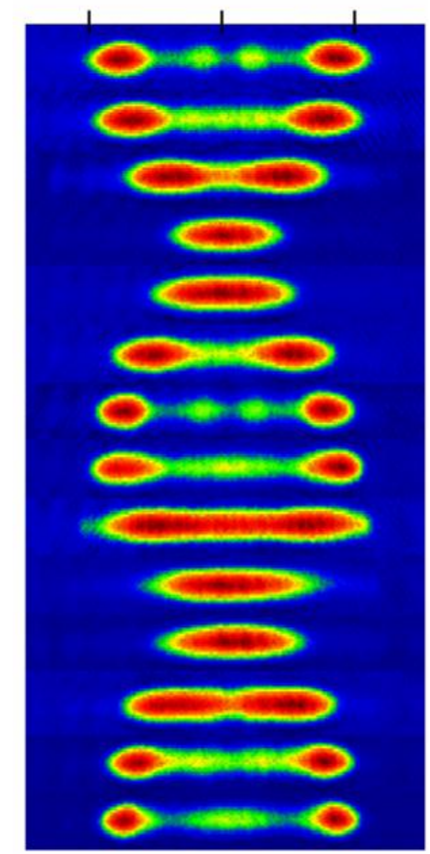
非平衡1次元ボース気体
Integrability and Thermalization



Toshiya Kinoshita

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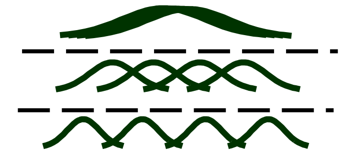
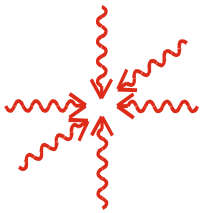
Work at Penn State University with
Trevor Wenger
Prof. David S. Weiss

PENNSTATE

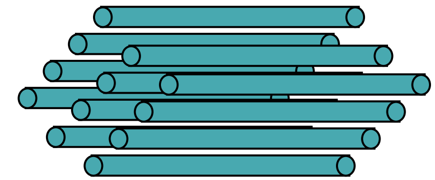
Outline



1D Bose gas theory



Equilibrium 1D Bose gas experiments



- Total energy
- 1D Cloud Size
- Local Pair Correlations

強相関系の物理

Non-Equilibrium 1D Bose gas experiments

- the Quantum Newton's cradle

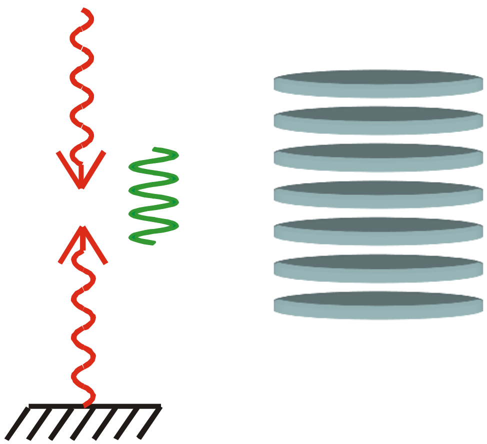
可積分系、非平衡物理

Optical Lattices

Calculable, versatile atom traps

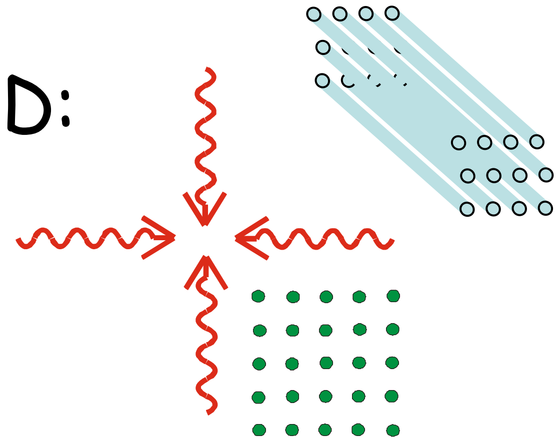
Depth $\propto$ Intensity

1D:

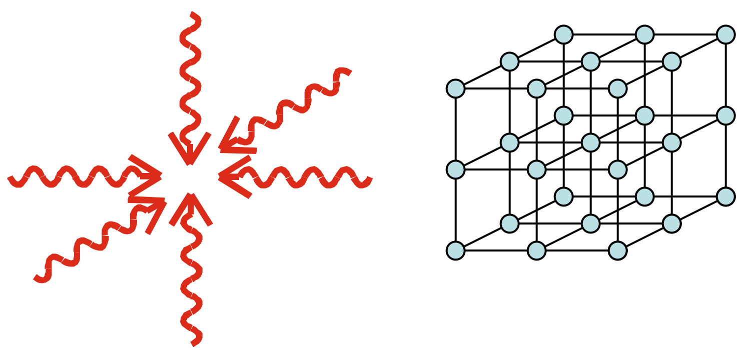


1D gases

2D:

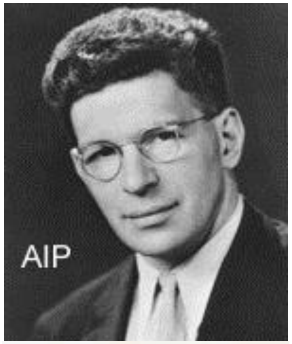


3D:

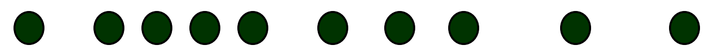


16μK

1D Bose gases with infinite hard core interactions



Lewi Tonks, 1936: Eq. of state of a 1D classical gas of hard spheres

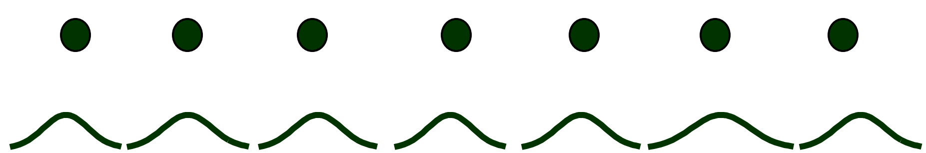


Marvin Girardeau, 1960: 1D Bose gases with infinite hard core repulsion

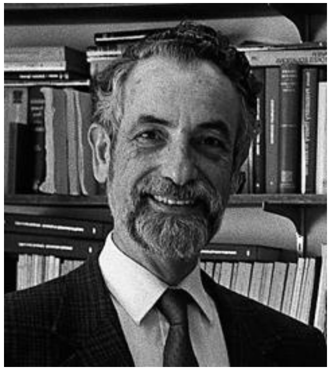
In 1D, if no two single particle wavefunctions overlap

$$\Rightarrow \quad \psi_{bosons} = \left|\psi_{fermions}\right|$$

"Fermionization"



1D Bose gases with variable point-like interactions



Elliot Lieb and Werner Liniger, 1963: Exact solutions for 1D Bose gases with arbitrary $\delta(z)$ interactions

$$H_{1D} = -\sum_{\text{all atoms}} \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \sum_{\text{all pairs}} g_{1D}\,\delta(z)$$

$(g_{1D} > 0)$

Solutions parameterized by

$$\gamma = \frac{m}{\hbar^2}\frac{g_{1D}}{n_{1D}}$$

$\gamma >> 1$
Tonks-Girardeau gas

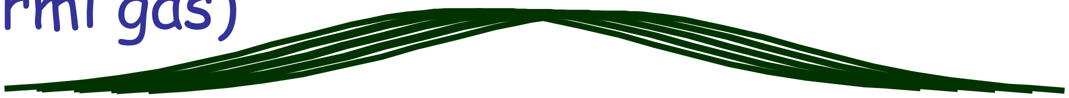
kinetic energy dominates

large g_{1D}
low density

$\gamma << 1$
mean field theory (Thomas-Fermi gas)

mean field energy dominates

small g_{1D}
high density



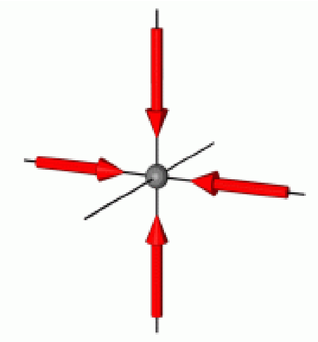
1D Bose atomic gases

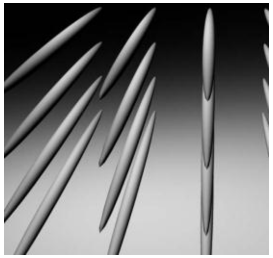
Maxim Olshanii, 1998: Adaptation to real atoms

$$\gamma \approx \frac{2 \ a_{3D}}{a_\perp^2 \ n_{1D}}$$

a_{3D} = 3D scattering length

$a_\perp$ = transverse size of wavefunction





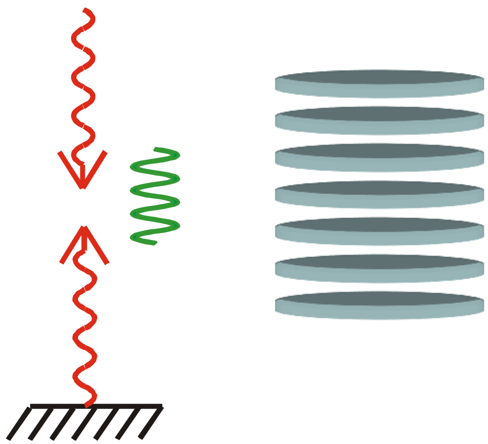
$\gamma\uparrow$ when $a_{3D}\uparrow$, $\underline{n_{1D}\downarrow}$ or $\underline{a_\perp\downarrow}$

Optical Lattices

Calculable, versatile atom traps

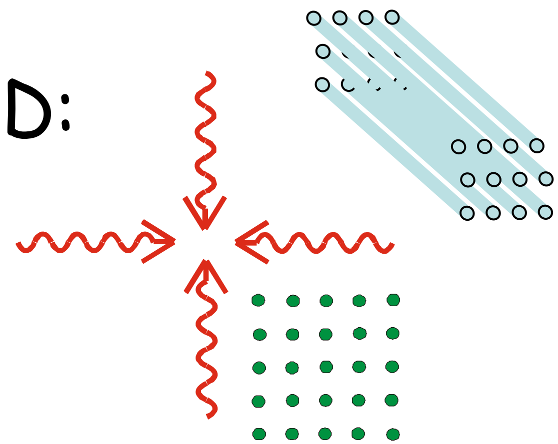
Depth $\propto$ Intensity

1D:

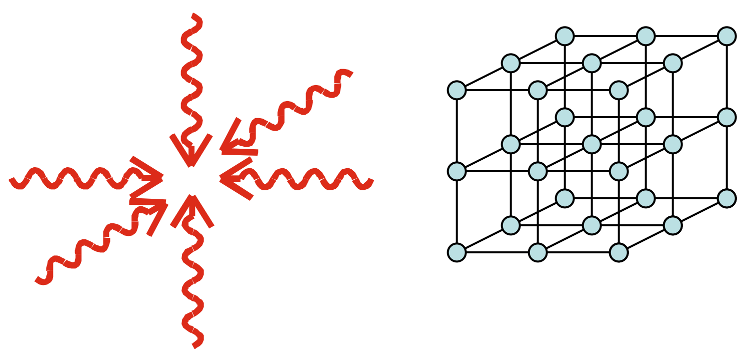


1D gases

2D:

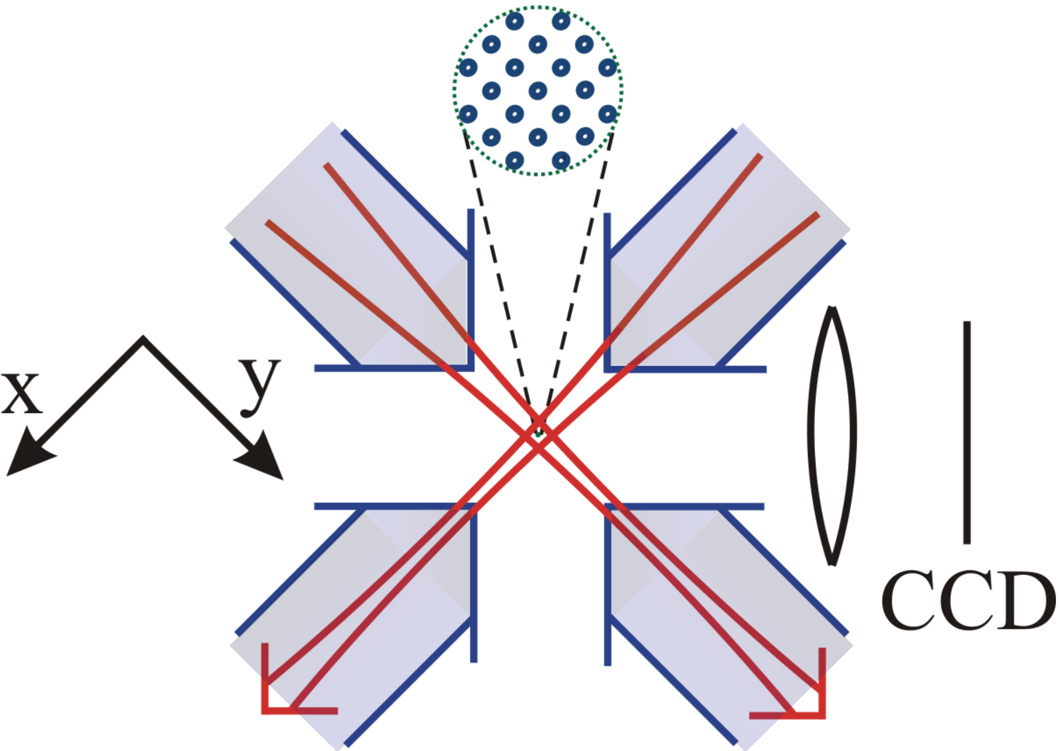


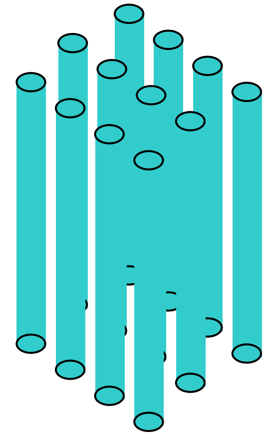
3D:



16µK

Bundles of 1D Systems

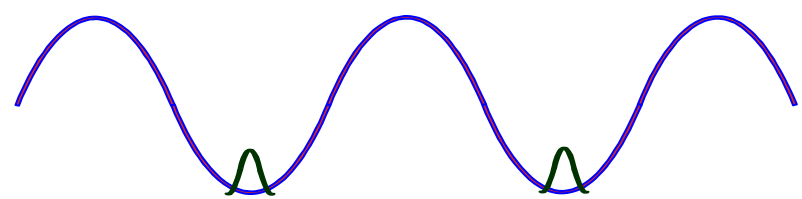




50-300 atoms/tube

1000-8000 tubes

aspect ratio
150 ~ 700



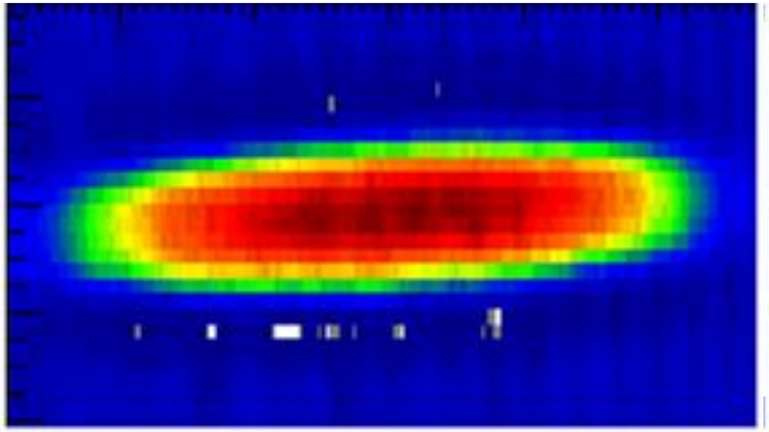
For 1D: negligible tunneling;
all energies << $\hbar\omega_\perp$

