$K^{bar}N$ system studied with coupled-channel Complex Scaling Method

A. Doté (KEK Theory center / IPNS/ J-PARC branch)
T. Inoue (Nihon univ.)
T. Myo (Osaka Inst. Tech. univ.)

1. **Introduction**

2. **Scattering problem solved with Complex Scaling Method**
   - *Formalism / Test calculation with AY $K^{bar}N$ potential*

3. **Set up for the calculation of $K^{bar}N$-πY system**
   - *Kinematics / Non-rela. approximation*

4. **Results**
   - *I=0 channel: Scattering amplitude / Resonance pole*
   - *I=1 channel: Scattering amplitude*

5. **Summary and future plans**

arXiv:1207.5279

基研研究会「ハドロン物質の諸相と状態方程式 ---中性子星の観測に照らして---」
'12.08.29-09.01 @ YITP, Kyoto, Japan

1. Introduction

1. Introduction

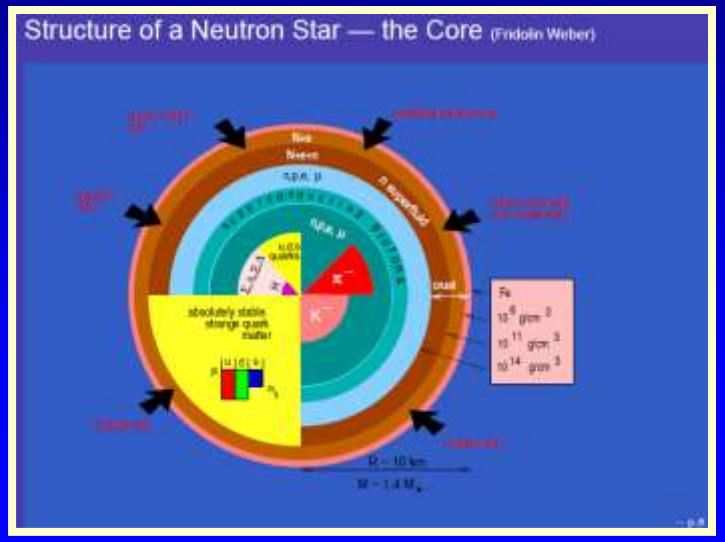
Neutron star in universe

Radius ~ 10km
Mass ~ M_{sun}

Maybe …

High dense at core ~ > 2 ρ_0

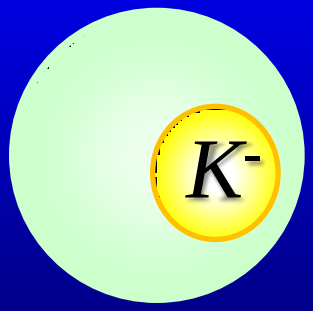
Involving strangeness



K^{bar} nuclear system on the Earth

… light nucleus with K^- mesons

✓ *Involving strangeness via K^- meson*



- *Self-bound K^{bar}-nuclear system*

1. Introduction

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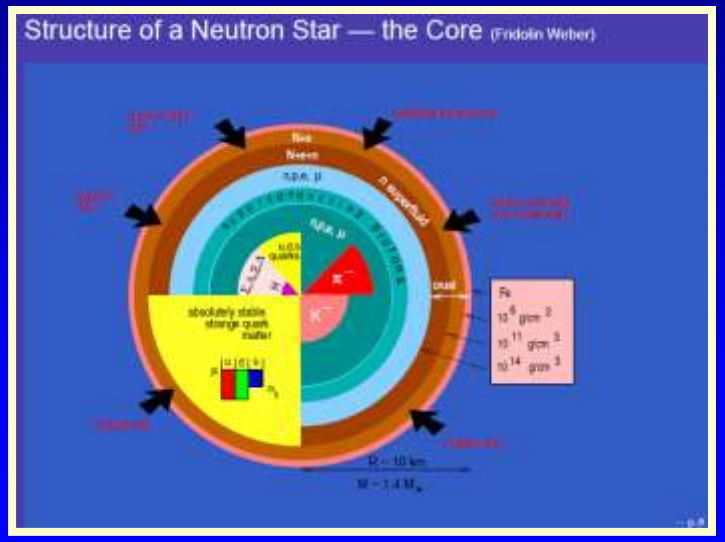
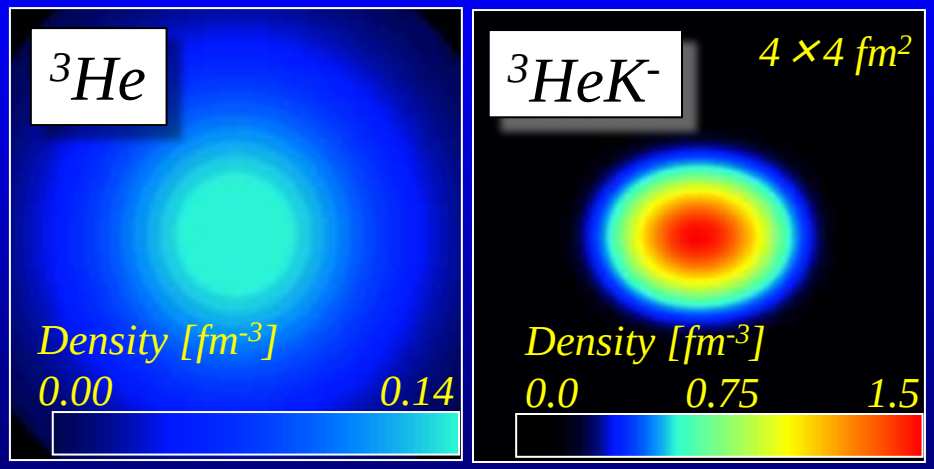
Involving strangeness



K^{bar} nuclear system on the Earth



… light nucleus with K^- mesons

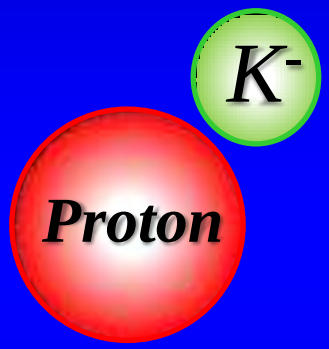
✓ Involving strangeness via K^- meson

Strong attraction by K^- meson
⇒ **Dense** nuclear system

A phenomenological $K^{bar}N$ potential†
⇒ B(K) ~ 100MeV, **2~4ρ_0**

† A. D., H. Horiuchi, Y. Akaishi and T. Yamazaki, PLB 590 (2004) 51; PRC 70 (2004) 044313.

These days, K^-pp has been focused in both of theoretical and experimental studies!



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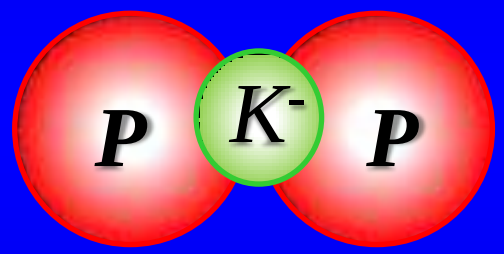
$\Lambda(1405)$



Most essential kaonic nucleus!

$^3HeK^-$, $pppK^-$,
$^4HeK^-$, $pppnK^-$,
…
$^8BeK^-$,
…

Complicated nuclear system with K^-

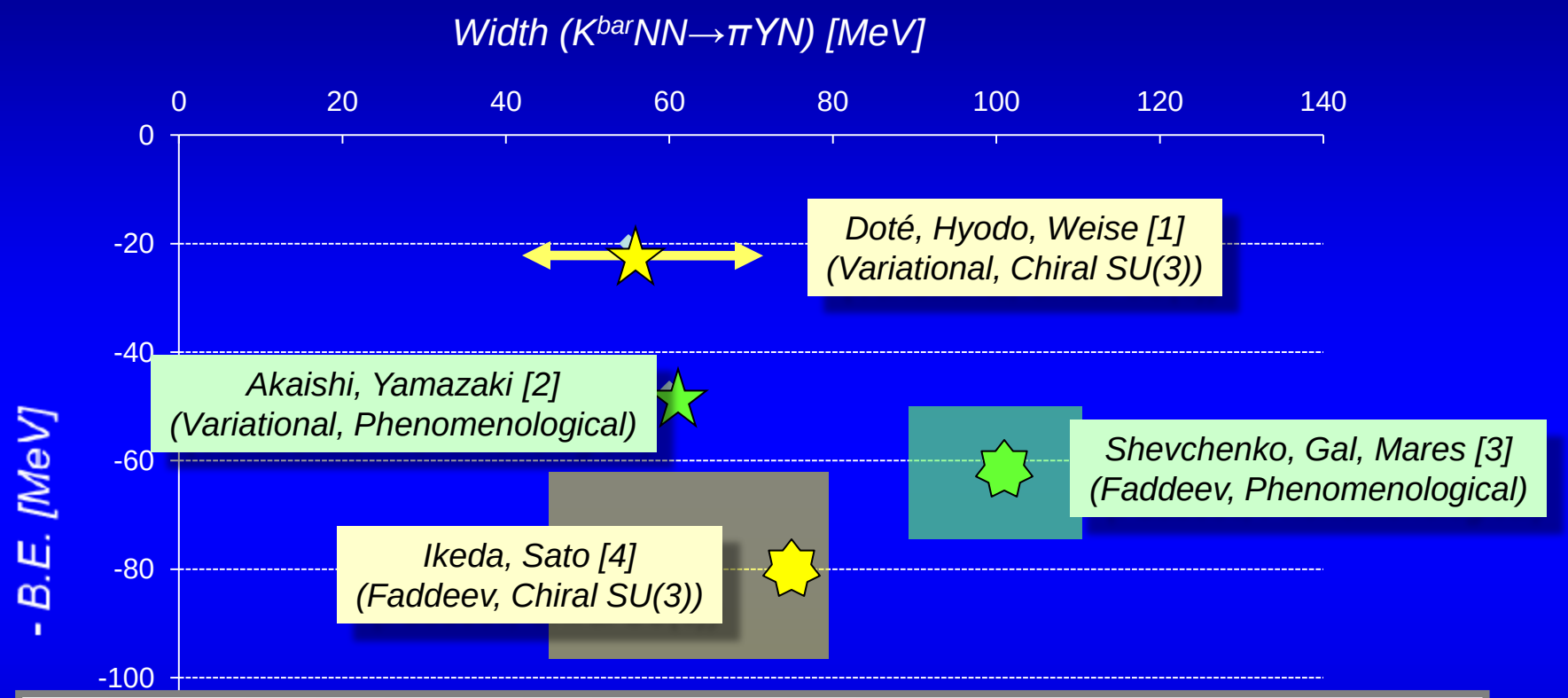
Theoretical studies of K^-pp

Since K^-pp is a three-body system,
it can be studied with various methods:

- *Doté, Hyodo, Weise* — *PRC79, 014003(2009)*
  Variational with a chiral SU(3)-based $K^{bar}N$ potential
- *Akaishi, Yamazaki* — *PRC76, 045201(2007)*
  ATMS with a phenomenological $K^{bar}N$ potential
- *Ikeda, Sato* — *PRC76, 035203(2007)*
  Faddeev with a chiral SU(3)-derived $K^{bar}N$ potential
- *Shevchenko, Gal, Mares* — *PRC76, 044004(2007)*
  Faddeev with a phenomenological $K^{bar}N$ potential
- *Wycech, Green* — *PRC79, 014001(2009)*
  Variational with a phenomenological $K^{bar}N$ potential (with p-wave)
- *Arai, Yasui, Oka* — *PTP119, 103(2008)*
  Uchino, Hyodo, Oka — *PTP119, 103(2008)*
  Λ^* nuclei model
- *Nishikawa, Kondo* — *PRC77, 055202(2008)*
  Skyrme model

All studies predict that K^-pp can be bound!

Theoretical studies of K^-pp



「100MeV以下の束縛、かなり広い崩壊幅」

しかし手法・相互作用によって計算結果はバラけている。。。

[1] PR
[2] PR
[3] PR
[4] PRC76, 035203 (2007)

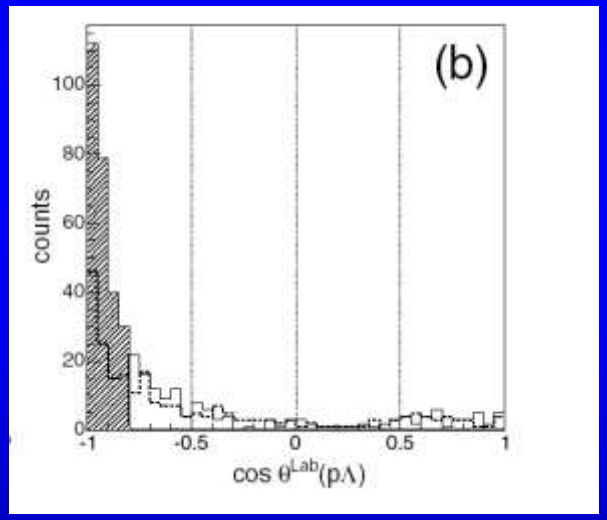
data.

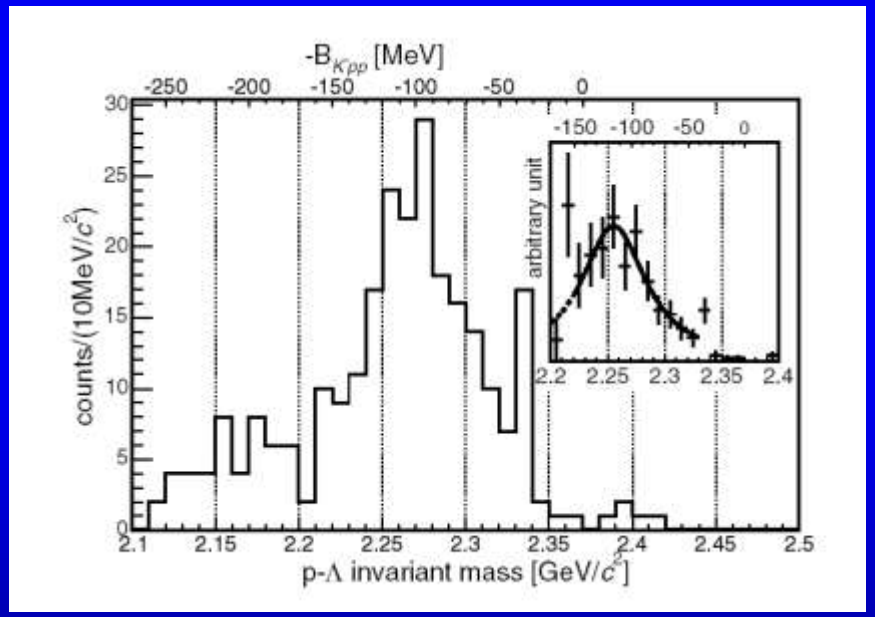
ta,

"Λ(1405)" etc.

