回転するカラー超伝導体:
渦の構造とその電磁特性

ハドロン物質の諸相と状態方程式
ー 中性子星の観測に照らして ー
2012年9月1日@京大基研

Muneto Nitta(新田宗土) (Keio U./慶應大学)

Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Keio University
1858
CALAMVS GLADIO FORTIOR

共同研究者

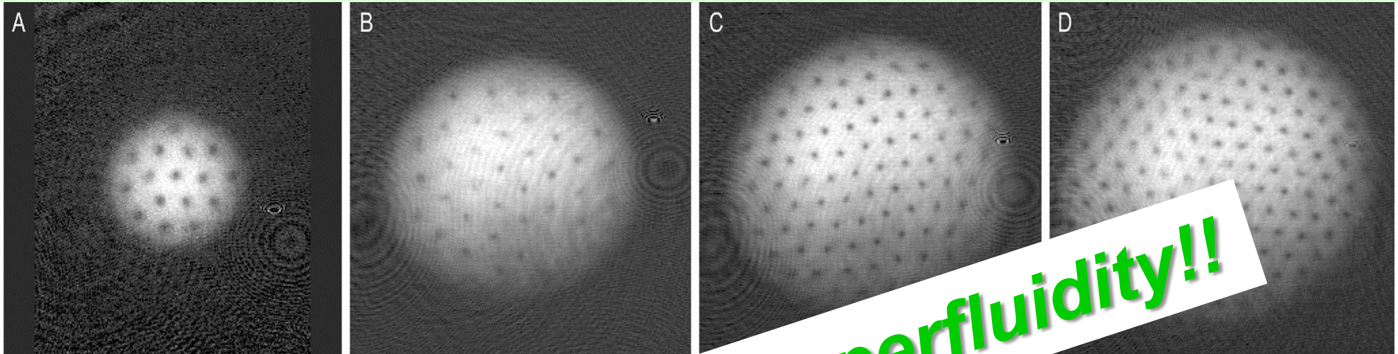
青＝ハドロン･原子核、緑＝素粒子、赤＝物性

仲野英司（高知大）, 松浦妙子（北大）, 衛藤稔（山形大）,
山本直希（INS, Seatle）, 安井繁宏（KEK）, 板倉数記（KEK),
広野雄士（東大/理研）, 金澤拓也, 藤原高徳（茨城大),
福井隆裕（茨城大）, Walter Vinci（Pisa）, Mattia Cipriani（Pisa）

1. with Nakano,Matsuura, Phys.Rev.D78:045002,2008 [arXiv:0708.4096]
2. with Eto, Phys.Rev.D80:125007,2009 [arXiv:0907.1278]
3. with Eto,Nakano, Phys.Rev.D80:125011,2009 [arXiv:0908.4470]
4. with Eto,Yamamoto, Phys.Rev.Lett.104:161601,2010 [arXiv:0912.1352]
5. with Yasui,Itakura, Phys.Rev.D81:105003,2010 [arXiv:1001.3730] Last talk
6. with Hirono,Kanazawa, Phys.Rev.D83:085018,2011 [arXiv:1012.6042]
7. with Eto,Yamamoto, Phys.Rev.D83:085005,2011 [arXiv:1101.2574]
8. with Fujiwara,Fukui,Yasui, Phys.Rev.D84:076002,2011 [arXiv:1105.2115] Last talk
9. with Hirono, Phys.Rev.Lett.109:062501,2012 [arXiv:1203.5059] Next talk
10. with Vinci,Cipriani, arXiv:1206.3535
11. with Cipriani,Vinci, arXiv:1208.5704

This talk

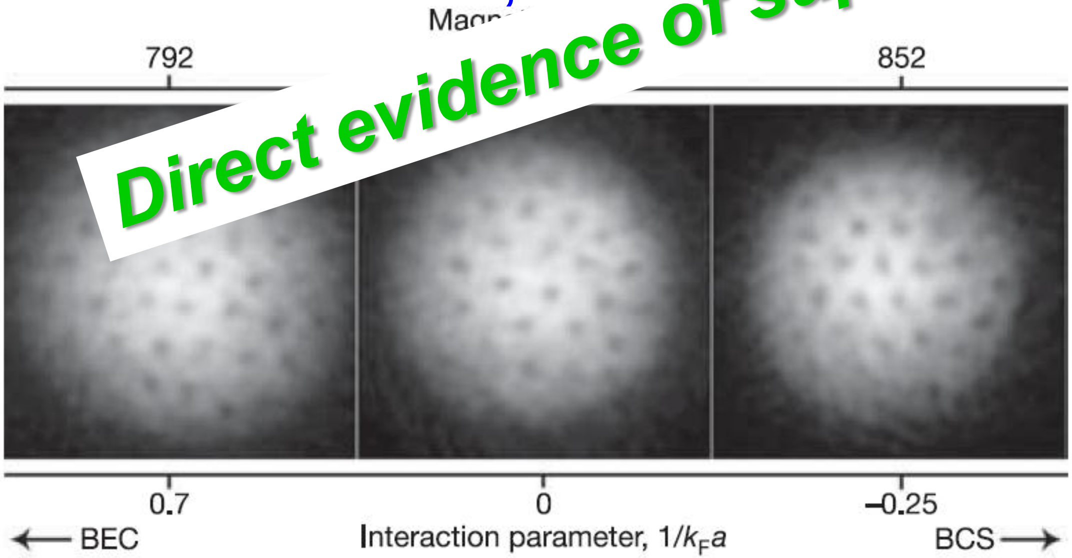
Vortex nucleation in superfluid under rotation



Ultracold atomic BEC

Direct evidence of superfluidity!!

Abo-Shaeer, Ra… …etterle, Science 292, 476-479 (2001)



BEC/BCS Crossover

Zwierlein, Abo-Shaeer, Schirotzek, Schunck & Ketterle
Nature 435, 1047-1051 (23 June 2005)

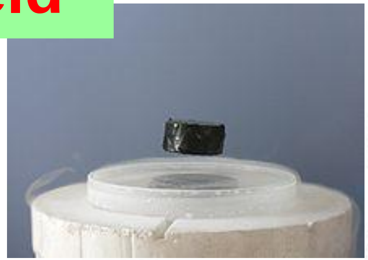
Superconductors in magnetic field

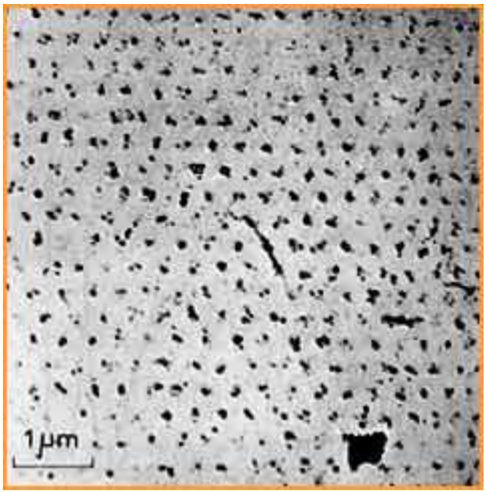
Vortex Matter

http://ltp.phys.titech.ac.jp/vormat.htm

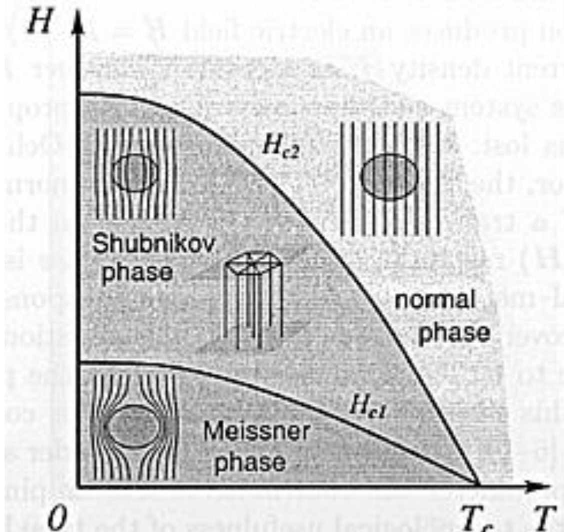
1. Vortex Core States
2. Vortex Lattice、Glass, Liquid
3. Vortex Dynamics