So $\gamma\uparrow$ when the lattice power $\uparrow$

Expansion in the 1D tubes

0 ms

7 ms

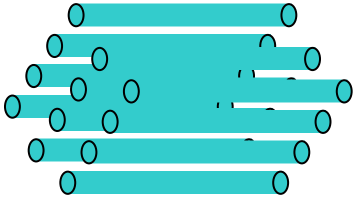


17 ms

← up

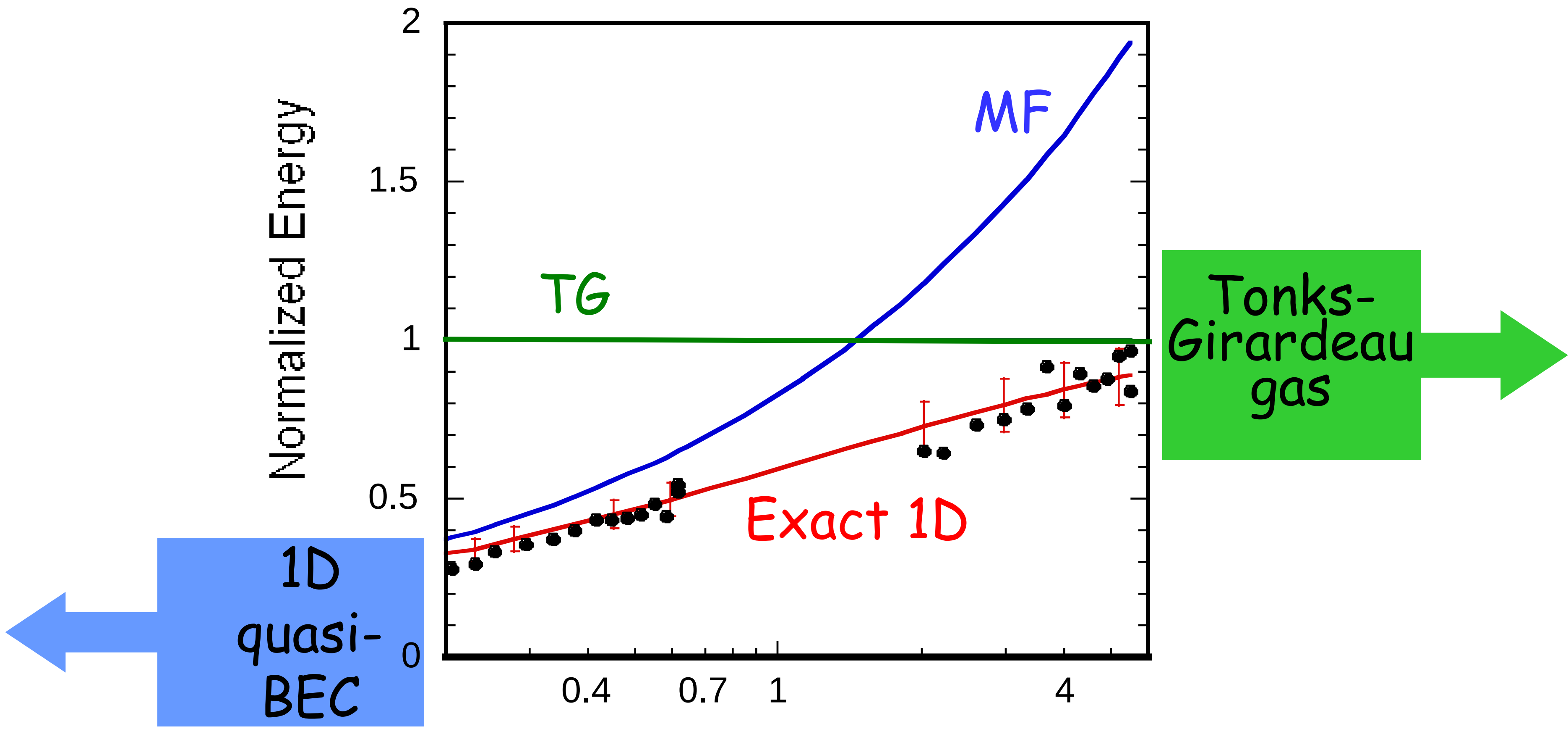
50-300 atoms/tube

1000-8000 tubes



aspect ratio 150 ~ 700

Family of curves parameterized by γ



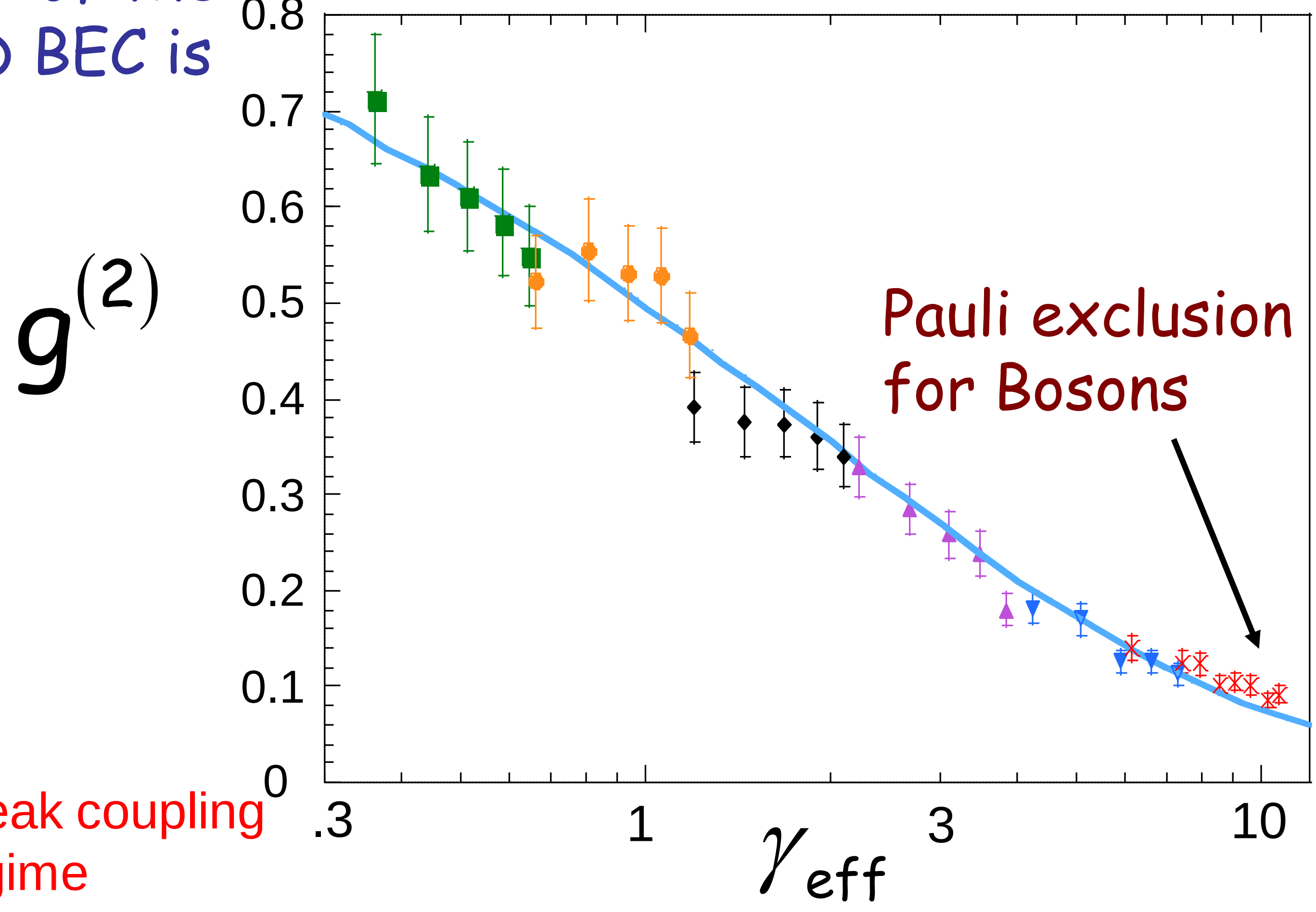
weak coupling γ strong coupling

Normalized Local Pair Correlations

By photo-association Theory: Gangardt & Shlyapnikov, PRL **90** 010401 (2003)

Expt: Kinoshita, Wenger, DSW, PRL **95** 190406 (2005)

$g^{(2)}$ of the 3D BEC is 1.

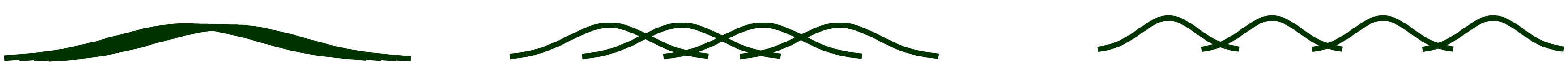


Strong coupling regime

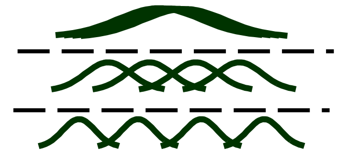
Fermionized Bosons !

$g^{(3)}$, higher order correlation also decreases

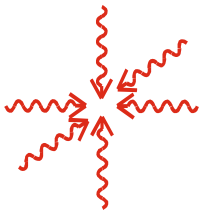
Weak coupling regime



Outline

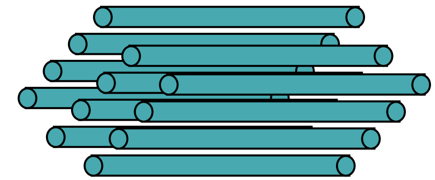


1D Bose gas theory



Equilibrium 1D Bose gas experiments

- Total energy
- 1D Cloud Size
- Local Pair Correlations



強相関系の物理

Non-Equilibrium 1D Bose gas experiments

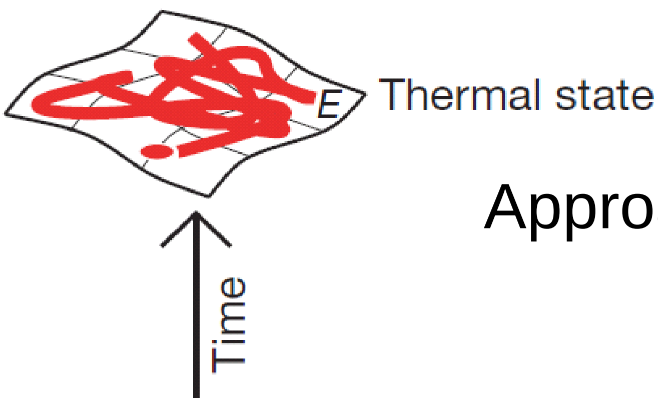
- the Quantum Newton's cradle

可積分系、非平衡物理

Collisions in 1D

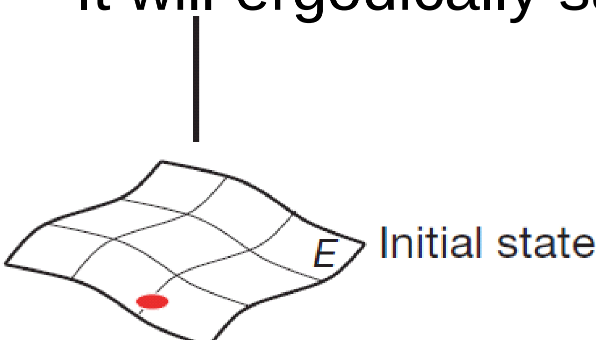
For identical particles, reflection looks just like transmission !

→ Two-body collisions between distinct bosons cannot change their momentum distribution.



Approach to a Thermal Equilibrium

It will ergodically sample the entire phase space (E = const.)

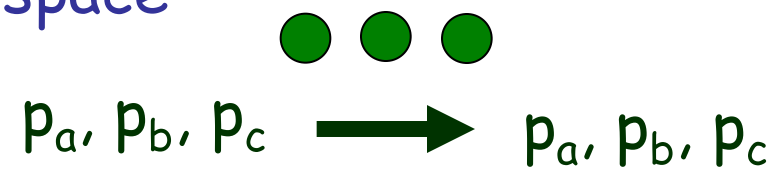


Integrable systems never reach a thermal equilibrium (too many constrains)

Does a Real 1D Gas Thermalize?

1D Bose gases with δ-fn interactions are integrable systems ➔ they do not:

- ergodically sample phase space
- ≈ become chaotic
- ≈ thermalize



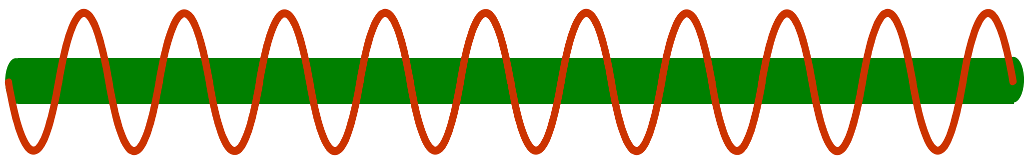
Thermalization in a real 1D Bose gas has been a somewhat open question.

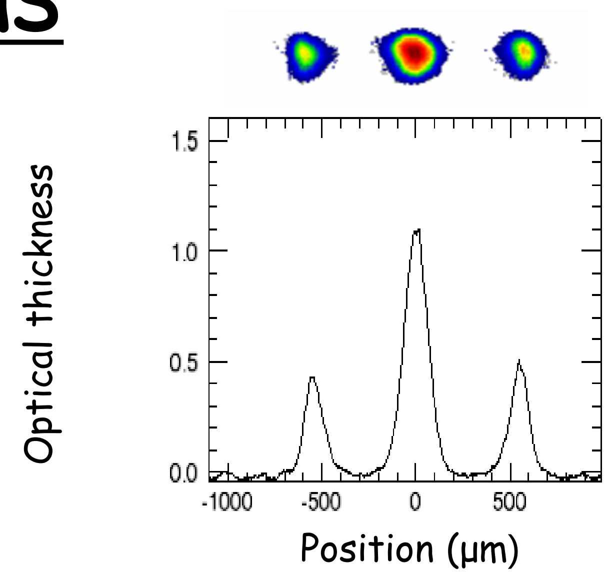
Do imperfectly δ-fn interactions lift integrability enough to allow the atoms to thermalize?

Do longitudinal potentials matter?

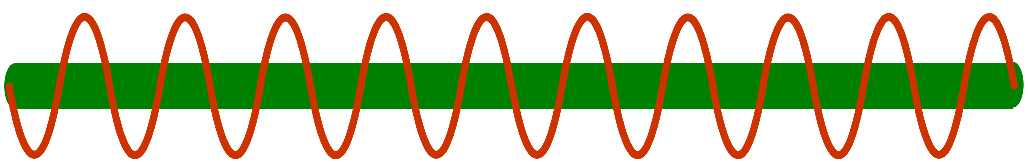
Procedure: take the 1D gas out of equilibrium and see how it evolves.