Experimental studies of K^-pp

- *FINUDA collaboration (DAΦNE, Frascati)*
  - *K^- absorption at rest on various nuclei (^6Li, ^7Li, ^{12}C, ^{27}Al, ^{51}V)*
  - *Invariant-mass method*



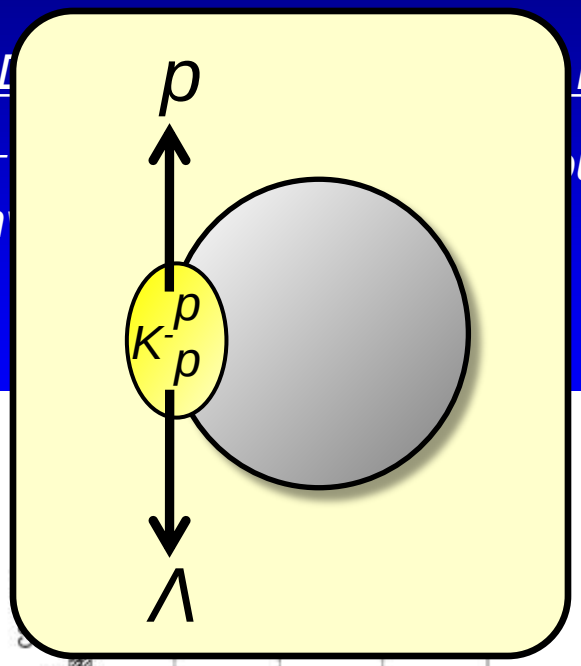


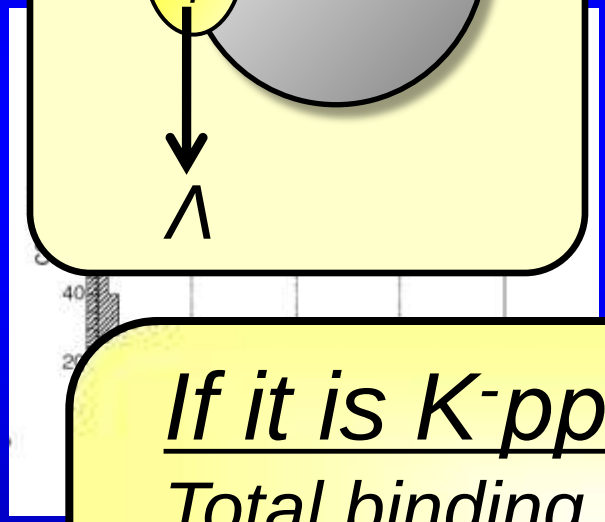
Strong correlation between emitted p and Λ (back-to-back)

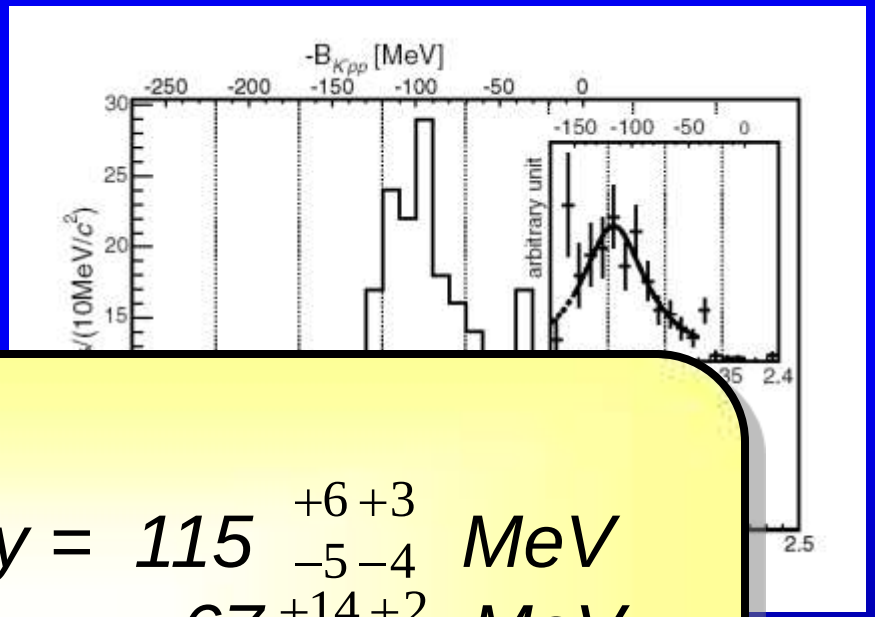
Invariant mass of p and Λ

Experimental studies of K^-pp

- FINUD … Frascati)
  - K^- … us nuclei (^{6}Li, ^{7}Li, ^{12}C, ^{27}Al, ^{51}V)
  - In…







Stron…
emitt…

If it is K^-pp, …

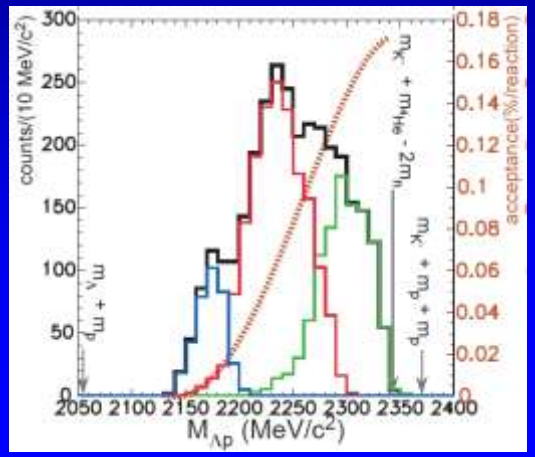
Total binding energy = $115\,^{+6\,+3}_{-5\,-4}$ MeV

Decay width = $67\,^{+14\,+2}_{-11\,-3}$ MeV

PRL 94, 212303 (2005)

Experimental studies of K^-pp

- Re-analysis of KEK-PS E549

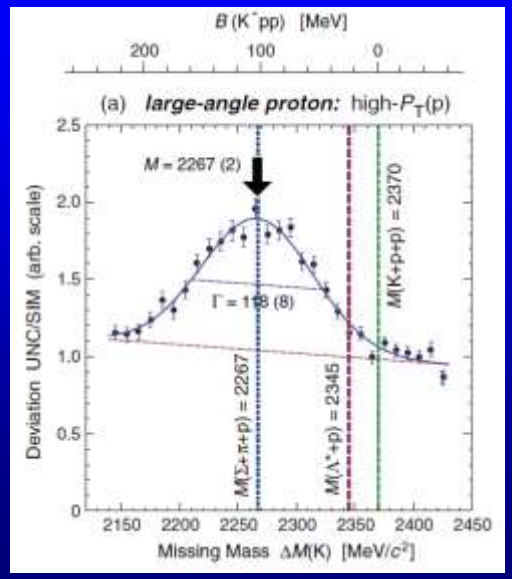


- K^- stopped on ^4He target
- Λp invariant mass

Strong Λp back-to-back correlation is confirmed.
Unknown strength is there
in the same energy region as FINUDA.

T. Suzuki et al (KEK-PS E549 collaboration), arXiv:0711.4943v1[nucl-ex]

- DISTO collaboration



- $p + p \rightarrow K^+ + \Lambda + p$ @ 2.85GeV
- Λp invariant mass
- Comparison with simulation data

K^-pp???
B. E.= 103 ±3 ±5 MeV
Γ = 118 ±8 ±10 MeV

T. Yamazaki et al. (DISTIO collaboration), PRL104, 132502 (2010)

as of 2011

Experiments of kaonic nuclei at J-PARC

E15: A search for deeply bound kaonic nuclear states
by ^{3}He(inflight K^-, n) reaction
(M. Iwasaki (RIKEN), T. Nagae (Kyoto))

K^-pp

E17: Precision spectroscopy of kaonic ^{3}He atom
3d→2p X-rays
(R. Hayano (Tokyo), H. Outa (Riken))

$K^{bar}N$ interaction

E27: Search for a nuclear Kbar bound state K^-pp in the d(π^+, K^+) reaction
(T. Nagae (Kyoto))

K^-pp

E31: Spectroscopic study of hyperon resonances below KN threshold via the (K^-, n) reaction on deuteron
(H. Noumi (Osaka))

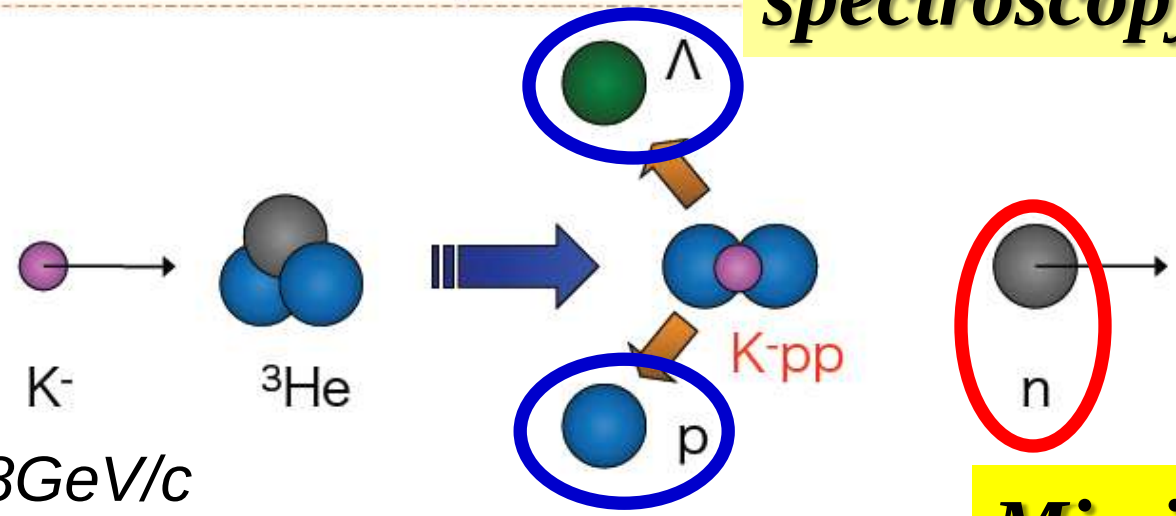
$\Lambda(1405)$

as of 2011

Experiments of kaonic nuclei at J-PARC

J-PARC E15 Experiment

Invariant mass spectroscopy



$$K^- + {}^3\mathrm{He} \rightarrow K^- pp + n$$
$$K^- pp \rightarrow \Lambda + p$$

Missing mass spectroscopy

All emitted particles will be measured.

「完全実験」

rkshop, 7-9. Aug. 08)

as of 2011

Experiments of kaonic nuclei at J-PARC

E15: A search for deeply bound kaonic nuclear states by ^{3}He(inflight K^-, n) reaction (M. Iwasaki (RIKEN), T. Nagae (Kyoto))

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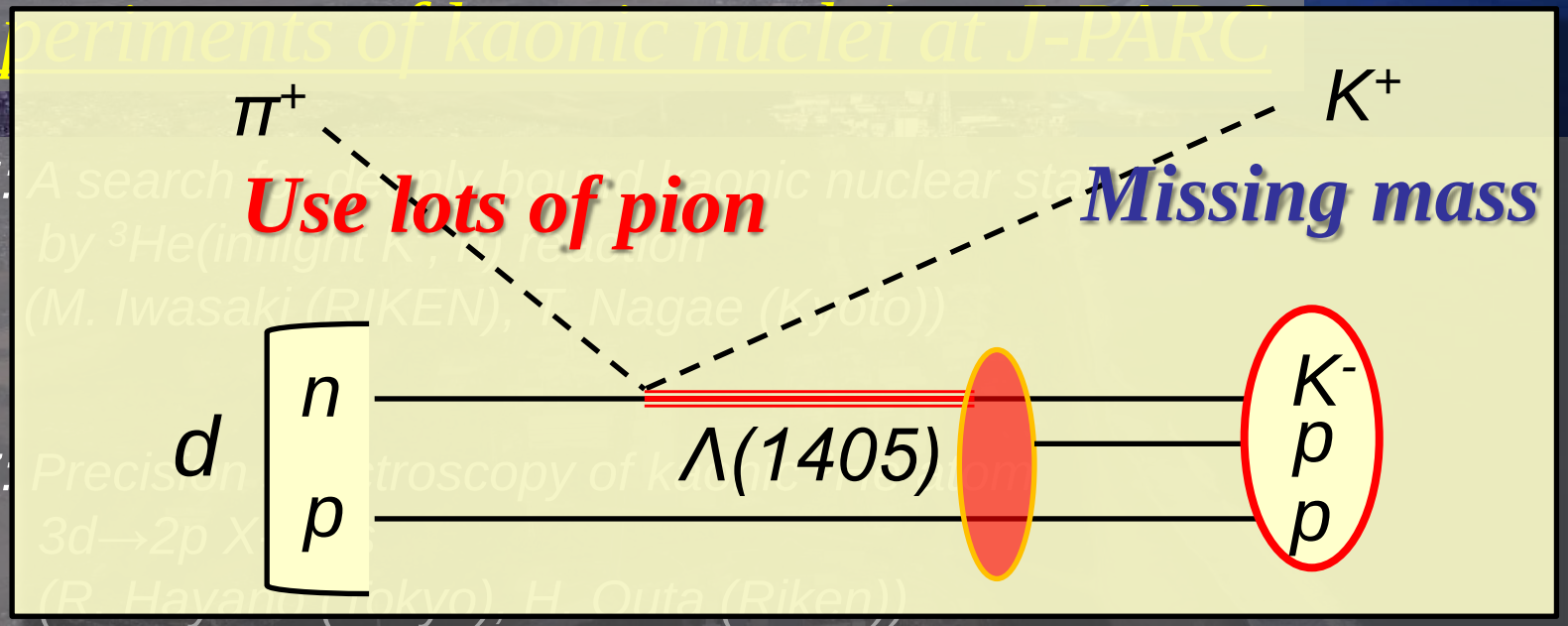


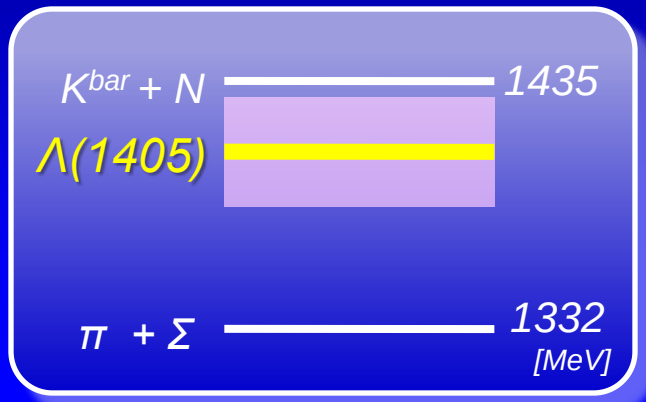
E27: Search for a nuclear Kbar bound state K^-pp in the $d(\pi^+, K^+)$ reaction
(T. Nagae (Kyoto))

E31: Spectroscopic study of hyperon resonances below KN threshold (H. Noumi (Osaka))

Preceding E15,
E27 experiment was performed in June.
Analysis is now undergoing!

