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"Nonlinear massive gravity theory and its observational test"

Tunneling Field in Massive Gravity

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paper to appear

Outline

- Motivation
- Setup of model
- Equations of motion
- Hawking-Moss solutions
- Colemann-DeLuccia solutions
- Conclusion

Motivation of our work

Question in GR: why cosmological constant is so small?

In context of massive gravity(MG), the effective cosmological constant consists of a contribution from mass of graviton.

A. E. Gumrukcuoglu, C. Lin and S. Mokohyama JCAP 106, 231101(2011);

→ it is interesting to study the landscape in the context of MG

→ our work

- setup the model and find the tunneling solutions;
- estimation of Hawking-Moss solution;
- evaluate the Colemann-De Luccia solution.

I. Setup of model

- Action

$$S = I_g + I_m,$$

$$I_g \equiv \int d^4x\,\sqrt{-g}\left[\frac{R}{2} + m_g^2(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)\right],$$

$$I_m \equiv -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma)\right],$$

where

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),$$

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{g^{\mu\sigma}G_{ab}(\phi)\partial_\nu\phi^a\partial_\sigma\phi^b}.$$

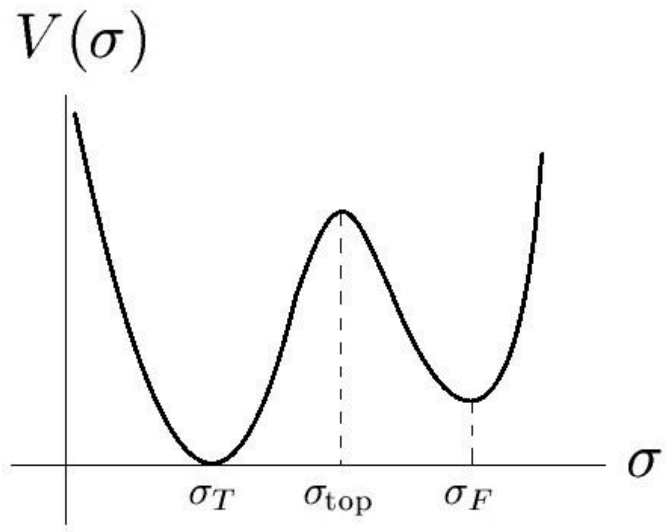
fiducial metric

- potential $V(\sigma)$

  local minima: σ_F

  global minima: σ_T

  local max: $\sigma_{\rm top}$



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ae^{-B},$$

$$B = S_E[\bar{g}_{\mu\nu,B}, \bar{\phi}_B] - S_E[\bar{g}_{\mu\nu,F}, \bar{\phi}_F],$$

bounce solution ‘false vacuum’

usually, bounce solutions are explored by assuming an O(4) symmetry

Lowest action

- spacetime metric: FLRW

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2 dt^2 + a(t)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2 \Omega_{ij} d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

fiducial Hubble parameter

$\longrightarrow$ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(t), \quad \phi^i = x^i.$$

Euclidean action

$$S_E = iS[x^0 \to ix^0_E]$$

Inserting the ansatz, the Euclidean action can be written as

$$I_{gE} = \int d^4x_E\ \sqrt{\Omega}\left[-3KNa - \frac{3\dot{a}^2 a}{N} - m_g^2\left(L_{2E} + \alpha_3 L_{3E} + \alpha_4 L_{4E}\right)\right],$$

$$I_{mE} = \int d^4x_E\ a^3\sqrt{\Omega}\left[\frac{1}{2N}\dot{\sigma}^2 + NV(\sigma)\right],$$

where $\dot{} \equiv d/dt_E$

$$L_{2E} = 3a(a-b)(2Na + i\dot{f}a - Nb),$$

$$L_{3E} = (a-b)^2(4Na + 3i\dot{f}a - Nb),$$

$$L_{4E} = (a-b)^3(N + i\dot{f}).$$

II. Equations of motion

- Constraint equation

$$(i\dot{a}+Nb_{,f})\left[\left(3-\frac{2b}{a}\right)+\alpha_3\left(1-\frac{b}{a}\right)\left(3-\frac{b}{a}\right)+\alpha_4\left(1-\frac{b}{a}\right)^2\right]=0,$$

$$b_{,f}\equiv\frac{db}{df}=\sqrt{K}\sinh(Ff)$$

$\rightarrow$ Branch I $\quad Nb_{,f}=-i\dot{a},\quad$ (similar to branch II)

Branch II $\quad \left(3-\frac{2b}{a}\right)+\alpha_3\left(1-\frac{b}{a}\right)\left(3-\frac{b}{a}\right)+\alpha_4\left(1-\frac{b}{a}\right)^2=0.$

$$\rightarrow \boxed{b=X_\pm a,\qquad X_\pm\equiv\frac{1+2\,\alpha_3+\alpha_4\pm\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}.}$$