Vortex
Flux tube

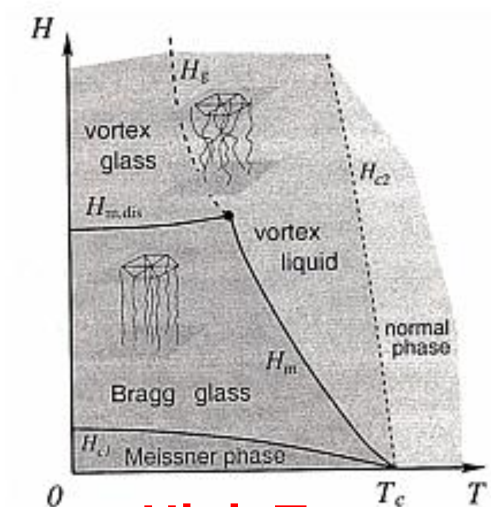




Abrikosov vortex lattice



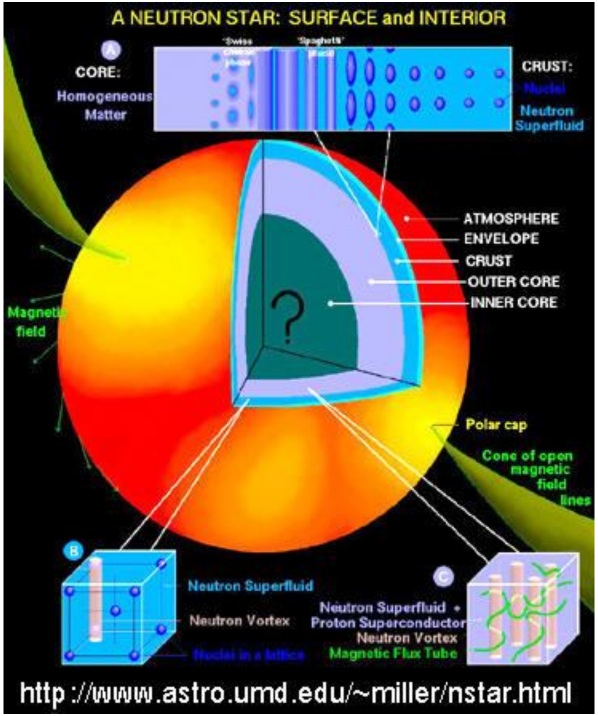
Conventional supercond



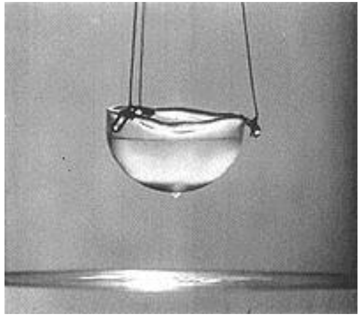
High Tc supercond

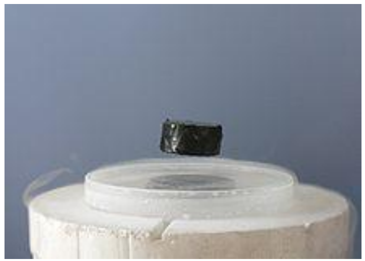
Neutron Stars

Core Nuclear matter



Neutron
superfluid





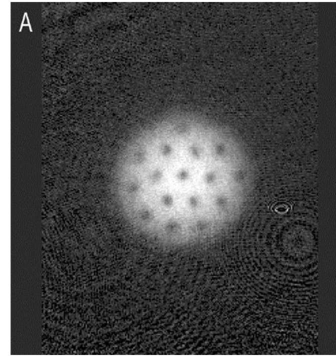
Proton
super-
conductor

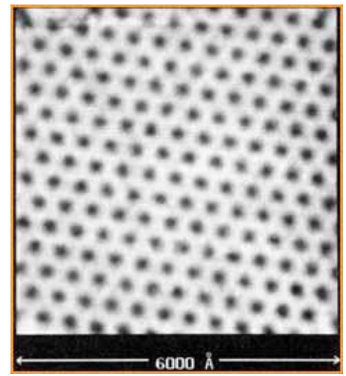
Rotation

Magnetic
field

Baym&Pines ('60s)
Anderson&Itoh ('75)

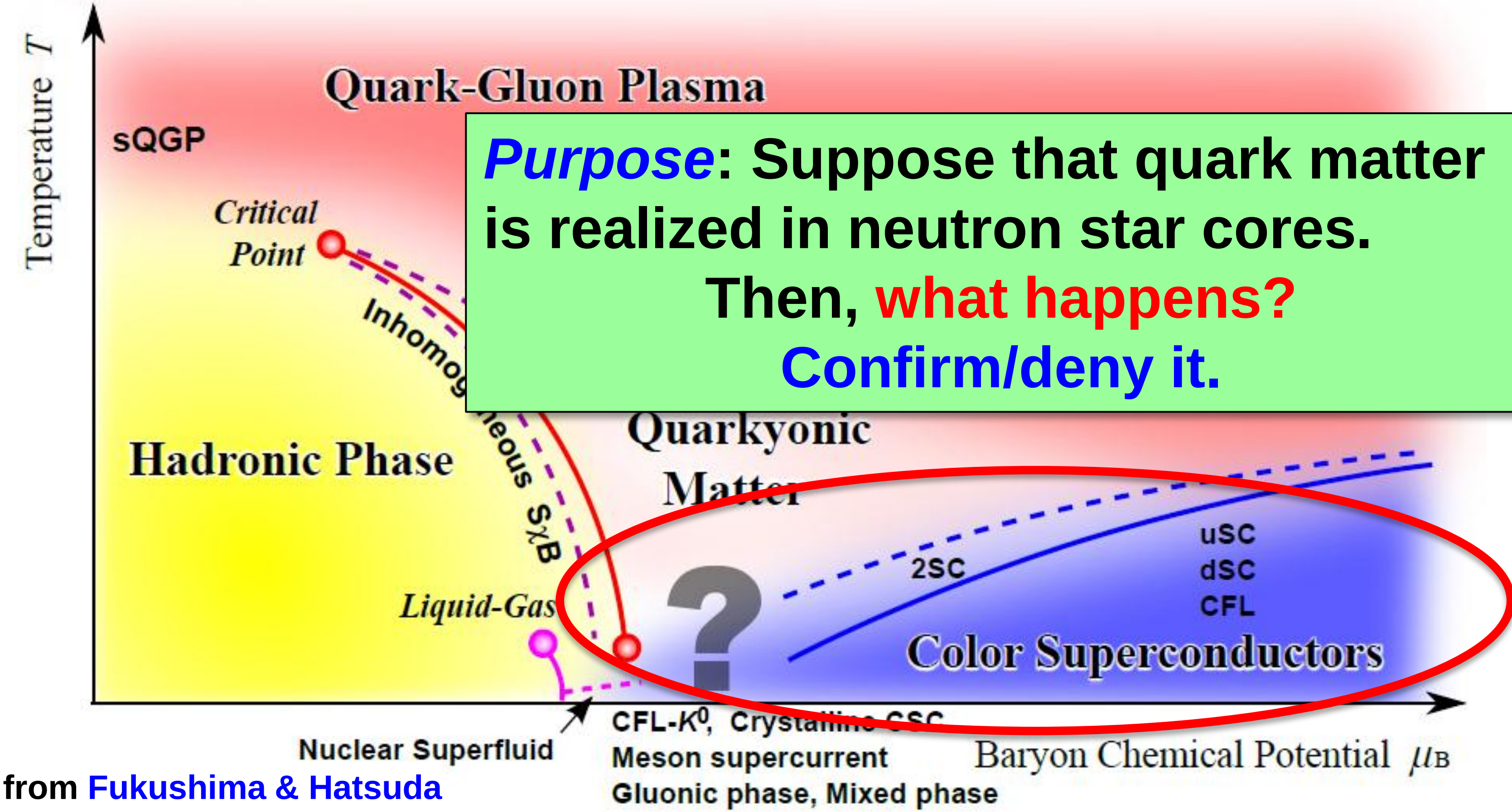
Superfluid
vortices





vortices
(Flux tubes)

Topic in this talk



Purpose: **Suppose that quark matter is realized in neutron star cores.**

Then, what happens?