1. Introduction

Λ(1405) = Important building block of K^{bar} nuclei



$J^{\pi}=1/2^-$, I=0
$K^{bar}N$-πΣ coupled system

I=0 $K^{bar}N$ potential … very attractive
→ *Interesting properties of K^{bar} nuclei !*

coupled-channel Complex Scaling Method

- *Investigate a resonant state, based on the variational approach*
- *Explicit treatment of the πΣ channel*

Importance of πΣN in K^-pp
Y. Ikeda and T. Sato, PRC 79, 035201(2009)

Chiral SU(3) potential (KSW)

N. Kaiser, P. B. Siegel and W. Weise, NPA 594, 325 (1995)

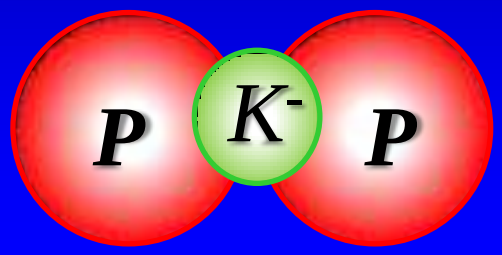
- *Based on Chiral SU(3) theory*
  → ***Energy dependence***

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}\left(\omega_i + \omega_j\right)\sqrt{\frac{M_i M_j}{s\,\omega_i\,\omega_j}}\; g_{ij}(r)$$

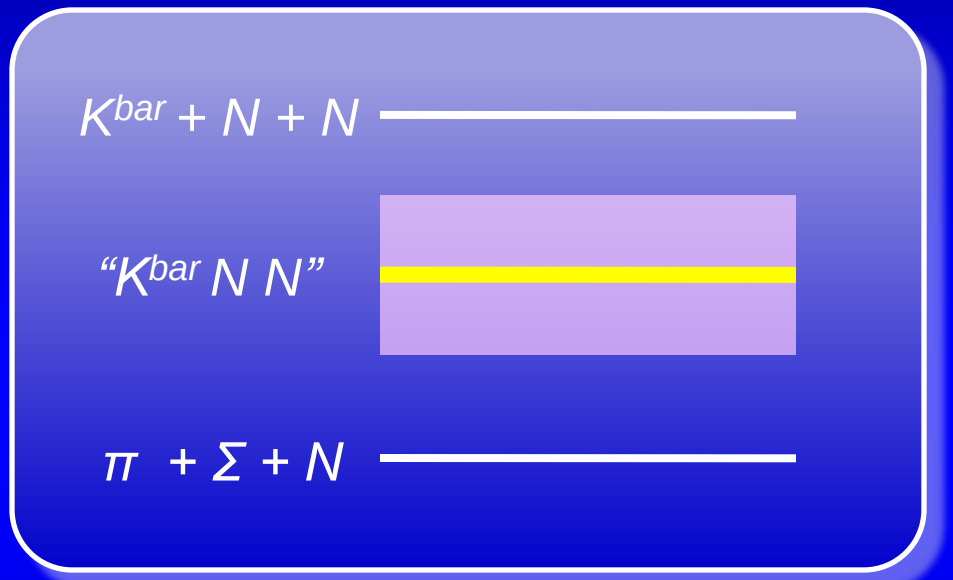
+ r-space, Gaussian form

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3}\exp\left[-\left(r/d_{ij}\right)^2\right]$$

1. Introduction



Most essential kaonic nucleus!



coupled-channel Complex Scaling Method

- *Based on the variational approach*
- *Explicitly treatment of the πYN channel*
- *Proper treatment of resonant state*

Complex Scaling Method

Complex rotation of coordinate

$$U(\theta):\quad \mathbf{r} \to \mathbf{r}\,e^{i\theta},\quad \mathbf{k} \to \mathbf{k}\,e^{-i\theta}$$

$$H_\theta \equiv U(\theta) H U^{-1}(\theta),\quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

$$E = \langle\Phi|H|\Phi\rangle = \langle\Phi_\theta|H_\theta|\Phi_\theta\rangle$$

- ✓ Resonant wave function is transformed from a divergent function to a dumping function.

$$\Phi_R \sim e^{ik_R r} = e^{i(\kappa - i\gamma)r}$$

$$U(\theta)\Phi_R \sim \exp\{(\gamma + i\kappa) r e^{i\theta}\}$$
$$= \exp\left[\left(\underline{\gamma\cos\theta - \kappa\sin\theta}\right) r\right] \cdot \exp\left[i\left(\gamma\sin\theta + \kappa\cos\theta\right) r\right]$$

Negative when $\tan\theta > \gamma / \kappa$

- ✓ Resonant and bound states are independent of a scaling angle θ. (ABC theorem†)

- ✓ Resonant states can be obtained by diagonalizing H_θ with Gaussian basis, in the same way as calculating bound state.

† J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

2. Scattering problem solved with Complex Scaling Method

Calculation of $K^{bar}N$ scattering amplitude

- *Formalism*
- *Test with AY potential*

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō, PRC 75, 044602 (2007)

$$H_l \psi_{l,k}^{(+)}(r) = E \psi_{l,k}^{(+)}(r)$$

$$\psi_{l,k}^{(+)}(r) = \hat{j}_l(kr) + \psi_{l,k}^{sc}(r).$$

$$\psi_{l,k}^{sc}(r) \underset{r\to\infty}{\to} k f_l(k) \hat{h}_l^{(+)}(kr),$$

- *Unknown*
- *Non square-integrable*

$$(E - H_l)\psi_{l,k}^{sc}(r) = V(r)\hat{j}_l(kr).$$

Complex scaling

$$r \to r\, e^{i\Theta}$$

$$(E - H_l(\theta))\psi_{l,k}^{sc,\theta}(r) = e^{i\theta/2} V(re^{i\theta})\hat{j}_l(kre^{i\theta}).$$

Expanding with square-integrable basis function (ex: Gaussian basis)

$$\psi_{l,k}^{sc,\theta}(r) \approx \sum_{i=1}^{N} c_i(\theta)\phi_i(r)$$

Point 1

Separate the incoming wave $j_l(kr)$ *!*

$$h_l^{(+)}(x) \to \exp\left[i\left(kr - \frac{l\pi}{2}\right)\right]$$

Point 2

$$\psi_{l,k}^{sc,\theta}(r) \underset{r\to\infty}{\to} e^{i\theta/2} k f_l(k) i^{-l} e^{ikr\cos\theta - kr\sin\theta}$$

square-integrable for $0 < \theta < \pi$

Calc. of scattering amplitude with CSM

A. T. Kruppa, R. Suzuki and K. Katō,
PRC 75, 044602 (2007)

- Calculation of scattering amplitude

$$f_l(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\, \hat{j}_l(kr) V(r) \psi_{l,k}^{(+)}(r)$$

$$f_l^{Born}(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\, \hat{j}_l(kr) V(r) \hat{j}_l(kr)$$

$$f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2}\int_0^\infty dr\, \hat{j}_l(kr) V(r) \psi_{l,k}^{sc}(r)$$

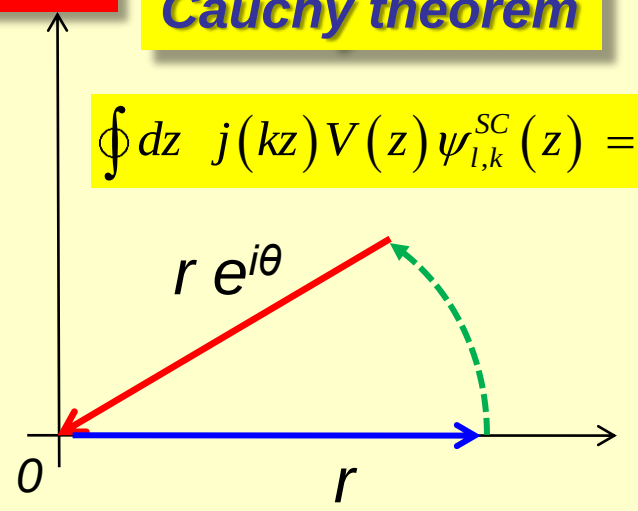
We don't have $\psi_{l,k}^{SC}(r)$
which is a solution along r,

but we have $\psi_{l,k}^{SC,\theta}(r)$
which is a solution along $re^{i\theta}$.

Point 3

Cauchy theorem

$$\oint dz \;\; j(kz) V(z) \psi_{l,k}^{SC}(z) = 0$$



$$f_l^{sc}(k) = -\frac{2m}{\hbar^2 k^2} e^{i\theta/2}\int_0^\infty dr\, \hat{j}_l(kre^{i\theta}) V(re^{i\theta}) \psi_{l,k}^{sc,\theta}(r)$$

expressed with Gaussian base

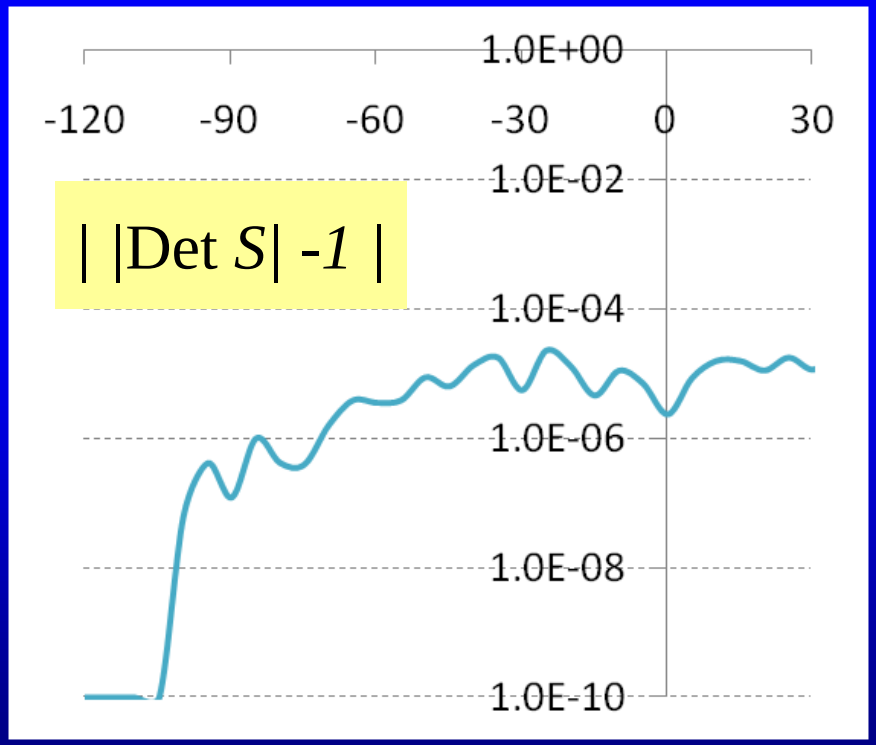
$f_l^{SC}(k)$ ***is independent of θ.***

Test calculation with AY potential

Phenomenological, E-independent

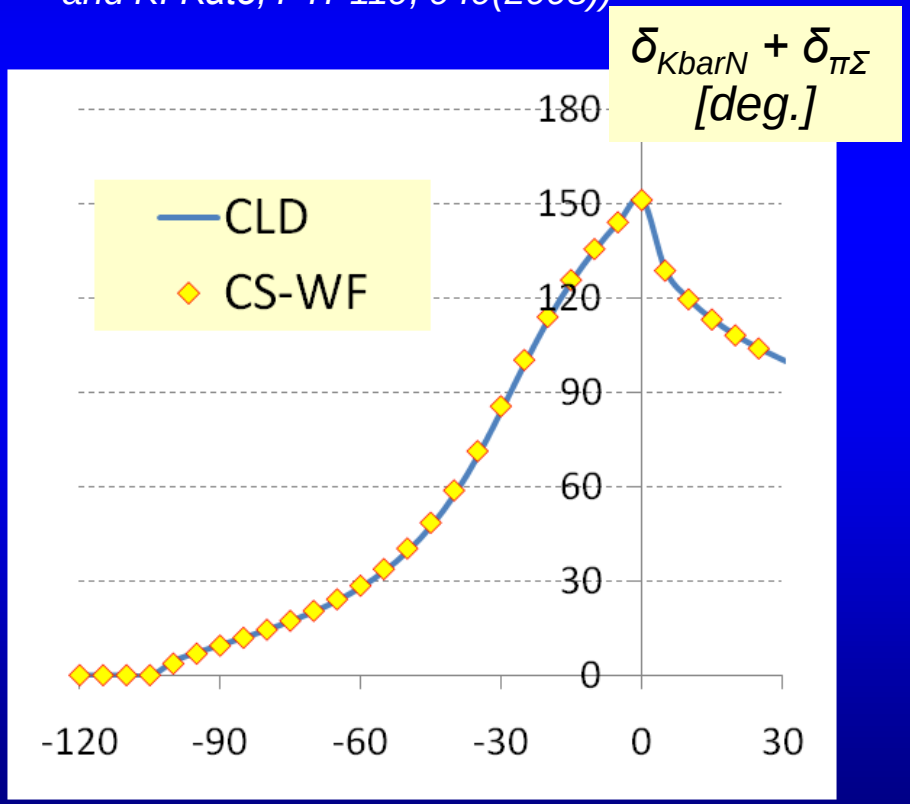
Unitarity violation of S-matrix

E_{KbarN} [MeV]