- Friedmann equation & EOM for tunneling field

$$
\begin{cases}
\dfrac{3}{a^2}\left(\dfrac{da}{d\tau}\right)^2 - \dfrac{3K}{a^2} = \dfrac{1}{2}\left(\dfrac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\[2ex]
\dfrac{d^2\sigma}{d\tau^2} + 3\mathcal{H}\dfrac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0
\end{cases}
$$

where $\mathrm{d}\tau \equiv N\mathrm{d}t,$

$$
\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],
$$

III. Hawking-Moss(HM) solutions

- HM solutions can be found at the local maximum of the potential

$$\sigma = \sigma_{\rm top}$$

$$\rightarrow \quad \frac{3}{a^2}\left(\frac{da}{d\tau}\right)^2 - \frac{3K}{a^2} = -\Lambda_{\pm,\rm eff} \equiv -V(\sigma_{\rm top}) - \Lambda_{\pm},$$

$$\rightarrow \quad a_{\rm HM}(\tau) = H_{\rm HM}^{-1}\sqrt{K}\cos\left(H_{\rm HM}\tau\right) \qquad H_{\rm HM} \equiv \sqrt{\frac{\Lambda_{\pm,\rm eff}}{3}}$$

$$\Downarrow \quad b_{\rm HM} = F^{-1}\sqrt{K}\cosh(F f_{\rm HM}) = X_{\pm} a_{\rm HM}$$

$$f_{\rm HM}(\tau) = \frac{1}{F}\ln\left[\alpha\cos(H_{\rm HM}\tau) \pm \sqrt{\alpha^2\cos^2(H_{\rm HM}\tau) - 1}\right],$$

$$\uparrow$$

$$\alpha \equiv X_{\pm}\frac{F}{H_{\rm HM}}$$

• inserting these solutions into the Euclidean action and evaluate by integrating in the range $H_{\rm HM}\tau = -\pi/2 \longrightarrow \pi/2$, we find when $|\alpha| > 1$ the action becomes complex which is uninteresting here.

$\rightarrow$ Let $\boxed{|\alpha| \leq 1}$

$$Y_\pm \equiv 3(1 - X_\pm) + 3\alpha_3(1 - X_\pm)^2 + \alpha_4(1 - X_\pm)^3\,,$$

$$\boxed{S_E[a_{\rm HM}, \sigma_{\rm top}] = -\frac{8\pi^2}{H_{\rm HM}^2}\left[1 - \frac{Y_\pm}{6\alpha^2 X_\pm}\left(\frac{m_g}{F}\right)^2\left[2 - \sqrt{1-\alpha^2}(2+\alpha^2)\right]\right]\,,}$$

• inserting these solutions into the Euclidean action, we find when $|\alpha| > 1$ the action becomes complex which is physically uninteresting here.

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stadard HW solution

Correction due to the mass of graviton

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→ Let $\boxed{|\alpha| \leq 1}$ → $\left[2 - \sqrt{1-\alpha^2}(2+\alpha^2)\right] \geq 0$

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stadard HW solution

determines HM action suppressed or enhanced

Correction due to the mass of graviton

Case 1. $\alpha_3 = \alpha_4 = 0$

$$\Lambda_\pm = \begin{cases} -\infty\,, & \text{for } \Lambda_+ \\ \frac{3}{4}m_g^2\,, & \text{for } \Lambda_- \end{cases}$$

- so we just have the minus branch;

$$X_- = \frac{3}{2}\,, \qquad Y_- = -\frac{3}{2}\,, \qquad \Longrightarrow \qquad \frac{Y_-}{X_-} = -1 < 0$$

- HM action is smaller than the standard one.

$$\Downarrow$$

- the corresponding tunneling rate is suppressed.

Case 2. $\alpha_3 = 0$

(a). X_+ branch

$$X_+ = \frac{1}{\alpha_4}\left(1+\alpha_4+\sqrt{1-\alpha_4}\right)\ , \qquad Y_+ = -\frac{2}{\alpha_4^2}(1+\sqrt{1-\alpha_4})(1+\alpha_4+\sqrt{1-\alpha_4})$$

$$\Downarrow$$

$$\frac{Y_+}{X_+} = \frac{2}{\sqrt{1-\alpha_4}-1}$$

- When $\alpha_4 < 0$ HM action is larger than the standard one, so the tunneling rate is enhanced;

- When $0 < \alpha_4 \leq 1$ HM action is smaller than the standard one, so the tunneling rate is suppressed;

- $\alpha_4 \leq 1$ and $\alpha_4 = 0$ is singular.