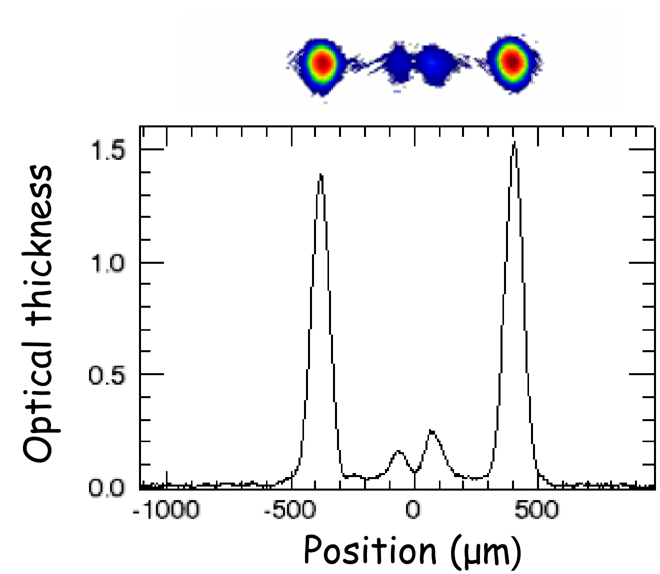
Creating Non-Equlibrium Distributions





1 standing
wave pulse

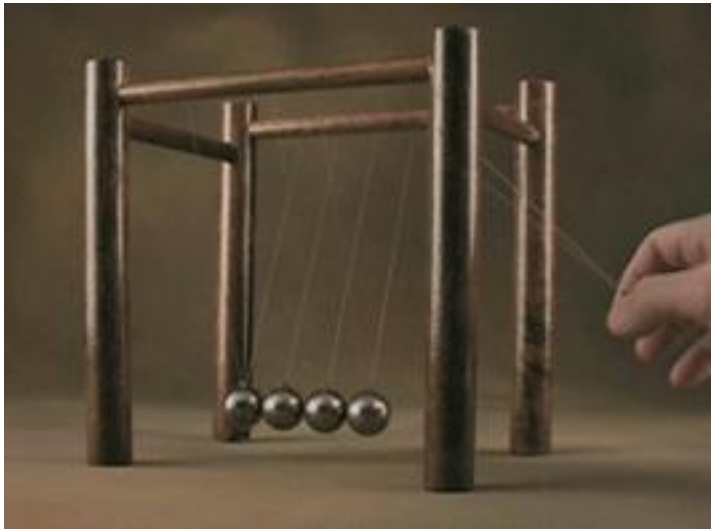




2 standing
wave pulses

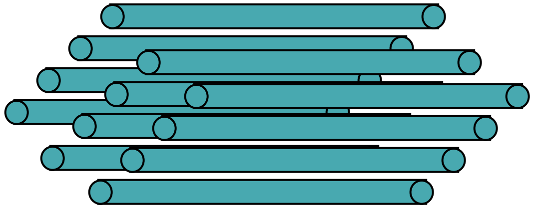
Wang, et al., PRL **94**, 090405 (2005)

Harmonic Trap Motion



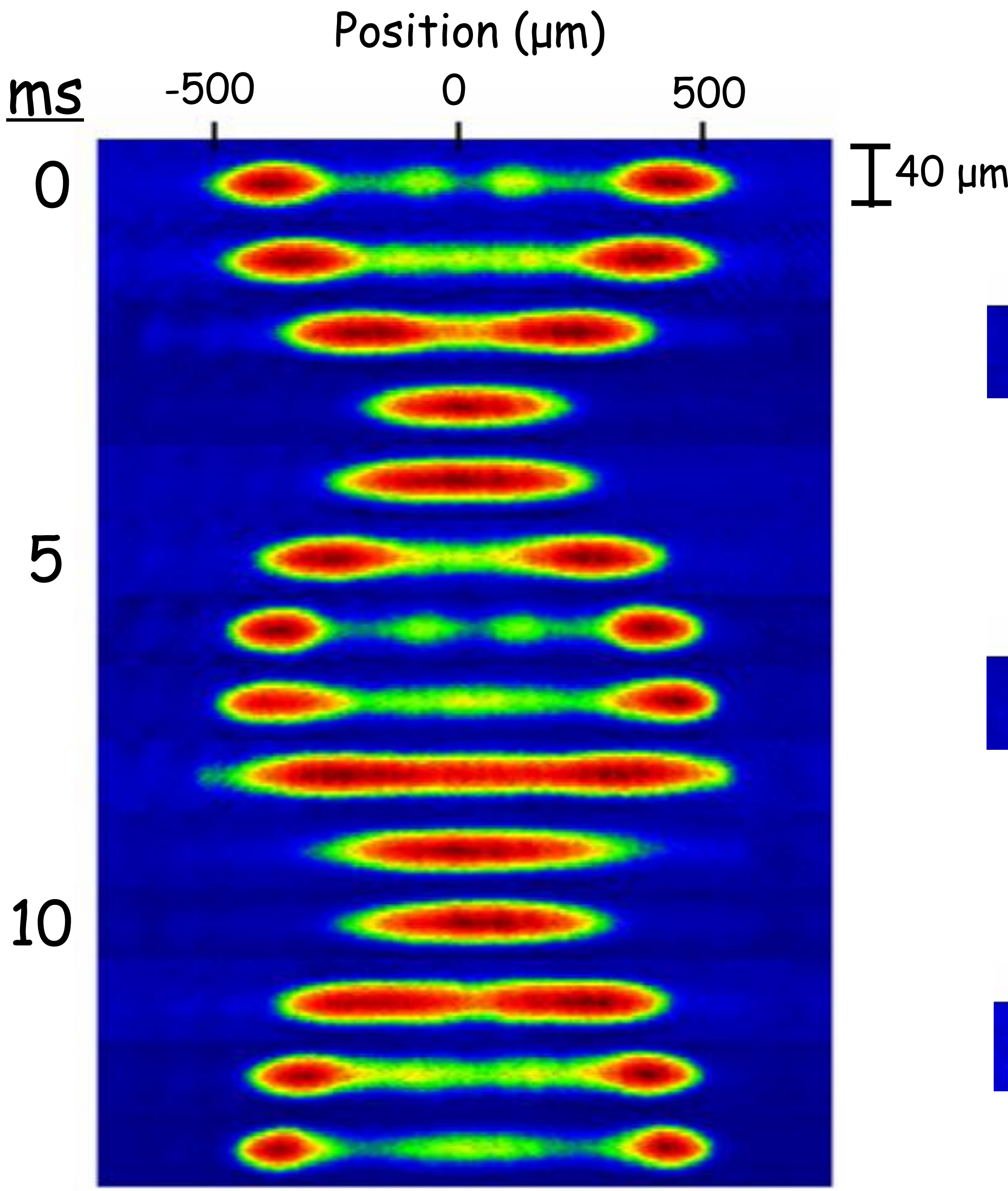
A classical Newton's cradle

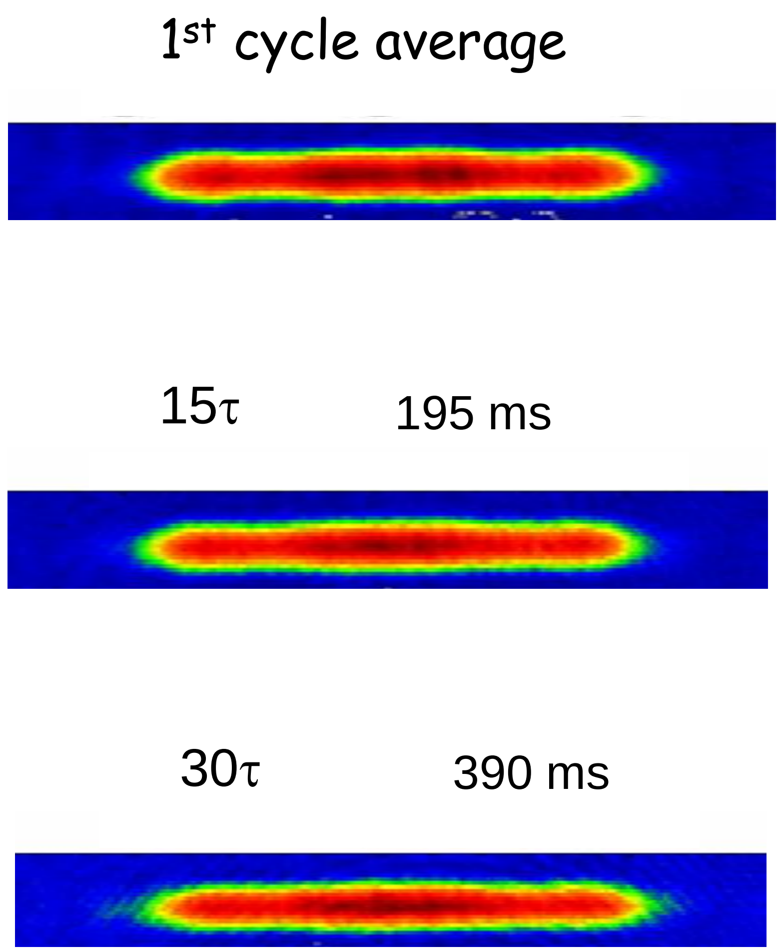
We make thousands of parallel **quantum Newton's cradles**, each with 50-300 oscillating atoms.