Phase shift sum

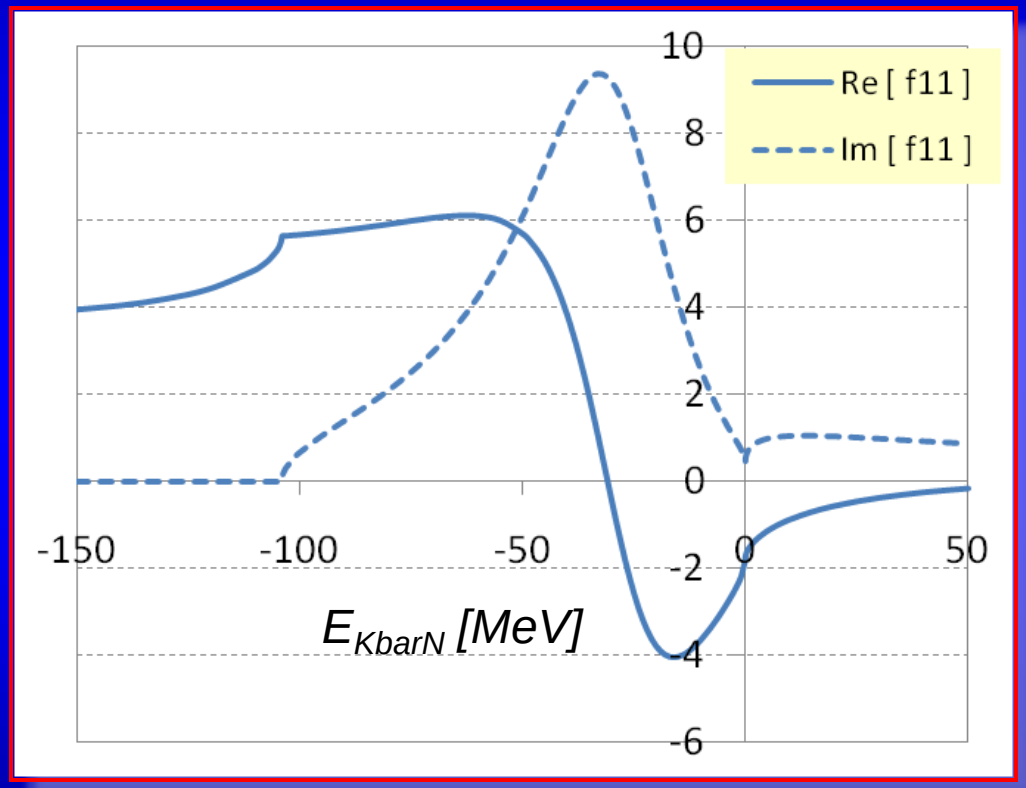
Checked by
Continuum Level Density method
(R. Suzuki, A. T. Kruppa, B. G. Giraud, and K. Katō, PTP119, 949(2008))



E_{KbarN} [MeV]

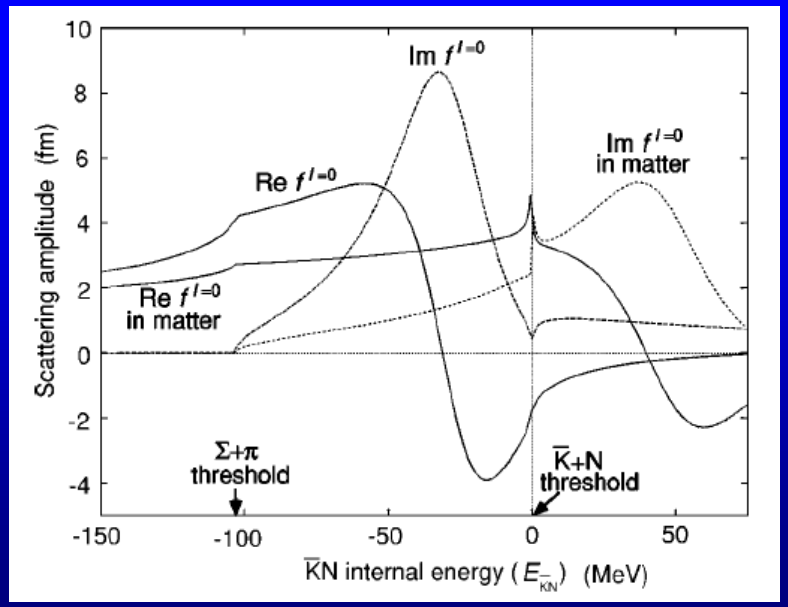
Test calculation with AY potential

$K^{bar}N$ (I=0) scattering amplitude $f^{I=0}_{\bar{K}N\to\bar{K}N}$ [fm]



Y. Akaishi and T. Yamazaki, PRC65, 044005 (2002)

Scattering length
= -1.76 + i 0.46 fm



Scattering length
(Scatt. amp. @ E_{KbarN}=0)
= -1.77 + i 0.47 fm

3. Set up for the calculation of $I=0$ $K^{bar}N-\pi\Sigma$ system

- *Kinematics*
- *Non-rela. approximation of KSW potential*

Kinematics

1. Non-relativistic

$$H_{NR} = \sum_{c=K^{bar}N,\pi\Sigma} \left[m_c + M_c + \frac{\mathbf{p}^2}{2\mu_c} \right] |c\rangle\langle c| + V_{KSW}$$

$$E = M_c + \frac{\hbar^2 k_c^2}{2M_c} + m_c + \frac{\hbar^2 k_c^2}{2m_c}$$

$$\mu_c = \frac{M_c m_c}{M_c + m_c}$$ Reduced mass

$$\hbar k_c = \left[2\mu_c \left(E - M_c - m_c \right) \right]^{1/2}$$

2. Semi-relativistic

$$H_{SR} = \sum_{c=K^{bar}N,\pi\Sigma} \left[\sqrt{m_c^2 + \mathbf{p}^2} + \sqrt{M_c^2 + \mathbf{p}^2} \right] |c\rangle\langle c| + V_{KSW}$$

$$E = \Omega_c + \omega_c = \sqrt{M_c^2 + \hbar^2 k_c^2} + \sqrt{m_c^2 + \hbar^2 k_c^2}$$

$$\varepsilon_c = \frac{\Omega_c \omega_c}{\Omega_c + \omega_c}$$ Reduced energy

$$\hbar k_c = \left[\omega_c^2 - m_c^2 \right]^{1/2}$$

Non-rela. approximation of KSW potential

A. Original

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i\,\omega_j}}\; g_{ij}(r)$$

- *Normalized Gaussian*

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^{\,3}} \exp\left[-\left(r/d_{ij}\right)^2\right]$$

- *Range parameter for coupling potential*

$$d_{ij} = \left(d_{ii} + d_{jj}\right)/2$$

B. Non-rela. approx. version 1

$$\omega_i \sim \mu_i$$

$$\Rightarrow \quad V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i\,\mu_j}}\; g_{ij}(r)$$

Comparison of the flux factor for differential cross section between non-rela. and rela.

C. Non-rela. approx. version 2

$m_i/\omega_i\,,\; M_i/\Omega_i \to 1$ *@ non-rela. limit (small p^2)*

$$\Rightarrow \quad V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i\,m_j}}\; g_{ij}(r)$$

Kinematics

Potential

Semi-rela.

Non-rela.

A. Original

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i\,\omega_j}}\; g_{ij}(r)$$

B. Non-rela. approx. version 1

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i\,\mu_j}}\; g_{ij}(r)$$

C. Non-rela. approx. version 2

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i\,m_j}}\; g_{ij}(r)$$

4. Result

Using Chiral SU(3) potential (KSW)

… r-space, Gaussian form, Energy-dependent

- *I=0 $K^{bar}N$ scattering length and the range parameters of the KSW potential*
- *Scattering amplitude*
- *Resonance pole … wave function and size*

I=0 $K^{bar}N$ scattering length and the range parameters of KSW potential

Potential:

$$V_{ij}^{(I=0)}(r) = V_{ij}^{(I=0)}\left(\sqrt{s}\right) \frac{1}{\pi^{3/2} d_{ij}^{\,3}} \exp\left[-\left(r/d_{ij}\right)^2\right]$$

$$a^{I=0}_{K^{bar}N} = f^{I=0}_{K^{bar}N}\left(\sqrt{s}_{K^{bar}N\ thr.}\right)$$

Data : $a^{I=0}_{KbarN}$ = -1.70 + i 0.68 fm (A. D. Martin)†

Range parameters:

$$d_{ij} = \begin{pmatrix} d_{K^{bar}N,K^{bar}N} & d_{K^{bar}N,\pi\Sigma} \\ & d_{\pi\Sigma,\pi\Sigma} \end{pmatrix}$$

$$d_{K^{bar}N,\pi\Sigma} \equiv \left(d_{K^{bar}N,K^{bar}N} + d_{\pi\Sigma,\pi\Sigma}\right)/2$$

† A. D. Martin, Nucl. Phys. B 179, 33 (1981)

I=0 $K^{bar}N$ scattering length and the range parameters of KSW potential

f_π=90 MeV

Data : $a^{I=0}_{KbarN}$ = -1.70 + i 0.68 fm (A. D. Martin)

	Kinematics	Non-rela.			Semi-rela.
	KSW potential	Original	Non-rela. v1	Non-rela. v2	Original
Range parameter [fm]	d_{KbarN}	0.593	0.576	0.574	0.487
	$d_{\pi\Sigma}$	0.541	0.725	0.751	0.457
KbarN Scatt. leng. [fm]	Re	−1.701	−1.700	−1.700	−1.703
	Im	0.679	0.677	0.687	0.677

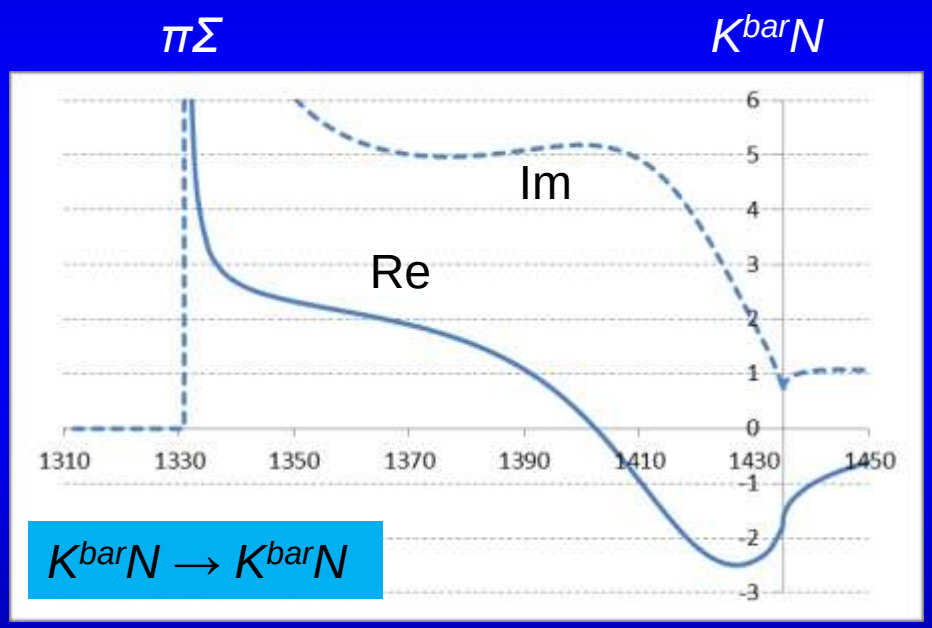
In all cases, a set of range parameters (d_{KbarN},$d_{\pi\Sigma}$) is found to reproduce the Martin's value.

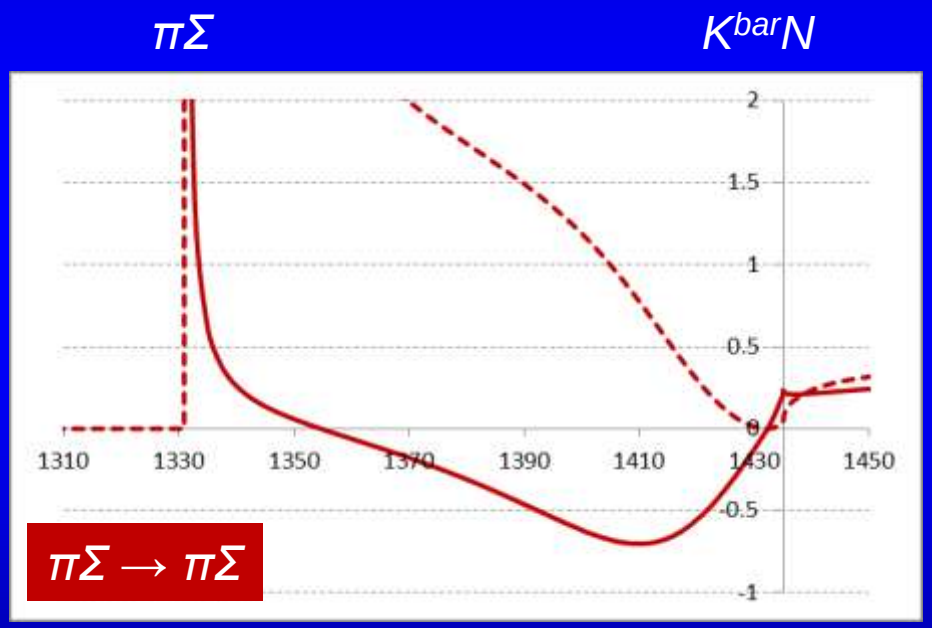
Scattering amplitude

… KSW org. – Non. rela.

$f_\pi = 90\ MeV$

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i\,\omega_j}}\; g_{ij}(r)$$





Singular behavior at πΣ threshold

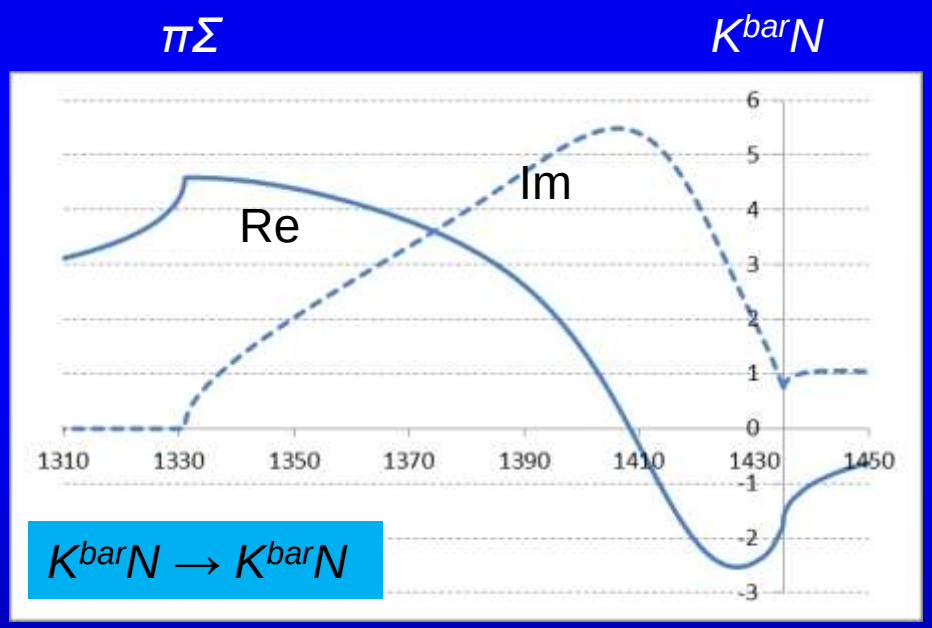
… due to mismatch of potential and kinematics.