Confirm/deny it.

from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Quantum Chromo Dynamics (QCD)

Quark matter

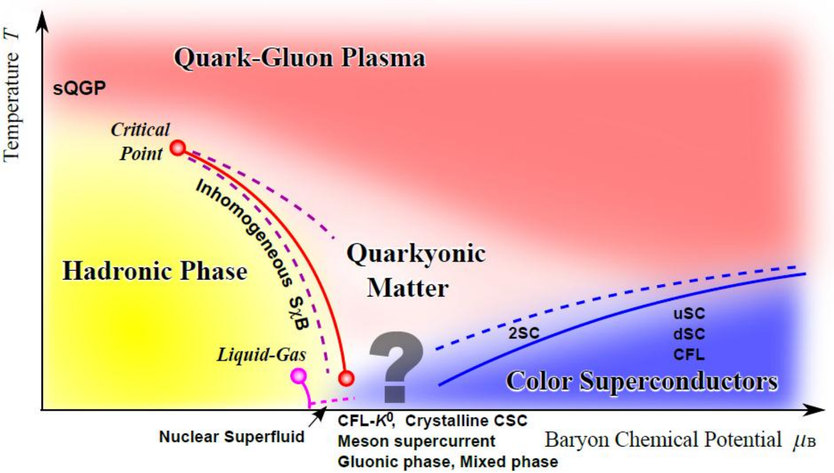
quarks

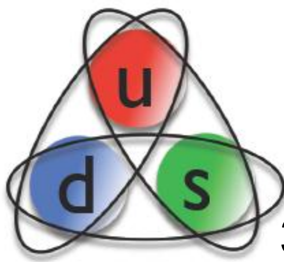
q^i_α i = u,d,s flavor(global) SU(3)

α = r,g,b color(gauge) SU(3)

Color-flavor locked (CFL) phase

"Color superconductor"





@ high density

Alford-Rajagopal-Wilczek('98)

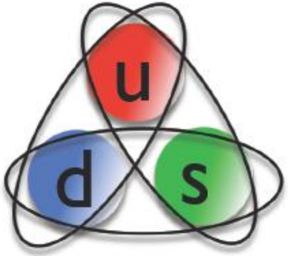
3x3 matrix

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q^j_\beta q^k_\gamma \sim \mathbf{1}_{\alpha i}$$

from Fukushima & Hatsuda Rept.Prog.Phys. 74 (2011) 014001

Color superconductivity as well as *superfluidity*

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3 \ (\mathrm{r, g, b}) \quad i = 1,2,3 \ (\mathrm{u, d, s})$$

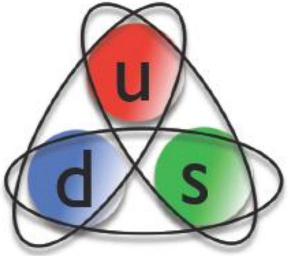
$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}}$$

$$\boxed{\Phi_{\alpha i} \to e^{i\alpha} g_{\mathrm{color}} \Phi_{\alpha i} g_{\mathrm{flavor}}}$$

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

Ground state

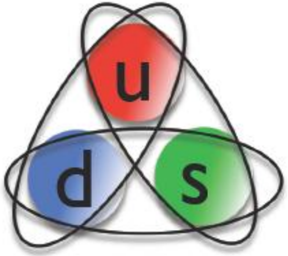
$$\Phi_{\alpha i} = \begin{array}{cc} \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \\ \begin{array}{ccc} \bar{u} = ds & \bar{d} = sb & \bar{s} = ud \end{array} & \end{array}$$

color-flavor locked (CFL)

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}} \quad \cancel{U(1)_{\mathrm{B}}}$$ **superfluidity**

$$\to H = SU(3)_{\mathrm{C+F}} \quad g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1} \quad \cancel{SU(3)_{\mathrm{C}}}$$ **color superconductivity**

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

Integer quantized superfluid vortex

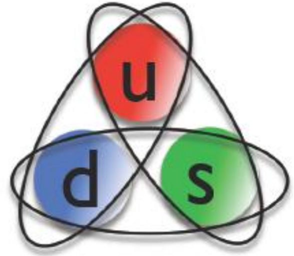
$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r) e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

$$\bar{u} = ds \qquad \bar{d} = sb \qquad \bar{s} = ud$$

Iida & Baym, Forbes & Zhitnitsky('02)

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3 \ (\mathrm{r,g,b}) \quad i = 1,2,3 \ (\mathrm{u,d,s})$$

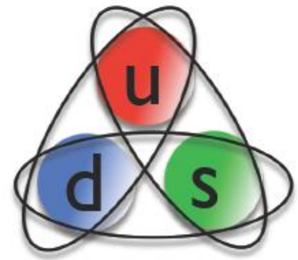
1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

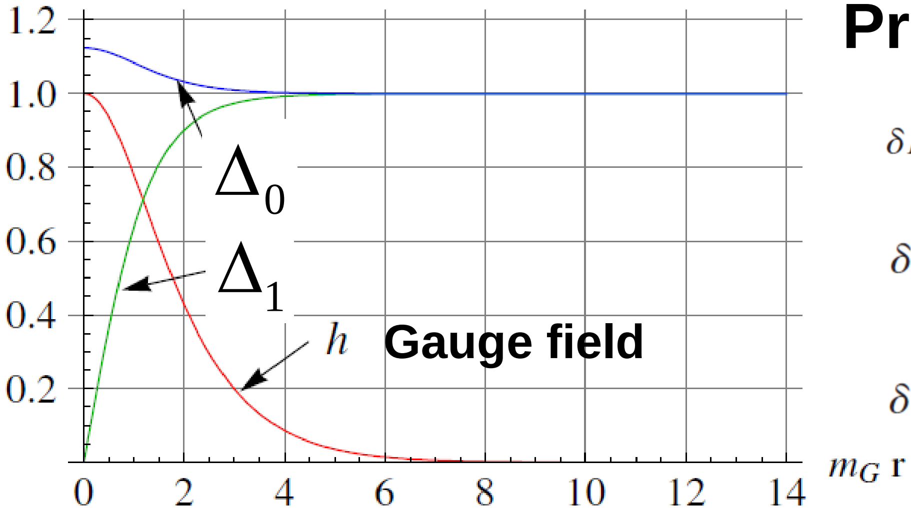
$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

Balachandran, Digal & Matsuura (BDM) ('05)
Nakano, MN & Matsuura ('07), Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$