1D Evolution in a Harmonic Trap

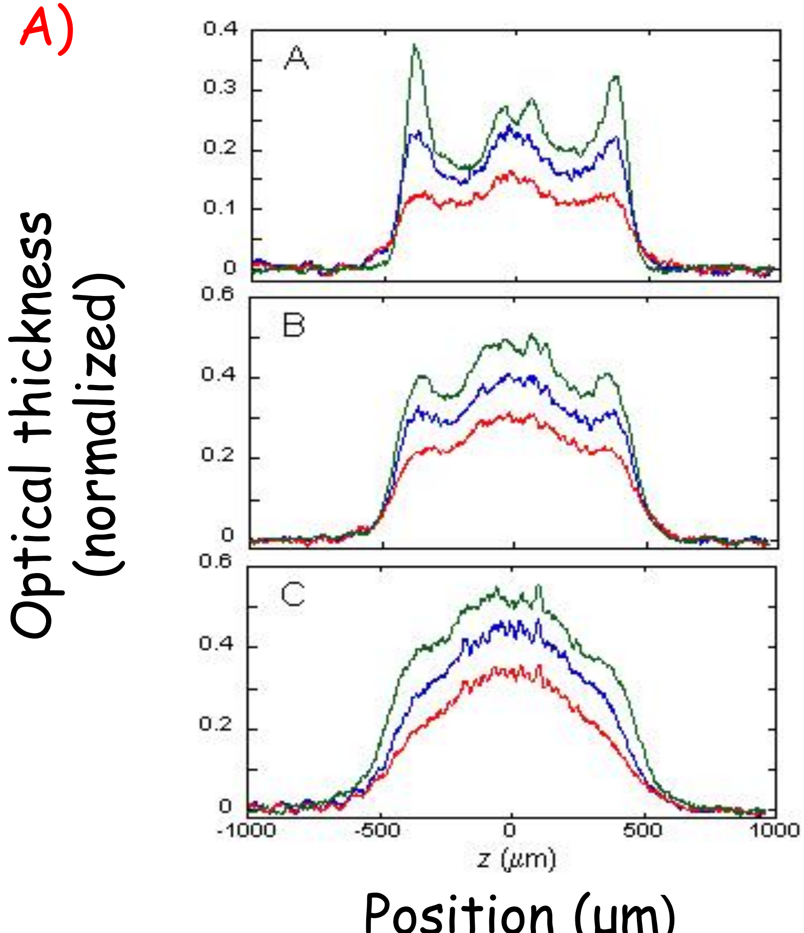
Kinoshita, Wenger, Weiss
Nature **440**, 900 (2006)





Dephased Momentum Distributions

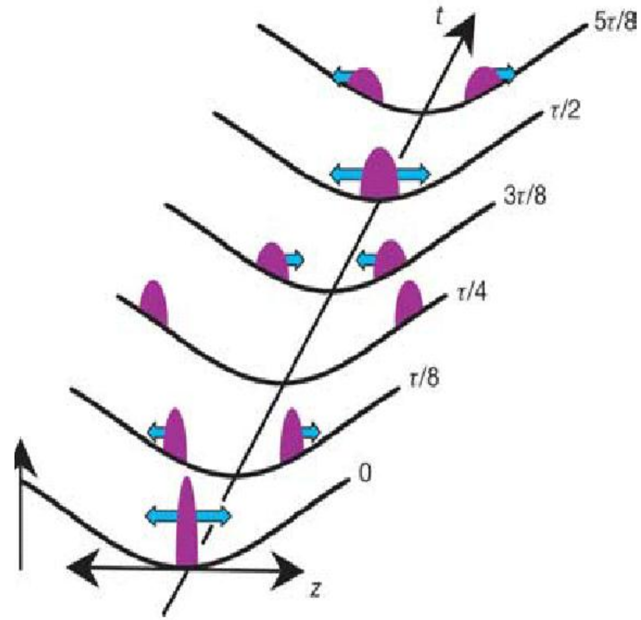
1st cycle average
15τ distribution
40τ distribution
(30τ in *A*)



γ=18

γ=3.2

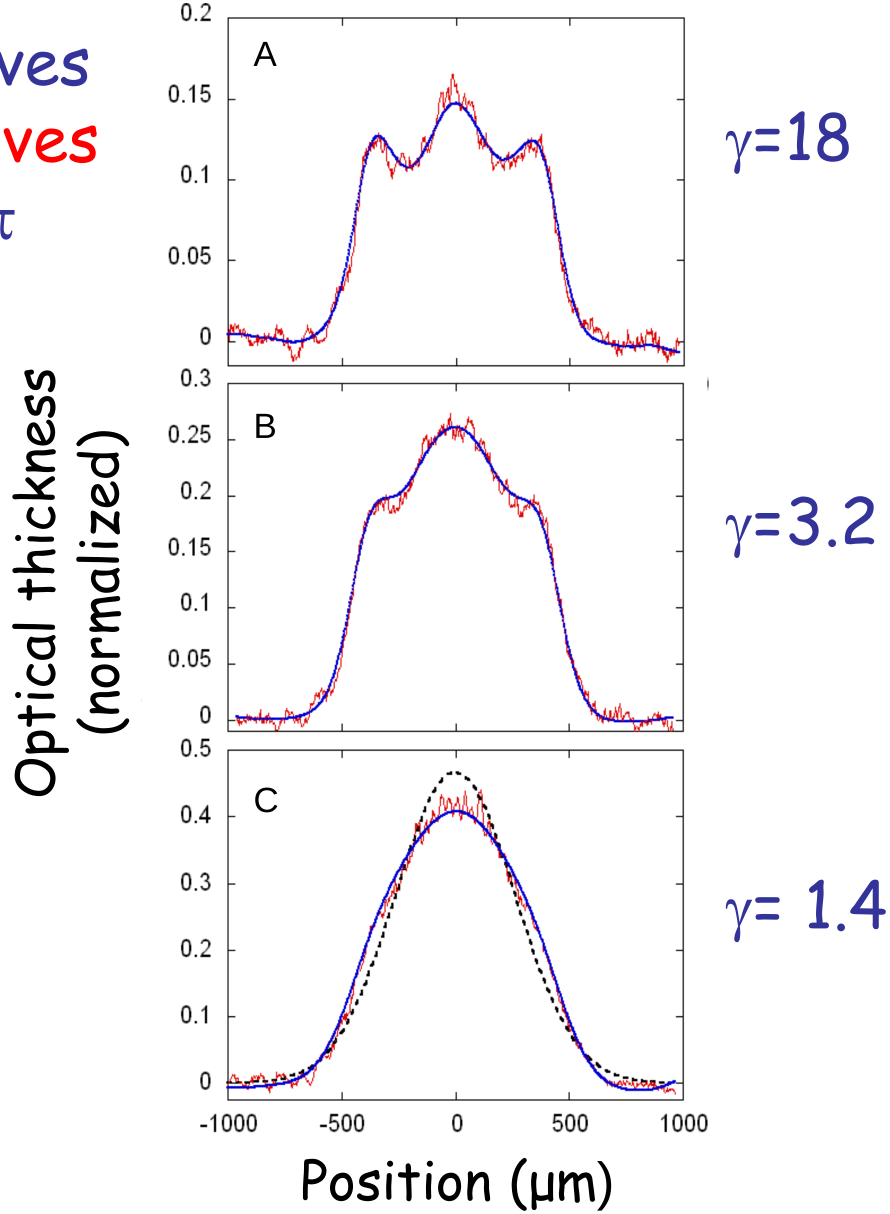
γ= 1.4



Project the evolution

Negligible Thermalization

Projected curves and actual curves at 30 τ or 40 τ

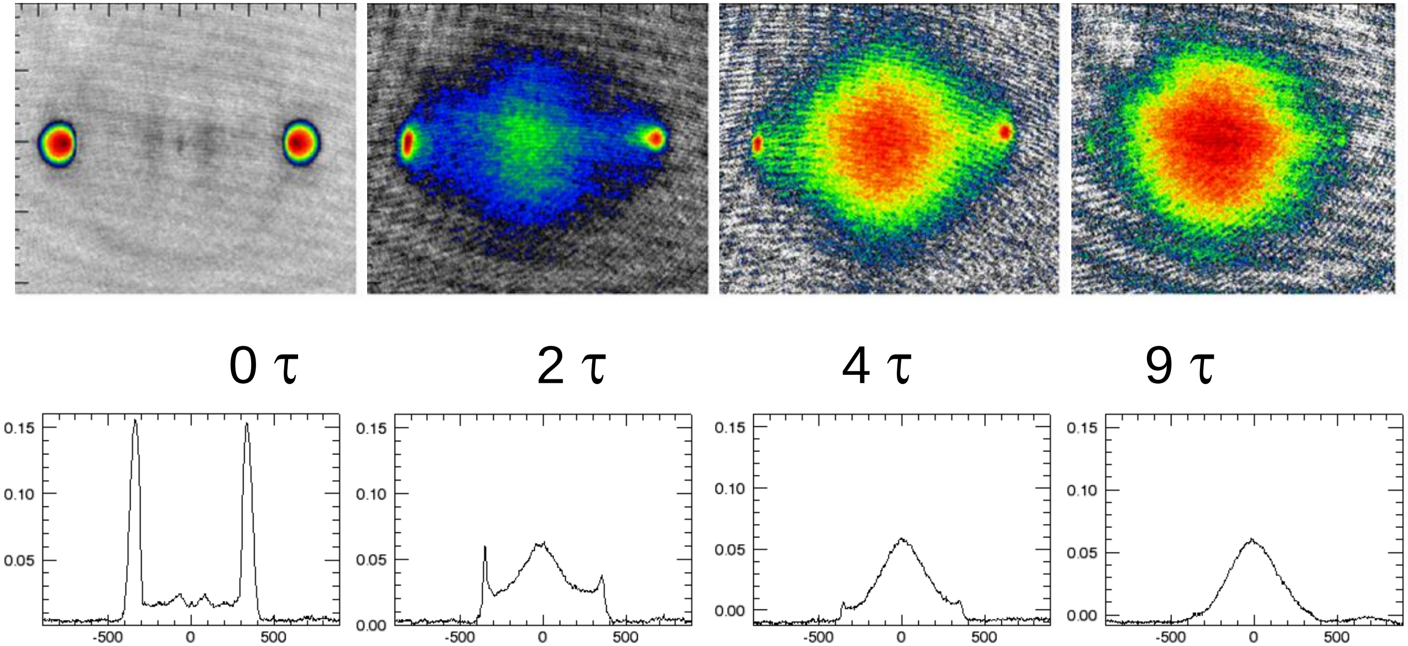


After dephasing, the 1D gases reach a steady state that is not thermal equilibrium

Each atom continues to oscillate with its original amplitude

What happens in 3D?

Thermalization occurs in ~3 collisions.



These collisions occur well above the Landau critical velocity for the 3D BEC.