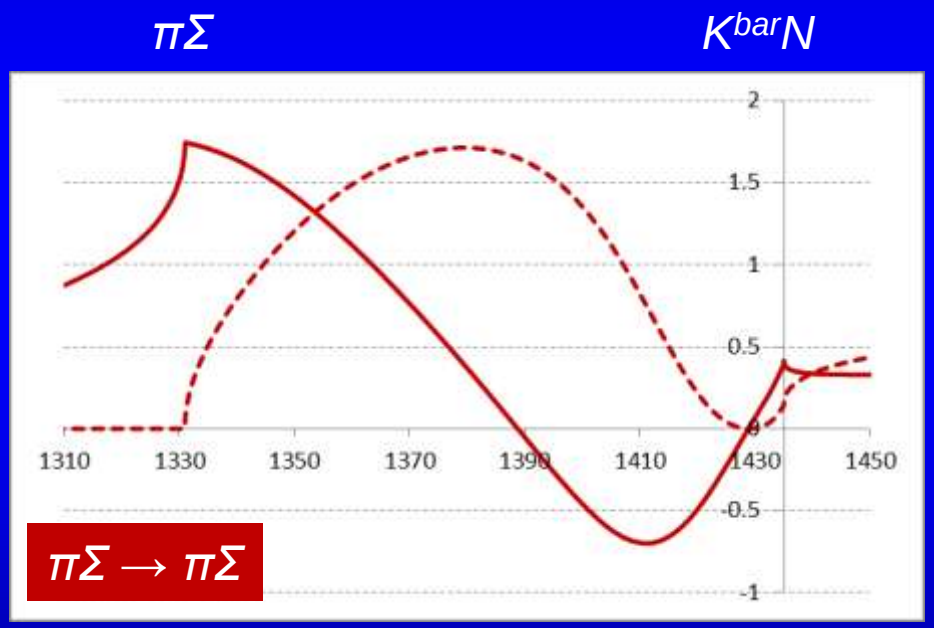
Scattering amplitude

… KSW NRv1 – Non. rela.

$f_\pi = 90\ MeV$

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\frac{1}{E_{Tot}}\sqrt{\frac{M_i M_j}{\mu_i\ \mu_j}}\ g_{ij}(r)$$





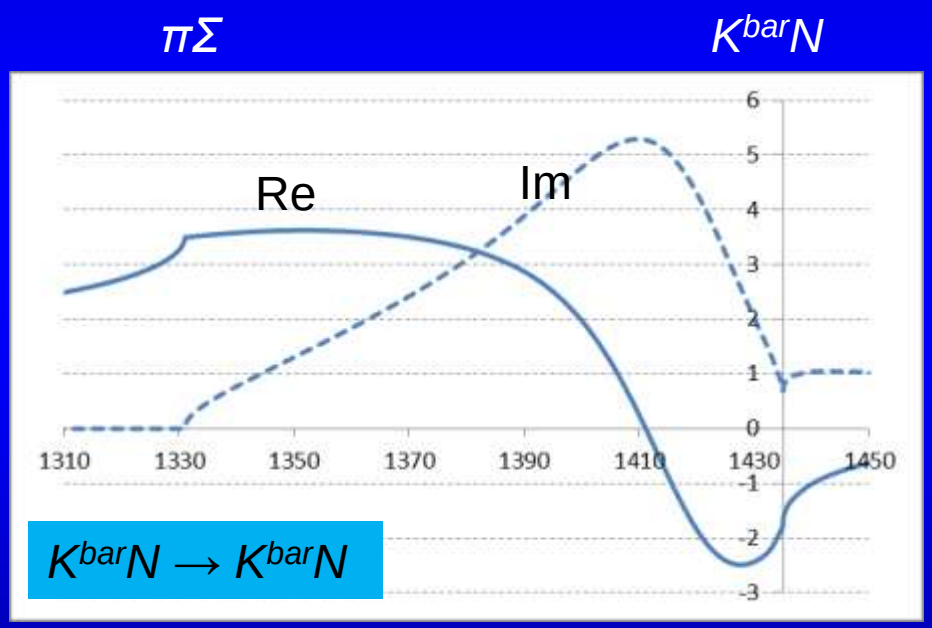
Singular behavior at πΣ threshold disappears.

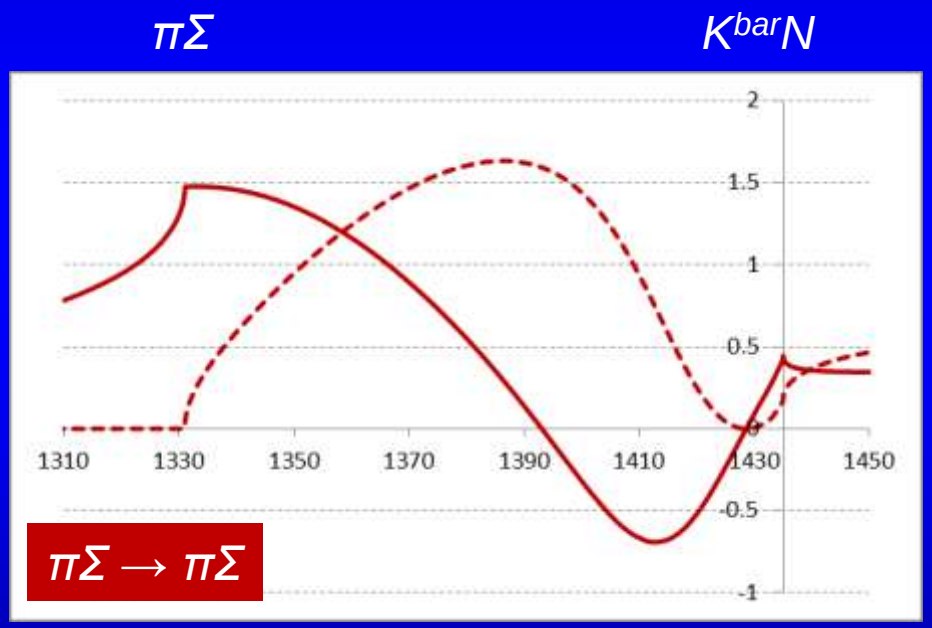
Scattering amplitude

… KSW NRv2 – Non. rela.

$f_\pi = 90\ MeV$

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{1}{m_i\, m_j}}\, g_{ij}(r)$$





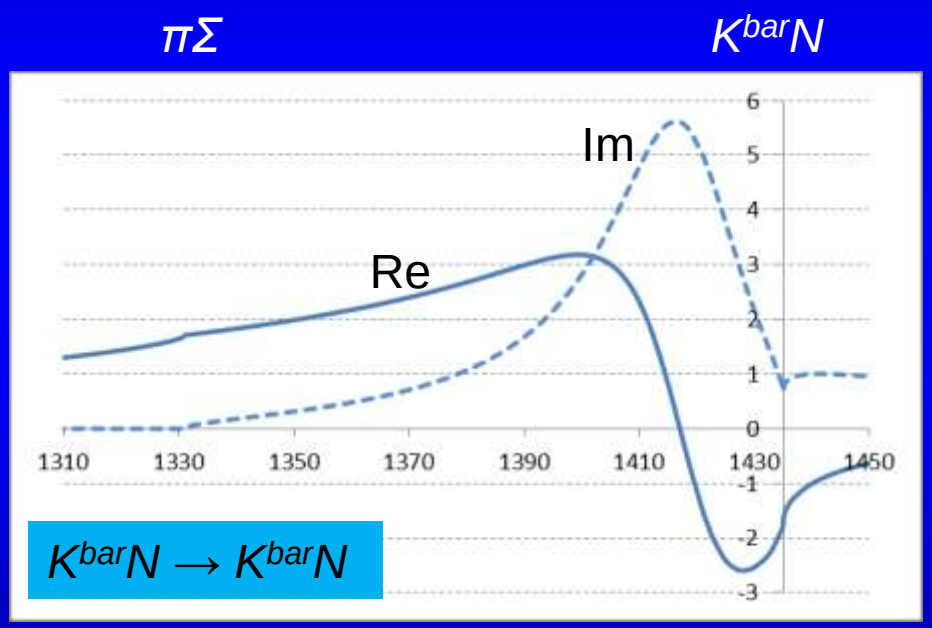
Essentially same as NRv1 case.

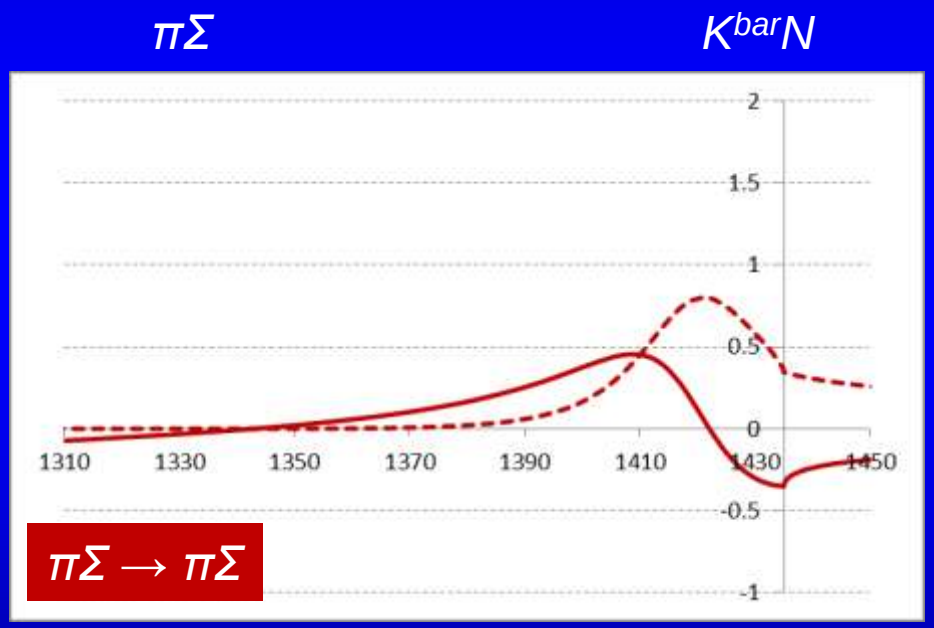
Scattering amplitude

… KSW org. – Semi rela.

$f_\pi = 90\ MeV$

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8 f_\pi^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\omega_i \omega_j}}\, g_{ij}(r)$$

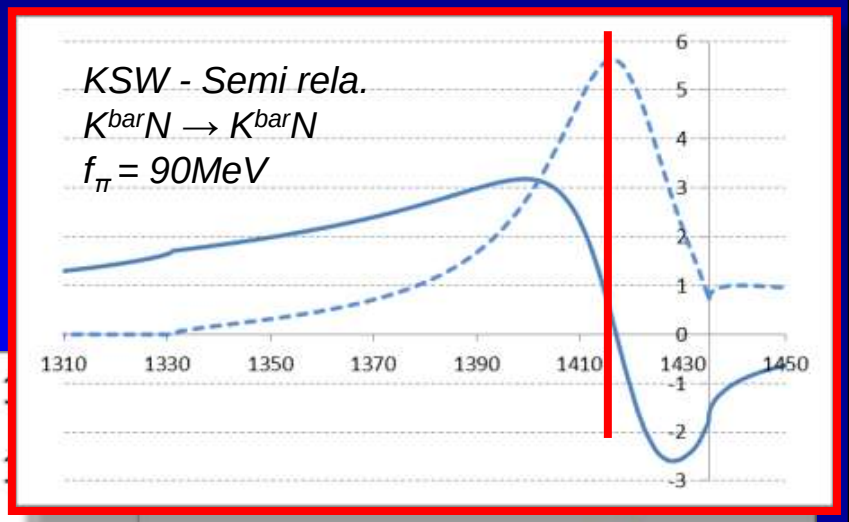




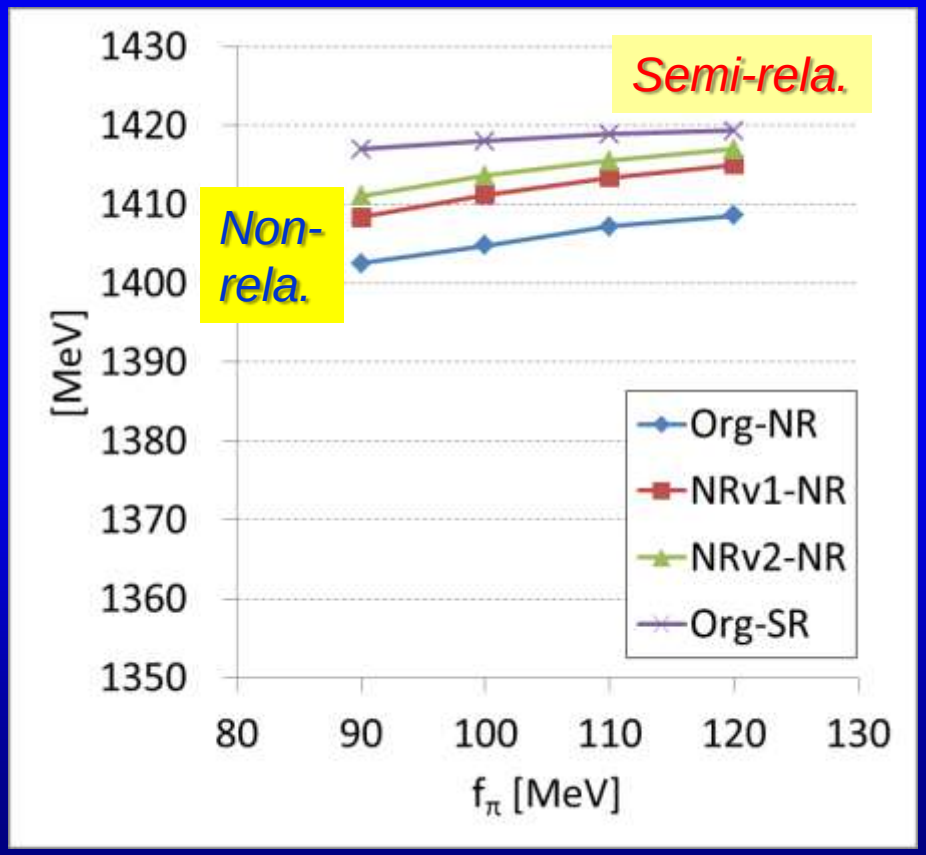
Compared with NR cases,
less attractive far below $K^{bar}N$ threshold.