Profiles

$F \equiv \Delta_1 + 2\Delta_0,$ **trace**

$G \equiv \Delta_1 - \Delta_0,$ **traceless**

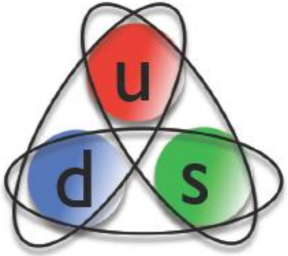
$$\delta F = q_\phi \sqrt{\frac{\pi}{2m_\phi r}} e^{-m_\phi r} + \left(-\frac{1}{3m_\phi^2 r^2} + \mathcal{O}\left(\frac{1}{(m_\phi r)^4}\right)\right)$$

$$\delta G \simeq q_\chi K_{1/3}(m_\chi r) \simeq q_\chi \sqrt{\frac{\pi}{2m_\chi r}} e^{-m_\chi r},$$

$$\delta h \simeq q_G m_G r K_1(m_G r) \simeq q_G \sqrt{\frac{\pi m_G r}{2}} e^{-m_G r}$$

Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \exp \frac{i\theta}{3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

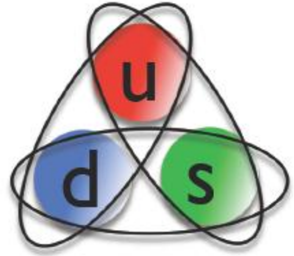
SU(3) color

→ color flux tube

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underline{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

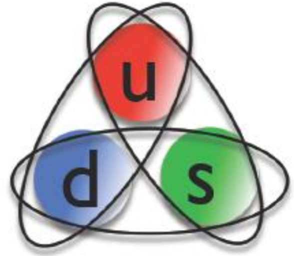
SU(3) color

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \underbrace{\boxed{\exp\left(\frac{i\theta}{3}\right)}}_{\text{1/3 quantized}} \underbrace{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

→ color flux tube

Comment

Non-Abelian vortices were discovered earlier in the context of Supersymmetry and String theory ('03)

Superfluid vortex

Non-Abelian vortex

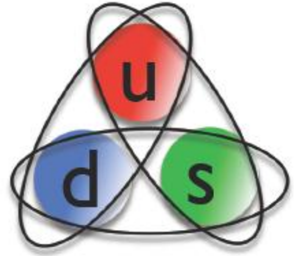
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Which are energetically favored?

Non-Abelian vortices

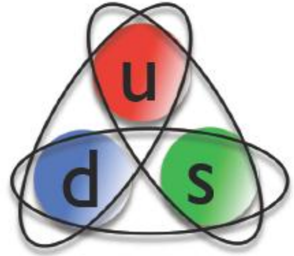
Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Split



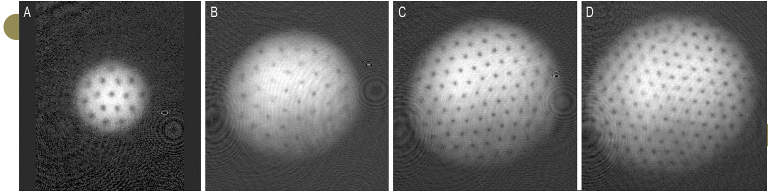
Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

$$E[\;] = E[\;] = E[\;] = \frac{1}{9}E[\;]$$

Nakano, MN & Matsuura ('07)



Abrikosov vortex lattice


Colorful vortex lattice

Colorful vortex lattice

Google画像検索
“カラフル　水玉”

$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2) \times U(1)]_{\mathrm{C+F}}$$

@ vortex core

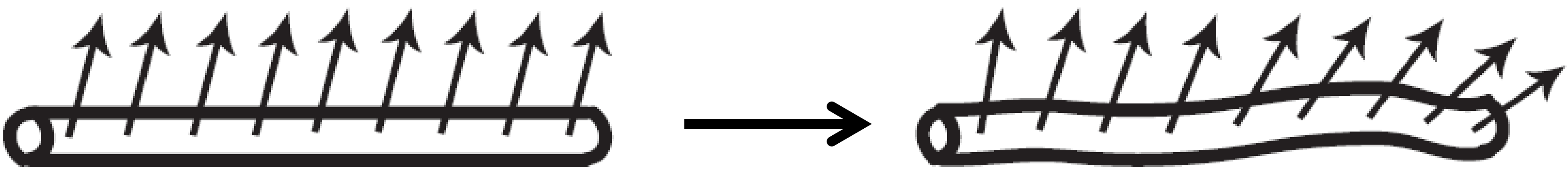
Nambu-Goldstone modes localized around the vortex

$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2) \times U(1)} = \mathbf{C}P^2$$

Continuous family of solutions exists

Eto,Nakano&MN('09)

= Gapless modes propagating along the vortex line



"ground state" **1+1 dim effective theory** **fluctuations**

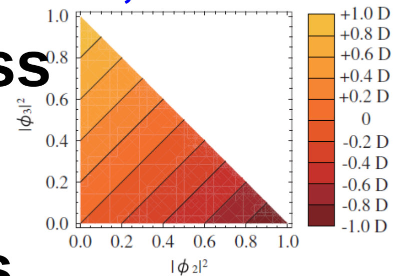
What we have found so far

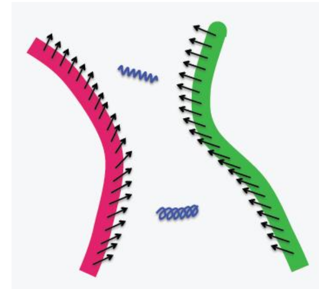
1. **Interaction between vortices** Nakano,MN&Matsuua ('07)

2. **Inclusion of strange quark mass**

   decay into one kind

   Eto,MN&Yamamoto('09)





3. **Interaction with quasi-particles**

   ($U(1)_B$ phonon, gluons) Hirono,Kanazawa&MN ('11)

4. **Quark bound states** Yasui,Itakuta&MN ('10)

   Index theorem Fujiwara,Fukui,MN&Yasui ('11)

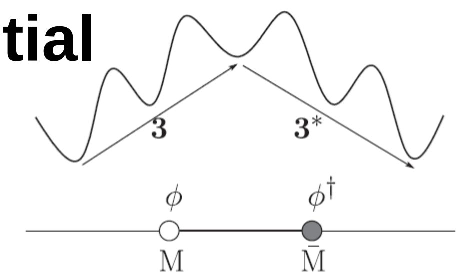
   Non-Abelian statistics Yasui,Itakuta&MN('11), Hirono,Yasui,Itakura&MN('12)

5. **Quantum mechanically induced potential**

   monopole-anti-monopole confined by vortex

   quark-hadron duality

   Eto,MN&Yamamoto, Shifman&Yung ('11)



Electric & Magnetic U(1)$_{EM}$ interaction

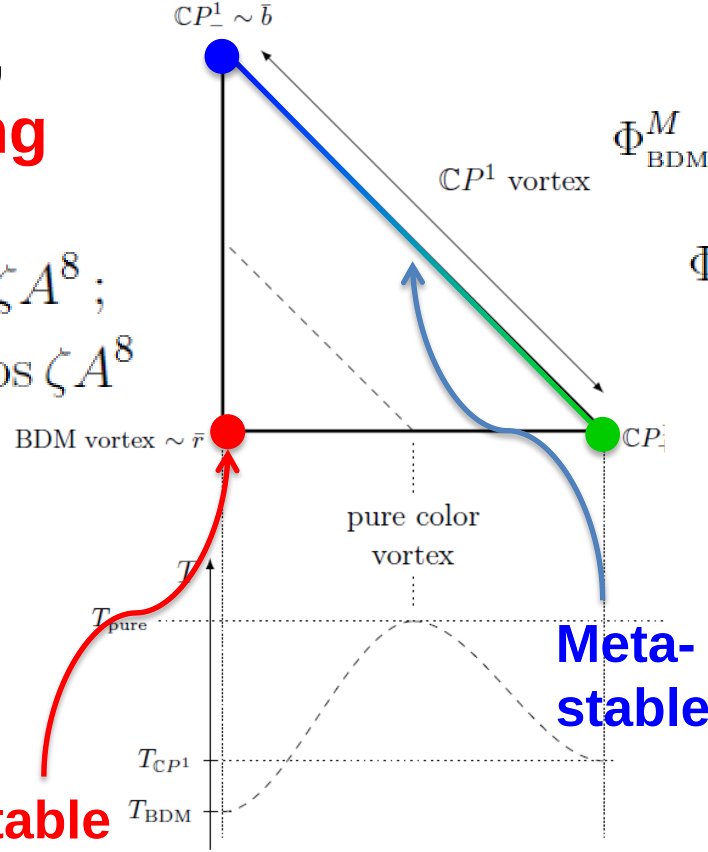
In the ground state,
photon-gluon mixing
occurs:

$$A_M = \cos\zeta A^{em} + \sin\zeta A^8 ;$$
$$A_0 = -\sin\zeta A^{em} + \cos\zeta A^8$$

massive

massless

$$\cos\zeta = \sqrt{\frac{e^2}{e^2 + 3g_s^2/2}} \equiv \frac{e}{g_M}$$

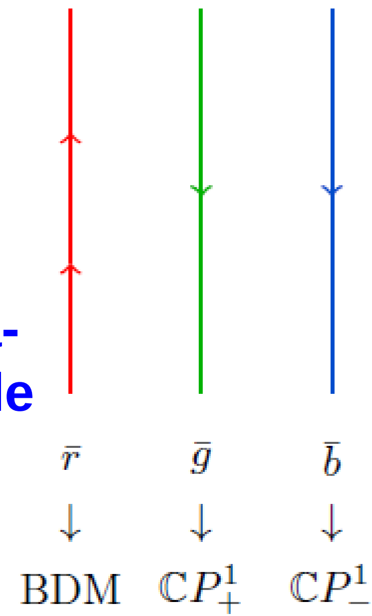


E&M induced potential

U(1)$_{EM}$ flux

$$\Phi^M_{\rm BDM} = \oint \vec{A}^M \cdot d\vec{l} = \frac{2\pi}{g_M}$$

$$\Phi^M_{\mathbb{C}P^1} = -\frac{1}{2}\Phi^M_{\rm BDM}$$



Vinci,Cipriani &MN
arXiv:1206.3535

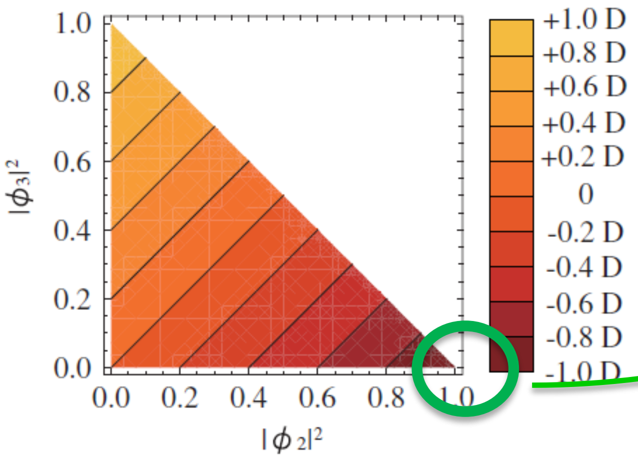
Electric & Magnetic U(1)$_{EM}$ interaction

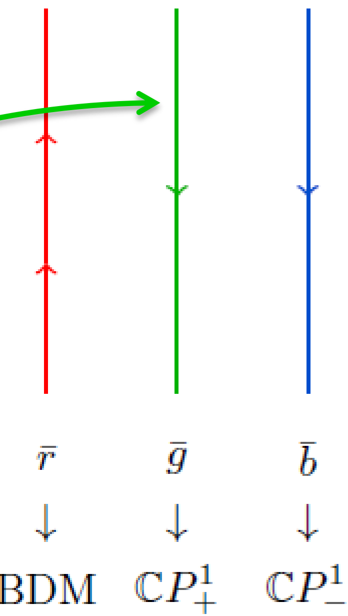
However, this potential is washed out by strange quark mass.

U(1)$_{EM}$ flux

$$\Phi^M_{\rm BDM} = \oint \vec{A}^M \cdot d\vec{l} = \frac{2\pi}{g_M}$$

$$\Phi^M_{\mathbb{C}P^1} = -\frac{1}{2}\Phi^M_{\rm BDM}$$





"Spontaneous magnetization"

However, very small~O(1)G,
which cannot be used for NS

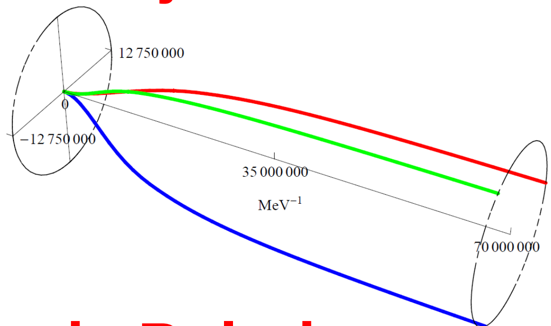
Vinci, Cipriani &MN
arXiv:1206.3535

Electric & Magnetic U(1)$_{EM}$ interaction

Other interesting phenomena

1. Colored Monopoles at Boojums

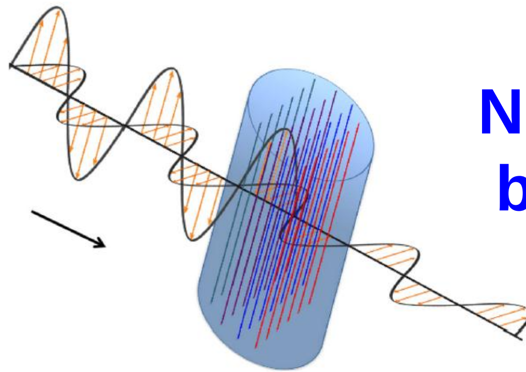
Cipriani,Vinci & MN,
arXiv:1208.5704



Here, I'll explain

2. Compact Stars as Cosmic Polarizers

Hirono &MN,
Phys.Rev.Lett.109:062501,2012
[arXiv:1203.5059]



Next talk
by Hirono

What is Boojum?

Boojum trees in Arizona

Boojum

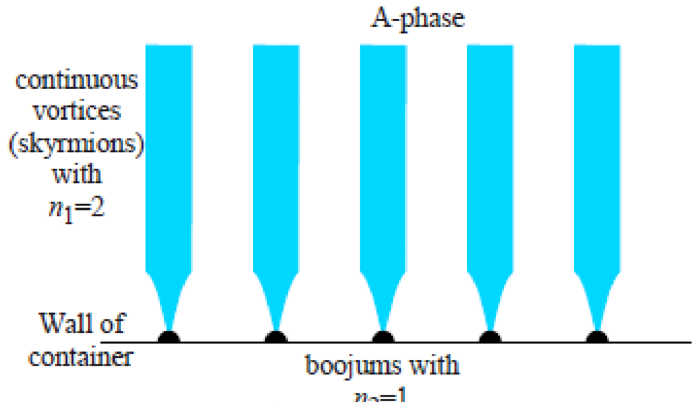
A particularly dangerous kind of Snark

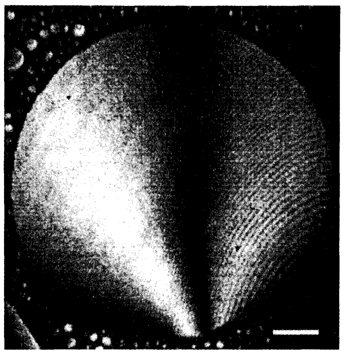
"The Hunting Of Snark"
Lewis Carroll

What is Boojum?