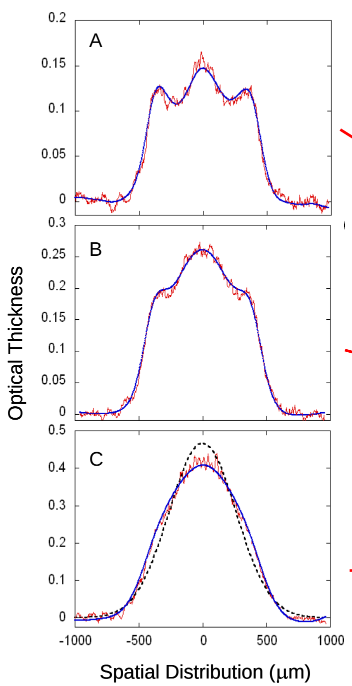
How many collisions have occurred ?

N_{tube} collisions per cycle.

$R \approx (2ka_{1D})^{-2} = \frac{1}{22}$ for $2\hbar k$ collisions

Olshanii, *PRL* **81**, 938 (1998)

We set lower limits to the number of reflections required for thermalization



>710

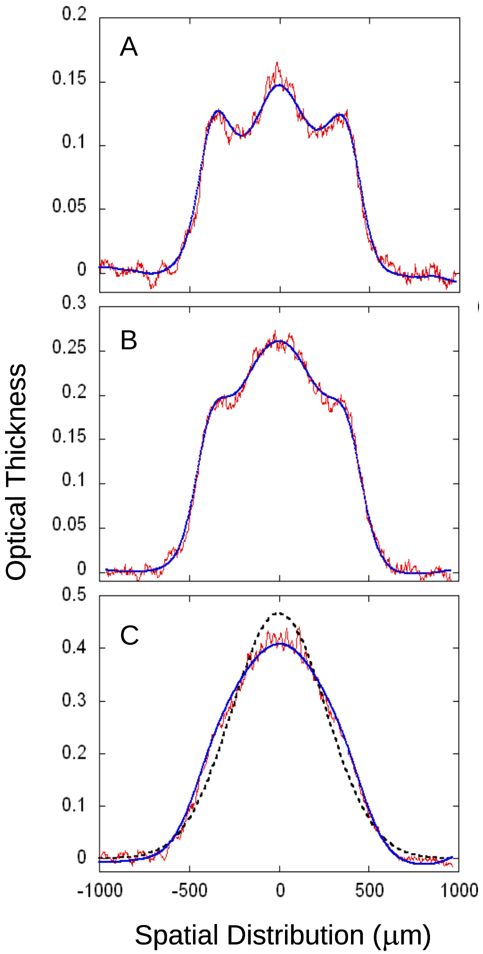
>9600 >>3

>2300

Lack of thermalization

This many-body 1D system is nearly integrable.

Lack of Thermalization



初期に与えられた、平衡から大きく離れた運動量分布を再分布させる機構が存在しない。

軸方向の弱いトラップポテンシャルは可積分性を崩すものの、熱平衡を引き起こすほどには十分でない。

This many-body 1D system is nearly integrable.

A New Type of Experiment : Direct Control of Non-Integrability

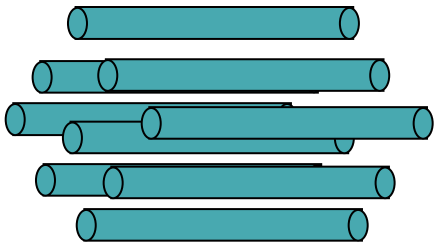
Is there a non-integrability threshold for thermalization?

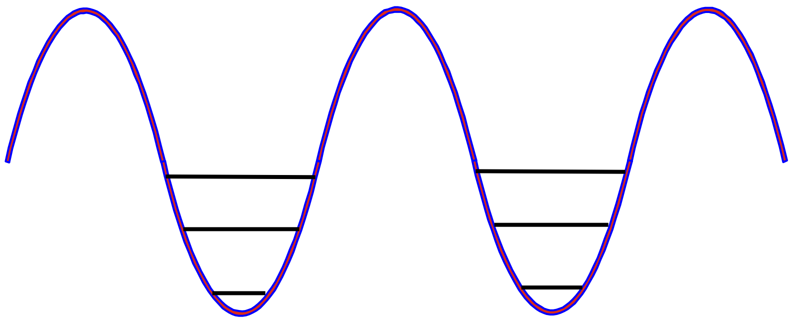
The classical KAM theorem shows that if a non-integrable system is sufficiently close to integrable, it will not ergodically sample phase space.

Is there a quantum mechanical analog?

Procedure:

controllably lift integrability and measure thermalization.

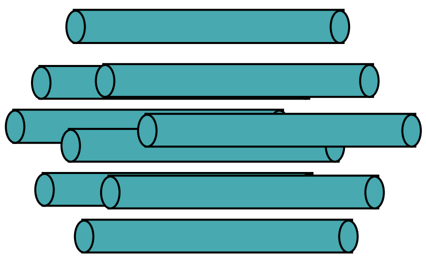


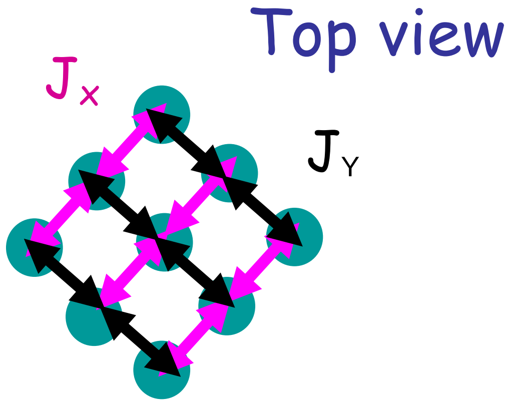


Ways to lift integrability

Allow tunneling among tubes (1D ➔ 2D and 3D behavior);
Finite range 1D interactions; Add axial potentials

Making 1D gases thermalize

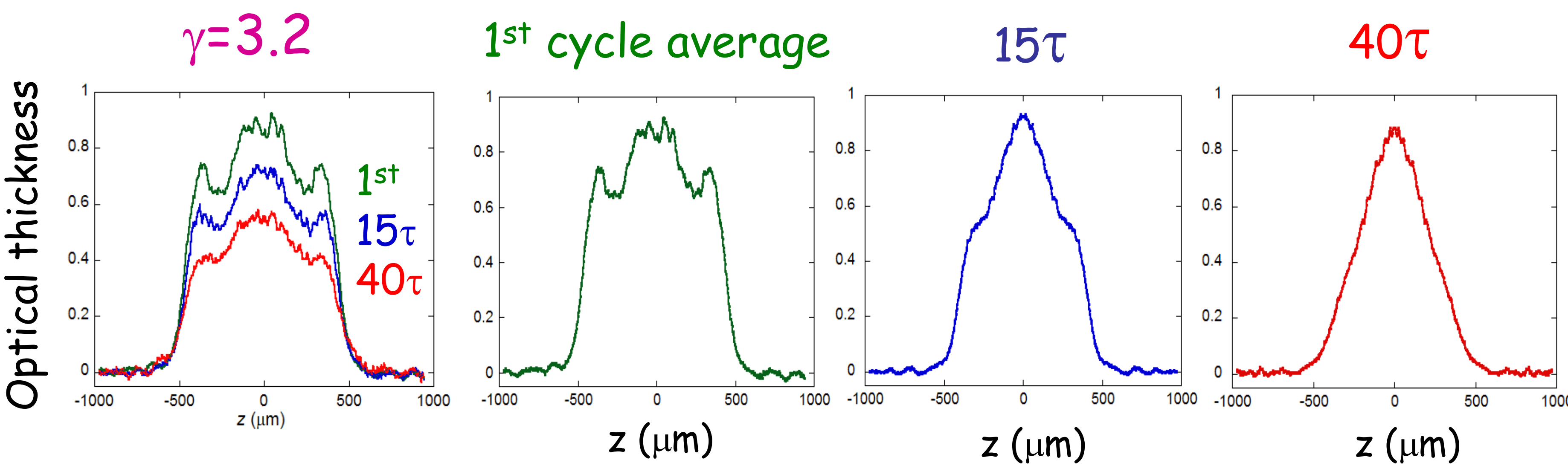




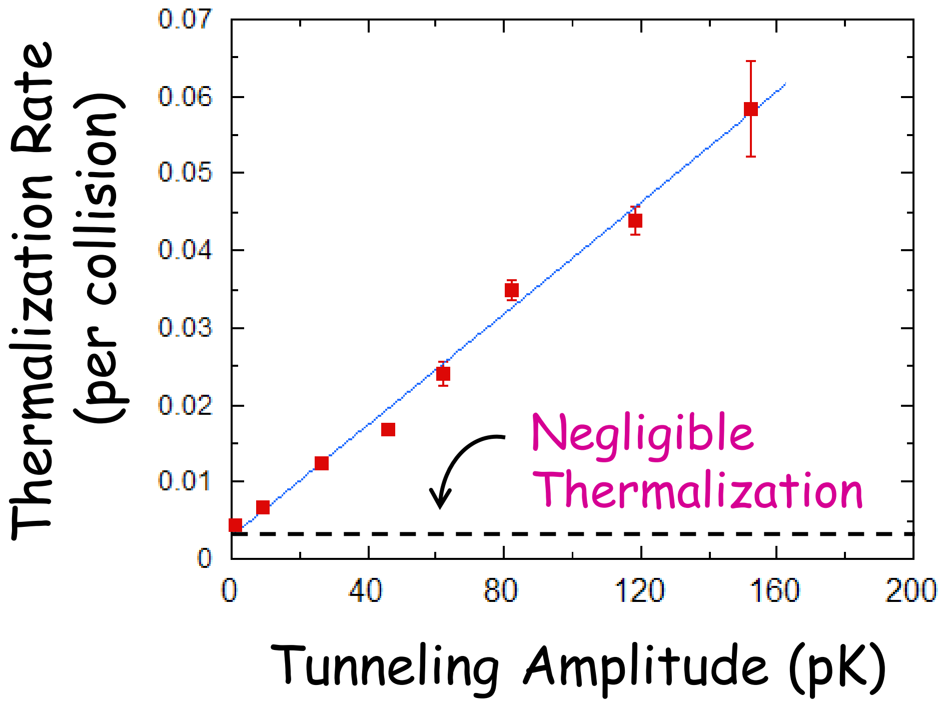
Allow tunneling among tubes ⇒ 1D ➔ 2D and 3D behavior

$U_x = U_Y = 16\mu K$

e. g. $U_x = U_Y = 4\mu K$



Controlled non-integrability



What is the final state ?

Detailed studies are in progress ....