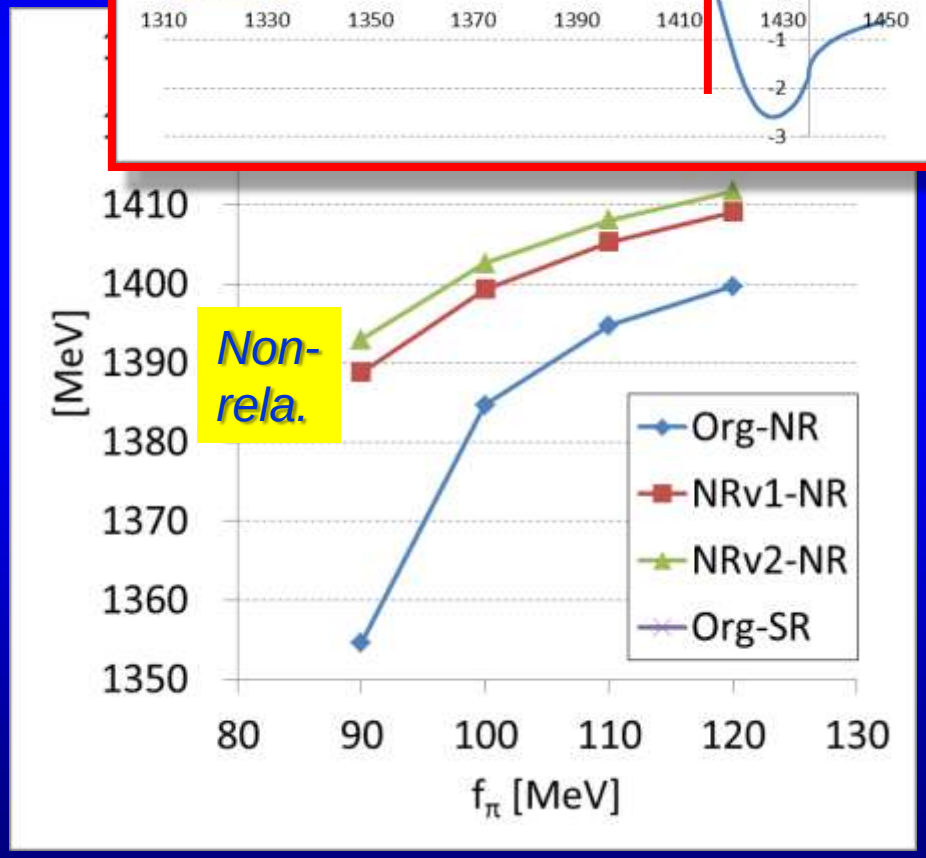
Position of resonant structure in the scattering amplitude

f_π=90 - 120MeV



Energy @ Re $f_{KbarN \rightarrow KbarN}$ = 0

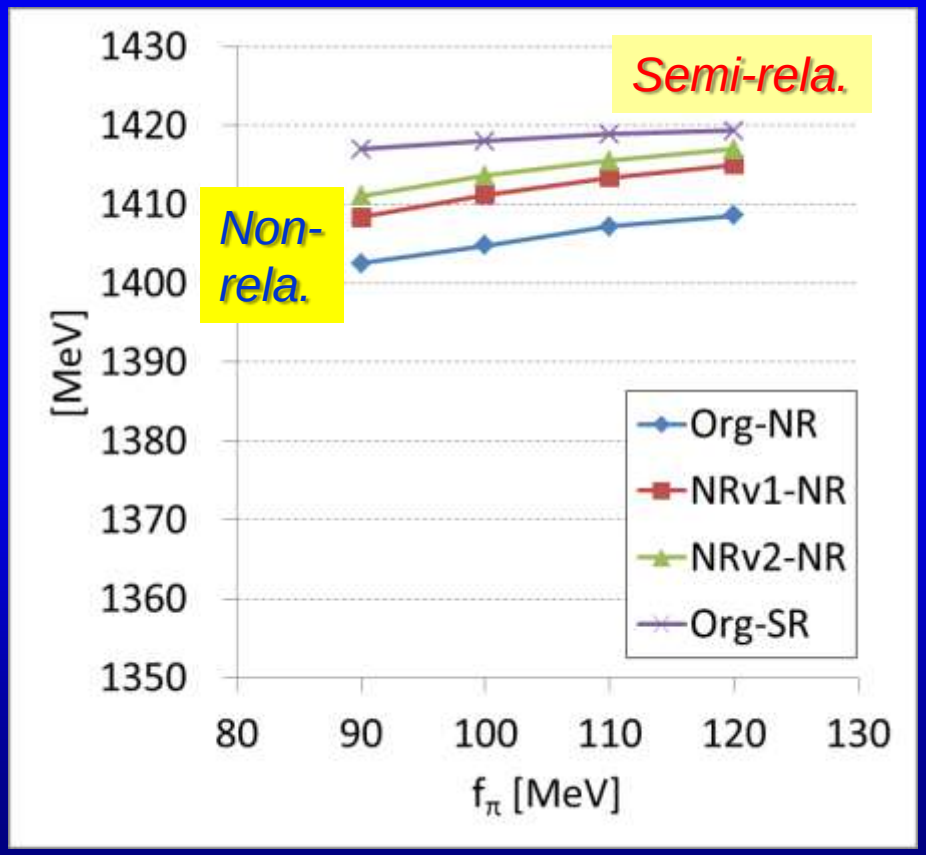




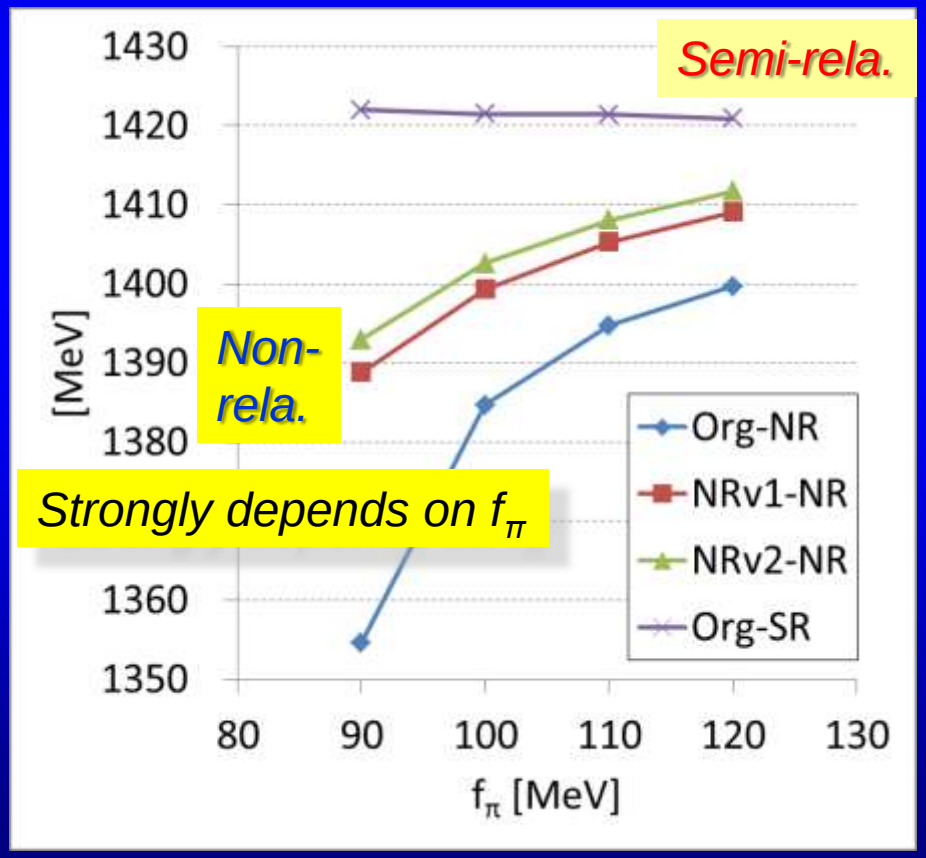
Position of resonant structure in the scattering amplitude

f_π=90 - 120MeV

Energy @ Re $f_{KbarN \to KbarN}$ = 0



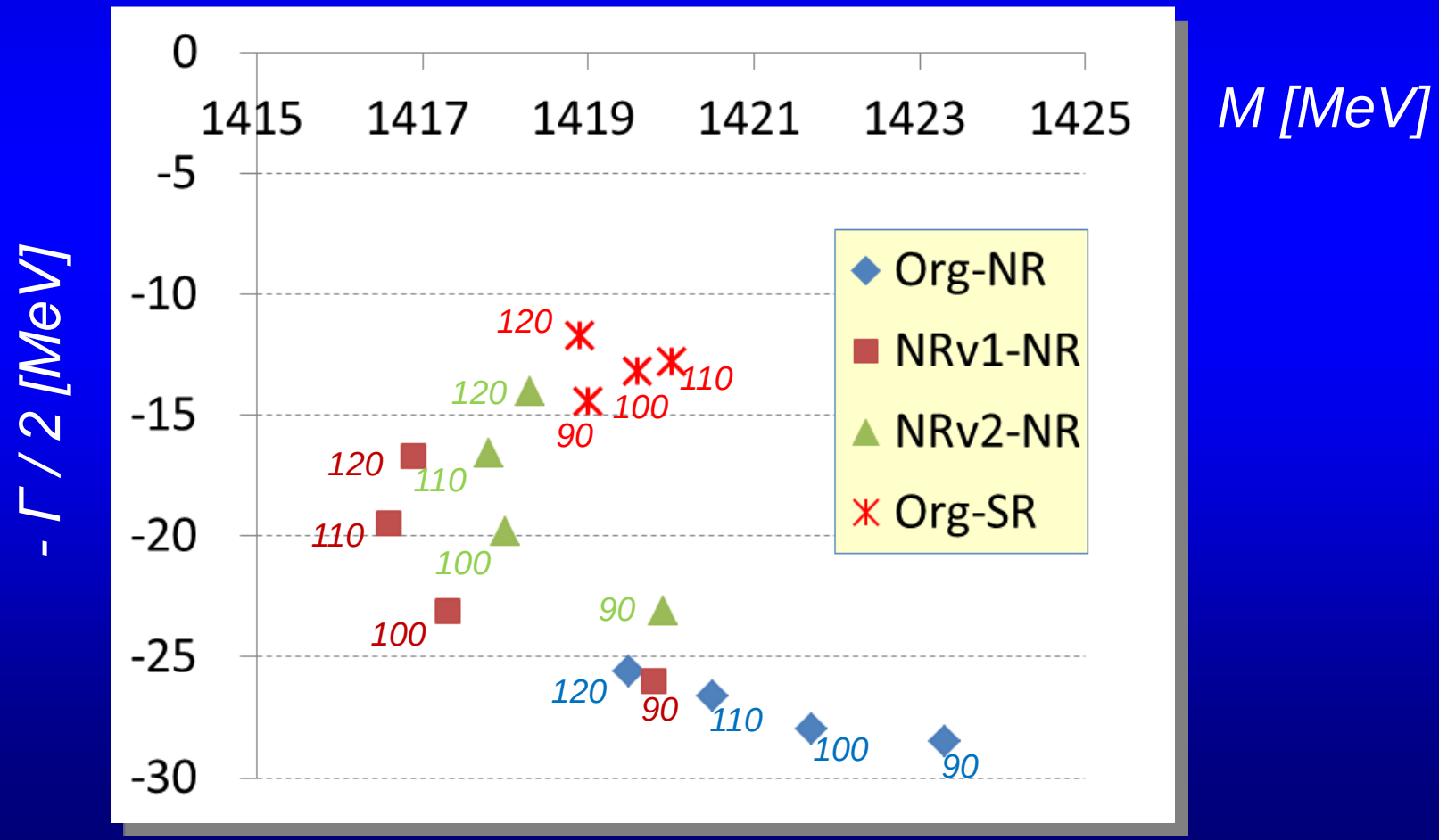
Energy @ Re $f_{\pi\Sigma \to \pi\Sigma}$ = 0



Pole position of the resonance

f_π=90 - 120MeV

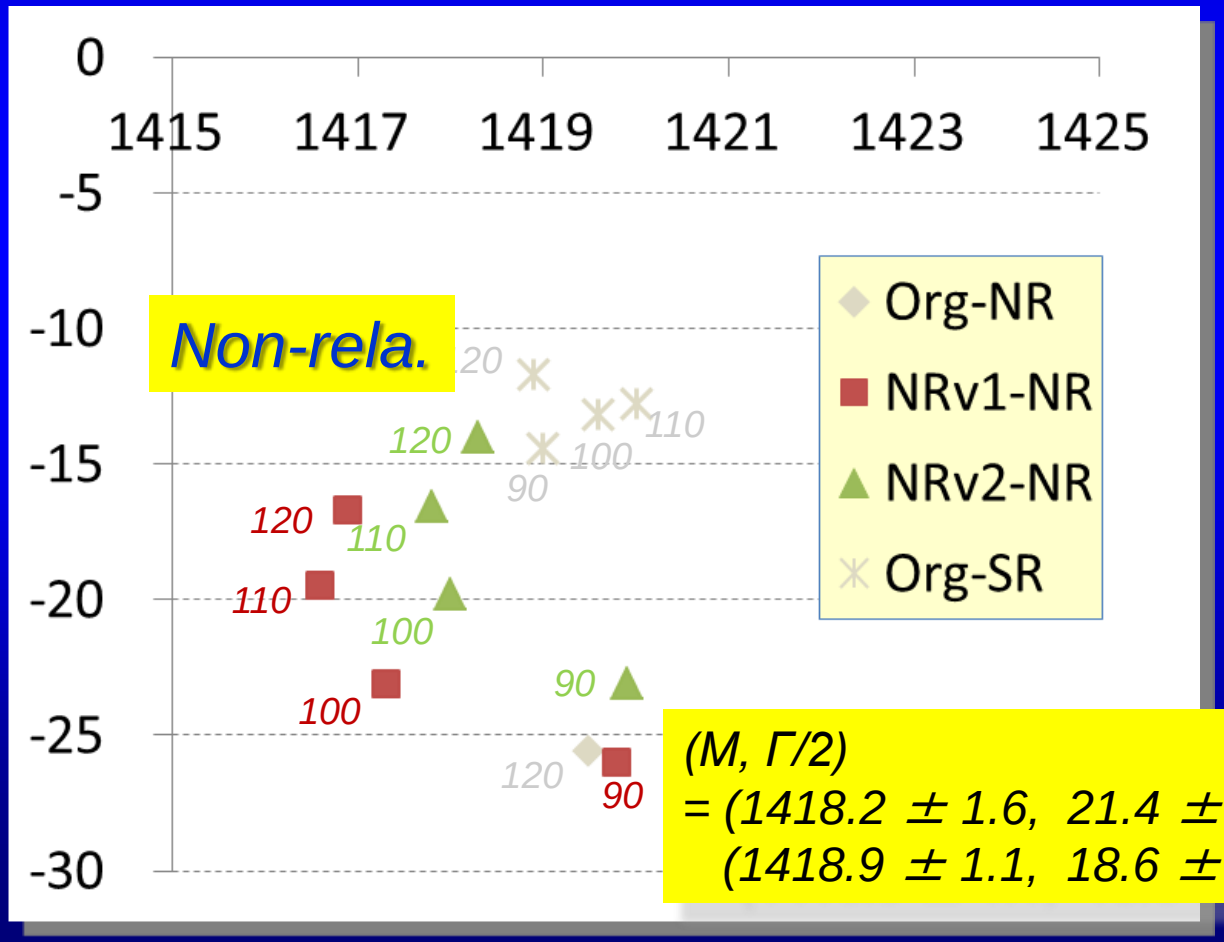
fpi	Org−NR		NRv1−NR		NRv2−NR		Org−SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
120	1419.5	−25.6	1416.9	−16.7	1418.3	−14.0	1418.9	−11.7



Pole position of the resonance

f_π=90 - 120MeV

fpi	Org−NR		NRv1−NR		NRv2−NR		Org−SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
120	1419.5	−25.6	1416.9	−16.7	1418.3	−14.0	1418.9	−11.7