(b). X_- branch

$$X_- = \frac{1}{\alpha_4}\left(1+\alpha_4-\sqrt{1-\alpha_4}\right)\ , \qquad Y_- = -\frac{2}{\alpha_4^2}(1-\sqrt{1-\alpha_4})(1+\alpha_4-\sqrt{1-\alpha_4})$$

$$\Downarrow$$

$$\frac{Y_-}{X_-} = -\frac{2}{\sqrt{1-\alpha_4}+1}$$

- $\alpha_4 \le 1$ and HM action is smaller than the standard one,

  so the tunneling rate is suppressed.

Case 3. $m_g \longrightarrow 0, \qquad m_g^2\alpha_3 = A_3 \neq 0, \qquad m_g^2\alpha_4 = A_4 \neq 0$

$$X_\pm = 1 + \frac{A_3}{A_3 + A_4}\left(1 \pm \mathrm{sgn}(\mathrm{A}_3)\right)\,, \qquad \mathrm{sgn}(\mathrm{A}_3) \equiv |\mathrm{A}_3|/\mathrm{A}_3$$

** without loss of generality, we assume $A_3 > 0$ in the analysis.

(a). X_+ branch

$$m_g^2\frac{Y_+}{X_+} = (1 - X_+)^2 A_3 > 0\,,$$

- HM action is larger than the standard one,

  so the tunneling rate is enhanced;

(b). X_- branch

$$\Lambda_- = 0\,, \qquad X_- = 1\,, \qquad m_g^2 Y_- = 0\,,$$

$\Downarrow$

reduces to the standard GR case, no contribution

** for $A_3 < 0$, conclusion is inverse, i.e.

X_+ branch reduces to the GR case;

X_- gives rise to a smaller HM action,
so the tunneling rate is suppressed.

IV. Coleman-De Luccia(CDL) solutions

Strategy: Consider the CDL solution near the HM limit

T. Tanaka and M. Sasaki, Prog. Theor. Phys. 88 (3) 1992

Expand the potential $V(\sigma)$ around $\sigma = \sigma_{\rm top}$ to the 4th order:

$$\boxed{V(\sigma) = V(\sigma_{\rm top}) - \frac{M^2}{2}(\sigma_{\rm top} - \sigma)^2 + \frac{m}{3}(\sigma_{\rm top} - \sigma)^3 + \frac{\nu}{4}(\sigma_{\rm top} - \sigma)^4 \,,}$$

$$\boxed{M^2 \equiv 4H_{\rm HM}^2(1+\chi^2) \,, \quad \chi^2 \ll 1}$$

Inserting this into EOM and expand the action to 4th order in χ

$$a(\tau) = \frac{\cos\left(\tilde{H}_{\rm HM}\tau\right)}{\tilde{H}_{\rm HM}}\left[1 + \frac{\epsilon^2 H_{\rm HM}^2}{8}\cos^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\epsilon^3) \,,$$

$$\boxed{\tilde{H}_{\rm HM} \equiv H_{\rm HM}(1 + H_{\rm HM}^2\epsilon^2/24)}$$

$$\sigma(\tau) = \sigma_{\rm top} + \epsilon H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right) + \frac{\epsilon^2 m}{12}\left[1 - 2\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right]$$

$$-\epsilon^3 H_{\rm HM}\sin\left(\tilde{H}_{\rm HM}\tau\right)\left[\frac{3H_{\rm HM}^2 - 4\nu'}{56}\cos^2\left(\tilde{H}_{\rm HM}\tau\right) - \frac{m^2}{36H_{\rm HM}^2}\sin^2\left(\tilde{H}_{\rm HM}\tau\right)\right] + \mathcal{O}(\epsilon^4) \,,$$

$$\boxed{\epsilon^2 \equiv 84\chi^2/(16H_{\rm HM}^2 + 9\nu')}$$

$$\boxed{\nu' \equiv \nu + m^2/18H_{\rm HM}^2}$$

Inserting this result into the action and evaluate it on the sphere, we obtain the derivation from HM solution:

$$\delta S_E^{(4)} = -\frac{\pi^2 \epsilon^4}{40} \left(11 H_{\mathrm{HM}}^2 + 4\nu'\right) < 0$$

$$\boxed{\epsilon^2 \equiv 84\chi^2/(16H_{\mathrm{HM}}^2 + 9\nu') > 0}$$

which shows a smaller value of action when compared with the HW one.

$$\Downarrow$$

CDL mode dominates the process of tunneling

Conclusion

- We set up a massive gravity model with tunneling field;
- corrections of HW solution from mass term is found, which implies suppression or enhancement of tunneling rate, depending on the choices of parameters;
- we evaluate CDL action and found that it is smaller than the HW one.