熱平衡化が始まる閾値はあるのか？

最終的な状態はカノニカル分布か

The approach to equilibrium can be controlled

At the moment we have found;

1) In the strong coupling region where 3-body collisions are suppressed,

   Thermalization is negligible (≒ Integrable System)

2) In a very week coupling regime, 2-body collisions excite the atoms to the second vibrational states and the following tunneling makes thermalization (thermalization time is proportional to the tunneling rate).

3) In the intermediate regime, the momentum distribution changes, but the initial memory is partially preserved.

These Experiments stimulate theoretical studies ...

Control Non-Equilibrium process

Understanding of Non-Equilibrium Dynamics is very important for Condensed Matter Physics and Statistical Physics

Integrable System + Perturbation to control dynamics

1D Bosons (ongoing project)
1D Fermions (p-wave of Hard Core particles)
Fermions on a Lattice (Fermi and Habbard)

Non-Integrable system, but some constrains

what a kind of constrains, magnitude
how to lift integrability

quenched by suddenly changing parameters

Cold Atom Experiments provide nice stages to study non-equilibrium dynamics.

Non-Equilibrium Process
(Isolated Quantum System)

1. *Are there Quantum analogs of Classical KAM Theory ?*

2. *What are the Quantum Effects ?*

Quantum Fluctuation, Correlations,,,,

FPU experiment in Quantum regime using 1D Bosons in the Optical Lattice is proposed by ;
I. Danshita, R. Hipolito, A. Polkovnikov, and V. Oganesyan,

Non-Equilibrium Process
(Isolated Quantum System)

3. *量子系特有のダイナミクス*

- <u>Non-integrable systems</u>

Thermalization is expected to occurr at the level of individual eigenstates (*Eigenstate Thermalization Hypothesis*)

---> expectation value of few-body observable in a given eigenstate with energy *E* equals the *microcanonical average* at the mean energy *E*

Deutsch, *PRA (1991)*
Srednicki, *PRE (1994)*
Rigol, Dunjko, & Olshanii, *Nature (2008)*

Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009)*, *PRA (2009)*
.....

4. *Integrable system may relax to a steady state (not a thermal equilibrium, but something else)*

- <u>Integrable systems</u>

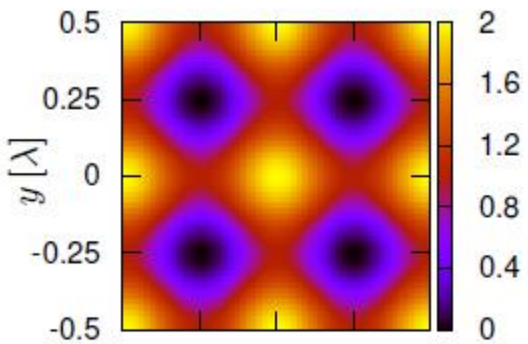
Thermalization does **<u>not</u>** occur: steady states usually carry memory of the initial conditions and are not canonical (*generalized Gibbs ensembles*)

Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)*
Rigol, Muramatsu, & Olshanii, *PRA (2006)*

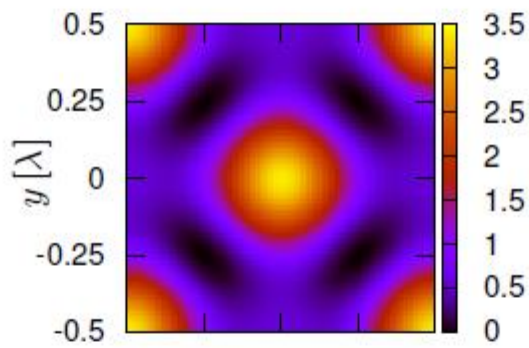
Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006)*, *JSTAT (2007)*
Gangardt & Pustilnik, *PRA (2008)*
Eckstein *& Kollar, PRL (2008)*, *PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
.....

アンチドット型光格子

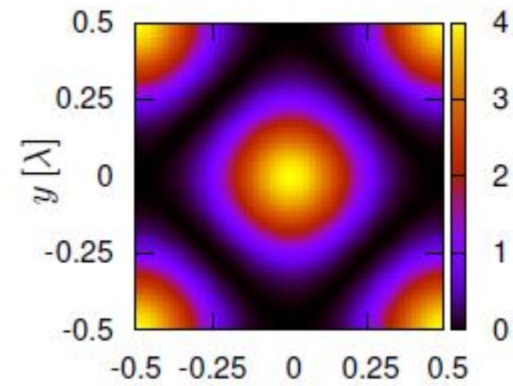
$$U(x,y)=U_0[\cos^2(kx)+\cos^2(ky)\ {\color{red}+2\cos(kx)\cos(ky)\cos\Phi}]$$