(M, Γ/2)
= (1418.2 ± 1.6, 21.4 ± 4.7) … NRv1
(1418.9 ± 1.1, 18.6 ± 4.6) … NRv2

Pole position of the resonance

f_π=90 - 120MeV

fpi	Org−NR		NRv1−NR		NRv2−NR		Org−SR	
	M	−Γ /2	M	−Γ /2	M	−Γ /2	M	−Γ /2
90	1423.3	−28.5	1419.8	−26.0	1419.9	−23.1	1419.0	−14.4
100	1421.7	−28.0	1417.3	−23.1	1418.0	−19.8	1419.6	−13.2
110	1420.5	−26.6	1416.6	−19.5	1417.8	−16.6	1420.0	−12.8
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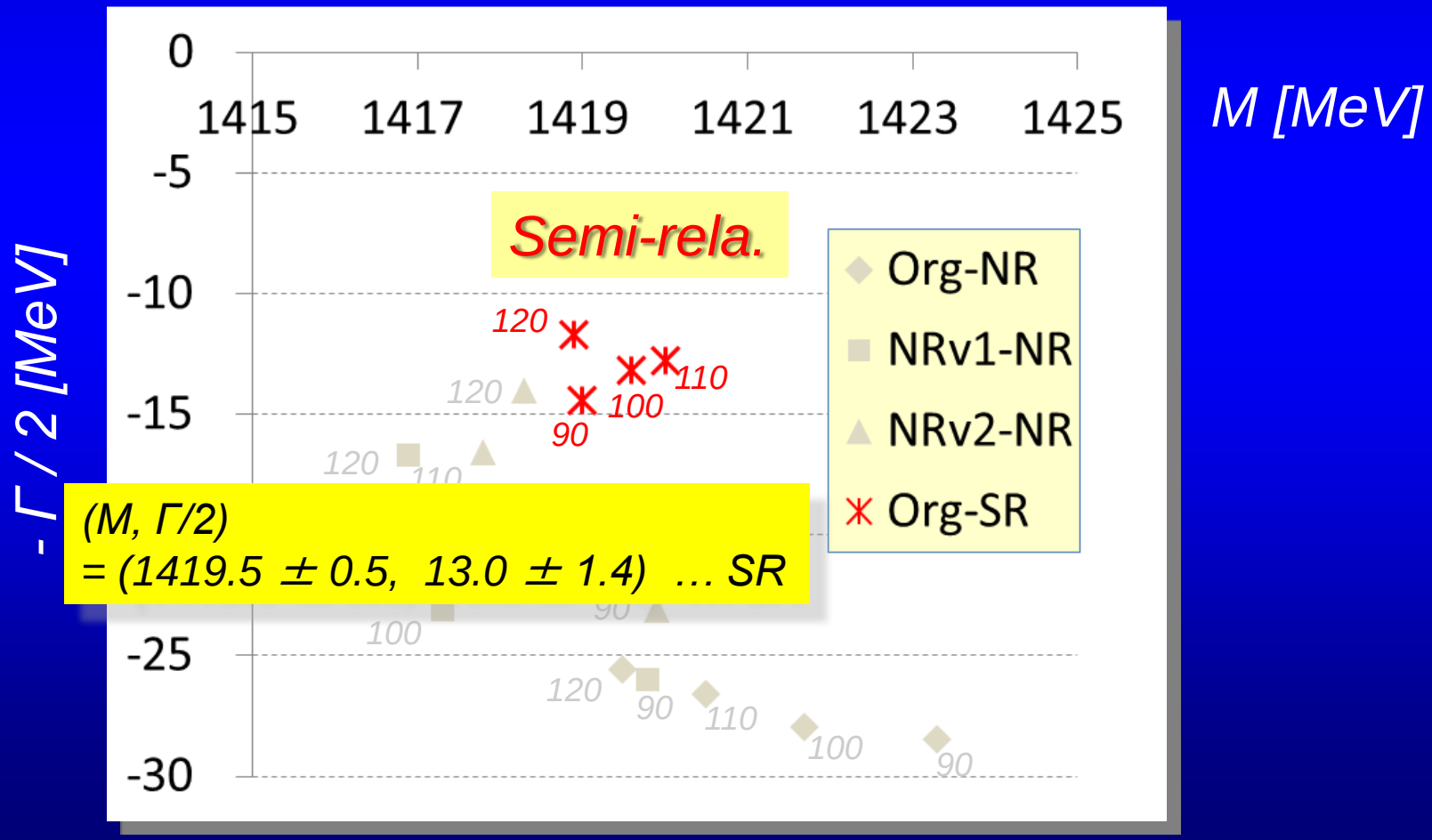
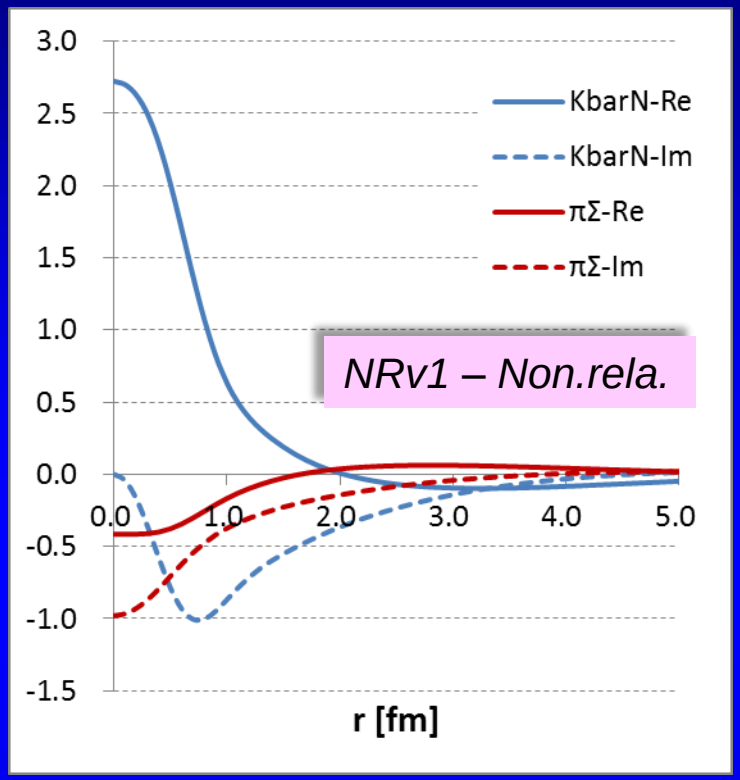
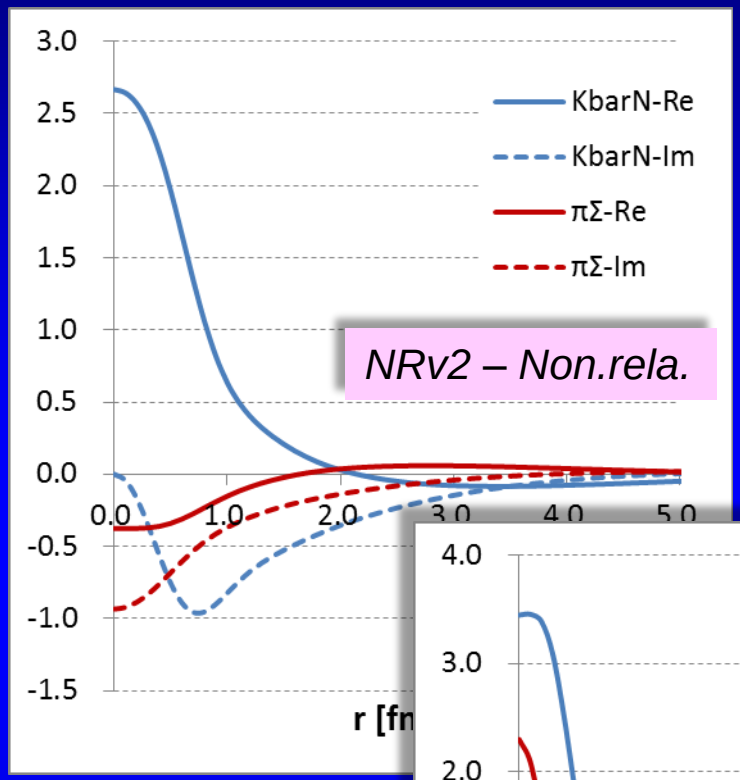
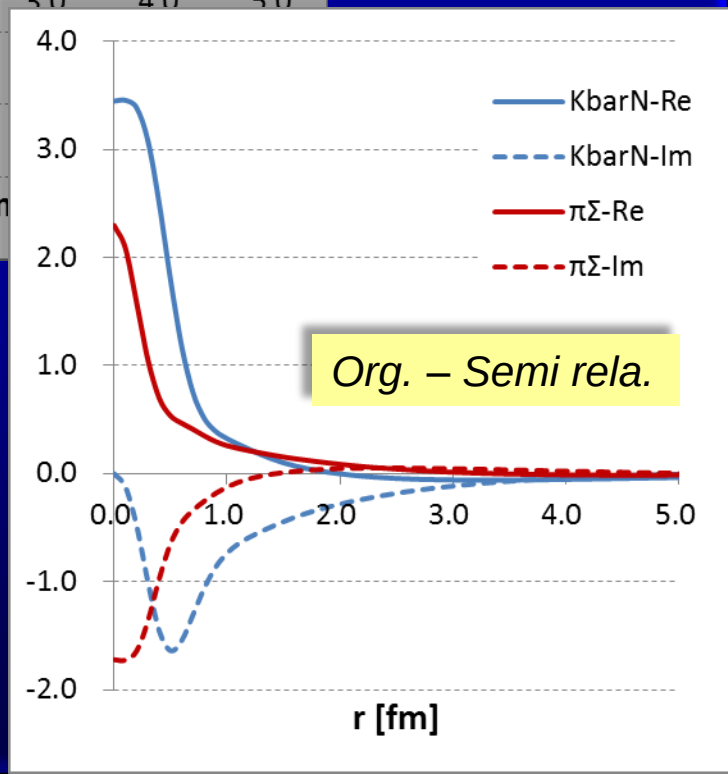


"Wave function" of the resonance pole

f_π=90 MeV
θ=30 °







πΣ component also localized due to Complex scaling.

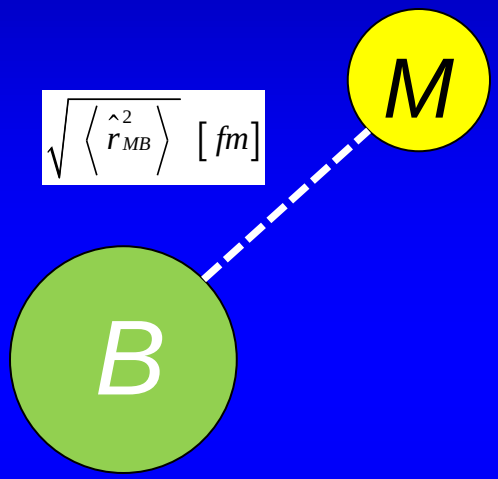
** The wave functions shown above are multiplied by a phase factor so that the $K^{bar}N$ wfn. becomes real at r=0.*

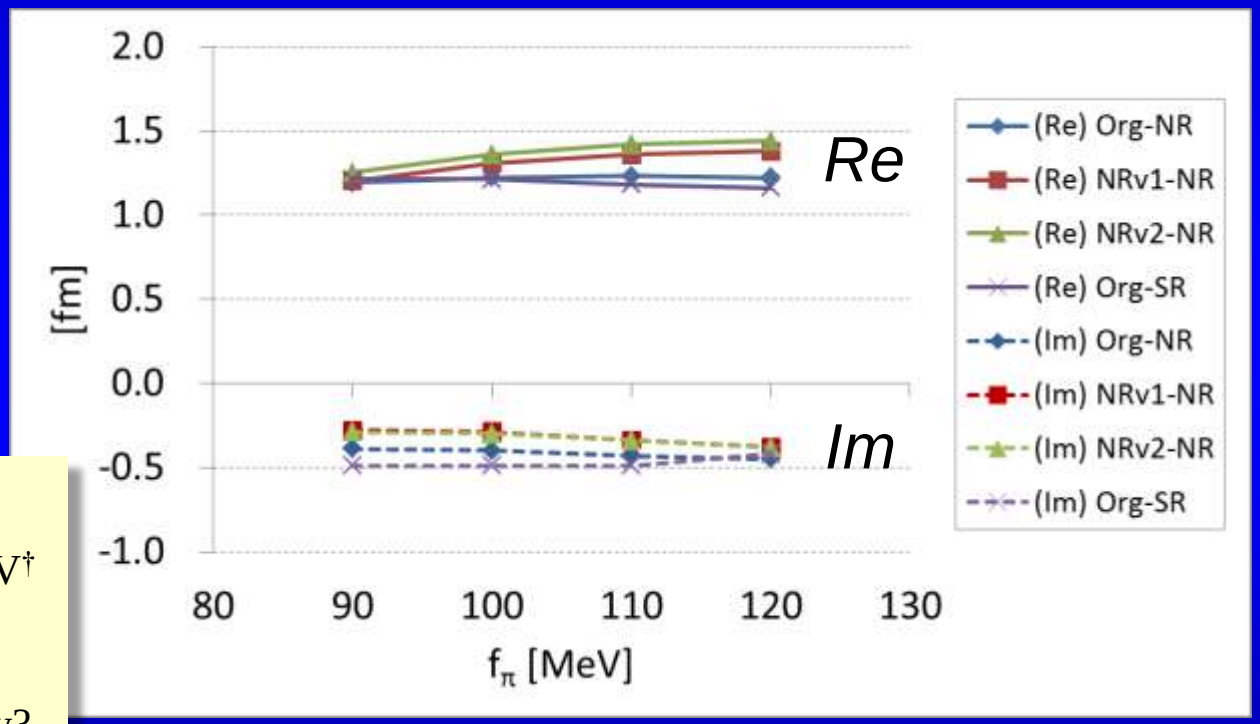
"Size" of the resonance pole state

f_π=90 - 120MeV

Mean distance between meson and baryon

$\sqrt{\langle \hat{r}^2_{MB} \rangle}$ [fm]





? *Somehow small.*

cf) 1.9 fm @ M = 1423 MeV†
(B = 12 MeV)

- Difference of binding energy?
- Different definition of pole?
  Gamow state or bound state

† A. D., T. Hyodo, W. Weise, PRC79, 014003 (2009)

NR: ～1.3 - 0.3 i [fm]
SR: ～1.2 - 0.5 i [fm]

4. Result

Using Chiral SU(3) potential (KSW)

… r-space, Gaussian form, Energy-dependent

- *<u>I=1 case</u>*
  - *The range parameters of the KSW potential*
  - *Scattering amplitude*