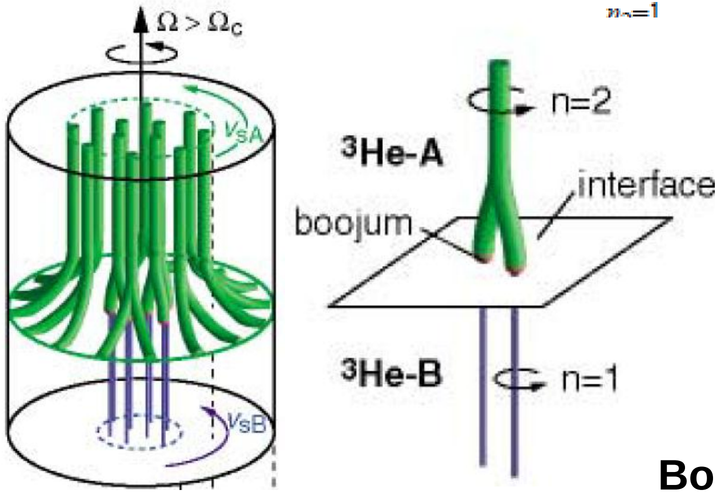
In physics, named by
D.Mermin (1976)

Boojums on the container wall in superfluid ^{3}He-A

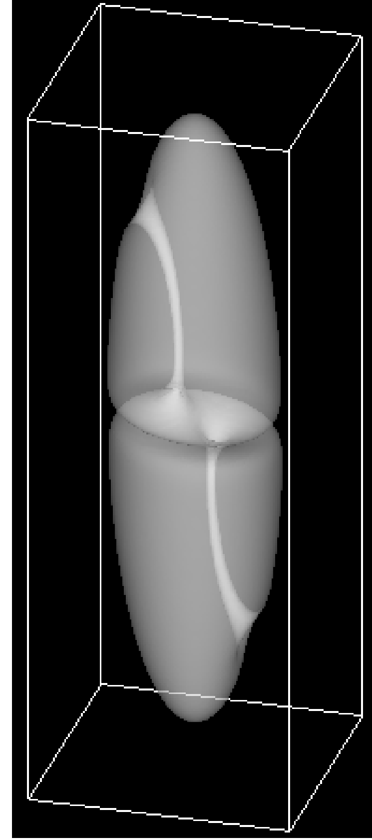




Boojum in liquid crystal



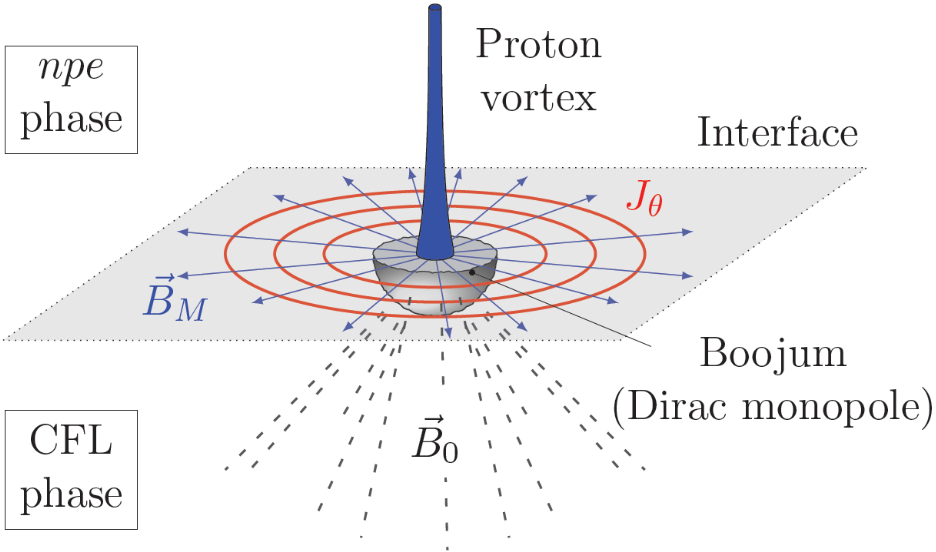
Boojums in ^{3}He A-B phase boundary



Boojum in two component BECs
Kasamatsu-Takeuchi-MN-Tsubota, JHEP1011:068,2010 [arXiv:1002.4265]

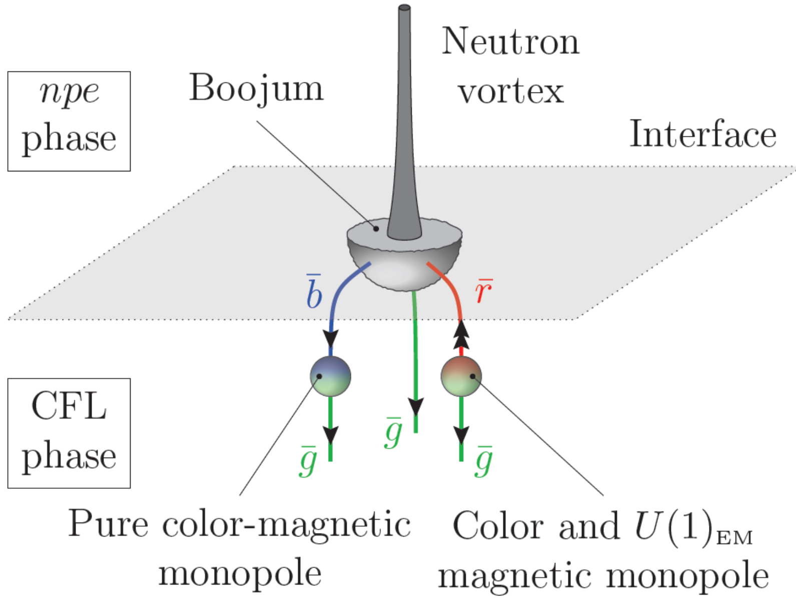
Boojums at interface of nuclear matter/quark matter