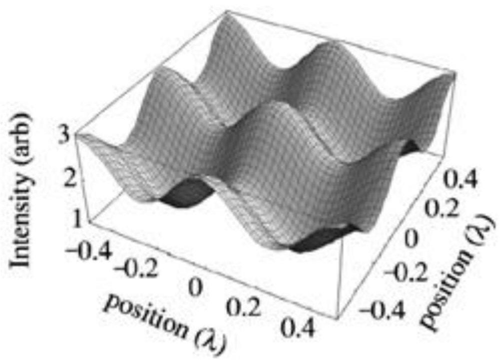
Φ=π/2

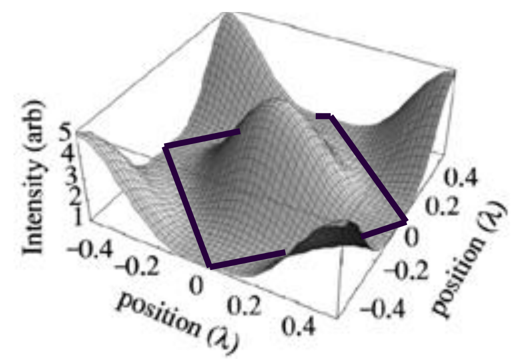


Φ=π/4



Φ=0

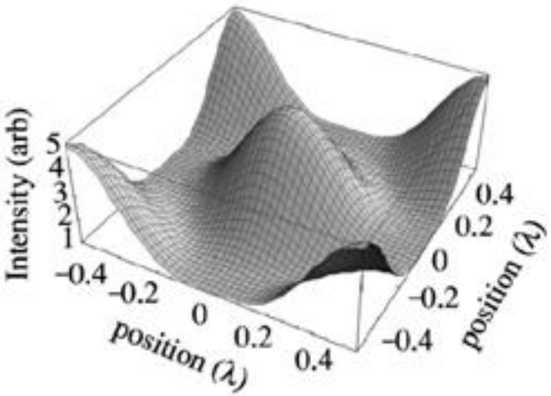


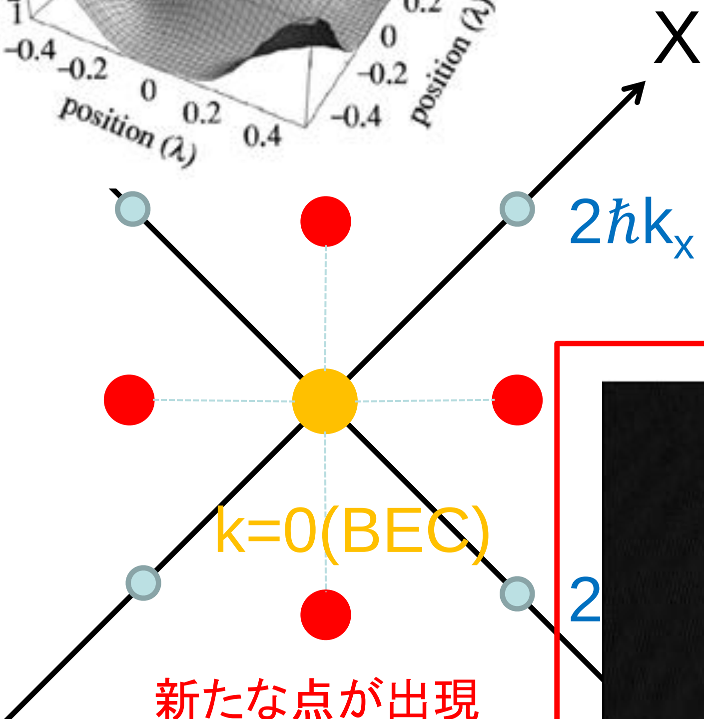


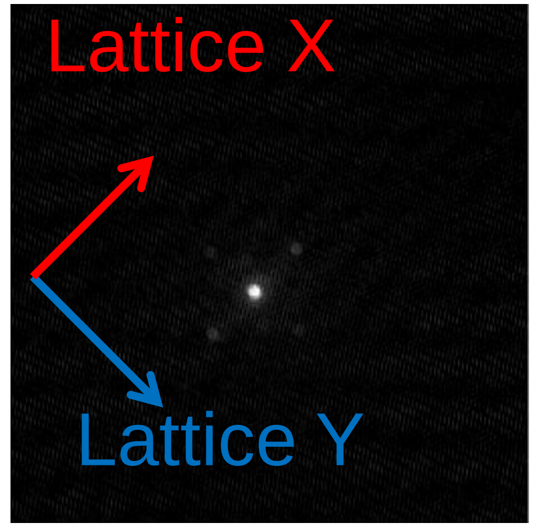
アンチドット型光格子

相対位相Φ=0の場合、
互いのポテンシャル極小が連結

光格子と原子波干渉パターン

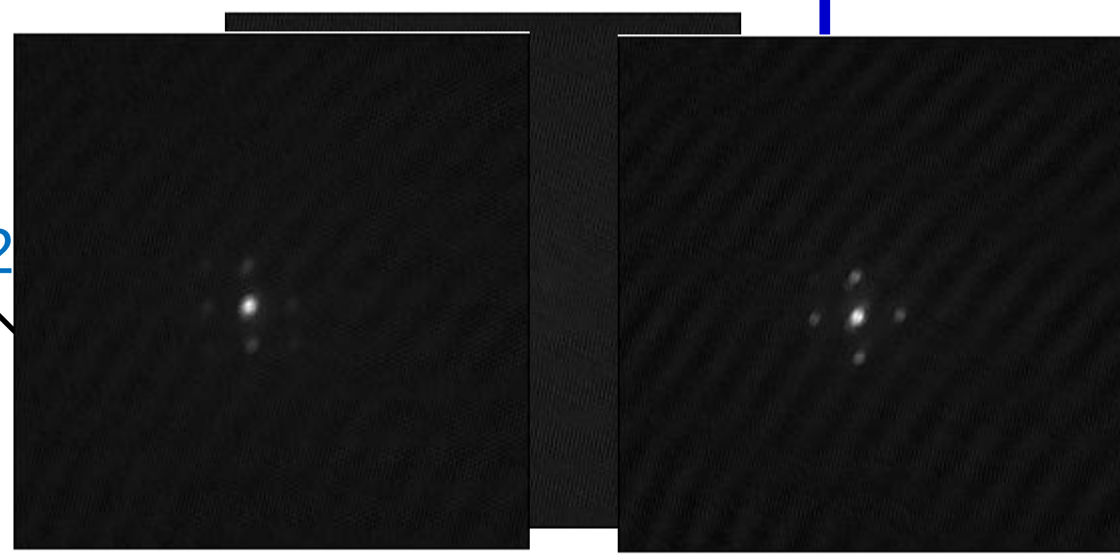






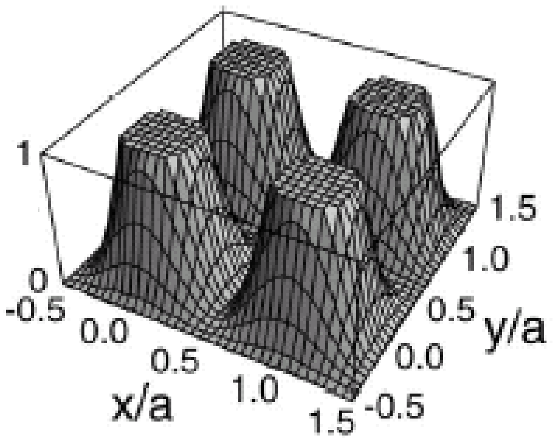
XYで周波数差のある通常の光格子
($U=6.5E_{rec}$) TOF25ms

同様のパターン



Φ=0に近い→アンチドット型光格子を形成している

Quantum Gases Flowing in 2D Anti-Dot Lattices……



Quantum Chaos
(Billiard of Quantum Gas)

Quantum Turbulence

Non-Equilibrium Phenomena

Control Non-Equilibrium process

Understanding of Non-Equilibrium Dynamics is very important for Condensed Matter Physics and Statistical Physics

Integrable System + Perturbation to control dynamics

1D Bosons (ongoing project)
1D Fermions (p-wave of Hard Core particles)
Fermions on a Lattice (Fermi and Habbard)

Non-Integrable system, but some constrains

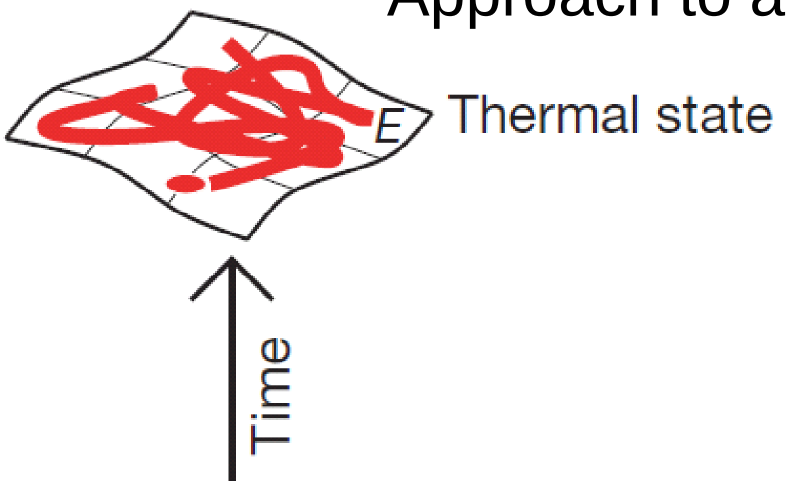
what a kind of constrains, magnitude
how to lift integrability

quenched by suddenly changing parameters

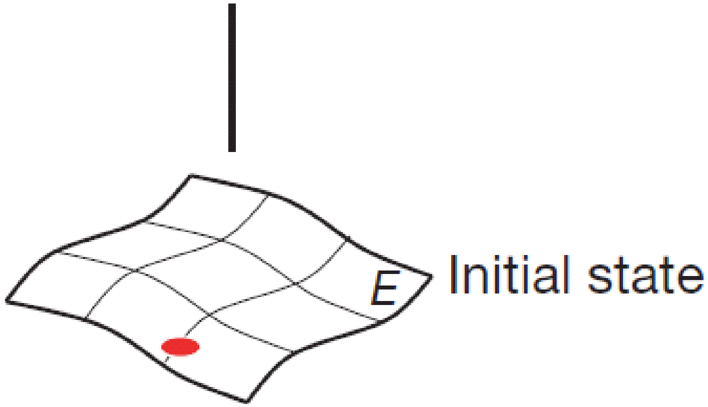
Cold Atom Experiments provide nice stages to study non-equilibrium dynamics.

Non-Equilibrium Process (Classical System)

Approach to a Thermal Equilibrium



It will ergodically sample the entire phase space (E = const.)



Integrals of Motions (conserved quantities) other than

the energy strongly restrict the sampling regions.

Integrable systems never reach a thermal equilibrium (too many constrains)

KAM Theory:

There is a threshold above which the interaction breaking Integrability makes systems thermalization

Non-Equilibrium Process (Isolated Quantum System)

- <u>Non-integrable systems</u>

Thermalization is expected to occurr at the level of individual eigenstates (*Eigenstate Thermalization Hypothesis*)

---> expectation value of few-body observable in a given eigenstate with energy *E* equals the *microcanonical average* at the mean energy *E*

Deutsch, *PRA (1991)*
Srednicki, *PRE (1994)*
Rigol, Dunjko, & Olshanii, *Nature (2008)*

Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009)*, *PRA (2009)*
.....

- <u>Integrable systems</u>

Thermalization does **<u>not</u>** occur: steady states usually carry memory of the initial conditions and are not canonical (*generalized Gibbs ensembles*)

Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)*
Rigol, Muramatsu, & Olshanii, *PRA (2006)*

Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006)*, *JSTAT (2007)*
Gangardt & Pustilnik, *PRA (2008)*
Eckstein *& Kollar, PRL (2008)*, *PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
.....

They may relax to a steady state
(not a thermal equilibrium, but something else)

1次元ボゾン

Theory

- Exactly solvable from weakly interacting
  to strongly correlated regimes
- **Integrable system**

Experiment

- Better understanding of strongly correlated system
  for condensed matter physics,
  for atom entanglement schemes
- Direct comparison to Theory
- Test ground for other (more complicated) correlated systems
  fundamental properties
  method to extract correlation properties
- **Process from Non-equilibrium states**

1次元ボゾン

Theory

- Exactly solvable from weakly interacting
  to strongly correlated regimes
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  method to extract correlation properties
- Process from Non-equilibrium states

Maximizing Entropy $S = k_B \mathrm{Tr}[\rho \ln(1/\rho)]$

*Rigol, Dunjko, Yurovsky and Olshanii, PRL, **98**, 050405 (2007)*

Grand Canonical Distribution →

Thermal equilibrium

$$\hat{\rho} = Z^{-1} \exp\left[-\left(\hat{H} - \mu \hat{N}_b\right)/k_B T\right]$$

$$Z = \mathrm{Tr}\left\{\exp\left[-\left(\hat{H} - \mu \hat{N}_b\right)/k_B T\right]\right\}$$

$$E = \mathrm{Tr}\left\{\hat{H}\hat{\rho}\right\}, \quad N_b = \mathrm{Tr}\left\{\hat{N}_b \hat{\rho}\right\}$$

For Integrable system

Maximize entropy S, subject to the constrains imposed by a full set of conserved quantities.

Constrained equilibrium

$$\hat{\rho}_c = Z_c^{-1} \exp\left[-\sum_m \lambda_m \hat{I}_m\right]$$

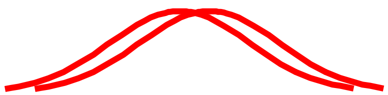
$$Z_c = \mathrm{Tr}\left\{\exp\left[-\sum_m \lambda_m \hat{I}_m\right]\right\}$$

$$\langle \hat{I}_m \rangle (t=0) = \mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_c\right\}$$

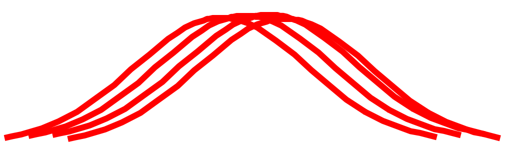
Generalized Gibbs ensemble with many Lagrange multipliers.

How to make strongly correlated system with a dilute gas.....

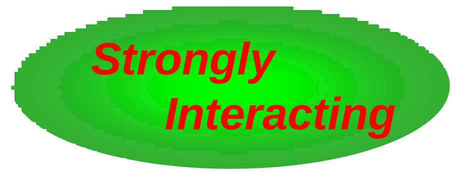
$E_{kinetic}$ ~~vs~~ $E_{interaction}$

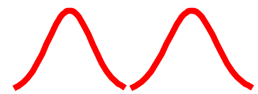


重なりによる相互作用
エネルギーの上昇

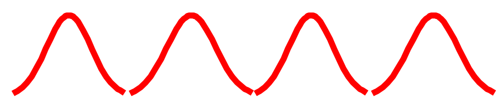


平均場近似





局在による力学的
エネルギーの上昇



強相関系

Many Questions remain unsolved;

- ETH has a finite threshold value for breakdown
  above which thermalization starts to occur ?

- Validity of Generalized Gibbs Ensemble

  which observables ?

  under the conditions that what a kind of constrains
  (conserved quantities) exist ?

- "Partial Thermalization"
  - keep some memory of the initial states -

1D Quantum Gas is probably the best test bench for

Studying Non-equilibrium quantum dynamics

(eventually) Controlling the dynamics

Tools;

Perturbations for breaking Integrability

tunneling, periodic potentials, mixture of different atoms….

System Size (Number of atoms), Interaction strength, Observables

Degree of Non-equilibrium (initially quite far from equilibrium)

Summary

Non-equilibrium 1D Bose gases: quantum Newton's cradle.
Independent δ-int. 1D Bose gases do not thermalize!

- Relaxed conditions allow 1D Bose gases do thermalize.

We have a theory to test.

We can also lift integrability in other ways. Is there universal behavior?