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

Potential:

$$V_{ij}^{(I=1)}(r) = V_{ij}^{(I=1)}\left(\sqrt{s}\right) \frac{1}{\pi^{3/2} d_{ij}^{3}} \exp\left[-\left(r/d_{ij}\right)^{2}\right]$$

$$a_{K^{bar}N}^{I=1} = f_{K^{bar}N}^{I=1}\left(\sqrt{s}_{K^{bar}N\ thr.}\right)$$

Data : $a^{I=1}_{KbarN}$ = 0.37 + i 0.60 fm (A. D. Martin)†

Range parameters:

$$d_{ij} = \begin{pmatrix} d_{K^{bar}N,K^{bar}N} & d_{K^{bar}N,\pi\Sigma} & d_{K^{bar}N,\pi\Lambda} \\ & d_{\pi\Sigma,\pi\Sigma} & d_{\pi\Sigma,\pi\Lambda} \\ & & d_{\pi\Lambda,\pi\Lambda} \end{pmatrix}$$

† A. D. Martin, Nucl. Phys. B 179, 33 (1981)

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

Potential:

$$V_{ij}^{(I=1)}(r) = V_{ij}^{(I=1)}\left(\sqrt{s}\right) \frac{1}{\pi^{3/2} d_{ij}^{\,3}} \exp\left[-\left(r/d_{ij}\right)^2\right]$$

$$a_{K^{bar}N}^{I=1} = f_{K^{bar}N}^{I=1}\left(\sqrt{s}_{K^{bar}N\,thr.}\right)$$

Data : $a^{I=1}_{KbarN}$ = 0.37 + i 0.60 fm (A. D. Martin)†

Range parameters:

$$d_{ij} = \begin{pmatrix} d_{K^{bar}N,K^{bar}N} & d_{K^{bar}N,\pi\Sigma} & d_{K^{bar}N,\pi\Lambda} \\ & d_{\pi\Sigma,\pi\Sigma} & \cancel{d_{\pi\Sigma,\pi\Lambda}} \\ & & \cancel{d_{\pi\Lambda,\pi\Lambda}} \end{pmatrix}$$

$$d_{K^{bar}N,\pi\Sigma} \equiv \left(d_{K^{bar}N,K^{bar}N} + d_{\pi\Sigma,\pi\Sigma}\right)/2$$

SU(3) C.G. coefficients for I=1 channel

$$C^{(I=1)}{}_{ij} = \begin{pmatrix} 1 & -1 & -\sqrt{3/2} \\ & 2 & 0 \\ & & 0 \end{pmatrix}$$

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

Potential:

$$V_{ij}^{(I=1)}(r) = V_{ij}^{(I=1)}\left(\sqrt{s}\right) \frac{1}{\pi^{3/2} d_{ij}^{3}} \exp\left[-\left(r/d_{ij}\right)^{2}\right]$$

$$a_{K^{bar}N}^{I=1} = f_{K^{bar}N}^{I=1}\left(\sqrt{s}_{K^{bar}N\,thr.}\right)$$

Data : $a^{I=1}_{KbarN}$ = 0.37 + i 0.60 fm (A. D. Martin)†

Range parameters:

$$d_{ij} = \begin{pmatrix} d_{K^{bar}N,K^{bar}N} & d_{K^{bar}N,\pi\Sigma} & d_{K^{bar}N,\pi\Lambda} \\ & d_{\pi\Sigma,\pi\Sigma} & \cancel{d_{\pi\Sigma,\pi\Lambda}} \\ & & \cancel{d_{\pi\Lambda,\pi\Lambda}} \end{pmatrix}$$

$$d_{K^{bar}N,\pi\Sigma} \equiv \left(d_{K^{bar}N,K^{bar}N} + d_{\pi\Sigma,\pi\Sigma}\right)/2$$

SU(3) C.G. coefficients for I=1 channel

$$C^{(I=1)}{}_{ij} = \begin{pmatrix} 1 & -1 & -\sqrt{3/2} \\ & 2 & 0 \\ & & 0 \end{pmatrix}$$

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

Data … 2 real values

$a^{I=1}_{KbarN} = 0.37 + i\ 0.60$ fm

Unknown parameters … 3 real values

$$\left\{ d_{K^{bar}N,K^{bar}N},\quad d_{\pi\Sigma,\pi\Sigma},\quad d_{K^{bar}N,\pi\Lambda} \right\}$$

$d_{KN,KN}$ is fixed to that for I=0 channel.

- ✓ *In a chiral unitary model, isospin symmetric subtraction constants have been often used.*
- ✓ *In a study with separable potential, the cutoff parameter for $K^{bar}N$ is not so different between I=0 and 1 channels.†*

$$\left\{ d^{(I=0)}_{K^{bar}N,K^{bar}N},\quad d_{\pi\Sigma,\pi\Sigma},\quad d_{K^{bar}N,\pi\Lambda} \right\}$$

$d_{\pi\Sigma,\ \pi\Sigma}$ and $d_{KN,\ \pi\Lambda}$ are searched to reproduce the complex value $a^{I=1}_{KbarN}$.

† Y. Ikeda and T. Sato, Phys. Rev. C 76, 035203 (2007)

I=1 channel … $K^{bar}N$ - $\pi\Sigma$ - $\pi\Lambda$

f_π=90 MeV

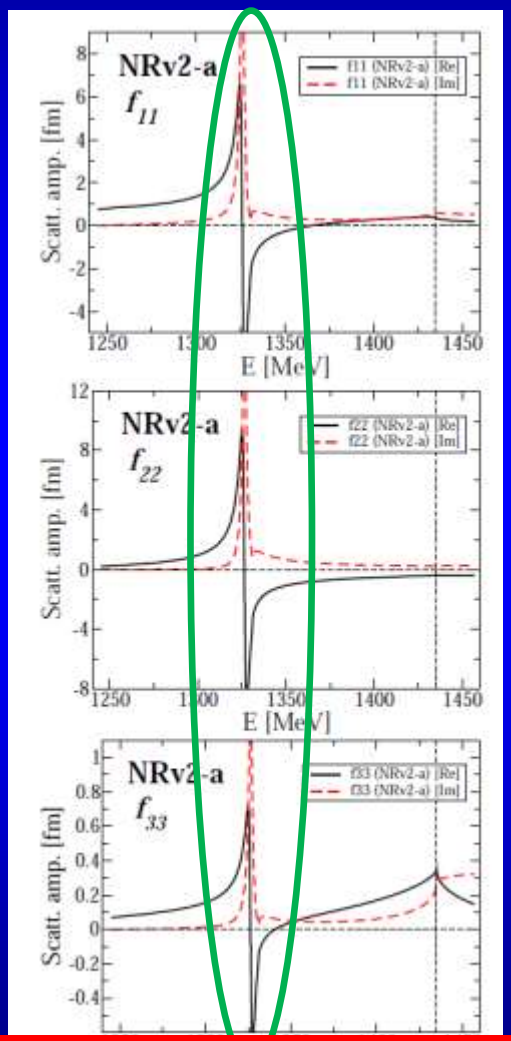
NRv2

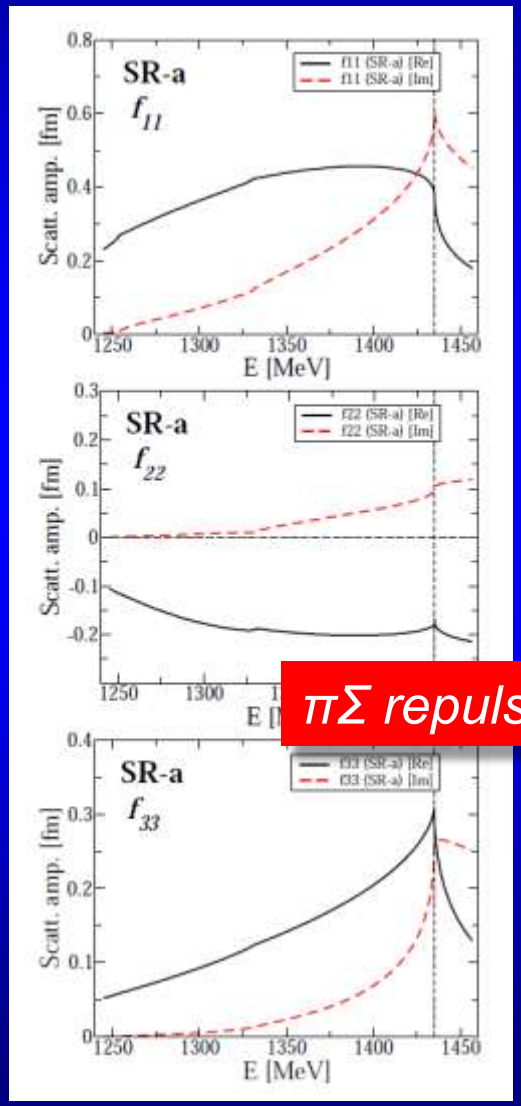
SR

$K^{bar}N$

$\pi\Sigma$

$\pi\Lambda$





$\pi\Sigma$ repulsive ???

A narrow resonance exists
at a few MeV below $\pi\Sigma$ threshold

I=1 channel : $K^{bar}N$, $\pi\Sigma$, $\pi\Lambda$

f_π=90 MeV

Give up the reproduction of Re $a^{I=1}_{KbarN}$.

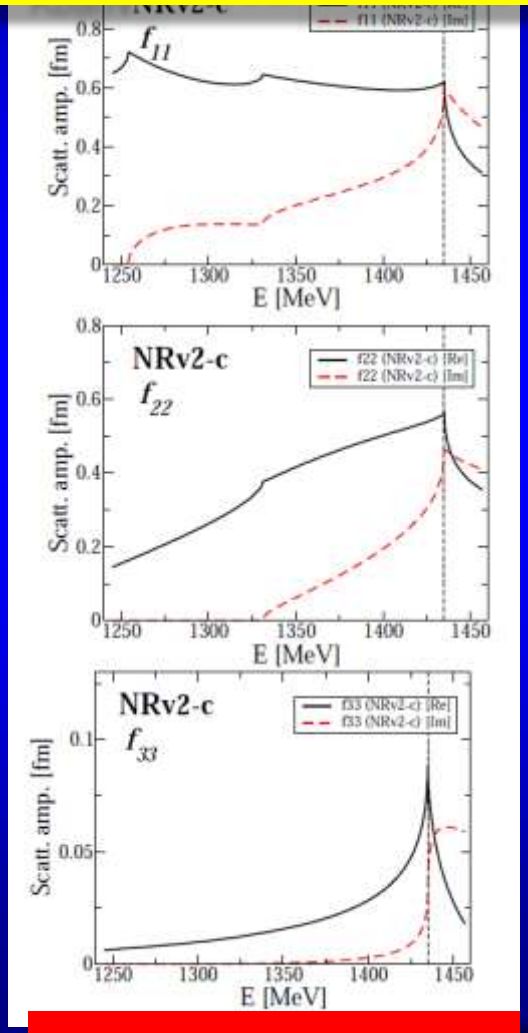
$K^{bar}N$

$\pi\Sigma$

$\pi\Lambda$

NRv2

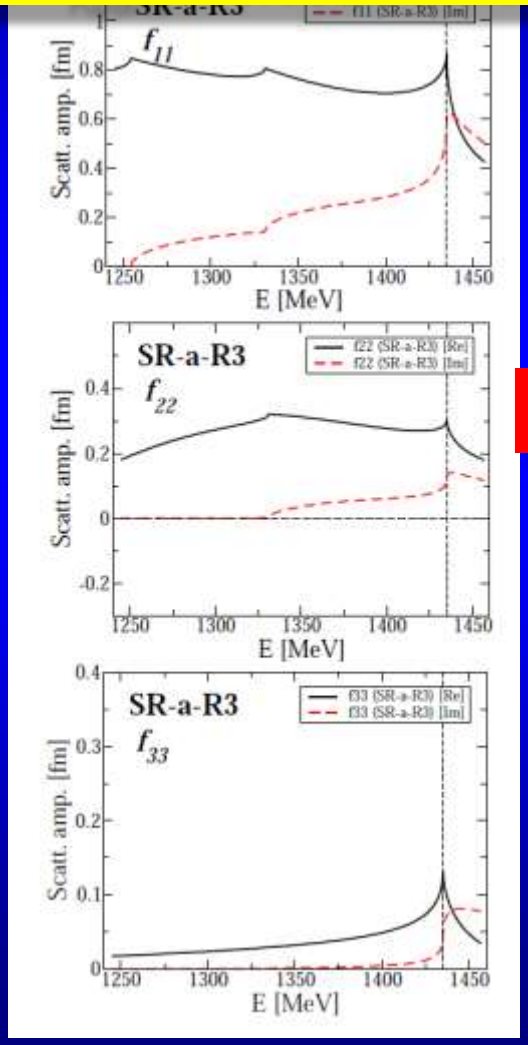
$a^{I=1}_{KbarN}$ = 0.621 + i 0.600 fm



No resonance exists.

SR

$a^{I=1}_{KbarN}$ = 0.869 + i 0.600 fm



$\pi\Sigma$ attractive

5. Summary and Future plan

5. Summary

K^{bar} nuclear system is one of nuclear systems with strangeness. It may be related to dense matter such as neutron star.

Scattering and resonant states of $K^{bar}N$-πY system is studied with a coupled-channel Complex Scaling Method using a chiral SU(3) potential

- Coupled Channel problem = $K^{bar}N + \pi Y$
- Solved with *Gaussian base*
- A Chiral SU(3) potential (KSW) … r-space, Gaussian form, energy dependence
- ***Calculated scattering amplitude with help of CSM***

Non-rela. / Semi-rela. kinematics and two types of non-rela. approximation of KSW potential are tried.

- The KSW potential can be determined by the $K^{bar}N$ scattering length obtained by Martin's analysis of old data.

5. Summary and future plans

Non-rela. / Semi-rela. kinematics and two types of non-rela. approximation of KSW potential are tried.

- f_π dependence (f_π = 90 ~ 120MeV) … Small for Semi-Rela. case
- Constrained by *I=0* $K^{bar}N$ scattering length …
  - Pole position

$(M, \Gamma/2)$		
NRv1 :	(1418.2 ± 1.6,	21.4 ± 4.7)
NRv2 :	(1418.9 ± 1.1,	18.6 ± 4.6)
SR :	(1419.5 ± 0.5,	13.0 ± 1.4)

? ccCSM finds the upper pole of the two poles of Λ(1405), if the present potential has the double pole.

  - "Size" of the *I=0* pole state

NR: ~ 1.3 – 0.3i fm
SR: ~ 1.2 – 0.5i fm

- *A potential for I=1 channel ($K^{bar}N$-πΣ-πΛ) is also constructed.*

Future plans

Three-body system ($K^{bar}NN$-πYN); Updated data of K^-p scattering length by SHIDDARTA

Thank you very much!

A. D. is thankful to Prof. Katō for his advice on the scattering-amplitude calculation in CSM, and to Dr. Hyodo for useful discussion.