PG form :

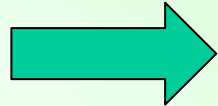
$$ds^2 = -\kappa^2 dt^2 + \frac{\tilde{\alpha}^2}{1 - K\tilde{\alpha}^2 r^2/a^2(t)} \left( dr - \frac{\dot{a}}{a} r dt \right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2.$$

$$K = 0, \pm 1. \quad \longleftrightarrow \quad \text{All types of Friedmann Universe}$$

- Perfect fluid :  $T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$

$$\rho = \rho(t), \quad P = P(t). \quad u_\mu = (-\kappa, 0, 0, 0)$$

**EOM**



$$\left\{ \begin{array}{l} \frac{3}{\kappa^2} \tilde{H}^2 = \frac{\rho}{M_{\text{pl}}^2} + \frac{m^2}{\alpha} - \frac{3K}{a^2}, \\ -\frac{1}{\kappa^2} (3\tilde{H}^2 + 2\dot{\tilde{H}}) = \frac{p}{M_{\text{pl}}^2} - \frac{m^2}{\alpha} + \frac{K}{a^2}. \end{array} \right. \quad (\tilde{H}(t) := d \ln a / dt)$$


Rescaling time coordinate  $t \rightarrow \tau = \kappa t$  with  $H := d \ln a / d \tau$  gives  
the standard cosmological equation with effective C.C.  $\Lambda_{\text{eff}} = m^2/\alpha$ .

Thus, our solution can accommodate spatially flat, open, and closed models.

# More familiar form


$$ds^2 = -d\tau^2 + \frac{\tilde{\alpha}^2}{1 - K\tilde{\alpha}^2 r^2/a^2(\tau)} \left( dr - \frac{\dot{a}}{\kappa a} r d\tau \right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2.$$

**Coordinate transformation:**  $r \rightarrow \tilde{r} = \frac{\tilde{\alpha} r}{a(\tau)}.$


$$ds^2 = -d\tau^2 + a^2 \left( \frac{d\tilde{r}^2}{1 - K\tilde{r}^2} + \tilde{r}^2 d\Omega^2 \right).$$

**with Stuckelberg fields, which do not respect the same symmetry,**

$$\phi^0 = \frac{\tau}{\kappa}, \quad \phi^i = \frac{a(\tau)\tilde{r}}{\tilde{\alpha}} \hat{n}^i.$$


$$\Sigma_{\mu\nu} dx^\mu dx^\nu = - \left( \frac{1}{\kappa^2} - \frac{a^2 H^2 \tilde{r}^2}{\tilde{\alpha}^2} \right) d\tau^2 + 2 \frac{a^2 H \tilde{r}}{\tilde{\alpha}^2} d\tau d\tilde{r} + \frac{a^2}{\tilde{\alpha}^2} (d\tilde{r}^2 + \tilde{r}^2 d\Omega^2).$$

# Relation to the work of Gratia, Hu, and Wyman

arXiv:1205.4241

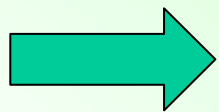
Try to find spatially isotropic solution:

$$ds^2 = -b^2(t, r)dt^2 + a^2(r, t) (dr^2 + r^2 d\Omega^2).$$

with Stuckelberg fields, which respects the same symmetry,

$$\phi^0 = f(t, r), \quad \phi^i = g(t, r) \frac{x^i}{r}.$$

(N.B.  $b(t, r) = 1$  &  $a(t, r) = a(t)$   $\rightarrow$  Flat FRW metric)



**Potential:** 
$$-\mathcal{U} = P_0 \left( \frac{g}{ar} \right) + \sqrt{X} P_1 \left( \frac{g}{ar} \right) + W P_2 \left( \frac{g}{ar} \right).$$

$$\begin{cases} P_0(x) = -12 - 2x(x - 6) - 12(x - 1)(x - 2)\alpha_3 - 24(x - 1)^2\alpha_4, \\ P_1(x) = 2(3 - 2x) + 6(x - 1)(x - 3)\alpha_3 + 24(x - 1)^2\alpha_4, \\ P_2(x) = -2 + 12(x - 1)\alpha_3 - 24(x - 1)^2\alpha_4. \end{cases}$$

$$X = \left( \frac{\dot{f}}{b} + \mu \frac{g'}{a} \right)^2 - \left( \frac{\dot{g}}{b} + \mu \frac{f'}{a} \right)^2, \quad W = \frac{\mu}{ab} (\dot{f}g' - \dot{g}f'), \quad \mu = \text{sgn}(\dot{f}g' - \dot{g}f'), \quad \dot{\phantom{x}} = \frac{\partial}{\partial t}, \quad ' = \frac{\partial}{\partial r}.$$

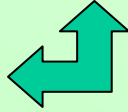


# Relation to the work of Gratia, Hu, and Wyman II

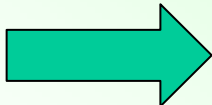
**EOMs for f & g :**

$$\left\{ \begin{array}{l} \partial_t \left[ \frac{a^3 r^2}{\sqrt{X}} \left( \frac{\dot{f}}{b} + \mu \frac{g'}{a} \right) P_1 + \mu a^2 r^2 g' P_2 \right] - \partial_r \left[ \frac{a^2 b r^2}{\sqrt{X}} \left( \mu \frac{\dot{g}}{b} + \frac{f'}{a} \right) P_1 + \mu a^2 r^2 \dot{g} P_2 \right] = 0, \\ -\partial_t \left[ \frac{a^3 r^2}{\sqrt{X}} \left( \frac{\dot{g}}{b} + \mu \frac{f'}{a} \right) P_1 + \mu a^2 r^2 f' P_2 \right] + \partial_r \left[ \frac{a^2 b r^2}{\sqrt{X}} \left( \mu \frac{\dot{f}}{b} + \frac{g'}{a} \right) P_1 + \mu a^2 r^2 \dot{f} P_2 \right] = a^2 b r \left[ P'_0 + \sqrt{X} P'_1 + W P'_2 \right]. \end{array} \right.$$

The solution to the first EOM is given by  $P_1(x_0) = 0$  &  $g(t,r)=x_0$  a r.

$$x_0 = \frac{1 + 6\alpha_3 + 12\alpha_4 \pm \sqrt{1 + 3\alpha_3 + 9\alpha_3^2 - 12\alpha_4}}{3(\alpha_3 + 4\alpha_4)}.$$


The second EOM reduces to



$$\sqrt{X} P'_1(x_0) = \left[ \frac{2P_2(x_0)}{x_0} - P'_2(x_0) \right] W - P'_0(x_0).$$

Our parameter choice with  $\alpha_3 = \frac{1}{3}(\alpha - 1)$ ,  $\alpha_4 = \frac{1}{12}(\alpha^2 - \alpha + 1)$  automatically satisfies this equation.

$$\left( P'_0(x_0) = P'_1(x_0) = 0, \quad P'_2(x_0) = \frac{2P_2(x_0)}{x_0} \right)$$

# Summary and comments

- We have presented a **spatially flat, open, and closed Friedmann Universe** in Massive Gravity, though the Stuckelberg fields are inhomogeneous.
- Our analysis is based on the observation that **any PG metric with the Stuckelberg fields in the unitary gauge generates an effective cosmological constant** for a choice of one parameter family.
- Our choice of parameter is special in that **fluctuation modes become non-dynamical at quadratic order**. However, recently, it is suggested that they may **acquire kinetic term at cubic order, signaling the ghost instabilities**, though they use a different fiducial metric. I am also not sure what happens if we take into account quantum corrections.