Proton boojum



Dirac monopole

Neutron boojum = Colorful boojum

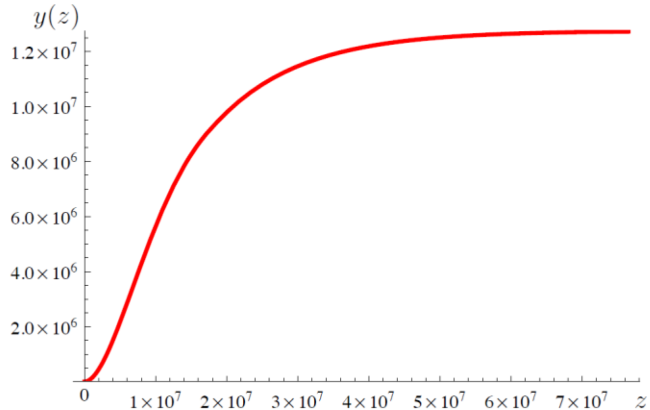


Confined color monopoles

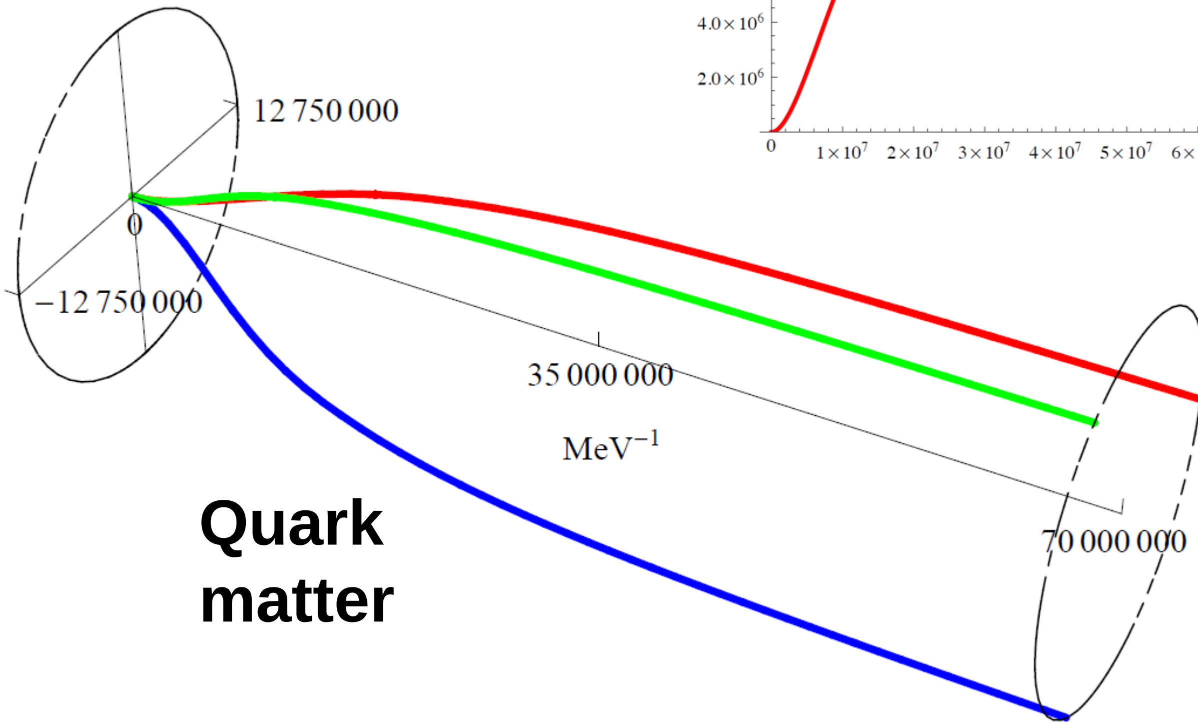
Cipriani,Vinci & MN, arXiv:1208.5704

Neutron boojum

= Colorful boojum



Nuclear matter



Quark matter

Cipriani,Vinci & MN, arXiv:1208.5704

Conclusion

If **quark matter (CFL phase) is realized in neutron star core, non-Abelian vortices (color flux tubes) *must* be created and constitute a colorful lattice.**

Discussion

What is an observational signal?
Confirm/deny quark matter.

1.The existence of vortex lattice implies:

*** Lattice oscillation: *Tkachenko, Keivin* modes**

*** Transportation of particles, anisotropic neutrino emission?**

*** EOS more stiff (?): anisotropic pressure?**

2. Its rigid rotation yields gravitational wave radiation(?)

Questions, discussions, or seminar: **nitta@phys-h.keio.ac.jp**

Landau-Ginzburg model from QCD

Iida&Baym('01)
Giannakis&Ren('02)
Iida,Matsuura,
Tachibana&Hatsuda('04)

$$\mathcal{L} = \mathrm{Tr}(K_0 \mathcal{D}_0 \Phi^\dagger \mathcal{D}_0 \Phi - K_3 \mathcal{D}_i \Phi^\dagger \mathcal{D}_i \Phi), + \frac{\varepsilon}{2} F_{0i}^2 - \frac{1}{4\lambda} F_{ij}^2 - V,$$

$$V_{\mathrm{GL}} = \mathrm{Tr}\left[\Phi^\dagger \left\{\left(\alpha + \frac{2\epsilon}{3}\right)\mathbf{1}_3 + \epsilon X_3\right\}\Phi\right] + \beta_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 + \beta_2 \mathrm{Tr}[(\Phi^\dagger \Phi)^2],$$

$$\alpha = 4N(\mu)\log\frac{T}{T_c}, \quad \beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2} N(\mu) \equiv \beta,$$

$$K_3 = \frac{1}{3}K_0 = \frac{7\zeta(3)}{12(\pi T_c)^2} N(\mu), \quad \epsilon = N(\mu)\frac{m_s^2}{\mu^2}\log\frac{\mu}{T_c},$$

Effective action on a vortex

Eto,Nakano&MN('09)

$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2$$

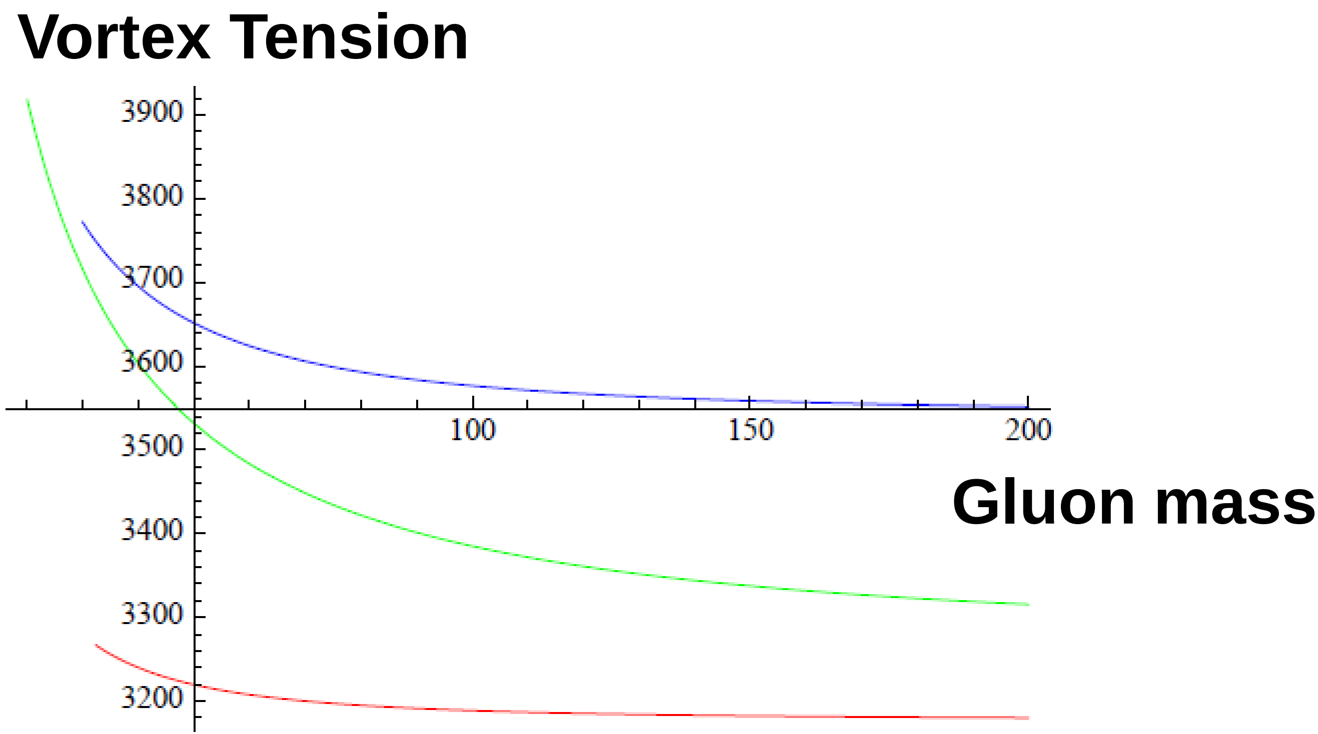
1+1 dim. world-sheet theory

$$\mathcal{L}_{\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha [\partial^\alpha \phi^\dagger \partial_\alpha \phi + (\phi^\dagger \partial^\alpha \phi)(\phi^\dagger \partial_\alpha \phi)],$$

$$\mathcal{L}_{g\mathbb{C}P^2} = C \sum_{\alpha=0,3} K_\alpha [\mathcal{D}^\alpha \phi^\dagger \mathcal{D}_\alpha \phi + (\phi^\dagger \mathcal{D}^\alpha \phi)(\phi^\dagger \mathcal{D}_\alpha \phi)],$$

Nambu-Goldstone modes localized around the vortex

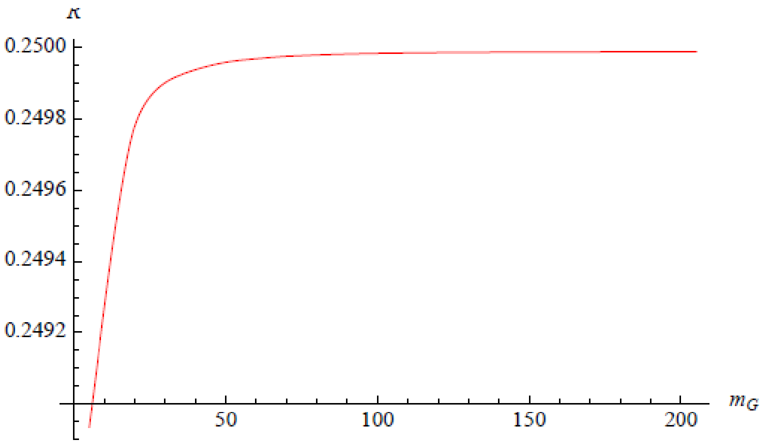
= Gapless modes propagating along the vortex line

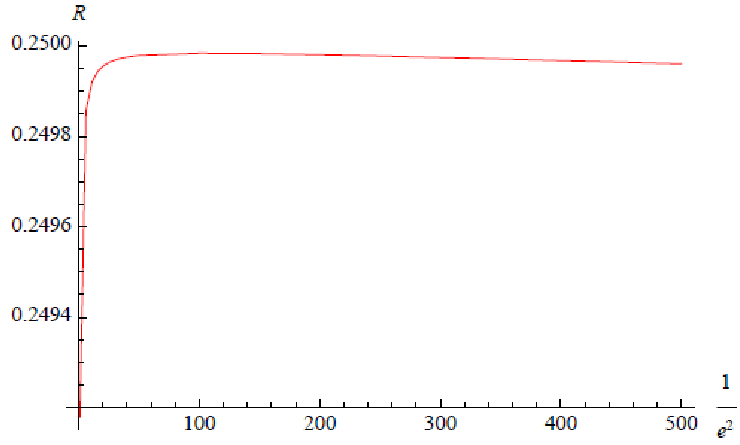


Vortex Tension
3900
3800
3700
3600
3500
3400
3300
3200
100
150
200
Gluon mass

Decay probability of metastable vortices

$$P \sim e^{-\beta R}, \quad R \equiv \frac{\mathcal{T}_{\rm pure} - \mathcal{T}_{\mathbb{C}P^1}}{\mathcal{T}_{\rm pure} - \mathcal{T}_{\rm BDM}} \qquad \beta \sim K_1^2/\lambda_1 \sim \mu^2/T_c^2;$$





Fermions trapped inside a vortex core

$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$

$$\downarrow$$

$$K = [SU(2)\times U(1)]_{\mathrm{C+F}}$$

@ vortex core

$$q = \begin{pmatrix} u_r & d_r & s_r \\ u_g & \boxed{\begin{matrix} d_g & s_g \\ d_b & s_b \end{matrix}} \\ u_b & & \end{pmatrix}$$

Triplet Majorana fermion

protected by SO(3)

Yasui-Itakura-MN,
Phys.Rev.D81,105003(2010)
[arXiv:1001.3730]

Index Theorem

T.Fujiwara, T.Fukui, MN, S.Yasui,
PRD84,076002 (2011) [arXiv:1105.2115]

$$q = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix} \longleftarrow \Delta = \left(\begin{array}{cc|c} \Delta_Q & & \\ & \Delta_Q & \\ \hline & & \Delta_q \end{array}\right)$$

index	
q	triplet
Q	doublet
$2Q$	singlet

Majorana zero modes

$\boxed{Q=0 \quad q=1}$

1	triplet
0	doublet
0	singlet

Exchange statistics of NA Majorana fermion

$$\tau_k^{\mathbf{S},\mathcal{P}} = \sigma_k^{\mathbf{S}} \otimes h_k^{\mathcal{P}} \qquad \mathbf{S} = \mathbf{1},\ \mathbf{3},\ \mathbf{5},\ \mathcal{P} = \mathcal{E},\ \mathcal{O}$$

Ivanov's matrices

$$h_2^{\mathcal{E}} = h_2^{\mathcal{O}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad h_1^{\mathcal{E}} = h_1^{\mathcal{O}} = \begin{pmatrix} e^{i\frac{\pi}{4}} & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{pmatrix}$$

$$h_3^{\mathcal{E}} = h_3^{\mathcal{O}\dagger} = h_1^{\mathcal{E}}$$

NEW

$$\sigma_1^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_2^{\mathbf{1}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}, \quad \sigma_3^{\mathbf{1}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_1^{\mathbf{3}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \sigma_2^{\mathbf{3}} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}, \quad \sigma_3^{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

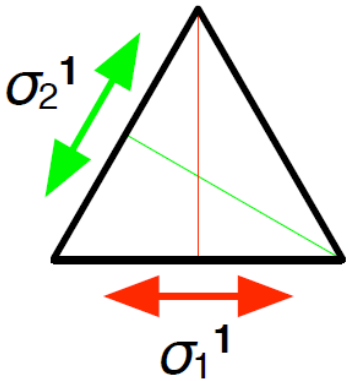
$$\sigma_1^{\mathbf{5}} = \sigma_2^{\mathbf{5}} = \sigma_3^{\mathbf{5}} = 1$$

Coxeter group

(n=4,Type A_3)

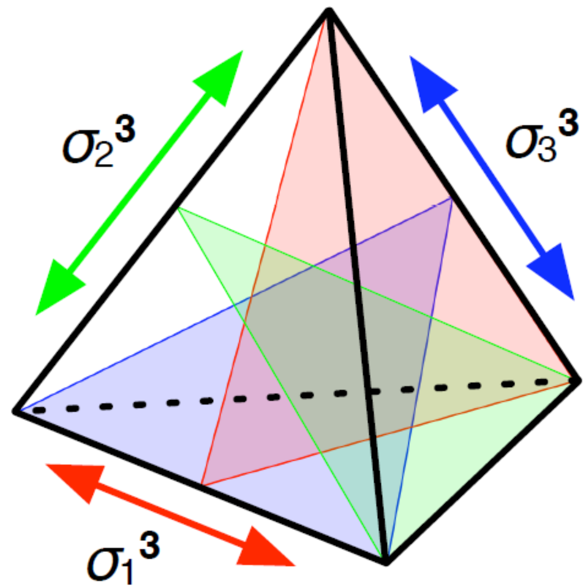
N-simplex and Coxeter group are hidden in the Hilbert space !!

singlet (**1**)

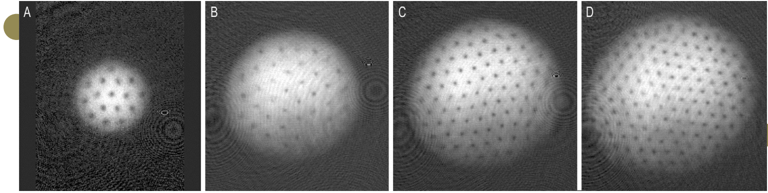


2-simplex (triangle)

triplet (**3**)



3-simplex (tetrahedron)



Abrikosov vortex lattice